

**Algorithm:** precise, unambiguous, step by step procedure for carrying out some calculation or more generally for solving some problem

**Algorithmics:** study of algorithms (design & analysis)

**Algorithm Properties:** Input, Output, Precision, Determinism, Finiteness, Correctness, Generality

**Algorithm Creation Process:** understand, design, analyse (possibly back to design), implement

**Analysis:** correctness, termination, simplicity, generality, time, space

**Logarithms:**  $b^e = x$  iff  $\log_b x = e$ ;  $\log xy = \log x + \log y$ ,  $x, y > 0$ ;

$\log \frac{x}{y} = \log x - \log y$ ,  $x, y > 0$ ;  $\log_b x^y = y \log_b x$ ;  $\log_a x = \frac{\log_b x}{\log_b a}$ ,  $a > 0, a \neq 1$ ;

$\log_b x > \log_b y$ ,  $b > 1, x > y > 0$

**Series:**  $\sum_{i=1}^n i = \frac{n(n+1)}{2} = \Theta(n^2)$ ;  $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6} = \Theta(n^3)$ ;

$\sum_{i=1}^n i^3 = (\sum_{i=1}^n i)^2 = \Theta(n^4)$ ;  $\sum_{i=1}^n i^k \approx \frac{n^{k+1}}{k+1} = \Theta(n^{k+1})$ ;  $\sum_{i=0}^n a^i = \frac{a^{n+1}-1}{a-1} = \Theta(a^n)$ ;  $\sum_{i=1}^n i2^i = (n-1)2^{n+1} + 2 = \Theta(n2^n)$ ;  $\sum_{i=1}^n \frac{1}{i} \approx$

$\ln n + 0.57 = \Theta(\lg n)$ ;  $\sum_{i=1}^n \lg i = \Theta(n \lg n)$ ;  $\sum_{i=1}^n i \lg i = \Theta(n^2 \lg n)$

**Theorem:** mathematical statement that has been proved true

**Lemma:** ‘small’ theorem, usually used in proof of a more important mathematical statement

**Corollary:** mathematical statement which easily follows from a theorem

**Proof:** logical argument that a mathematical statement is true

**Proof by Construction:** mathematical statement about the existence of an object can be proved by constructing the object

**Proof by Contradiction:** assume that a mathematical statement is false and show that the assumption leads to a contradiction

**Polynomial Degree:** highest power

**Intervals:** closed ( $[a, b] = x | a \leq x \leq b$ ), open ( $(a, b) = x | a < x < b$ ), half-open (either side)

**Subsequence:** consists of only certain terms in the same order as the full sequence

**Substring:** assume string index start from 1, then for  $t[i, j]$  if  $i < j$  then substring is from  $i$  to  $j$  inclusive, if  $i = j$  then substring is only  $i$ , else then empty string

**Boolean Expression:** containing boolean variables, operators, parentheses

**Normal Forms:** conjunctive (clause linked with  $\wedge$ , inside has  $\vee$ ), disjunctive (opposite)

**Upper Bound:**  $u$  such that  $x \leq u$  for all  $x \in X$ ,  $X$ : all reals

**Lower Bound:**  $l$  such that  $x \geq l$  for all  $x \in X$

**Supremum:** least upper bound

**Infimum:** greatest lower bound

**Graph:** consists of set of vertices and edges, edge is unordered (unless directed)

pair of vertices, simple if without loops or multiple edges

**Degree:** number of edges incident on the vertex

**Path:** alternating sequence of vertices and edges, starting and ending with vertices, simple has no repeated vertices

**Diameter:** maximum distance between any two vertices

**Cycle:** path starting and ending at the same vertex with actual length, simple if without repeated vertices

**Hamiltonian Cycle:** cycle that contains each vertex exactly once

**Euler Cycle:** cycle with no repeated edges that contains all edges and vertices, exists iff connected and degree of every vertex is even

**Complement:** of simple graph, denoted as  $\bar{G}$ , same vertices, edge in  $\bar{G}$  iff not in  $G$

**Tree:** connected and acyclic; connected and has  $n - 1$  edges, acyclic and has  $n - 1$  edges, level of vertex is simple path length from root, height is max length

**Homogeneous Recurrence:** characteristic equation ( $a_0 t_n + a_1 t_{n-1} + \dots + a_k t_{n-k} = 0$ ) linear, homogeneous (combination of  $t_i = 0$ ), constant coefficients, guess  $t_n = x^n$ , unknown  $x$ , so  $a_0 x^n + a_1 x^{n-1} + \dots + a_k x^{n-k} = 0$ , factor out  $x^{n-k}$ , so  $p(x) = a_0 x^k + a_1 x^{k-1} + \dots + a_k x^0 = 0$ , so the general solution is  $\sum_{i=1}^k c_i r_i^n$ , where  $r$  is the roots of the equation (if distinct)

**HR Example:**  $T(n) = 2T(n-1), T(1) = 1, T(n) - 2T(n-1) = 0, T(n) = x^n, T(n-1) = x^{n-1}$ , solve  $x^n - 2x^{n-1} = 0, x = 2, T(n) = c_1 2^n, T(1) = 1 = c_1 2^1, c_1 = 0.5$

**HR with Non Distinct Roots:** for each non distinct root include it but multiply with  $n$  each time ( $c_1 r^n + c_2 n r^n + c_3 n^2 r^n + \dots$ )