the higher vapor pressures required for high current operation. Thus, it seems that cesium is most effectively utilized by high-temperature converters. This fact is further confirmed by the results given in Fig. 8, which shows that at high temperatures the resistivity is lower for higher vapor pressures and suggests that high-power devices are practical in this range.

Apparently then, if a low-temperature, high-power thermionic converter is to operate efficiently, some material other than cesium must be used for an ion source unless a method can be devised for only injecting ions into the interelectrode space and subsequently condensing out the neutral atoms formed by recombination. This might be accomplished by allowing the cesium atoms to enter the device through a hot, high work function mesh or screen so dimensioned that only ions generated by resonance ionization could emerge. The neutrals then formed by recombination might be condensed on a cold surface and thus be purged from the device. In this manner, the advantages of a low-pressure converter might be obtained without suffering the losses inherent in high-pressure, low-temperature devices.

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Young's Modulus, Shear Modulus, and Poisson's Ratio in Silicon and Germanium

J. J. WORTMAN AND R. A. EVANS

Research Triangle Institute, Durham, North Carolina

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The elastic coefficients for an arbitrary rectangular coordinate system are calculated as a function of direction cosines in the crystal. Young's modulus, shear modulus, and Poisson's ratio are defined in general and values tabulated for some of the more important directions in the crystal. Graphs of these moduli are also plotted as a function of crystal direction for orientations in the (100) and (110) planes as well as planes determined by the [110] direction and any perpendicular direction.

I. INTRODUCTION

THE recent use of semiconductors as stress and strain transducers has required that detailed calculations of the stress and strain in the semiconductor be made.[1] It is common in such calculations to resolve the stresses into components along the crystal axes. In many cases, however, it is more convenient to work in an arbitrarily oriented coordinate system. In the latter case, a generalized Hooke's law must be used and the elastic coefficients determined from a tensor transformation for the particular orientation. These calculations are long and laborious; as a result, inadequate approximations of the elastic coefficients are often made. It is the purpose of this paper to give some of the more important elastic moduli of germanium and silicon as a function of crystal direction for common orientations.

II. STIFFNESS AND COMPLIANCE COEFFICIENTS

The compliance coefficients s_{pq}' and the stiffness coefficients c_{qp}' are defined as the proportionality constants between stress and strain by the generalized Hooke's law:

$$\sigma_q'=\Sigma_p c_{qp}'\epsilon_p', \quad \epsilon_p'=\Sigma_q s_{pq}'\sigma_q', \quad (p, q=1,2, \ldots, 6),$$

where σ_q' and ϵ_p' are the engineering stresses and strains, respectively.[2] The prime is used to indicate an arbitrary rectangular coordinate system. The lack of a prime indicates the crystal axis coordinate system. When the crystal-axis coordinate system is used for a cubic crystal (Ge and Si), the compliance coefficients reduce to the following matrix[3]:

$$s_{pq}=\begin{pmatrix} s_{11} & s_{12} & s_{12} & 0 & 0 & 0 \\ s_{12} & s_{11} & s_{12} & 0 & 0 & 0 \\ s_{12} & s_{12} & s_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & s_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & s_{44} \end{pmatrix}.$$

A similar matrix can be written for c_{pq}.

Table I lists the three independent components of the stiffness and compliance coefficients for germanium and silicon.

The c_{qp}' and s_{pq}' matrices will in general each contain 36 terms, none of which are necessarily zero. Since c_{qp}' and s_{pq}' are symmetric matrices, there will be at most 21 independent terms. In order to calculate the coefficients for an arbitrary rectangular system (rotated axes), one must revert to tensor notation (fourth-order

[1] W. G. Pfann and R. N. Thruston, J. Appl. Phys. 32, 2008 (1961).

[2] "Standards on Piezoelectric Crystals," Proc. IRE 37, 1378 (1949).

[3] See J. F. Nye's *Physical Properties of Crystals* (Oxford University Press, London, 1957).

