Virtual Wave Lab

INSTITUTO
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EXPERIMENTS:

1. Wave length and celerity
2. Wave-induced velocity and particle trajectories
3. Wave reflection from beaches
4. Wave reflection from a partial vertical barrier
5. Wave shoaling and breaking
6. Irregular wave breaking
7. Wave setup and set down

Note: Experiments (1) to (5) are based in Chapter 12 of the book by Dean and Dalrymple (1991).

Experiment 1. "Wave length and celerity"

1. **Background theory**

The linear dispersion equation is obtained from the Laplace equation considering the dynamic free surface boundary condition, kinematic free surface and bottom boundary condition, and spatial and temporal lateral boundary conditions for small amplitude waves propagating on a uniform water depth. The dispersion equation provides a relationship between the wave period T, the water depth h, and the wave length, L. The equation describes how a field of waves consisting of different periods propagate with different celerity (speed),

$$\sigma^2 = gk \tanh(kh) \quad (1)$$

where $k = 2\pi/L$ is the wave number, $\sigma = 2\pi/T$ is the angular frequency, and $C = L/T$ is the wave celerity. The dispersion equation needs to be solved for L using a numerical method such as the Newton-Raphson method. On the other hand, the dispersion equation can be solved explicitly in shallow ($kh < \pi/10$)) and deep ($kh > \pi$) waters where tanh (kh) = kh and tanh (kh) = 1 (see Figure 1), respectively.

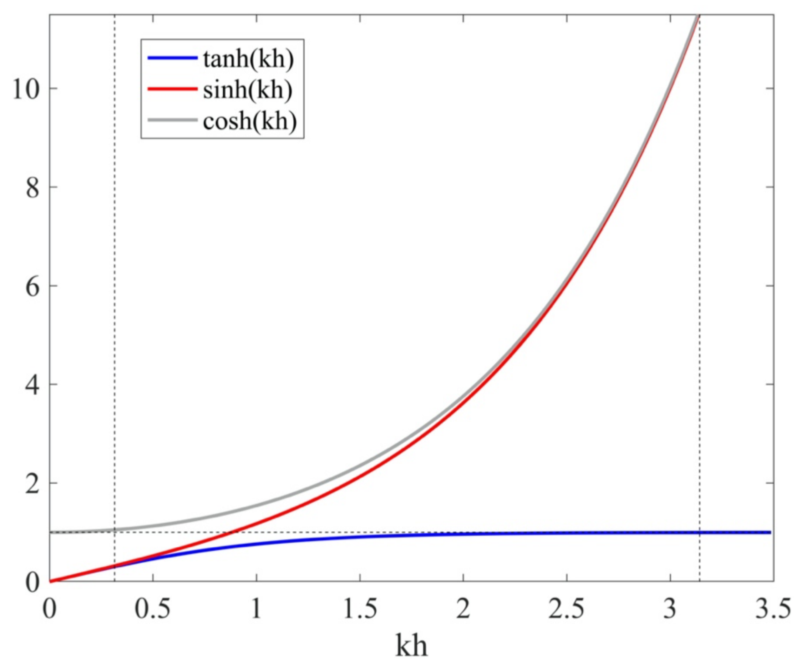

Figure 1.- Relative depth and asymptotes to hyperbolic functions.

2. **Objective**

To compare measured wave length and celerity with respect to linear wave theory estimates.

3. **Instructions**
   - Use the virtual wave flume for h = 0.60 m.

- Run two different tests (Test 1: H = 0.15 m, and T = 1 s; Test 2: H = 0.15 m and T = 4 s).
- Employ two water level sensors by adjusting the separation between them until the two free surface elevation time series are in phase (i.e., overlap each other).
- Change the water depth to h = 0.40 m.
- Run two tests (Test 3: H = 0.10 m and T = 1 s; Test 4: H = 0.15 m and T = 4 s).
- Repeat the procedure to measure the wave length for each case.

4. Assignment

Solve the linear dispersion equation using the Newton-Raphson method to estimate the wave L and C for the four tests. Compare the L and C wave flume observations with linear theory, quantify the differences, and discuss the sources of error.

Table 1.- Summary of results.

Test	h (m)	H (m)	T (s)	$L_{measured}$ (m)	$L_{computed}$ (m)	error	h/L
1	0.6	0.15	1				
2			4				
3	0.4	0.10	1				
4			4				

Reference:

Dean, R.G., and Dalrymple, R.A., 1991, Water wave mechanics for engineers and scientists. Advanced series on ocean engineering, Vol. 2. World Scientific.

Experiment 2. "Wave-induced velocities and particle trajectories"

1. **Background theory**

The velocity components under progressive or standing waves can be estimated from the velocity potential ϕ from the solution of the Laplace equation which is given in terms of the wave height, *H,* water depth, *h,* the wave number, $k = 2\pi/L$, and the angular frequency, $\sigma = 2\pi/T$,

$$\phi(x, y, z) = -\frac{H}{2}\frac{g}{\sigma}\frac{\cosh k(h+z)}{\cosh kh}\sin(kx - \sigma t) \tag{1a}$$

$$\phi(x, y, z) = \frac{H}{2}\frac{g}{\sigma}\frac{\cosh k(h+z)}{\cosh kh}\cos kx \sin \sigma t \tag{1b}$$

where t is the time, g is the gravity acceleration, and x and z represent the horizontal and vertical position of the water particle, respectively. Therefore, velocity components u and w can be estimated for a progressive (eqn. 2a) and standing (eqn. 2b) wave as,

$$u = -\frac{\partial\phi}{\partial x} = \frac{H}{2}\sigma\frac{\cosh k(h+z)}{\sinh kh}\cos(kx - \sigma t)$$

$$w = -\frac{\partial\phi}{\partial z} = \frac{H}{2}\sigma\frac{\sinh k(h+z)}{\sinh kh}\sin(kx - \sigma t) \tag{2a}$$

$$u = \frac{H}{2}\sigma\frac{\cosh k(h+z)}{\sinh kh}\sin kx \sin \sigma t$$

$$w = -\frac{H}{2}\sigma\frac{\sinh k(h+z)}{\sinh kh}\cos kx \sin \sigma t \tag{2b}$$

The velocity associated with either progressive or stationary waves decreases with distance from the free surface (Figure 1) due to dependence on the elevation z (eqn. 2a and 2b). For standing waves, the maximum values for u and w occur underneath the nodes and antinodes (Figure 1b), whereas u and w are zero below the antinodes and nodes, respectively.

