Statistical Quality Control

BY: ANKITA, REENA, RAVIRAJ & CHETAN.

What is SQC ?

- **Statistical quality control (SQC)** is the term used to describe the set of statistical tools used by quality professionals.

History

- SQC was pioneered by Walter A. Shewhart at Bell Laboratories in the early 1920s.

- Shewhart developed the control chart in 1924 and the concept of a state of statistical control.

- Shewhart consulted with Colonel Leslie E. Simon in the application of control charts to munitions manufacture at the Army's Picatinney Arsenal in 1934.

History

- W. Edwards Deming invited Shewhart to speak at the Graduate School of the U.S. Department of Agriculture, and served as the editor of Shewhart's book *Statistical Method from the Viewpoint of Quality Control* (1939) which was the result of that lecture.

- Deming was an important architect of the quality control short courses that trained American industry in the new techniques during WWII.

- Deming traveled to Japan during the Allied Occupation and met with the Union of Japanese Scientists and Engineers(JUSE)in an effort to introduce SQC methods to Japanese industry

SQC Categories

- Descriptive statistics
- Statistical process control (SPC)
- Acceptance sampling

Descriptive Statistics

- **Descriptive statistics** are used to describe quality characteristics and relationships.

"Data don't make any sense, we will have to resort to statistics."

Descriptive Statistics

- **The Mean-** measure of central tendency

- **The Range-** difference between largest/smallest observations in a set of data

- **Standard Deviation** measures the amount of data dispersion around mean

The Mean

- To compute the mean we simply sum all the observations and divide by the total no. of observations.

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

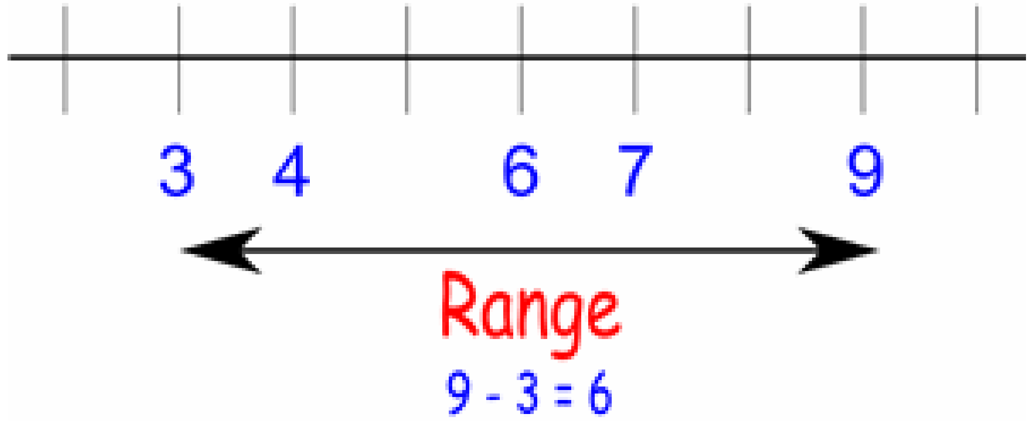
where $\bar{x}$ = the mean

x_i = observation i, $i = 1, \ldots, n$

n = number of observations

The Range

- Range, which is the difference between the **largest** and **smallest** observations.



Standard Deviation

- Standard deviation is a measure of dispersion of a curve.
- It measures the extent to which these values are scattered around the central mean.

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \overline{x})^2}{n - 1}}$$

where σ = standard deviation of a sample
$\overline{x}$ = the mean
x_i = observation $i, i = 1, \ldots, n$
n = the number of observations in the sample

Statistical process control

- Extend the use of descriptive statistics to monitor the quality of the product and process
- Statistical process control help to determine the amount of variation
- To make sure the process is in a *state of control*

Variation In Quality

- No two items are exactly alike.
- Some sort of variations in the two items is bound to be there. In fact it is an integral part of any manufacturing process.
- This difference in characteristics known as variation.
- This variation may be due to substandard quality of raw material, carelessness on the part of operator, fault in machinery system etc..

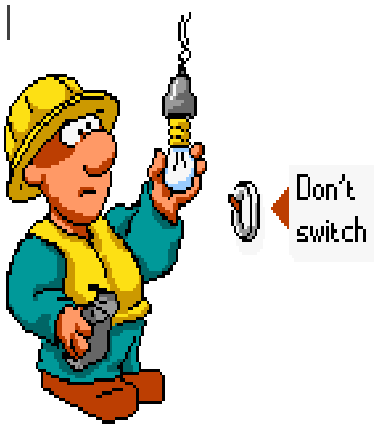
Types Of Variations

Variation due to "CHANCE CAUSES"

Variation due to "ASSIGNABLE CAUSES"

Variation due to chance causes/common causes

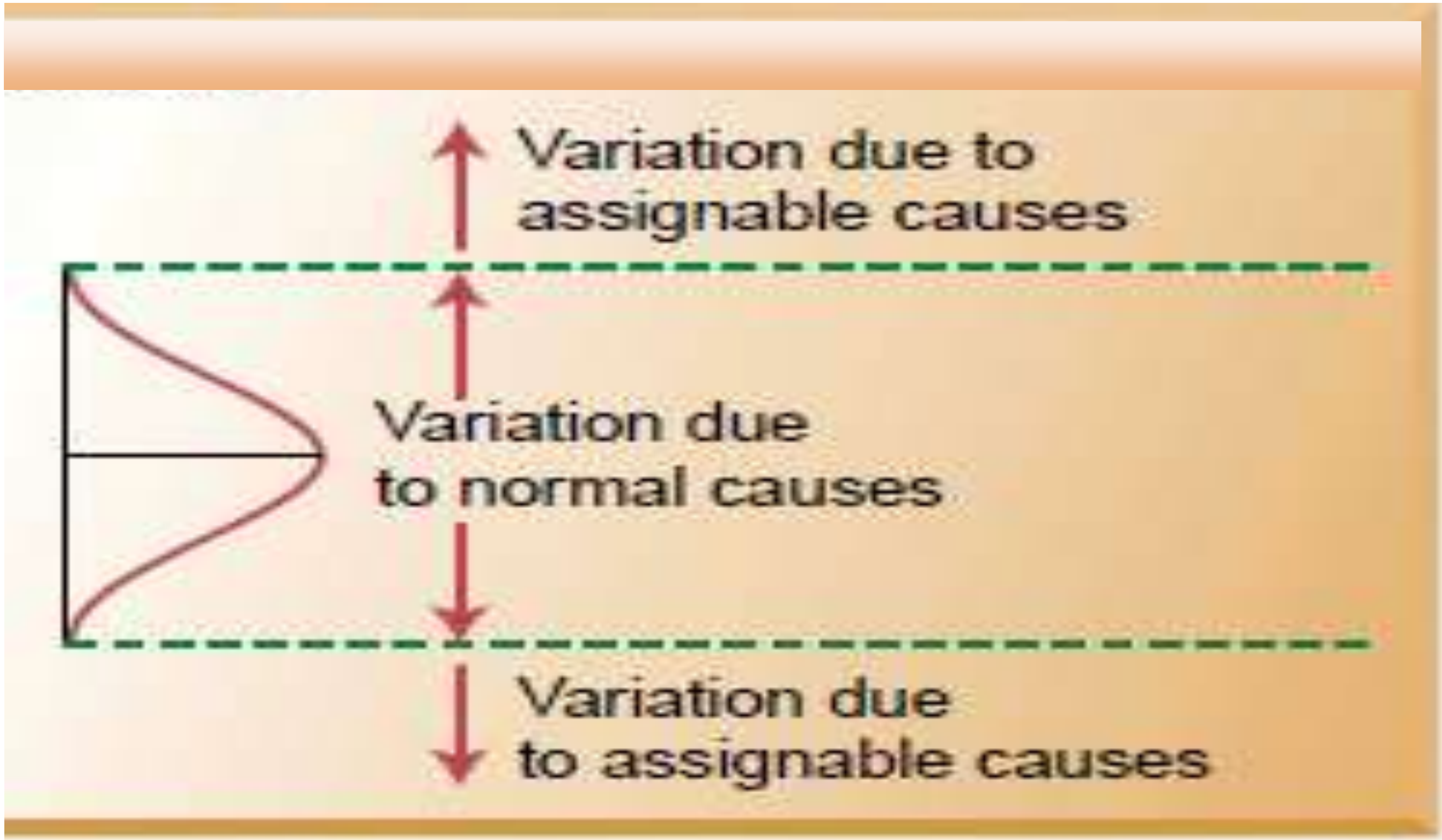
- Variation occurred due to chance.
- This variation is **NOT** due to defect in machine, Raw material or any other factors.
- Behave in “random manner”.
- Negligible but Inevitable
- The process is said to be under the state of statistical control.



