CHAPTER 10

ACCEPTANCE SAMPLING

10.1 INTRODUCTION

Acceptance sampling is the process of evaluating a portion of the product/material in a lot for the purpose of accepting or rejecting the lot as either conforming or not conforming to quality specifications.

Inspection for acceptance purpose is carried out at many stages in manufacturing. There are generally two ways in which inspection is carried out : (*i*) 100% inspection (*ii*) Sampling inspection.

In 100% inspection all the parts or products are subjected to inspection, whereas in sampling inspection only a sample is drawn from the lot and inspected.

A sample may be defined as the number of items drawn from a lot, batch or population for inspection purposes.

Sampling inspection can be defined as a technique to determine the acceptance or rejection of a lot or population on the basis of number of defective parts found in a random sample drawn from the lot. If the number of defective items does not exceed a predefined level, the lot is accepted, otherwise it is rejected.

Sampling inspection is not a new concept. In our daily life we use sampling inspection in selecting certain consumable items. For example, while purchasing our annual or monthly requirements of wheat, rice or such other food grains we naturally take a handful of grains to judge its quality for taking purchasing decision. If we are not satisfied we take another sample and after two or three samples from the same or different sources we take purchasing decision. Or, let us take another example, suppose we want to purchase mangoes we normally take one or two mangoes from the lot and taste its quality, if the samples taken are found good we decide to purchase the required quantity.

Similarly, in engineering sampling inspection is preferred because it is more practical, quick and economical as compared to 100% inspection. The main purpose of acceptance sampling is to distinguish between good lots and bad lots, and to classify the lots according to their acceptability or non-acceptability.

Advantages of Sampling Inspection

The advantages of sampling inspection are as follows :

1. The items which are subjected to destructive test must be inspected by sampling inspection only.
2. The cost and time required for sampling inspection is quite less as compared to 100% inspection.
3. Problem of inspection fatigue which occurs in 100% inspection is eliminated.
4. Smaller inspection staff is necessary.
5. Less damage to products because only few items are subjected to handling during inspection.
6. The problem of monotony and inspector error introduced by 100% inspection is minimised.
7. The most important advantage of sampling inspection is that, it exerts more effective pressure on quality improvement. Since the rejection of entire lot on the basis of sampling brings much stronger pressure on quality improvement than the rejection of individual articles.

Limitations of Sampling Inspection

1. **Risk of making wrong decisions.** However, in sampling inspection, since only a part is inspected, it is inevitable that the sample may not always represent the exact picture obtaining in the lot and hence, there will be likelihood or risk of making wrong decisions about the lot. This wrong decision can be made in two ways. Firstly, a really good lot (that is, containing less proportion of defectives than specified) may be rejected because the sample drawn may be bad. Secondly, a really bad lot (that is, a lot containing greater proportion of defectives than specified) may be accepted because the sample drawn may be good. In the former case, the producer has to suffer a risk of his good lots being rejected and hence the associated risk (chance) is called as the producer's risk. In the latter case, the consumer runs the risk of accepting bad lots and hence the associated risk is called as consumer's risk.

2. The sample usually provides less information about the product than 100 per cent inspection.

3. Some extra planning and documentation is necessary.

However, in scientific sampling plans, these risks are quantified and the sampling criteria are adjusted to balance these risks, in the light of the economic factors involved.

The success of a sampling scheme depends upon the following factors :

(*i*) Randomness of samples
(*ii*) Sample size
(*iii*) Quality characteristic to be tested
(*iv*) Acceptance criteria
(*v*) Lot size

Industrial Uses of Acceptance Sampling

1. To determine the quality and acceptability of incoming raw materials, component parts, products etc.
2. To decide the acceptability of semifinished products for further processing as it undergoes the operations from machine to machine or section to section within the factory.
3. To determine the quality of outgoing products.
4. For improving, maintaining and controlling the quality of the products manufactured.

Lot Formation

A lot is a collection of items from which a sample of two or more articles is drawn and inspected to determine its acceptability.

The lot formation greatly influences the outgoing quality and inspection costs. The following points should be taken into consideration as guidelines for the formation of a lot :

1. The products from different sources (processes, production shifts, input materials, etc.) should not be mixed together, unless there is an evidence that the lot-to-lot variation is small enough to be ignored.
2. For lot formation the products should not be accumulated over extensive period of time.
3. The extraneous information such as process capability, poor inspection etc. should not be used in lot formation.
4. The lot should be as large as possible consistent with the above to take advantage of low proportionate sampling costs.

10.2 SAMPLING METHODS

The sampling methods can be classified as :

- Simple Random sampling
- Systematic sampling
- Two stage sampling.
- Stratified sampling
- Cluster sampling

10.2.1 Simple Random Sampling

Selecting a sample in such a way that each item in a lot has an equal chance of being selected, is called random sampling. Since, a judgement about the lot is to be made on the basis of only a sample it is very important that the sample truely represents the universe from which it is drawn. This requires that the sample size be large enough and the sampling procedure such as to avoid bias. Parts resting on the bottom or in the middle of a group must be selected as well as those lying conveniently on top.

According to this method, the sample of the requisite size n is drawn from a lot of size N, in such a manner ; that while selecting an item, the chance for any item of the lot being included in the sample is the same. An item once drawn is not placed back in the lot.

10.2.2 Stratified Sampling

In large lots, the difficulties of random selection may be so great that it may be advisable to adopt stratified (proportional) sampling. For stratified sampling the following rules should be followed :

1. *Draw proportional samples.* According to this rule, inspection lots should, whenever possible, be divided into sub-lots on the basis of homogenity (into certain number of homogeneous groups or strata).
2. Draw sample items from all parts of such sub-lots of the inspection lot.
3. Draw sample items blind (without any bias).

From each sub-lot into which the inspection lot is divided a sub-sample should be selected. The size of the sub-sample should be proportional to the size of the sub-lot.

This method of sampling may be generally more efficient than the simple random sampling, as the simple random sampling may not always result in the selection of the items from such stratum of the lot, thereby affecting the representativeness of the sample drawn.

It would be advisable to ensure that a minimum of two items are selected from each sub-lot.

10.2.3 Systematic Sampling

When the items in a lot are presented in an orderly manner, (such as piles of mild steel sheets or stacks of cement bags) it is possible to considerably simplify the selection of a random sample of the required size.

In systematic sampling one item is chosen at random from the lot and thereafter, the items are selected regularly at predetermined intervals. It has been established that this method of systematic sampling is quite good approximation to the simple random sampling provided there is no deliberate attempt to manipulate the sequence of the items in the lot in any desired manner while the lot is presented for inspection.

10.2.4 Cluster Sampling

When the lot submitted for inspection consists of certain groups of clusters of items, it may be advantageous and economical to select a few clusters of items and then examine all the items in the selected clusters.

For example, when the lot consists of items packed in cartons and it is either impracticable or costly to repack the cartons opened for selecting sample items, then only few cartons are selected at random without replacement in the first instance and all the items in the selected cartons are inspected.

10.2.5 Two Stage Sampling

When the lot submitted for inspection consists of larger number of packages each consisting of a number of items, it may not be economical to select few packages and inspect all the items in these packages (as in case of cluster sampling). In such cases, the sample is selected in two stages. In the first stage a desired number of packages (primary units) are selected at random and in the second stage, the required number of items are chosen at random from the selected primary units.

Sampling inspection may be carried out as

(i) Sampling by attributes

(ii) Sampling by variables.

In Sampling by attributes, the items are classified as non-defectives or defectives according to one or more characteristics. The decision on acceptance or rejection is made depending upon the number of defectives found in the sample (or in some cases in more than one sample). As the degree of defectiveness is not taken into account, sampling by attributes can be applied to both measurable or non-measurable characteristics. Go and no-go gauges can be used for the inspection of measurable characteristics it is not necessary to register the measurements, but to detect only the number of defective items found in the sample.

In sampling by variables, it is necessary to take measurements on the characteristics inspected. A statistical treatment will then be used to decide the acceptance or rejection of the lot.

Symbols used in Relation to Sampling Acceptance

Following symbols and terms are used in connection with acceptance sampling :

N = number of pieces in a given lot (Lot size)

n = number of pieces in a sample (Sample size)

M = number of defective pieces in a given lot of size N.

m = number of defective pieces in a given sample of size n.

c = acceptance number, the maximum allowable number of defective pieces in a sample of size n.

p = fraction defective. In a given submitted lot $p = M/N$ in a given sample $p = m/n$.

p' = true process average fraction defective of a product submitted for inspection.

