Binomial distribution

1) the probability that a pen manufactured by a company will be defective is $1/10$. If 12 such pens are manufactured, find the probability that
(a) exactly 2 will be defective
(b) none will be defective
(c) atleast 2 will be defective

Sol:- Total no. of pen = $n = 12$

probability of defective pen $p = 1/10 = 0.1$

probability of a non defective pen $q = 1 - p = 0.9$

Probability of r success is

$$P(r) = {}^nC_r \, p^r \, q^{n-r}$$

n = no. of repeated trials
p = probability of a success
q = failure

$$P(2) = {}^{12}C_2 \, 0.1^2 . (0.9)^{12-2}$$

$$= \frac{\lfloor 12}{\lfloor 2 \lfloor 10} \times 0.1^2 \times 0.9^{10}$$

$$= \frac{\cancel{12}^{6} \times 11 \times \cancel{10}}{\cancel{2} \times \cancel{10}}$$

${}^nC_0 = 1$

${}^nC_1 = n$

${}^nC_n = 1$

$$= 66 \times 0.1^2 \times 0.9^{10}$$

$$= 0.2301$$

b) none will be defective

$$P(0) = {}^{12}C_0 \, (0.1)^0 \, (0.9)^{12} = 1 \times 1 \times 0.9^{12} =$$

c) atleast 2 will be defective

$$P(2) = 1 - [P(0) + P(1)] = [{}^{12}C_0 (0.1)^0 \times 0.9^{12} + {}^{12}C_1 \, 0.1^1 \times 0.9^{12-1}]$$

$$= [0.2824 + 12 (0.1)^1 (0.9)^{11}) = 1 - [0.2824 + 0.3766] = 0.3410$$

If ten coins are thrown simultaneously. Find the probability of getting (a) exactly 3 heads (b) at least 3 head (c) at most 3 head (d) at least 7 heads

Soln:-

$n = 10$

$p = \frac{1}{2}$ success in each toss

$q = 1 - p = 0.5$

Let 'x' no of success (heads)

$$P(X=x) = {}^{n}C_{r}\, p^{r}\, q^{n-r}$$

$$= {}^{10}C_{r} (0.5)^{r} (0.5)^{10-r}$$

$r = 0, 1, 2, \ldots 10$

$$P(X=x) = \frac{1}{1024} \times {}^{10}C_{x}\ ;\ x = 0, 1, \ldots 10$$

(a) $P(X=3)$

$$= \frac{1}{1024} \times {}^{10}C_{3} = \frac{120}{1024}$$

(b) $P(X \geq 3) = P(X=3) + 4 + 5 \ldots$

$$= 1 - [P(X=0) + P(X=1 + P(X=2)]$$

$$= 1 - \left[\frac{1}{1024}\,{}^{10}C_{0} + \frac{1}{1024}\,{}^{10}C_{1} + \frac{1}{1024}\,{}^{10}C_{2}\right]$$

$$= 968/1024$$

(c) $P(X \leq 3) = P(X=0) + P(X=1) + P(X=3)$

$$= 176/1024$$

(d) $P(X \geq 7) = X = 7 + 8 + 9 + 10$

$$= \cancel{15}\ 176/1024$$

poission distribution

$$P(X=x) = \frac{e^{-m} m^x}{x!} \qquad x = 0, 1, 2 \ldots$$

$m = np =$ mean

Q A mfg who produce medicine bottles, find that 0.1% of the bottles are defective. The bottles are placed in a boxes containing 500 bottles. A drug manufacturer buys 100 boxes from the producer of bottles using p.D find how many boxes will contain (i) none of defective (2) at least 2 defect (3) atmost two defects

$n = 500$

probability of dey $p = 0.1\% = 0.001$

$m = np = 500 \times 0.001 = 0.5$

$$P(X=x) = \frac{e^{-m} m^x}{x!} = \frac{e^{-0.5} (0.5)^x}{x!} = \frac{0.6065 (0.5)^x}{x!}$$

(i) No defective $x = 0$

$$P(X=0) = \frac{0.6065 (0.5)^0}{0!} = 0.6065$$

$= 100 \times 0.6065 = 61$ boxes

ii) at least 2 defective

$$P(X \geq 2) = 1 - [P(X=0) + P(X=1)]$$
$$= 1 - (0.6065 + 0.30325)$$

$P(X \geq 2) = 0.09025$

iii) p at most 2 defective

$$P(X \leq 2) = P(X=0) + P(X=0) + (P(X=2)$$

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