Hand-in Assignment 1

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Task I: The FIAT–Chrysler Case

Decision Variables

$$x_i \geq 0 \quad \text{number of cars produced of model } i \in \{\text{Panda, 500, Musa, Giulia}\}.$$

Objective Function

Maximize net profit:

$$\text{maximize } Z = \sum_i (1 - t_i)\big[p_i x_i - m_i p_i x_i - \text{sal}_i x_i\big]$$

where t_i is income tax, p_i is car price, m_i is material fraction, and sal_i is salary per car.

Constraints

$$\begin{aligned}
& x_i \geq x_i^{\min} && \forall i \quad \text{(minimum production)} \\
& x_{\text{Panda}} + x_{\text{Musa}} \leq 300{,}000 && \text{(marketing limit)} \\
& \sum_i \big(m_i p_i x_i + \text{sal}_i x_i\big) \leq B && \text{(budget constraint)}
\end{aligned}$$

Results (from Gurobi)

Model	Units Produced
Panda	120,000
500	100,000
Musa	80,000
Giulia	80,135

Net profit: $\approx$ 24.05 billion SEK.

Task II: The Global Market

The manager must determine production and shipment quantities for four car models (Panda, 500, Musa, Giulia) across four markets (Poland, Italy, US, Sweden). Constraints include:

- Meeting market demand.
- Forbidden exports: Musa $\rightarrow$ US, Giulia $\rightarrow$ Poland.
- Budget limit: 40 billion SEK, covering material, salaries, and delivery costs.
- Home-price material costs: salary and delivery costs per car.
- Export tax: 2.5% on US shipments.

Decision Variables

$x_{i,j}$ cars of model i shipped to market $j, i \in \{\text{Panda, 500, Musa, Giulia}\}, j \in \{\text{Poland, Italy, US, Sweden}\}$

Objective Function

Maximize net profit:

Where:

- $\text{profit}_{i,j} = p_{i,j} - \text{salary}_i - m_i \text{p}_{i,home} - d_{i,j} - e_{i,j} p_{i,j}$: profit of each car i sold to country j.
- t_i: tax on profit for model i.
- $p_{i,j}$: price of model i in market j.
- salary_i: salary per car produced.
- m_i: material cost fraction of home price.
- $d_{i,j}$: delivery cost per car.
- $e_{i,j}$: export tax on price for model i sold in country j.

Constraints

Demand satisfaction:	$x_{i,j} \geq D_{i,j}, \quad \forall i, j$
Minimum production number:	$\sum_j x_{i,j} \geq x_i^{\min}, \quad \forall i$
Forbidden routes:	$x_{\text{Musa,US}} = 0, \quad x_{\text{Giulia,Poland}} = 0$
Budget:	$\sum_i \sum_j \left(\text{salary}_i - m_i \text{p}_{i,home} - d_{i,j}\right) \leq 40\text{B SEK}$
Non-negativity:	$x_{i,j} \geq 0$

Results

Feasible Solution (Gurobi)

Model	Produce	Poland	Italy	US	Sweden
Panda	117,000	75,000	0	40,000	2,000
500	495,743	0	40,000	450,743	5,000
Musa	11,000	10,000	0	0	1,000
Giulia	4,000	0	0	3,000	1,000

Net profit: 47,304,988,496 Kr

Task III: Multi-line Foam Production

Problem Description

Fiat produces foam for car seats by mixing three components. Each component can be processed on four production lines with different capacities and production rates. Each unit of foam requires one unit of each component. The goal is to determine how many hours each line should work on each component to **maximize total foam production**.

Decision Variables

$$
\begin{aligned}
x_{ij} &\geq 0 \quad \text{hours line } i \text{ spends on component } j, \quad i = 1..4, j = 1..3 \\
F &\geq 0 \quad \text{total foam units produced (bottleneck output)}
\end{aligned}
$$

Note: F represents the number of foam units produced. Since 1 unit of foam requires 1 unit of each component, F is limited by the component with the smallest total production. The constraints ensure F does not exceed the available units of any component.

