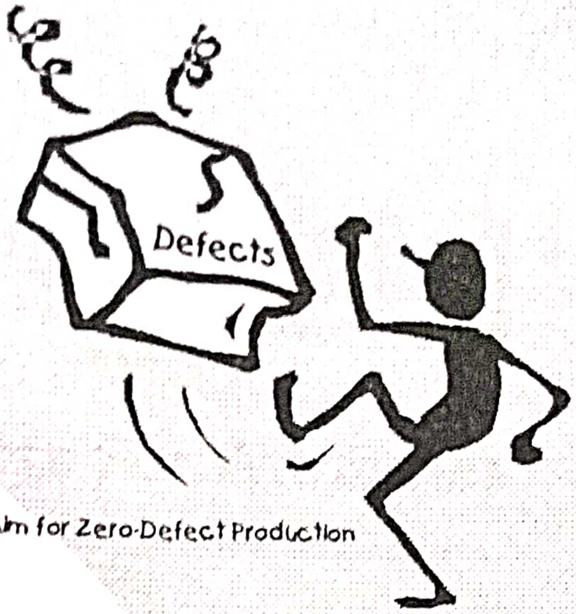


Aim for Zero-Defect Production

Control Charts For Attributes

CHAPTER - 9

Learning Objectives

While reading and after studying this chapter, you will be able to:

- ✓ Appreciate the need for control charts for attributes
- ✓ Appreciate the purposes of p, np, c, and u charts
- ✓ Learn how to construct p, np, c and u charts
- ✓ Select appropriate charts for different situations
- ✓ Adapt control charts to troubleshoot quality problems
- ✓ Compare between variable and attribute control charts

9

Control Charts For Attributes

"Your most unhappy customers are your greatest source of learning"

– Bill Gates

9.1. NEED FOR CONTROL CHARTS FOR ATTRIBUTES (Practical Limitations of Control Charts for Variables)

Though variable control charts *i.e.*, $\overline{X}$ and R charts are powerful tools for the diagnosis of quality problems, they do have certain limitations. The limitations of $\overline{X}$ and R charts are as follows :

1. $\overline{X}$ and R charts cannot be used for quality characteristics that are attributes. For example, quality characteristics such as incorrect colour, blow holes, cracks are not measurable ; and a variable control chart is not applicable.
2. $\overline{X}$ and R charts can be used only for one measurable characteristic at a time. For example, an automatic turning may have a 50 dimensions and therefore 50 $\overline{X}$ and R charts would be required. Thus this method is too expensive and time consuming.

In order to avoid the above limitations, control charts for attributes are used.

9.2. ATTRIBUTES

✓ ***What is meant by attribute ?***

An attribute refers to those quality characteristics that conform to specifications or do not conform to specifications.

✓ ***Two types of attributes are :***

(*i*) Attributes where measurement are not possible. For example, visually suspected items such as colour, missing parts, scratches and damage.

(*ii*) Attributes where measurements can be made but are not made because of economy of time, cost or need. In common practice, instead of using measuring instruments such as micrometer, vernier caliper, *etc.* we use 'Go – No Go Gauges' to conform it good or bad.

9.3. CONTROL CHARTS FOR ATTRIBUTES

✓ Control charts for attributes monitor the number of defects or fraction defect rate present in the sample.

✓ ***Types of attributes control charts used are :***

1. *The p chart :* The chart for fraction rejected as non-conforming to specifications.
2. *The np chart :* The control chart for number of non-conforming items.
3. *The c chart :* The control chart for number of non-conformities.
4. *The u chart :* The control chart for number of non-conformities per unit.

9.4. p CHART

✓ In the *p* chart, the quality characteristic of interest is the proportion (*p* for proportion) of non-conforming or defective units.

✓ ***Fraction defective, p,*** may be defined a the ratio of the number of defective articles found in any inspection to the total number of articles actually inspected.

Generally fraction defective is expressed as a decimal fraction.

Mathematically, $\boxed{p = \dfrac{np}{n}}$

where p = Fraction defective,

np = Number of defectives, and

n = Number of items inspected in the sub-group.

✓ ***Per cent defective*** is 100 p, that is 100 times the fraction defective.

For actual calculation of the control limits fraction defective is used. However for charting percent defective is used.

✓ ***Control Limits :***

Let $\bar{p}$ = Average fraction defective $= \dfrac{\Sigma np}{\Sigma n}$

Control line or central line, $CL_p = \bar{p}$

Upper control limit, $UCL_p = \bar{p} + 3\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}}$

and Lower control limit, $LCL_p = \bar{p} - 3\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}}$

9.4.1. Purpose of the p chart

The purposes of *p* chart are :

✓ To discover, identity and correct causes of bad quality.

✓ To discover, identity and correct the erratic causes of quality improvement.

✓ To discover the average proportion of defective articles submitted for inspection, over a period of time.

✓ To suggest where it i...

✓ To d...

9.5. np CHART

✓ Generally p and np charts are quiet same.

✓ Whenever subgroup size is variable, p chart is used. ***If subgroup size is constant,*** then np (also known as pn) chart is used.

✓ ***When and why np chart is preferred over p chart ?***

❖ When subgroup size is constant, the np chart is preferred over p charts.

❖ ***Reasons :*** (*i*) The np chart saves one calculation for each subgroup, the division of number of rejects by subgroup size to get p.

(*ii*) Some people may understand the np chart more readily.

✓ ***Control Limits for np chart :***

Control limit or central line, $CL_{np} = n\bar{p}$

Upper control limit, $UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}\,(1-\bar{p})}$

and Lower control limit, $LCL_{np} = n\bar{p} - 3\sqrt{n\bar{p}\,(1-\bar{p})}$

where $\bar{p}$ = Average fraction defective = $\dfrac{\Sigma np}{\Sigma n}$, and

n = Number of items inspected in subgroup.

Example 9.1 ***Ten samples of equal size are taken to prepare a p-chart. The total number of parts in these ten samples was 1200, and the total number of defects counted was 155. Determine the centre, UCL and LCL for the p-chart.***

To find : Centre, UCL and LCL for the p-chart.

☺ ***Solution :*** Total number of parts sampled, $\Sigma n = 1200.$

Total number of defects, $\Sigma np = 155$

Then, Average fraction defective = $\dfrac{\text{Total number of defects}}{\text{Total number of parts sampled}}$

$$\bar{p} = \frac{\Sigma np}{\Sigma n} = \frac{155}{1200} = 0.129$$

Average sample size, $n = \dfrac{1200}{10} = 120$

$\therefore$ Centre line, $CL_p = \bar{p} = \mathbf{0.129}$ **Ans.** ☜

Upper control limit, $UCL_p = \bar{p} + 3\sqrt{\dfrac{\bar{p}\,(1-\bar{p})}{n}}$

$$= 0.129 + 3\sqrt{\frac{0.129\,(1-0.129)}{120}}$$

$= \mathbf{0.221}$ **Ans.** ☜

$$\text{Lower control limit, } LCL_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.129 - 3\sqrt{\frac{0.129(1-0.129)}{120}}$$

$$= \mathbf{0.037} \quad \textbf{Ans.}$$

Example 9.2 *15 samples of 25 parts each have been collected. In two samples there were no defects ; in four samples there was one defect ; in five samples there were two defects ; and in four samples there were three defects. Determine the centre, UCL, and LCL for the p chart.*

To find : Centre, UCL, and LCL for the p-chart.