Variation due to assignable causes

Non – random causes like:

- Difference in quality of raw material
- Difference in machines
- Difference in operators
- Difference of time


Variation due to
assignable causes
Variation due
to normal causes
Variation due
to assignable causes

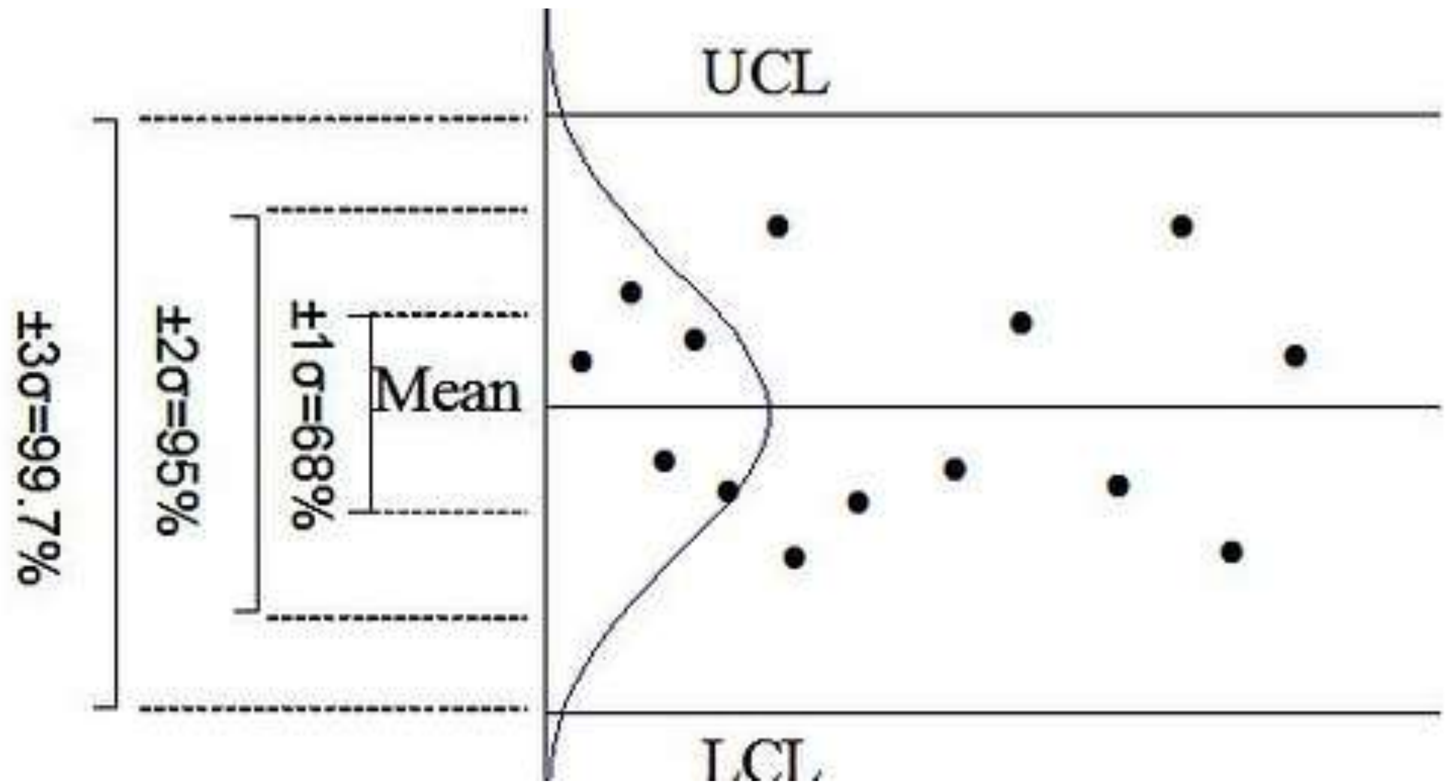
Specification and control limits

- No item in the world can be a true copy of another item.
- It is not expressed in absolute values but in terms of a range.
- For Eg:

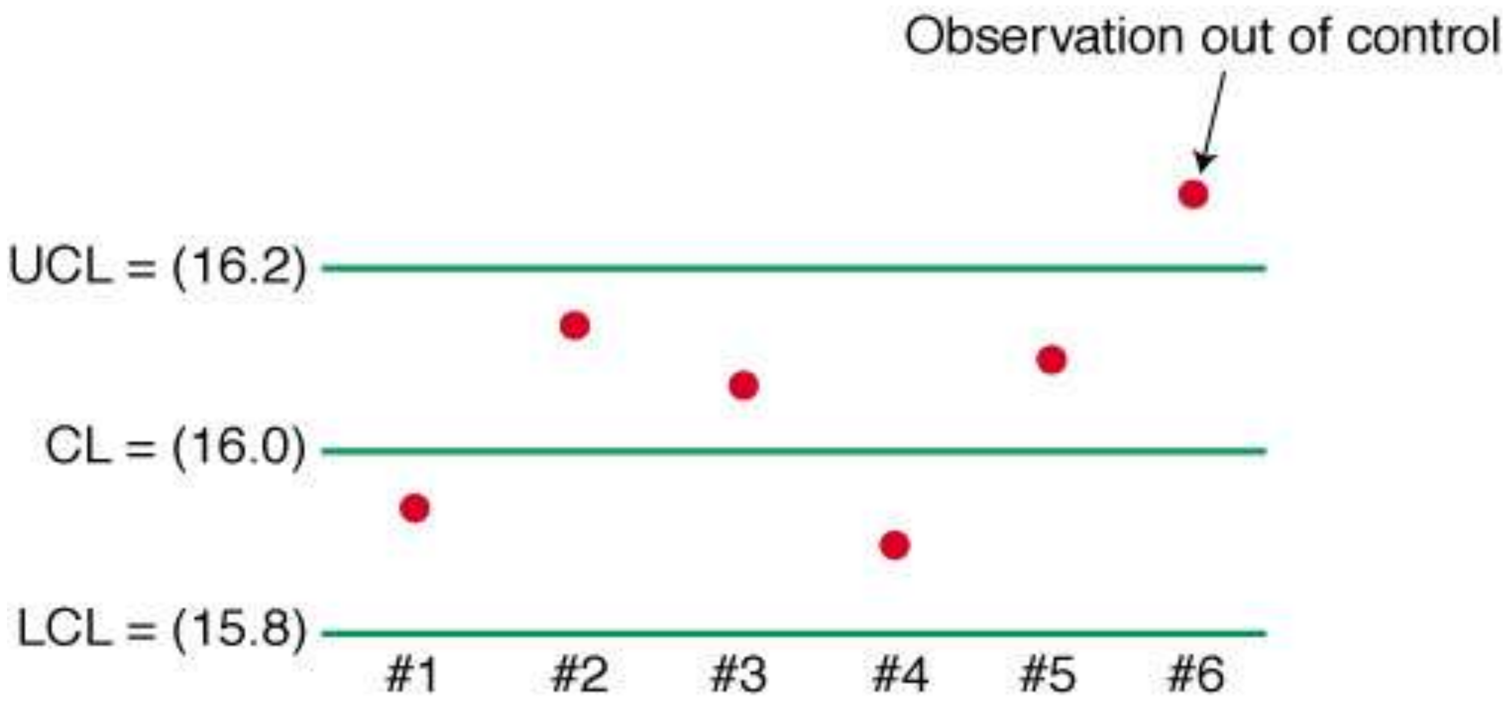
The diameter of a pen is expected by its manufacturer not as 7mm but as 7mm ± 0.05.