TABLE I. The stiffness and compliance coefficients of Ge and Si (after Mason[a]).

	s_{11}	s_{12}	s_{44}	c_{11}	c_{12}	c_{44}
	($cm^2/10^{12}$ dyn)			(10^{12} dyn/cm^2)		
Si	0.768	−0.214	1.26	1.657	0.639	0.796
Ge	0.964	−0.260	1.49	1.292	0.479	0.670

[a] W. P. Mason, *Physical Acoustics and the Properties of Solids* (D. Van Nostrand Company, New York, 1958).

tensor) and perform a transformation.[4] The results of such a transformation are shown in Table II. The transformation from the crystal axes x_i (unprimed) to the arbitrary system x_i' (primed) is described by:

$$x_i' = l_i x_1 + m_i x_2 + n_i x_3, \quad i = 1,2,3,$$

where l, m, n are the direction cosines of the transformation. By using the relationship for the elastic coefficients in Table II and Hooke's generalized law, calculations for any crystal orientation are possible—one need only substitute the direction cosines into the elastic coefficients.

TABLE II. Compliance and stiffness coefficients for rotated axes in cubic crystals.

Coefficient	Expression[a]	Analogous[b] coefficients
c_{11}'	$=c_{11}+c_c(l_1^4+m_1^4+n_1^4-1)$	c_{22}', c_{33}'
c_{12}'	$=c_{12}+c_c(l_1^2l_2^2+m_1^2m_2^2+n_1^2n_2^2)$	c_{13}', c_{23}'
c_{14}'	$=c_c(l_1^2l_2l_3+m_1^2m_2m_3+n_1^2n_2n_3)$	$c_{36}', c_{25}', c_{24}', c_{35}'$ $c_{26}', c_{34}', c_{16}', c_{15}'$ $c_{46}', c_{45}', c_{56}'$
c_{44}'	$=c_{44}+c_c(l_2^2l_3^2+m_2^2m_3^2+n_2^2n_3^2)$	c_{55}', c_{66}'
s_{22}'	$=s_{11}+s_c(l_2^4+m_2^4+n_2^4-1)$	s_{11}', s_{33}'
s_{13}'	$=s_{12}+s_c(l_1^2l_3^2+m_1^2m_3^2+n_1^2n_3^2)$	s_{12}', s_{23}'
s_{14}'	$=2s_c(l_1^2l_2l_3+m_1^2m_2m_3+n_1^2n_2n_3)$	$s_{36}', s_{25}', s_{24}', s_{35}'$ $s_{26}', s_{34}', s_{16}'$
s_{56}'	$=4s_c(l_1^2l_2l_3+m_1^2m_2m_3+n_1^2n_2n_3)$	s_{46}', s_{15}'
s_{55}'	$=s_{44}+4s_c(l_3^2l_1^2+m_3^2m_1^2+n_3^2n_1^2)$	s_{44}', s_{66}'

[a] $c_{ij}' = c_{ji}'$ $\quad s_{ij}' = s_{ji}'$

$c_c = c_{11} - c_{12} - 2c_{44}$, $\quad s_c = s_{11} - s_{12} - \frac{1}{2}s_{44}$.

[b] The direction cosines for a particular coefficient are determined by the subscripts on the term. It will be recalled that each subscript on the coefficient represents two subscripts

$(1 \to 11, 2 \to 22, 3 \to 33, 6 \to 12, 5 \to 13, 4 \to 23)$.

These are used in determining the direction cosines. For example, if the coefficient subscript is 36, which expands to 3312, then the geometrical factor is

$(l_3l_3l_1l_2 + m_3m_3m_1m_2 + n_3n_3n_1n_2) = (l_1l_2l_3^2 + m_1m_2m_3^2 + n_1n_2n_3^2)$.

III. YOUNG'S MODULUS, SHEAR MODULUS, AND POISSON'S RATIO

Although germanium and silicon crystals are highly anisotropic, it is often desirable to define elastic moduli as is common in isotropic media. These moduli depend in general on the direction in the crystal. Consider a uniaxial stress σ_i' acting in the i' direction. Young's modulus is defined as

$$Y_i' \equiv \sigma_i'/\epsilon_i';$$

and Poisson's ratio is defined as

$$\nu_{ij}' \equiv -\epsilon_j'/\epsilon_i', \quad i,j = 1',2',3', \quad i \neq j.$$

It can readily be shown that

$$Y_i' = 1/s_{ii}',$$

and

$$\nu_{ij}' = -s_{ji}'/s_{ii}' = -s_{ij}'/s_{ii}'.$$

Note from Table II that $\nu_{ij}' \neq \nu_{ji}'$.

