大数据计算及应用(三)

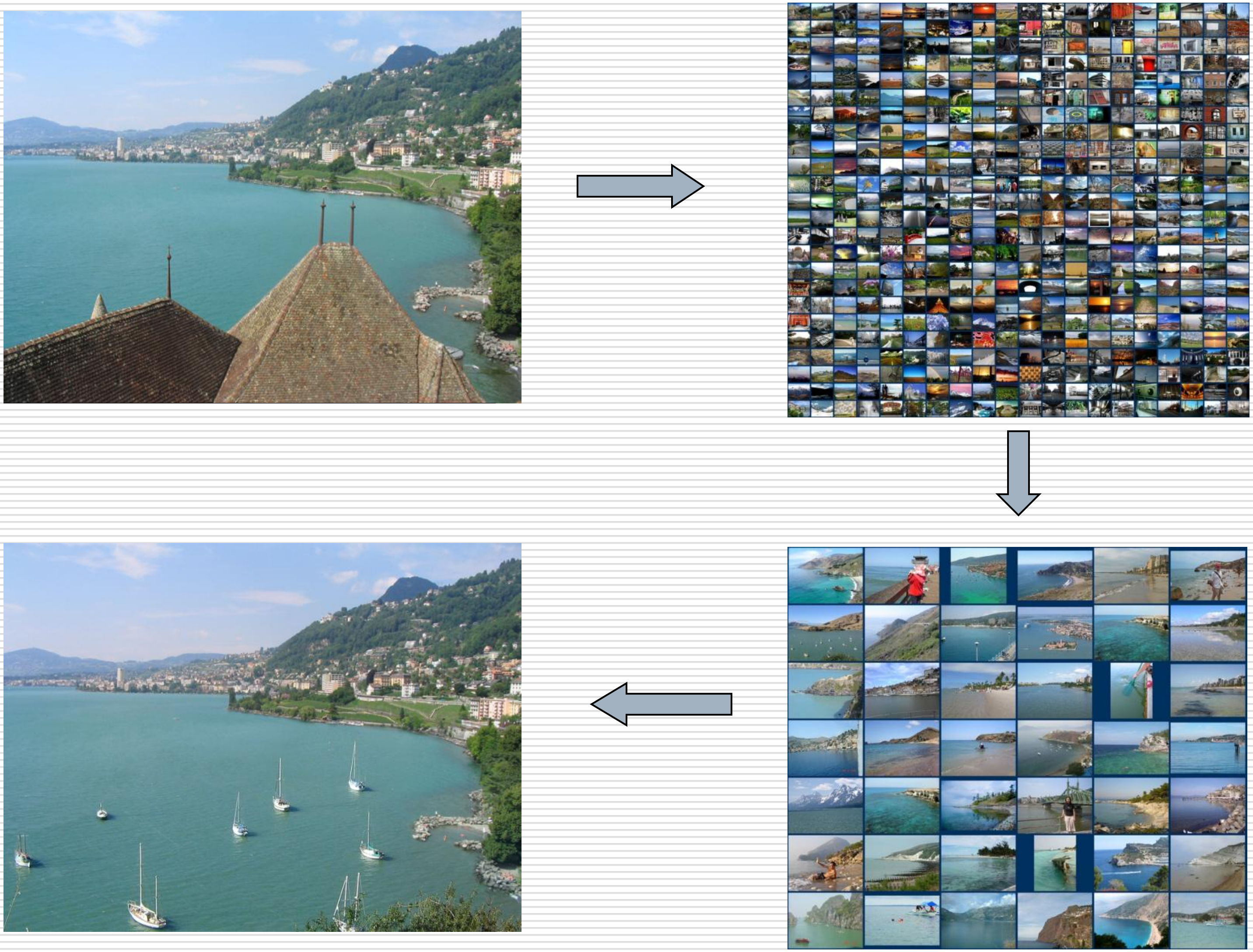
Finding Similar Items: Locality Sensitive Hashing

Slides adapted from http://www.mmds.org

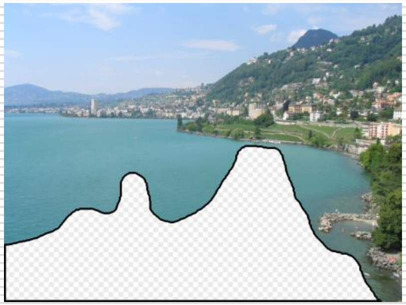
Agenda

High dim. data	Graph data	Infinite data	Machine learning	Apps
Locality sensitive hashing	PageRank, SimRank	Filtering data streams	SVM	Recommender systems
Clustering	Community Detection	Web advertising	Decision Trees	Association Rules
Dimensionality reduction	Spam Detection	Queries on streams	Perceptron, kNN	Duplicate document detection

