

**Algorithm:** precise, unambiguous, step by step procedure for carrying out some calculation or more generally for solving some problem

**Algorithmics:** study of algorithms (design & analysis)

**Algorithm Properties:** Input, Output, Precision, Determinism, Finiteness, Correctness, Generality

**Algorithm Creation Process:** understand, design, analyse (possibly back to design), implement

**Analysis:** correctness, termination, simplicity, generality, time, space

**Logarithms:**  $b^e = x$  iff  $\log_b x = e$ ;  $\log xy = \log x + \log y$ ,  $x, y > 0$ ;

$\log \frac{x}{y} = \log x - \log y$ ,  $x, y > 0$ ;  $\log_b x^y = y \log_b x$ ;  $\log_a x = \frac{\log_b x}{\log_b a}$ ,  $a > 0, a \neq 1$ ;

$\log_b x > \log_b y$ ,  $b > 1, x > y > 0$

**Series:**  $\sum_{i=1}^n i = \frac{n(n+1)}{2} = \Theta(n^2)$ ;  $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6} = \Theta(n^3)$ ;

$\sum_{i=1}^n i^3 = (\sum_{i=1}^n i)^2 = \Theta(n^4)$ ;  $\sum_{i=1}^n i^k \approx \frac{n^{k+1}}{k+1} = \Theta(n^{k+1})$ ;  $\sum_{i=0}^n a^i = \frac{a^{n+1}-1}{a-1} = \Theta(a^n)$ ;  $\sum_{i=1}^n i2^i = (n-1)2^{n+1} + 2 = \Theta(n2^n)$ ;  $\sum_{i=1}^n \frac{1}{i} \approx$

$\ln n + 0.57 = \Theta(\lg n)$ ;  $\sum_{i=1}^n \lg i = \Theta(n \lg n)$ ;  $\sum_{i=1}^n i \lg i = \Theta(n^2 \lg n)$

**Theorem:** mathematical statement that has been proved true

**Lemma:** ‘small’ theorem, usually used in proof of a more important mathematical statement

**Corollary:** mathematical statement which easily follows from a theorem

**Proof:** logical argument that a mathematical statement is true

**Proof by Construction:** mathematical statement about the existence of an object can be proved by constructing the object

**Proof by Contradiction:** assume that a mathematical statement is false and show that the assumption leads to a contradiction

**Polynomial Degree:** highest power

**Intervals:** closed ( $[a, b] = x | a \leq x \leq b$ ), open ( $(a, b) = x | a < x < b$ ), half-open (either side)

**Subsequence:** consists of only certain terms in the same order as the full sequence

**Substring:** assume string index start from 1, then for  $t[i, j]$  if  $i < j$  then substring is from  $i$  to  $j$  inclusive, if  $i = j$  then substring is only  $i$ , else then empty string

**Boolean Expression:** containing boolean variables, operators, parentheses

**Normal Forms:** conjunctive (clause linked with  $\wedge$ , inside has  $\vee$ ), disjunctive (opposite)

**Upper Bound:**  $u$  such that  $x \leq u$  for all  $x \in X$ ,  $X$ : all reals

**Lower Bound:**  $l$  such that  $x \geq l$  for all  $x \in X$

**Supremum:** least upper bound

**Infimum:** greatest lower bound

**Graph:** consists of set of vertices and edges, edge is unordered (unless directed) pair of vertices, simple if without loops or multiple edges

**Degree:** number of edges incident on the vertex

**Path:** alternating sequence of vertices and edges, starting and ending with vertices, simple has no repeated vertices

**Diameter:** maximum distance between any two vertices

**Cycle:** path starting and ending at the same vertex with actual length, simple if without repeated vertices

**Hamiltonian Cycle:** cycle that contains each vertex exactly once

**Euler Cycle:** cycle with no repeated edges that contains all edges and vertices, exists iff connected and degree of every vertex is even

**Complement:** of simple graph, denoted as  $\bar{G}$ , same vertices, edge in  $\bar{G}$  iff not in  $G$

**Tree:** connected and acyclic; connected and has  $n - 1$  edges, acyclic and has  $n - 1$  edges, level of vertex is simple path length from root, height is max length

**Homogeneous Recurrence:** characteristic equation ( $a_0 t_n + a_1 t_{n-1} + \dots + a_k t_{n-k} = 0$ ) linear, homogeneous (combination of  $t_i = 0$ ), constant coefficients, guess  $t_n = x^n$ , unknown  $x$ , so  $a_0 x^n + a_1 x^{n-1} + \dots + a_k x^{n-k} = 0$ , factor out  $x^{n-k}$ , so  $p(x) = a_0 x^k + a_1 x^{k-1} + \dots + a_k x^0 = 0$ , so the general solution is  $\sum_{i=1}^k c_i r_i^n$ , where  $r$  is the roots of the equation (if distinct)

**HR Example:**  $T(n) = 2T(n-1)$ ,  $T(1) = 1$ ,  $T(n) - 2T(n-1) = 0$ ,  $T(n) = x^n$ ,  $T(n-1) = x^{n-1}$ , solve  $x^n - 2x^{n-1} = 0$ ,  $x = 2$ ,  $T(n) = c_1 2^n$ ,  $T(1) = 1 = c_1 2^1$ ,  $c_1 = 0.5$