Thus, the diameter of a pen produced by the manufacturer can vary from 6.95 mm to 7.05 mm.

Setting Control Limits



HOW CONTROL LIMITS ARE USEFUL.....?



SPC Methods-Control Charts

- Control Charts show sample data plotted on a graph with CL, UCL, and LCL
- Control chart for variables are used to monitor characteristics that can be measured, e.g. length, weight, diameter, time
- Control charts for attributes are used to monitor characteristics that have discrete values and can be counted, e.g. % defective, number of flaws in a shirt, number of broken eggs in a box

Control Charts for Variables

▶ x-bar charts

It is used to monitor the changes in the mean of a process (central tendencies).

▶ R-bar charts

It is used to monitor the dispersion or variability of the process

Constructing a X-bar chart (sigma is not given)

- A factory produces 50 cylinders per hour. Samples of 10 cylinders are taken at random from the production at every hour and the diameters of cylinders are measured. Draw X-bar and R charts and decide whether the process is under control or not.

(For n=4 A2= 0.73 D3= 0, D4=2.28)

Sample no.	x1	x2	x3	x4
1	230	238	242	250
2	220	230	218	242
3	222	232	236	240
4	250	240	230	225
5	228	242	235	225
6	248	222	220	230
7	232	232	242	242
8	236	234	235	237
9	231	248	251	271
10	220	222	224	231

Sample no.	x1	x2	x3	x4	Sigma Xi	Mean X-bar	Range R
1	230	238	242	250	960	240.00	20
2	220	230	218	242	910	227.50	24
3	222	232	236	240	930	232.50	18
4	250	240	230	225	945	236.25	25
5	228	242	235	225	930	232.50	17
6	248	222	220	230	920	230.00	28
7	232	232	242	242	948	237.00	10
8	236	234	235	237	942	235.50	3
9	231	248	251	271	1001	250.25	40
10	220	222	224	231	897	224.25	11
Total						2345.75	196

Calculation of x-bar and R-bar

- Now,

$$\overline{\overline{x}} = \frac{\sum \overline{x}}{\mathrm{m}} = \frac{2345.75}{10} = 234.75$$

$$\overline{R} = \frac{\sum R}{m} = \frac{196}{10} = 19.6$$

Sample Size (n)	Factor for x-Chart A2	Factors for R-Chart D3	D4
2	1.88	0.00	3.27
3	1.02	0.00	2.57
4	0.73	0.00	2.28
5	0.58	0.00	2.11
6	0.48	0.00	2.00
7	0.42	0.08	1.92
8	0.37	0.14	1.86
9	0.34	0.18	1.82
10	0.31	0.22	1.78
11	0.29	0.26	1.74
12	0.27	0.28	1.72
13	0.25	0.31	1.69
14	0.24	0.33	1.67
15	0.22	0.35	1.65

Control limits of X-Bar Chart

- Central line C.L = $\bar{\bar{x}} = 234.75$

- U.C.L = $\bar{\bar{x}} + A2 * \bar{R}$

  =234.75 + (0.73) (19.6)

