Algorithm: precise, unambiguous, step by step procedure for carrying out some calculation or more generally for solving some problem

Algorithmics: study of algorithms (design & analysis)

Algorithm Properties: Input, Output, Precision, Determinism, Finiteness, Correctness, Generality

Algorithm Creation Process: understand, design, analyse (possibly back to design), implement

Analysis: correctness, termination, simplicity, generality, time, space

Logarithms: $b^e = x$ iff $\log_b x = e$; $\log xy = \log x + \log y$, $x, y > 0$; $\log \frac{x}{y} = \log x - \log y$, $x, y > 0$; $\log_b x^y = y \log_b x$; $\log_a x = \frac{\log_b x}{\log_b a}$, $a > 0, a \neq 1$; $\log_b x > \log_b y, b > 1, x > y > 0$

Series: $\sum_{i=1}^n i = \frac{n(n+1)}{2} = \Theta(n^2)$; $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6} = \Theta(n^3)$; $\sum_{i=1}^n i^3 = (\sum_{i=1}^n i)^2 = \Theta(n^4)$; $\sum_{i=1}^n i^k \approx \frac{n^{k+1}}{k+1} = \Theta(n^{k+1})$; $\sum_{i=0}^n a^i = \frac{a^{n+1}-1}{a-1} = \Theta(a^n)$; $\sum_{i=1}^n i2^i = (n-1)2^{n+1} + 2 = \Theta(n2^n)$; $\sum_{i=1}^n \frac{1}{i} \approx \ln n + 0.57 = \Theta(\lg n)$; $\sum_{i=1}^n \lg i = \Theta(n \lg n)$; $\sum_{i=1}^n i \lg i = \Theta(n^2 \lg n)$

Theorem: mathematical statement that has been proved true

Lemma: 'small' theorem, usually used in proof of a more important mathematical statement

Corollary: mathematical statement which easily follows from a theorem

Proof: logical argument that a mathematical statement is true

Proof by Construction: mathematical statement about the existence of an object can be proved by constructing the object

Proof by Contradiction: assume that a mathematical statement is false and show that the assumption leads to a contradiction

Polynomial Degree: highest power

Intervals: closed ($[a, b] = x | a \leq x \leq b$), open ($(a, b) = x | a < x < b$), half-open (either side)

Subsequence: consists of only certain terms in the same order as the full sequence

Substring: assume string index start from 1, then for $t[i, j]$ if $i < j$ then substring is from i to j inclusive, if $i = j$ then substring is only i, else then empty string

Boolean Expression: containing boolean variables, operators, parentheses

Normal Forms: conjunctive (clause linked with $\wedge$, inside has $\vee$), disjunctive (opposite)

Upper Bound: u such that $x \leq u$ for all $x \in X$, X: all reals

Lower Bound: l such that $x \geq l$ for all $x \in X$

Supremum: least upper bound

Infimum: greatest lower bound

Graph: consists of set of vertices and edges, edge is unordered (unless directed) pair of vertices, simple if without loops or multiple edges

Degree: number of edges incident on the vertex

Path: alternating sequence of vertices and edges, starting and ending with vertices, simple has no repeated vertices

Diameter: maximum distance between any two vertices

Cycle: path starting and ending at the same vertex with actual length, simple if without repeated vertices

Hamiltonian Cycle: cycle that contains each vertex exactly once

Euler Cycle: cycle with no repeated edges that contains all edges and vertices, exists iff connected and degree of every vertex is even

Complement: of simple graph, denoted as $\bar{G}$, same vertices, edge in $\bar{G}$ iff not in G

Tree: connected and acyclic; connected and has $n - 1$ edges, acyclic and has $n - 1$ edges, level of vertex is simple path length from root, height is max length