$\overline{p}$ = average fraction defective in observed samples.

p_a = probability of acceptance.

p_c = consumer's risk, the probability of accepting product of some stated quality. The probability of rejecting product of some stated quality is referred to as producer's risk, $(1 - P_a)$. $P_{0.95}, P_{0.50}, P_{0.15}$ etc.

= fraction defective having a probability of acceptance of 0.95, 0.50, 0.15 etc. under any given acceptance criteria.

10.3 THE OPERATING CHARACTERISTIC (OC) CURVE

The operating characteristic curve for an attribute sampling plan is a graph of fraction defective in a lot against the probability of acceptance. For any fraction defective p' in a submitted lot, the OC curve shows the probability P_a that such a lot will be accepted by the sampling plan. In a single sampling plan three parameters are specified.

N = lot size from which the samples are drawn

n = sample size

C = acceptance number

If the sampling plan is $\begin{cases} N = 100 \\ n = 5 \\ C = 2 \end{cases}$

It means take a random sample of 5 from a lot of 100, if the sample contains more than 2 defectives, reject the lot otherwise accept the lot. By changing the parameters N, n and C different sampling plans can be obtained. For different sampling plans the OC curve will differ. To construct an OC curve, we should know the mathematical probability of accepting lots with varying per cent defectives. This can be obtained from the table of Poisson's distribution (given in Appendix).

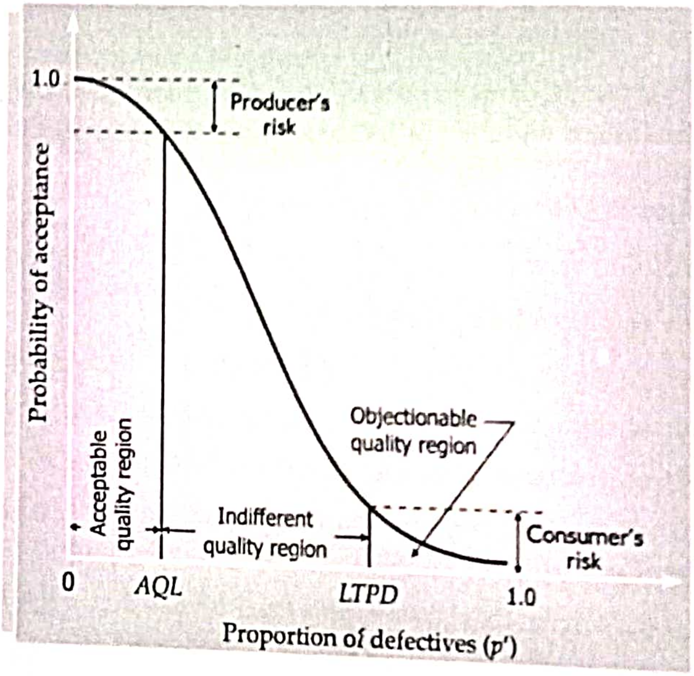


Fig. 10.1 *OC* curve.

10.4 PRODUCER'S RISK AND CONSUMER'S RISK

There are always two parties to an acceptance procedure, the party submitting the product for acceptance and the party for whom the decision is made regarding acceptance or rejection (*i.e.*, producer and consumer). Figure 10.2 shows an ideal '*OC*' curve for a case where it is desired to accept all lots of 2% defectives. All lots greater than 2% defectives have a probability of acceptance of zero. Actually, however, no sampling plan exists that can discriminate perfectly, there always remains some risk that a good lot will be rejected or that a bad lot will be accepted.

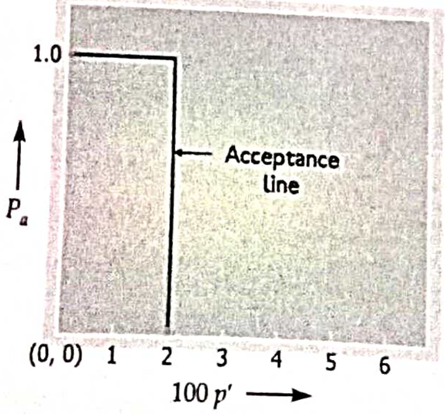


Fig. 10.2 Ideal *OC* curve.

10.4.1 Producer's Risk

Since, ideal sampling plan which will satisfy both the consumer and producer is not possible, some compromise has to be made and they have to tolerate certain risk.

If the quality is good still from sampling plan some lots are rejected the producer has to suffer. The producer's risk is the probability to rejecting a good lot which otherwise would have been accepted. So the producer should be protected against the rejection of relatively better products. The producer can decrease his risk by producing products at a better quality level than the specified *AQL* (explained latter) depending on other economical considerations.

Saying a producer's risk $\alpha = 0.05$ means that in the long run about 1 lot in 20 will be rejected provided that the lots are coming from a process controlled at AQL quality level.

10.4.2 Consumer's Risk

If the quality is bad still from the sampling plan some lots are to be accepted the consumer will suffer. Consumer's risk is the probability of defective lots being accepted which otherwise would have been rejected. Saying that $P_{0.10} = 2.5\%$ means the consumer does not want a worse quality containing more than 2.5% defectives and he would at the most accept 10% of lots containing 2.5% defectives.

At first impression, it appears that the producer and consumer should have completely opposite viewpoints towards the selection of sampling plans (*i.e.*, the intersects are conflicting). But, more critical consideration will show that, there is a continuing relationship between producer and consumer ; substantial rejection of good products in the effort to exclude bad products are not necessarily in the interest of consumer. The consumer is interested in quality, he is also interested in cost. In the long run the costs incident to the rejection of good products tend to be passed on by the producer to the consumer. Secondly, any good product that he rejects is not available for his immediate use.

10.5 QUALITY INDICES FOR ACCEPTANCE SAMPLING PLANS

1. Acceptable Quality Level (AQL)

It represents the maximum proportion of defectives which the consumer finds definitely acceptable. AQL can also be defined as the maximum per cent defectives that for the purpose of sampling inspection can be considered satisfactory as a process average. It is the fraction defective that can be tolerated without any serious effect upon further processing or an customer relations.

As an AQL is an acceptable quality level, the probability of acceptance for an AQL lot should be high. In fact the producer's safe point is termed as AQL.

2. Rejectable Quality Level (RQL)

It is also called as Lot Tolerance Percent Defective (LTPD). This is a definition of unsatisfactory quality. It represents the proportion of defectives which the consumer finds definitely unacceptable. As RQL is an unacceptable quality level, the probability of acceptance for an RQL lot should be low. The probability of accepting a lot at RQL level represents consumer's risk.

3. Indifference Quality Level (IQL)

This is a quality level somewhere between the AQL and RQL. It is frequently defined as the quality level having a probability of acceptance of 0.50 for a given sampling plan.

4. Average Outgoing Quality (AOQ)

It represents the average % defective in the outgoing products after inspection, including all accepted and all rejected lots which have been 100% inspected and defectives replaced by non-defectives.

So, for a given fraction defectives, the lot accepted as a result of first sampling inspection will have a fraction defective p', the rejected lots are subjected to 100% inspection and rectification (defective articles are either replaced or corrected) the AOQ will therefore be less than p'.

Let, $n =$ sample size,

$N =$ lot size,

$k =$ number of lots submitted for acceptance,

$p' =$ fraction defective,

$p_a =$ probability of acceptance

$(1 - P_a) =$ probability of rejection.

Then, the proportion of the lots accepted $= P_a k$.

Proportion of the lots rejected $= k(1 - P_a)$ which are subjected to 100% inspection and rectification.

Number defects originally present in k lots $= k.N.P'$.

Number of defectives in the outgoing lots

$$= p'.P_a.k(N - n)$$

Total number of defectives

$$= p'.P_a k(N - n) + k(n)(0)$$

$$\therefore \quad \text{AOQ} = \frac{P_a p'(N - n) + k(n)(0)}{k.N} = P_a \cdot p'\left(\frac{N - n}{N}\right)$$

If the sample size is much less as compared to lot size

$$\text{AOQ} = P_a \cdot p'$$

The calculation of average outgoing quality gives the expected quality in the long run. Over a short period, the outgoing quality may be better or worse than the long-run average.

10.6 AVERAGE OUTGOING QUALITY LIMIT (AOQL)

For any given n and c there is a maximum value of AOQ beyond which the average fraction of defective passed forward will not rise, no matter how bad the quality of the lots when they arrive at inspection. Because, when incoming quality is perfect, outgoing quality must likewise be perfect. However, when incoming quality is very bad, outgoing quality will also be perfect, because the sampling plan will cause all lots to be rejected, which are subjected at 100% inspection and rectification. Thus at either extreme – incoming quality very good or very bad – the outgoing quality will tend to be very good. Between these extremes is the point at which the percent of defectives in the outgoing product will reach its maximum. This point is known as the average outgoing limit (AOQL). Therefore, AOQL is the maximum possible value of the average percentage defectives in the outgoing products after inspection and rectification.

