8

Control Charts For Variables & Process Capability

"Variation is sometimes called the root cause of poor quality; but it is also a fact of life."

– Hilario, L.Oh., General Motors Corp.

8.1. VARIATION – THE AXIOM OF MANUFACTURE

✓ Before describing the various control charts, it is appropriate to have the knowledge of process variations. One of the axioms or truisms of manufacturing is that no two objects are ever made exactly alike. When variations are very small, it may appear that items are identical; but precision instruments will show differences.

✓ ***Four sources of variations :*** There are four factors that contribute to these variations. They are processes, materials, operators and miscellaneous factors. The source of miscellaneous variations includes environmental factors such as heat, light, radiation and humidity.

✓ ***Types of variations :*** There are two kinds of variations. They are :

1. Assignable causes of variations, and
2. Chance causes of variations.

8.1.1. Assignable Causes of Variations

✓ ***Assignable causes of variations*** are larger in magnitude and can be easily traced and detected.

✓ The reasons for assignable causes of variation are due to :

(*a*) Differences among machines,

(*b*) Differences among materials,

(*c*) Differences among processes,

(*d*) Differences in each of these factors overtime, and

(*e*) Differences in their relationship to one another.

✓ The prime objective of a control chart is detecting assignable causes of variation by analyzing data (say in length, diameter, weight or a part).

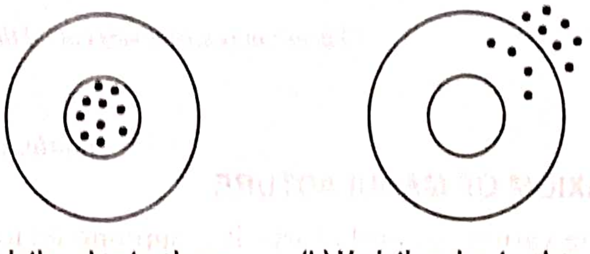
8.1.2. Chance (or Random) Causes of Variations

✓ ***Chance causes of variations*** are inevitable in any process. These are difficult to trace and control even under best conditions of production. All occur at random.

✓ Random variations cannot be avoided. They are caused by factors such as human variability from one operation cycle to the next, minor variations in raw materials, and machine vibration.

8.2. Distinction Between Chance and Assignable Causes of Variation

The distinction between chance and assignable causes of variations are depicted in Fig.8.1.



(a) Variation due to chance causes only

(b) Variation due to chance plus assignable causes of variations

Fig. 8.1. Distinction between chance and assignable causes of variation

✓ When only chance causes are present in a process, the process is considered to be in control, as shown in Fig.8.1(*a*).

✓ When an assignable causes of vibration are also present, then the variation will be excessive and the process is classified as out of control *i.e.*, beyond the expected normal variation. Refer Fig.8.1(*b*).

✓ Therefore, ***the objective of control charts is to restrict the chance causes of variation by detecting and eliminating the assignable causes.***

8.3. CONTROL CHART

8.3.1. What is it ?

✓ ***A control chart*** is a graph that displays data taken over time and the variations of this data.

✓ It is a tool to distinguish between chance and assignable causes of variations in a process.

✓ The control chart is used to check whether the process is controlled statically or not.

✓ Control chart makes possible the diagnosis and correction of the many production troubles.

✓ With the help of a control chart, one can find out the natural capability of a production process.

✓ The control chart is used to evaluate process stability and to decide when to adjust the process.

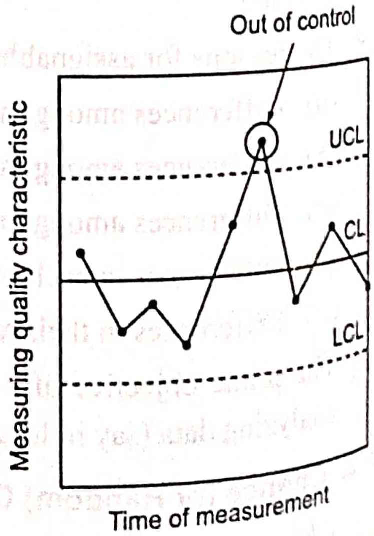


Fig. 8.2. General form of control chart

✓ The general form of the control chart is illustrated in Fig.8.2.

8.4. TYPES OF CONTROL CHARTS

The two basic types of control charts used are :

1. Control charts for variables, and
2. Control charts for attributes

✓ ***Control charts for variables*** require a measurement of the quality characteristic of interest.