The maximum particle displacement $|\zeta|$ and $|\xi|$, for x and z directions, can be expressed as a function of the wave height H, the mean particle position underneath the wave, the wave period (T), and the water depth (h) (Figure 2).

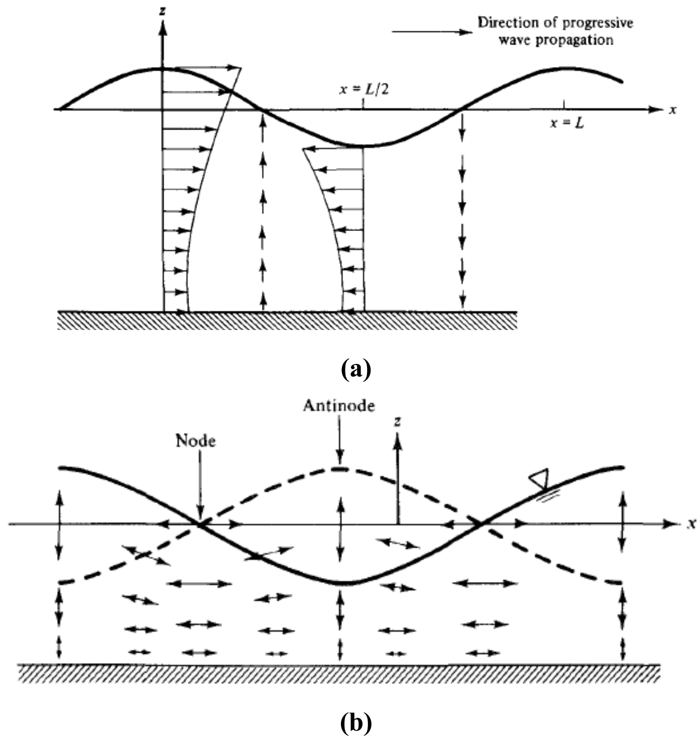

Figure 1. Wave-induced water particle velocities underneath a (a) progressive and (b) standing water wave. (Taken from Dean & Dalrymple, 1991).

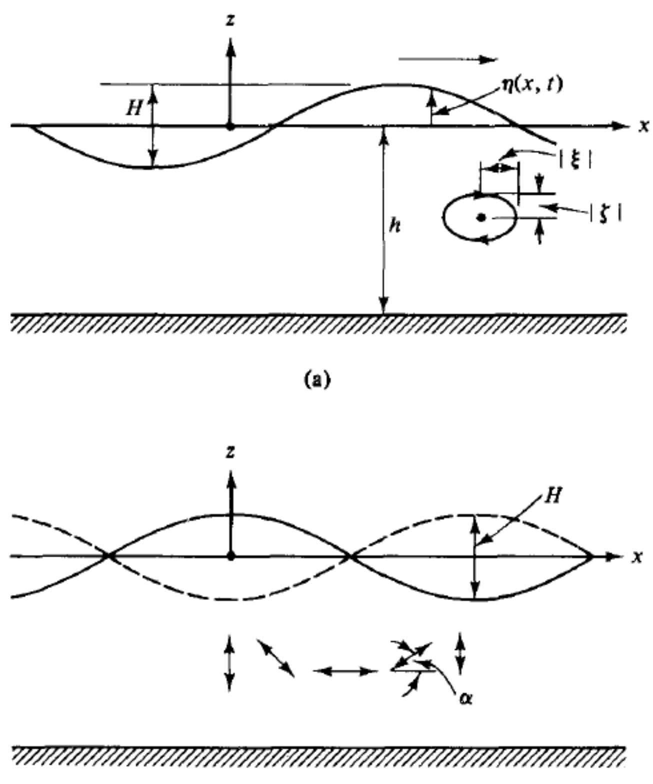

Figure 2. Water particles trajectories underneath a (a) progressive and (b) standing wave. (Taken from Dean & Dalrymple, 1991).

The maximum horizontal and vertical displacements of the water particles can be estimated theoretically, using linear wave theory, by integration of the velocity components (u and w) with respect to time. Hence, we obtain the following expression for progressive (eqn. 3a) and standing (eqn. 3b) waves:

$$|\zeta| = \frac{H}{2}\frac{\cosh k(h+z)}{\sinh kh}$$
$$|\xi| = \frac{H}{2}\frac{\sinh k(h+z)}{\sinh kh} \tag{3a}$$

$$|\zeta| = \frac{H}{2}\frac{\cosh k(h+z)}{\sinh kh}\sin kx$$
$$|\xi| = \frac{H}{2}\frac{\sinh k(h+z)}{\sinh kh}\cos kx \tag{3b}$$

2. **Objectives**

Measured the wave-induced velocities and particle trajectories under progressive and standing waves and compared to linear wave theory

3. **Instructions**

Progresive waves:

i. Set the x-z location of the neutrally buoyant particle. The free-surface elevation sensor will correspond to the x location, whereas the velocity sensor to the z location.
ii. Start progressive wave generation and observe the free surface, wave-induced velocity, and particle movement at the original position of the neutrally buoyant particle.
iii. Export the measured variables to estimate the magnitudes of u and w and the particle maximum displacement $|\zeta|$ y $|\xi|$ as observed in the flume.
iv. Follow (i) to (iii) for two different water depths (suggestion: $z = -h/2$ and $z = -h$).