For acceptance/rectification scheme the OC curve is used but for rectification scheme the curve of AOQ plotted against p' is used.

Any acceptance/rectification plan guarantees that regardless of the incoming quality submitted, the outgoing quality in the long run will not be worse than the AOQL.

Table 10.1 shows a common type of approximate calculation to determine the AOQL. This table refers to the plan, $n = 75$ and $c = 1$ when N is large in comparison with n. The right hand column gives average outgoing quality (AOQ) for each assumed per cent defective in submitted lots. The maximum value of AOQ is 1.12% occurring when submitted lots are 2.2% defective. This maximum value of AOQ is the AOQL.

Table 10.1 : *AOQ from $n = 75$, $c = 1$ for Acceptance/Rectification plan*

Percent defectives in submitted lots	Probability of acceptance	AOQ = 100 $p'.P_a$
0.4	0.963	0.385
1.0	0.827	0.827
1.4	0.718	1.005
1.6	0.663	1.061
1.8	0.610	1.098
2.0	0.558	1.116
2.2	0.509	1.120
2.5	0.441	1.102
3.0	0.343	1.029
3.5	0.262	0.917
4.0	0.199	0.796

The above computed figures are plotted in Fig. 10.3. The curve plotted is known as AOQ curve.

The line AOQ $= p'$ represents what would happen if there were no rectification. For any given values of 'n' and 'c', the AOQ curve falls below the line AOQ $= p'$. As p' increases, the proportion of rectified lots increases and hence the AOQ curve falls below the line AOQ $= p'$.

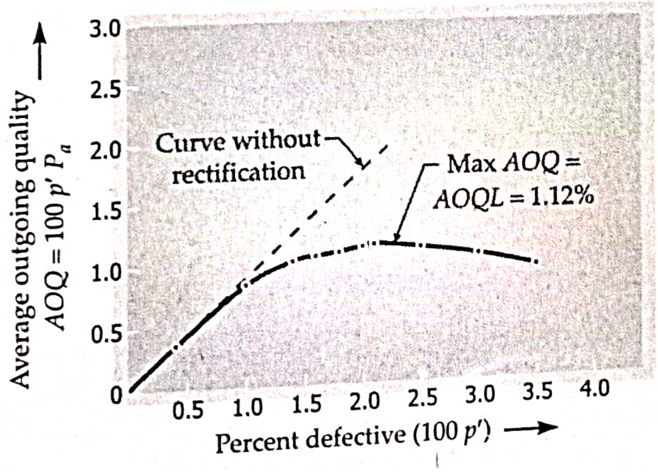


Fig. 10.3 Average outgoing quality for acceptance/rectification plan.

10.7 STEPS IN THE DESIGN OF AN ACCEPTANCE PLAN

The protection provided to the consumer and the producer is completely described by the OC curve of the plan. When the acceptance plan is designed, it calls for designing a plan whose OC curve will pass through two stipulated points agreed upon by the consumer and producer. The first of these is the acceptable quality level (AQL) representing the maximum proportion of defectives which the consumer finds definitely acceptable. The second, is the Lot Tolerance Percent Defective (LTPD) which represents the proportion of defectives which the consumer finds definitely unacceptable as described earlier.

Accordingly, an OC curve can be divided into three regions, (zones) as shown in Fig. 10.1.

1. Acceptable Quality Region
2. Indifferent Quality Region
3. Objectionable Quality Region

The consumer is willing to accept all lots which have a fraction defective represented by AQL or less and reject all lots which have a fraction defectives represented by LTPD or more. So far, as the lots with a fraction defective between AQL and LTPD are concerned, in quality control terminology, the consumer is said to be "indifferent" to them. But to take advantage of the economies made available by acceptance sampling, the consumer must be willing to assume some risk of accepting some lots which he finds definitely unacceptable, that is, whose proportion of defectives equals or exceeds the LTPD. Also, the producer must be willing to assume some risk of having some of definitely acceptable lots rejected that is some of the lots whose proportion of defectives equals or is less than the AQL. Usually the consumer's and producer's risks are agreed upon and explicitly stated in quantitative terms. For example, the producer may agree to accept the risk that 10 per cent of the lots with a fraction defective of say 0.01 (AQL) will be rejected, and the consumer may agree to accept the risk that 5 per cent of the lots with a fraction defective of 0.25 (LTPD) will be accepted.

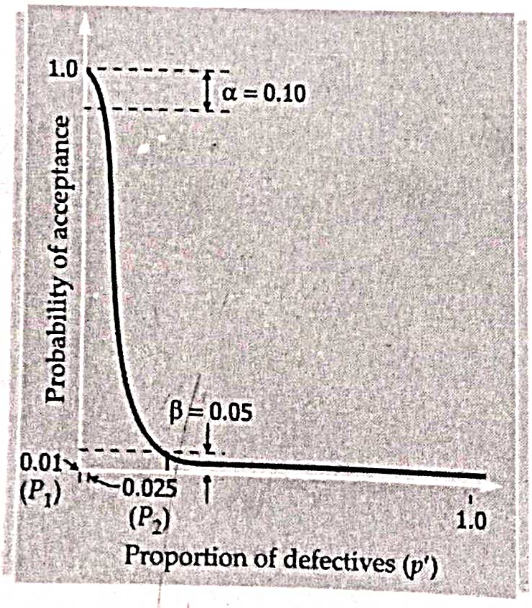


Fig. 10.4 OC curve ($\alpha = 0.10, \beta = 0.05$).

Let us discuss the hypothetical case we have presented in light of Fig. 10.4. In our case, the consumer defined the AQL as being 0.01 and the LTPD as being 0.25. The producer is willing to accept a 10 per cent risk that lots with a proportion of defectives of 0.01 would be rejected and the consumer is willing to accept a 5% risk that lots with a proportion of defectives of 0.25 would be accepted. These conditions and the symbols commonly used to represent them are :

$$p_1 = \text{AQL} = 0.01$$
$$p_2 = \text{LTPD} = 0.25$$
$$\alpha = \text{producer's risk} = 0.10$$
$$\beta = \text{consumer's risk} = 0.05.$$

When we say that the producer's risk is to be 0.10, this means that the probability of accepting a lot which contains 1 percent defectives should be 1 minus 0.10 or 0.90. Also, a consumer's risk of 0.05 means that there should be probability of 0.05 of accepting a lot which contains 25 per cent defectives.

The OC curve for this set of conditions would have the general form shown in Fig. 10.4.

In effect, what the consumer and producer agree upon is a set of two points for the OC curve ; these points are $(p_1, 1-\alpha)$ and (p_2, β). In other words, they agree that any acceptance plan whose OC curve passes through these pre-selected points will be satisfactory. A trial and error approach must be employed in this design. To illustrate this, the question may be ; given a lot size of 100, what should be the sample size and acceptance number ? The only way in which this can be determined is by arbitrary selecting a sample size and an acceptance number, calculating the respective probabilities of acceptance for the AQL and LTPD, and hoping that they coincide with $(1-\alpha)$ and β. Chances are that they will not. Another combination of sample size and acceptance number must then be selected, and the procedure repeated until they coincide.

The problem of design is further complicated by the fact that even if p_1, p_2, α and β remain fixed, a different plan would be required for each lot size because the lot size enters into the probability computations.

Characteristics of OC Curve

1. The Operating Characteristic (OC) Curve of an Acceptance Sampling plan shows the ability of the plan to distinguish between good lots and bad lots.

2. Sampling acceptance plans with same per cent samples gives very different quality protection. For example, the curves (Fig. 10.5) show that lots which are 4% defective will be accepted 81% of the time using a 10% sample from a lot of 100, 42% of the time using a 10% sample from a lot of 200, and less than 2% of the time by a 10% sample from a lot of 1000, assuming an acceptance number of zero in all cases.

   Obviously a producer making a product 4% defective would have a strong motive for trying to have his product inspected in lots of 50 rather than in lots of 100.

3. Fixed sample size tends towards constant quality protection. It is the absolute size of the sample rather than its relative size that determines the quality protection given by an acceptance sampling plan.

