Homework 1

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1 Read the Symbols into the Array one by one

The task is to read an unknown-length non-empty string $s \in \Sigma^+$ of symbols into a list stored as contiguous array, one symbol at a time. The array may be over-allocated, subject to the restriction that the array may never be larger than $2|s|$. Reading a symbol has a basic cost of 1. When the currently allocated array is full, a new larger array has to be allocated and the contents must be copied, incurring a cost of 1 for each symbol already stored in the array. In other words, the cost for the k_{th} symbol is either 1 (not full) or k (full).

Theorem 1. ***DOUBLE SIZE*** *that starts with an array of size* 1 *and doubles the array size each time when the array is full has a competitive ratio of 3.*

Proof. Consider the worst case:
the length of s is $2^j + \epsilon$, where $0 < \epsilon < 2^j, \epsilon \in Z^+$
then $OPT = 2^j + \epsilon$.
$ALG = (1 + 2 + ... + 2^j) + 2^j + \epsilon$
$= 2^{j+1} - 1 + 2^j + \epsilon$
$< 3 \cdot 2^j + \epsilon - 1$
$= 3 \cdot OPT - 1 - 2\epsilon$
$< 3 \cdot OPT$
Hence, the *DOUBLE SIZE* algorithm has a competitive ratio of 3.

2 Job Scheduling Problem

We have m identical machines and a sequence of jobs characterized by their running times. The jobs are presented one by one, and we have to schedule each job before we see the next one.
Performance is measured by the makespan. Each job is assigned to a single machine. There are no additional constraints, preemption is not allowed, all the machines have the same speed, and our objective is to minimize the makespan.

Theorem 2. *For this problem, the lower bound of the competitive ratio among all deterministic online algorithms is* $2 - \frac{1}{m}$*, where* m *is the number of machines.*

Proof. In case of a deterministic online algorithm $\mathcal{A}$:
Consider the job sequence $\mathcal{I}$ consisting of m jobs, each having a processing time

of 1, if $\mathcal{A}$ schedules all the jobs in different machines, then its competitive ratio is 1. Otherwise, its competitive ratio is at least 2(2>$2-\frac{1}{m}$).
On the basis of the first case:
Consider the job sequence $\mathcal{I}$ consisting of m(m−1) jobs(each having a processing time of 1),followed by one job having a processing time of m. because $\mathcal{A}$ is a deterministic online algorithm, $\mathcal{A}$ schedules the first m(m-1) jobs in different machines so a load of m-1 is generated. The final job then make a makespan of 2m-1.
On the other hand, OPT reserves a machine for the final job and averagely shechdules the first m(m-1) jobs on other machines. This gives a makespan of m.
In this case $ALG = 2m - 1$ and $OPT = m$. the competitive ratio is $2 - \frac{1}{m}$.
So we conclude that for all deterministic online algorithms, the lower bound of competitive ratio is $2 - \frac{1}{m}$.