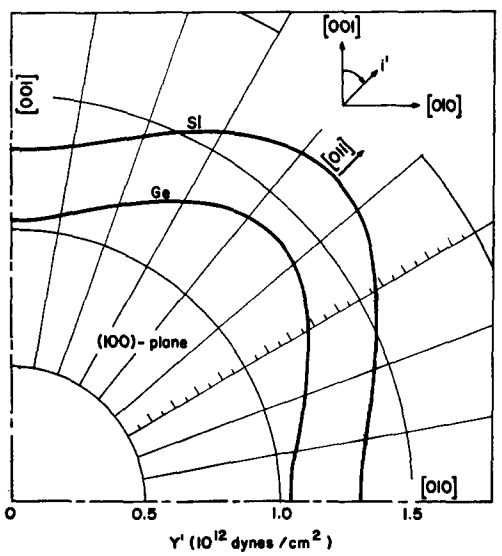

FIG. 1. Young's modulus as a function of direction in the (100) plane.

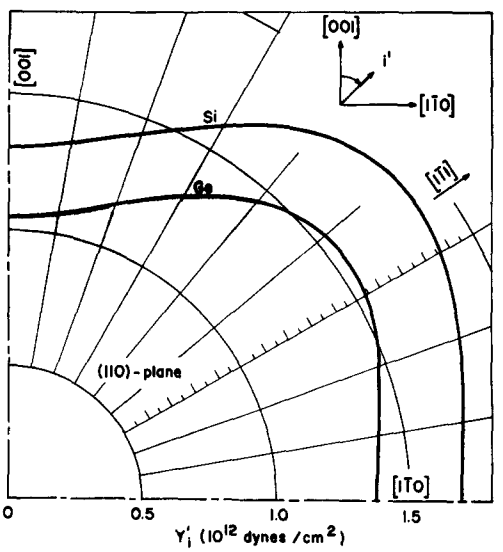

FIG. 2. Young's modulus as a function of direction in the (110) plane.

[4] W. P. Mason, *Physical Acoustics and the Properties of Solids* (D. Van Nostrand Company, Inc., Princeton, New Jersey, 1958).

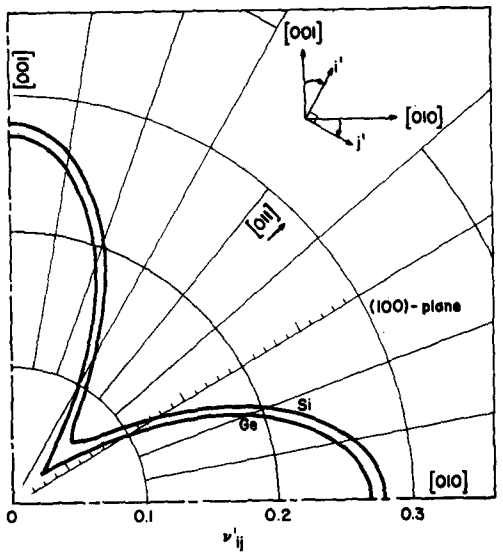

FIG. 3. Poisson's ratio as a function of direction in the (100) plane. Both i' and j' are in the (100) plane.

A shear modulus G_r' can be defined as

$$G_r' \equiv \sigma_r'/\epsilon_r'; \qquad \begin{matrix} r'=4',5',6' \\ \sigma_q'=0,\ q'\neq r'. \end{matrix}$$

It can be shown that $G_r' = 1/s_{rr}'$.

Values of Y_i', ν_{ij}', and G_r' have been worked out for some of the more common crystal orientations These data are shown in Figs. 1–8 and Table III. The graphs are constructed in two different ways. The first method is to let both the i' and j' directions rotate together in a specified plane. The second method is to hold i' fixed in a specified direction and to let j' rotate in the i' plane. Table III lists values of Y_i', ν_{ij}', G_r' which remain constant for certain directions. Only the first quadrant of each plane is shown; the other quadrants are obtained by symmetry. Figures 1 and 2 are plots of Young's modulus; Poisson's ratio is plotted in Figs. 3–5; and shear modulus is plotted in Figs. 6–8.