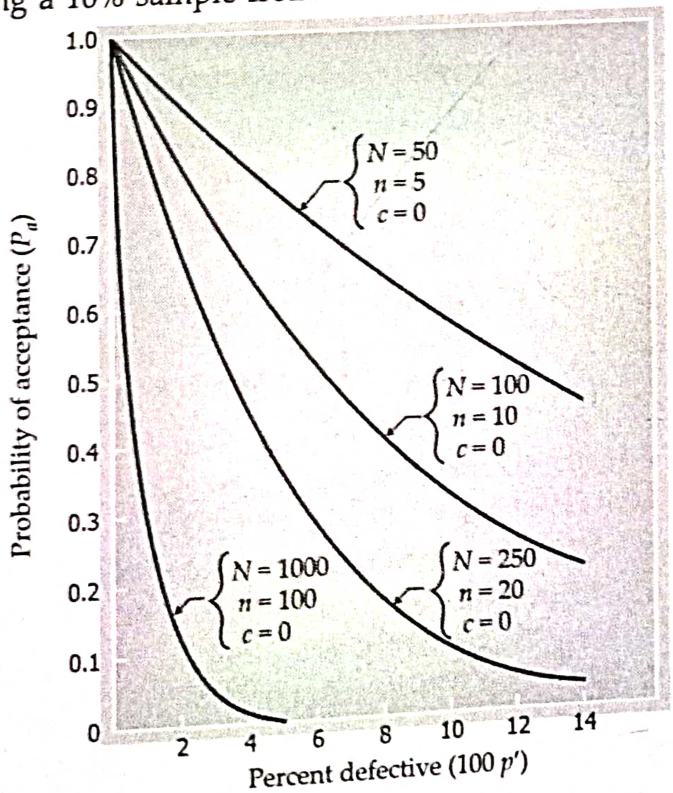


Fig. 10.5 Comparison of OC curve.

Figure 10.6 illustrates this point.

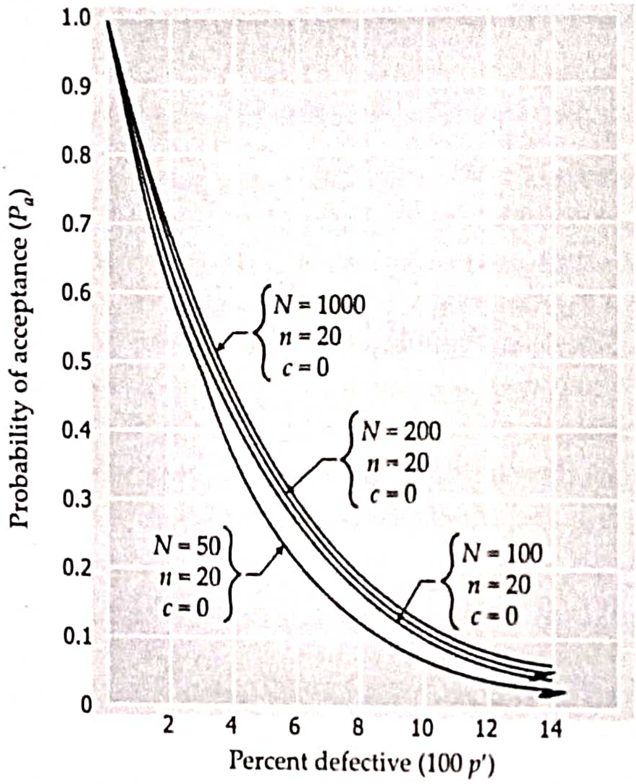


Fig. 10.6 Comparison of OC curve for four sampling plan.

4. The OC curves of plans with acceptance numbers greater than zero are superior t those of comparable plans with acceptance number of zero.

   With fixed value of N and $c \propto n$, larger the value of n, th e better is the ability of th plan to discriminate between good and bad lots. Figure 10.7 illustrates this point.

5. The larger the sample size and acceptance number, the steeper the slope of the OC curve. Figure 10.7 shows that the larger sample size which protects the consumer against the acceptance of relatively bad lots also gives better protection to the producer against rejection of relatively good lots.

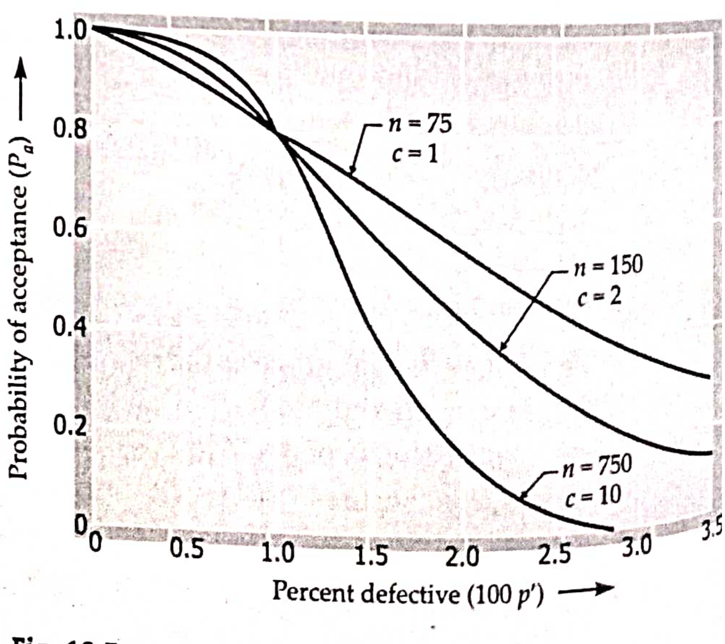


Fig. 10.7

10.8 SAMPLING PLANS

Sampling plans may be grouped into *three* categories :

- Single sampling plan
- Double sampling plan
- Multiple sampling plan

10.8.1 Single Sampling Plan

When a decision on acceptance or rejection of the lot is made on the basis of only one sample, the acceptance plan is known as a single sampling plan.

In a single sampling plan three numbers are specified

N = lot size, from which the sample is drawn.

n = sample size.

c = acceptance number.

If the sampling plan is

$$\left.\begin{aligned} N &= 50 \\ n &= 5 \\ c &= 1 \end{aligned}\right\}$$

It means, take a sample of 5 items from a lot of 50, if the sample contains more than 1 defective reject the lot, otherwise accept the lot.

Inspect a sample of n pieces

⇩

If number of defectives

Does not exceed 'c'	Exceeds 'c'
⇩	⇩
Accept the Lot	Reject the Lot

Single Sampling Plan

10.8.2 Double Sampling Plan

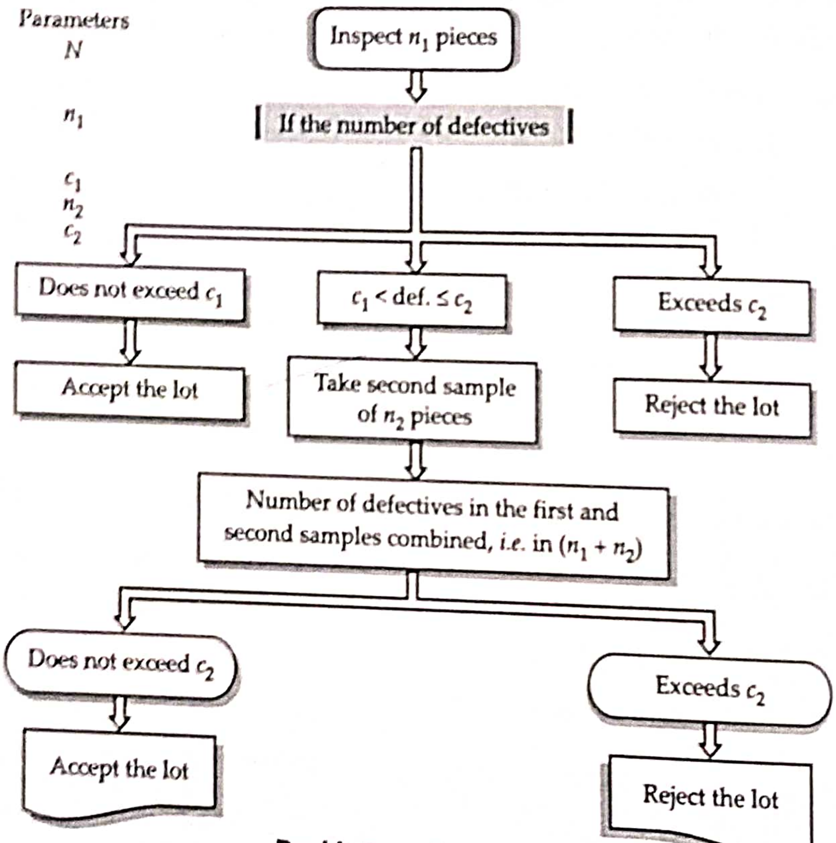
In double sampling plan the decision on acceptance or rejection of the lot is based on two samples. A lot may be accepted at once if the first sample is good enough or rejected at once if the first sample is bad enough. If the first sample is neither good enough nor bad enough, the decision is based on the evidence of first and second sample combined.

