

11. (10 points) Prove that if R is a symmetric, transitive relation on A and the domain of R is A , then R is reflexive on A .

If $(x, y) \in R$

And $x \in A \wedge y \in A$

Then $(y, x) \in R$ and $(x, x) \in R$ and $(y, y) \in R$

And $\forall x \in A \exists y \in A [(x, y)] \in R$

$\therefore \forall x \in A [(x, x) \in R]$ & R is reflexive

12. (10 points) Suppose that R and S are equivalence relations on a set A . Prove that $R \cap S$ is an equivalence relation on A .

Let $a \in A$ be arbitrary

then $(a, a) \in R$ and $(a, a) \in S$ ($R \cap S$ = reflexive)

$(a, a) \in R \cap S$

if $(a, b) \in A$ so $(a, b) \in R \cap S$. Then $(a, b) \in R$ & $(a, b) \in S$

$\therefore (b, a) \in R$ and $(b, a) \in S$ $\therefore (b, a) \in R \cap S$ & $R \cap S$ is symmetric

Let $(a, b, c) \in A$ and $(a, b), (b, c) \in R \cap S$ then $(a, b), (b, c) \in R$ and $(a, b), (b, c) \in S$.

R is transitive so $(a, c) \in R$
 S is transitive so $(a, c) \in S$ } $(a, c) \in R \cap S$ = transitive

$\therefore R \cap S$ is equivalence because its reflexive, symmetric, & trans