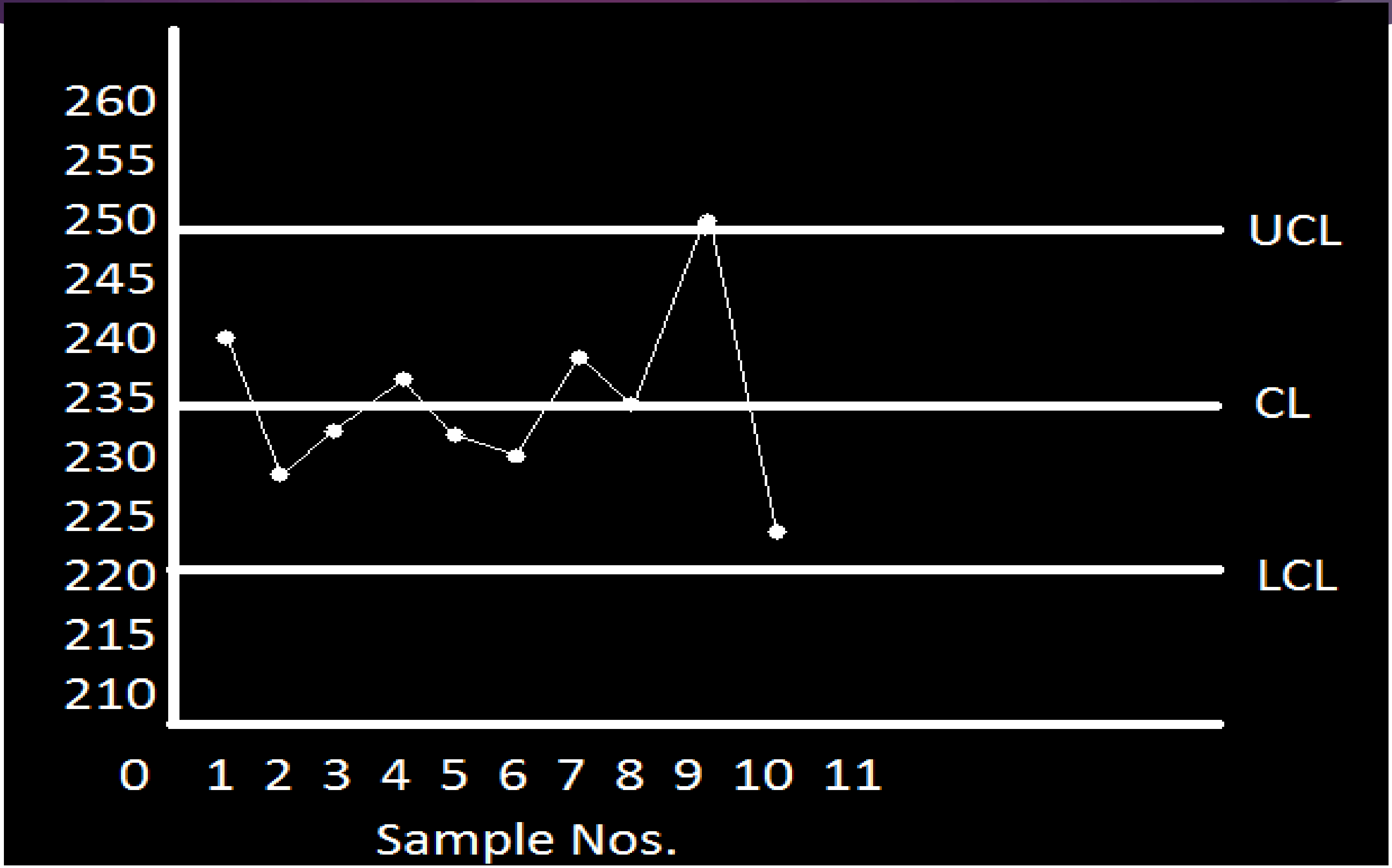
  =**249.06**

- L.C.L= $\bar{\bar{x}} - A2 * \bar{R}$

  =234.75- (0.73) (19.6)

  =**220.72**

X-Bar Chart


260
255
250
245
240
235
230
225
220
215
210
UCL
CL
LCL
0 1 2 3 4 5 6 7 8 9 10 11
Sample Nos.

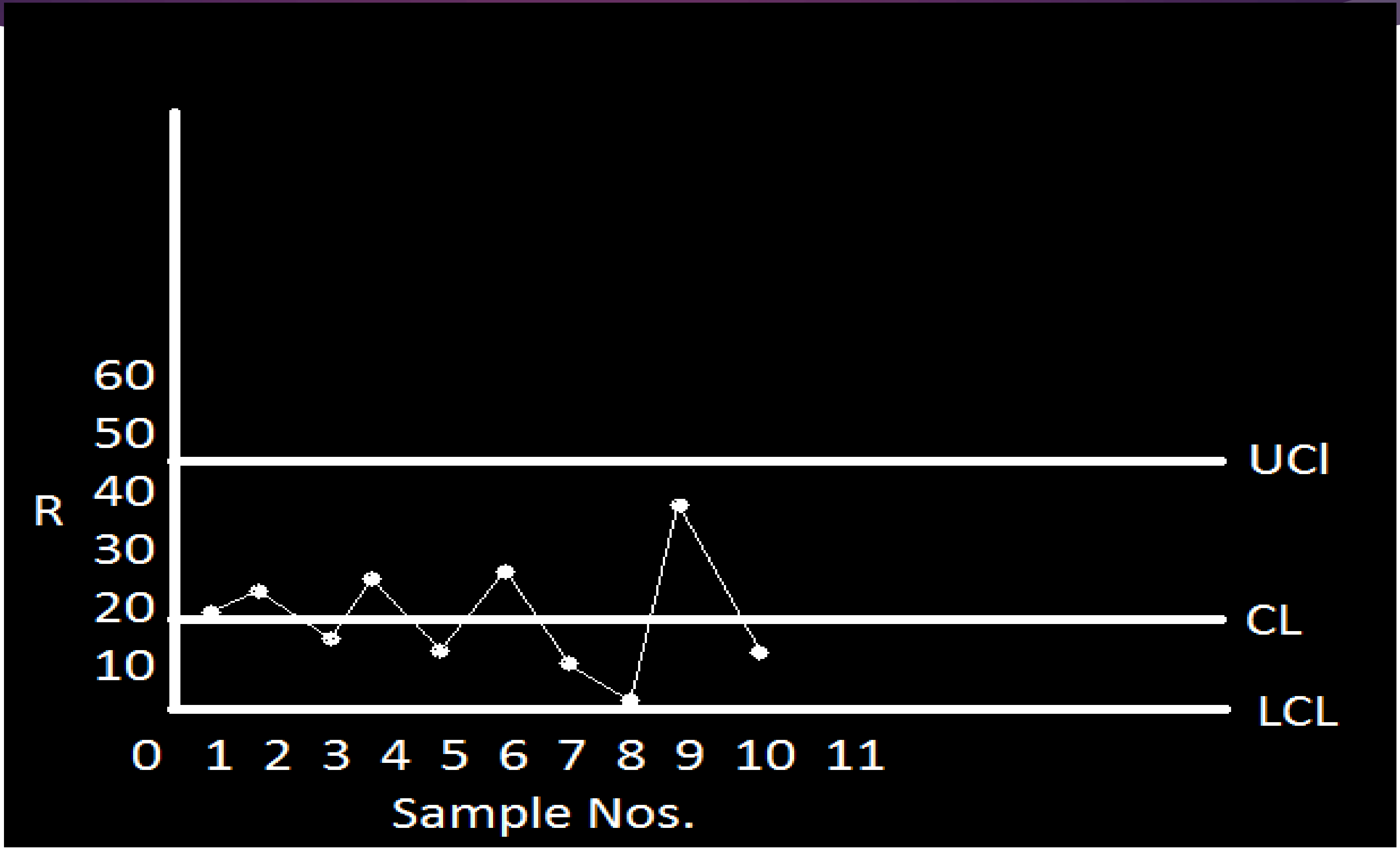
Control limits of R-Bar Chart

- Central Line = $\bar{R} = 19.6$
- U.C.L = $D4\bar{R} = (2.28)*(19.96)$

  =45.50
- L.C.L = $D3\bar{R} = (0)*(19.96)$

  =0

R-Bar Chart



Constructing a X-bar Chart (Sigma is given)

- A quality control inspector at the Coca-Cola soft drink company has taken twenty-five samples with four observations each of the volume of bottles filled. The data and the computed means are shown in the table. If the standard deviation of the bottling operation is 0.14 ounces, use this information to develop control limits of three standard deviations for the bottling operation.

Sample Number	(bottle volume in ounces) 1	2	3	4	Average $\bar{x}$	Range R
1	15.85	16.02	15.83	15.93	15.91	0.19
2	16.12	16.00	15.85	16.01	15.99	0.27
3	16.00	15.91	15.94	15.83	15.92	0.17
4	16.20	15.85	15.74	15.93	15.93	0.46
5	15.74	15.86	16.21	16.10	15.98	0.47
6	15.94	16.01	16.14	16.03	16.03	0.20
7	15.75	16.21	16.01	15.86	15.96	0.46
8	15.82	15.94	16.02	15.94	15.93	0.20
9	16.04	15.98	15.83	15.98	15.96	0.21
10	15.64	15.86	15.94	15.89	15.83	0.30
11	16.11	16.00	16.01	15.82	15.99	0.29
12	15.72	15.85	16.12	16.15	15.96	0.43
13	15.85	15.76	15.74	15.98	15.83	0.24
14	15.73	15.84	15.96	16.10	15.91	0.37
15	16.20	16.01	16.10	15.89	16.05	0.31
16	16.12	16.08	15.83	15.94	15.99	0.29
17	16.01	15.93	15.81	15.68	15.86	0.33
18	15.78	16.04	16.11	16.12	16.01	0.34
19	15.84	15.92	16.05	16.12	15.98	0.28
20	15.92	16.09	16.12	15.93	16.02	0.20
21	16.11	16.02	16.00	15.88	16.00	0.23
22	15.98	15.82	15.89	15.89	15.90	0.16
23	16.05	15.73	15.73	15.93	15.86	0.32
24	16.01	16.01	15.89	15.86	15.94	0.15
25	16.08	15.78	15.92	15.98	15.94	0.30
Total					398.75	7.17