Standing waves:

i. Select a standing wave and observe the location of the nodes and antinodes.
ii. Set the neutrally buoyant particle at the x location of the antinode and run the experiment for two different elevations (suggestion: $z = -h/2$ and $z = -h$.).
iii. Start the standing wave generation and observe the free surface, wave-induced velocity, and particle movement at the original position of the neutrally buoyant particle.

iv. Export the measured variables to estimate the magnitudes of u and w and the particle maximum displacement $|\zeta|$ y $|\xi|$ as observed in the flume.
v. Repeat (ii-iv) at the x corresponding to a node.

4. **Assignment**

- Compute the wave-induced particle velocities and the particle trajectory displacement for both progressive and standing waves using wave theory (eqns. 2a, 2b, 3a, and 3b).
- Compare the measured velocities and trajectories with respect to linear wave theory results. Compute the relative error for each test.
- Discuss differences between theory and observations in terms of wave characteristics.

References:

Dean, R.G., and Dalrymple, R.A., 1991, Water wave mechanics for engineers and scientists. Advanced series on ocean engineering, Vol. 2. World Scientific.

Experiment 3. "Wave reflection from beaches"

1. **Background theory**

A system of incident and reflected waves in front of a beach or structure can be represented as shown in Figures 1 and 2. According to linear wave theory, incident and reflected waves can be expressed as,

$$\eta_i = \frac{H_i}{2}\cos(kx - \sigma t) \tag{1}$$

$$\eta_r = \frac{H_r}{2}\cos(kx + \sigma t + \delta) \tag{2}$$

where η_i is the incident wave, η_r is the reflected wave, δ is the phase lag induced by the reflection process, x is the cross-shore location, t is the time, and H_i and H_r are the incident and reflected wave, respectively, and

$$k = \frac{2\pi}{L} \tag{3}$$

$$\sigma = \frac{2\pi}{T} \tag{4}$$

The combined wave η_c is given by,

$$\eta_c = \eta_i + \eta_r \tag{5}$$

where the total vertical displacement, $2|\eta_c|$, can be expressed as,

$$2|\eta_c| = \sqrt{H_i^2 + 2H_iH_r\cos(2kx + \delta) + H_r^2} \tag{6}$$

The equation above refers to the wave envelope which is function of the cross-shore distance. The maximum and minimum of equation (6) are given by,

$$2|\eta_c|_{\max} = H_i + H_r \tag{7}$$

and

$$2|\eta_c|_{\min} = H_i - H_r \tag{8}$$

separated by $L/4$. Thus, reflection can be defined by the reflection coefficient given by,

$$\kappa_r = \frac{H_r}{H_i} = \frac{2|\eta_c|_{\max} - 2|\eta_c|_{\min}}{2|\eta_c|_{\max} + 2|\eta_c|_{\min}} \tag{9}$$

where the minimum and maximum reflection coefficient values are, 0 and 1, respectively.

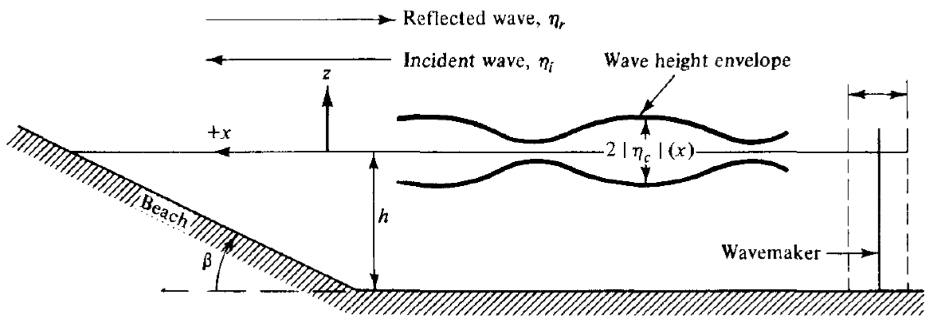

Figure 1. Experimental setup (Taken from Dean & Dalrymple, 1991).

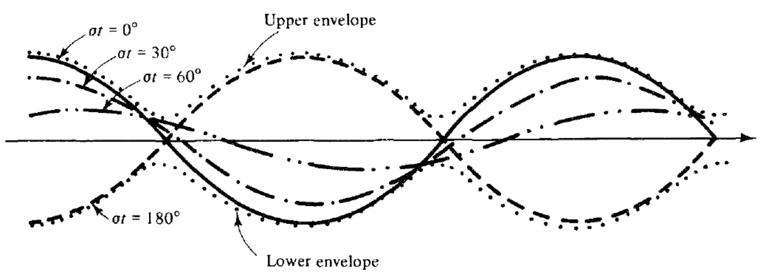

Figure 2. Instantaneous free-surface displacement and wave envelope for a partially standing wave system. (Taken from Dean & Dalrymple, 1991).

An approximate theory to estimate the reflection coefficient from a planar beach was developed by Miche (1947). Miche defined a critical wave steepness at deep water $(H_o/L_o)_{\text{crit}}$:

$$\left(\frac{H}{L_0}\right)_{\text{crit}} = \left(\frac{2\beta}{\pi}\right)^{1/2} \frac{\sin^2\beta}{\pi} \tag{10}$$

where β is the beach slope. He found that reflection coefficients depend on the deep-water wave steepness, H_o/L_o, as:

$$\kappa_r = 1, \qquad \frac{H_0}{L_0} \leq \left(\frac{H}{L_0}\right)_{\text{crit}}$$

$$\kappa_r = \frac{(H_0/L_0)_{\text{crit}}}{H_0/L_0}, \qquad \frac{H_0}{L_0} \geq \left(\frac{H}{L_0}\right)_{\text{crit}} \tag{11}$$

where the incident wave height is related to deep water wave height by,

$$H_0 = \sqrt{\frac{2C_G}{C_0}} H_i \tag{12}$$

where $C_G = nC, \quad n = \frac{1}{2}\left(1 + \frac{2kh}{\sinh 2kh}\right)$ and $C_0 = \frac{L_0}{T}$

2. **Objective**

Estimate the reflection coefficients for different incident wave conditions in the virtual wave flume and compare with theory.

