Statistical Quality Control

CHAPTER OUTLINE

Learning Objectives for Chapter

After careful study of this chapter, you should be able to do the following:

1. Understand the role of statistical tools in quality improvement.
2. Understand the different types of variability, rational subgroups, and how a control chart is used to detect assignable causes.
3. Understand the general form of a Shewhart control chart and how to apply zone rules (such as the Western Electric rules) and pattern analysis to detect assignable causes.
4. Construct and interpret control charts for variables such as *X-bar*, *R*, *S*, and individuals charts.
5. Construct and interpret control charts for attributes such as *P* and *U* charts.
6. Calculate and interpret process capability ratios.
7. Calculate the ARL performance for a Shewhart control chart.
8. Construct and interpret a cumulative sum and exponentially-weighted moving average control chart.
9. Use other statistical process control problem-solving tools.

Quality Improvement and Statistics

- **Definitions of Quality**

Quality means fitness for use

- quality of design
- quality of conformance

Quality is inversely proportional to variability.

Quality Improvement and Statistics

- **Quality Improvement**

Quality improvement is the reduction of variability in processes and products.

Alternatively, ***quality improvement*** is also seen as “waste reduction”.

Statistical Process Control

- Statistical process control is a collection of tools that when used together can result in process stability and variance reduction

Statistical Process Control

The seven major tools are

1) Histogram
2) Pareto Chart
4) Cause and Effect Diagram
5) Defect Concentration Diagram
6) Control Chart
7) Scatter Diagram
8) Check Sheet

Introduction to Control Charts

15-2.1 Basic Principles

- A process that is operating with only ***chance causes of variation*** present is said to be in statistical control.
- A process that is operating in the presence of ***assignable causes*** is said to be out of control.
- The eventual goal of SPC is the ***elimination of variability*** in the process.

Introduction to Control Charts

15-2.1 Basic Principles

A typical control chart has control limits set at values such that if the process is in control, nearly all points will lie within the upper control limit (UCL) and the lower control limit (LCL).

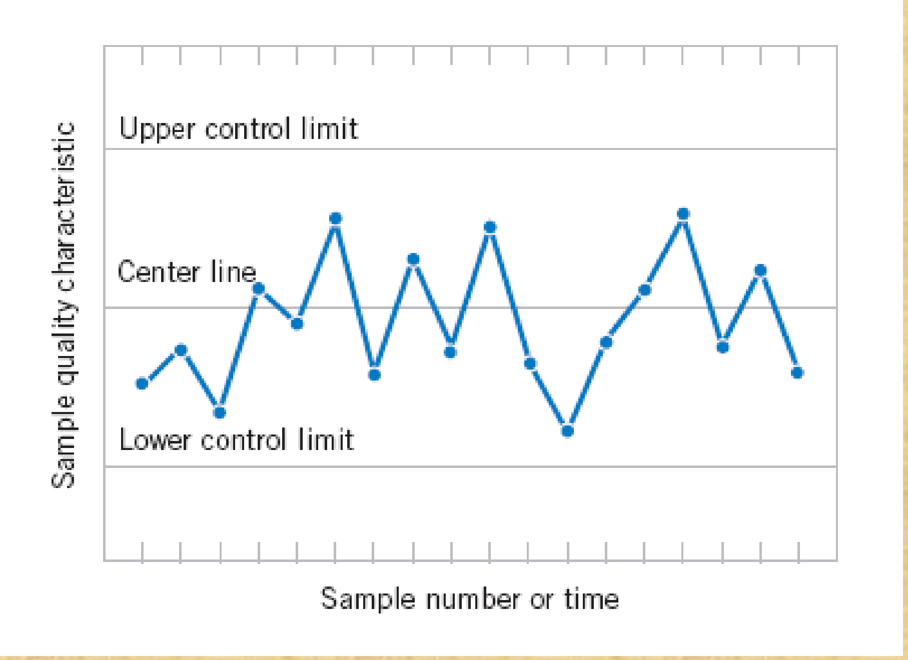

Figure 15-1 A typical control chart.

Introduction to Control Charts

15-2.1 Basic Principles

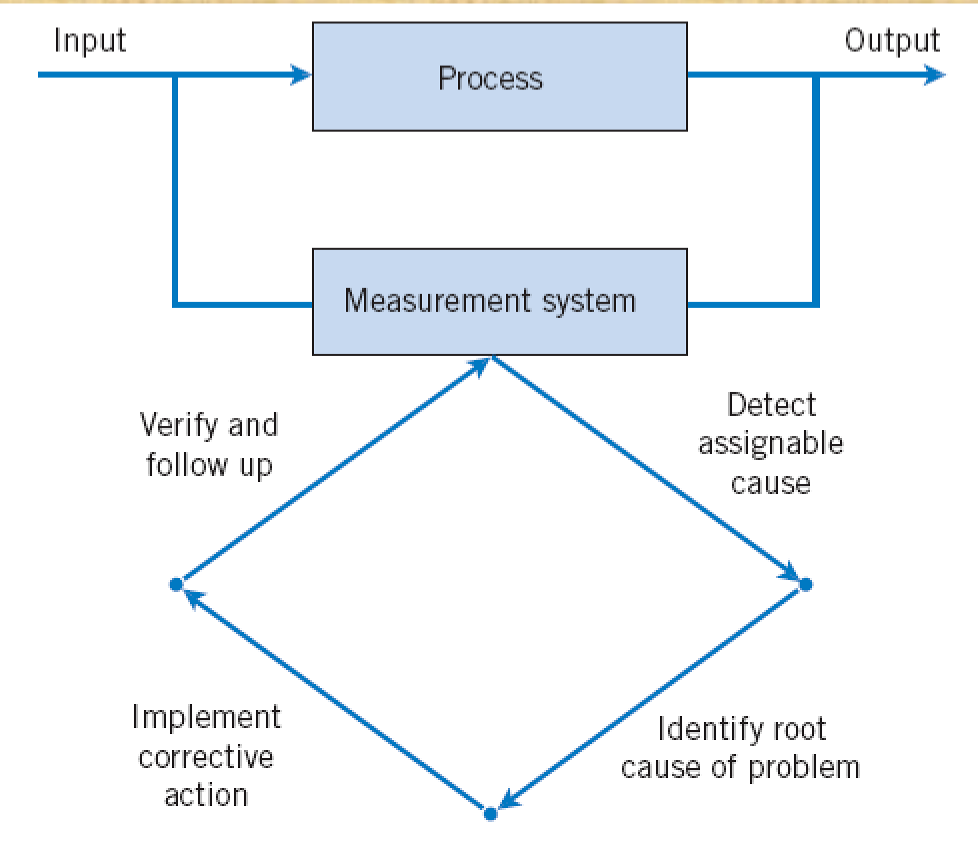

Figure 15-2 Process improvement using the control chart.

Introduction to Control Charts

15-2.1 Basic Principles

$$
\begin{aligned}
UCL &= \mu_W + k\sigma_W \\
CL &= \mu_W \\
LCL &= \mu_W - k\sigma_W
\end{aligned}
\qquad (15\text{-}1)
$$

where

k = distance of the control limit from the center line

μ_w = mean of some sample statistic, W.

σ_w = standard deviation of some statistic, W.

Introduction to Control Charts

15-2.1 Basic Principles

Important uses of the control chart

1. Most processes do not operate in a state of statistical control
2. Consequently, the routine and attentive use of control charts will identify assignable causes. If these causes can be eliminated from the process, variability will be reduced and the process will be improved
3. The control chart only detects assignable causes. Management, operator, and engineering action will be necessary to eliminate the assignable causes.

Introduction to Control Charts

15-2.1 Basic Principles

Types of control charts

- Variables Control Charts
  - These charts are applied to data that follow a continuous distribution.
- Attributes Control Charts
  - These charts are applied to data that follow a discrete distribution.

Introduction to Control Charts

15-2.1 Basic Principles

Popularity of control charts

1) Control charts are a proven technique for improving productivity.
2) Control charts are effective in defect prevention.
3) Control charts prevent unnecessary process adjustment.
4) Control charts provide diagnostic information.
5) Control charts provide information about process capability.

Introduction to Control Charts

15-2.2 Design of a Control Chart

Suppose we have a process that we assume the true process mean is $\mu = 74$ and the process standard deviation is $\sigma = 0.01$. Samples of size 5 are taken giving a standard deviation of the sample average, is

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.01}{\sqrt{5}} = 0.0045$$

Introduction to Control Charts

15-2.2 Design of a Control Chart

- Control limits can be set at 3 standard deviations from the mean in both directions.
- "3-Sigma Control Limits"

UCL = 74 + 3(0.0045) = 74.0135

CL= 74

LCL = 74 - 3(0.0045) = 73.9865

Introduction to Control Charts

15-2.2 Design of a Control Chart

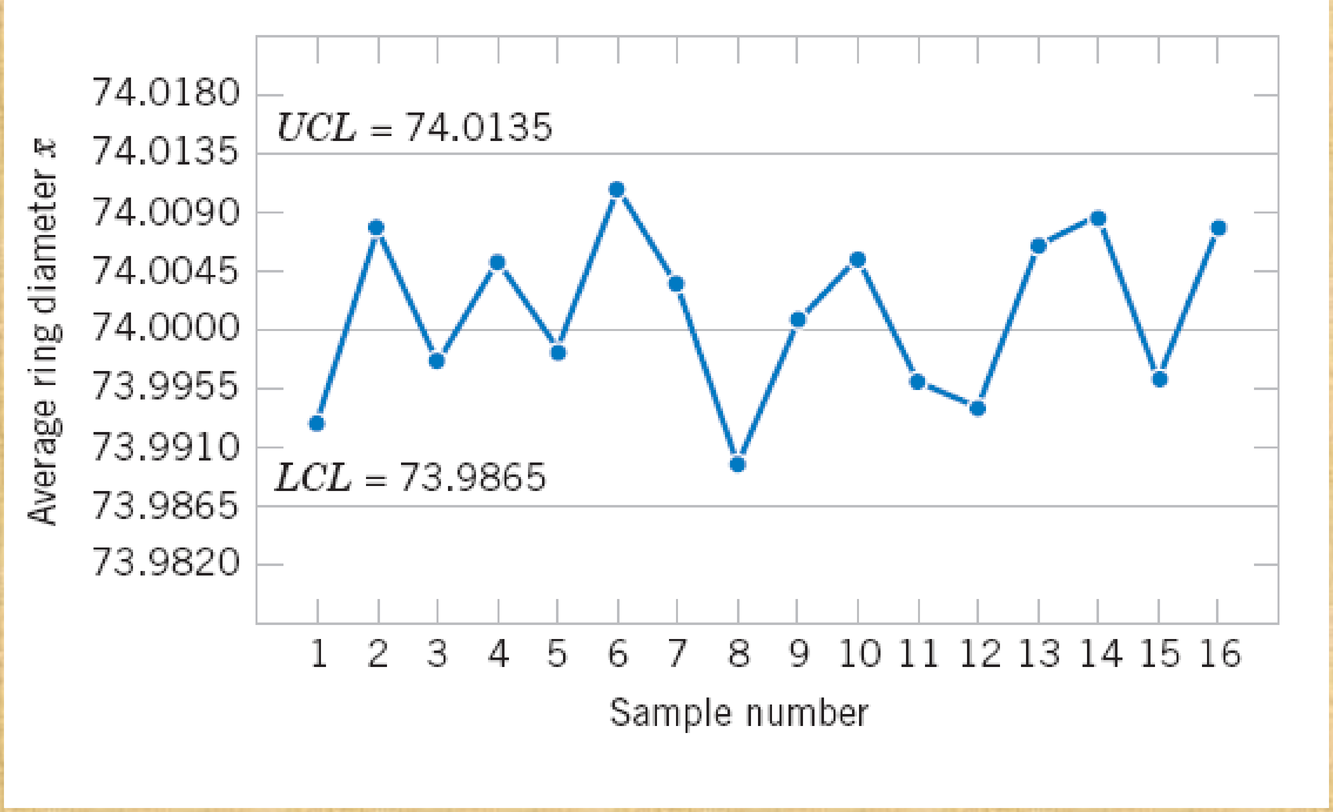

Figure 15-3 *X-bar* control chart for piston ring diameter.

15-2.2 Design of a Control Chart

- Choosing the control limits is equivalent to setting up the critical region for hypothesis testing

$$H_0: \mu = 74$$
$$H_1: \mu \neq 74$$

Introduction to Control Charts

15-2.3 Rational Subgrouping

- Subgroups or samples should be selected so that if assignable causes are present, the chance for differences between subgroups will be maximized, while the chance for differences due to these assignable causes within a subgroup will be minimized.

Introduction to Control Charts

15-2.3 Rational Subgrouping

Constructing Rational Subgroups

- Select consecutive units of production.
  - Provides a “snapshot” of the process.
  - Good at detecting process shifts.
- Select a random sample over the entire sampling interval.
  - Good at detecting if a mean has shifted
  - out-of-control and then back in-control.