Scene Completion Problem

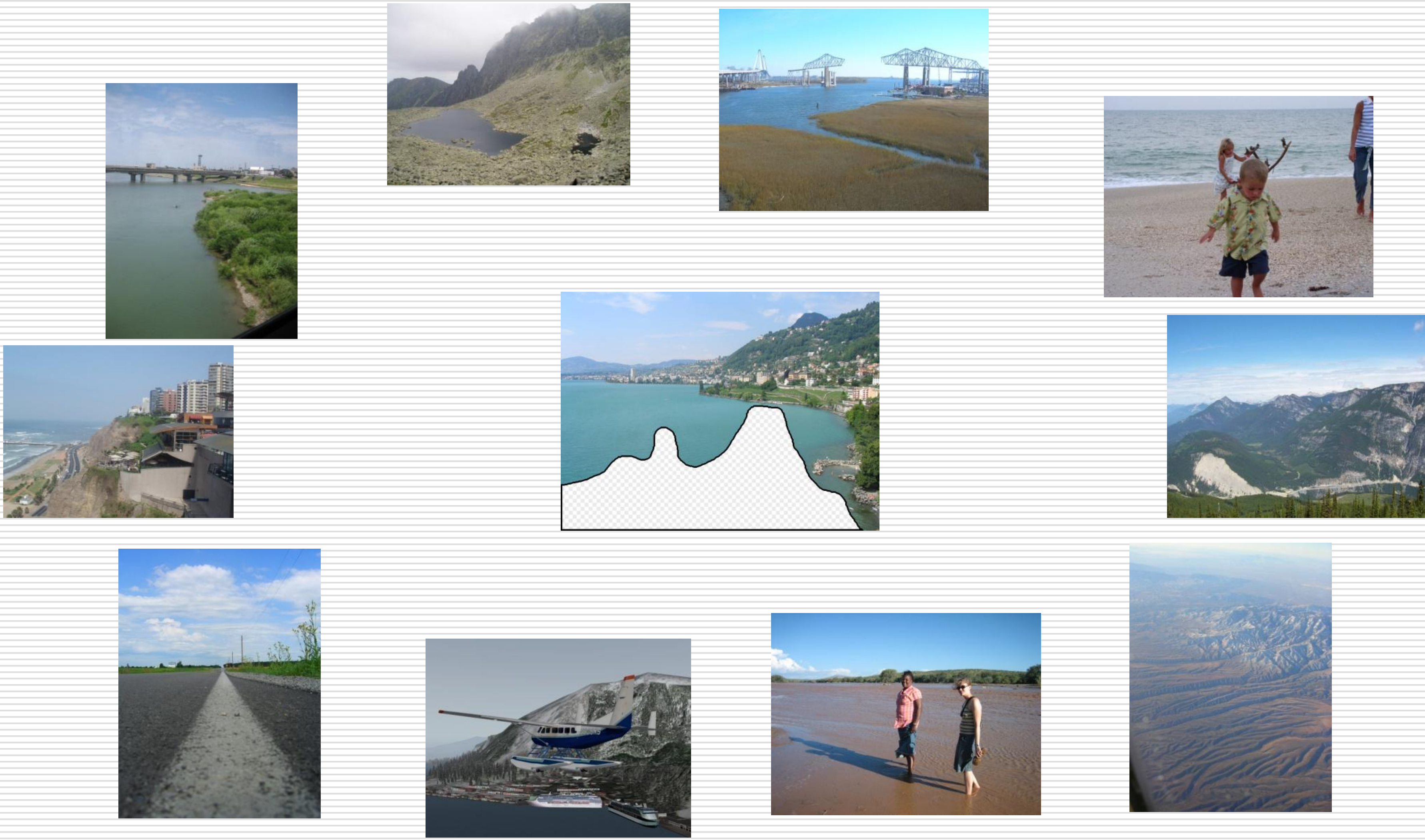


Scene Completion Problem



[Hays and Efros, SIGGRAPH 2007]