Equations

$$UCL = \overline{\overline{X}} + z\sigma\overline{x}$$

$$LCL = \overline{\overline{X}} - z\sigma\overline{x}$$

$$\sigma\overline{x} = \frac{s}{\sqrt{n}}$$

$$s = \frac{\overline{R}}{d2}$$

- **Solution**

The center line of the control data is the average of the samples:

$$\bar{\bar{x}} = \frac{398.75}{25}$$

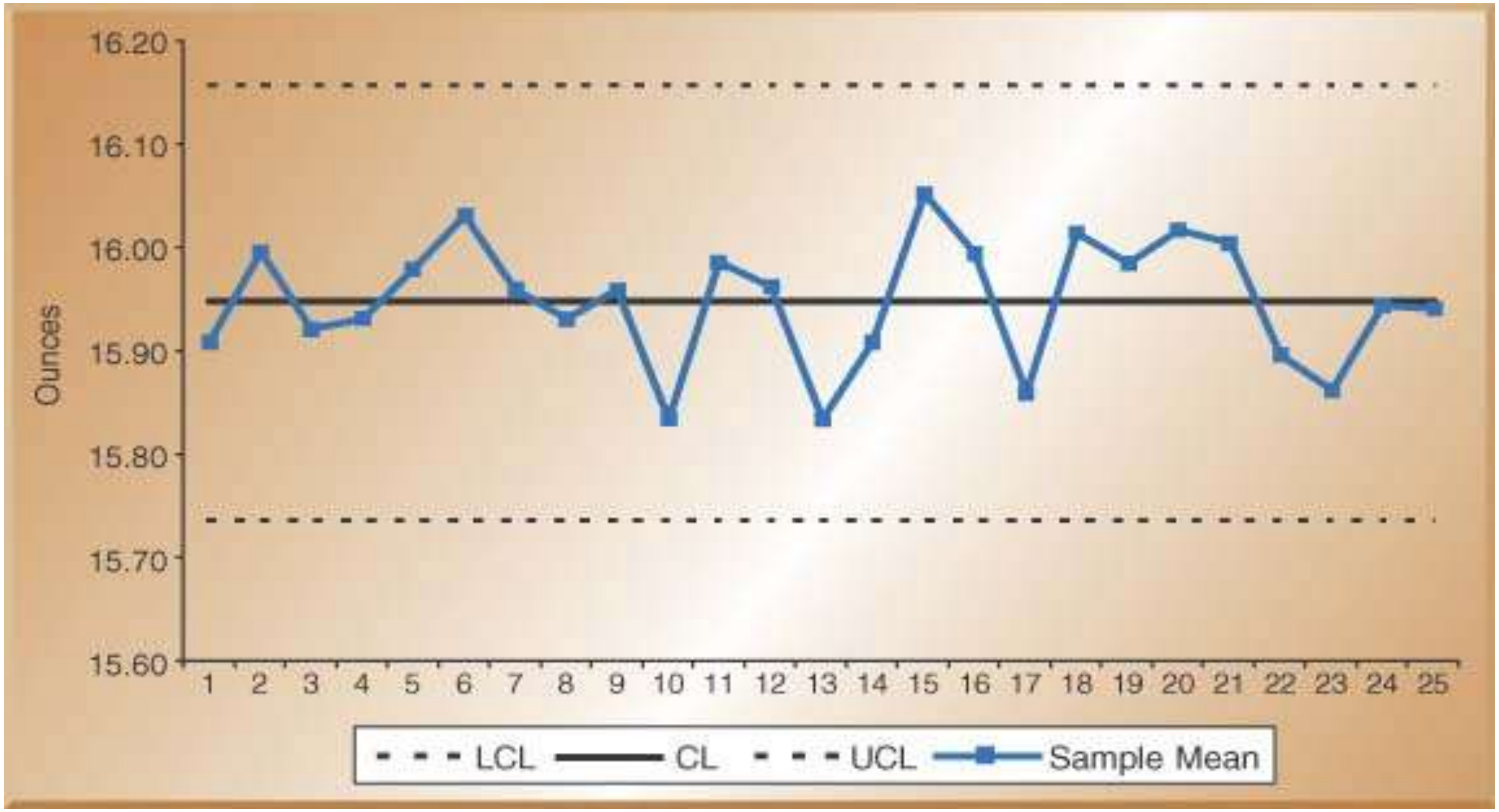
$$\bar{\bar{x}} = 15.95$$

The control limits are

$$\text{UCL} = \bar{\bar{x}} + z\sigma_{\bar{x}} = 15.95 + 3\left(\frac{.14}{\sqrt{4}}\right) = 16.16$$

$$\text{LCL} = \bar{\bar{x}} - z\sigma_{\bar{x}} = 15.95 - 3\left(\frac{.14}{\sqrt{4}}\right) = 15.74$$

X-Bar Control Chart



Control Charts for Attributes

- **Attributes are discrete events; yes/no, pass/fail**

Use P-Charts for quality characteristics that are discrete and involve <u>yes/no or good/bad</u> decisions

- Number of leaking caulking tubes in a box of 48
- Number of broken eggs in a carton

Use C-Charts for discrete defects when there can be more than one defect per unit

- Number of flaws or stains in a carpet sample cut from a production run
- Number of complaints per customer at a hotel

P-Chart Example

- A Production manager of a BKT tire company has inspected the number of defective tires in five random samples with 20 tires in each sample. The table below shows the number of defective tires in each sample of 20 tires. Calculate the control limits.

Sample Number	Number of Defective Tires	Number of Observations Sampled	Fraction Defective
1	3	20	.15
2	2	20	.10
3	1	20	.05
4	2	20	.10
5	1	20	.05
6	3	20	.15
7	3	20	.15
8	2	20	.10
9	1	20	.05
10	2	20	.10
11	3	20	.15
12	2	20	.10
13	2	20	.10
14	1	20	.05
15	1	20	.05
16	2	20	.10
17	4	20	.20
18	3	20	.15
19	1	20	.05
20	1	20	.05
Total	40	400	

- **Solution**

The center line of the chart is

$$\text{CL} = \overline{p} = \frac{\text{total number of defective tires}}{\text{total number of observations}} = \frac{40}{400} = .10$$