Introduction to Control Charts

15-2.4 Analysis of Patterns on Control Charts

- Look for “runs” - this is a sequence of observations of the same type (all above the center line, or all below the center line)
- Runs of say 8 observations or more could indicate an out-of-control situation.
  - Run up: a series of observations are increasing
  - Run down: a series of observations are decreasing

Introduction to Control Charts

15-2.4 Analysis of Patterns on Control Charts

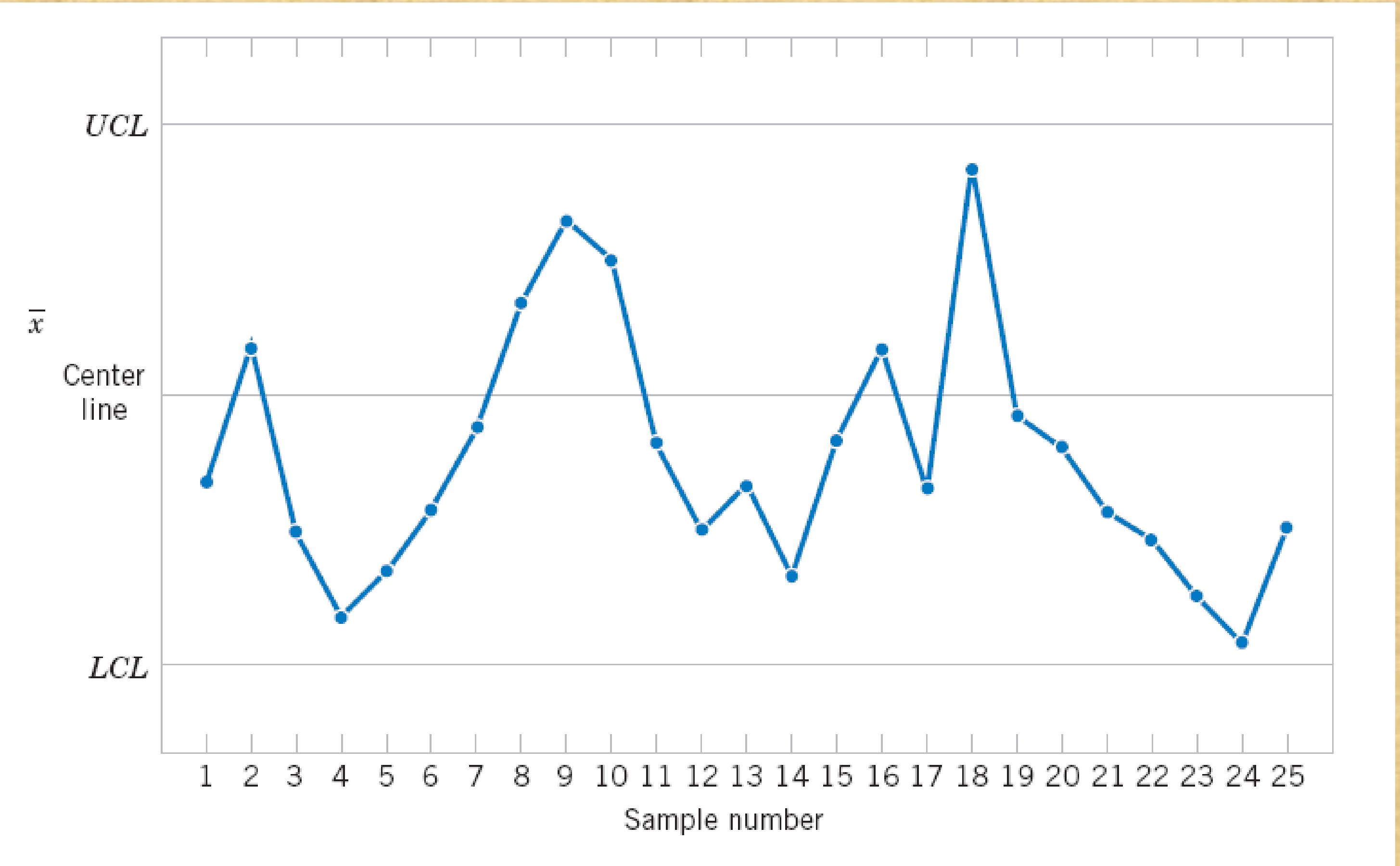

Figure 15-4 *X-bar* control chart.

Introduction to Control Charts

15-2.4 Analysis of Patterns on Control Charts

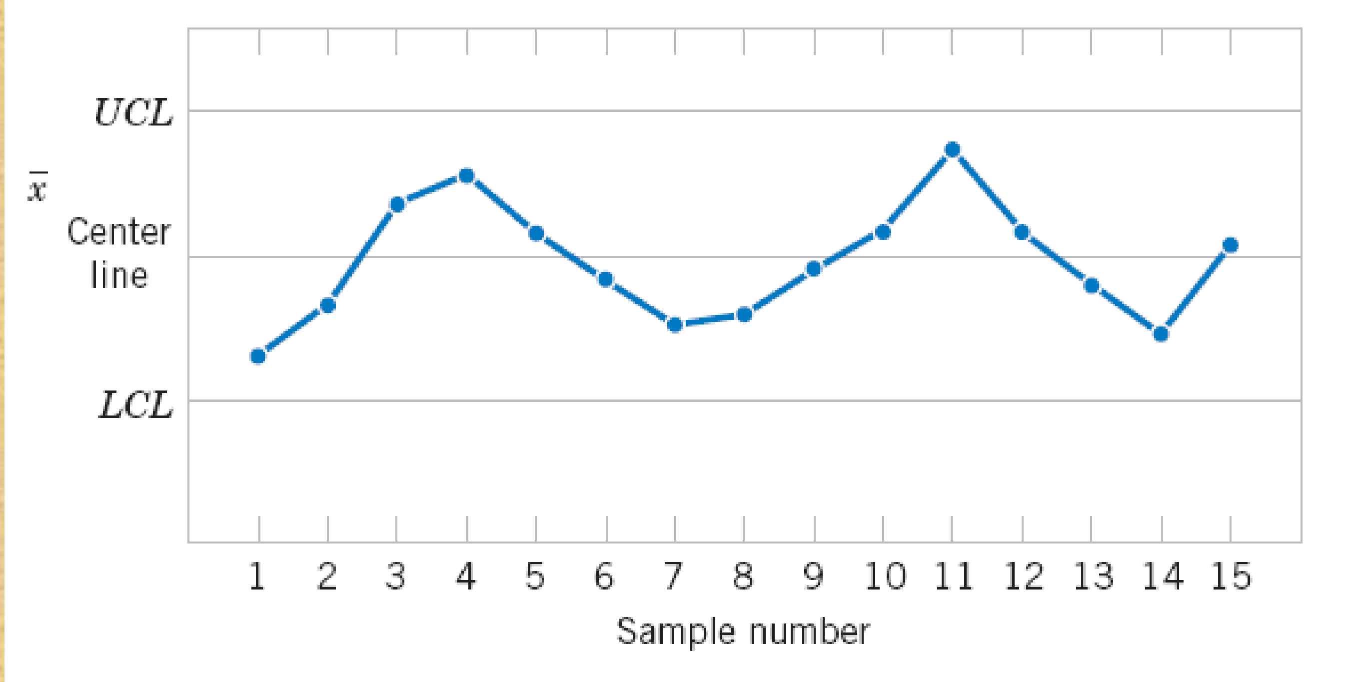

Figure 15-5 An *X-bar* chart with a cyclic pattern.

Introduction to Control Charts

15-2.4 Analysis of Patterns on Control Charts

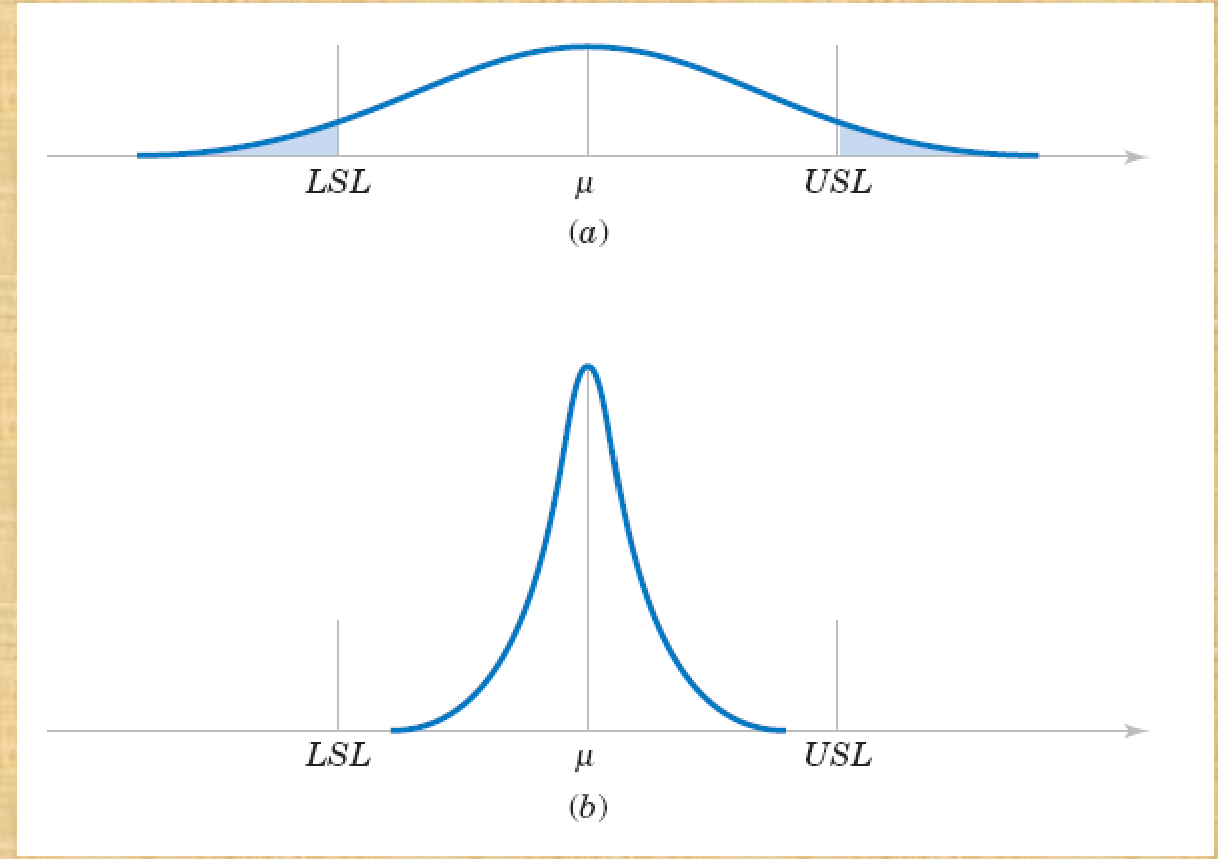

Figure 15-6 (a) Variability with the cyclic pattern. (b) Variability with the cyclic pattern eliminated.

Introduction to Control Charts

15-2.4 Analysis of Patterns on Control Charts

Western Electric Handbook Rules

A process is considered out of control if any of the following occur:

1) One point plots outside the 3-sigma control limits.
2) Two out of three consecutive points plot beyond the 2-sigma warning limits.
3) Four out of five consecutive points plot at a distance of 1-sigma or beyond from the center line.
4) Eight consecutive points plot on one side of the center line.

Introduction to Control Charts

15-2.4 Analysis of Patterns on Control Charts

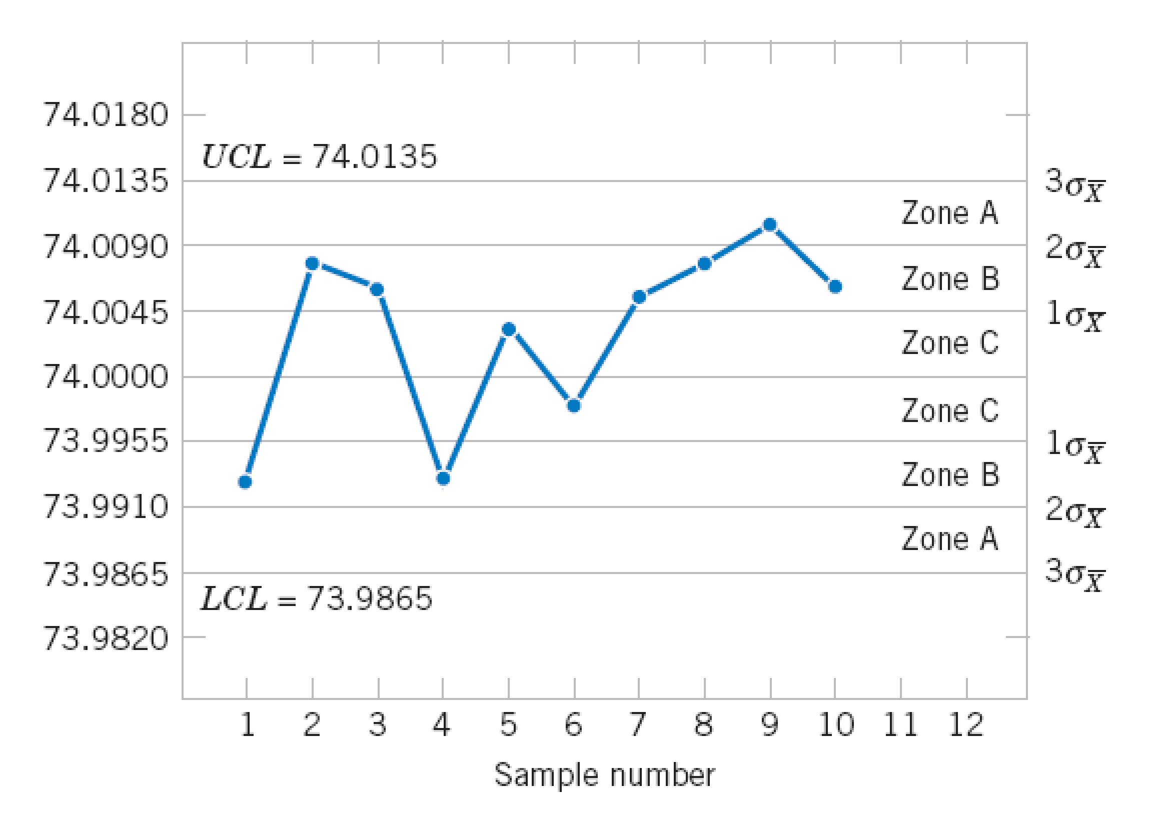

Figure 15-7 The Western Electric zone rules.

$\bar{X}$ and R or S Control Charts

3-sigma control limits:

$$
\begin{aligned}
UCL &= \mu + 3\sigma/\sqrt{n} \\
LCL &= \mu - 3\sigma/\sqrt{n} \\
CL &= \mu
\end{aligned}
\qquad (15\text{-}2)
$$

The grand mean:

$$\hat{\mu} = \overline{\overline{X}} = \frac{1}{m}\sum_{i=1}^{m} \overline{X}_i \qquad (16\text{-}3)$$

X-bar and *R* or *S* Control Charts

The average range:

$$\overline{R} = \frac{1}{m}\sum_{i=1}^{m} R_i \qquad (16\text{-}5)$$

An unbiased estimator of σ:

$$\hat{\sigma} = \frac{\overline{R}}{d_2} \qquad (15\text{-}6)$$

where the constant d_2 is tabulated for various sample sizes in Appendix Table XI.

X-bar and *R* or *S* Control Charts

$\overline{\text{X}}$ Control Chart (from $\overline{R}$):

The center line and upper and lower control limits for an $\overline{X}$ control chart are

$$UCL = \overline{\overline{x}} + A_2\overline{r} \qquad CL = \overline{\overline{x}} \qquad LCL = \overline{\overline{x}} - A_2\overline{r} \tag{15-9}$$

where the constant A_2 is tabulated for various sample sizes in Appendix Table XI.

