

Algorithm: precise, unambiguous, step by step procedure for carrying out some calculation or more generally for solving some problem

Algorithmics: study of algorithms (design & analysis)

Algorithm Properties: Input, Output, Precision, Determinism, Finiteness, Correctness, Generality

Algorithm Creation Process: understand, design, analyse (possibly back to design), implement

Analysis: correctness, termination, simplicity, generality, time, space

Logarithms: $b^e = x$ iff $\log_b x = e$; $\log xy = \log x + \log y$, $x, y > 0$;

$\log \frac{x}{y} = \log x - \log y$, $x, y > 0$; $\log_b x^y = y \log_b x$; $\log_a x = \frac{\log_b x}{\log_b a}$, $a > 0, a \neq 1$;

$\log_b x > \log_b y$, $b > 1, x > y > 0$

Series: $\sum_{i=1}^n i = \frac{n(n+1)}{2} = \Theta(n^2)$; $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6} = \Theta(n^3)$;

$\sum_{i=1}^n i^3 = (\sum_{i=1}^n i)^2 = \Theta(n^4)$; $\sum_{i=1}^n i^k \approx \frac{n^{k+1}}{k+1} = \Theta(n^{k+1})$; $\sum_{i=0}^n a^i =$

$\frac{a^{n+1}-1}{a-1} = \Theta(a^n)$; $\sum_{i=1}^n i2^i = (n-1)2^{n+1} + 2 = \Theta(n2^n)$; $\sum_{i=1}^n \frac{1}{i} \approx$

$\ln n + 0.57 = \Theta(\lg n)$; $\sum_{i=1}^n \lg i = \Theta(n \lg n)$; $\sum_{i=1}^n i \lg i = \Theta(n^2 \lg n)$

Theorem: mathematical statement that has been proved true

Lemma: ‘small’ theorem, usually used in proof of a more important mathematical statement

Corollary: mathematical statement which easily follows from a theorem

Proof: logical argument that a mathematical statement is true

Proof by Construction: mathematical statement about the existence of an object can be proved by constructing the object

Proof by Contradiction: assume that a mathematical statement is false and show that the assumption leads to a contradiction

Polynomial Degree: highest power

Intervals: closed ($[a, b] = x | a \leq x \leq b$), open ($(a, b) = x | a < x < b$), half-open (either side)

Subsequence: consists of only certain terms in the same order as the full sequence

Substring: assume string index start from 1, then for $t[i, j]$ if $i < j$ then substring is from i to j inclusive, if $i = j$ then substring is only i , else then empty string

Boolean Expression: containing boolean variables, operators, parentheses

Normal Forms: conjunctive (clause linked with \wedge , inside has \vee), disjunctive (opposite)

Upper Bound: u such that $x \leq u$ for all $x \in X$, X : all reals

Lower Bound: l such that $x \geq l$ for all $x \in X$

Supremum: least upper bound

Infimum: greatest lower bound

Graph: consists of set of vertices and edges, edge is unordered (unless directed) pair of vertices, simple if without loops or multiple edges

Degree: number of edges incident on the vertex

Path: alternating sequence of vertices and edges, starting and ending with vertices, simple has no repeated vertices

Diameter: maximum distance between any two vertices

Cycle: path starting and ending at the same vertex with actual length, simple if without repeated vertices

Hamiltonian Cycle: cycle that contains each vertex exactly once

Euler Cycle: cycle with no repeated edges that contains all edges and vertices, exists iff connected and degree of every vertex is even

Complement: of simple graph, denoted as \bar{G} , same vertices, edge in \bar{G} iff not in G

Tree: connected and acyclic; connected and has $n-1$ edges, acyclic and has $n-1$ edges, level of vertex is simple path length from root, height is max length

Homogeneous Recurrence: characteristic equation ($a_0 t_n + a_1 t_{n-1} + \dots + a_k t_{n-k} = 0$) linear, homogeneous (combination of $t_i = 0$), constant coefficients,

guess $t_n = x^n$, unknown x , so $a_0 x^n + a_1 x^{n-1} + \dots + a_k x^{n-k} = 0$, factor out x^{n-k} , so $p(x) = a_0 x^k + a_1 x^{k-1} + \dots + a_k x^0 = 0$, so the general solution is

$\sum_{i=1}^k c_i r_i^n$, where r is the roots of the equation (if distinct)

HR Example: $T(n) = 2T(n-1)$, $T(1) = 1$, $T(n) - 2T(n-1) = 0$, $T(n) = x^n$, $T(n-1) = x^{n-1}$, solve $x^n - 2x^{n-1} = 0$, $x = 2$, $T(n) = c_1 2^n$, $T(1) = 1 = c_1 2^1$, $c_1 = 0.5$

HR with Non Distinct Roots: for each non distinct root include it but multiply with n each time ($c_1 r^n + c_2 n r^n + c_3 n^2 r^n + \dots$)

Asymptotic Upper Bound: $f(n = O(g(n)))$ if there exist $C_1 > 0$ and N_1 such that $f(n) \leq C_1 g(n)$, $n \geq N_1$

Asymptotic Lower Bound: $f(n) = \Omega(g(n))$ if there exist $C_2 > 0$ and N_2 such that $f(n) \geq C_2 g(n)$, $n \geq N_2$

Master Theorem: $T(n) = aT(n/b) + f(n)$, $f(n) = \Theta(n^k)$, $T(n) = \Theta(n^k)$ if $a < b^k$, $T(n) = \Theta(n^k \log n)$ if $a = b^k$, $T(n) = \Theta(n^{\log_b a})$ if $a > b^k$

Smoothness: if the function is non decreasing from N inclusive to infinity, and for every integer $b \geq 2$, $f(bn) = O(f(n))$

Formal Smoothness: for any integer $b \geq 2$, $C > 0$, $N > 0$, $f(bn) \leq C f(n)$, $f(n) \leq f(n+1)$, for all $n \geq N$