It can be shown that for a cubic crystal, Y has a maximum in the ⟨111⟩ directions and a minimum in the ⟨100⟩ directions. Poisson's ratio varies over a very large range of values in the crystal as shown in Figs. 3–5. The extreme values of Poisson's ratio have not been de-

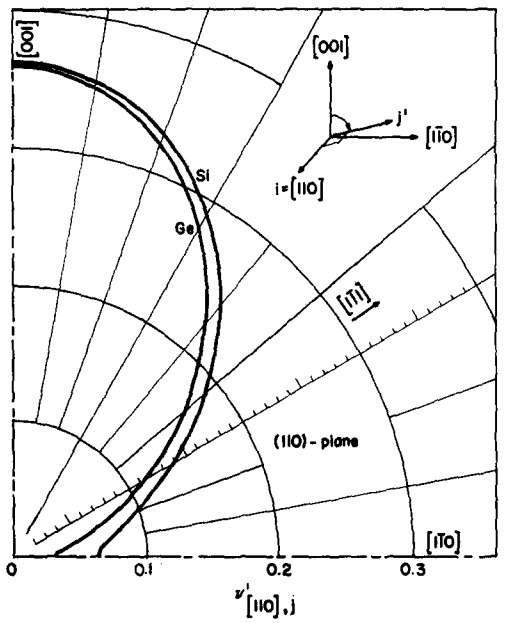

FIG. 5. Poisson's ratio for i' fixed in the [110] direction and j' varying in the (110) plane.

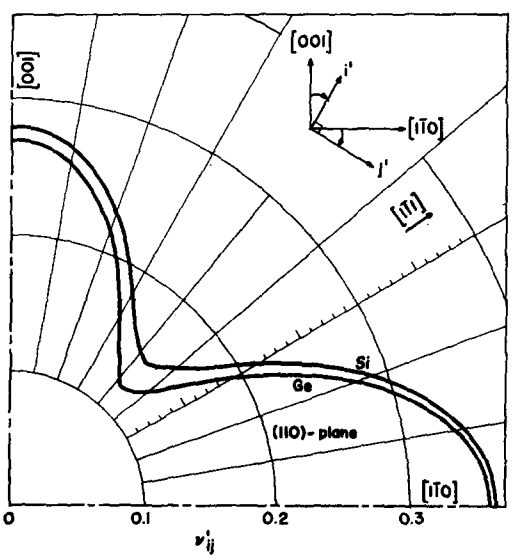

FIG. 4. Poisson's ratio as a function of direction in the (110) plane. Both i' and j' are in the (110) plane.

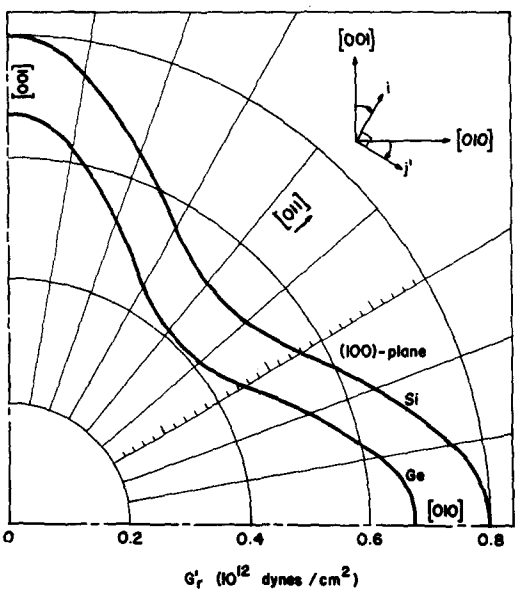

FIG. 6. Shear modulus as a function of direction in the (100) plane.

termined here; however, upper and lower bounds on ν are $0.048<\nu<0.403$ for silicon and $0.022<\nu<0.403$ for germanium.

In cases where the primed axes are principal axes of stress, Hooke's law can be written as

$$Y_1'\epsilon_1'=\sigma_1'-\nu_{12}'\sigma_2'-\nu_{13}'\sigma_3$$
$$Y_2'\epsilon_2'=-\nu_{21}'\sigma_1'+\sigma_2'-\nu_{23}'\sigma_3'$$
$$Y_3'\epsilon_3'=-\nu_{31}'\sigma_1'-\nu_{32}'\sigma_2'+\sigma_3'.$$

TABLE III. Directions for which Y_i', ν_{ij}', G_r' are constant.