✓ ***Control charts for attributes*** require a determination of whether a part is defective or how many defects are there in the sample.

8.5. CONTROL CHARTS FOR VARIABLES

✓ The quality characteristics which can be measured and expressed in specific units of measurements are called ***variables.***

✓ Control charts based upon measurements of quality characteristics are called as ***control charts for variables.***

✓ ***Types of variable control charts :*** The most commonly used variable control charts are :

(*i*) $\overline{X}$ or average charts,

(*ii*) R or range charts, and

(*iii*) σ or standard deviation chart.

✓ The $\overline{X}$ chart is used to monitor the ***centering*** of the process to control its accuracy.

✓ The R-chart monitors the ***dispersion*** or precision of the process.

✓ The σ-chart shows the variation of the process.

8.6. CONSTRUCTION OF $\overline{X}$ AND R CHARTS

Step 1 : Select the characteristics for applying a control chart.

Step 2 : Select the appropriate type of control chart.

Step 3 : Collect the data.

Step 4 : Choose the rational sub-group *i.e.*, sample.

Step 5 : Calculate the average ($\overline{X}$) and range (R) for each sample.

For example, if a sub-group contains 5 items whose dimensions (say diameter or length or weight or *etc.*) are x_1, x_2, x_3, x_4 and x_5, then

$$\text{Sub-group average,} \quad \boxed{\overline{X} = \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}}$$

$$\text{and} \quad \text{subgroup range,} \quad \boxed{R = \text{Maximum value} - \text{Minimum value}}$$

Step 6 : Calculate the average of the averages ($\overline{\overline{X}}$) and average of range ($\overline{R}$).

Let N = Number of sub-groups

Then, average of averages (or grand average) is given by

$$\boxed{\overline{\overline{X}} = \frac{\Sigma \overline{X}}{N}}$$

Similarly average of range ($\overline{R}$) is given by

$$\boxed{\overline{X} = \frac{\Sigma R}{N}}$$

Step 7 : Calculate the control limits for $\overline{X}$ and R charts.

(a) Control limits of $\overline{X}$ chart :

Control limit or centre line, $CL_{\overline{X}} = \overline{\overline{X}}$

Upper control limit, $UCL_{\overline{X}} = \overline{\overline{X}} + A_2 \overline{R}$

and Lower control limit, $LCL_{\overline{X}} = \overline{\overline{X}} - A_2 \overline{R}$

where A_2 = Factor or constant for $\overline{X}$ chart, taken from the Table 2. (see Appendix).

(b) Control limits for R chart :

Control limit or centre line, $CL_R = \overline{R}$

Upper control limit, $UCL_R = D_4\overline{R}$

and Lower control limit, $LCR_R = D_3\overline{R}$

where D_3 and D_4 are statistical factors, taken from Table 2. (see Appendix)

Step 8 : Plot CL, UCL and LCL on the chart.

Step 9 : Plot individual $\overline{X}$ and R values on the chart.

Step 10 : Check whether the process is in control or not.

Step 11 : Revise the control limits if the points are outside.

$\boxed{\text{Note}}$ ***1. Control limits for $\overline{\overline{X}}$ charts in terms of $\overline{\sigma}$:***

If sample standard deviation ($\overline{\sigma}$) is known, then limits of $\overline{X}$ chart can be defined as

Control limit, $CL_{\overline{X}} = \overline{\overline{X}}$

Upper control limit, $UCL_{\overline{X}} = \overline{\overline{X}} + A_1 \cdot \overline{\sigma}$

and Lower control limit, $LCL_{\overline{X}} = \overline{\overline{X}} - A_1 \cdot \overline{\sigma}$

where A_1 = Statistical factor, taken from Table 4 (see Appendix).

2. Control limits on σ chart :

Let $\bar{\sigma}$ = Sample standard deviation, and

σ' = Population (universe) standard deviation or standard deviation of process distribution.

$$\text{Control limit on } \sigma \text{ chart, } CL_{\sigma} = \bar{\sigma}$$
$$\text{Upper control limit, } UCL_{\sigma} = B_4 \bar{\sigma}$$
and
$$\text{Lower control limit, } LCL_{\sigma} = B_3 \bar{\sigma}$$

When σ' *i.e.*, universe or population standard deviation is known, then the control limits may be

$$\text{Upper control limit, } UCL_{\sigma} = B_2 \sigma'$$
and
$$\text{Lower control limit, } LCL_{\sigma} = B_1 \sigma'$$

where B_1, B_2, B_3 and B_4 are statistical factors, taken from Table 3 and 4 (see Appendix).