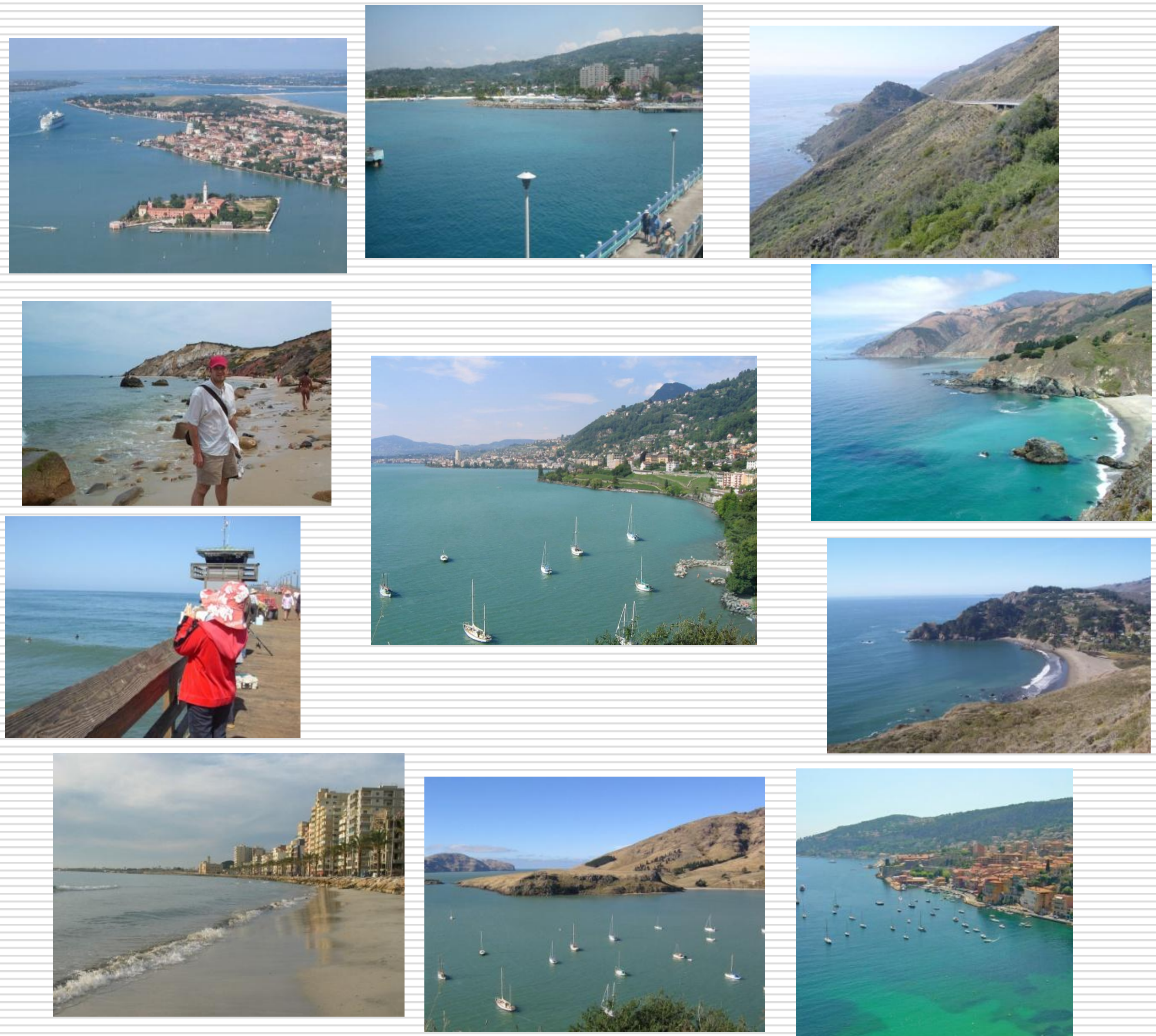
Scene Completion Problem



10 nearest neighbors from a collection of 2.3M images

[Hays and Efros, SIGGRAPH 2007]

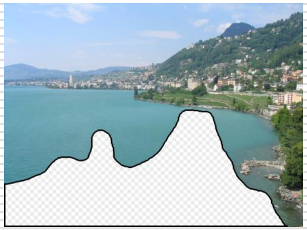
Scene Completion Problem



10 nearest neighbors from a collection of 2.3M images

A Common Metaphor

- Many problems can be expressed as finding "similar" sets:
  - Find near-neighbors in high-dimensional space
- Examples:
  - Pages with similar words
    - For duplicate detection, classification by topic
  - Customers who purchased similar products
    - Products with similar customer sets
  - Images with similar features
    - Users who visited similar websites



Problem for Today's Lecture

- Given: High dimensional data points $x_1, x_2, \ldots$
  - For example: Image is a long vector of pixel colors

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \rightarrow [1\ 0\ 1\ 0\ 0\ 1\ 0\ 1\ 0]$$

img1	*img2*
1	1
0	1
1	1
0	1
0	0
1	1
0	1
1	1
0	0

Is *img1* similar to *img2*?