Parameters n_1 = number of pieces in the first sample

c_1 = acceptance number for the first sample, *i.e.*, the maximum number of defectives that will permit the acceptance of the lot on the basis of the first sample.

$$n_2 = \text{number of pieces in the second sample.}$$
$$n_1 + n_2 = \text{number of pieces in the two samples combined.}$$
$$c_2 = \text{acceptance number for the two samples combined.}$$

i.e., the maximum number of defectives that will permit the acceptance of the lot on the basis of the first and second sample combined.



Double Sampling Plan

The following example illustrates the use of a double sampling plan. Let,

and
$$\left.\begin{array}{ll} N = 500 & n_1 = 35 \\ c_1 = 1 & n_2 = 50 \\ c_2 = 4 & \end{array}\right\}$$

This may be interpreted as follows :

1. Take a first sample of 35 items from a lot of 500 and inspect.
2. Accept the lot on the basis of first sample, if it contains 0 and 1 defective.
3. Reject the lot on the basis of first sample if it contains more than 4 defectives.
4. Take a second sample of 50 items if the first sample contains 2, 3 or 4 defectives.

5. Accept the lot on the basis of first and second sample combined, if the combined sample of 85 items contains 4 or less defectives.
6. Reject the lot on the basis of combined sample if the combined sample contains more than 4 defectives.

The lot thus may be accepted in the following ways :

(*a*) 0 or 1 defective in the first sample (without taking second sample).
(*b*) 2 defectives in the first sample followed by 0, 1 or 2 defectives in the second sample.
(*c*) 3 defectives in the first sample followed by 0 or 1 defective in the second sample.
(*d*) 4 defectives in the first sample followed by 0 defective in the second sample.

The probability of accepting the lot is the sum of the probabilities of these different ways in which it may be accepted.

10.8.3 Multiple Sampling Plan

The phrase multiple sampling is generally used when three or more samples of stated size are permitted and when the decision on acceptance or rejection must be reached after a stated number of samples.

The phrase sequential sampling is generally used when a decision is possible after each item has been inspected and when there is no specified limit on the total number of units to be inspected. However, some writers use the two phrases interchangeably.

A multiple sampling procedure can be represented on a table such as the following :

Sample	Sample size	Combined samples		
		Size	Acceptance number	Rejection number
First	n_1	n_1	c_1	r_1
Second	n_2	n_1+n_2	c_2	r_2
Third	n_3	$n_1+n_2+n_3$	c_3	r_3
Fourth	n_4	$n_1+n_2+n_3+n_4$	c_4	r_4
Fifth	n_5	$n_1+n_2+n_3+n_4+n_5$	c_5	c_5+1

Multiple Sampling Plan

A first sample of n_1 is drawn, the lot is accepted if there are no more than c_1 defectives, the lot is rejected if there are more than r_1 defectives. Otherwise a second sample of n_2 is drawn the lot is accepted if there are no more than c_2 defectives in the combined sample of n_1+n_2. The lot is rejected if there are more than r_2 defectives in the combined sample of n_1+n_2. The procedure is continued in accordance with the above table. If by the end of the fourth sample, the lot is neither accepted nor rejected, a sample n_5 is drawn. The lot is accepted if the number of defectives in the combined sample of $n_1+n_2+n_3+n_4+n_5$ does not exceed C_5. Otherwise the lot is rejected. Note that $c_1 < c_2 < ... < c_5$ and $c_i < r_i$ for all *i*.

A multiple sampling plan will generally involve less total inspection, than the corresponding single or double sampling plan guranteeing the same protection. But they usually require higher administrative costs and higher calibre inspection personnel may be necessary to guarantee proper use of the plans.

10.9 DESIGN OF ITEM BY ITEM SEQUENTIAL SAMPLING PLANS

A certain product is subjected to lot by lot acceptance or rejection on the basis of a destructive test applied to a sample. The conditions of the test are considerably more severe than it is expected in practice. All items tested are damaged to such an extent that they are of no further use. In order to keep the number of items tested to a minimum consistent with the desired quality protection, it is necessary to design an item by item sequential sampling plan.

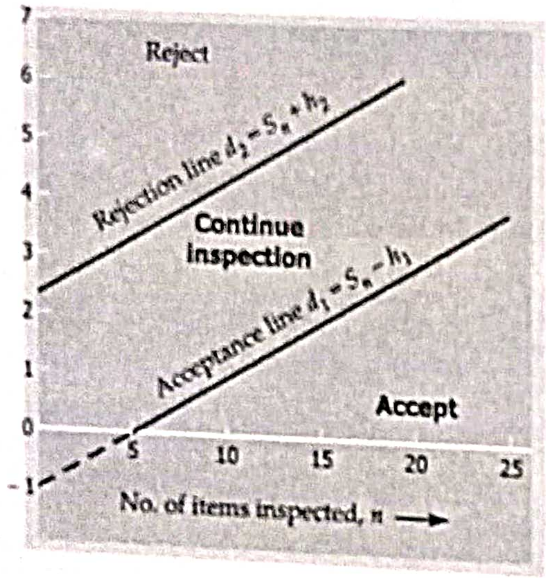


Fig. 10.8 Graphical representation of item by item of a sequential plan.

Such a plan may be designed, so that OC curve passes through any two points desired.

Figure 10.8 gives a graphical representation of an item by item sequential plan. The plan is fully defined by the equation of the rejection line, $d_2 = S_n + h_2$, and the acceptance line, $d_1 = S_n - h_1$. To compute S, the slope of these lines and h_1 and h_2, the intercepts, certain auxiliary symbols, g_1, g_2 a and b, are used.

The symbol α represents $(1 - p_a)$ for a lot quality p_1.

β represents P_a for a lot quality p_2.

The necessary computations are as follows :

$$g_1 = \log \frac{p_2}{p_1} \qquad S = \frac{g_2}{g_1 + g_2}$$

$$g_2 = \log \frac{1 - p_1}{1 - p_2} \qquad h_1 = \frac{b}{g_1 + g_2} \text{ and}$$

$$a = \log \frac{1 - \beta}{\alpha} \qquad h_2 = \frac{a}{g_1 + g_2}$$

$$b = \log \frac{1 - \alpha}{\beta}$$

The purpose of adopting such an item-by-item plan is to reduce the average amount of inspection below the amount that would be obtain with a multiple sampling plan giving about the same quality protection.

Characteristics of a Good Acceptance Plan

An acceptance sampling plan should have these characteristics :

1. The index (AQL, AOQL, etc.) used to define 'quality' should reflect the needs of the consumer and producer and not be chosen primarily for statistical convenience.
2. The sampling risks should be known in quantitative terms (the OC curve). The producer should have adequate protection against the rejection of good lots ; the consumer should be protected against the acceptance of bad lots.
3. The plan should minimize the total cost of inspection.
4. The plan should be flexible enough to reflect changes in lot size, quality of product submitted and any other pertinent factors.
5. The measurements required by plan should provide information useful in estimating individual lot quality and long run quality.

Comparison between Single, Double and Multiple Sampling Plans

	Single sampling plan	*Double sampling plan*	*Multiple sampling plan*
1. *Average number of pieces inspected per lot.*	Largest	In between single and multiple plans	Lowest
2. *Cost of administration*	Lowest	In between single and multiple plans	Largest
3. *Information available regarding prevailing quality level.*	Largest	In between single and multiple plans	Least
4. *Acceptability to producers.*	Less (gives only one chance of passing the lot)	Most acceptable	Indecision is continued for a long term.

Compared with regards to all the four aspects double sampling plan is best. Where the cost of inspection is high, we choose multiple sampling plans. If the decision is taken on single sampling plan the lot has not to wait, but in multiple sampling plan indecision is continued for a long period and lot has to wait, therefore, more storage space is necessary to store the items until the decision is taken.

Designing Single Sampling Attribute Plans Having a Stated Value of $P_{0.50}$

The quality $P_{0.50}$, for any lot-by-lot acceptance sampling plan is the lot or process quality that has a probability of acceptance of 0.50. $P_{0.50}$ is called as the indifference quality.