3. Factors for estimating σ' from $\overline{R}$ or $\bar{\sigma}$:

$$\text{Standard deviation of the process, } \sigma = \frac{\overline{R}}{d_2}$$

or

$$\text{Standard deviation of the process, } \sigma = \frac{\bar{\sigma}}{c_2}$$

where $\overline{R}$ = Average of ranges,

$\bar{\sigma}$ = Standard deviation of sample means,

d_2 and c_2 = Statistical factors, taken from Table 5 (see Appendix).

4. If the standard deviation of the process distribution (σ) is known, then limits of $\overline{X}$ chart can be defined as

$$\text{Upper control limit, } UCL_{\overline{X}} = \overline{\overline{X}} + 3\,\sigma_{\overline{X}}$$

$$\text{and Lower control limit, } LCL_{\overline{X}} = \overline{\overline{X}} - 3\,\sigma_{\overline{X}}$$

where $\overline{\overline{X}}$ = Average of averages,

$\sigma_{\overline{X}} = \frac{\sigma}{\sqrt{n}}$ = Standard deviation of sample means,

σ = Standard deviation of the process distribution, and

n = Sample size.

Example 8.1 ***In the production of certain rods a process is said to be under control if the outside diameters have a mean of 60 mm and a standard deviation of 0.05 mm. Construct a control chart for the mean of random samples of size 4, showing the central line, the upper and lower control limits on the graph paper.***

What will be the value of 2-sigma (warning) limits for this process ?

Given Data : $\overline{\overline{X}} = 60$ mm ; $\bar{\sigma} = 0.05$ mm ; $n = 4$.

Solution : ***(i) To construct a control chart for the mean i.e., $\overline{X}$ chart :***

For a subgroup size of 4, Table 4 gives the following factors :

$$A_1 = 1.88 \; ; \; B_3 = 0 \; ; \; B_4 = 2.57$$

Control limit or centre line, CL $= \bar{\bar{X}} = 60$ **mm** **Ans.**

Upper control limit, $UCL_{\bar{X}} = \bar{\bar{X}} + A_1\,\bar{\sigma} = 60 + 1.88\,(0.05) =$ **60.094 mm** **Ans.**

Lower control limit, $LCL_{\bar{X}} = \bar{\bar{X}} - A_1\,\bar{\sigma} = 60 - 1.88\,(0.05) =$ **59.905 mm** **Ans.**

The control chart can be constructed as shown in Fig.8.3.

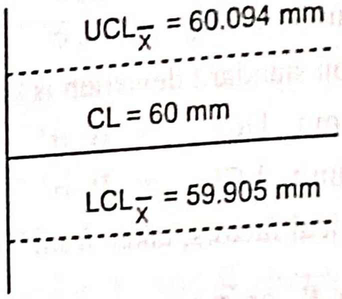


Fig. 8.3. $\overline{X}$ chart

(ii) To find the value of 2-sigma limits :

We know that, $\sigma' = \dfrac{\bar{\sigma}}{c_2}$

where $c_2 = 0.7979$, for $n = 4$ from Table 5.

$$\therefore \sigma' = \frac{0.05}{0.7979} = 0.0626$$

Then 2-sigma limits is given by $\bar{\bar{X}} \pm 2\,\sigma'$.

$\therefore$ 2-sigma limits $= 60 \pm 2\,(0.0626) =$ **60 ± 0.125 mm** **Ans.**

$\therefore$ Upper warning limit $=$ **60.125 mm** **Ans.**

and Lower warning limit $=$ **59.875 mm** **Ans.**

Example 8.2 ***Control charts for $\overline{X}$ and R are maintained on the breaking strength in a certain destructive type of particular type of ceramic insulator used in vacuum tubes. The subgroup size is 15. The values of $\overline{X}$ and σ are computed for each subgroup. After 12 subgroups, $\Sigma\overline{X} = 1307$ and $\Sigma R = 191.5$. Compute the value of the 3-sigma limits for the $\overline{X}$ and R charts, and estimate the value of σ' on the assumption that the process is in control.***

Given Data : Subgroup size, $n = 15$; $N = 12$; $\Sigma\overline{X} = 1307$; $\Sigma R = 191.5$.

To find : *(i)* Control limits for $\overline{X}$ and R chart, and

(ii) Value of σ'.