☺ ***Solution :*** Number of samples = 15

Sample size, n = 25

$$\text{Average fraction defective, } \bar{p} = \frac{\text{Total number of defects}}{\text{Total number of parts sampled}}$$

$$= \frac{2(0) + 4(1) + 5(2) + 4(3)}{15(25)} = 0.069$$

$$\therefore \text{Centre line, } CL_p = \bar{p} = \mathbf{0.069} \quad \textbf{Ans.}$$

$$\text{Upper control limit, } UCL_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.069 + 3\sqrt{\frac{0.069(1-0.069)}{25}}$$

$$= \mathbf{0.221} \quad \textbf{Ans.}$$

$$\text{Lower control limit, } LCL_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.069 - 3\sqrt{\frac{0.069(1-0.069)}{25}} = -\mathbf{0.083} \quad \textbf{Ans.}$$

Since the fraction defective cannot be negative, therefore $LCL_{\bar{p}}$ *is taken as zero.*

Example 9.3 ***A p-chart is to be constructed. Ten samples of 20 parts each have been collected, and the average number of defects per sample = 1.75. Determine the centre, UCL, and LCL for p-chart.***

To find : CL_p, UCL_p and LCL_p.

☺ ***Solution :*** Number of samples = 10

Sample size, n = 20

Average number of defects per sample $= 2.75$

$\therefore$ Total number of defects $= 1.75 \times 10 = 17.5$

Then, Average fraction defective, $\bar{p} = \dfrac{17.5}{20} = 0.875$

Centre line, $CL_p = \bar{p} =$ **0.875 Ans.**

$$\text{Upper control limit, } UCL_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.875 + 3\sqrt{\frac{0.875\,(1-0.875)}{20}}$$

$$= \mathbf{1.097} \quad \textbf{Ans.}$$

and
$$\text{Lower control limit, } LCL_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.875 - 3\sqrt{\frac{0.875\,(1-0.875)}{20}}$$

$$= \mathbf{0.653} \quad \textbf{Ans.}$$

Example 9.4 ***The yield of good chips during a certain step in silicon processing of integrated circuits averages 91%. The number of chips per wafer is 200. Determine the centre, UCL and LCL for p-chart that might be used for this process.***

To find : CL_p, UCL_p and LCL_p.

☺ *Solution :* Percentage defective $= 100 - 91 = 9\%$

or $\bar{p} = 9\% = 0.09$

Sample size, $n = 200$

$\therefore$ Centre line, $CL_p = \bar{p} =$ **0.09 Ans.**

$$\text{Upper control limit, } UCL_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.09 + 3\sqrt{\frac{0.09\,(1-0.09)}{200}}$$

$$= \mathbf{0.151} \quad \textbf{Ans.}$$

and
$$\text{Lower control limit, } LCL_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.09 - 3\sqrt{\frac{0.09\,(1-0.09)}{200}}$$

$$= \mathbf{0.029} \quad \textbf{Ans.}$$

Example 9.5 ***For the above Example, find the 100 p or percent p control limits.***

Solution : In Example 9.4, the calculated values are

$$CL_p = 0.09 ; \quad UCL_p = 0.151 ; \quad LCL_p = 0.029$$

Then control limits for $100p$ chart is given as

Control limit or centre line, $CL_{100p} = 100 \times CL_p = 100 \times 0.09 = 9$ **Ans.**

Upper control limit, $UCL_{100p} = 100 \times UCL_p = 100 \times 0.151 = 15.1$ **Ans.**

and Lower control limit, $LCL_{100p} = 100 \times LCL_p = 100 \times 0.029 = 2.9$ **Ans.**

Note It may be noted from Example 9.5 that percent control limits or $100p$ control limits have been calculated by first calculating UCL_p and LCL_p and then multiplied by 100.

Example 9.6 ***For the Example 9.4, find the p-chart control limits using a 95 percent confidence interval.***

Solution : From Example 9.4, the calculated values are $\bar{p} = 0.09$; $n = 200$.

We know that the 95% confidence interval fall within 2σ (to be precise $1.96\,\sigma$) limits.

$$CL_p = \bar{p} = 0.09$$

Upper control limit, $UCL_p = \bar{p} + 1.96\,\sigma = \bar{p} + 1.96\sqrt{\dfrac{\bar{p}\,(1-\bar{p})}{n}}$

$$= 0.09 + 1.96\sqrt{\frac{0.09\,(1-0.09)}{200}} = 0.129 \text{ Ans.}$$

Lower control limit, $LCL_p = \bar{p} - 1.96\,\sigma$

$$= 0.09 - 1.96\sqrt{\frac{0.09\,(1-0.09)}{200}} = 0.05 \text{ Ans.}$$

Note Generally control limits using 2σ values are called ***warning control limits.***

i.e., Warning control limits $= \bar{p} \pm 2\sigma = \bar{p} \pm 2\sqrt{\dfrac{\bar{p}\,(1-\bar{p})}{n}}$

Example 9.7 ***The LCL and UCL for a p-chart are : LCL = 0.10 and UCL = 0.25. Determine the sample size 'n' that is compatible with this control chart.***

Given Data : LCL = 0.10 ; UCL = 0.25.

To find : Sample size 'n'.

Solution : We know that,

$$UCL = \bar{p} + 3\sqrt{\frac{\bar{p}\,(1-\bar{p})}{n}} = 0.25$$

and

$$LCL = \bar{p} - 3\sqrt{\frac{\bar{p}\,(1-\bar{p})}{n}} = 0.10$$

Adding (*i*) and (*ii*), we get

$$2\bar{p} = 0.35 \quad \text{or} \quad \bar{p} = 0.175$$

Substituting $\bar{p} = 0.175$ in equation (*i*), we get

$$0.175 + 3\sqrt{\frac{0.175\,(1-0.175)}{n}} = 0.25$$

Sample size, n = **231 Ans.**

Example 9.8 ***Following is the record of the defectives observed during the inspection process of an automatic machine producing small capacitors of standard size.***

(i) Find the 99.7% control limits for the process.

(ii) Draw the control chart and comment on the state of control in the process.

