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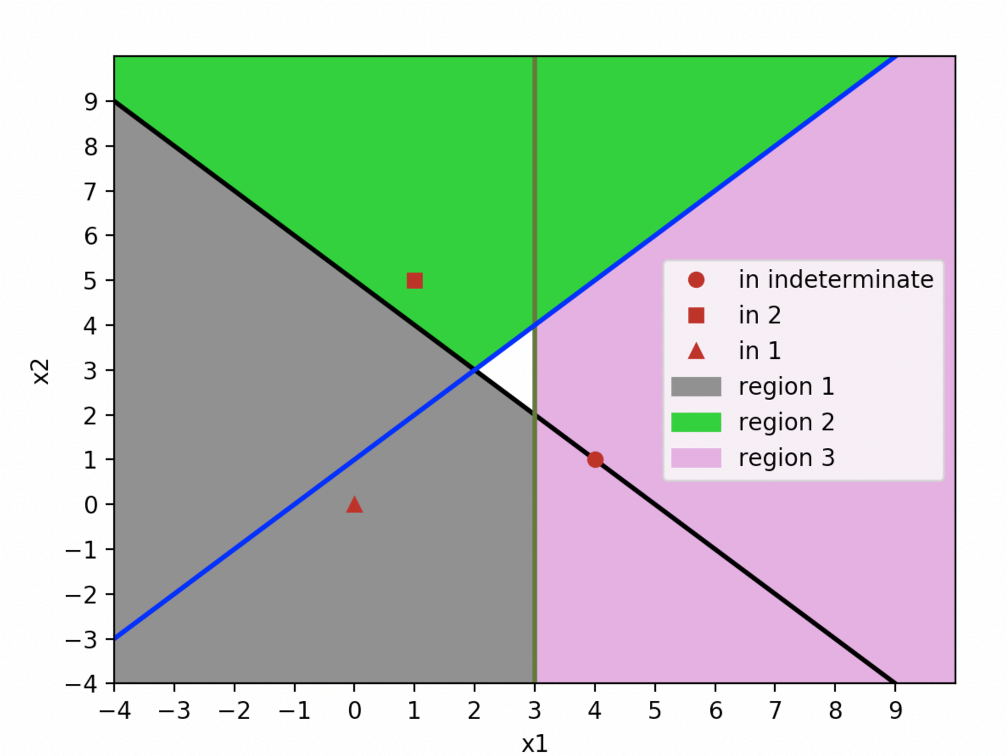
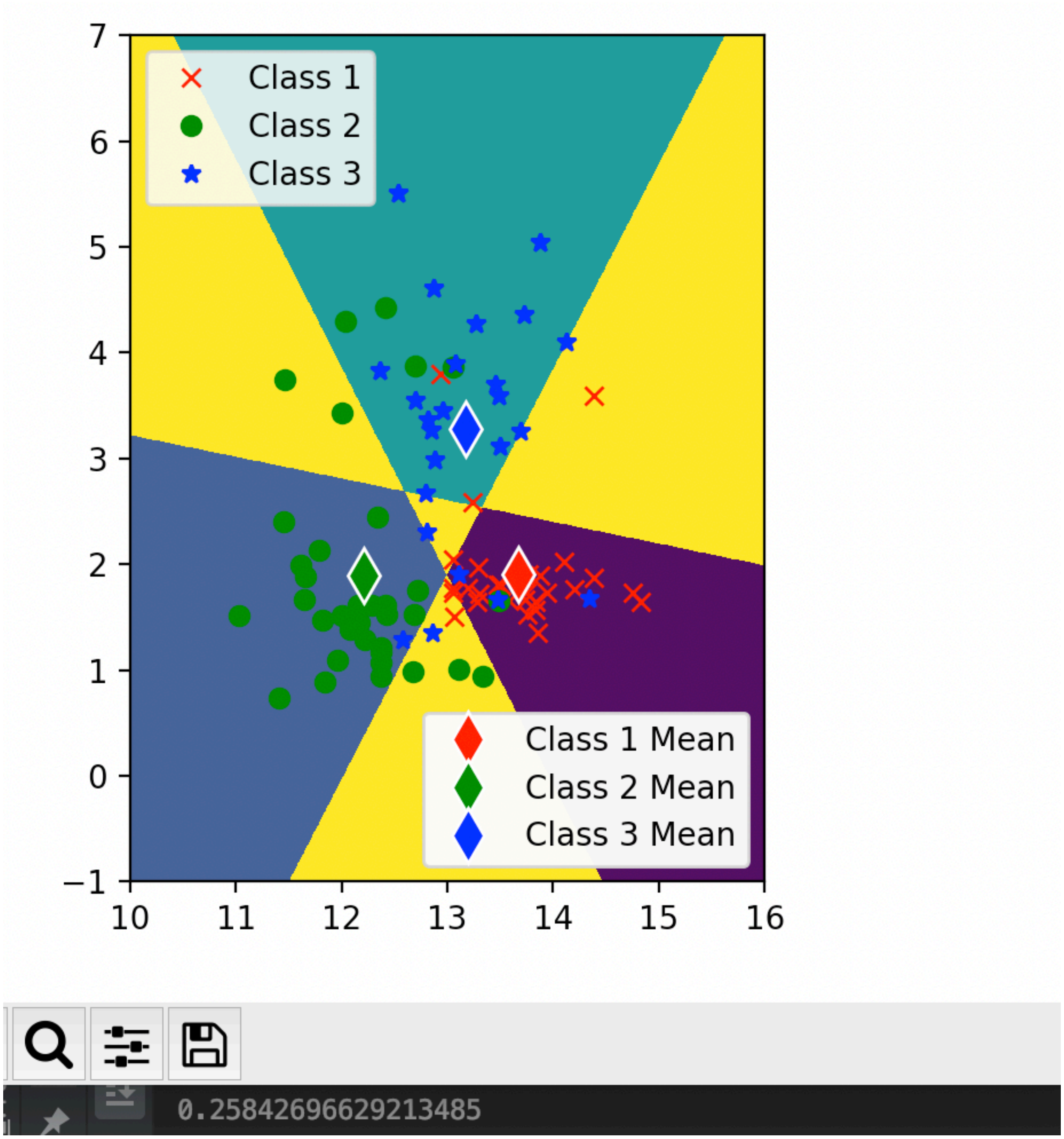
Problem1