The single acceptance criteria is designed for any desired $P_{0.50}$ using the following approximate formula :

$$n = \frac{c+0.67}{P_{0.50}}$$

Suppose that lots 2.5% defective or better are considered to be acceptable but that it is desired to reject lots that are any worse. With this quality standard, we can find a set of single sampling plans for which a lot 2.5% defective will have a P_a of 0.50. Assuming value of acceptance number c from 0 up to any desired number and solving for sample size n, using $P_{0.50}$ as 0.025. The resulting family of acceptance plan will be as follows :

c	n	c	n	c	n
0	27	4	187	8	347
1	67	5	227	9	387
2	107	6	267	10	427
3	147	7	307	11	467

The OC curves of these 12 single sampling plans will pass through the point $P_a = 0.50$, $p=0.025$, but the plans having the larger sample size gives better discrimination between lots that are somewhat better than 2.5% defective and lots that are somewhat worse.

A choice among the various plans requires balancing the extra inspection costs of larger samples against the advantage of better assurance that a lot meeting the quality standard will be accepted and one failing to meet it will be rejected.

10.10 THE AVERAGE TOTAL INSPECTION CURVE (ATI CURVE) [FIG. 10.9]

The average total amount of inspection called under a programme of 100% inspection of rejected lots will depend on the quality of the material submitted. It the material contains no defective items, there will be no rejections and the amount of inspection per lot will be n. If the items are all defective every lot will be subjected to 100% inspection, in which case the amount of inspection per lot will be N, the size of the lot. If material is between zero defective and 100% defective, the average amount of inspection per lot will be between n and N.

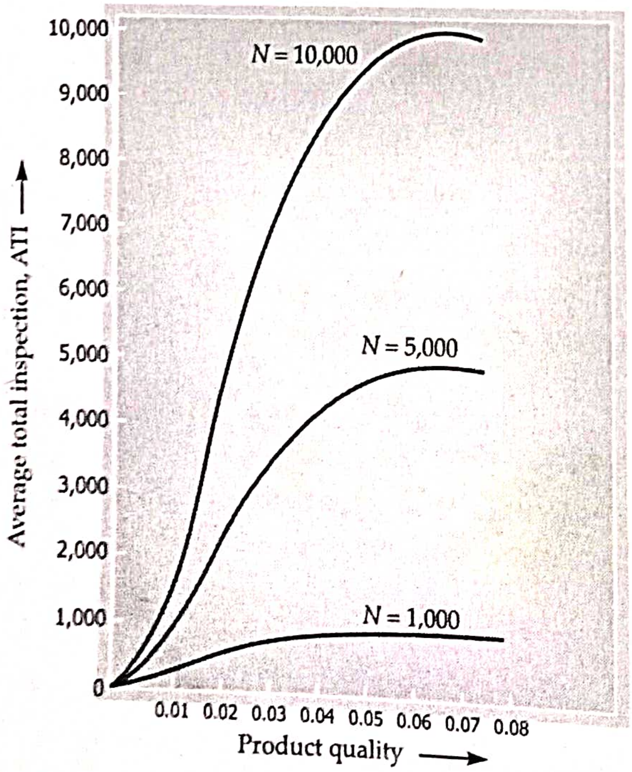


Fig. 10.9 Average total inspection curve for the sampling plans $n = 100$, $c = 2$, lots of 1,000, 5,000 and 10,000.

If the product is of quality p' and the probability of lot acceptance is P_a, then, on the average, the amount of inspection per lot will be, $\text{ATI} = n + (1 - P_a)(N - n)$.

Where, ATI stands for Average Total Inspection.

For example, if $N = 10{,}000$, $n = 100$, $c=2$ and if the fraction defective of material submitted for inspection $(p') = 0.05$ then, for all the OC curves, $P_a = 0.12$ and $(1 - P_a) = 0.88$. Hence,

$$\text{ATI} = 100 + (0.88)(10{,}000 - 100) = 8{,}812$$

Figure 10.9 shows the Average Total Inspection curves for the sampling plan $n=100$, $c=2$, lots of 1,000, 5,000 and 10,000.

The Average Total Inspection (ATI) for Double Sampling Plans

Average total inspection under a double sampling plan in which rejected lots are inspected 100 per cent is given by the formula

$$ATI = n_1 P_{a1} + (n_1 + n_2) P_{a2} + N(1 - P_a)$$

in which P_{a1} is the probability of acceptance on the first sample, P_{a2} is the probability of acceptance on the second sample. P_a is the probability of final acceptance and $(1 - P_a)$ is the probability of final rejections since $P_a = P_{a1} + P_{a2}$, the above formula can be written as,

$$ATI = n_1 + n_2(1 - P_{a1}) + (N - n_1 - n_2)(1 - P_a)$$

The reasoning underlying the above formula is as follows :

1. n_1 items only will be inspected if the lot is accepted on the first sample and the chance of this is P_{a1}.
2. $(n_1 + n_2)$ items will be inspected if the lot is accepted on the second sample, and the chance of this is P_{a2}.
3. N items will be inspected if the lot is rejected and the chance of this is $(1 - P_a)$.

To illustrate the use of the above formula for computing average total inspection, consider the computation of average total inspection for the plan $n_1 = 50$, $n_2 = 100$, $c_1 = 2$ and $c_2 = 6$ and lot size $N = 1600$. Take $p' = 0.06$. Then from Molina's tables we find that the probability of acceptance on the first sample (*i.e.*, probability of 2 or less defective items out of 50) is 0.423 and the probability of rejection on the first sample (*i.e.*, probability of more than 6 defective items out of 50) is 0.0034. The probability of acceptance on the combined samples is the sum of

1. the probability of 3 defective units in the first sample times the probability of 3 or less defective units in the second. For a lot fraction defective of 0.06, this is equal to $0.22 \times 0.15 = 0.033$.
2. the probability of 4 defective units in the first sample times the probability of 2 or less defective units in the second sample $= 0.17 \times 0.06 = 0.010$.
3. the probability of 5 defective units in the first sample times the probability of 1 or less defective units in the second sample
$$= 0.010 \times 0.02 = 0.002.$$
4. the probability of 6 defective units in the first sample times the probability of zero defective units in the second sample
$$= 0.05 \times 0.002 = 0.0001.$$

The total of these four probabilities $= 0.045$.

Therefore, the total probability of acceptance for a lot fraction defective of $0.06 = 0.423 + 0.045 = 0.468 = P_a$.

Hence, for $p' = 0.06$.

$$ATI = 50 + 100(1 - 0.423) + (1600 - 150)(1 - 0.468)$$
$$= 50 + 58 + 722 = 880.$$

Solved Problems

Problem 10.1 *A single sampling plan uses a sample size of 15 and an acceptance number 1. Using hypergeometric probabilities, compute the probability of acceptance of lots of 50 articles 2% defective.*

Solution. Given $N = 50, \quad n = 15, \quad c = 1, \quad p' = 0.02$ (2%)

Number of defective articles

$$= 0.02 \times 50 = 1$$

Number of non-defective articles

$$= 50 - 1 = 49.$$

$$P_0 = \frac{^{49}C_{15}}{^{50}C_{15}}$$

$$= \frac{49!}{34! \times 15!} \times \frac{35! \times 15!}{50!} = 0.7$$

$$P_1 = \frac{^{1}C_1 \times {}^{49}C_{14}}{^{50}C_{15}}$$

$$= \frac{49!}{35! \times 14!} \times \frac{35! \times 15!}{50!} = 0.3$$

$\therefore$ Probability of acceptance $= 0.7 + 0.3 = 1.$

Problem 10.2 *The single sampling plan in problem 1 is used with relatively large lot, compute the probability of acceptance using Poisson's approximation.*

Solution. Now, $n.p' = 15 \times 0.02 = 0.3$

Using Poisson's approximation from Table G (Appendix)

P_1 or less $= 0.963 =$ probability of acceptance.

Problem 10.3 *A double sampling plan is as follows :*

(*a*) *Select a sample of 2 from a lot of 20. If both articles inspected are good, accept the lot. If both are defective, reject the lot. If 1 is good and 1 defective, take a second sample of one article.*

(*b*) *If the article in the second sample is good, accept the lot. If it is defective reject the lot.*

If a lot 25% defective is submitted, what is the probability of acceptance ? Compute this by the method that is theoretically correct rather than an approximate method.

Solution. Since the lot is 25% defective,

Number of defective articles in the lot $= 20 \times 0.25 = 5$

and Number of non-defective articles $= 20 - 5 = 15.$

Accept the lot,

(*a*) When the number of defectives in the first sample is zero.

(b) Number of defectives in the first sample $=1$ and non-defective in the second sample is zero.

