. (*10 points*) Use induction to prove that $\frac{n^3}{3} + \frac{n^5}{5} + \frac{7n}{15}$ is an integer for all natural numbers n.

Let $n = 1$

$\frac{1^3}{3} + \frac{1^5}{5} + \frac{7}{15} = 1$ = true

Let $n = x+1$

$$\frac{(x+1)^3}{3} + \frac{(x+1)^5}{5} + \frac{7(x+1)}{15}$$

$$= \left(\frac{x^3}{3} + \frac{x^5}{5} + \frac{7x}{15}\right) + \frac{7}{15} + \left(\frac{3x^2 + 3x + 1}{3}\right) + \left(\frac{5x^4 + 10x^3 + 10x^2 + 5x + 1}{5}\right)$$

$n = 1$ = integer

$n = x+1$ = integer $\wedge$ = integer

so $\frac{n^3}{3} + \frac{n^5}{5} + \frac{7n}{15}$ = integer for all n