We can find that the white area is indeterminate regions, and Point(0,0) belongs to region1. Point(1,5) belongs to region 2.

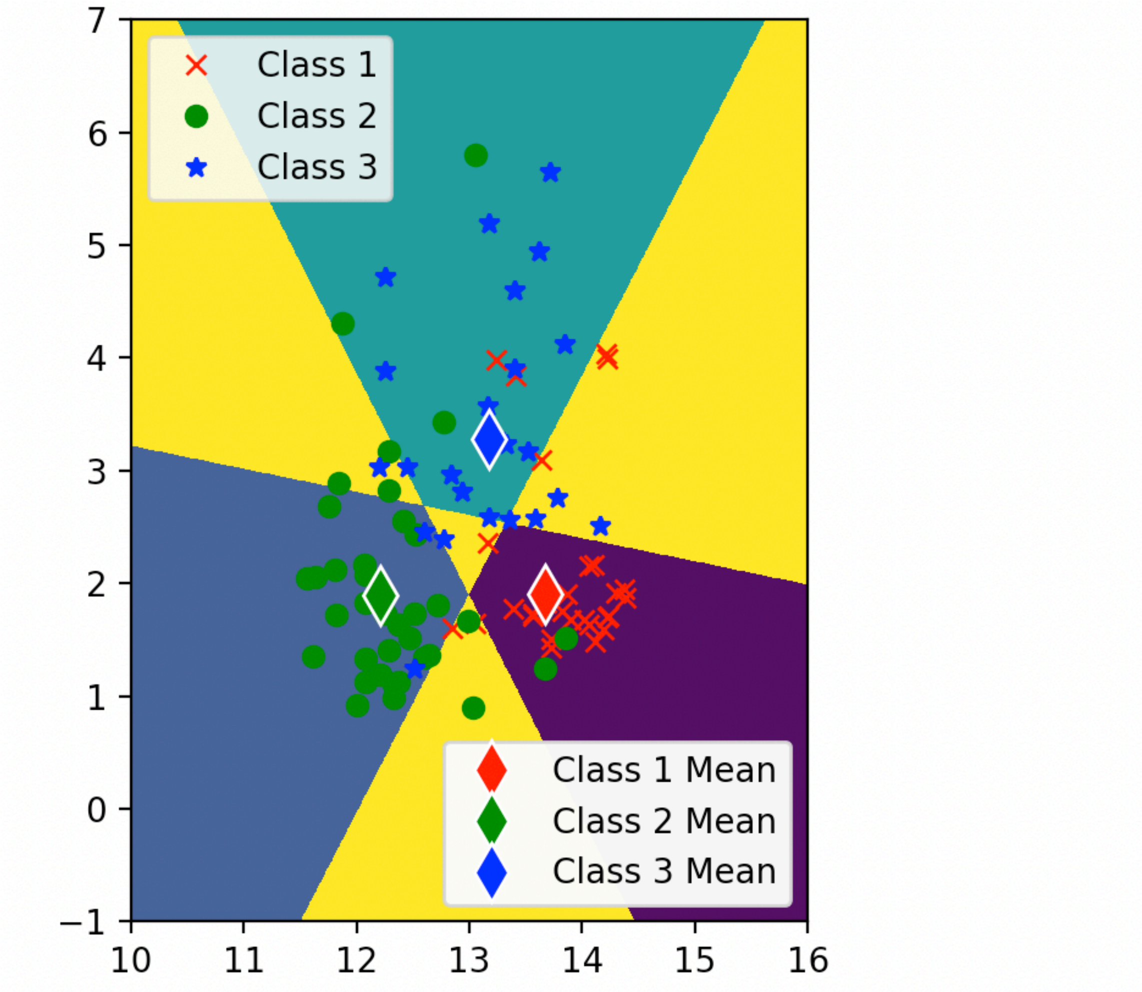
Point(4,1) cannot be classified because it is exactly on the decision boundaries.