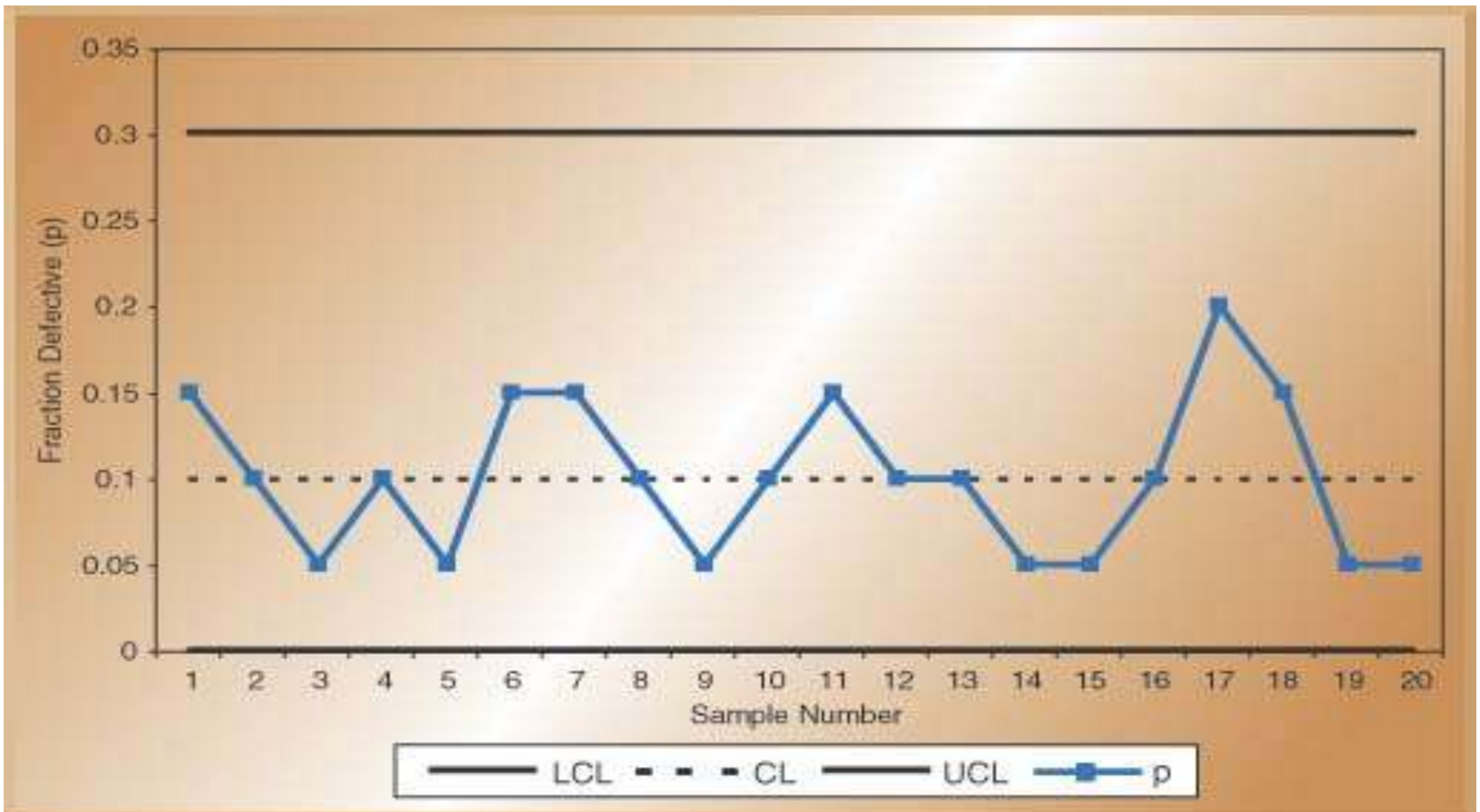
$$\sigma_p = \sqrt{\frac{\overline{p}(1-\overline{p})}{n}} = \sqrt{\frac{(.10)(.90)}{20}} = .067$$

$$\text{UCL} = \overline{p} + z(\sigma_p) = .10 + 3(.067) = .301$$

$$\text{LCL} = \overline{p} - z(\sigma_p) = .10 - 3(.067) = -.101 \longrightarrow 0$$

In this example the lower control limit is negative, which sometimes occurs because the computation is an approximation of the binomial distribution. When this occurs, the LCL is rounded up to zero because we cannot have a negative control limit.

P- Control Chart



C - Chart Example

- The number of weekly **customer complaints** are monitored in a large hotel using a c-chart. Develop **three sigma control limits** using the data table below.

The number of weekly customer complaints are monitored at a large hotel using a c-chart. Complaints have been recorded over the past twenty weeks. Develop three-sigma control limits using the following data:

Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	Tota
No. of Complaints	3	2	3	1	3	3	2	1	3	1	3	4	2	1	1	1	3	2	2	3	44

- Solution

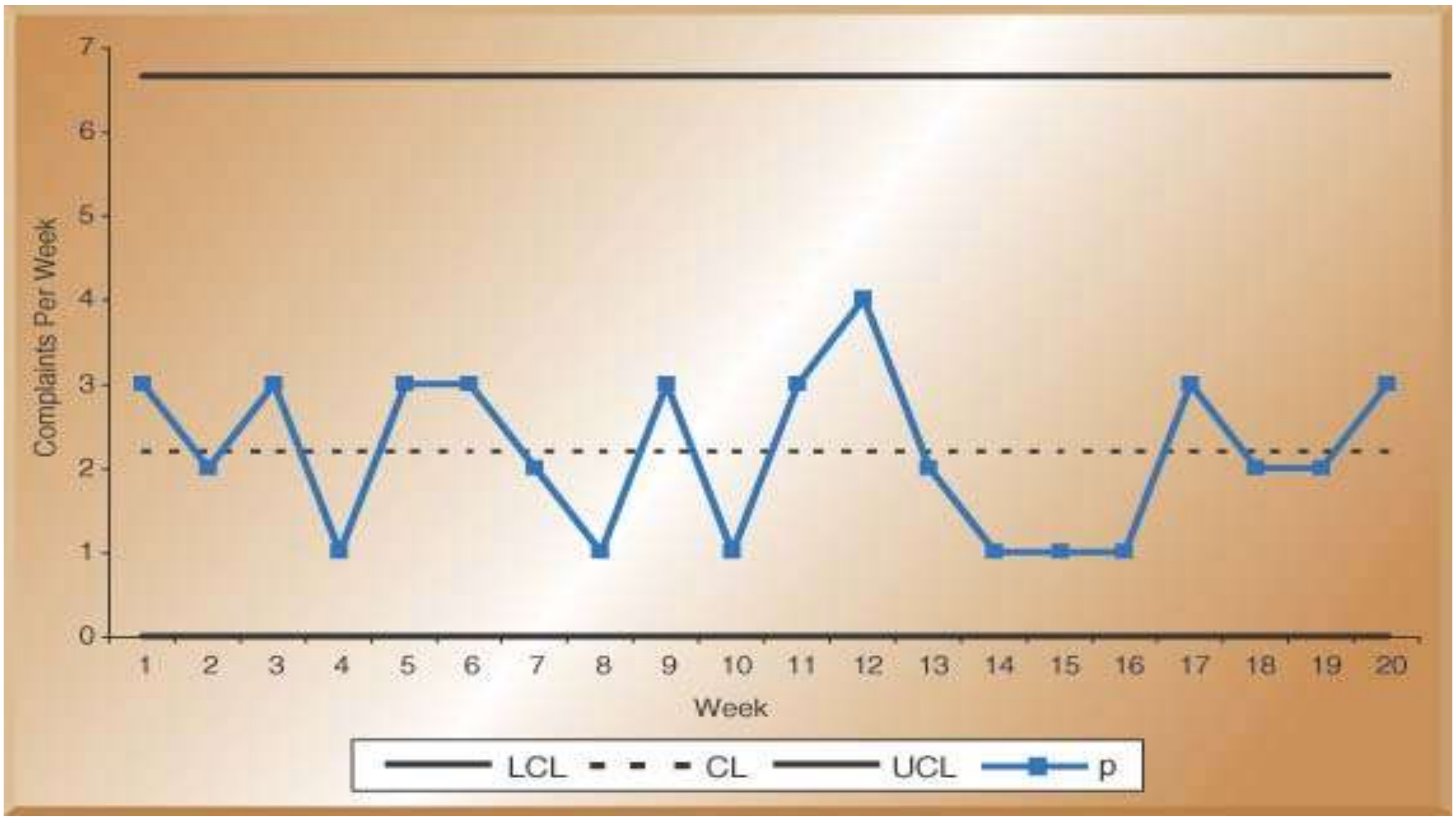
The average number of complaints per week is $\frac{44}{20} = 2.2$. Therefore, $\bar{c} = 2.2$.

