13. (*10 points*) Let n be a positive integer and r an integer such that $0 \leq r \leq n$.

Prove that $C(n,r) = C(n, n-r)$.

$$ {}^{n}C_r = \frac{n!}{r!\,(n-r)!} = \frac{n!}{(n-r)!\,(n-(n-r))!} = {}^{n}C_{n-r} $$

14. (*10 points*) Let n be a positive integer and r an integer such that $0 \leq r \leq n$.

Prove that $C(n,r) = C(n-1,r) + C(n-1,r-1)$

$$ C(n-1,r) + C(n-1,r-1) = \frac{(n-1)!}{r!\,(n-1-r)!} + \frac{(n-1)!}{(r-1)!\,[n-1-(r-1)]!} $$

$$ = \quad " \quad " \quad + \frac{(n-1)!}{(r-1)!\,(n-r)!} $$

$$ = \frac{(n-1)!\,(n-r)}{r!\,(n-r-1)!\,(n-r)} + \frac{(n-1)!\,r}{(r-1)!\,(n-r)!\,r} $$

$$ = \frac{(n-1)!\,(r+n-r)}{r!\,(n-r)!} $$

$$ = \frac{(n-1)!\,n}{r!\,(n-r)!} $$

$$ = \frac{n!}{r!\,(n-r)!} $$

$$ = \binom{n}{r} = C(n,r) $$