Experiment 7: “Wave set-down and set-up”

1. Background theory

Wave breaking induces a depression in the mean sea level (*set-down*) near the breaking point and an increase in the mean water level (*set-up)* that increases toward the shoreline where it reaches a maximum. Longuet-Higgins and Stewart (1964) show that *set-down* and del *set-up* are associated to the gradient of the wave radiation stress defined as the excess of momentum flux due to waves. These authors derived the depth integrated and time-averaged momentum equation assuming stationary flow, beach with straight and parallel contours, negligible Reynolds stresses, and no wind, given by:

$$\frac{\partial S_{xx}}{\partial x} = -\rho g(h+\bar{\eta})\frac{\partial \bar{\eta}}{\partial x} \tag{1}$$

where S_{xx} is the cross-shore component of the wave radiation stress, h is the water depth, and $\bar{\eta}$ is the change in the mean sea level due to waves. Therefore, equation (1) represents the balance between the radiation stress gradient and the sea level gradient inside the surf zone (Figure 1).

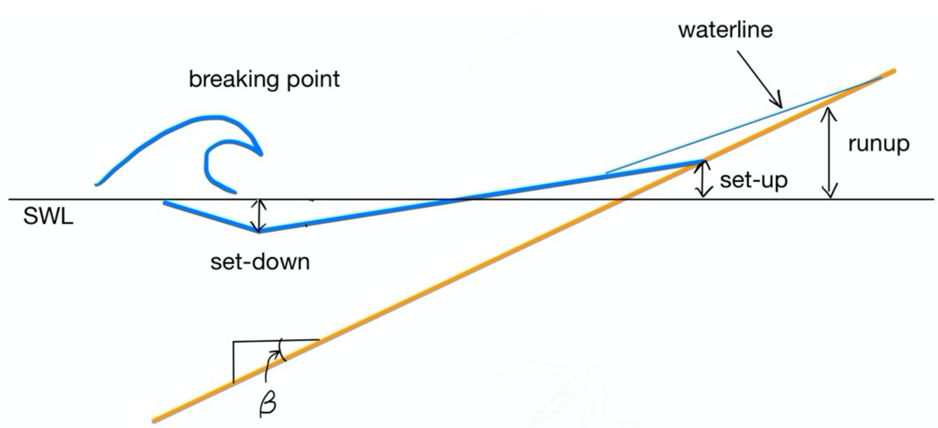

Figure 1.- Mean sea level variation inside the surf zone.

To evaluate the cross-shore component of the radiation stress we can employ second order linear theory approximation (Longuet-Higgins and Stewart, 1964),

$$S_{xx} = \frac{E}{2}[1+2G] = E\left[\frac{1}{2}+G\right] = E\left[\frac{1}{2}(1+G)+\frac{G}{2}\right] = \frac{ECg}{c}+\frac{EG}{2} \tag{2}$$

where,

$$G = \frac{2kh}{\sinh 2kh} \quad ; \qquad Cg = \frac{c}{2}(1+G) \quad ; \qquad n = \frac{Cg}{G}; \qquad E = \frac{1}{8}\rho g H^2$$

2. **Objective**

To investigate the mean sea level variability inside the surf zone in the virtual wave flume and compare the measurement with linear wave theory.

3. **Instructions**

- In the virtual wave flume Main Menu select the experiment "Wave set-down and set-up".
- Run the irregular wave tests (Test 1: $H_s = 0.125$ m, $T_p = 2.5$ s, $\gamma = 3.3$; Test 2: $H_s = 0.1$ m, $T_p = 1.5$ s, $\gamma = 3.3$) during 300 s to visually identify the offshore limit of the surf zone.
- Deploy 10 wave gauges (WG) from offshore to the shore with an increasing resolution inside the surf zone.
- Run the two tests and export the data for further analysis.

4. **Assignment**

- Calculate the mean sea level at the different cross-shore locations by taking the mean of the free-surface to identify the *set-up* and *set-down* cross-shore locations.
- Estimate the wave parameters for each wave in the irregular wave train (H_i, T_i) using the zero down-crossing method.
- Compute $\overline{H}$ and $\overline{T}$ from the individual waves.
- Plot $\bar{\eta}$ and $\overline{H}$ vs cross-shore location x (see Figure 2).
- Calculate the cross-shore radiation stress component S_{xx} using $\overline{H}$ and $\overline{T}$ and plot with respect to x.
- Express equation (1) in finite difference and solve for $\bar{\eta}_{i+1}$.
- Compute the mean sea level at the different cross-shore locations using the finite difference scheme employing the measured $\bar{\eta}_i$ at the offshore location as boundary condition
- Compare the measured $\bar{\eta}$ vs the one computed by equation (1).

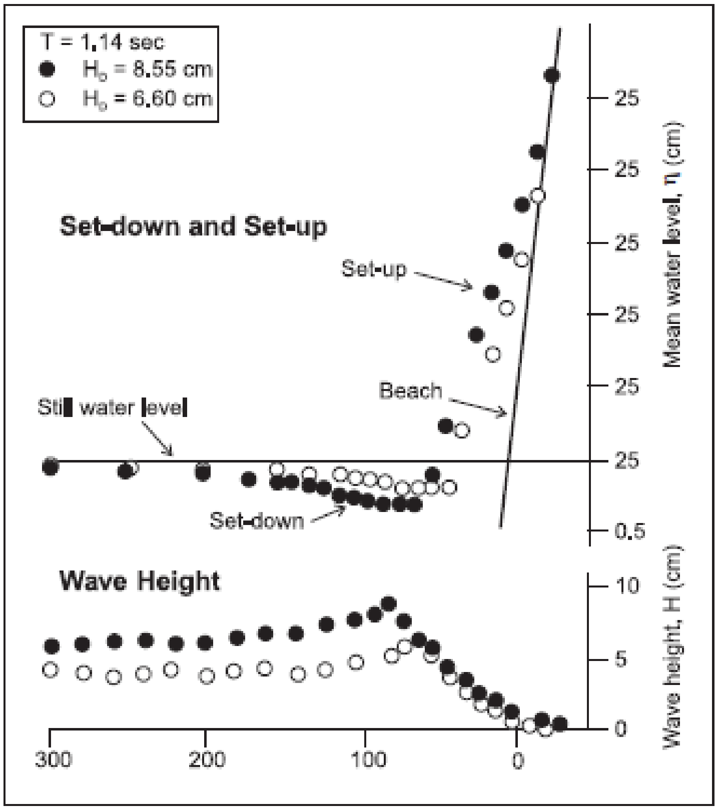

Figure 2. Wave height and set-down/set-up across-shore variation for two different wave conditions. Taken from Davidson-Arnott (2009).

References:

Davidson-Arnott, R. (2009). Introduction to Coastal Processes and Geomorphology. Cambridge: Cambridge University Press. doi:10.1017/CBO9780511841507

Dean, R.G., and Dalrymple, R.A. (2002) Coastal Processes with Engineering Applications, Cambridge University Press, 475 pp.

Longuet-Higgins, M.S., and Stewart, R.W. (1964) Radiation stresses in water waves; a physical discussion with applications. *Deep-Sea Res* 11:529–562