$$UCL = \bar{c} + z\sqrt{\bar{c}} = 2.2 + 3\sqrt{2.2} = 6.65$$

$$LCL = \bar{c} - z\sqrt{\bar{c}} = 2.2 - 3\sqrt{2.2} = -2.25 \longrightarrow 0$$

As in the previous example, the LCL is negative and should be rounded up to zero. Following is the control chart for this example:

C - Control Chart



Process Capability

- Evaluating the ability of a production process to meet or exceed preset specifications. This is called **process capability.**

- **Product specifications,** often called *tolerances,* are preset ranges of acceptable quality characteristics, such as product dimensions.

Two parts of process capability

- 1) Measure the variability of the output of a process, and
- 2) Compare that variability with a proposed specification or product tolerance.

Measuring Process Capability

- To produce an acceptable product, the process must be *capable* and *in control* before production begins.

$$C_p = \frac{USL - LSL}{6\sigma}$$

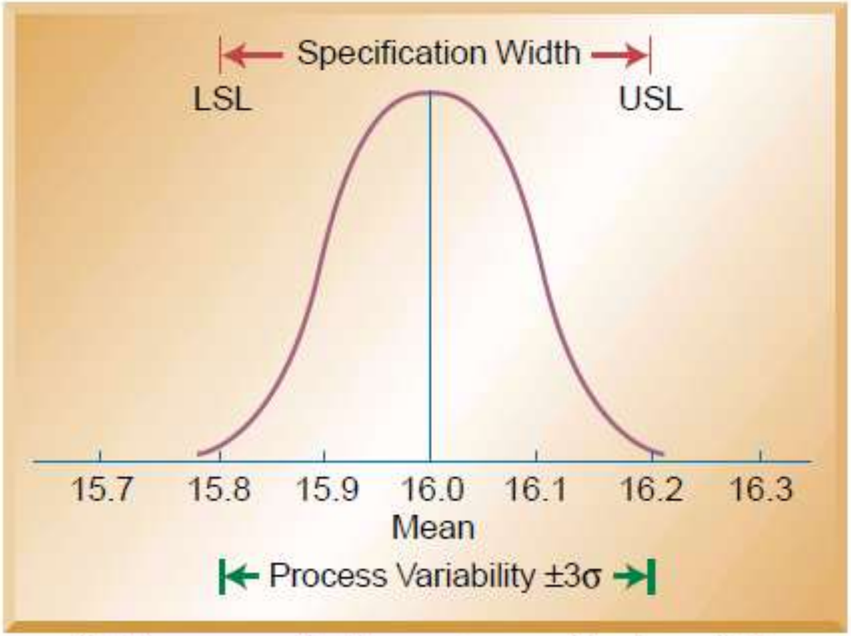
Example

- Let’s say that the specification for the acceptable volume of liquid is preset at 16 ounces ±.2 ounces, which is 15.8 and 16.2 ounces.

Figure (a)

- The process produces 99.74 percent (three sigma) of the product with volumes between 15.8 and 16.2 ounces.

$$C_p = 1$$

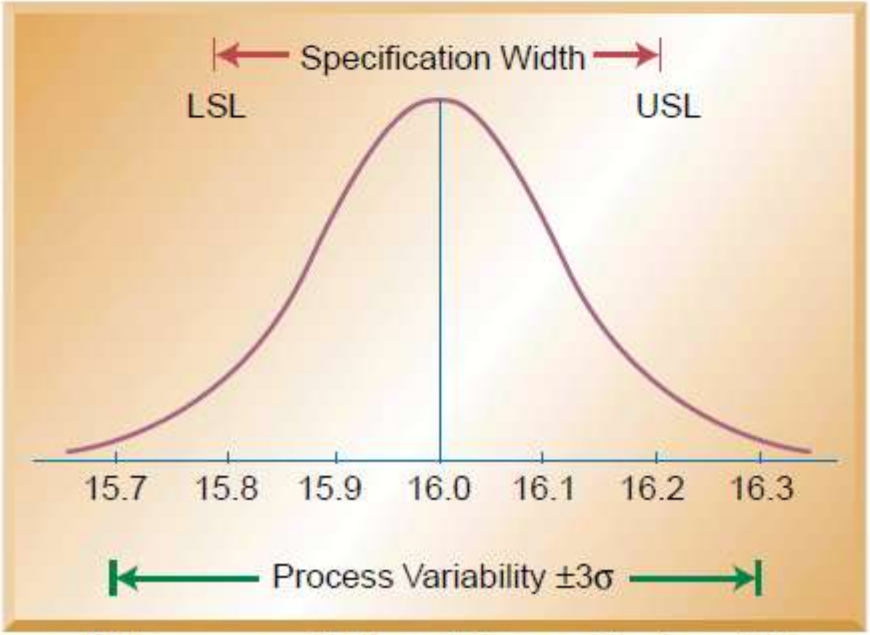


(a) Process variability meets specification width

Figure (b)

- The process produces 99.74 percent (three sigma) of the product with volumes between 15.7 and 16.3 ounces.