Homogeneous Recurrence: characteristic equation ($a_0 t_n + a_1 t_{n-1} + \cdots + a_k t_{n-k} = 0$) linear, homogeneous (combination of $t_i = 0$), constant coefficients, guess $t_n = x^n$, unknown x, so $a_0 x^n + a_1 x^{n-1} + \cdots + a_k x^{n-k} = 0$, factor out x^{n-k}, so $p(x) = a_0 x^k + a_1 x^{k-1} + \cdots + a^k x^0 = 0$, so the general solution is $\sum_{i=1}^k c_i r_i^n$, where r is the roots of the equation (if distinct)

HR Example: $T(n) = 2T(n-1), T(1) = 1, T(n) - 2T(n-1) = 0, T(n) = x^n, T(n-1) = x^{n-1}$, solve $x^n - 2x^{n-1} = 0, x = 2, T(n) = c_1 2^n, T(1) = 1 = c_1 2^1, c_1 = 0.5$

HR with Non Distinct Roots: for each non distinct root include it but multiply with n each time ($c_1 r^n + c_2 n r^n + c_3 n^2 r^n + \cdots$)

Asymptotic Upper Bound: $f(n = O(g(n)))$ if there exist $C_1 > 0$ and N_1 such that $f(n) \leq C_1 g(n), n \geq N_1$

Asymptotic Lower Bound: $f(n) = \Omega(g(n))$ if there exist $C_2 > 0$ and N_2 such that $f(n) \geq C_2 g(n), n \geq N_2$

Master Theorem: $T(n) = aT(n/b) + f(n), f(n) = \Theta(n^k)$, $T(n) = \Theta(n^k)$ if $a < b^k$, $T(n) = \Theta(n^k \log n)$ if $a = b^k$, $T(n) = \Theta(n^{\log_b a})$ if $a > b^k$

Smoothness: if the function is non decreasing from N inclusive to infinity, and for every integer $b \geq 2, f(bn) = O(f(n))$

Formal Smoothness: for any integer $b \geq 2, C > 0, N > 0, f(bn) \leq Cf(n), f(n) \leq f(n+1)$, for all $n \geq N$

Binary Tree Traversal: preorder (root, left, right), inorder (left, root, right), postorder (left, right, root)

Binary Search Tree: to insert search through and put in spot where it would be, to delete node with 0 or 1 child simply delete like linked list, else find minimum in right subtree, swap with target, and delete

Heap: if array start from 1 then left child is $2i$, right child is $2i+1$, to siftdown maxheap get larger child, if child is larger than current move child up, look at next child, to delete swap last in array with target, siftdown swapped, to insert do so at end, then siftup (if current is greater than parent move parent down, look at next parent), to heapify do siftdown from $i = n/2 \to 1$

Heap Complexities: lg n for delete, insert (O only), & siftdown (tree height), n for heapify

Indirect Heap: key, into (key index into heap index), outof (heap index into key index), whenever setting outof[x] = y also set into[y] = x, for increase/decreaseKey just siftup the modified node

Heapsort: heapify (maxheap), largest element is in the first, swap with last in the heap, decrease the heap length, siftdown on root, repeat until heap has only 1 element (which would be minimum)

Disjoint Sets: stored as array of parents, parent[i] is the parent of element i, makeset (constructs new set with one element from parameter, set parent to itself), findset (returns the marked member of the set containing parameter, keep going through parent, while doing so check the node parent, if not root make it root and continue to next parent), union (replace sets containing the parameters with the union, find the sets of both and make one of the sets the parent of the other, use the deepest tree as the parent)

Interpolation Search: estimates where the search key is in the array, assumes values in array increase linearly with index, worst case n but with random keys average case is lg lg n

Hashing: average $1 + \frac{n}{2m}$

Topological Sort: DFS, but for every dead end (all incident vertices visited already) add to head of list

Depth First Search: set visit to false for all, then for each unvisited node run recursive (in case of disconnected graph), in recursive set visit to true for the vertex and recurse on all adjacent (if not visited), complexity is $\Theta(|V| + |E|)$ for adjacency list, with adjacency matrix is $\Theta(|V|^2)$ (to get adjacent vertices need to go through whole row/column)