**HR with Non Distinct Roots:** for each non distinct root include it but multiply with  $n$  each time ( $c_1 r^n + c_2 n r^n + c_3 n^2 r^n + \dots$ )

**Asymptotic Upper Bound:**  $f(n) = O(g(n))$  if there exist  $C_1 > 0$  and  $N_1$  such that  $f(n) \leq C_1 g(n)$ ,  $n \geq N_1$

**Asymptotic Lower Bound:**  $f(n) = \Omega(g(n))$  if there exist  $C_2 > 0$  and  $N_2$  such that  $f(n) \geq C_2 g(n)$ ,  $n \geq N_2$

**Master Theorem:**  $T(n) = aT(n/b) + f(n)$ ,  $f(n) = \Theta(n^k)$ ,  $T(n) = \Theta(n^k)$  if  $a < b^k$ ,  $T(n) = \Theta(n^k \log n)$  if  $a = b^k$ ,  $T(n) = \Theta(n^{\log_b a})$  if  $a > b^k$

**Smoothness:** if the function is non decreasing from  $N$  inclusive to infinity, and for every integer  $b \geq 2$ ,  $f(bn) = O(f(n))$

**Formal Smoothness:** for any integer  $b \geq 2$ ,  $C > 0$ ,  $N > 0$ ,  $f(bn) \leq C f(n)$ ,  $f(n) \leq f(n+1)$ , for all  $n \geq N$

**Binary Tree Traversal:** preorder (root, left, right), inorder (left, root, right), postorder (left, right, root)

**Binary Search Tree:** to insert search through and put in spot where it would be, to delete node with 0 or 1 child simply delete like linked list, else find minimum in right subtree, swap with target, and delete

**Heap:** if array start from 1 then left child is  $2i$ , right child is  $2i+1$ , to sift down maxheap get larger child, if child is larger than current move child up, look at next child, to delete swap last in array with target, sift down swapped, to insert do so at end, then sift up (if current is greater than parent move parent down, look at next parent), to heapify do sift down from  $i = n/2 \rightarrow 1$

**Heap Complexities:**  $\lg n$  for delete, insert ( $O$  only), & sift down (tree height),  $n$  for heapify

**Indirect Heap:** key, into (key index into heap index), outof (heap index into key index), whenever setting outof[x] = y also set into[y] = x, for increase/decreaseKey just sift up the modified node

**Heapsort:** heapify (maxheap), largest element is in the first, swap with last in the heap, decrease the heap length, sift down on root, repeat until heap has only 1 element (which would be minimum)

**Disjoint Sets:** stored as array of parents, parent[i] is the parent of element i, makeset (constructs new set with one element from parameter, set parent to itself), findset (returns the marked member of the set containing parameter, keep going through parent, while doing so check the node parent, if not root make it root and continue to next parent), union (replace sets containing the parameters with the union, find the sets of both and make one of the sets the parent of the other, use the deepest tree as the parent)

**Interpolation Search:** estimates where the search key is in the array, assumes values in array increase linearly with index, worst case  $n$  but with random keys average case is  $\lg \lg n$

**Hashing:** average  $1 + \frac{n}{2m}$

**Topological Sort:** DFS, but for every dead end (all incident vertices visited already) add to head of list

**Depth First Search:** set visit to false for all, then for each unvisited node run recursive (in case of disconnected graph), in recursive set visit to true for the vertex and recurse on all adjacent (if not visited), complexity is  $\Theta(|V| + |E|)$  for adjacency list, with adjacency matrix is  $\Theta(|V|^2)$  (to get adjacent vertices need to go through whole row/column)

**DFS Applications:** find spanning trees, connected components, cut vertices/articulation points, exploring graphs, topological sort, backtracking

**Breadth First Search:** same as DFS for starting, but instead of recursive set current visit to true, add current to queue, for all in queue dequeue, then for each adjacent to dequeued if was not visited then visit and enqueue, complexity same as DFS

**BFS Applications:** find spanning trees, connected components, shortest path, exploring graphs, web crawling, social networking, garbage collection, puzzles, games

**Mergesort:** to merge just compare first element of both lists, get smallest one and put into new list, if one emptied then just grab all from the other, main method simply splits the list into 2, recurse on both, then merge, is stable

**Bubblesort:** start from end, keep swapping if out of order, after each pass will get smallest at the start, keep going while ignoring the ones at the start (already sorted),  $n^2$ , stable

**Insertion sort:** start with sorted list of 1 element (the first), then insert a[2] into the sorted list (only a[1]), then insert a[3] into the sorted list of a[1], a[2], and so on,  $n^2$ , stable

**Quicksort:** partition (values before partition is less than, values after partition is greater than or equal to, goes through the whole portion), then recurse on the halves before and after the partition, worst  $n^2$ , but average with random partition  $n \log n$ , not stable

**Counting Sort:** init count array, count how many times index appears in actual array, convert count array to cumulative, get cumulative count per element and use it as index in new array, subtract the count (for duplicates), copy new array to original, complexity  $n + m$ , range from 0 to  $m$ , stable

**Radix Sort:** use counting sort on each digit, complexity is  $kn$ ,  $k$  is max number of digits, stable

**Random Select:** worst case  $n^2$ , average  $n$