R Chart:

The center line and upper and lower control limits for an R chart are

$$UCL = D_4\overline{r} \qquad CL = \overline{r} \qquad LCL = D_3\overline{r} \tag{15-12}$$

where $\overline{r}$ is the sample average range, and the constants D_3 and D_4 are tabulated for various sample sizes in Appendix Table XI.

X-bar and *R* or *S* Control Charts

3-sigma control limits for *S*:

$$LCL = c_4\sigma - 3\sigma\sqrt{1 - c_4^2} \qquad CL = c_4\sigma. \qquad UCL = c_4\sigma + 3\sigma\sqrt{1 - c_4^2} \qquad (15\text{-}13)$$

$$\overline{S} = \frac{1}{m}\sum_{i=1}^{m} S_i \qquad (16\text{-}14)$$

An unbiased estimator of σ:

$$\hat{\sigma} = \overline{S}/c_4 \qquad (15\text{-}15)$$

where the constant c_4 is tabulated for various sample sizes in Appendix Table XI.

X-bar and *R* or *S* Control Charts

S Chart:

$$UCL = \bar{s} + 3\frac{\bar{s}}{c_4}\sqrt{1 - c_4^2} \qquad CL = \bar{s} \qquad LCL = \bar{s} - 3\frac{\bar{s}}{c_4}\sqrt{1 - c_4^2} \qquad (15\text{-}16)$$

$\bar{\text{X}}$ Control Chart (from $\bar{S}$):

$$UCL = \bar{\bar{x}} + 3\frac{\bar{s}}{c_4\sqrt{n}} \qquad CL = \bar{\bar{x}} \qquad LCL = \bar{s} - 3\frac{\bar{s}}{c_4\sqrt{n}} \qquad (15\text{-}17)$$

X-bar and *R* or *S* Control Charts

Example 15-1

A component part for a jet aircraft engine is manufactured by an investment casting process. The vane opening on this casting is an important functional parameter of the part. We will illustrate the use of $\overline{X}$ and R control charts to assess the statistical stability of this process. Table 16-1 presents 20 samples of five parts each. The values given in the table have been coded by using the last three digits of the dimension; that is, 31.6 should be 0.50316 inch.

The quantities $\overline{\overline{x}} = 33.3$ and $\overline{r} = 5.8$ are shown at the foot of Table 16-1. The value of A_2 for samples of size 5 is $A_2 = 0.577$. Then the trial control limits for the $\overline{X}$ chart are

$$\overline{x} \pm A_2\overline{r} = 33.32 \pm (0.577)(5.8) = 33.32 \pm 3.35$$

or

$$UCL = 36.67 \qquad LCL = 29.97$$

Table 15-1 Vane-Opening Measurements

Sample Number	x_1	x_2	x_3	x_4	x_5	$\bar{x}$	r	s
1	33	29	31	32	33	31.6	4	1.67332
2	33	31	35	37	31	33.4	6	2.60768
3	35	37	33	34	36	35.0	4	1.58114
4	30	31	33	34	33	32.2	4	1.64317
5	33	34	35	33	34	33.8	2	0.83666
6	38	37	39	40	38	38.4	3	1.14018
7	30	31	32	34	31	31.6	4	1.51658
8	29	39	38	39	39	36.8	10	4.38178
9	28	33	35	36	43	35.0	15	5.43139
10	38	33	32	35	32	34.0	6	2.54951
11	28	30	28	32	31	29.8	4	1.78885
12	31	35	35	35	34	34.0	4	1.73205
13	27	32	34	35	37	33.0	10	3.80789
14	33	33	35	37	36	34.8	4	1.78885
15	35	37	32	35	39	35.6	7	2.60768
16	33	33	27	31	30	30.8	6	2.48998
17	35	34	34	30	32	33.0	5	2.00000
18	32	33	30	30	33	31.6	3	1.51658
19	25	27	34	27	28	28.2	9	3.42053
20	35	35	36	33	30	33.8	6	2.38747
						$\bar{\bar{x}} = 33.32$	$\bar{r} = 5.8$	$\bar{s} = 2.345$

X-bar and *R* or *S* Control Charts

Example 15-1

For the R chart, the trial control limits are

$$UCL = D_4\bar{r} = (2.115)(5.8) = 12.27$$
$$LCL = D_3\bar{r} = (0)(5.8) = 0$$

The $\overline{X}$ and R control charts with these trial control limits are shown in Fig. 16-8. Notice that samples 6, 8, 11, and 19 are out of control on the $\overline{X}$ chart and that sample 9 is out of control on the R chart. (These points are labeled with a "1" because they violate the first Western Electric rule.)

X-bar and *R* or *S* Control Charts

Example 15-1

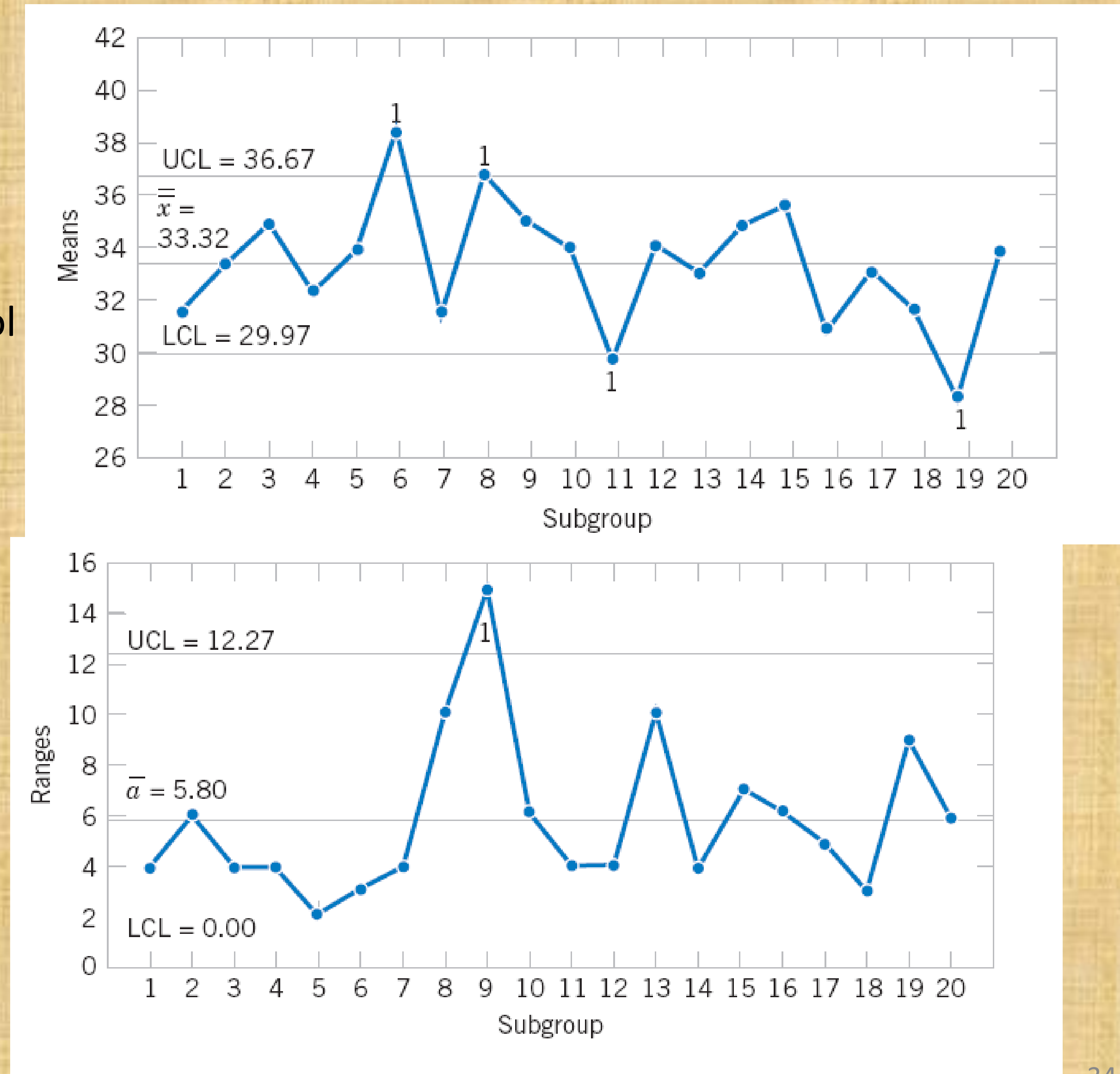

Figure 15-8 $\overline{X}$ and R control charts for vane opening.

X-bar and *R* or *S* Control Charts

Example 15-1

For the S chart, the value of $c_4 = 0.94$.

Therefore,

$$\frac{3\bar{s}}{c_4}\sqrt{1 - c_4^2} = \frac{3(2.345)}{0.94}\sqrt{1 - 0.94^2} = 2.553$$

and the trial control limits are

$$UCL = 2.345 + 2.553 = 4.898$$
$$LCL = 2.345 - 2.553 = -0.208$$

The LCL is set to zero. If $\bar{s}$ is used to determine the control limits for the $\overline{X}$ chart,

$$\bar{\bar{x}} \pm \frac{3\bar{s}}{c_4\sqrt{n}} = 33.32 \pm \frac{3(2.345)}{0.94} = 33.32 \pm 3.35$$

X-bar and *R* or *S* Control Charts

Example 15-1

and this result is nearly the same as from $\bar{r}$. The S chart is shown in Fig. 16-9. Because the control limits for the $\overline{X}$ chart calculated from $\bar{s}$ are nearly the same as from $\bar{r}$, the chart is not shown.

Suppose that all of these assignable causes can be traced to a defective tool in the wax-molding area. We should discard these five samples and recompute the limits for the $\overline{X}$ and R charts. These new revised limits are, for the $\overline{X}$ chart,

$$UCL = \bar{\bar{x}} + A_2\bar{r} = 33.21 + (0.577)(5.0) = 36.10$$

$$LCL = \bar{\bar{x}} - A_2\bar{r} = 33.21 - (0.577)(5.0) = 30.33$$

and for the R chart,

$$UCL = D_4\bar{r} = (2.115)(5.0) = 10.57$$

$$LCL = D_3\bar{r} = (0)(5.0) = 0$$

X-bar and *R* or *S* Control Charts

Example 15-1

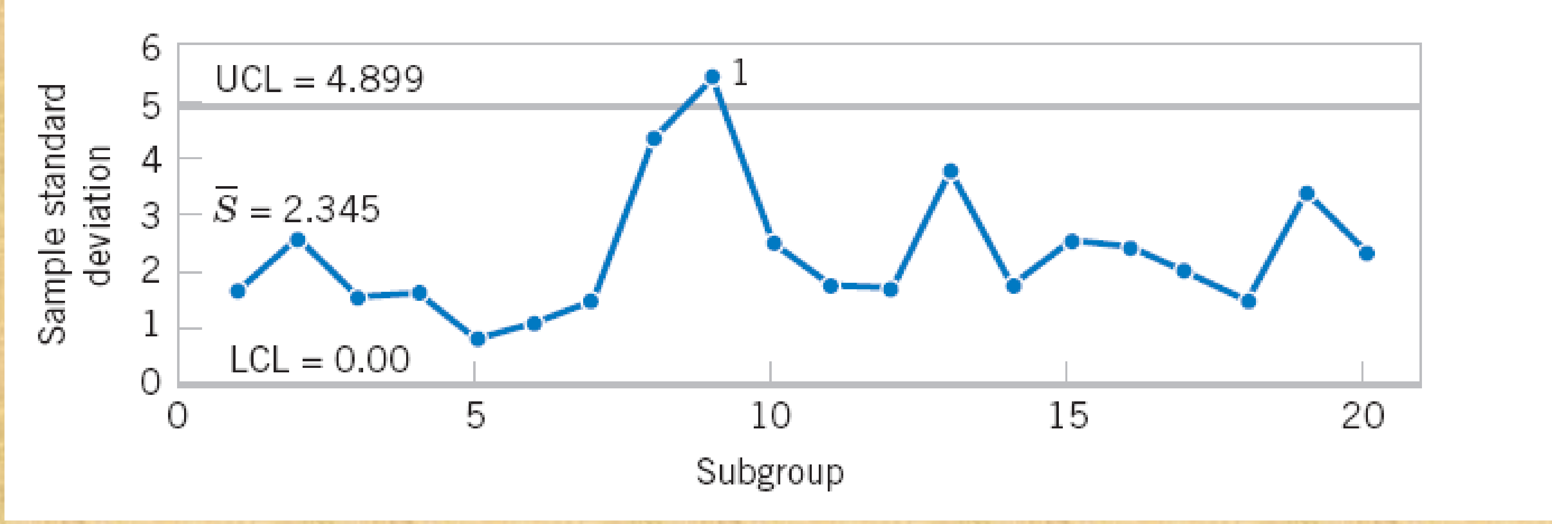

Figure 15-9 The *S* control chart for vane opening.

X-bar and *R* or *S* Control Charts

Example 15-1

The revised control charts are shown in Fig. 16-10. Notice that we have treated the first 20 preliminary samples as **estimation data** with which to establish control limits. These limits can now be used to judge the statistical control of future production. As each new sample becomes available, the values of $\overline{x}$ and r should be computed and plotted on the control charts. It may be desirable to revise the limits periodically, even if the process remains stable. The limits should always be revised when process improvements are made.