3. **Instructions**

For three different wave conditions:

- Estimate the wave length using linear wave theory.
- In the virtual wave flume set the moving cart velocity with the wave gauge to measure the wave envelop over a distance of at least one wave length.
- Export the measured data

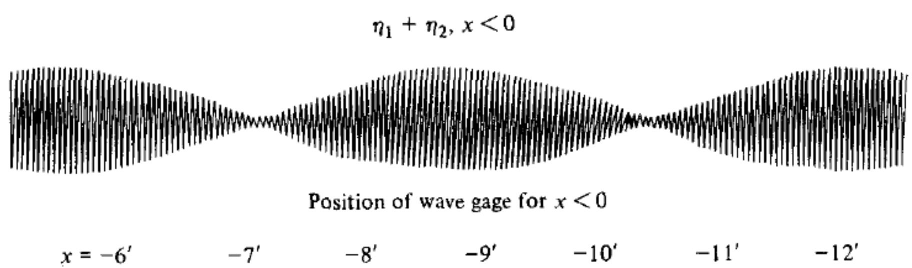

Figure 3. Example of the free-surface elevation measured by the moving cart (Taken from Dean & Dalrymple, 1991).

4. **Assignment**

For each test calculate the reflection coefficient, κ_r, using the maximum ($2|\eta_c|_{\max}$) and minimum ($2|\eta_c|_{\min}$) values of the wave envelope. Compare the reflection coefficients against Miche's theory. Discuss differences in terms of the wave characteristics.

<u>Reference</u>:

Dean, R.G., and Dalrymple, R.A., 1991. Water wave mechanics for engineers and scientists. Advanced Series in Ocean Engineering, vol.2. World Scientific, 353 pp.

Experiment 4: "Wave reflection from a partial vertical barrier"

1. Background theory

As water waves propagate toward a vertical partially submerged barrier (Figure 1) part of the incident wave energy will be reflected and part will be transmitted beneath the vertical barrier. Assuming that no energy is lost in the reflection process,

$$H_r^2 + H_t^2 = H_i^2, \tag{1}$$

where H_r is the reflected wave height, H_t is the transmitted wave height, and H_i is the incident wave height. The transmission coefficient, κ_t, is defined as,

$$\kappa_t = \frac{H_t}{H_i} \tag{2}$$

where $\kappa_r^2 + \kappa_t^2 = 1$; and κ_r is the reflection coefficient defined as $\kappa_r = H_r/H_i$.

Different wave theories have been developed to predict regular wave transmission past vertical barriers. Wiegel (1960) power transmission theory states that to determine the wave height of the transmitted wave component, we can assume that all the incident wave power propagated below the lower edge of the barrier is transmitted. The wave power is depth-integrated from the bottom at $z = -h$ to the lower edge of the barrier at $z = -h + \Delta$ and time averaged over one wave period T,

$$\frac{1}{T}\int_0^T \int_{-h}^{-h+\Delta} p_i\, u_i dzdt = \frac{1}{T}\int_0^T \int_{-h}^{0} p_t\, u_t dzdt \tag{3}$$

where p is the wave-induced dynamic pressure and u is the wave-induced horizontal velocity and subscripts i and t refer to the incident and transmitted components, respectively. Assuming linear wave theory, Wiegel (1960) obtained,

$$k_t = T_F^{1/2}, \tag{4}$$

where the transmission function, T_F, is given by,

$$T_F = \frac{2k\Delta + \sinh 2k\Delta}{2kh + \sinh 2kh}. \tag{5}$$

This theory was modified by Kriebel and Bollmann (1996) to account for the wave reflection effects on the pressure and the horizontal fluid velocity below the barrier that results on a greater pressure $p_i + p_r$ and a reduced velocity $u_i - u_r$ (subscript *r* refers to reflected). For the modified power transmission theory, the pressure and horizontal velocity in the left-hand side of equation (3) was replaced with the total contribution from the incident and reflected wave components finding,

$$\kappa_t = \frac{2T_F}{1+T_F} \, . \tag{6}$$

These two theories, based on linear wave theory, have been widely employed as an engineering solution despite their limitations. The aim of this experiment is to employ an approximate theory to predict the transmission coefficient and compare it with numerical experiments conducted in the virtual wave flume.

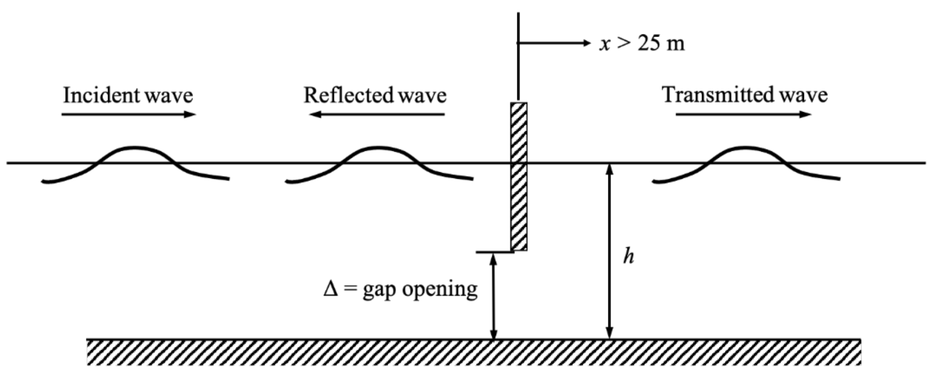

Fig. 1. Wave interaction with a partial vertical barrier problem. Adapted from Dean and Dalrymple (1991).

2. Objective

To measure wave reflection and transmission from a partial vertical barrier and to compare with approximate theory.

3. Instructions

- Use the virtual wave flume to conduct a series of experiments considering different wave periods (Table 1) for a fixed water depth h = 0.6 m and a fixed barrier gap Δ = 0.3 m.
- For each test, measure the wave envelope for x < 25 m by moving a wave cart sensor along the wave flume over a distance of at least one wave length.
- From the wave envelope determine the incident and reflected wave height to compute the reflection coefficient as explained in the wave reflection experiment (see Experiment 3, Supplementary material).
- Use a fixed sensor to measure the transmitted wave and determine the transmission coefficient.