(i) Probability of zero defective in the first sample

$$P_I(0) = \frac{{}^{15}C_2 \times {}^{5}C_0}{{}^{20}C_2} = 0.553$$

(ii) Probability of 1 defective in the first sample

$$P_I(1) = \frac{{}^{15}C_1 \times {}^{5}C_1}{{}^{20}C_2} = 0.395$$

Now for second sample,

$$N = 1$$

$n_2 = 1$ Defective articles $= 4$

$c_2 = 1$ Non-defective articles $= 14.$

(iii) Probability of 0 defective in the second sample

$$P_{II}(0) = \frac{{}^{14}C_1 \times {}^{4}C_0}{{}^{18}C_1} = 0.777$$

Now, probability of zero defective in the first sample

$$= P_I(0) = 0.553$$

Probability of 1 defective in the first sample, with zero defective in second sample

$$= 0.395 \times 0.777$$

$\therefore$ Probability of acceptance

$$P_a = 0.553 + 0.395 \times 0.0777 = 0.859.$$

Problem 10.4 *In a double sampling plan, $N = 5{,}000$, $n_1 = 100$, $c_1 = 0$, $n_2 = 100$ and $c_2 = 1$.*

(a) *Use Poisson's table to compute the probability of acceptance of a 1% defective lot.*

(b) *Assume that a lot rejected by this sampling plan will be 100% inspected. What will be the AOQ if the submitted product is 1% defective ? Considering both the inspection of rejected lots, what will be the average number of articles inspected per lot if the submitted product is 1% defective ?*

Solution. The double sampling plan is,

$$N = 5{,}000\,; \quad n_1 = 100\,; \quad c_1 = 0\,; \quad n_2 = 100\,; \quad c_2 = 1$$

Number of defective articles

$$= 5{,}000 \times 0.01 = 50$$

Number of non-defective articles

$$= 5000 - 50 = 4950.$$

(a) By using Poisson's table

$$n_1 \times p' = 100 \times 0.01 = 1$$

$$\therefore \quad P_I(0) = 0.368 \text{ (from table G)}$$

Now, accept the lot, when

1. there is zero defective in the first sample.
2. there is one defective in first sample and zero defective in the second sample.

$$P_I(1) = 0.736 - 0.368 = 0.368$$

Now, for second sampling plan

$$N = 5000 - 100 = 4900$$

Number of defectives = 49

$$\therefore \quad p' = \frac{49}{4900} = 0.01$$

$$n_2 . p' = 100 \times 0.01 = 1$$

$$P_{II}(0) = 0.368$$

$\therefore$ Probability of acceptance

$$= P_a = P_1(0) + P_I(1) \times P_{II}(0)$$

$$= 0.368 + 0.368 \times 0.368$$

$$= 0.368 + 0.1354 = 0.5034$$

(b) $$\text{AOQ} = P_a . p'\left(\frac{N-n}{N}\right) \cong P_a . p'$$

$$= 0.5034 \times 0.01 = 0.005034 = 0.5034\%.$$

Average number of articles inspected

= Articles accepted on the basis of first sample + Articles accepted on the basis of first and second samples combined + Rejected articles

$= P_1(0) \times n_1 + P_1(0) \times P_1(1)(n_1 + n_2) + (1 - P_a) \times$ lot size

$= 0.368 \times 100 + 0.368 \times 0.368 \times 200 + (1 - 0.5034) \times 5{,}000$

$= 2547.$

Problem 10.5 *The lot size N is 2,000 in a certain AOQL inspection procedure. The desired AOQL of 2% can be obtained with any one of the three sampling plans. These are :*

(i) $n = 65, c = 2$ *(ii)* $n = 41, c = 1$ *and* *(iii)* $n = 18, c = 0$.

If a large number of lots 0.3% defective are submitted for acceptance, what will be the average number of units inspected per lot under each of these sampling plans ?

Solution. (i) Now

$$np' = \frac{65 \times 0.3}{100} = 0.195$$

From table G

for $\qquad np' = 0.195$

and $\qquad c = 2\ ;\ P_a = 0.999$

$\therefore$ Average number of items inspected per lot

$$= P_a \cdot n + (1 - P_a) N$$

$$= 0.999 \times 65 + 0.001 \times 2000 = 66.935 \text{ (say 67)}$$

(ii) $np' = \dfrac{41.0.3}{100} = 0.123$

From table G,

for $\qquad np' = 0.123$

and $\qquad c = 1,\ P_a = 0.993.$

$\therefore$ Average number of items inspected per lot

$$= 0.993 \times 41 + 0.007 \times 2{,}000 = 54.959 \text{ (say 55)}$$

(iii) $np' = \dfrac{18 \times 0.3}{100} = 0.054$

for $\qquad np' = 0.054$

and $\qquad c = 0,\ P_a = 0.947$

$\therefore$ Average number of items inspected per lot

$$= 0.947 \times 18 + 0.053 \times 2000 = 123.046 \text{ (say 124)}$$

Problem 10.6 *(a) Explain the step by step procedure for constructing the OC curve for a single sampling plan.*

(b) Draft the OC curve of the single sampling plan : $n = 300,\ c = 5$

Solution. The step by step method of constructing the OC curve for a single sampling plan is as follows :

1. Step up table headings and the P_a column as follows :

n	np'	p'	P_a	$P_a \cdot p'$
			0.98	
			0.95	
			0.70	
			0.50	
			0.20	
			0.05	
			0.02	

where, $\quad n$ = sample size, $\quad np'$ = number of defectives

p' = fraction defective, $\quad P_a$ = probability of acceptance

$P_a \cdot p'$ = AOQ = Average Outgoing Quality.

The chosen values of P_a will give ordinate values which, when co-ordinated with p' values to be derived, will facilitate construction of an OC curve.

2. Search Table G (*Appendix*) under the given np' value until the desired P_a (or closest value to desired P_a) is located.

(If the exact value is not found, the value in the P_a column should be changed to correspond with the one selected).

3. Place the np' value associated with the selected P_a in the np' column.

4. Divide the np' value by n. This will give the p' co-ordinate of P_a for the OC curve.

(*b*) To draft the OC curve for the single sampling plan, $n = 300$, $c = 5$.

1. Table construction

n	np'	p'	P_a	$P_a \cdot p'$
300			0.98	
300			0.95	
300			0.70	
300			0.50	
300			0.20	
300			0.05	
300			0.02	

2. Finding np' and p'

Search through Table G (*Appendix*) under $np' = 5$ discloses P_a value of 0.983. This is the closest value to 0.98. The np' value associated with a P_a value of 0.983 is 2.0. This value of np', when divided by $n = 300$ gives p' value of 0.0067. The same procedure is followed for each of the other P_a values until the table is computed.

n	np'	p'	P_a	$P_a \cdot p' = AOQ$
300	2.0	0.0067	0.983	0.0065
300	2.6	0.0087	0.951	0.00827
300	4.4	0.0147	0.72	0.0106
300	5.6	0.0187	0.512	0.00957
300	7.8	0.025	0.210	0.00526
300	10.5	0.035	.05	0.00175
300	12.0	0.04	0.02	0.0008

The $P_a \cdot p'$ column is provided to give the necessary values for the graphing of an AOQ curve with p' being the abscissa and $P_a \cdot p'$ the ordinate.

Figures 10.10 (*a*) and (*b*) show both of these curves as plotted from the data of the table above.