X-bar and *R* or *S* Control Charts

Example 15-1

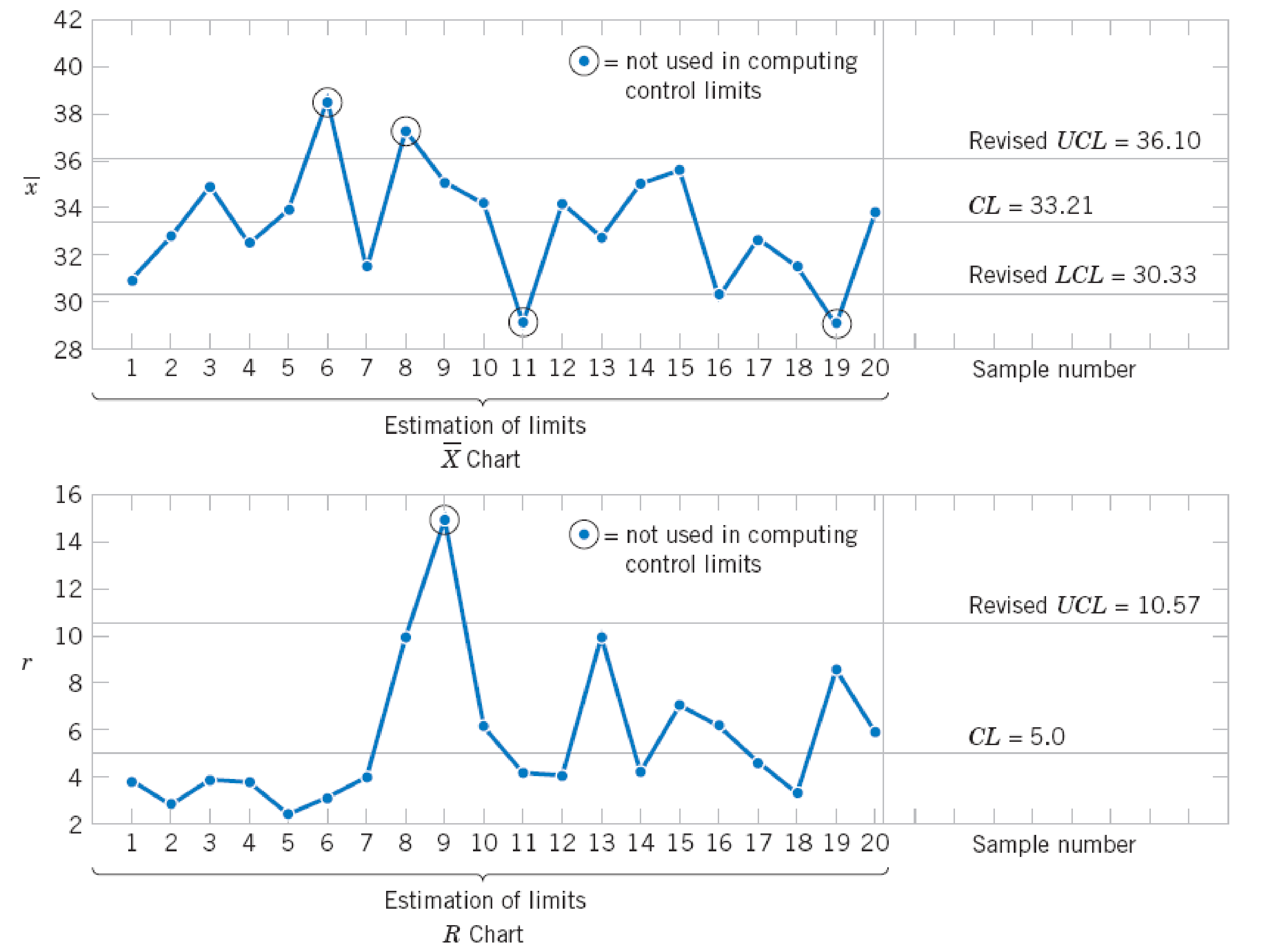

Figure 15-10
The *X-bar*
and *R* control charts
for vane opening.

Control Charts for Individual Measurements

- What if you could not get a sample size greater than 1 ($n = 1$)? Examples include
  - Automated inspection and measurement technology is used, and every unit manufactured is analyzed.
  - The production rate is very slow, and it is inconvenient to allow samples sizes of $N > 1$ to accumulate before analysis
  - Repeat measurements on the process differ only because of laboratory or analysis error, as in many chemical processes.
- The individual control charts are useful for samples of sizes $n = 1$.

Control Charts for Individual Measurements

- The ***moving range*** (MR) is defined as the absolute difference between two successive observations:

$$MR_i = |x_i - x_{i-1}|$$

which will indicate possible shifts or changes in the process from one observation to the next.

Control Charts for Individual Measurements

Individuals Control Chart

The center line and upper and lower control limits for a control chart for individuals are

$$
\begin{aligned}
UCL &= \overline{x} + 3\frac{\overline{mr}}{d_2} = \overline{x} + 3\frac{\overline{mr}}{1.128} \\
CL &= \overline{x} \\
LCL &= \overline{x} - 3\frac{\overline{mr}}{d_2} = \overline{x} - 3\frac{\overline{mr}}{1.128}
\end{aligned}
\tag{15-19}
$$

and for a control chart for moving ranges

$$
\begin{aligned}
UCL &= D_4\overline{mr} = 3.267\overline{mr} \\
CL &= \overline{mr} \\
LCL &= D_3\overline{mr} = 0
\end{aligned}
$$

Control Charts for Individual Measurements

Interpretation of the Charts

- X Charts can be interpreted similar to *X-bar* charts. MR charts cannot be interpreted the same as *X-bar* or R charts.
- Since the MR chart plots data that are "correlated" with one another, then looking for patterns on the chart does not make sense.
- MR chart cannot really supply useful information about process variability.
- More emphasis should be placed on interpretation of the X chart.

Process Capability

- **Process capability** refers to the performance of the process when it is operating in control.
- Two graphical tools are helpful in assessing process capability:
  - **Tolerance chart** (or tier chart)
  - **Histogram**

Process Capability

Tolerance Chart

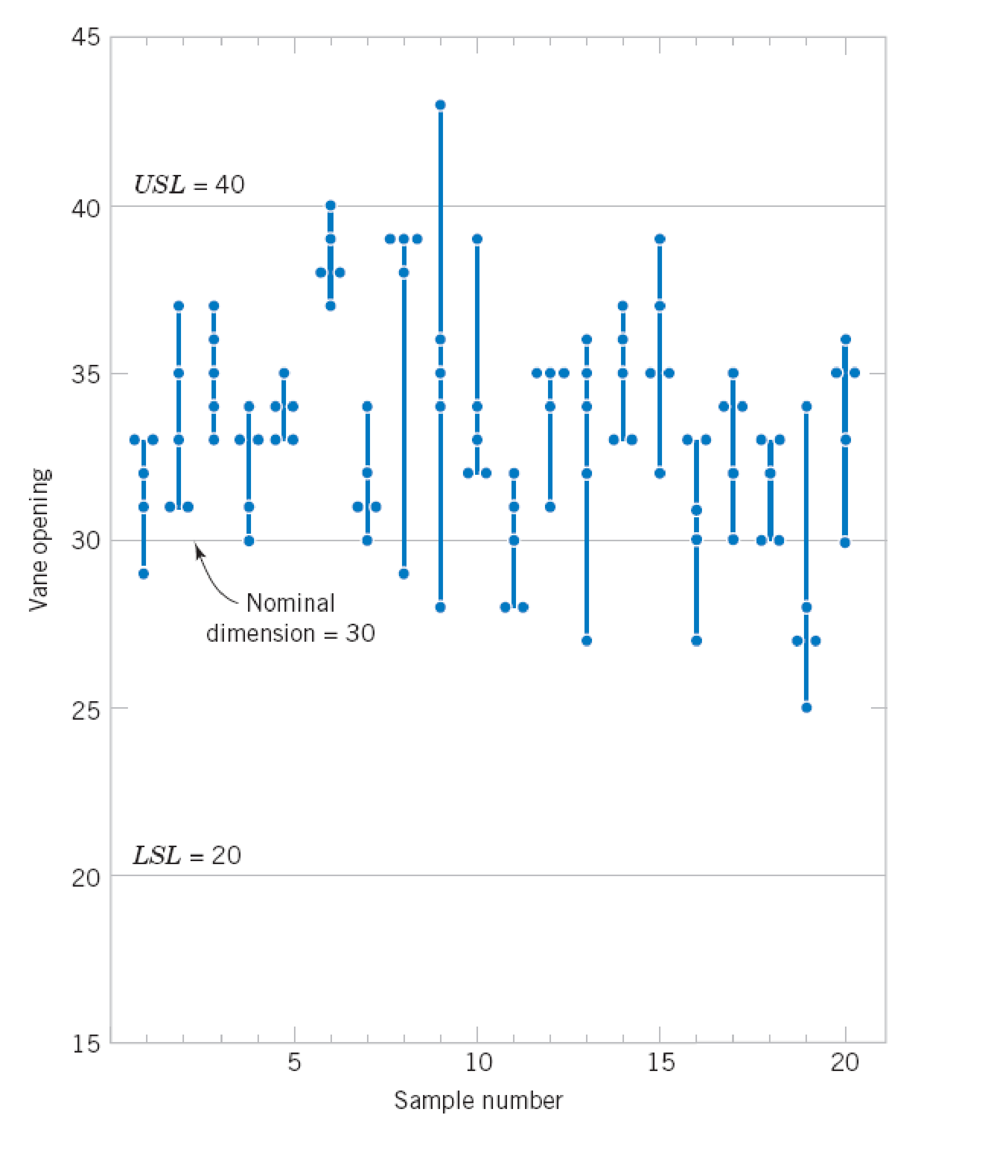

Figure 16-12 Tolerance diagram of vane openings.

Process Capability

Histogram

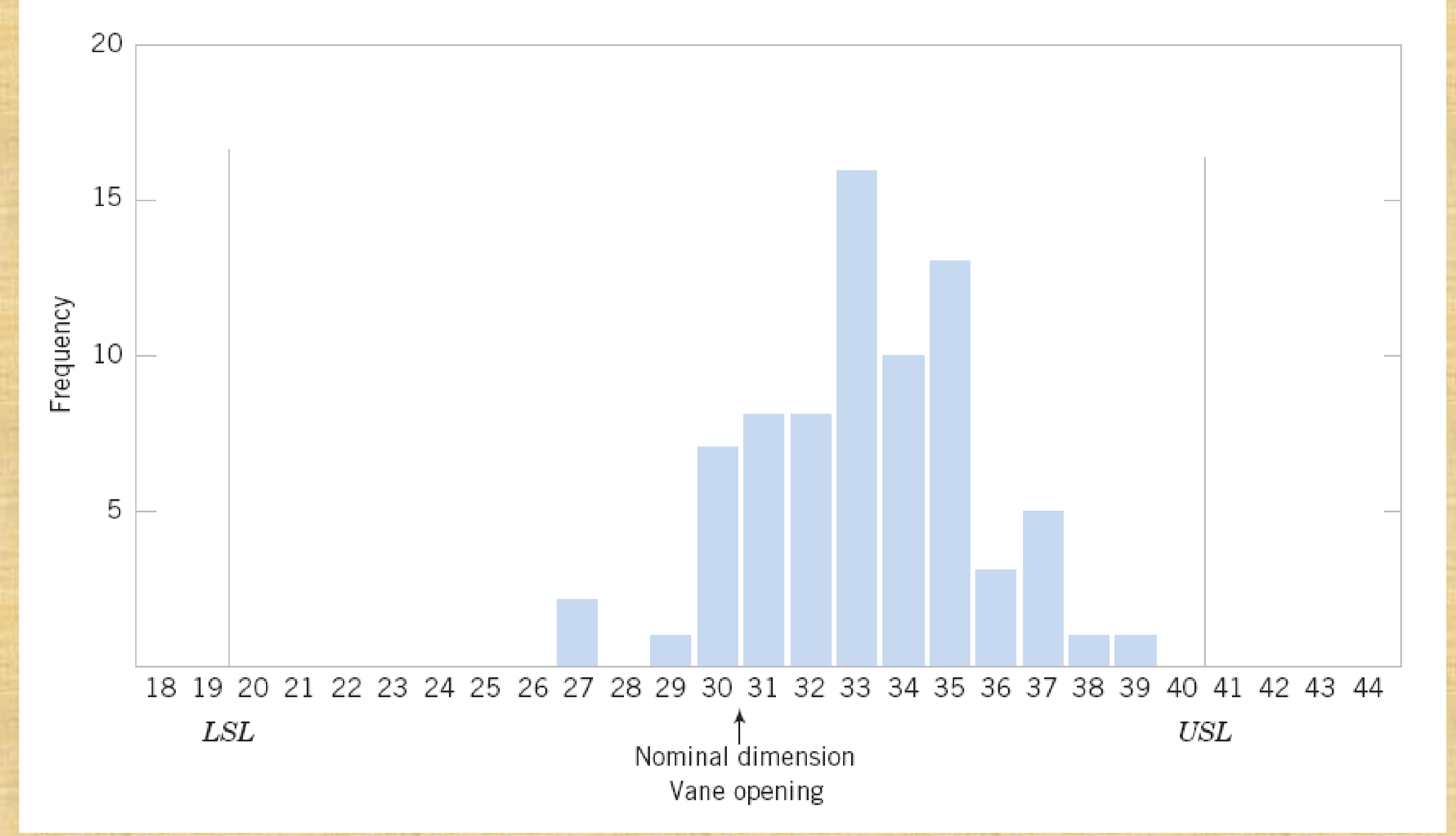

Figure 15-13 Histogram for vane openings.

Process Capability

Process Capability Ratio

The **process capability ratio** (*PCR*) is

$$PCR = \frac{USL - LSL}{6\sigma} \qquad (15\text{-}20)$$

PCR$_k$

$$PCR_k = \min\left[\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right] \qquad (15\text{-}21)$$

Process Capability

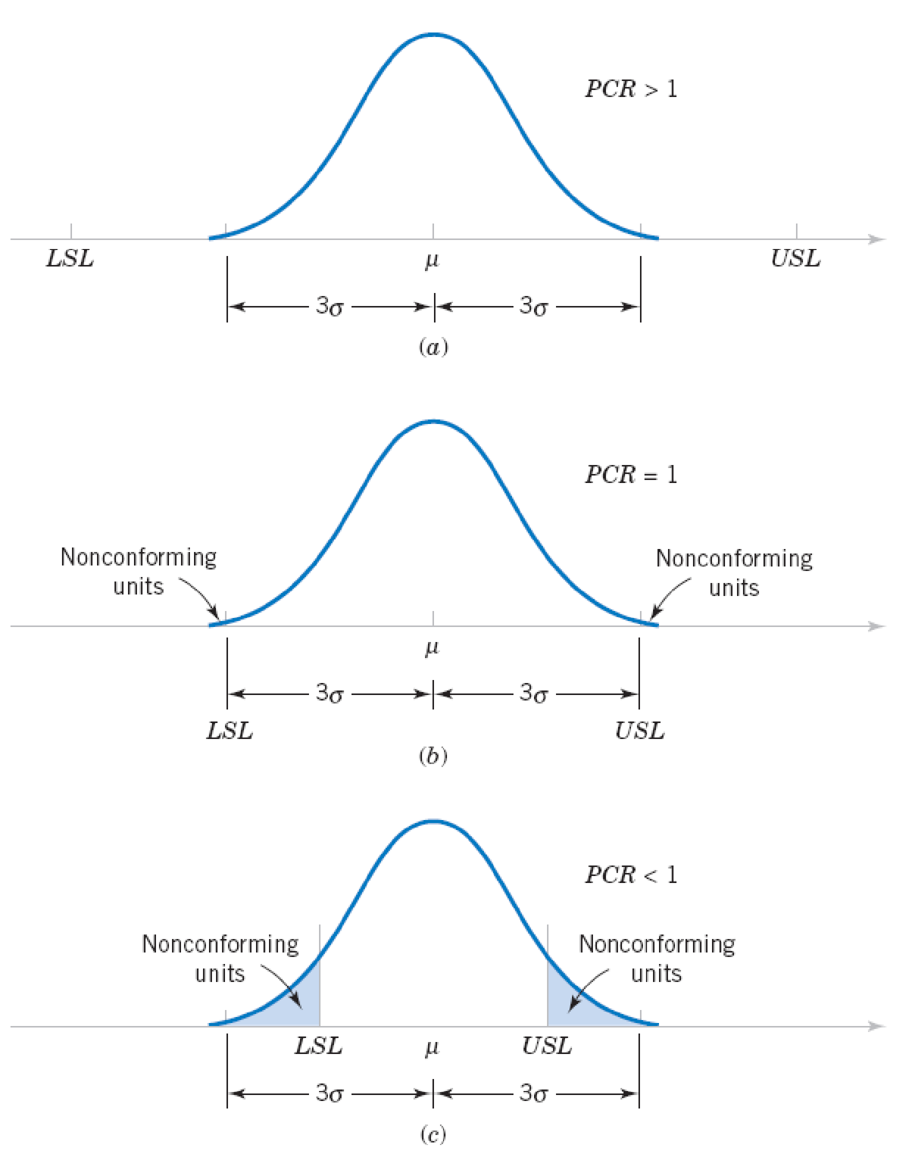

Figure 15-14 Process Fallout and the process capability ratio (PCR).