Sample Number	Sample Size	Number of defective capacitors	Sample Number	Sample Size	Number of Defective Capacitors
1	25	3	9	30	3
2	50	5	10	25	2
3	45	1	11	55	5
4	55	2	12	40	4
5	35	–	13	50	3
6	40	1	14	25	2
7	50	9	15	40	2
8	65	2			

Solution : ***(i) To find CL_p, UCL_p and LCL_p :***

The 99.7% control limits means 3σ control limits. As the size of the sample is non uniform, so we have to calculate the average sample size first.

$$\text{Average sample size, } n = \frac{\text{Sum of size of each sample}}{\text{Total number of samples}}$$

$$= \frac{25+50+45+55+35+40+50+65+30+25+55+40+50+25+40}{15}$$

$$= \frac{630}{15} = 42$$

$$\text{Average fraction defective, } \bar{p} = \frac{\text{Total number of defectives in all samples}}{\text{Total number of items inspected}}$$

$$\bar{p} = \frac{3+5+1+2+0+1+9+2+3+2+5+4+3+2+2}{15\,(42)}$$

$$= \frac{44}{630} \approx 0.070$$

Centre line or control limit, $CL_p = \bar{p} = \mathbf{0.07}$ **Ans.**

$$\text{Upper control limit, } UCL_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.07 + 3\sqrt{\frac{0.07(1-0.07)}{42}} = 0.188 \text{ Ans.}$$

$$\text{and Lower control limit, } LCL_p = \bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.07 - 3\sqrt{\frac{0.07(1-0.07)}{42}} = -0.048 = 0 \text{ Ans.}$$

(ii) To draw the control chart :

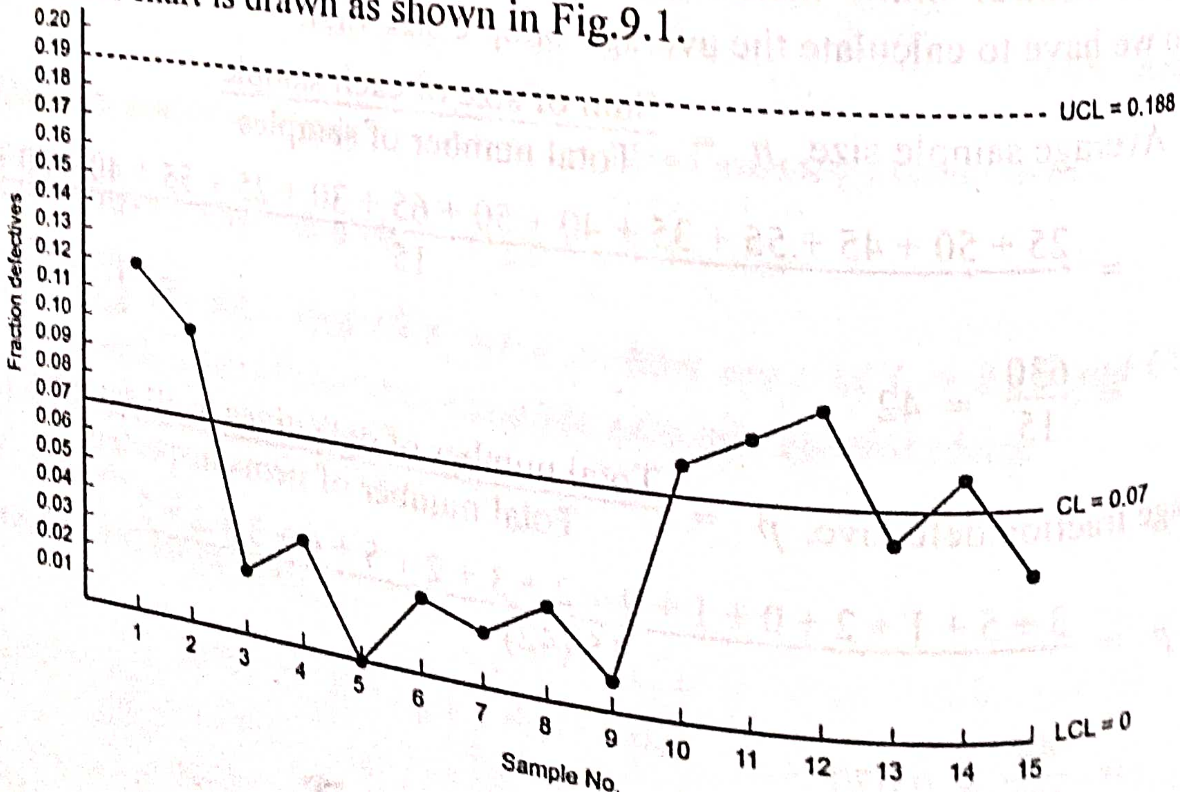
In order to draw the control chart, we need to calculate the fraction defective for each sample, as shown in Table 9.1.

$$\text{Fraction defective} = \frac{\text{Number of defectives in the sample}}{\text{Corresponding sample size}}$$

Table 9.1. Fraction defectives

Sample Number	Fraction defectives	Sample Number	Fraction defectives
1	$\left(\frac{3}{25}=\right)$ 0.12	9	0.010
2	0.10	10	0.08
3	0.022	11	0.09
4	0.036	12	0.10
5	0	13	0.06
6	0.025	14	0.08
7	0.018	15	0.05
8	0.03		

Now the control chart is drawn as shown in Fig.9.1.



Comment : It can be seen from the chart that all points lies inside the control limits. Hence it can be said that the **process is 'in control'. Ans.**

Example 9.9 ***In the manufacture of certain special duty transformers, the number of defectives found in the inspection of 20 lots of 100 samples are given below :***

Lot Number	*Number of defectives*	*Lot Number*	*Number of defectives*
1	5	11	7
2	4	12	6
3	3	13	3
4	5	14	5
5	4	15	4
6	6	16	2
7	9	17	8
8	15	18	7
9	11	19	6
10	6	20	4

(a) Determine the control limits of p-chart and state whether the process is in control.

(b) Determine the new value of mean fraction defective if some points are out of control. Compute the corresponding control limits and state whether the process is still in control or not.

(c) Determine the sample size when a quality limit not worse than 9% is desirable and a 10% bad product will not be permitted more than three times in thousand.

☺ ***Solution : (a) To find control limits of p-chart :***

We know that,
$$\bar{p} = \frac{\text{Total number of defectives}}{\text{Total number of items inspected}}$$

$$= \frac{120}{20 \times 100} = 0.06$$

∴ Control limit or center line, $CL_p = \bar{p} = \mathbf{0.06}$ **Ans.**

Upper control limit, $UCL_{\bar{p}} = \bar{p} + 3\sqrt{\frac{\bar{p}\,(1-\bar{p})}{n}}$

$$= 0.06 + 3\sqrt{\frac{0.06\,(1-0.06)}{100}} = \mathbf{0.1309}$$ **Ans.**

and Lower control limit, $LCL_{\bar{p}} = \bar{p} - 3\sqrt{\frac{\bar{p}\,(1-\bar{p})}{n}}$

$$= 0.06 - 3\sqrt{\frac{0.06\,(1-0.06)}{100}}$$

$$= \mathbf{-0.0109 = 0}$$ **Ans.**

After observing all the given values of defectives, we can find that only 8^{th} lot having fraction defective $\frac{15}{100} = 0.15$ will lie above UCL_p.