☺ ***Solution : (i) To find control limits for $\overline{X}$ and R chart :***

Average of averages, $\bar{\bar{X}} = \dfrac{\Sigma\overline{X}}{N}$

where N = Number of subgroups

$$\therefore \quad \overline{\overline{X}} = \frac{1307}{12} = 108.92$$

and Average of range, $\overline{R} = \frac{\Sigma R}{N} = \frac{191.5}{12} = 15.96$

For a subgroup size (n) of 15, Table 2 and 5 give the following factors :

Factor for $\overline{X}$ chart, $A_2 = 0.22$

Factor for R chart, $D_3 = 0.35$ and $D_4 = 1.65$

For estimate from R, $d_2 = 3.472$

For $\overline{\overline{X}}$ chart :

Control limit, $CL_{\overline{X}} = \overline{\overline{X}} = \mathbf{108.92}$ **Ans.**

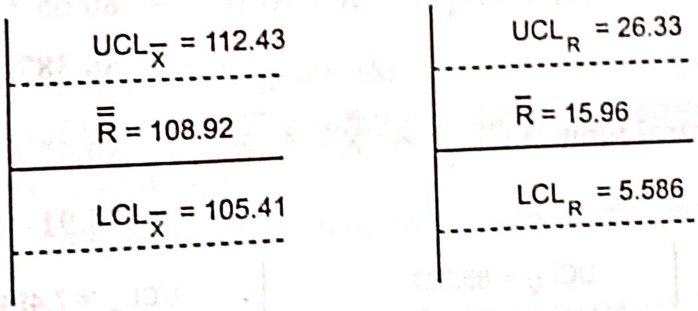
Upper control limit, $UCL_{\overline{X}} = \overline{\overline{X}} + A_2 \cdot \overline{R}$

$= 108.92 + 0.22\,(15.96) = \mathbf{112.43}$ **Ans.**

and Lower control limit, $LCL_{\overline{X}} = \overline{\overline{X}} - A_2\,\overline{R}$

$= 108.92 - 0.22\,(15.96) = \mathbf{105.41}$ **Ans.**

Now the $\overline{X}$ chart can be constructed as shown in Fig.8.4(a).



(a) $\overline{X}$ chart (b) R chart

Fig. 8.4.

For R chart :

Control limit, $CL_R = \overline{R} = \mathbf{15.96}$ **Ans.**

Upper control limit, $UCL_R = D_4\,\overline{R} = 1.65\,(15.96) = \mathbf{26.33}$ **Ans.**

and Lower control limit, $LCL_R = D_3\,\overline{R} = 0.35\,(15.96) = \mathbf{5.586}$ **Ans.**

Now the R chart can be constructed as shown in Fig.8.4(b).

(ii) To find the value of σ' :

We know that, population standard deviation, $\sigma' = \frac{\overline{R}}{d_2}$

$$\therefore \quad \sigma' = \frac{15.96}{3.472} = \mathbf{4.596} \text{ Ans.}$$

✓ *Why do we need process capability analysis ?*

Fig. 8.6 shows the relationship of limits, specifications and distributions.