Process Capability

Example 15-3

EXAMPLE 15-3 Electrical Current

For an electronic manufacturing process a current has specifications of 100 ± 10 milliamperes. The process mean μ and standard deviation σ are 107.0 and 1.5, respectively. The process mean is nearer to the *USL*. Consequently,

$$PCR = \frac{110 - 90}{6 \cdot 5} = 2.22$$

and

$$PCR_k = \frac{110 - 107}{3 \cdot 15} = 0.67$$

The small PCR_k indicates that the process is likely to produce currents outside of the specification limits. From the normal distribution in Appendix Table II,

$$\begin{aligned} P(X < LSL) &= P(Z < (90 - 107)/1.5) \\ &= P(Z < -11.33) \cong 0 \\ P(X > USL) &= P(Z > (110 - 107)/1.5) \\ &= P(Z > 2) = 0.023 \end{aligned}$$

Process Capability

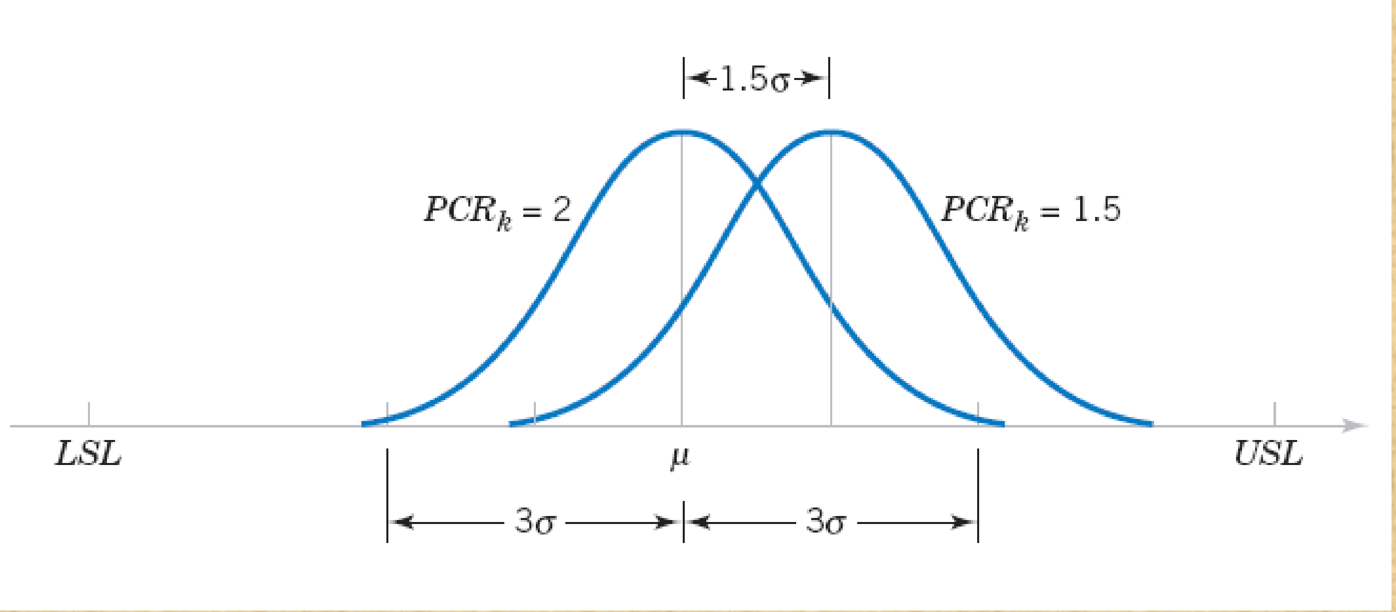

Figure 15-15 Mean of a six-sigma process shifts by 1.5 standard deviations.

Attribute Control Charts

15-6.1 *P* Chart (Control Chart for Proportions)

The center line and upper and lower control limits for the P chart are

$$UCL = \overline{p} + 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} \quad CL = \overline{p} \quad LCL = \overline{p} - 3\sqrt{\frac{\overline{p}(1-\overline{p})}{n}} \qquad (15\text{-}24)$$

where $\overline{p}$ is the observed value of the average fraction defective.

Attribute Control Charts

Example 15-4

EXAMPLE 15-4 Ceramic Substrate

Suppose we wish to construct a fraction-defective control chart for a ceramic substrate production line. We have 20 preliminary samples, each of size 100; the number of defectives in each sample is shown in Table 15-4. Assume that the samples are numbered in the sequence of production. Note that $\overline{p} = (800/2000) = 0.40$; therefore, the trial parameters for the control chart are

$$UCL = 0.40 + 3\sqrt{\frac{(0.40)(0.60)}{100}} = 0.55 \qquad CL = 0.40$$

$$LCL = 0.40 - 3\sqrt{\frac{(0.40)(0.60)}{100}} = 0.25$$

The control chart is shown in Fig. 15-16. All samples are in control. If they were not, we would search for assignable causes of variation and revise the limits accordingly. This chart can be used for controlling future production.

Attribute Control Charts

Example 15-4

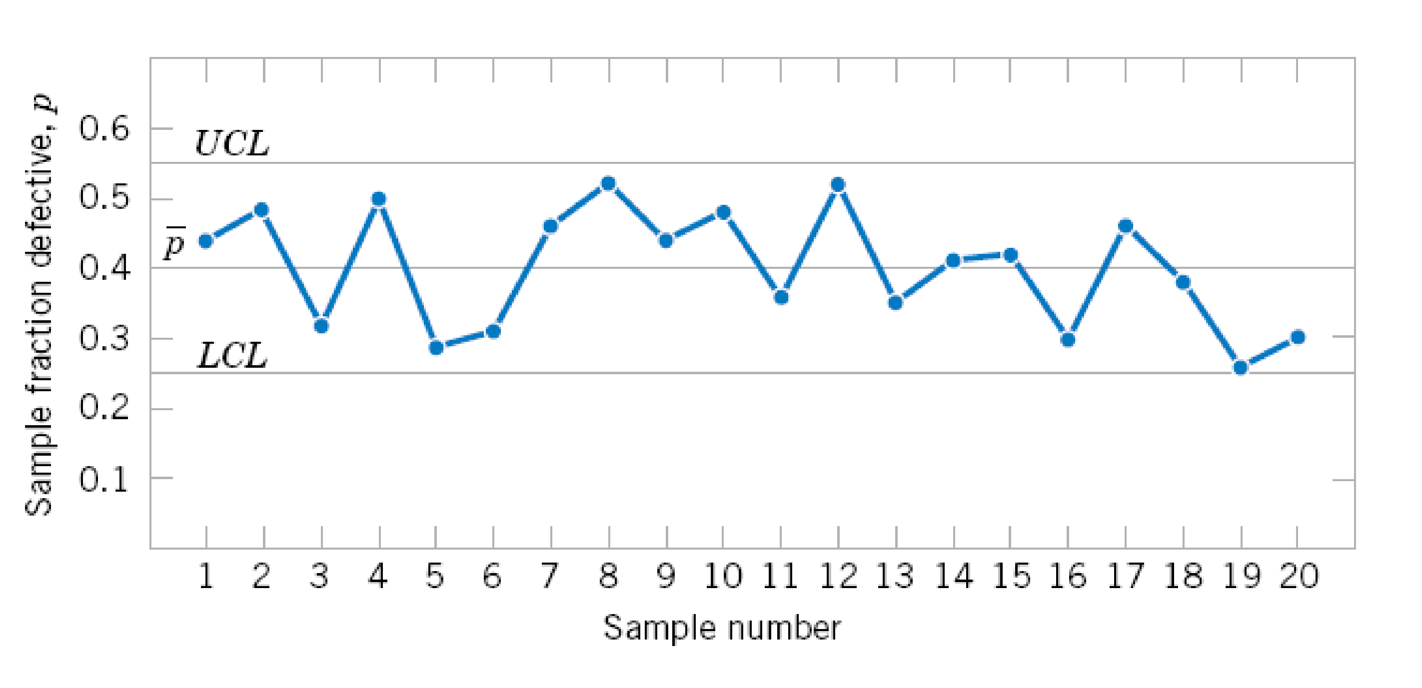

Figure 15-16 *P* chart for a ceramic substrate.

Attribute Control Charts

Example 15-4

Practical Interpretation: Although this process exhibits statistical control, its defective rate ($\overline{p} = 0.40$) is very poor. We should take appropriate steps to investigate the process to determine why such a large number of defective units is being produced. Defective units should be analyzed to determine the specific types of defects present. Once the defect types are known, process changes should be investigated to determine their impact on defect levels. Designed experiments may be useful in this regard.

Attribute Control Charts

15-6.2 *U* Chart (Control Chart for Defects per Unit)

The center line and upper and lower control limits on the U chart are

$$UCL = \overline{u} + 3\sqrt{\frac{\overline{u}}{n}} \qquad CL = \overline{u} \qquad LCL = \overline{u} - 3\sqrt{\frac{\overline{u}}{n}} \qquad (15\text{-}27)$$

where $\overline{u}$ is the average number of defects per unit.

Attribute Control Charts

Example 15-5

EXAMPLE 15-5 Printed Circuit Boards

Printed circuit boards are assembled by a combination of manual assembly and automation. Surface Mount Technology (SMT) is used to make the mechanical and electrical connections of the components to the board. Every hour, five boards are selected and inspected for process-control purposes. The number of defects in each sample of five boards is noted. Results for 20 samples are shown in Table 15-5.

Table 15-5 Number of Defects in Samples of Five Printed Circuit Boards

Sample	Number of Defects	Defects per Unit u_i	Sample	Number of Defects	Defects per Unit u_i
1	6	1.2	11	9	1.8
2	4	0.8	12	15	3.0
3	8	1.6	13	8	1.6
4	10	2.0	14	10	2.0
5	9	1.8	15	8	1.6
6	12	2.4	16	2	0.4
7	16	3.2	17	7	1.4
8	2	0.4	18	1	0.2
9	3	0.6	19	7	1.4
10	10	2.0	20	13	2.6

Attribute Control Charts

Example 15-5

The center line for the U chart is

$$\overline{u} = \frac{1}{20}\sum_{i=1}^{20} u_i = \frac{32.0}{20} = 1.6$$

and the upper and lower control limits are

$$UCL = \overline{u} + 3\sqrt{\frac{\overline{u}}{n}} = 1.6 + 3\sqrt{\frac{1.6}{5}} = 3.3$$

$$LCL = \overline{u} - 3\sqrt{\frac{\overline{u}}{n}} = 1.6 - 3\sqrt{\frac{1.6}{5}} < 0$$

The control chart is plotted in Fig. 15-17. Because LCL is negative, it is set to 0. From the control chart in Fig. 15-17, we see that the process is in control.

Practical Interpretation: Eight defects per group of five circuit boards are too many (about $8/5 = 1.6$ defects/board), and the process needs improvement. An investigation needs to be made of the specific types of defects found on the printed circuit boards. This will usually suggest potential avenues for process improvement.