Problem2



Using x1 and x2, the classification error on the training set is about 0.258426966

Therefore, the accuracy on training set is about 0.741573034



Using x1 and x2, the classification error on the testing set is about 0.292134

Therefore, the accuracy on testing set is about 0.7078651

Problem3

（a）

let $\vec{u_1} = [u_{11}, u_{12}, \cdots u_{1n}]$

$\vec{u_2} = [u_{21}, u_{22} \cdots u_{2n}]$

$\vec{x} = [x_1, \cdots x_n]$

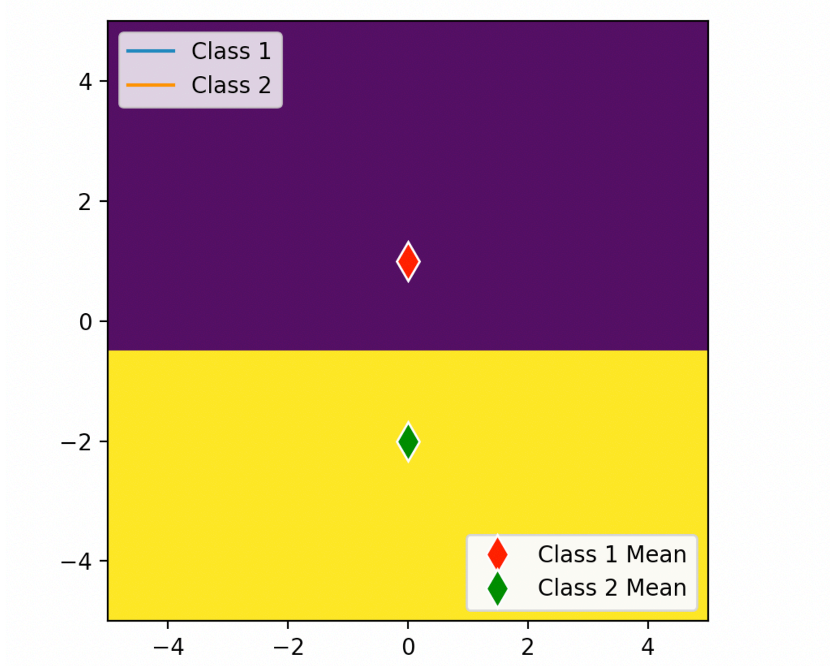
Euclidean distance will be $g(x) = \|\vec{x} - \vec{u_2}\|^2 - \|\vec{x} - \vec{u_1}\|^2$

we can get that $g(x) = 2(u_1 - u_2)^T x + (u_2^T u_2 - u_1^T u_1)$

just like $g(x) = wx + b$

we can know that $g(x) = 2(u_1 - u_2)^T x + (u_2^T u_2 - u_1^T u_1)$ is linear

（b）



(c)

from part(a) we can get that

for each 2-class classifier

$$\begin{cases} g(x) = 2(\mu_1-\mu_2)^T x + (\mu_2^T\mu_2 - \mu_1^T\mu_1) & \overset{1}{\underset{2}{\gtrless}} \\ g(x) = 2(\mu_1-\mu_3)^T x + (\mu_3^T\mu_3 - \mu_1^T\mu_1) & \overset{1}{\underset{3}{\gtrless}} \\ g(x) = 2(\mu_2-\mu_3)^T x + (\mu_3^T\mu_3 - \mu_2^T\mu_2) & \overset{2}{\underset{3}{\gtrless}} \end{cases}$$

so we can get that

$$g_1(x) = \begin{cases} 2(\mu_1-\mu_2)^T x + (\mu_2^T\mu_2 - \mu_1^T\mu_1) \\ 2(\mu_1-\mu_3)^T x + (\mu_3^T\mu_3 - \mu_1^T\mu_1) \end{cases}$$