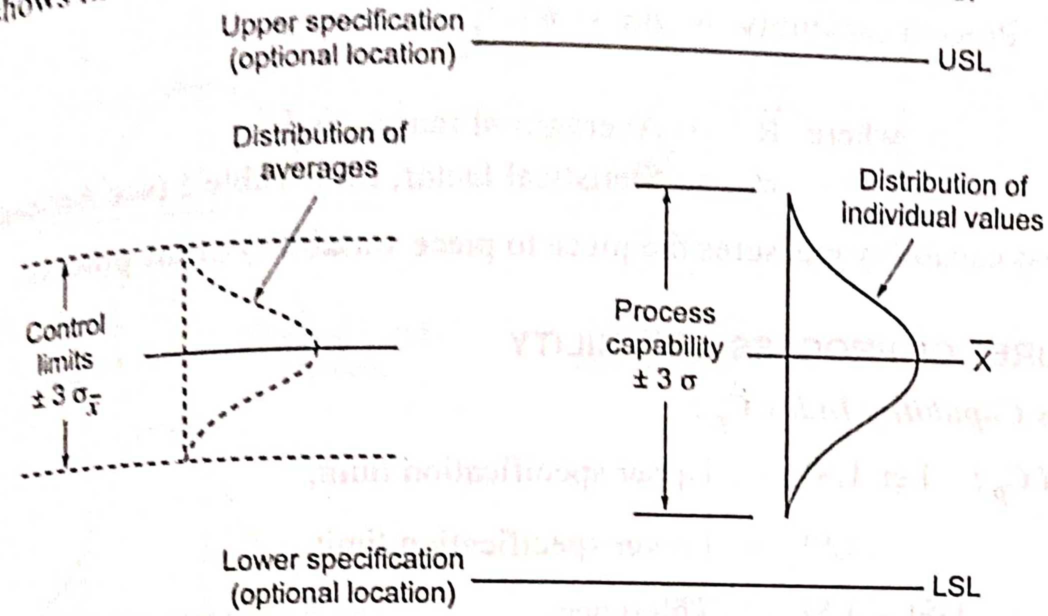


Fig. 8.6. Relationship of limits, specifications, and distributions

From Fig.8.6, it can be concluded that even the process (*i.e.*, average value of items) is in control, the individual item may not be within the specified limits. So it is necessary to see whether the process is capable of producing the items within the specified limits. This can be achieved by carrying out the process capability.

8.8. PROCESS CAPABILITY

✓ *Definition :* Process capability may be defined as the ***"minimum spread of a specific measurement variation which will include 99.7% of the measurements from the given process."***

✓ In other words, $\boxed{\text{Process capability} = 6\sigma}$. Since 99.7% area in the normal curve is between -3σ to $+3\sigma$, therefore process capability is equal to 6σ.

✓ Process capability ($= 6\sigma$), is also called as ***natural tolerance.***

✓ ***The purpose of process capability analysis are :***

(*i*) To find out whether the process is inherently capable of meeting the specified tolerance limits.

(*ii*) To identify why a process 'capable' is failing to meet specifications.

8.9. METHOD OF DOING PROCESS CAPABILITY ANALYSIS

The procedure to do process capability analysis is as follows :

1. Calculate the average $\overline{X}$ and range R of each sample.
2. Calculate the grand average $\overline{\overline{X}}$. This measures the centering of the process.
3. Calculate the control limits and plot $\overline{X}$ and R charts. These control charts measure the stability of the process.

4. Calculate the process capability.

$$\text{Process capability} = 6\sigma = 6\left(\frac{\overline{R}}{d_2}\right)$$

where $\overline{R}$ = Averages of range, and

d_2 = Statistical factor, from Table 5 (see Appendix)

This process capability measures the piece to piece variability of the process.

8.10. MEASURES OF PROCESS CAPABILITY

(i) Process Capability Index C_p :

✓ ***To find C_p :*** Let USL = Upper specification limit,

LSL = Lower specification limit,

USL – LSL = Tolerance,

σ = Population standard deviation,

6σ = Process capability, and

C_p = Capability index.

Then the capability index is defined as

$$C_p = \frac{\text{Total specification tolerance}}{\text{Process capability}}$$

or

$$\boxed{C_p = \frac{\text{USL} - \text{LSL}}{6\,\sigma}}$$

✓ ***Interpretation of C_p :***

(*i*) If $C_p > 1$ means that the process variation is less than the specification. That is the process is capable of meeting the specifications.

(*ii*) If $C_p < 1$ means that the process is not capable of meeting the specifications.

(*iii*) If $C_p = 1$ means that the process is just meeting specifications.

The larger the capability index (C_p), the better the quality. So one has to improve the C_p value by improving process capability and having realistic specifications.

Drawback of C_p : The capability index does not measure process performance in terms of the nominal or target value.

(ii) Process capability Index C_{pk} :

✓ C_{pk} measures not only the process variation with respect to allowable specifications, it also considers the location of the process average.

✓ ***To find C_{pk} :*** $$\boxed{C_{pk} = \frac{\min\ \{(\text{USL} - \overline{X}) \text{ or } (\overline{X} - \text{LSL})\}}{3\,\sigma}}$$

✓ The relationship between process capability and capability index is shown in Fig.8.7.