Table 1.- Simulated cases for the wave interaction with a partial vertical barrier

Case	H (m)	T (sec)	h (m)	Δ (m)
1	0.10	1	0.60	0.30
2	0.10	1.5	0.60	0.30
3	0.10	2	0.60	0.30
4	0.10	3	0.60	0.30
5	0.10	4	0.60	0.30
6	0.10	5	0.60	0.30

4. Assignment

For each test, plot κ_t as a function of kh using the approximate theory and also plot the experimental results. Compute the differences between wave flume data and theory and discuss.

References:

Dean, R.G., and Dalrymple, R.A., 1991. Water wave mechanics for engineers and scientists. Advanced Series in Ocean Engineering, vol.2. World Scientific, 353 pp.

Kriebel, D. L., & Bollmann, C. A. (1996). Wave transmission past vertical wave barriers. Coastal Engineering Proceedings, 1(25). https://doi.org/10.9753/icce.v25.%p

Wiegel, R.L. (1960). Transmission of waves past a rigid vertical thin barrier. J.of the Waterways and Harbors Division, ASCE, 86 (WW1), 1-12.

Experiment 5: “Wave shoaling and breaking”

1. Background theory

The wave shoaling coefficient can be estimated by the conservation of energy flux equation. In general, this coefficient increases as the wave propagates in shallow water and the conservation of energy flux equations predicts that it goes to infinite as the water depth *h* approaches to 0. However, in nature this condition does not occur owing to wave breaking. As the water waves propagate on a water depth similar to its wave height, it becomes unstable due to the mismatch of the wave celerity and water particle velocity near the crest. The wave breaking type depends on the beach slope and wave characteristics. For instance, for mild slopes spilling breaking, characterized by a wide surf zone, is more common. On the other hand, plunging breakers often happen when the beach slope is steep and it is characterized by the impinging jet on the wave trough. Collapsing breakers require a very steep beach and consist of narrow or inexistent surf zone (Dean & Dalrymple, 1991).

Beach slope and wave steepness are related through the *surf similarity parameter* (Battjes, 1974) or Iribarren (Iribarren and Nogales, 1949) number given by:

$$\varepsilon_0 = \frac{\tan\beta}{\sqrt{H_0/L_0}} \quad \text{or} \quad \varepsilon_b = \frac{\tan\beta}{\sqrt{H_b/L_0}}$$

where $\tan\beta$ is the beach slope, *H* is the wave height, and *L* is the wave length, and subscripts *b* and 0 denotes breaking or deep-water conditions, respectively. The different breaking types (Figure 1) can be determined by the Iribarren number (Table 1).

Tabla 1. Breaking types according to the *surf similarity parameter* (Battjes, 1974) given by deep water or breaking wave conditions.

Breaking type	ξ_0	ξ_b
Surging or collapsing	$\xi_0 > 3.3$	$\xi_b > 2.0$
Plunging	$0.5 < \xi_0 < 3.3$	$0.4 < \xi_b < 2.0$
Spilling	$\xi_0 < 0.5$	$\xi_b < 0.4$