Attribute Control Charts

Example 15-5

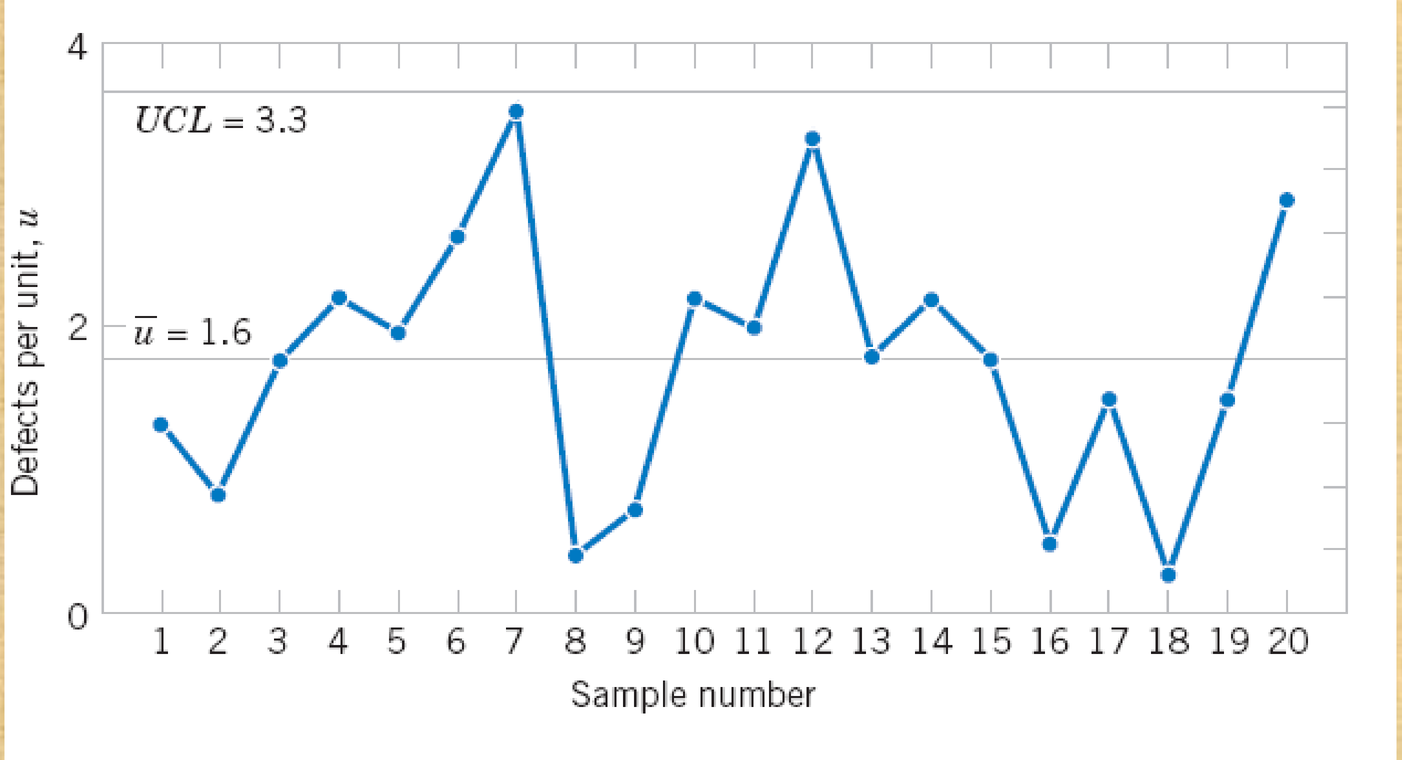

Figure 15-17 *U* chart of defects per unit on printed circuit boards.

Control Chart Performance

Average Run Length

- The average run length (ARL) is a very important way of determining the appropriate sample size and sampling frequency.
- Let p = probability that any point exceeds the control limits. Then,

$$\text{ARL} = \frac{1}{p} \qquad (15\text{-}28)$$

Control Chart Performance

Table 15-6 Average Run Length (ARL) for an $\overline{X}$ Chart with 3-Sigma Control Limits

Magnitude of Process Shift	ARL $n = 1$	ARL $n = 4$
0	370.4	370.4
0.5σ	155.2	43.9
1.0σ	43.9	6.3
1.5σ	15.0	2.0
2.0σ	6.3	1.2
3.0σ	2.0	1.0

Control Chart Performance

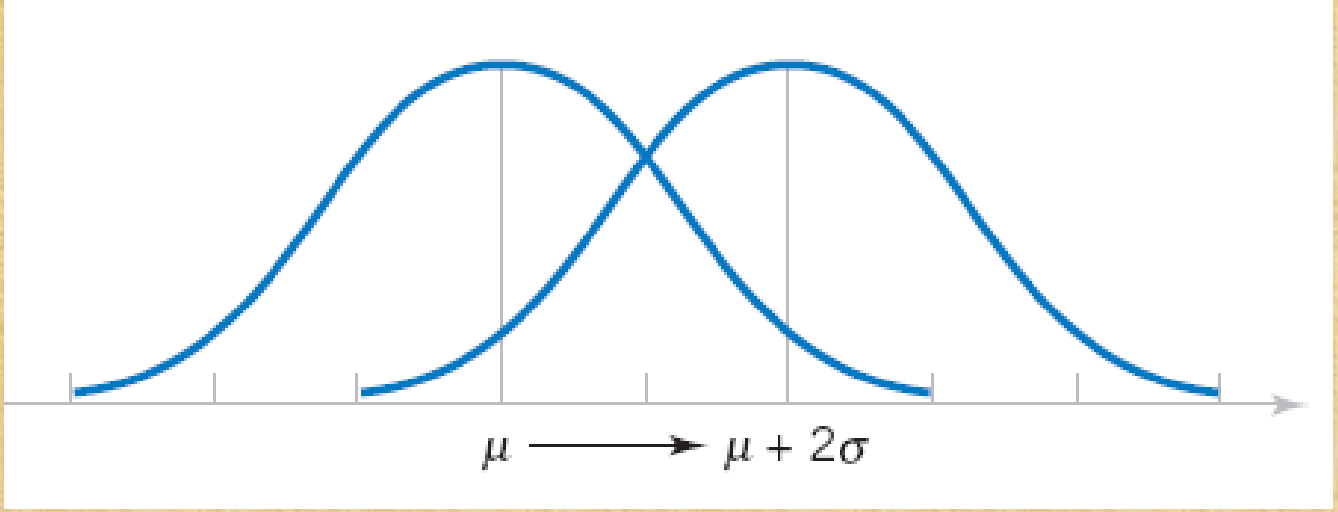

Figure 15-18 Process mean shift of 2σ.

Time-Weighted Charts

15-8.1 Cumulative Sum Control Chart

- The cusum chart incorporates all information in the sequence of sample values by plotting the *cumulative sums* of the deviations of the sample values from a target value.
- If μ_0 is the target for the process mean, $\overline{X}_j$ is the average of the jth sample, then the cumulative sum control chart is formed by plotting the quantity

$$S_i = \sum_{j=1}^{i} (\overline{X}_j - \mu_0)$$

Table 15-7 CUSUM Computations for the Chemical Process Concentration Data in Table 15-3

Observation, i	x_i	$x_i - 99$	$s_i = (x_i - 99) + s_{i-1}$
1	102.0	3.0	3.0
2	94.8	−4.2	−1.2
3	98.3	−0.7	−1.9
4	98.4	−0.6	−2.5
5	102.0	3.0	0.5
6	98.5	−0.5	0.0
7	99.0	0.0	0.0
8	97.7	−1.3	−1.3
9	100.0	1.0	−0.3
10	98.1	−0.9	−1.2
11	101.3	2.3	1.1
12	98.7	−0.3	0.8
13	101.1	2.1	2.9
14	98.4	−0.6	2.3
15	97.0	−2.0	0.3
16	96.7	−2.3	−2.0
17	100.3	1.3	−0.7
18	101.4	2.4	1.7
19	97.2	−1.8	−0.1
20	101.0	2.0	1.9

Time-Weighted Charts

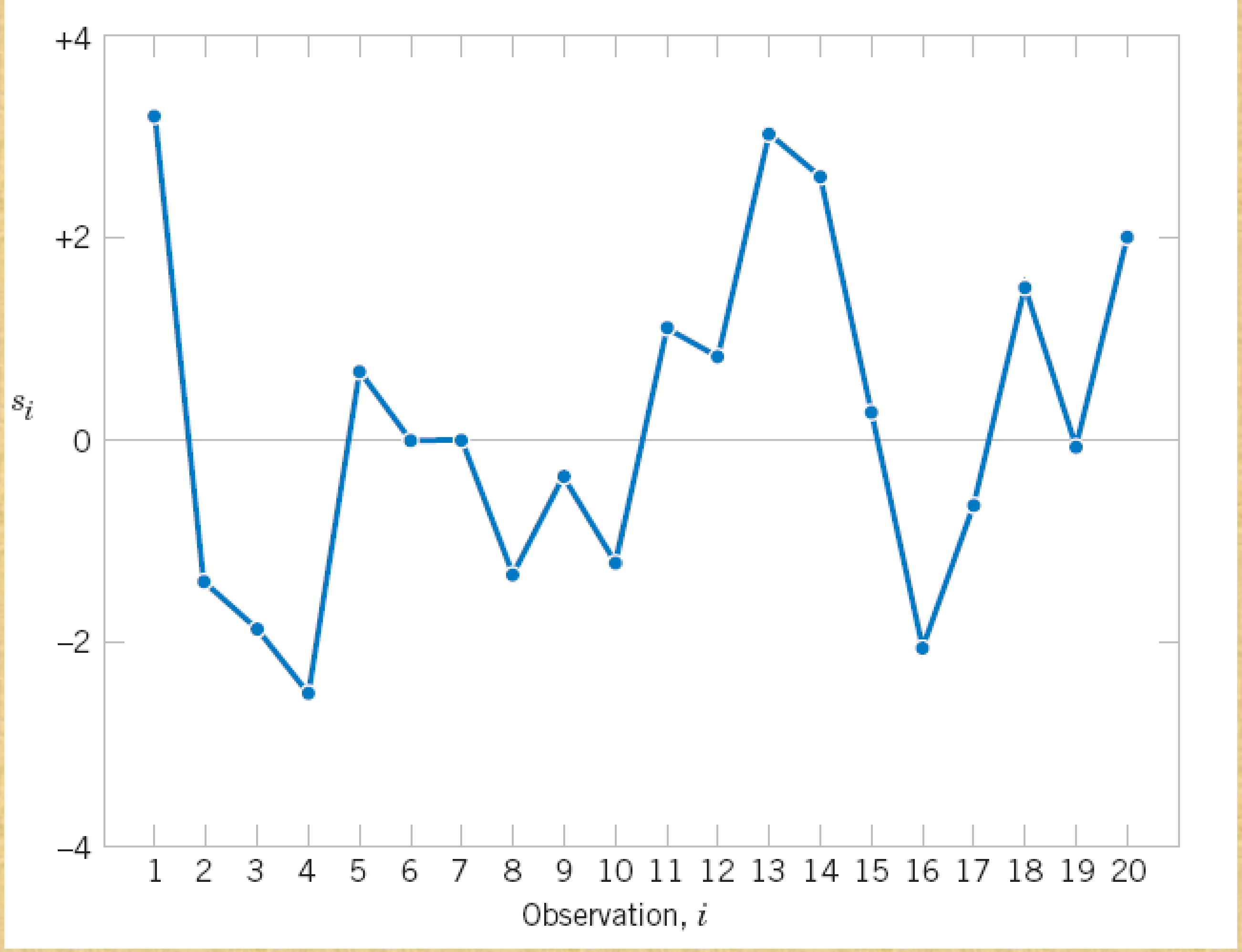

Figure 15-19 Plot of the cumulative sum for the concentration data, Table 15-7.

Time-Weighted Charts

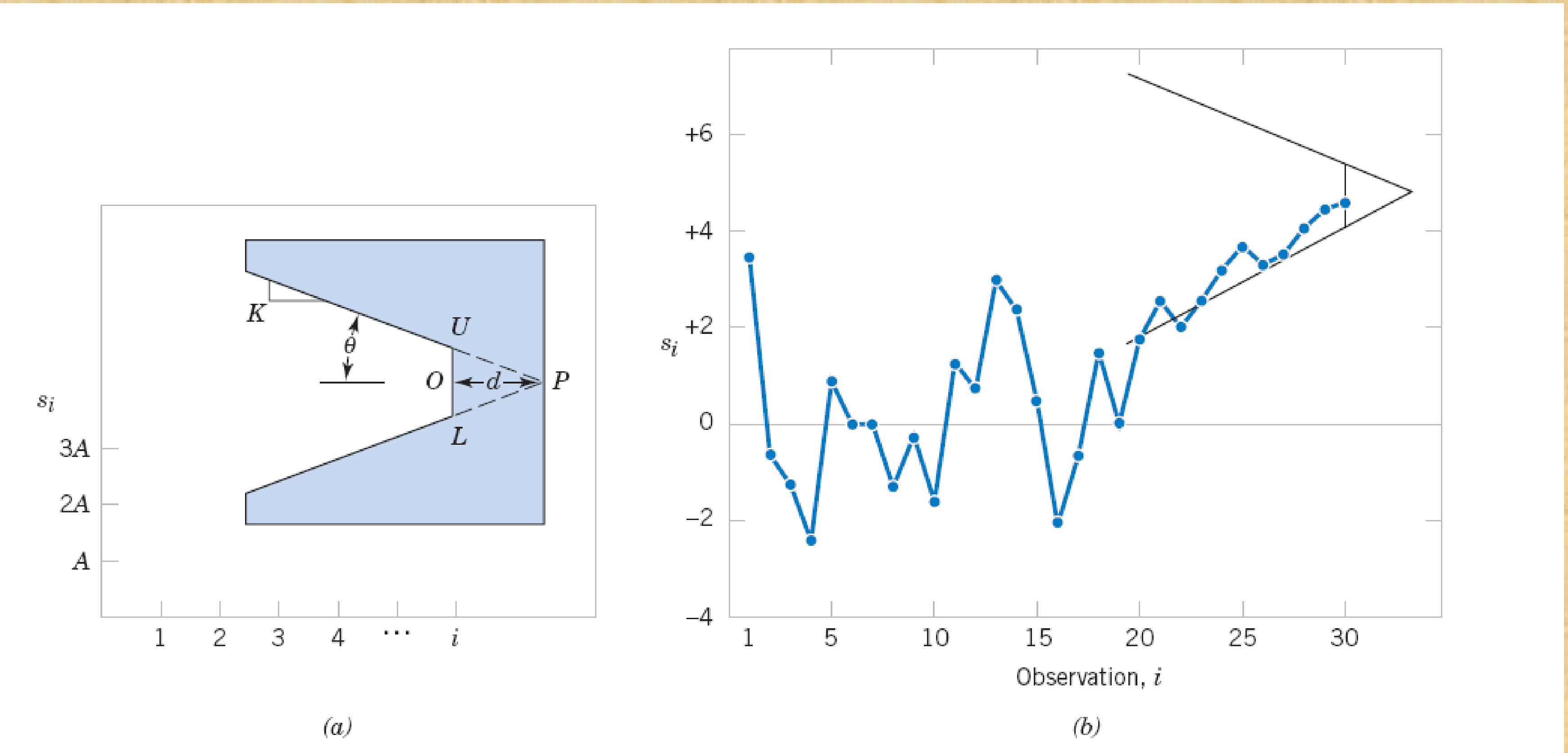

Figure 15-20 The cumulative sum control chart. (a) The V-mask and scaling. (b) The cumulative sum control chart in operation.

15-8.1 Cumulative Sum Control Chart

CUSUM Control Chart

$$s_H(i) = \max[0, \overline{x}_i - (\mu_0 + K) + s_H(i-1)] \tag{15-30}$$

and

$$s_L(i) = \max[0, (\mu_0 - K) - \overline{x}_i + s_L(i-1)] \tag{15-31}$$

where the starting values $s_H(0) = s_L(0) = 0$.