Problem for Today's Lecture

- Given: High dimensional data points $x_1, x_2, \ldots$
  - Another example:

    Document is a long vector of words existence

	doc1	*doc2*
angle	1	1
before	0	1
country	1	1
end	0	1
food	0	0
good	1	1
house	0	1
live	1	1
use	0	0

Is *doc1* similar to *doc2*?

Problem for Today's Lecture

- Goal: Find all pairs of documents (x_i, x_j) that are within some distance threshold $d(x_i, x_j) \leq s$

n high time complexity: O(n^2) ⇨ O(n)

precision?

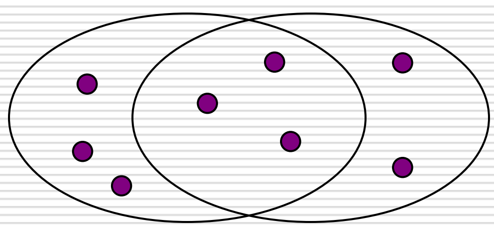
large space requirement

i

doc1	doc2		docN
1	1		1
0	1		1
1	1		1
0	1		1
0	0	......	0
1	1		1
0	1		1
1	1		1
0	0		0
......	......		......

Distance Measures

- Goal: Find near-neighbors in high-dim. space
  - We formally define "near neighbors" as points that are a "small distance" apart
- For each application, we first need to define what "distance" means
- Today: Jaccard distance/similarity
  - The Jaccard similarity of two sets is the size of their intersection divided by the size of their union: $sim(C_1, C_2) = |C_1 \cap C_2| / |C_1 \cup C_2|$
  - Jaccard distance: $d(C_1, C_2) = 1 - |C_1 \cap C_2| / |C_1 \cup C_2|$



3 in intersection
8 in union
Jaccard similarity= 3/8
Jaccard distance = 5/8

Encoding Sets as Bit Vectors

- Encode sets using 0/1 (bit, boolean) vectors
  - One dimension per element in the universal set
- Interpret set intersection as bitwise AND, and set union as bitwise OR

- Example: $C_1 = 10111$; $C_2 = 10011$
  - Size of intersection = 3; size of union = 4,
  - Jaccard similarity (not distance) = 3/4
  - Distance: $d(C_1,C_2) = 1 – (\text{Jaccard similarity}) = 1/4$

Task: Finding Similar Documents

- **Goal:** Given a large number (N in the millions or billions) of documents, find "near duplicate" pairs
- **Applications:**
  - Mirror websites, or approximate mirrors
    - Don't want to show both in search results
  - Similar news articles at many news sites
    - Cluster articles by "same story"
- **Problems:**
  - Many small pieces of one document can appear out of order in another
  - Documents are so large or so many that they cannot fit in main memory
  - Too many documents to compare all pairs

3 Essential Steps for Similar Docs

1. *Shingling:* Convert documents to sets

2. *Min-Hashing:* Convert large sets to short signatures, while preserving similarity

3. *Locality-Sensitive Hashing:* Focus on pairs of signatures likely to be from similar documents
   - Candidate pairs!

Problem for Today's Lecture

precision? Shingling

n

high time complexity: O(n^2) ⇨ O(n)