Therefore the process is **out of control. Ans.**

(b) To find revised value of control limits of p-chart :

After eliminating the 8^{th} lot *i.e.*, out of control point, we get the revised $\bar{p}$ as

$$\bar{p} = \frac{120-15}{19 \times 100} = \frac{105}{1900} = 0.056$$

Then the revised control limits are :

Control limit or centre line, $CL_p = \bar{p} = \mathbf{0.056}$ **Ans.**

$$\text{Upper control limit, } UCL_p = 0.056 + 3\sqrt{\frac{0.056\,(1-0.056)}{100}}$$

$$= \mathbf{0.125} \quad \textbf{Ans.}$$

$$\text{and} \quad \text{Lower control limit, } LCL_p = 0.056 - 3\sqrt{\frac{0.056\,(1-0.056)}{100}}$$

$$= \mathbf{-0.013 = 0} \quad \textbf{Ans.}$$

Now we can find that all the points are ***within control limits.***

(c) To find new sample size (n′) :

We know that the probability that a defective worse than 9% defective quality will not be permitted is more than 3 times in thousand (0.3%) is corresponding to 3σ limits.

$$\therefore \quad \bar{p} + 3\,\sigma_p = 0.09$$

$$\text{or} \quad 0.056 + 3\sqrt{\frac{0.056\,(1-0.056)}{n}} = 0.09$$

or

$$n = \mathbf{333} \quad \textbf{Ans.}$$

Example 9.10 ***The ABC company purchases small steel bearing balls in cartons that usually contain several thousand balls. Each shipment consists of a number of cartons. As part of the acceptance procedure for these balls, 375 balls are selected at random from each carton and are subjected to visual inspection for certain non-conformities. In a shipment of 10 cartons, the respective percentages of rejectable balls in the sample from each carton are 0.4, 0, 0.75, 2, 0, 0.5, 0.25, 0, 0.5 and 1.3. Does this shipment of balls appear to exhibit statistical control with respect to the quality characteristic examined in this inspection ?***

☺ **Solution :** Average fraction defective, $\bar{p} = \dfrac{\text{Total number of defectives}}{\text{Total number of items inspected}}$

or $$\bar{p} = \frac{(0.4 + 0 + 0.75 + 2 + 0 + 0.5 + 0.25 + 0 + 0.5 + 1.3) \times \frac{375}{100}}{10 \times 375} = 0.0057$$

$$\text{Centre line, } CL_p = \bar{p} = \mathbf{0.0057}$$

$$\text{Upper control limit, } UCL_p = \bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

$$= 0.0057 + 3\sqrt{\frac{0.0057(1-0.0057)}{375}}$$

$$= \mathbf{0.01736}$$

and $$\text{Lower control limit, } LCL_p = 0.0057 - 3\sqrt{\frac{0.0057(1-0.0057)}{375}}$$

$$= \mathbf{-0.00596 = 0}$$

In terms of percentage, $UCL_p = 1.73$ and $LCL_p = 0$.

After observing the given readings, we can notice that 4th subgroup *i.e.*, 2% falls above UCL. Therefore **the shipment does not exhibit statistical control. Ans.**

PROBLEMS ON *np*-CHART

Example 9.11 *A large number of samples of 300 items each taken from a process that has a percentage non-conforming of 12%.*

(a) What is the expected average number of non-conforming units per sample ?

(b) Find the 3σ control limits for a np-chart to control this process.

Given Data : $n = 300$; $\bar{p} = 12\% = 0.12$.

☺ ***Solution : (a) To find average number of non-conforming units per sample (np) :***

$$\text{Average number of non-conforming units per sample} = n\bar{p}$$

$$= 300 \times 0.12 = \mathbf{36} \quad \textbf{Ans.}$$

(b) To find np-chart control limits :

$$\text{Centre line, } CL_{np} = n\bar{p} = 20$$

$$\text{Upper control limit, } UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$= 36 + 3\sqrt{36(1-0.12)} = \mathbf{52.88} \quad \textbf{Ans.}$$

and $$\text{Lower control limit, } LCL_{np} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$$

$$= 36 - 3\sqrt{36(1-0.12)} = \mathbf{19.11} \quad \textbf{Ans.}$$

Example 9.12 *Data were collected for a large box of bolts containing about 10% non-conforming items. Plot the np-chart based on the data given below :*

Subgroup Number	*Number inspected*	*Number non-conforming*	*Subgroup Number*	*Number inspected*	*Number non-conforming*
1	200	28	14	200	23
2	200	20	15	200	28
3	200	24	16	200	28
4	200	19	17	200	15
5	200	17	18	200	23
6	200	25	19	200	17
7	200	25	20	200	22
8	200	22	21	200	25
9	200	22	22	200	20
10	200	16	23	200	18
11	200	22	24	200	14
12	200	18	25	200	13
13	200	20			

If a p-chart is plotted instead of np-chart, will it give a different indication ?

☺ ***Solution :*** Since the subgroup size is constant, therefore *np*-chart can be used.

From the given table, sum of non-conforming bolts = 524.

$$\text{Average fraction defective, } \bar{p} = \frac{\text{Total number of defectives}}{\text{Total number of items inspected}}$$

$$= \frac{520}{25\,(200)} = 0.1084$$

$$\text{Centre line, } CL_{np} = n\,\bar{p} = 200 \times 0.1084 = \mathbf{20.96} \quad \textbf{Ans.}$$

$$\text{Upper control limit, } UCL_{np} = n\,\bar{p} + 3\sqrt{n\,\bar{p}\,(1-\bar{p})}$$

$$= 20.96 + 3\sqrt{20.96\,(1-0.1084)} = \mathbf{33.96} \quad \textbf{Ans.}$$

$$\text{Lower control limit, } LCL_{np} = n\,\bar{p} - 3\sqrt{n\,\bar{p}\,(1-\bar{p})}$$

$$= 20.96 - 3\sqrt{20.96\,(1-0.1084)}$$

$$= \mathbf{7.96} \quad \textbf{Ans.}$$

The *np*-chart can be drawn as shown in Fig.9.2.