Production Data

Line	Capacity (h)	Rate C1	Rate C2	Rate C3
1	100	10	15	5
2	150	15	10	5
3	80	20	5	10
4	200	10	15	20

Table 1: Line capacities and production rates (units/hour)

Constraints

1. **Line capacity:**

$$\sum_{j=1}^{3} x_{ij} \leq \text{Capacity}_i, \quad \forall i$$

2. **Component production limits (bottleneck constraint):**

$$F \leq \sum_{i=1}^{4} \text{Rate}_{ij} \cdot x_{ij}, \quad \forall j = 1, 2, 3$$

3. **Non-negativity:**

$$x_{ij} \geq 0, \quad F \geq 0$$

Objective Function

Maximize total foam units:

$$\text{maximize } F$$

Optimal Solution (Gurobi)

Line	Component 1 (h)	Component 2 (h)	Component 3 (h)
1	0	100	0
2	88	61	0
3	80	0	0
4	0	54	146

Total foam units produced: 2,920 units.

Task IV: Retailers Localization

Problem Description

Fiat wants to open new retailers in a set of possible locations I in Puglia, using a part of last year's revenues W. Each location $i \in I$ has:

- Fixed cost: F_i (land, administrative)
- Variable cost: C_i per 100 m^2
- Minimum and maximum retailer size: L_i, U_i (in hundreds of m^2)
- Revenue: R_i, per 100 m^2

Only **up to K retailers** can be opened. The objective is to select which locations to open and the size of each retailer to **maximize total profit**, under budget and size constraints.

Decision Variables

$y_i \in \{0,1\}$	1 if location i is opened, 0 otherwise
$s_i \geq 0$	size of retailer at location i (hundreds of m^2)

Constraints

1. **Link size to opening decision (Big-M constraints):**

$$L_i\, y_i \leq s_i \leq U_i\, y_i, \quad \forall i \in I$$

This ensures that if the retailer is not opened ($y_i = 0$), then $s_i = 0$; otherwise the size respects $[L_i, U_i]$.

2. **Maximum number of retailers:**

$$\sum_{i \in I} y_i \leq K$$

3. **Budget constraint (optional, if budget W is considered):**

$$\sum_{i \in I} (F_i\, y_i + C_i\, s_i) \leq W$$

4. **Non-negativity and integrality:**

$$y_i \in \{0,1\}, \quad s_i \geq 0, \quad \forall i \in I$$

Objective Function

Maximize total profit:

$$\text{maximize } Z = \sum_{i \in I} y_i (R_i s_i - F_i - C_i s_i)$$

Task V: Investments Optimization

Problem Description

FIAT wants to invest 1 billion crowns in five debt securities $I = \{A, B, C, D, E\}$ with the following properties: type (Private, Public, Government), Moody's rating, duration d_i, expected revenue r_i, and taxation t_i. The portfolio must satisfy budget, risk, duration, and logical constraints.

Decision Variables

$$x_i \geq 0 \quad \text{amount invested in security } i$$
$$y_i \in \{0, 1\} \quad \text{1 if security } i \text{ is chosen, 0 otherwise}$$

Constraints

$$\begin{aligned}
&\text{Budget:} && \sum_{i \in I} x_i \leq 1\,\text{B crowns} \\
&\text{Government/Public minimum allocation:} && \sum_{i \in I_{gov/public}} x_i \geq 0.4 \times 1\,\text{B} \\
&\text{Average duration:} && \sum_{i \in I} d_i x_i \leq 5 \sum_{i \in I} x_i \\
&\text{Average risk:} && \sum_{i \in I} \text{risk}_i x_i \leq 1.5 \sum_{i \in I} x_i \\
&\text{Mutual exclusivity (C and D):} && y_C + y_D \leq 1 \\
&\text{Conditional investment (E requires 1M in A):} && x_E \leq M y_E, \quad x_A \geq 1\,\text{M} \cdot y_E \\
&\text{Big-M linking:} && x_i \leq M y_i \quad \forall i \in I \\
&\text{Non-negativity:} && x_i \geq 0,\ y_i \in \{0, 1\} \quad \forall i \in I
\end{aligned}$$

Objective Function

$$\text{maximize } Z = \sum_{i \in I} x_i r_i (1 - t_i)$$

Optimal Solution (Gurobi)

The optimal investment portfolio obtained by solving the MILP model is summarized below:

Investment	Amount Invested (crowns)	Chosen (0/1)
A	227,272,727	1
B	0	1
C	704,545,455	1
D	0	0
E	68,181,818	1

Table 2: Optimal investment allocation

The total expected revenue after taxes is 39,538,636 crowns. All constraints including budget, government/public allocation, average duration and risk, mutual exclusivity, and conditional investment have been satisfied.