Experiment 1. "Wave length and celerity"

1. **Background theory**

The linear dispersion equation is obtained from the Laplace equation considering the dynamic free surface boundary condition, kinematic free surface and bottom boundary condition, and spatial and temporal lateral boundary conditions for small amplitude waves propagating on a uniform water depth. The dispersion equation provides a relationship between the wave period T, the water depth h, and the wave length, L. The equation describes how a field of waves consisting of different periods propagate with different celerity (speed),

$$\sigma^2 = gk \tanh(kh) \tag{1}$$

where $k = 2\pi/L$ is the wave number, $\sigma = 2\pi/T$ is the angular frequency, and $C = L/T$ is the wave celerity. The dispersion equation needs to be solved for L using a numerical method such as the Newton-Raphson method. On the other hand, the dispersion equation can be solved explicitly in shallow ($kh < \pi/10$)) and deep ($kh > \pi$) waters where tanh (kh) = kh and tanh (kh) = 1 (see Figure 1), respectively.

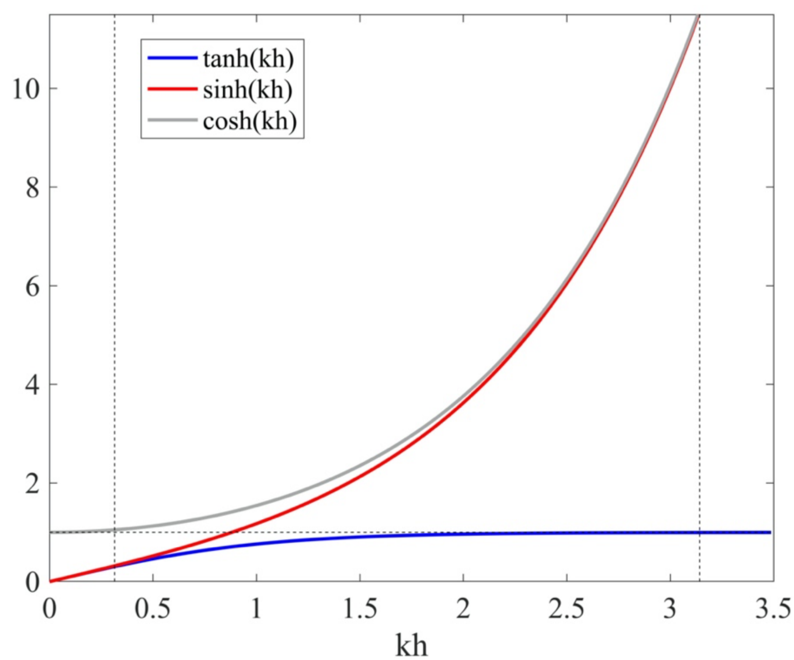

Figure 1.- Relative depth and asymptotes to hyperbolic functions.

2. **Objective**

To compare measured wave length and celerity with respect to linear wave theory estimates.

3. **Instructions**
    - Use the virtual wave flume for h = 0.60 m.

- Run two different tests (Test 1: $H = 0.15$ m, and $T = 1$ s; Test 2: $H = 0.15$ m and $T = 4$ s).
- Employ two water level sensors by adjusting the separation between them until the two free surface elevation time series are in phase (i.e., overlap each other).
- Change the water depth to $h = 0.40$ m.
- Run two tests (Test 3: $H = 0.10$ m and $T = 1$ s; Test 4: $H = 0.15$ m and $T = 4$ s).
- Repeat the procedure to measure the wave length for each case.

4. Assignment

Solve the linear dispersion equation using the Newton-Raphson method to estimate the wave L and C for the four tests. Compare the L and C wave flume observations with linear theory, quantify the differences, and discuss the sources of error.

Table 1.- Summary of results.

Test	h (m)	H (m)	T (s)	$L_{measured}$ (m)	$L_{computed}$ (m)	error	h/L
1	0.6	0.15	1				
2			4				
3	0.4	0.10	1				
4			4				

Reference:

Dean, R.G., and Dalrymple, R.A., 1991, Water wave mechanics for engineers and scientists. Advanced series on ocean engineering, Vol. 2. World Scientific.