(ii) If a *p*-chart were drawn for the same data, it would be exactly like the *np*-chart except for the graduations on the vertical scale. Each unit on the vertical scale would represent $\frac{1}{200}$ (*i.e.*, $\frac{1}{n}$) as much as it does on the chart for *np*. It is evident that there is **no fundamental difference in the appearance and information of the *np*-chart and the *p*-chart.** **Ans.**

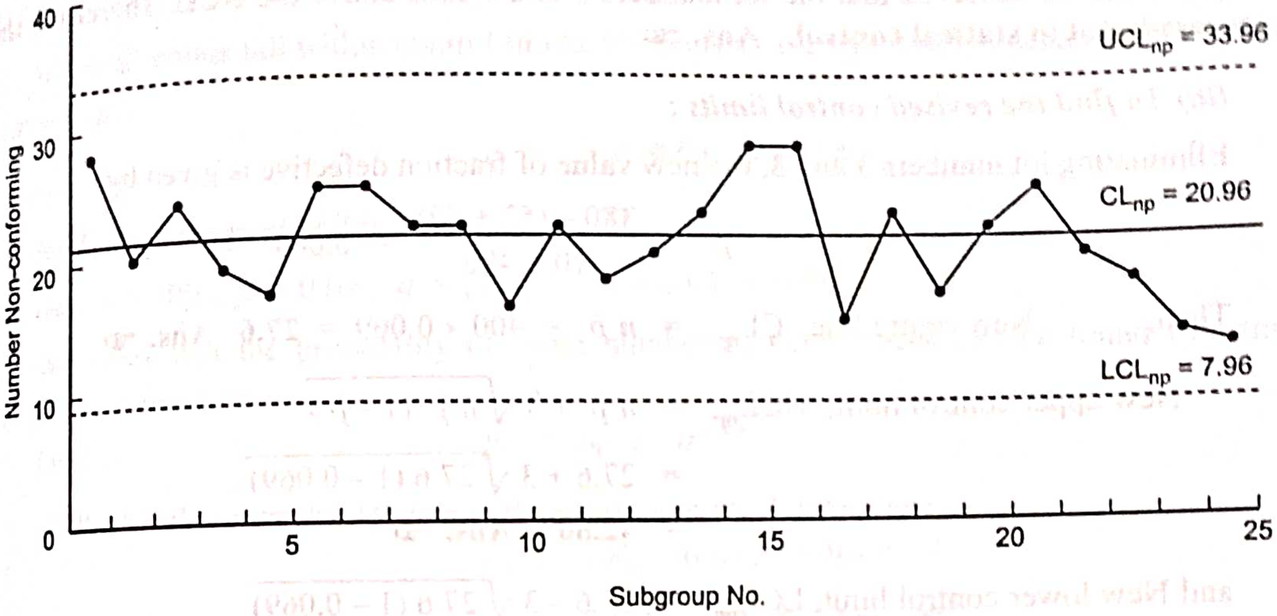


Fig. 9.2. np-chart

Example 9.13 *The number of defectives in lots for a particular item is shown below. Each lot consists of 400 items.*

Lot No.	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*
No. of defects per lot	*24*	*38*	*52*	*26*	*36*	*30*	*42*	*52*	*20*	*16*	*20*	*24*

(i) Compute trial control limits for np-chart.

(ii) State whether the process is in control or not ?

(iii) Calculate the revised control limits if you feel necessary.

☺ ***Solution : (i) To find trail control limits for np-chart :***

From the given table,

$$\text{Total number of defective} = \Sigma np = 24 + 38 + 52 + \ldots\ldots + 24 = 380$$

$$\text{Total number of items inspected} = \Sigma n = 12 \times 400 = 4800$$

$$\therefore \bar{p} = \frac{\Sigma np}{\Sigma n} = \frac{380}{4800} = 0.079$$

$$\text{Then centre line, } CL_{np} = n\bar{p} = 400 \times 0.079 = \mathbf{31.66} \quad \textbf{Ans.}$$

$$\text{Upper control limit, } UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}\,(1-\bar{p})}$$

$$= 31.66 + 3\sqrt{31.66\,(1-0.079)} = \mathbf{47.86} \quad \textbf{Ans.}$$

$$\text{and Lower control limit, } LCL_{np} = n\bar{p} - 3\sqrt{n\bar{p}\,(1-\bar{p})}$$

$$= 31.66 - 3\sqrt{31.66\,(1-0.079)} = \mathbf{15.46} \quad \textbf{Ans.}$$

(*ii*) It can be observed that the lot numbers 3 and 8 falls above the UCL. Therefore, the **process is not in statical control. Ans.**

(iii) To find the revised control limits :

Eliminating lot numbers 3 and 8, the new value of fraction defective is given by

$$\bar{p} = \frac{380-(52+52)}{10 \times 400} = 0.069$$

Then, New centre line, $CL_{np} = n\bar{p} = 400 \times 0.069 = 27.6$ **Ans.**

New upper control limit, $UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$

$= 27.6 + 3\sqrt{27.6(1-0.069)}$

$=$ **42.807 Ans.**

and New lower control limit, $LCL_{np} = 27.6 - 3\sqrt{27.6(1-0.069)}$

$=$ **22.531 Ans.**

The process is still out of control, as the number of defectives for lot numbers 9 and 11 falls below LCL.

Example 9.14 ***An item is made in lots of 100 each. The lots are subjected to 100% inspection. The record sheet for the first 20 lots inspected showed that a total of 80 items were defective.***

(i) Determine the total control limits for np-chart.

(ii) Assuming all points fall within the control limits, what is your estimate of the process average fraction defective p'?

(iii) If this p' remains unchanged, what is the probability that the 21st lot will contain exactly 2 defective ? That it will contain 2 or more defectives ?

☺ ***Solution : (i) To determine the total control limits for np-chart :***

Average fraction defective, $\bar{p} = \frac{80}{20 \times 100} = 0.04$

$\therefore$ Centre line, $CL_{np} = n\bar{p} = 100 \times 0.04 = 4$ **Ans.**

Upper control limit, $UCL_{np} = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$

$= 4 + 3\sqrt{4(1-0.04)} = 9.878$ **Ans.**

and Lower control limit, $LCL_{np} = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$

$= 4 - 3\sqrt{4(1-0.04)} = -1.878 = 0$ **Ans.**

(b) To find the estimate of the process average fraction defective (p') :

When all points fall within control limits, the process average fraction defective p' is taken equal to $\bar{p}$.

$$\therefore \quad p' = \bar{p} = 0.04 \quad \textbf{Ans.}$$ ☞

(c) To find the probability :

Here $n = 100$; $p' = 0.04$; $q' = (1 - p') = 1 - 0.04 = 0.96$

We know that the probability of exact number of occurrences can be found by using binomial expression

i.e.,
$$p_r = {}^{n}C_r \, p'^{r} \, q^{n-r}$$

(i) Probability that the 21st lost will contain exactly 2 defective :

$$p_2 = 100_{C_2} \, (0.04)^2 \, (0.96)^{100-2}$$
$$= 0.145 = 14.5\% \quad \textbf{Ans.}$$ ☞