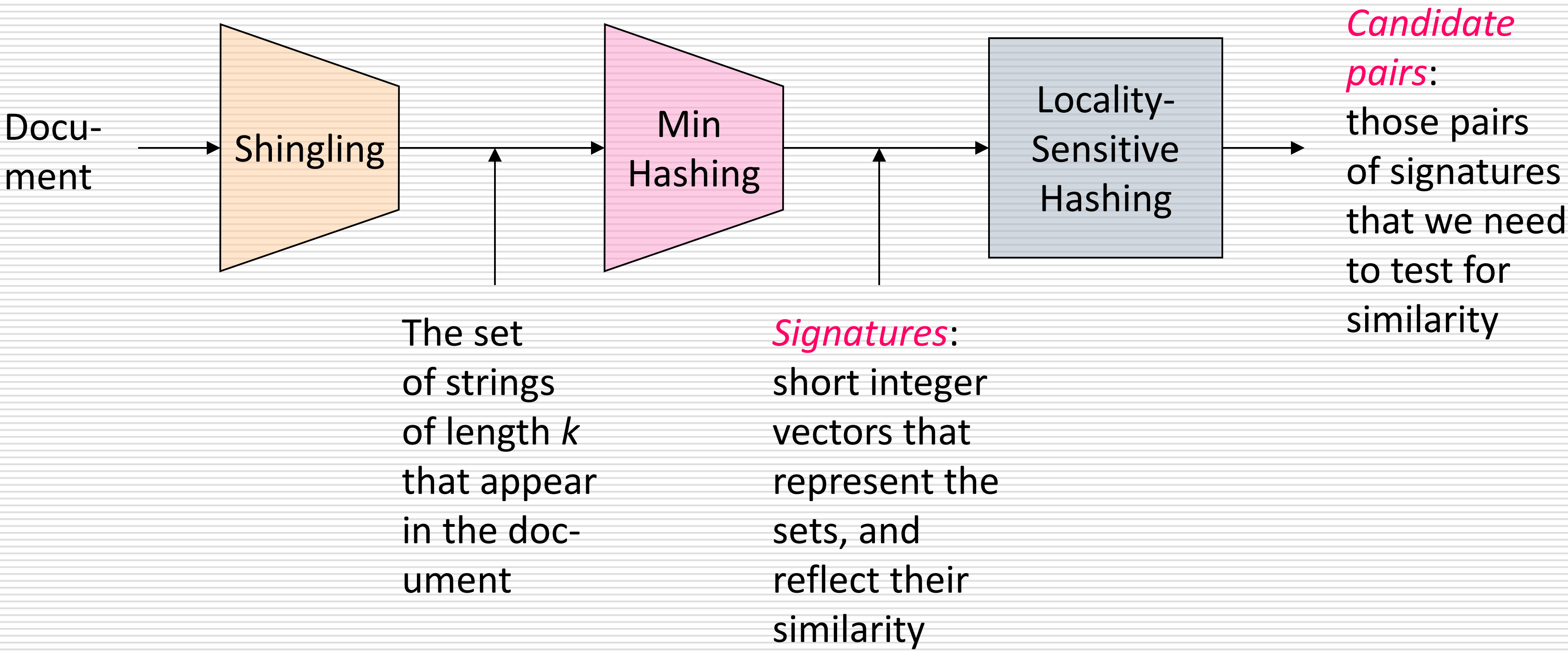
LSH

large space requirement: MinHash

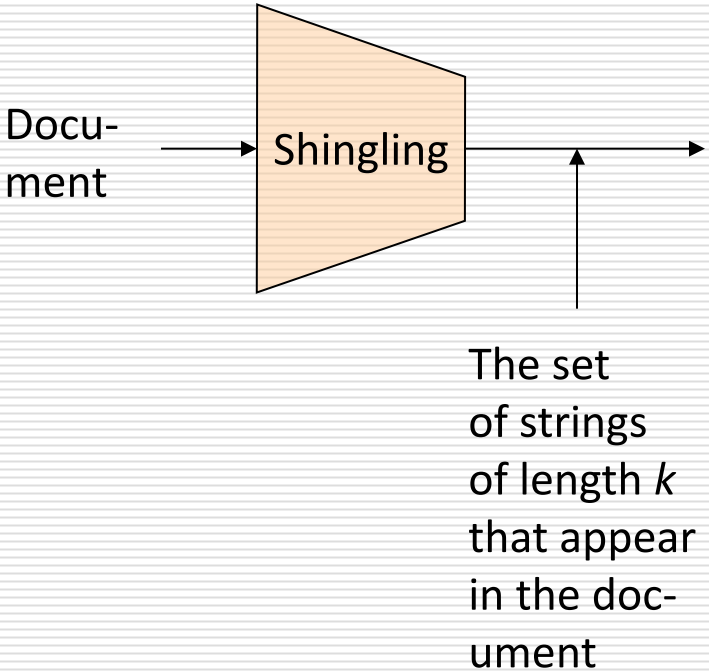
i

doc1	doc2		docN
1	1		1
0	1		1
1	1		1
0	1		1
0	0	......	0
1	1		1
0	1		1
1	1		1
0	0		0
......	......		......

The Big Picture



The Big Picture



- Step 1: *Shingling:* Convert documents to sets

Documents as High-Dim Data

- Step 1: *Shingling:* Convert documents to sets

- Simple approaches:
  - Document = set of words appearing in document
  - Document = set of "important" words
  - Don't work well for this application. Why?

