Experiment 3. "Wave reflection from beaches"

1. **Background theory**

A system of incident and reflected waves in front of a beach or structure can be represented as shown in Figures 1 and 2. According to linear wave theory, incident and reflected waves can be expressed as,

$$\eta_i = \frac{H_i}{2}\cos(kx - \sigma t) \tag{1}$$

$$\eta_r = \frac{H_r}{2}\cos(kx + \sigma t + \delta) \tag{2}$$

where η_i is the incident wave, η_r is the reflected wave, δ is the phase lag induced by the reflection process, x is the cross-shore location, t is the time, and H_i and H_r are the incident and reflected wave, respectively, and

$$k = \frac{2\pi}{L} \tag{3}$$

$$\sigma = \frac{2\pi}{T} \tag{4}$$

The combined wave η_c is given by,

$$\eta_c = \eta_i + \eta_r \tag{5}$$

where the total vertical displacement, $2|\eta_c|$, can be expressed as,

$$2|\eta_c| = \sqrt{H_i^2 + 2H_iH_r\cos(2kx + \delta) + H_r^2} \tag{6}$$

The equation above refers to the wave envelope which is function of the cross-shore distance. The maximum and minimum of equation (6) are given by,

$$2|\eta_c|_{\max} = H_i + H_r \tag{7}$$

and

$$2|\eta_c|_{\min} = H_i - H_r \tag{8}$$

separated by $L/4$. Thus, reflection can be defined by the reflection coefficient given by,

$$\kappa_r = \frac{H_r}{H_i} = \frac{2|\eta_c|_{\max} - 2|\eta_c|_{\min}}{2|\eta_c|_{\max} + 2|\eta_c|_{\min}} \tag{9}$$

where the minimum and maximum reflection coefficient values are, 0 and 1, respectively.

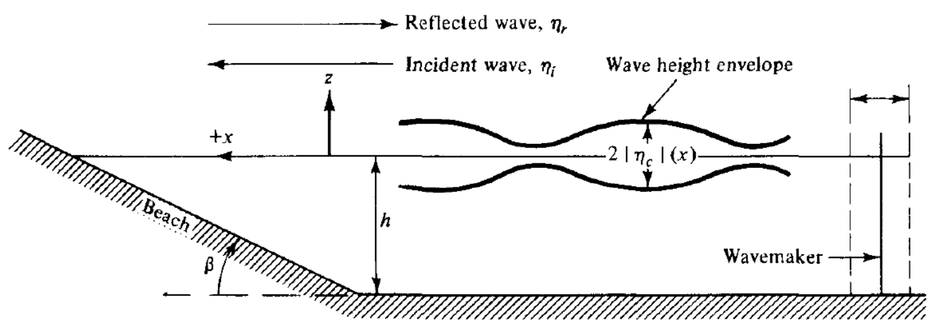

Figure 1. Experimental setup (Taken from Dean & Dalrymple, 1991).

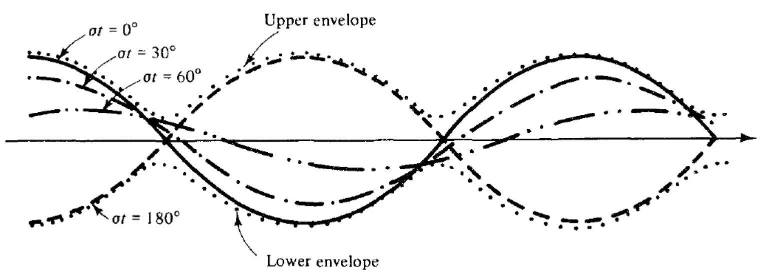

Figure 2. Instantaneous free-surface displacement and wave envelope for a partially standing wave system. (Taken from Dean & Dalrymple, 1991).

An approximate theory to estimate the reflection coefficient from a planar beach was developed by Miche (1947). Miche defined a critical wave steepness at deep water $(H_o/L_o)_{\text{crit}}$:

$$\left(\frac{H}{L_0}\right)_{\text{crit}} = \left(\frac{2\beta}{\pi}\right)^{1/2} \frac{\sin^2 \beta}{\pi} \tag{10}$$

where β is the beach slope. He found that reflection coefficients depend on the deep-water wave steepness, H_o/L_o, as:

$$\kappa_r = 1, \qquad \frac{H_0}{L_0} \leq \left(\frac{H}{L_0}\right)_{\text{crit}}$$

$$\kappa_r = \frac{(H_0/L_0)_{\text{crit}}}{H_0/L_0}, \qquad \frac{H_0}{L_0} \geq \left(\frac{H}{L_0}\right)_{\text{crit}} \tag{11}$$

where the incident wave height is related to deep water wave height by,

$$H_0 = \sqrt{\frac{2C_G}{C_0}} H_i \tag{12}$$

where $C_G = nC$, $n = \frac{1}{2}\left(1 + \frac{2kh}{\sinh 2kh}\right)$ and $C_0 = \frac{L_0}{T}$

2. **Objective**

Estimate the reflection coefficients for different incident wave conditions in the virtual wave flume and compare with theory.

3. **Instructions**

For three different wave conditions:

- Estimate the wave length using linear wave theory.
- In the virtual wave flume set the moving cart velocity with the wave gauge to measure the wave envelop over a distance of at least one wave length.
- Export the measured data

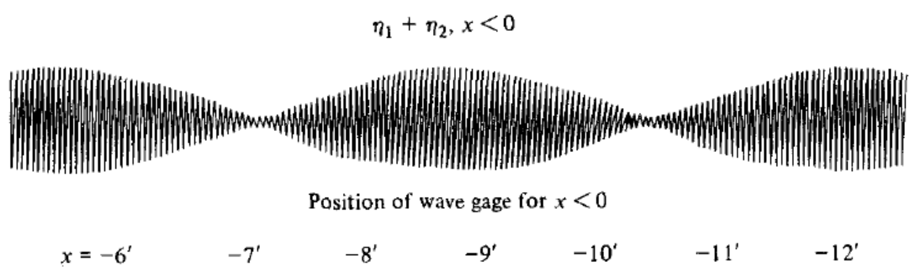

Figure 3. Example of the free-surface elevation measured by the moving cart (Taken from Dean & Dalrymple, 1991).

4. **Assignment**

For each test calculate the reflection coefficient, κ_r, using the maximum ($2|\eta_c|_{\max}$) and minimum ($2|\eta_c|_{\min}$) values of the wave envelope. Compare the reflection coefficients against Miche's theory. Discuss differences in terms of the wave characteristics.

<u>Reference</u>:

Dean, R.G., and Dalrymple, R.A., 1991. Water wave mechanics for engineers and scientists. Advanced Series in Ocean Engineering, vol.2. World Scientific, 353 pp.