(ii) Probability that it will contain 2 or more defectives :

$$p_{2 \text{ or more defective}} = 1 - (p_0 + p_1)$$

where
$$p_0 = 100_{C_0} \, (0.04)^0 \, (0.96)^{100} = 0.168, \text{ and}$$
$$p_1 = 100_{C_1} \, (0.04)^1 \, (0.96)^{100-1} = 0.07$$

$$\therefore \quad p_{2 \text{ or more defective}} = 1 - [0.0168 + 0.07]$$
$$= 0.913 \text{ or } 91.3\% \quad \textbf{Ans.}$$ ☞

9.6. c - CHART

- ✓ In the c-chart (*c* for count), the number of defects in the sample are plotted over time.
- ✓ The *c*-chart applies to the number of defects in a subgroup of constant size.
- ✓ The *p* and *np* charts control the fraction defective* in the product whereas the *c*-chart controls the number of defects* in the product. In other words, the control chart for defects is called as *c*-chart.
- ✓ The *c*-chart is based on Poisson distribution.
- ✓ ***The situations where c-chart can be used are :***

  (*i*) Number of typographical errors on the printed page.

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* *Difference between a defect and defective :*

- ✓ An item is said to be *defective* if it fails to conform to the specifications in any of the characteristics.
- ✓ Each characteristics that does not meet the specifications is a *defect.*
- ✓ For example, if a casting contains undesirable hard spots, blow holes *etc.*, the casting is defective and the hard spots, blow holes, *etc.*, are the defects.

(ii) Number of defective rivets in an automobile body.

(iii) Number of rust spots on steel sheets.

(iv) Number of defects such as cracks, block holes, undercuts, *etc.*, in a casting or a welded piece.

Control Limits for c - chart

Let c = Number of defects, and

n = Number of samples.

Then, Average number of defects, $\bar{c} = \dfrac{\text{Total number of defects in all samples}}{\text{Total number of samples}}$

$$\bar{c} = \frac{\Sigma c}{n}$$

i.e.,

Control limit or central line, $CL_c = \bar{c}$

Upper control limit, $UCL_c = \bar{c} + 3\sqrt{\bar{c}}$

and Lower control limit, $LCL_c = \bar{c} - 3\sqrt{\bar{c}}$

9.7. *u* - CHART

✓ When the subgroup size varies from sample to sample, then the *u*-chart is used.

✓ *u*-chart controls the number of defects per unit.

✓ ***Control limits for u-chart***

In *u*-chart, the control limits will vary from sample to sample.

$$\bar{u} = \frac{\text{Number of defects in a sample}}{\text{Number of units in a sample}} = \frac{c}{n}$$

∴ Control limit or centre line, $CL_u = \bar{u}$

Upper control limit, $UCL_u = \bar{u} + 3\sqrt{\dfrac{\bar{u}}{n}}$

and Lower control limit, $LCL_u = \bar{u} - 3\sqrt{\dfrac{\bar{u}}{n}}$

9.8. COMPARISON BETWEEN VARIABLE CHARTS AND ATTRIBUTE CHARTS

Table 9.2 compares the variable and attribute charts.

Table 9.2. Comparison of three types of control charts

Statistical measure plotted	Average $\bar{X}$ and range R	Percentage non-conforming (p)	Number of non-conformities (c)
Type of data required	Variable data (measured values of a characteristic)	Attribute data (number of defective units of product)	Attribute data (number of defects per unit of product)
General field of application	Control of individual characteristics	Control of overall fraction defective of a process	Control of overall number of defects per unit
Significant advantages	✓ Provides maximum utilization of information available from data. ✓ Provides detailed information on process average and variations for control of individual dimensions.	✓ Data required are often already available from inspection records. ✓ Easily understood by all personnel. ✓ Provides an overall picture of quality.	✓ Same advantages as p-chart but also provides a measure of defectiveness.
Significant disadvantages	✓ Not understood unless training is provided, can cause confusion between control limits and tolerance limits. ✓ Cannot be used with go / no go type of data.	✓ Does not provide detailed information for control of individual characteristics. ✓ Does not recognise different degrees of defectiveness in units of product.	✓ Does not provide detailed information for control of individual characteristics.
Sample size	Usually 4 or 5	Uses given inspection results for samples of 25, 50 or 100	Any convenient unit of product such as 50 m of wire or one computer set.

Note In general practice, initially p-chart is drawn. In case the p-chart shows out of control, then only $\bar{X}$ and R charts are plotted for detailed analysis.

9.9. FORMULA SUMMARY

Table 9.3. Different types of control charts for attributes

Type	Central Line	Control Limits
p	$\bar{p}$	$\text{UCL} = \bar{p} + 3\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}}$ $\text{LCL} = \bar{p} - 3\sqrt{\dfrac{\bar{p}(1-\bar{p})}{n}}$

Type	Central Line	
np	$n\bar{p}$	$UCL = n\bar{p} + 3\sqrt{n\bar{p}\,(1-\bar{p})}$ $LCL = n\bar{p} - 3\sqrt{n\bar{p}\,(1-\bar{p})}$
c	$\bar{c}$	$UCL = \bar{c} + 3\sqrt{\bar{c}}$ $LCL = \bar{c} - 3\sqrt{\bar{c}}$
u	$\bar{u}$	$UCL = \bar{u} + 3\sqrt{\dfrac{\bar{u}}{n}}$ $LCL = \bar{u} - 3\sqrt{\dfrac{\bar{u}}{n}}$

PROBLEMS ON c CHART

Example 9.15 *Ten airplane wings were inspected for non-conforming welds. Th number of non-conforming welds per wing ranged from 26 to 55. The total number of no conforming welds for the 12 units was 420. Determine the control limits for a c-chart.*

☺ *Solution :* Average number of non-conforming welds per wing is given by

$$\bar{c} = \frac{\text{Total number of non-conforming welds in 10 units}}{\text{Total units}}$$

$$= \frac{420}{12} = 35$$

$\therefore$ Control limit or centre line, $CL_c = \bar{c} = 35$ **Ans.**

Upper control limit, $UCL_c = \bar{c} + 3\sqrt{\bar{c}} = 35 + 3\sqrt{35} = 52.75$ **Ans.**

and Lower control limit, $LCL_c = \bar{c} - 3\sqrt{c} = 35 - 3\sqrt{35} = 17.25$ **Ans.**

Example 9.16 *Ten castings were inspected in order to locate defects in them. Eve casting was found to contain certain number of defects as given below. It is required to p a c-chart and draw the conclusion.*

Castings	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Number of defects	*2*	*4*	*1*	*5*	*5*	*6*	*3*	*4*	*0*	*7*

☺ *Solution :* $\bar{c} = \dfrac{\text{Total number of defects in all samples}}{\text{Total number of items inspected}}$

$$= \frac{2+4+1+5+5+6+3+4+0+7}{10} = 3.7$$

$\therefore$ Control limit or centre line, $CL_c = \bar{c} = 3.7$

Upper control limit, $UCL_c = \bar{c} + 3\sqrt{\bar{c}} = 3.7 + 3\sqrt{3.7} = 9.472$

and Lower control limit, $LCL_c = \bar{c} - 3\sqrt{\bar{c}}$

$$= 3.7 - 3\sqrt{3.7} = -2.072 = 0$$

The c-chart can be drawn as shown in Fig.9.3.