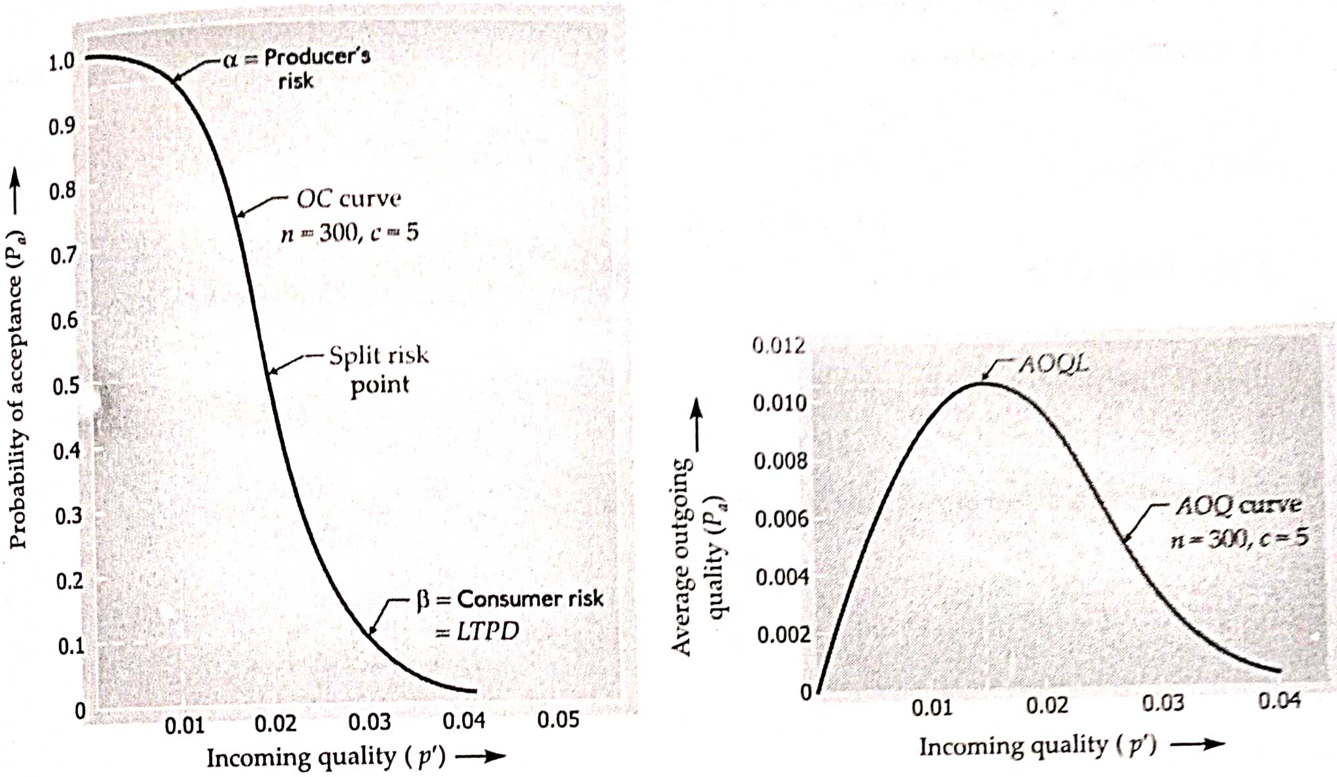


Fig. 10.10 (*a*) (*b*)

Problem 10.7 *A single sampling acceptance criterion for any desired $P_{0.50}$ is given by the approximate relation*

$$n = \frac{c + 0.67}{P_{0.50}}$$

where, n = *sample size*

c = *acceptance number*

$P_{0.50}$ = *fraction defective for which the probability of acceptance is 0.5.*

A vendor selects $P_{0.50} = 0.020$. The vendor has 2000 lots of which 1000 are of 1.5% quality and 1000 of 2.5% quality. The vendor chooses the acceptance number 1. Find how many lots are likely to pass vendor's inspection.

Solution. $$n = \frac{c + 0.67}{P_{0.50}}$$

In the above relation

$$P_{0.50} = 0.020 \quad \text{and} \quad c = 1$$

$$\therefore \quad n = \frac{1 + 0.67}{0.020} = 84$$

when $$p' = 0.015$$

$$np' = 84 \times 0.015 = 1.26$$

From Table G for $np' = 1.26$

and $c = 1.$

Probability of acceptance

$$P_a = 0.641$$

Also, when $p' = 0.025$

$$np' = 84 \times 0.025 = 2.1$$

From Table G for $np' = 2.1$

and $c = 1$

Probability of acceptance

$$P_a = 0.380.$$

Therefore, total number of lots accepted, when $c = 1$

$$= 0.641 \times 1000 + 0.380 \times 1000 = 1{,}021.$$

Problem 10.8 *Design a sequential sampling plan for the following specifications :*

$$\alpha = 0.05, \quad P_1 = 0.10$$

$$\beta = 0.20, \quad P_2 = 0.30$$

Also compute :

(a) *Average outgoing quality when* $p' = p_1$

(b) *Minimum number of items inspected for accepting the lot.*

(c) *Minimum number of defectives for rejection of the lot.*

(d) *Average number of items inspected when the quality of the lot is* p_1.

Solution. For sequential sampling plan

Acceptance line, $d_1 = S_n - h_1$

Rejection line, $d_2 = S_n + h_2$

$$S = \frac{g_2}{g_1 + g_2}$$

$$h_1 = \frac{b}{g_1 + g_2}$$

$$h_2 = \frac{a}{g_1 + g_2}$$

where,

$$g_1 = \log\frac{p_2}{p_1} = \log\frac{0.30}{0.10} = 0.4771$$

$$g_2 = \log\frac{1-p_1}{1-p_2} = \log\frac{0.90}{0.70} = 0.1091$$

$$a = \log\frac{1-\beta}{\alpha} = \log\frac{0.80}{0.05} = 1.2041$$

$$b = \log\frac{1-\alpha}{\beta} = \log\frac{0.95}{0.20} = 0.6767$$

$\therefore$
$$h_1 = \frac{0.6767}{0.5862} = 1.154$$

$$h_2 = \frac{1.2061}{0.5862} = 2.054$$

$$S = \frac{0.1091}{0.5862} = 0.186$$

Substituting the values, we have

$$d_1 = 0.186\,n - 1.154$$

and
$$d_2 = 0.186\,n + 2.054$$

(*a*) When $p' = p_1'$

$$\text{AOQ} = P_a \cdot p' = (1-\alpha) . p' = 0.95 \times 0.10 = 0.095$$

(*b*) Minimum number of items inspected for acceptance

$$= \frac{h_1}{S} = \frac{1.154}{0.186} \cong 7.$$

(*c*) Minimum number of defectives for rejecting the lot

$$h_2 = 2.054 \text{ say } 3.$$

(*d*) Average number of items inspected when the lot quality is p_1

$$= \frac{a(a+b)-b}{p_1(g_1+g_2)-g_2} = \frac{0.05\ (1.8808) - 0.6767}{0.1(0.5862) - 0.1091} = 12.$$

Problem 10.9 *Design a single-sampling inspection plan by attribute which will meet or nearly meet the following requirements :*

$$\alpha = 0.05, \quad p_1 = 0.008$$

$$\beta = 0.10 \quad p_2 = 0.04$$

Solution. Producer's risk

$$\alpha = 0.05$$

i.e., for quality of $p_1 = 0.008$

i.e., 0.8% of defectives, the expected probability of acceptance is

$$(1-\alpha) = 0.95.$$

Trying with acceptance number

$$c = 0$$

From Table G, corresponding to

$$c = 0 \text{ and } P_a = 0.95$$

$$np' = 0.05 \text{ (by interpolation)}$$

$$\therefore \quad n = \frac{np'}{p'} = \frac{0.05}{0.008} = 7$$

Now, it should satisfy the consumer's risk also for which

$$np' = 7 \times 0.04 = 0.28$$

and corresponding to $c=0$; $P_a = 0.756$ from Table G, but actually it should have been

$$\beta = 0.01$$

Trying with other acceptance numbers, results can be tabulated as follows :

c acceptance number	*np_1 corresponding to $P_a = 0.95$ from table G*	*n No. of items*	*np_2*	*$(P_a)_{0.10}$ Probability of acceptance for consumer's risk of 0.10 from Table G*
0	0.05	7	0.28	0.756
1	0.35	44	1.76	0.475
2	0.82	103	4.12	0.221
3	1.364	171	6.84	0.091

From the above table $c=3$ seems to be giving very close results and satisfying both the producer's and consumer's risk nearly.

$$\therefore \quad n = 171$$

and $\quad c=3$ is the required plan.

Problem 10.10 *A single sampling plan is given as $N = 10{,}000$, $n = 100$ and $c = 2$.*

(a) *Compute the approximate probability of acceptance of lots with 1% defective (use Poisson).*

(b) *Determine the AOQ value for the above lots.*

(c) *What will be the average inspection in per cent ? (Assume acceptance, rectification plan).*

Solution. $np' = 100 \times 0.01 = 1.$

From Table G, probability of 2 or less defective $= 0.920$

$\therefore$ Probability of acceptance of lot $= 92\%$.

(b)
$$\text{AOQ} = P_a \cdot p'$$
$$= 0.92 \times 0.01 = 0.0092.$$

(c) Total inspection of say 100 lots

$= 100$ articles each in 92 lots $+$ 10,000 articles in 8 lots

$= 9200 + 80{,}000 = 89{,}200$

$\therefore$ Total average percentage inspection

$$= \frac{89{,}200}{100 \times 10{,}000} \times 100 = 8.9\%.$$