**Experiment 2. "Wave-induced velocities and particle trajectories"**

1. **Background theory**

The velocity components under progressive or standing waves can be estimated from the velocity potential $\phi$ from the solution of the Laplace equation which is given in terms of the wave height, $H$, water depth, $h$, the wave number, $k = 2\pi/L$, and the angular frequency, $\sigma = 2\pi/T$,

$$\phi(x, y, z) = -\frac{H}{2}\frac{g}{\sigma}\frac{\cosh k(h+z)}{\cosh kh}\sin(kx - \sigma t) \tag{1a}$$

$$\phi(x, y, z) = \frac{H}{2}\frac{g}{\sigma}\frac{\cosh k(h+z)}{\cosh kh}\cos kx \sin \sigma t \tag{1b}$$

where $t$ is the time, $g$ is the gravity acceleration, and $x$ and $z$ represent the horizontal and vertical position of the water particle, respectively. Therefore, velocity components $u$ and $w$ can be estimated for a progressive (eqn. 2a) and standing (eqn. 2b) wave as,

$$u = -\frac{\partial\phi}{\partial x} = \frac{H}{2}\sigma\frac{\cosh k(h+z)}{\sinh kh}\cos(kx - \sigma t)$$

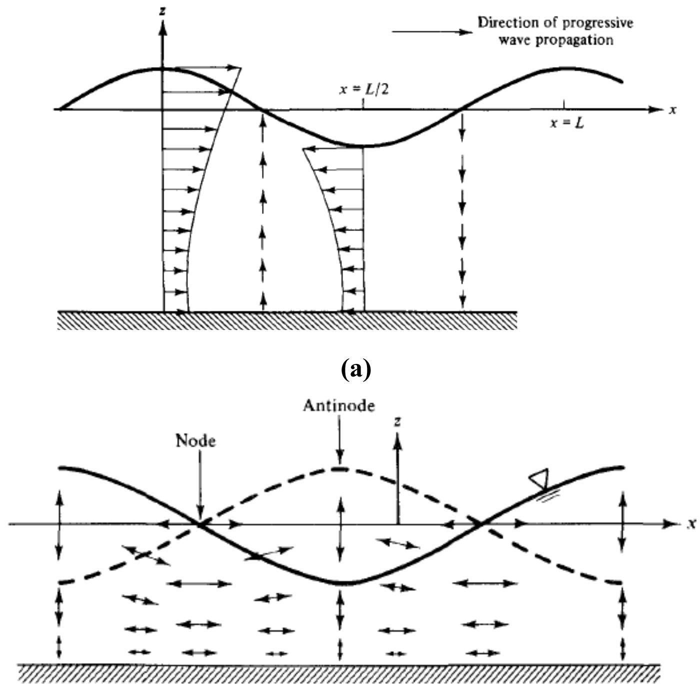
$$w = -\frac{\partial\phi}{\partial z} = \frac{H}{2}\sigma\frac{\sinh k(h+z)}{\sinh kh}\sin(kx - \sigma t) \tag{2a}$$

$$u = \frac{H}{2}\sigma\frac{\cosh k(h+z)}{\sinh kh}\sin kx \sin \sigma t$$

$$w = -\frac{H}{2}\sigma\frac{\sinh k(h+z)}{\sinh kh}\cos kx \sin \sigma t \tag{2b}$$