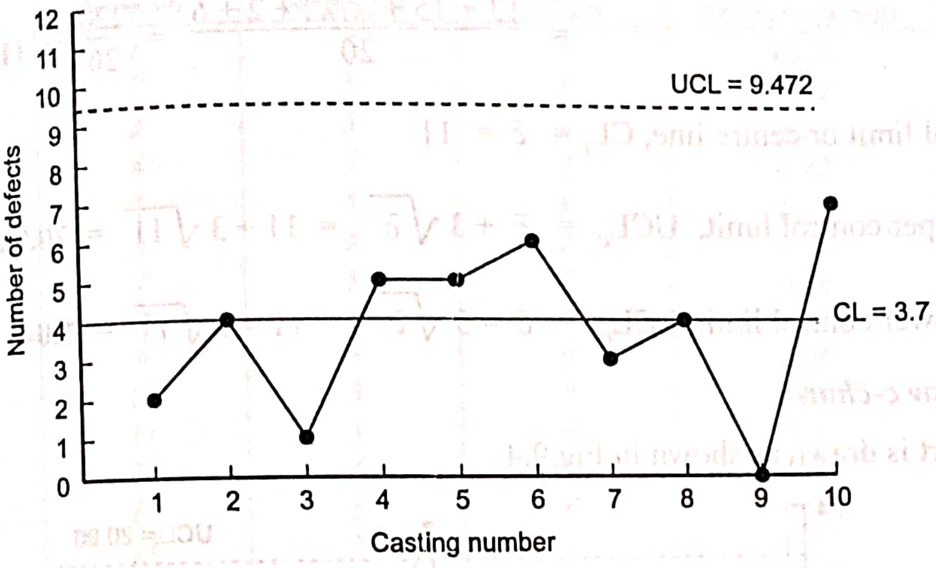

Fig. 9.3.

Conclusion : From Fig.9.3, it is clear that all the points lie within the control limits. Therefore *the process is in control.* **Ans.**

Example 9.17 ***The following table shows the number of point defects on the surface of a bus body on March 2003.***

Body Number	*Number of defects*	*Body Number*	*Number of defects*
1	13	11	17
2	15	12	11
3	19	13	7
4	8	14	11
5	6	15	14
6	17	16	6
7	7	17	16
8	9	18	10
9	3	19	2
10	23	20	6

(a) ***Compute the value of $\bar{c}$ and its control limits.***

(b) ***Draw c-chart.***

(c) ***Compute value of $\bar{c}$ and control limits for the future use, if you deem it necessary.***

Solution : (a) To find $\bar{c}$, UCL_c and LCL_c :

$$\bar{c} = \frac{\text{Total number of defects}}{\text{Total number of bodies}}$$

$$= \frac{13 + 15 + \ldots\ldots + 2 + 6}{20} = \frac{220}{20} = 11 \text{ Ans.}$$

$\therefore$ Control limit or centre line, $CL_c = \bar{c} = 11$

Upper control limit, $UCL_c = \bar{c} + 3\sqrt{\bar{c}} = 11 + 3\sqrt{11} = 20.96$ **Ans.**

and Lower control limit, $LCL_c = \bar{c} - 3\sqrt{\bar{c}} = 11 - 3\sqrt{11} = 1.04$ **Ans.**

(b) To draw c-chart :

The c-chart is drawn as shown in Fig.9.4.

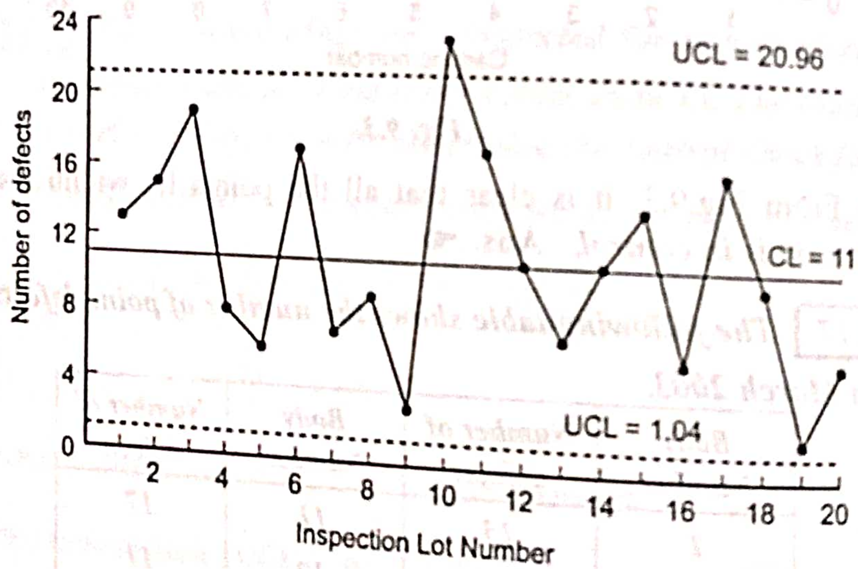

Fig. 9.4. c-chart for defects per lot for the data

It can be seen that body number 10 is out of control.

(c) To find the revised $\bar{c}$ and control limits :

$$\bar{c}_{\text{revised}} = \frac{220 - 23}{20 - 1} = 10.37 \text{ Ans.}$$

$\therefore$ Control limit or centre line, $CL_c = \bar{c} = 10.37$

Upper control limit, $UCL_c = 10.37 + 3\sqrt{10.37} = 20.03$ **Ans.**

and Lower control limit, $LCL_c = 10.37 - 3\sqrt{10.37} = 0.71$ **Ans.**

Now the process is in control.

PROBLEMS ON u-CHART

Example 9.18 ***The following data shows the number of defects per lot in 15 successive lots of 5 radio sets each.***

Lot Number	*Number Inspected*	*Number of defects*	*Number of defects per unit*
1	5	3	0.60
2	5	2	0.40
3	5	1	0.20
4	5	2	0.40
5	5	6	1.20
6	5	1	0.20
7	5	3	0.60
8	5	2	0.40
9	5	0	0.00
10	5	0	0.00
11	5	0	0.00
12	5	0	0.00
13	5	0	0.00
14	5	1	0.20
15	5	2	0.40
	$\Sigma n = 75$	$\Sigma c = 23$	

(i) Plot the u-chart based on the above data.

(ii) Suppose the reasons for out of control points are known and can be eliminated, then what will you suggest the control limits for future production.