Time-Weighted Charts

Example 15-6

EXAMPLE 15-6 Chemical Process Concentration CUSUM

A Tabular CUSUM

We will illustrate the tabular CUSUM by applying it to the chemical process concentration data in Table 15-7. The process target is $\mu_0 = 99$, and we will use $K = 1$ as the reference value and $H = 10$ as the decision interval. The reasons for these choices will be explained later.

Table 15-8 shows the tabular CUSUM scheme for the chemical process concentration data. To illustrate the calculations, note that

$$\begin{aligned} s_H(i) &= \max[0, x_i - (\mu_0 + K) + s_H(i-1)] \\ &= \max[0, x_i - (99 + 1) + s_H(i-1)] \\ &= \max[0, x_i - 100 + s_H(i-1)] \end{aligned}$$

$$\begin{aligned} s_L(i) &= \max[0, (\mu_0 - K) - x_i + s_L(i-1)] \\ &= \max[0, (99 - 1) - x_i + s_L(i-1)] \\ &= \max[0, 98 - x_i + s_L(i-1)] \end{aligned}$$

Table 15-8 The Tabular CUSUM for the Chemical Process Concentration Data

Observation	Upper CUSUM				Lower CUSUM		
i	x_i	$x_i - 100$	$s_H(i)$	n_H	$98 - x_i$	$s_L(i)$	n_L
1	102.0	2.0	2.0	1	−4.0	0.0	0
2	94.8	−5.2	0.0	0	3.2	3.2	1
3	98.3	−1.7	0.0	0	−0.3	2.9	2
4	98.4	−1.6	0.0	0	−0.4	2.5	3
5	102.0	2.0	2.0	1	−4.0	0.0	0
6	98.5	−1.5	0.5	2	−0.5	0.0	0
7	99.0	−1.0	0.0	0	−1.0	0.0	0
8	97.7	−2.3	0.0	0	0.3	0.3	1
9	100.0	0.0	0.0	0	−2.0	0.0	0
10	98.1	−1.9	0.0	0	−0.1	0.0	0
11	101.3	1.3	1.3	1	−3.3	0.0	0
12	98.7	−1.3	0.0	0	−0.7	0.0	0
13	101.1	1.1	1.1	1	−3.1	0.0	0
14	98.4	−1.6	0.0	0	−0.4	0.0	0
15	97.0	−3.0	0.0	0	1.0	1.0	1
16	96.7	−3.3	0.0	0	1.3	2.3	2
17	100.3	0.3	0.3	1	−2.3	0.0	0
18	101.4	1.4	1.7	2	−3.4	0.0	0
19	97.2	−2.8	0.0	0	0.8	0.8	1
20	101.0	1.0	1.0	0	−3.0	0.0	0

Time-Weighted Charts

Example 15-6

Therefore, for observation 1 the CUSUMs are

$$\begin{aligned} s_H(1) &= \max[0, x_1 - 100 + s_H(0)] \\ &= \max[0, 102.0 - 100 + 0] = 2.0 \end{aligned}$$

and

$$\begin{aligned} s_L(1) &= \max[0, 98 - x_1 + s_L(0)] \\ &= \max[0, 98 - 102.0 + 0] = 0 \end{aligned}$$

as shown in Table 15-8. The quantities n_H and n_L in Table 15-8 indicate the number of periods that the CUSUM $s_H(i)$ or $s_L(i)$ have been nonzero. Notice that the CUSUMs in this example never exceed the decision interval $H = 10$. We would therefore conclude that the process is in control.

Time-Weighted Charts

Example 15-6

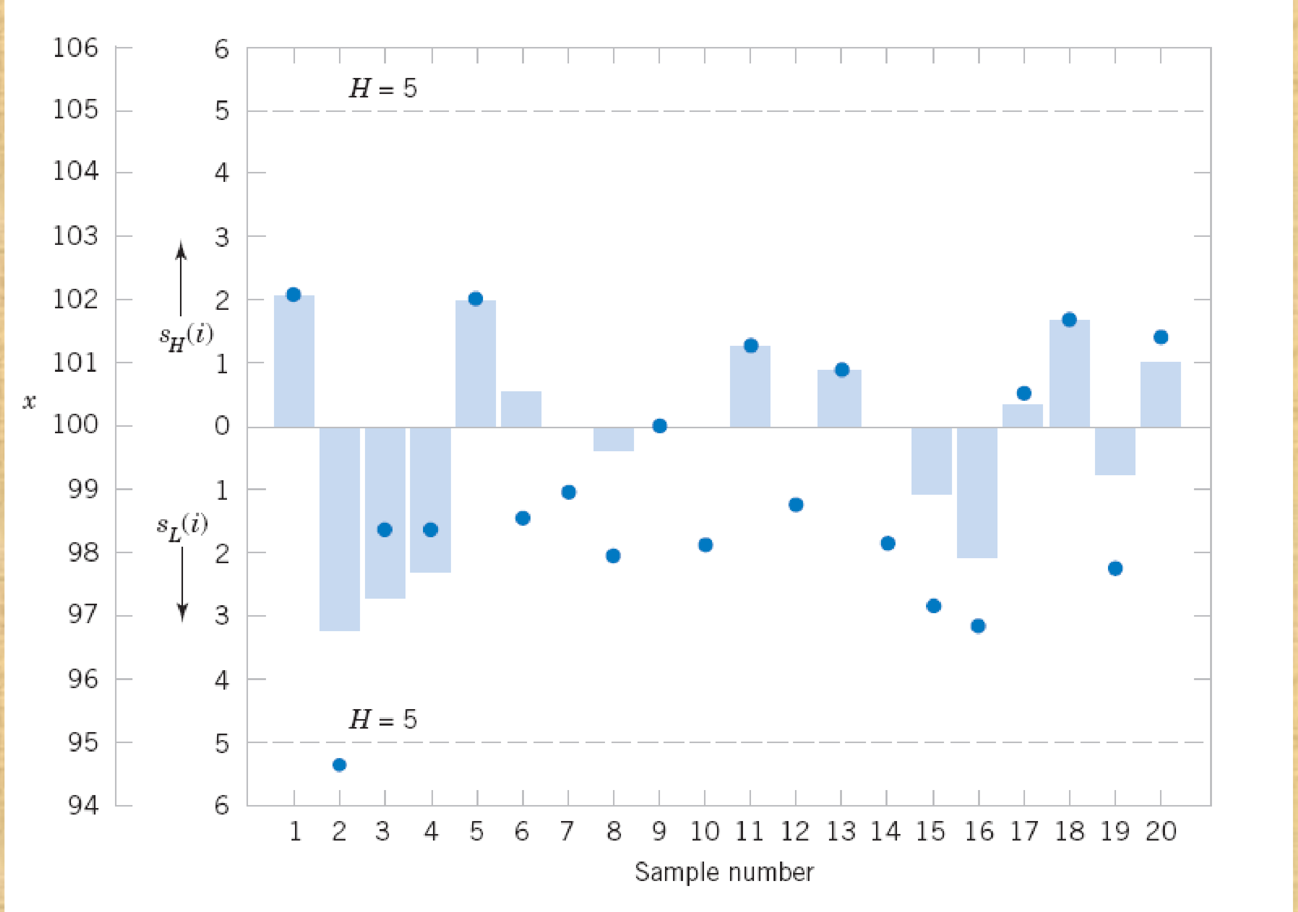

Figure 15-21 The CUSUM status chart for Example 15-6.

Time-Weighted Charts

Table 15-9 Average Run Lengths for a CUSUM Control Chart with $k = 1/2$

Shift in Mean (multiple of $\sigma_{\overline{X}}$)	$h = 4$	$h = 5$
0	168	465
0.25	74.2	139
0.50	26.6	38.0
0.75	13.3	17.0
1.00	8.38	10.4
1.50	4.75	5.75
2.00	3.34	4.01
2.50	2.62	3.11
3.00	2.19	2.57
4.00	1.71	2.01

Time-Weighted Charts

15-8.2 Exponential Weighted Moving Average Control Chart

Data collected in time order is often averaged over several time periods. For example, economic data is often presented as an average over the last four quarters. That is, at time t the average of the last four measurements can be written as

$$\bar{x}_t(4) = \frac{1}{4}x_t + \frac{1}{4}x_{t-1} + \frac{1}{4}x_{t-2} + \frac{1}{4}x_{t-3}$$

This average places weight of 1/4 on each of the most recent observations, and zero weight on older observations. It is called a **moving average** and in this case a *window* of size 4 is used.

Time-Weighted Charts

15-8.2 Exponential Weighted Moving Average Control Chart

For statistical process control, rather than use a fixed window size it is useful to place the greatest weight on the most recent observation or subgroup average, and then gradually decrease the weight on older observations. One average of this type can be constructed by a multiplicative decrease in the weights. Let $\lambda \leq 1$ denote a constant and μ_0 denote the process target or historical mean. Suppose that samples of size $n \geq 1$ are collected and $\overline{x}_t$ is the average of the sample at time t. The **exponentially weighted moving average (EWMA)** is

$$\begin{aligned} z_t &= \lambda \overline{x}_t + \lambda(1-\lambda)\overline{x}_{t-1} + \lambda(1-\lambda)^2\overline{x}_{t-2} + \cdots + \lambda(1-\lambda)^{t-1}\overline{x}_1 + (1-\lambda)^t\mu_0 \\ &= \sum_{k=0}^{t} \lambda(1-\lambda)^k \overline{x}_{t-k} + (1-\lambda)^t \mu_0 \end{aligned}$$

Each older observation has its weight decreased by the factor $(1-\lambda)$. The weight on the starting value μ_0 is selected so that the weights sum to one. Here z_t is also sometimes called a geometric average.

Time-Weighted Charts

15-8.2 Exponential Weighted Moving Average Control Chart

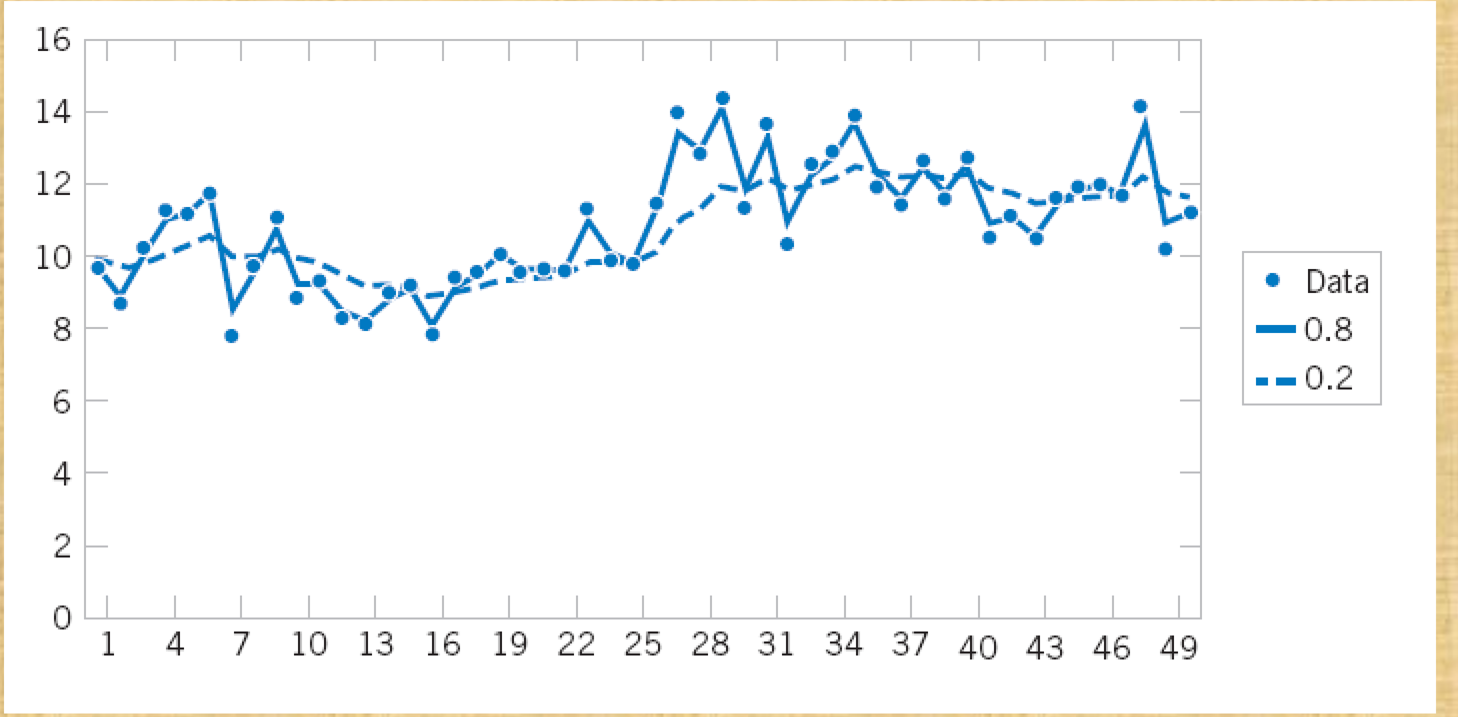

Figure 15-22 EWMAs with λ=0.8 and λ=0.2 show a compromise between a smooth curve and a response to a shift

Time-Weighted Charts

15-8.2 Exponential Weighted Moving Average Control Chart

EWMA Control Chart

$$LCL = \mu_0 - 3\frac{\sigma}{\sqrt{n}}\sqrt{\frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2t}]}$$

$$CL = \mu_0 \qquad (15\text{-}34)$$

$$UCL = \mu_0 + 3\frac{\sigma}{\sqrt{n}}\sqrt{\frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2t}]}$$

Time-Weighted Charts

Example 15-7

EXAMPLE 15-7 Chemical Process Concentration EWMA

Consider the concentration data shown in Table 15-3. Construct an EWMA control chart with $\lambda = 0.2$ with $n = 1$. It was determined that $\overline{x} = 99.1$ and $\overline{mr} = 2.59$. Therefore, $\hat{\mu}_0 = 99.1$ and $\hat{\sigma} = 2.59/1.128 = 2.30$. The control limits for z_1 are

$$LCL = 99.1 - 3(2.30)\sqrt{\frac{0.2}{2 - 0.2}[1 - (1 - 0.2)^2]} = 98.19$$

$$LCL = 99.1 + 3(2.30)\sqrt{\frac{0.2}{2 - 0.2}[1 - (1 - 0.2)^2]} = 100.01$$

The first few values of z_t along with the corresponding control limits are

t	1	2	3	4	5
x_t	102.0	94.8	98.3	98.4	102.0
z_t	99.68	98.70	98.62	98.58	99.26
LCL	97.72	97.33	97.12	97.00	96.93
UCL	100.48	100.87	101.08	101.20	101.27

The chart generated by Minitab is shown in Figure 15-23. Notice that the control limits widen as time increases but quickly stabilize. Each point is within its set of corresponding control limits so there are no signals from the chart.