Directions	Silicon Y_i' (10^{12} dyn/cm^2)	Silicon G_r' (10^{12} dyn/cm^2)	Silicon ν_{ij}'	Germanium Y_i' (10^{12} dyn/cm^2)	Germanium G_r' (10^{12} dyn/cm^2)	Germanium ν_{ij}'
Both i' and j' in the (111) plane	1.69	0.670	0.358	1.38	0.552	0.358
i' in the [100] direction, j' anywhere in the (100) plane	···	0.796	0.279	···	0.670	0.260
i' in the [111] direction, j' anywhere in the (111) plane	···	0.470	0.180	···	0.578	0.156

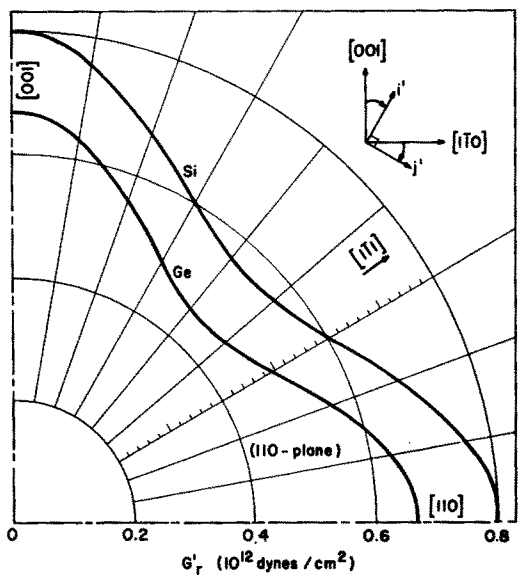

FIG. 7. Shear modulus as a function of direction in the (110) plane.

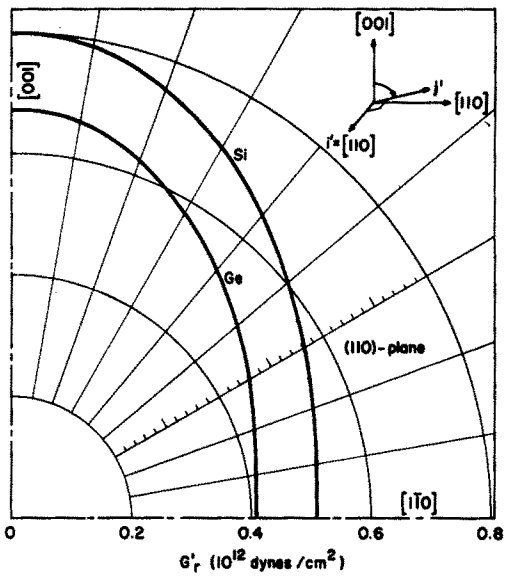

FIG. 8. Shear modulus for i' fixed in the [110] direction and j' varying in the (110) plane.

IV. EXAMPLE

As a simple example of how the figures and Table III are used, consider a germanium crystal. Assume that $i'=1'=[110]$-direction, and $j'=2'=[1\bar{1}1]$-direction; also assume that $\sigma_3'=0$ and ϵ_3' is not of interest

From Fig. 1 $Y_{[110]}'=1.38\times10^{12}$ dyn/cm^2,
Fig. 2 $Y_{[1\bar{1}1]}'=1.55\times10^{12}$ dyn/cm^2,
Fig. 5 $\nu_{[110],\,[1\bar{1}1]}'=0.139$,
Table III $\nu_{[1\bar{1}1],\,[110]}'=0.156$.

Then

$$1.38\times10^{12}\text{dyn/cm}^2\epsilon_1'=\sigma_1'-0.139\sigma_2'$$
$$1.55\times10^{12}\text{dyn/cm}^2\epsilon_2'=-0.156\sigma_1'+\sigma_2'.$$

These equations and data should prove useful for quick calculations of stress and strain in germanium and silicon crystals.

ACKNOWLEDGMENT

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