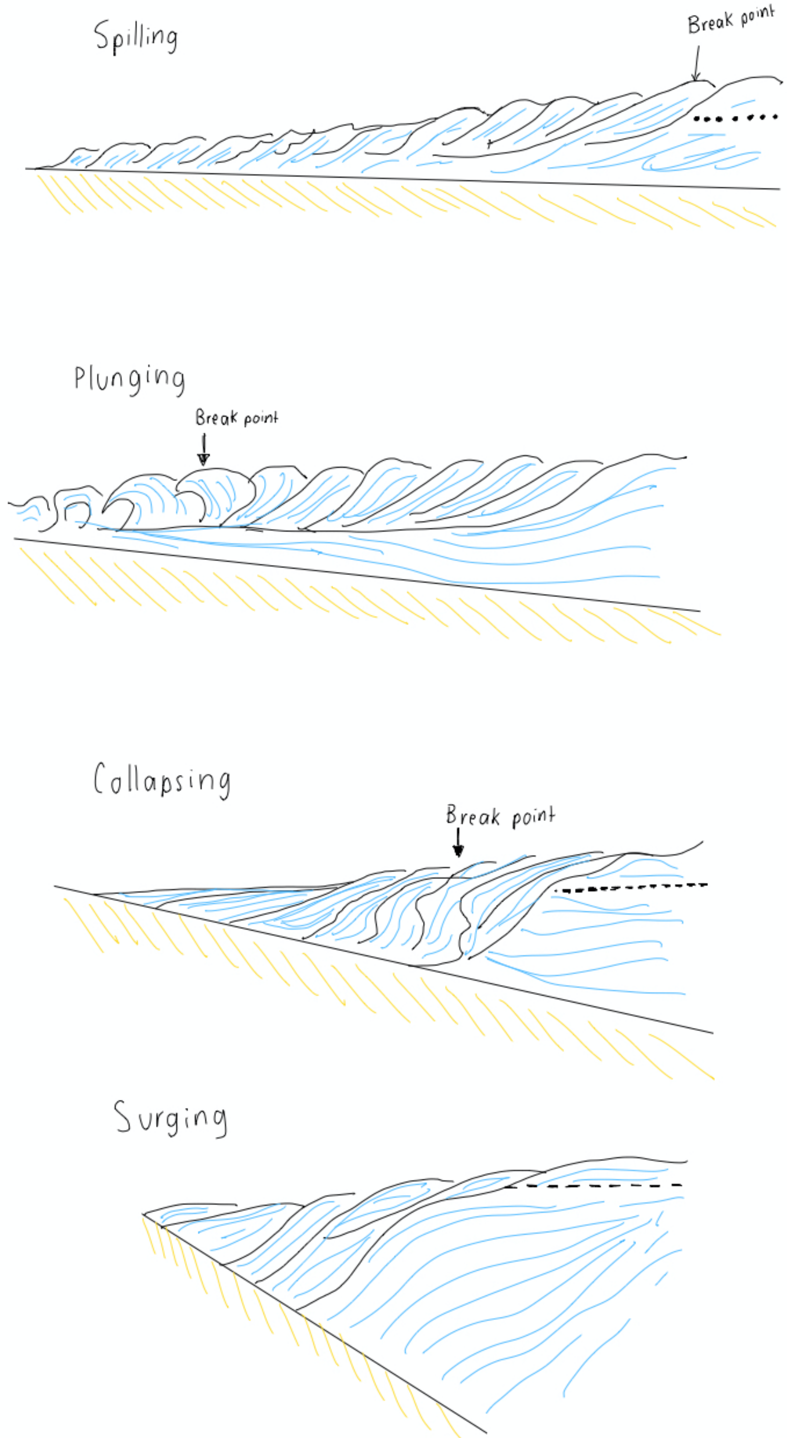

Figure 1. Wave breaking type according to beach slope: *Spilling*, *Plunging*, *Collapsing*, and *Surging*.

Field and laboratory observations show that the breaking wave height is proportional to the local water depth. There are different parametric wave breaking criteria and models reported in the literature (e.g., McCowan,1894; Dally et al., 1993). The simplest model was proposed by McCowan (1894) who determined that waves break when the wave height is a fraction of the water depth,

$$H_b = \kappa h_b$$

where κ = 0.78 for a solitary wave and the subindex *b* denotes breaking values. Therefore, when this criterion is exceeded, wave breaking occurs. Previous laboratory studies employing different beach slopes show a large scatter in the data (see Figure 2), suggesting there is no universal value for κ.

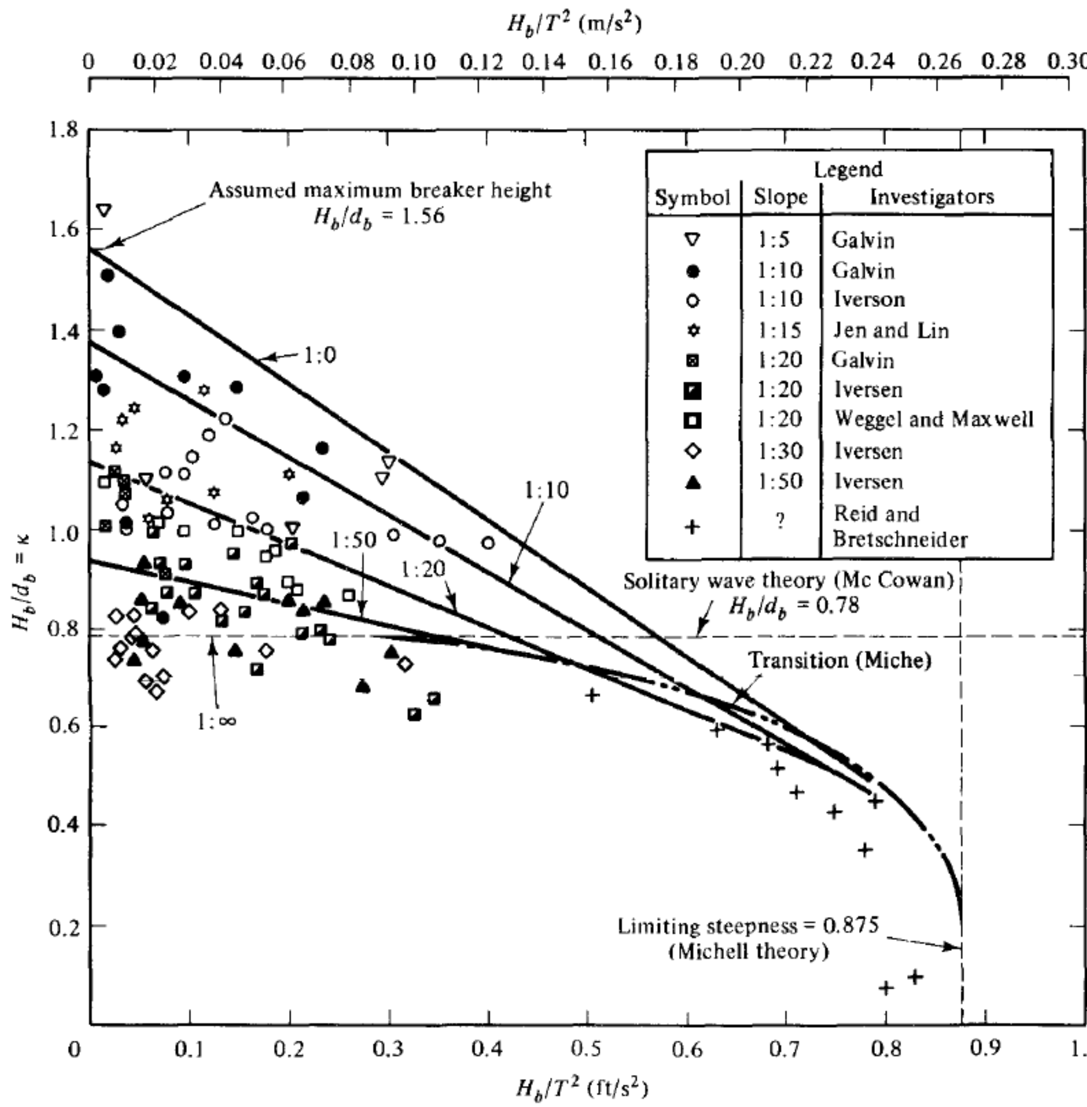

Figure 2. Experimental observations d_b/H_b with respect to H_b/T^2 (Weggel, 1972). Taken from Dean & Dalrymple, 1991.

2. Objective

To investigate progressive wave shoaling and breaking for monochromatic waves and to compare with linear wave theory.

3. Instructions

Employ the virtual wave flume using, three different wave conditions ($T = 2, 4, 6$ s), to:

- Observe the wave transformation and identify visually the breaking point location for each Test.
- Deploy wave gauge sensors along the flume with more resolution inside the surf zone to measure free surface elevation time series over 60 s. Export the data for further analysis.
- Calculate the wave height H from the free-surface elevation time series by means of the zero-down crossing method.
- Plot h vs H for each test.