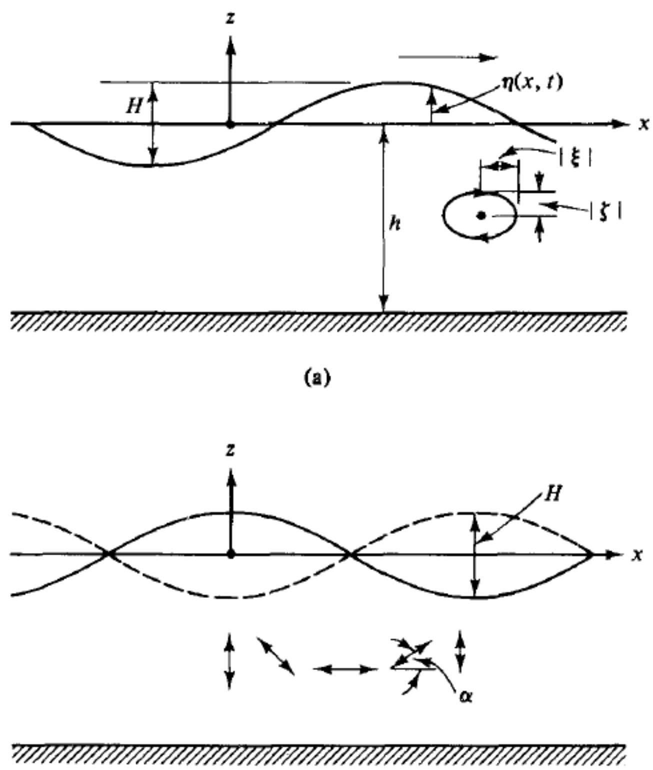
The velocity associated with either progressive or stationary waves decreases with distance from the free surface (Figure 1) due to dependence on the elevation $z$ (eqn. 2a and 2b). For standing waves, the maximum values for $u$ and $w$ occur underneath the nodes and antinodes (Figure 1b), whereas $u$ and $w$ are zero below the antinodes and nodes, respectively.

The maximum particle displacement $|\zeta|$ and $|\xi|$, for $x$ and $z$ directions, can be expressed as a function of the wave height $H$, the mean particle position underneath the wave, the wave period ($T$), and the water depth ($h$) (Figure 2).




**(b)**

Figure 1. Wave-induced water particle velocities underneath a (a) progressive and (b) standing water wave. (Taken from Dean & Dalrymple, 1991).




(b)

Figure 2. Water particles trajectories underneath a (a) progressive and (b) standing wave. (Taken from Dean & Dalrymple, 1991).

The maximum horizontal and vertical displacements of the water particles can be estimated theoretically, using linear wave theory, by integration of the velocity components ($u$ and $w$) with respect to time. Hence, we obtain the following expression for progressive (eqn. 3a) and standing (eqn. 3b) waves:

$$|\zeta| = \frac{H}{2}\frac{\cosh k(h+z)}{\sinh kh}$$
$$|\xi| = \frac{H}{2}\frac{\sinh k(h+z)}{\sinh kh} \tag{3a}$$

$$|\zeta| = \frac{H}{2}\frac{\cosh k(h+z)}{\sinh kh}\sin kx$$
$$|\xi| = \frac{H}{2}\frac{\sinh k(h+z)}{\sinh kh}\cos kx \tag{3b}$$

2. **Objectives**

Measured the wave-induced velocities and particle trajectories under progressive and standing waves and compared to linear wave theory

3. **Instructions**

Progresive waves:

i. Set the $x$-$z$ location of the neutrally buoyant particle. The free-surface elevation sensor will correspond to the $x$ location, whereas the velocity sensor to the $z$ location.
ii. Start progressive wave generation and observe the free surface, wave-induced velocity, and particle movement at the original position of the neutrally buoyant particle.
iii. Export the measured variables to estimate the magnitudes of $u$ and $w$ and the particle maximum displacement $|\zeta|$ y $|\xi|$ as observed in the flume.
iv. Follow (i) to (iii) for two different water depths (suggestion: $z = -h/2$ and $z = -h$).

Standing waves:

i. Select a standing wave and observe the location of the nodes and antinodes.
ii. Set the neutrally buoyant particle at the $x$ location of the antinode and run the experiment for two different elevations (suggestion: $z = -h/2$ and $z = -h$.).
iii. Start the standing wave generation and observe the free surface, wave-induced velocity, and particle movement at the original position of the neutrally buoyant particle.

iv. Export the measured variables to estimate the magnitudes of $u$ and $w$ and the particle maximum displacement $|\zeta|$ y $|\xi|$ as observed in the flume.

v. Repeat (ii-iv) at the $x$ corresponding to a node.

4. **Assignment**

- Compute the wave-induced particle velocities and the particle trajectory displacement for both progressive and standing waves using wave theory (eqns. 2a, 2b, 3a, and 3b).
- Compare the measured velocities and trajectories with respect to linear wave theory results. Compute the relative error for each test.
- Discuss differences between theory and observations in terms of wave characteristics.

References:

Dean, R.G., and Dalrymple, R.A., 1991, Water wave mechanics for engineers and scientists. Advanced series on ocean engineering, Vol. 2. World Scientific.