☺ ***Solution :*** In this case either a c-chart or a u-chart can be used. But this problem can be solved using a u-chart.

Average number of defects per unit, $\bar{u} = \dfrac{\Sigma c}{\Sigma n}$

$$\bar{u} = \frac{23}{75} = 0.31$$

$\therefore$ Control limit or centre line, $CL_u = \bar{u} = \mathbf{0.31}$ **Ans.**

Upper control limit, $UCL_u = \bar{u} + 3\sqrt{\dfrac{\bar{u}}{n}} = 0.31 + 3\sqrt{\dfrac{0.31}{5}}$

$= \mathbf{1.06}$ **Ans.**

and Lower control limit, $LCL_u = \bar{u} - 3\sqrt{\dfrac{\bar{u}}{n}} = 0.31 - 3\sqrt{\dfrac{0.31}{5}}$

$= \mathbf{-0.437 \approx 0}$ **Ans.**

Now the u-chart is drawn as shown in Fig.9.5.

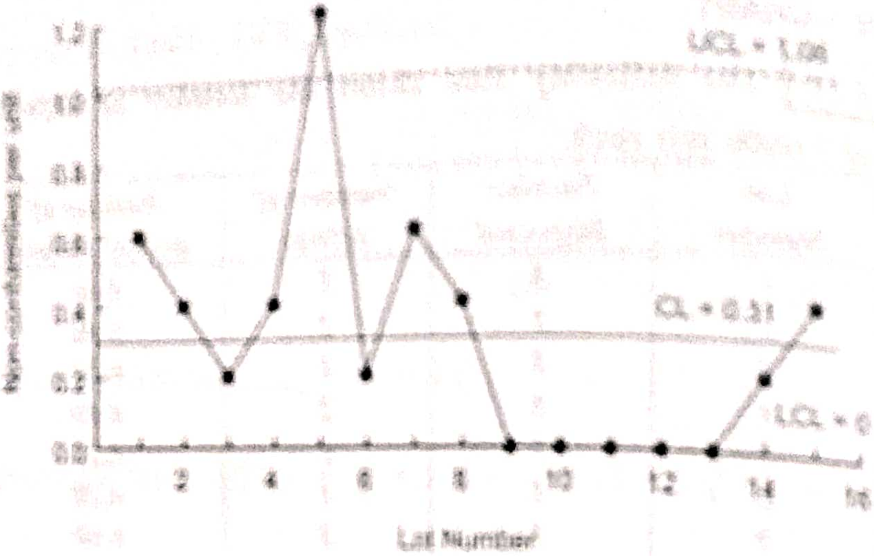

Fig. 9.5.

(b) To find the revised control limits :

From the control chart, it can be seen that lot number 5 is above UCL. So lot number 5 is out of control. Eliminating the lot number 5, the new $\bar{u}$ can be obtained for future production.

$$\therefore \bar{u} = \frac{23-6}{75-5} = \frac{17}{70} = 0.24$$

Then, Control limit or centre line, $CL_u = \bar{u} = \mathbf{0.24}$ **Ans.**

$$\text{Upper control limit, } UCL_u = \bar{u} + 3\sqrt{\frac{\bar{u}}{3}}$$

$$= 0.24 + 3\sqrt{\frac{0.24}{5}} = \mathbf{0.90} \quad \textbf{Ans.}$$

and

$$\text{Lower control limit, } LCL_u = \bar{u} - 3\sqrt{\frac{\bar{u}}{3}}$$

$$= 0.24 - 3\sqrt{\frac{0.24}{5}} = -0.42 = \mathbf{0} \quad \textbf{Ans.}$$

Example 9.19 ***The data taken on three samples inspection lots are shown in the following tabulation (in practice, a much larger number of lots would be used). A common basic lot size of n = 100 has been used.***

Inspection sample lot number	*Number of parts per inspection sample lot*	*Number of non-conformities per inspection sample lot*
1	*100*	*4*
2	*90*	*3*
3	*110*	*5*
Total	*300*	*12*

***Determine** (a) Central line on the u-chart, and*
(b) Control limits on the u-chart.

Solution : Since the quantity *i.e.,* sample size in each sample varies, therefore it is more appropriate to use the u-chart.

We know that, $\bar{u} = \dfrac{\text{Number of defects in a sample}}{\text{Number of units in a sample}} = \dfrac{c}{n}$

(a) To find central line on the u-chart :

$$k\,\bar{u} = \frac{12}{300} \times 100 = 4$$

Central line on the u-chart, $CL_u = k\,\bar{u} = 4$ **Ans.**

(b) To find control limits :

In u-chart, we know that the control limits will vary from sample to sample and they can be calculated as shown in table below.

Inspection lot number	$UCL_u = k\bar{u} + 3\sqrt{\dfrac{k\bar{u}}{n/k}}$	$LCL_u = k\bar{u} - 3\sqrt{\dfrac{k\bar{u}}{n/k}}$
1	$4 + 3\sqrt{\dfrac{4}{100/100}} = 10$	$4 - 3\sqrt{\dfrac{4}{100/100}} = -2 = 0$
2	$4 + 3\sqrt{\dfrac{4}{100/90}} = 9.69$	$4 - 3\sqrt{\dfrac{4}{100/90}} = -1.69 = 0$
3	$4 + 3\sqrt{\dfrac{4}{100/110}} = 10.29$	$4 - 3\sqrt{\dfrac{4}{100/110}} = -2.29 = 0$

Now the u-control chart is drawn as shown in Fig.9.6.

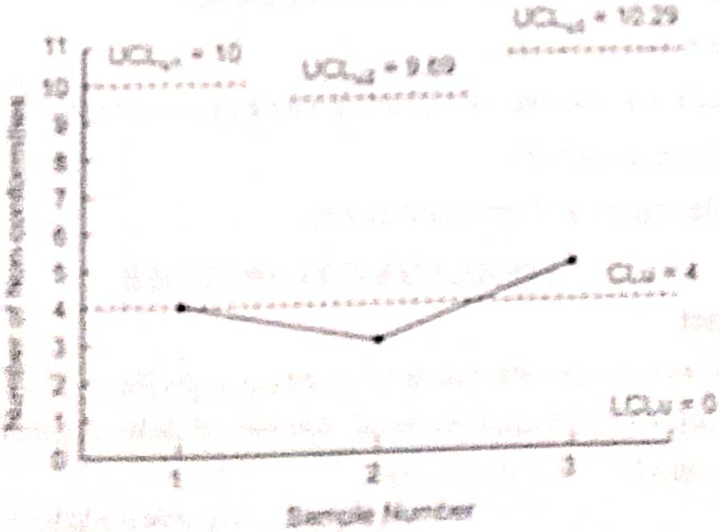

Fig. 9.6. u control chart

Since all the given points lie inside the control limits, therefore ***the process is in control.***