4. Assignment

Using the virtual wave flume results identify H_b and h_b and estimate the breaking wave index κ. Compare the virtual wave data results with respect to the value found by McCowan. For each case, estimate the *Surf Similarity Parameter* to classify the breaking wave type and compare with visual observations in the wave flume.

References:

Battjes, J.A., 1974. Surf Similarity, *Proc. 14th Intl. Conf. Coastal Eng.,* ASCE, Copenhagen, 466-480.

Dean, R.G., and Dalrymple, R.A., 1991. Water wave mechanics for engineers and scientists. Advanced Series in Ocean Engineering, vol.2. World Scientific, 353 pp.

Iribarren, C.R., Nogales, C. (1949), «Protection des ports», Proceedings XVIIth International Navigation Congress, Section II, Communication 4, Lisbon, pp. 31-80.

McCowan, J., 1894. On the highest wave of permanent type. *Philos. Mag. J. Sci*., Vol. 38.

Short, A. D., 1999. Handbook of beach and shoreface morphodynamics. Chichester, John Wiley, 392 pp.

Weggel, J.R., 1972. Maximum Breaker Height, *J. Waterways, Harbors Coastal Eng. Div., ASCE,* Vol. 98, WW4.

Experiment 6: "Irregular wave breaking"

1. **Background theory**

Wave breaking on natural beaches occurs over a wide area due to the variable wave heights and periods of the incoming irregular waves. Bigger waves will break farther offshore than smaller waves. The percentage of breaking waves inside the surf zone varies across the surf zone.

To identify breaking waves from a free surface elevation time series, $\eta(t)$, two parameters can be estimated (Xu *et al*, 1986): (i) the rate of temporal variation $R(t)$ and (ii) the free-surface slope $s(t)$ (Figure 1), given by,

$$R(t) = d\eta(t)/dt, \tag{1}$$

$$s(t) = R(t)/C \tag{2}$$

where C is the wave celerity. An increase of $R(t)$ is expected for breaking waves (Longuet-Higgins and Smith, 1983). The maximum slope for regular progressive waves is 0.586 (Longuet-Higiins and Fox, 1977), and hence,

$$s_{max} = 0.586, \quad R_{max} = 0.586c$$

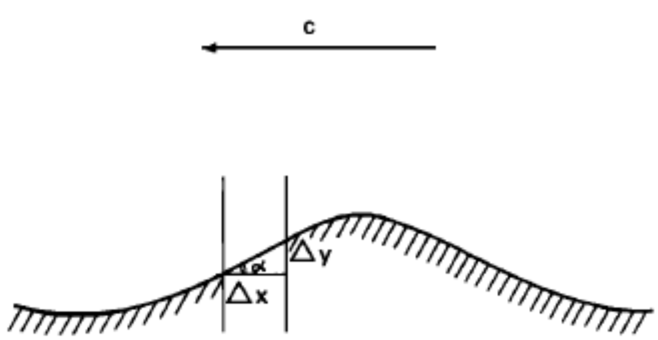

Figure 1. Interpretation of the rate of temporal variation R for a progressive wave (taken from Longuet-Higgins and Smith, 1983). R=c tanα; s = tanα.

Therefore, a wave is classified as broken if $R(t)>R_{max}$ at any point between the zero crossing. In some cases, this criterion is met in more than one place along the wave profile, but only one broken wave counts. This information is important as some breaking models (Thornton & Guza, 1983) predict the distribution of breaking and non-breaking waves in the breaking zone, requiring measurements for their calibration.

2. **Objective**

To investigate wave breaking characteristics of irregular progressive waves.

3. **Instructions**

Employ the virtual wave flume, testing two different wave conditions, to:

- Observe the irregular wave transformation for the two tests and identify the offshore limit of the surf zone.
- Deploy 10 wave gauge (WG) sensors along the flume with increasing resolution within the surf zone.
- Measure free-surface elevation time series during 300 s and export the measured data.

4. **Assignment**

For each case, use the zero-down crossing method to estimate the wave parameters (H and T) for each wave in the irregular wave train at the different cross-shore locations. Estimate the significant wave height H_s ($H_{1/3}$) and the root-mean square wave height H_{rms}. Plot H_{rms} and H_s vs h to determine the wave breaking index.

Compute R to separate between breaking and non-breaking waves at each cross-shore location using the linear dispersion equation to compute C. Plot the histograms of breaking and non-breaking waves at each cross-shore location for each test and the % of breaking and non-breaking waves vs x.

References:

Longuet-Higgins, M.S. and Fox, M.J.H., 1977. Theory of the almost-highest wave: the inner solution. *J. Fluid Mech.,* 80(4), 721-741.

Longuet-Higgins, M.S. and Smith, N.D., 1983. Measurement of breaking waves by a surface jump meter, *J. Geophys. Res*., 88(C14), 9823-9831.

Thornton, E.B., and Guza,R.T., 1982. Energy saturation and phase speeds measured on a natural beach, *J. Geophys. Res*., 87, 9499-9508.

Thornton, E.B. and Guza,R.T., 1983. Transformation of wave height distribution. *J. Geophys. Res*., 88 (C10), 5925-5938.

Xu, D., Hwamg, P.A., and Wu, J., 1986. Breaking of wind-generated waves. *J. Phys. Oceanogr.*, 16, 2172-2178.

Experiment 7: "Wave set-down and set-up"

1. **Background theory**

Wave breaking induces a depression in the mean sea level (*set-down*) near the breaking point and an increase in the mean water level (*set-up)* that increases toward the shoreline where it reaches a maximum. Longuet-Higgins and Stewart (1964) show that *set-down* and del *set-up* are associated to the gradient of the wave radiation stress defined as the excess of momentum flux due to waves. These authors derived the depth integrated and time-averaged momentum equation assuming stationary flow, beach with straight and parallel contours, negligible Reynolds stresses, and no wind, given by:

$$\frac{\partial S_{xx}}{\partial x} = -\rho g(h+\bar{\eta})\frac{\partial \bar{\eta}}{\partial x} \tag{1}$$

where S_{xx} is the cross-shore component of the wave radiation stress, h is the water depth, and $\bar{\eta}$ is the change in the mean sea level due to waves. Therefore, equation (1) represents the balance between the radiation stress gradient and the sea level gradient inside the surf zone (Figure 1).