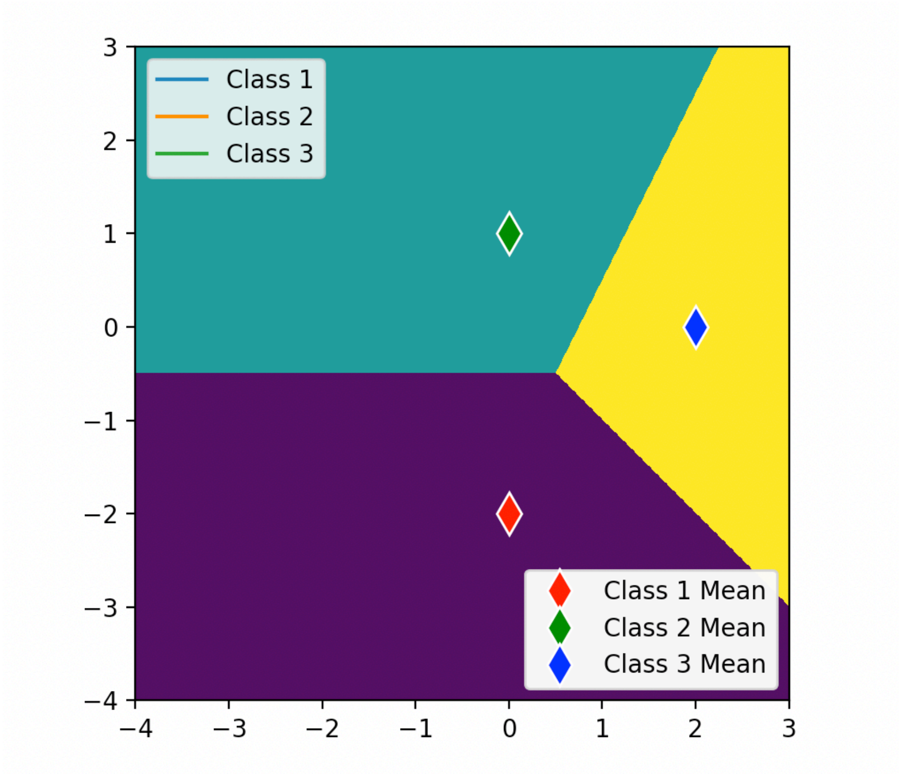
$$g_2(x) = \begin{cases} 2(\mu_1-\mu_2)^T x + (\mu_2^T\mu_2 - \mu_1^T\mu_1) \\ 2(\mu_2-\mu_3)^T x + (\mu_3^T\mu_3 - \mu_2^T\mu_2) \end{cases}$$

$$g_3(x) = \begin{cases} 2(\mu_1-\mu_3)^T x + (\mu_3^T\mu_3 - \mu_1^T\mu_1) \\ 2(\mu_2-\mu_3)^T x + (\mu_3^T\mu_3 - \mu_2^T\mu_2) \end{cases}$$

Because each discriminant function contains 2 linear functions

so, it is not linear

(d)



Class 1
Class 2
Class 3
Class 1 Mean
Class 2 Mean
Class 3 Mean
3
2
1
0
−1
−2
−3
−4
−4
−3
−2
−1
0
1
2
3

Problem4

for the points in convex hull of $\{x^n\}$, the linear discriminant is $y(x) = w^T x^n + w_0$

from $x = \sum_n \alpha_n x^n$

we can get that $y(x) = \sum_n \alpha_n (w^T x^n + w_0)$

And for other points that belongs to $\{z^m\}$ we have

$$y(z) = \sum_m \beta_m (w^T z^m + w_0) \quad \text{------ ------ --} \quad ①$$

If the convex hulls intersect, there must be at least 1 point in common between $\{x\}$ and $\{z\}$, so this point will be xz,

$$y(xz) = \sum_n \alpha_n (w^T x^n + w_0) = \sum_m \beta_m (w^T z^n + w_0) \quad \text{---} ②$$

Because of linearly separability, we have:

$$y(x^n) = w^T x^n + w_0 > 0$$
$$y(z^m) = w^T z^m + w_0 < 0 \quad \text{--------} ③$$

from ① and ③ we get contradiction. So, we can know that if they are intersect, they can not be linearly separable.

once they are linearly separable, we can know that

$$y(x^n) = w^T x^n + w_0 > 0$$
$$y(z^m) = w^T z^m + w_0 < 0 \quad \text{------} \quad ④$$

Assume that there is a intersect point of the convex hulls. we can not get equation ② from the fact that ④

So, After all, the two sets of vectors, either they are linearly separable or their convex hulls intersect