DFS Applications: find spanning trees, connected components, cut vertices/articulation points, exploring graphs, topological sort, backtracking

Breadth First Search: same as DFS for starting, but instead of recursive set current visit to true, add current to queue, for all in queue dequeue, then for each adjacent to dequeued if was not visited visit and enqueue, complexity same as DFS

BFS Applications: find spanning trees, connected components, shortest path, exploring graphs, web crawling, social networking, garbage collection, puzzles, games

Mergesort: to merge just compare first element of both lists, get smallest one and put into new list, if one emptied then just grab all from the other, main method simply splits the list into 2, recurse on both, then merge, is stable

Bubblesort: start from end, keep swapping if out of order, after each pass will get smallest at the start, keep going while ignoring the ones at the start (already sorted), n^2, stable

Insertion sort: start with sorted list of 1 element (the first), then insert a[2] into the sorted list (only a[1]), then insert a[3] into the sorted list of a[1], a[2], and so on, n^2, stable

Quicksort: partition (values before partition is less than, values after partition is greater than or equal to, goes through the whole portion), then recurse on the halves before and after the partition, worst n^2, but average with random partition n log n, not stable

Counting Sort: init count array, count how many times index appears in actual array, convert count array to cumulative, get cumulative count per element and use it as index in new array, subtract the count (for duplicates), copy new array to original, complexity n + m, range from 0 to m

Radix Sort: use counting sort on each digit, complexity is kn, k is max number of digits, stable

Random Select: worst case n^2, average n

Greedy Algorithms: on each step make choice that is feasible (satisfy constraint), locally optimal (best among all feasible choices), irrevocable (no backtracking)

Kruskal's Algorithm: greedy by edges, start with all vertices but no edges, sort edgelist, if adding smallest edge does not create cycle (the vertices being connected are not in the same disjoint set) then add (union the sets the vertices are in), otherwise skip it, stop when n-1 edges added, $\Theta|E| \log n$ (sort is $|E| \log |E|$, $\log |E| = \Theta(\log n)$, each findset/union takes $O(\log n)$)

Prim's Algorithm: start with 1 vertex and no edges, find minimum weight edge among all edges leaving the tree (can keep track of the distance to the tree per vertex and the nearest tree vertex), add it to the tree (after adding need to update vertices around the new one if the distance is now shorter, also update parent), stop when all vertices added, $O(|E| \log n)$ (if using adjacency list and heap, n for heapify, log n for every poll, also decreaseKey per edge possibly, which is where it comes from)

Dijkstra's Algorithm: init all priorities of vertices to infinity, put in priority queue, decreaseKey on the start with distance 0, dequeue vertex from queue, add it to tree, for every vertex adjacent to new one update distance where necessary, decreaseKey and set parent, do so until we have all vertices or we found the vertex we want

Principle of Optimality: required for dynamic programming, if we have a set of optimal solutions to a group of sub-problems, and we have an optimal method of combining solutions, then we have an optimal solution to a problem made up from the sub problems

Warshall's Algorithm: compute transitive closure of a directed graph (matrix where t[i][j] = 1 iff there is a directed nontrivial path from vertex i to vertex j), create series of matrices M_k from 1 to n, in M_k, m[i][j] = 1 iff there is a directed path from i to j such that no vertex on the path has label greater than k, so to get next matrix m[i][j] = 1 if it was 1 before or if both m[i][k] and m[k][j] is 1 (meaning there is a path if we allow going through k), complexity is $\Theta(n^3)$

Floyd's Algorithm: compute matrix where m[i][j] contains the shortest distance from vertex i to j (directed graph), similar to Warshall's with 3 loops, but when updating compare a[i][j] (current without k) and a[i][k] + a[k][j] (include k), store shorter one, when initializing go through edges and note down the distances, if no edge then set to infinity, same complexity as Warshall's