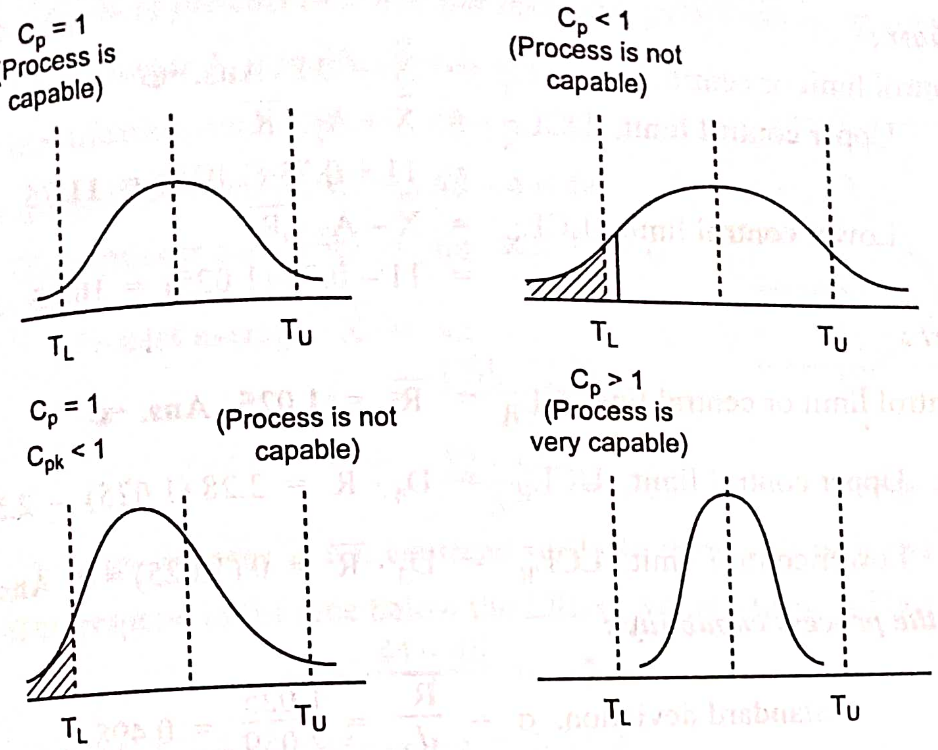


Fig. 8.7. Process capability and process capability index

✓ **Interpretation of C_{pk} :**

(i) C_{pk} value is always equal to or less than C_p value.

(ii) If $C_{pk} > 1$ means that the process confirms the specifications.

(iii) If $C_{pk} < 1$ means that the process does not conform to specifications.

(iv) If $C_{pk} = 1$ means that the process just conforms to specifications.

(v) If $C_p = C_{pk}$, then the process is centered.

Example 8.6 ***The following are the $\overline{X}$ and R values of 4 subgroups of readings :***
$\overline{X}$ = 10.2, 12.1, 10.8 and 10.9 ; R = 1.1, 1.3, 0.9 and 0.8.

The specification limits for the components are 10.7 ± 0.2.

(i) Establish the control limits for $\overline{X}$ and R charts.

(ii) Find the process capability.

(iii) Will the product able to meet its specification ?

Solution : ***(i) To find control limits for $\overline{X}$ and R charts :***

$$\overline{\overline{X}} = \frac{\Sigma \overline{X}}{N} = \frac{10.2 + 12.1 + 10.8 + 10.9}{4} = 11$$

and
$$\overline{R} = \frac{\Sigma R}{N} = \frac{1.1 + 1.3 + 0.9 + 0.8}{4} = 1.025$$

For subgroup size of 4, Table 2 and 5 give the following factors :

$$A_2 = 0.73 \; ; \quad D_3 = 0 \; ; \quad D_4 = 2.28 \; ; \quad d_2 = 2.059.$$

For $\overline{X}$ chart :

Control limit or central line, $CL_{\overline{X}} = \overline{\overline{X}} = 11$ Ans.

Upper control limit, $UCL_{\overline{X}} = \overline{\overline{X}} + A_2 \cdot \overline{R}$

$= 11 + 0.73\,(1.025) = 11.75$ Ans.

Lower control limit, $LCL_{\overline{X}} = \overline{\overline{X}} - A_2 \cdot \overline{R}$

$= 11 - 0.73\,(1.025) = 10.25$ Ans.

For R chart :

Control limit or central line, $CL_R = \overline{R} = 1.025$ Ans.

Upper control limit, $UCL_R = D_4 \cdot \overline{R} = 2.28\,(1.025) = 2.337$ Ans.

Lower control limit, $LCL_R = D_3 \cdot \overline{R} = 0\,(1.025) = 0$ Ans.

(ii) To find the process capability :

$$\text{Standard deviation, } \sigma = \frac{\overline{R}}{d_2} = \frac{1.025}{2.059} = 0.498$$

$$\therefore \text{ Process capability} = 6\sigma = 6\,(0.498) = 2.986 \text{ Ans.}$$

(iii) Will the product able to meet its specification ?