Time-Weighted Charts

15-8.2 Exponential Weighted Moving Average Control Chart

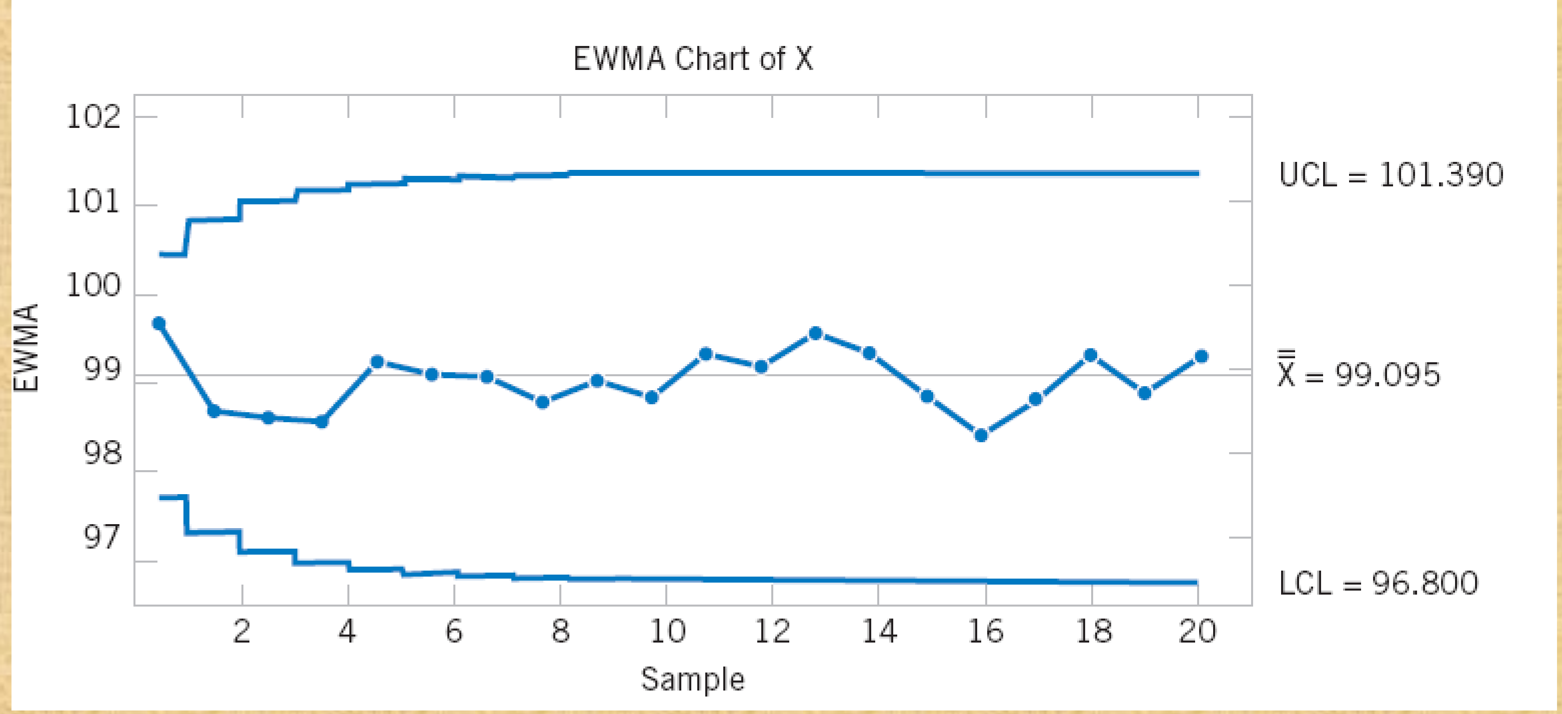

Figure 15-23 EWMA control chart for the Chemical Process Concentration Data from Minitab.

Other SPC Problem-Solving Tools

Pareto Diagram

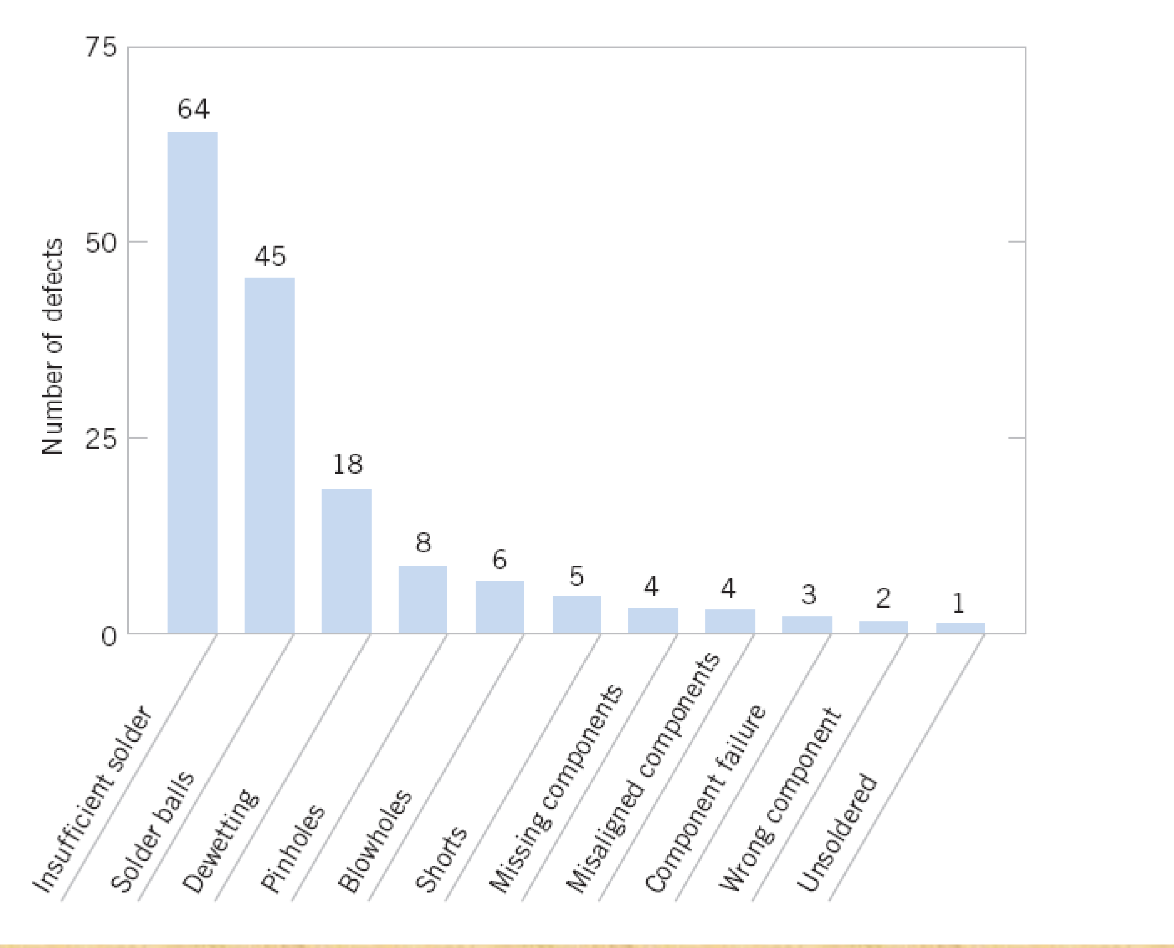

Figure 15-24 Pareto diagram for printed circuit board defects.

Other SPC Problem-Solving Tools

Cause-and-effect Diagram

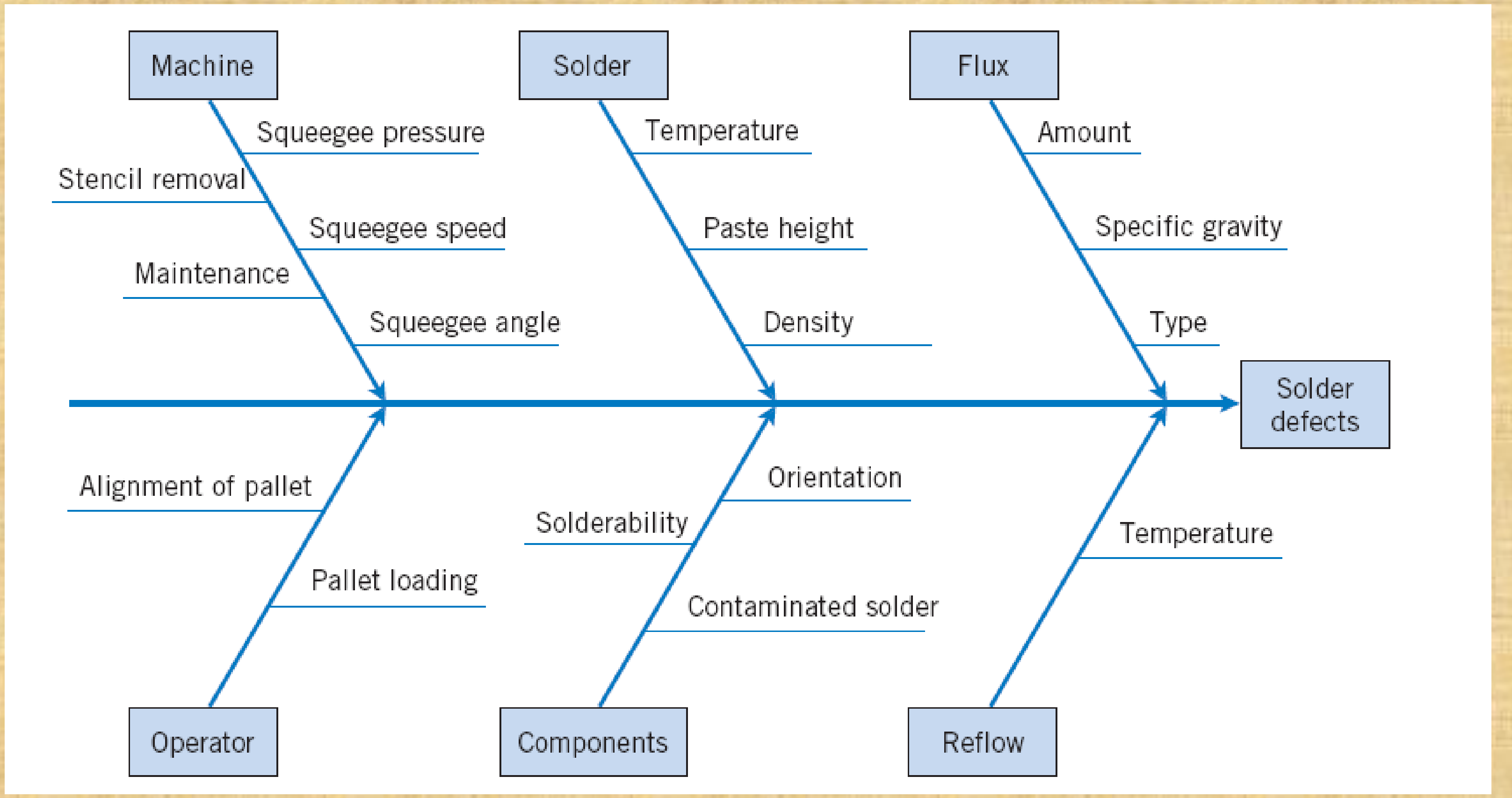

Figure 15-25 Cause-and-effect diagram for the printed circuit board flow solder process.

Other SPC Problem-Solving Tools

Defect Concentration Diagram

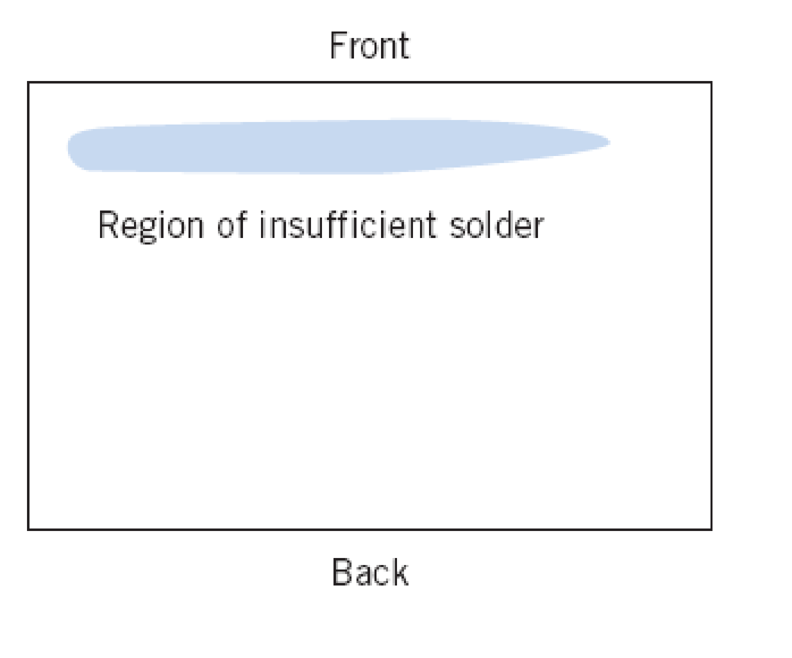

Figure 15-26 Defect concentration diagram for a printed circuit board.

Implementing SPC

- **Strategic Management of Quality**
- **Deming's 14 points**

Important Terms & Concepts of Chapter

ARL
Assignable causes
Attributes control charts
Average run length
C chart
Cause-and-effect diagram
Center line
Chance causes
Control chart
Control limits
Cumulative sum control chart
Defect concentration diagram
Defects-per-unit chart
Deming's 14 points
Exponentially-weighted moving average control chart (EWMA)
False alarm
Fraction-defective control chart
Implementing SPC
Individuals control chart (X chart)
Moving range
NP chart
P chart
Pareto diagram
PCR
PCR_k
Problem-solving tools
Process capability
Process capability ratio
Quality control
R chart
Rational subgroup
Run rule
S chart
Shewhart control chart
Six-sigma process
Specification limits