$$C_p \leq 1$$

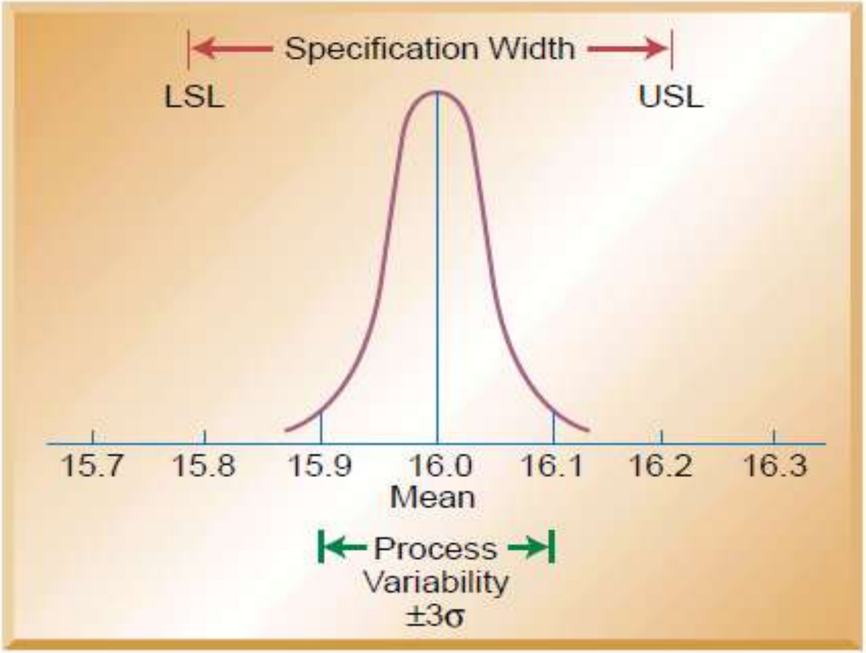


(b) Process variability outside specification width

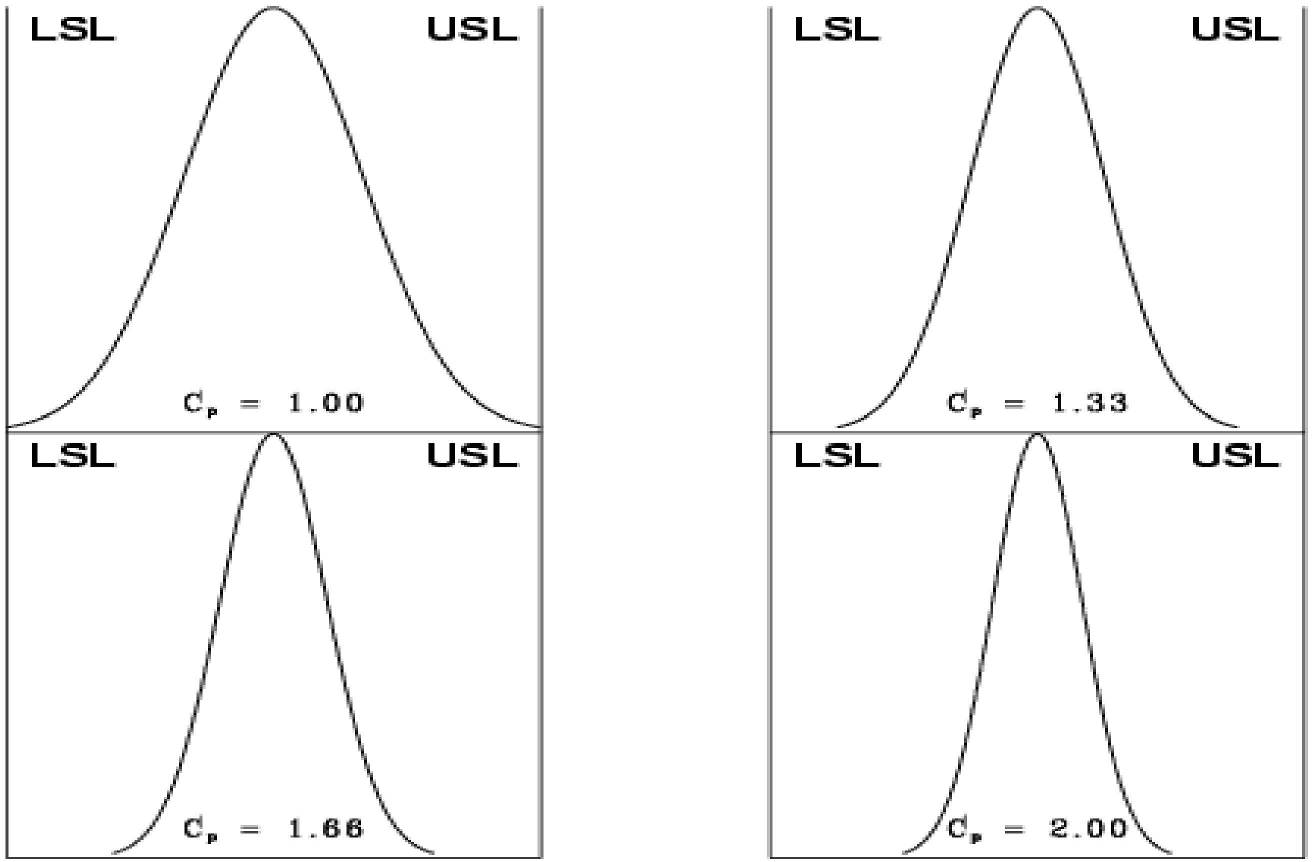
Figure (c)

- the production process produces 99.74 percent (three sigma) of the product with volumes between 15.9 and 16.1 ounces.

$$C_p \geq 1$$



(c) Process variability within specification width


LSL
USL
C_p = 1.00
LSL
USL
C_p = 1.33
LSL
USL
C_p = 1.66
LSL
USL
C_p = 2.00

Three bottling machines at Cocoa Fizz are being evaluated for their capability:

Bottling Machine	Standard Deviation
A	.05
B	.1
C	.2

If specifications are set between 15.8 and 16.2 ounces, determine which of the machines are capable of producing within specifications.

- **Solution**

To determine the capability of each machine we need to divide the specification width ($USL - LSL = 16.2 - 15.8 = .4$) by 6σ for each machine:

Bottling Machine	σ	USL−LSL	6σ	$C_p = \dfrac{USL - LSL}{6\sigma}$
A	.05	.4	.3	1.33
B	.1	.4	.6	.67
C	.2	.4	1.2	.33

Looking at the C_p values, only machine A is capable of filling bottles within specifications, because it is the only machine that has a C_p value at or above 1.

Process capability ratio **(off centering process)**

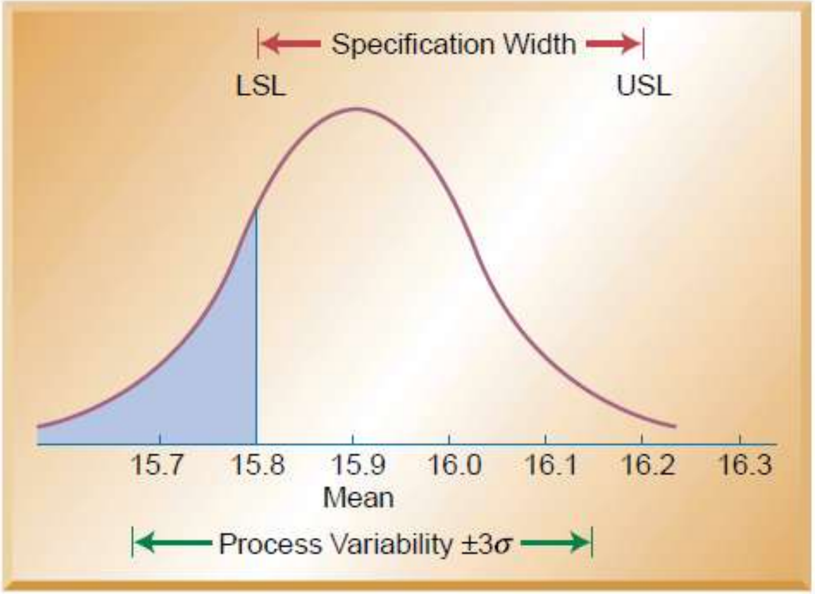
- There is a possibility that the process mean may shift over a period of time, in either direction, i.e., towards the USL or the LSL. This may result in more defective items then the expected. This shift of the process mean is called the off-centering of the process.

$$C_{pk} = \min\left(\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma}\right)$$

Example

- Process mean: $\mu = 15.9$
- Process standard deviation: $\sigma = 0.067$
- LSL = 15.8
- USL = 16.2

$$C_p = \frac{0.4}{6(0.067)} = 1$$



$$C_{p_k} = \min\left(\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right)$$

$$C_{p_k} = \min\left(\frac{16.2 - 15.9}{3(.1)}, \frac{15.9 - 15.8}{3(.1)}\right)$$

$$C_{pk} = \min\left(1.00, \quad 0.33\right)$$

$$C_{pk} = 0.33$$

Thank You…