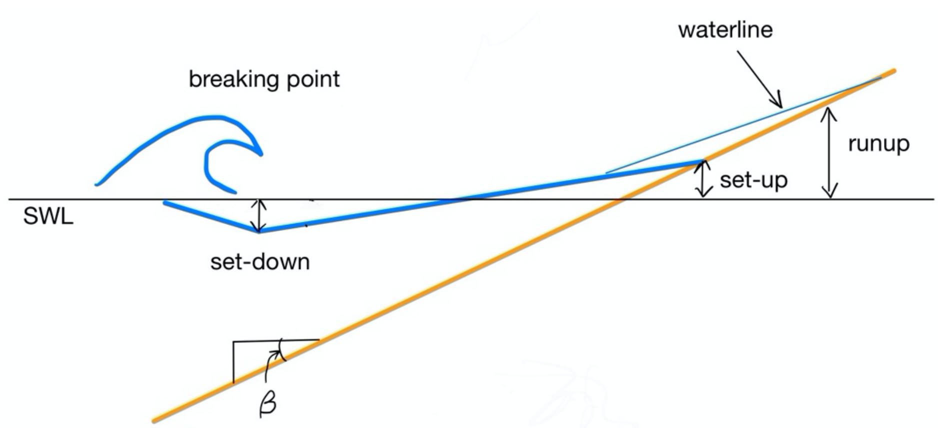

Figure 1.- Mean sea level variation inside the surf zone.

To evaluate the cross-shore component of the radiation stress we can employ second order linear theory approximation (Longuet-Higgins and Stewart, 1964),

$$S_{xx} = \frac{E}{2}[1+2G] = E\left[\frac{1}{2}+G\right] = E\left[\frac{1}{2}(1+G)+\frac{G}{2}\right] = \frac{ECg}{c}+\frac{EG}{2} \quad (2)$$

where,

$$G = \frac{2kh}{\sinh 2kh} \;; \qquad Cg = \frac{c}{2}(1+G) \;; \qquad n = \frac{Cg}{G}; \qquad E = \tfrac{1}{8}\rho g H^2$$

2. **Objective**

To investigate the mean sea level variability inside the surf zone in the virtual wave flume and compare the measurement with linear wave theory.

3. **Instructions**

- In the virtual wave flume Main Menu select the experiment "Wave set-down and set-up".
- Run the irregular wave tests (Test 1: $H_s = 0.125$ m, $T_p = 2.5$ s, $\gamma = 3.3$; Test 2: $H_s = 0.1$ m, $T_p = 1.5$ s, $\gamma = 3.3$) during 300 s to visually identify the offshore limit of the surf zone.
- Deploy 10 wave gauges (WG) from offshore to the shore with an increasing resolution inside the surf zone.
- Run the two tests and export the data for further analysis.

4. **Assignment**

- Calculate the mean sea level at the different cross-shore locations by taking the mean of the free-surface to identify the *set-up* and *set-down* cross-shore locations.
- Estimate the wave parameters for each wave in the irregular wave train (H_i, T_i) using the zero down-crossing method.
- Compute $\overline{H}$ and $\overline{T}$ from the individual waves.
- Plot $\bar{\eta}$ and $\overline{H}$ vs cross-shore location x (see Figure 2).
- Calculate the cross-shore radiation stress component S_{xx} using $\overline{H}$ and $\overline{T}$ and plot with respect to x.
- Express equation (1) in finite difference and solve for $\bar{\eta}_{i+1}$.
- Compute the mean sea level at the different cross-shore locations using the finite difference scheme employing the measured $\bar{\eta}_i$ at the offshore location as boundary condition
- Compare the measured $\bar{\eta}$ vs the one computed by equation (1).

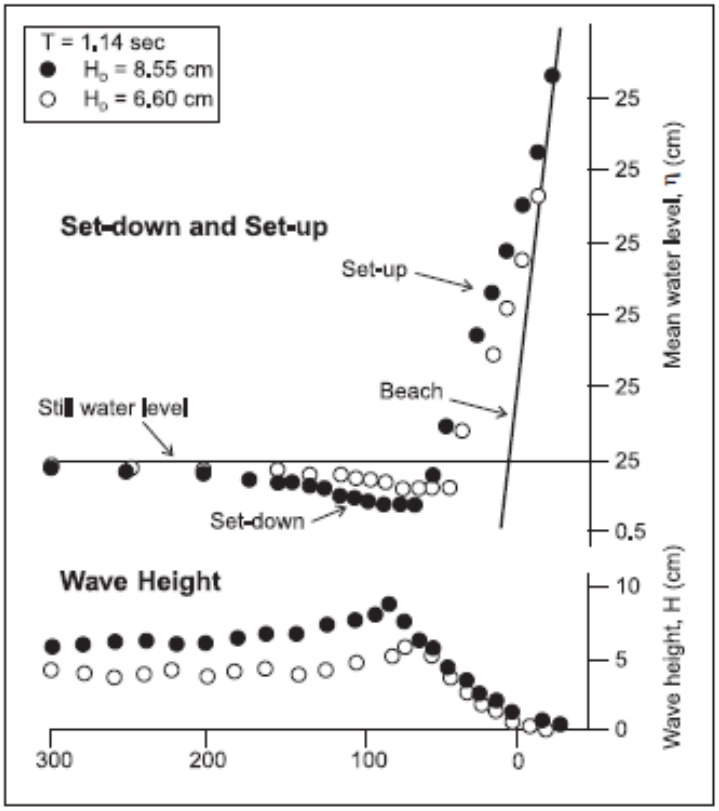

Figure 2. Wave height and set-down/set-up across-shore variation for two different wave conditions. Taken from Davidson-Arnott (2009).

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