Given that, Specification $= 10.7 \pm 0.2$

That is, $USL = 10.7 + 0.2 = 10.9$; and $LSL = 10.7 - 0.2 = 10.5$

$$\text{Capability index, } C_p = \frac{USL - LSL}{6\,\sigma} = \frac{10.9 - 10.5}{2.986} = 0.134$$

Since $C_p < 1$, therefore *the process is not capable of meeting the specifications.* That means, defective components will always be there.

Example 8.7 *A certain product has been statistically controlled at a process average of 46 and a standard deviation of 1.00. The product is presently being sold to two customers who have different specification requirements. Customer A has established a specification of 48 ±4.0 for the product, and customer B has specification of 46 ±4.0.*

(a) Based on the present process set up, what percent of the product produced will not meet the specifications set up by customer A ?

(b) What percent of the product will not meet the specifications of customer B ?

(c) Assuming that the two users' needs are equal, a suggestion is made to shift the process target to 47. At this suggested value, what percent of the product will not meet the specifications of customer A ?

(d) At the suggested process target, what percent of the product will not meet the specifications of customer B ?

(e) Do you think that this shift to a process target of 47 would be desirable ? Explain your answer.

Given Data : $\overline{\overline{X}} = 46$; $\sigma = 1$.

Solution : *(a) % of product that will not meet the specifications of customer A :*

Given that, for customer A, specification is 48 ± 4.0.

$\therefore$ Upper specification limit, USL $= 48 + 4 = 52$

and Lower specification limit, LSL $= 48 - 4 = 44$

Then, Process capability $= 6\sigma = 6 \times 1 = 6$

Process average, $\bar{\bar{X}} = 46$

$$C_p = \frac{USL - LSL}{6\sigma}$$

$$= \frac{52 - 44}{6} = 1.33$$

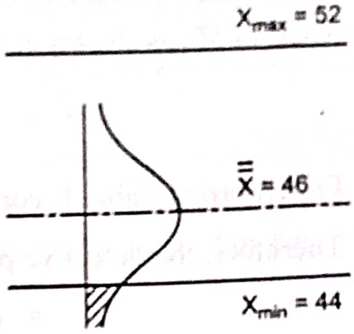


Fig. 8.8. (a)

Though $C_p > 1$, the process is not centered well. So there will be some defectives. The defectives will correspond to the area below the LSL of 44, as shown in Fig.8.8(a).

$$\therefore z = \frac{44 - 46}{1} = -2$$

From normal Table 1, corresponding area $= 0.0228$.

$\therefore$ **2.28% of the product will not meet the specifications. Ans.**

(b) % of product that will not meet the specifications of customer B :

Given that, for customer B, specification is 46 ± 4.

$\therefore$ Upper specification limit, USL $= 46 + 4 = 50$

and Lower specification limit, LSL $= 46 - 4 = 42$

Process capability $= 6\sigma = 6(1) = 6$

$$C_p = \frac{USL - LSL}{6\sigma}$$

$$= \frac{50 - 42}{6} = 1.33$$

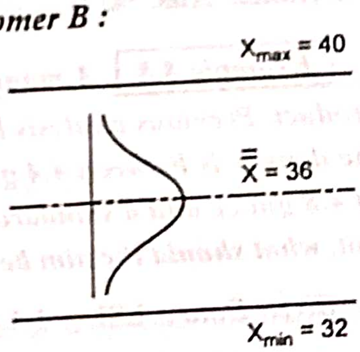


Fig. 8.8. (b)

Here $C_p > 1$ and also the process is centered mid-way as shown in Fig.8.8(b). Therefore **all products will meet the specifications for the customer B. Ans.**

(c) % of product that will not meet specifications of customer A when $\bar{\bar{X}} = 47$:

The defective products will correspond to the area below the LCL of 44, as shown in Fig.8.8(c).

$$\therefore z = \frac{44 - 47}{1} = -3$$

From normal Table 1, corresponding area $= 0.00135$.

$\therefore$ **0.135% of the products will not meet the specifications. Ans.**

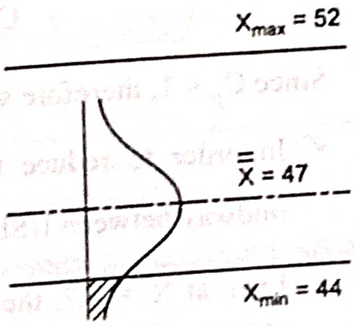


Fig. 8.8. (c)

(d) % of product that will not meet specifications of customer B when $\overline{\overline{X}} = 47$ *:*

The defective products will correspond to the area above the UCL of 50, as shown in Fig.8.8(*d*).

$$\therefore z = \frac{50-47}{1} = 3$$

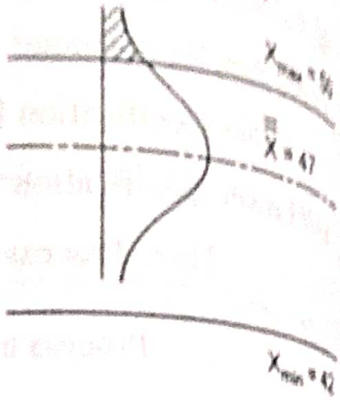


Fig. 8.8. (d)

From normal Table 1, corresponding area = 0.99865.

