Quadratic Modeling for 3-SAT Problem

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We assume that our function f(x) is a product of n sets, each set being the sum of 3 elements of our primary set:

$$f(x) = \prod_{i=1}^{n} s_i \tag{1}$$

where

$$s_i = \sum_{j=1}^{3} a_j. \tag{2}$$

For our model, we declare a variable x_i. If $x_i = 0$, it means we are using the variable a_i itself; but if $x_i = 1$, it means we are using the variable $\overline{a_i}$. we assume the number of x_is to be m.

There is another variable we might want to declare as $y_{i,j}$, where $y_{i,j} = 1$ if variable a_i or $\overline{a_i}$ exists in the j-th term; logically, if it does not, $y_{i,j} = 0$.

Also, the main variable taking random values due to quantum operations is $z_{i,j}$. We have $3n$ of them taking values according to the Ising model: 1 if a_i or $\overline{a_i}$ is 1 in the j-th term, and -1 if it is 0 .

So for every term, we have three combinations:

$$-1, -1, -1 \equiv 0, 0, 0 \tag{3}$$

$$-1, 1, -1 \equiv 0, 1, 0 \tag{4}$$

$$1, -1, 1 \equiv 1, 0, 1 \tag{5}$$

$$1, 1, 1 \equiv 1, 1, 1. \tag{6}$$

The first function we are trying to build is to charge the total cost if any term has its three variables taking boolean value 0 :

$$\sum_{i=1}^{n} \left(\left(\sum_{j=1}^{3} z_{i,j} \right)^2 - 3 \left\lfloor \frac{\sum_{j=1}^{3} z_{i,j}}{3} \right\rfloor \right). \tag{7}$$

the second part of our function checks whether our x_i takes the same value in every term or not. if not costs more.

$$-\sum_{k=1}^{m} (\sum_{i=1}^{k} \sum_{j=1}^{n} ((y_{k,j} - 2 * x_k) * z_{i,j}))^2 \tag{8}$$

the overall equation is:

$$A\sum_{i=1}^{n}\left(\left(\sum_{j=1}^{3} z_{i,j}\right)^{2} - 3\left\lfloor \frac{\sum_{j=1}^{3} z_{i,j}}{3} \right\rfloor\right) - B\sum_{k=1}^{m}\left(\sum_{i=1}^{k}\sum_{j=1}^{n}(y_{k,j} - 2x_k)z_{i,j}\right)^{2} \quad (9)$$