Therefore, the defective parts produced above the USL

$$= (1 - 0.99865) = 0.00135$$

Thus 0.135% of the products will not meet the specifications. Ans.

(e) When the process average is 46, total percentage defective

$$= 2.28\% + 0\% = 2.28\%$$

When the process average is 47, total percentage defective

$$= 0.135\% + 0.135\% = 0.27\%$$

Since the total percent defective is low, the shift to a **process average of 47** will be desirable. **Ans.**

Example 8.8 *A manufacturer of clusters chalk is concerned with the density of his product. Previous analysis has shown that his chalk has the required characteristics only if the density is between 4.4 gm/cc and 5.0 gm/cc. If a sample of 100 pieces gives an average of 4.8 gm/cc and a standard deviation of 0.2, is his process aimed at the proper density? If not, what should the aim be ? Is the process capable of meeting the density requirements.*

Given Data : LSL = 4.4 gm/cc ; USL = 5.0 gm/cc ; $\overline{\overline{X}} = 4.8$ gm/cc ; $\sigma = 0.2$.

☺ ***Solution :***

✓ Tolerance = USL − LSL = 5 − 4.4 = 0.6

Process capability = $6\sigma = 6(0.2) = 1.2$

$$C_p = \frac{USL - LSL}{6\sigma} = \frac{0.6}{1.2} = 0.5$$

Since $C_p < 1$, therefore some defective product is inevitable.

✓ In order to reduce the defective product, the process should be centered exactly midway between USL and LSL *i.e.*, 4.7. Hence **the aim is to fix $\overline{\overline{X}}' = 4.7$.**

✓ Even at $\overline{\overline{X}}' = 4.7$, the defective parts will be produced above USL and below LSL, as shown in Fig.8.9.

$$z_1 = \frac{5-4.7}{0.2} = 1.5$$

From normal Table 1, the corresponding area = 0.9332.

Therefore, the percent of the products produced above UCL

$$= (1 - 0.9332) = 0.0668$$

Thus 6.68% of the products above USL will not meet the specifications.

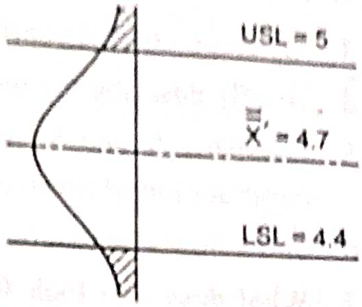


Fig. 8.9.

$$z_2 = \frac{4.4-4.7}{0.2} = -1.5$$

From normal Table 1, the corresponding area = 0.0668.

Thus 6.68% of the products below LSL will not meet the specifications.

Conclusion : Hence ***the process is not capable of meeting the density requirements.***

REVIEW AND SUMMARY

✓ *Two types of variations are : (i) Assignable causes of variation, and (ii) Chance causes of variation.*

✓ *A control chart is a graph that displays data taken over time and the variations of this data.*

✓ *The objective of control chart is to restrict the chance causes of variation by detecting and eliminating the assignable causes.*

✓ *Types of control charts :*

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(i) Control charts for variables – $\overline{X}$, R and σ charts.

(ii) Control charts for attributes.

✓ ***Process capability** may be defined as the 'minimum spread of a specific measurement variation which will include 99.7% of the measurements from the given process."*

∴ Process capability (or natural tolerance) = 6 σ

✓ ***Measures of process capability :***

$$\text{Process capability index, } C_p = \frac{USL - LSL}{6\,\sigma}$$

and

$$\text{Process capability index, } C_{pk} = \frac{Min\{(USL - \overline{X}) \text{ or } (\overline{X} - LSL)\}}{3\,\sigma;}$$

where notations have usual meanings.

SELF-ASSESSMENT QUESTIONS

1. Differentiate between the chance causes and assignable causes of variations giving suitable examples.
2. When a process is 'in control' or 'stable' ? What type of variations are present in the process ?