- Need to account for ordering of words!
- A different way: Shingles!

Define: Shingles (Grams)

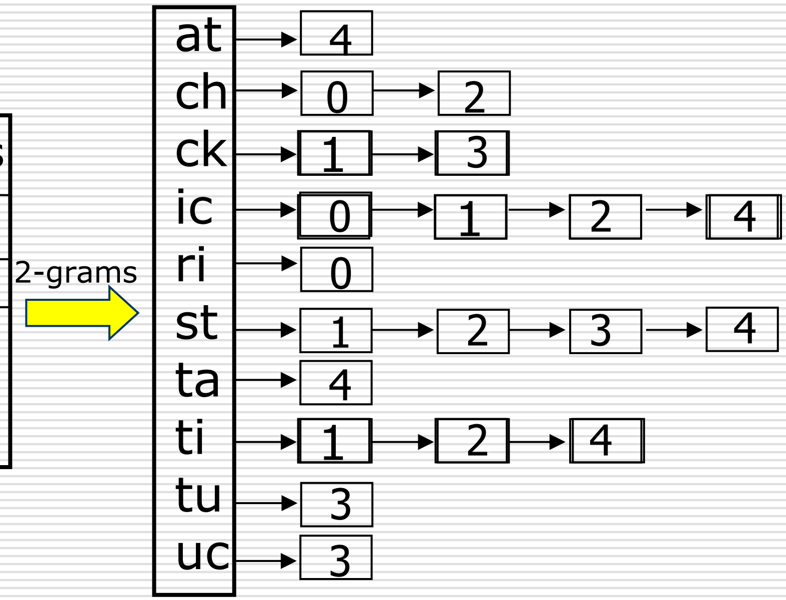
- A *k-shingle* (or *k-gram*) for a document is a sequence of k tokens that appears in the doc
  - Tokens can be characters, words or something else, depending on the application
  - Assume tokens = characters for examples

- Example: k=2; string S_1 = abcab
  Set of 2-shingles: $S(S_1)$ = {ab, bc, ca}

Define: Shingles (Grams)

- Application: similar string search

id	strings
0	rich
1	stick
2	stich
3	stuck
4	static



Inverted list

Define: Shingles (Grams)

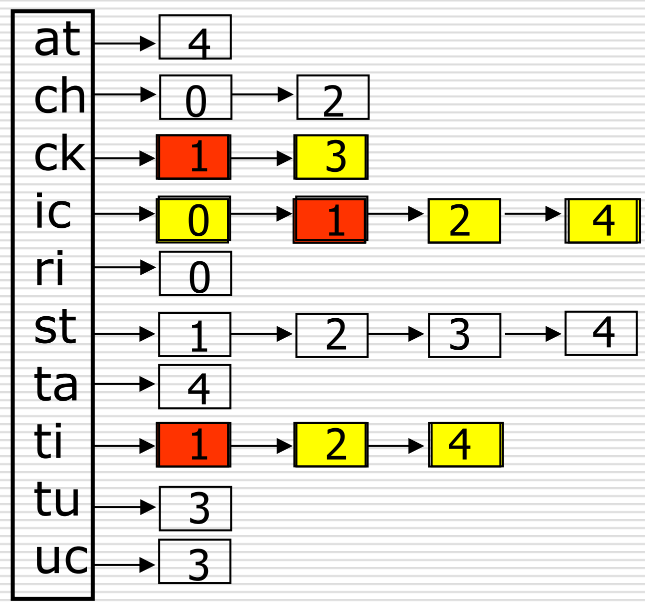
- Application: similar string search

Query: "shtick"

sh ht ti ic ck

id	strings
0	rich
1	stick
2	stich
3	stuck
4	static

2-grams



Inverted list

Compressing Shingles

- To compress long shingles, we can hash them to (say) 4 bytes
- Represent a document by the set of hash values of its *k*-shingles
  - Idea: Two documents could (rarely) appear to have shingles in common, when in fact only the hash-values were shared
- Example: k=2; document D_1= abcab
  Set of 2-shingles: $S(D_1)$ = {ab, bc, ca}
  Hash the singles: $h(D_1)$ = {1, 5, 7}

Similarity Metric for Shingles

- **Document D_1 is a set of its k-shingles $C_1=S(D_1)$**
- Equivalently, each document is a 0/1 vector in the space of *k*-shingles
  - Each unique shingle is a dimension
  - Vectors are very sparse

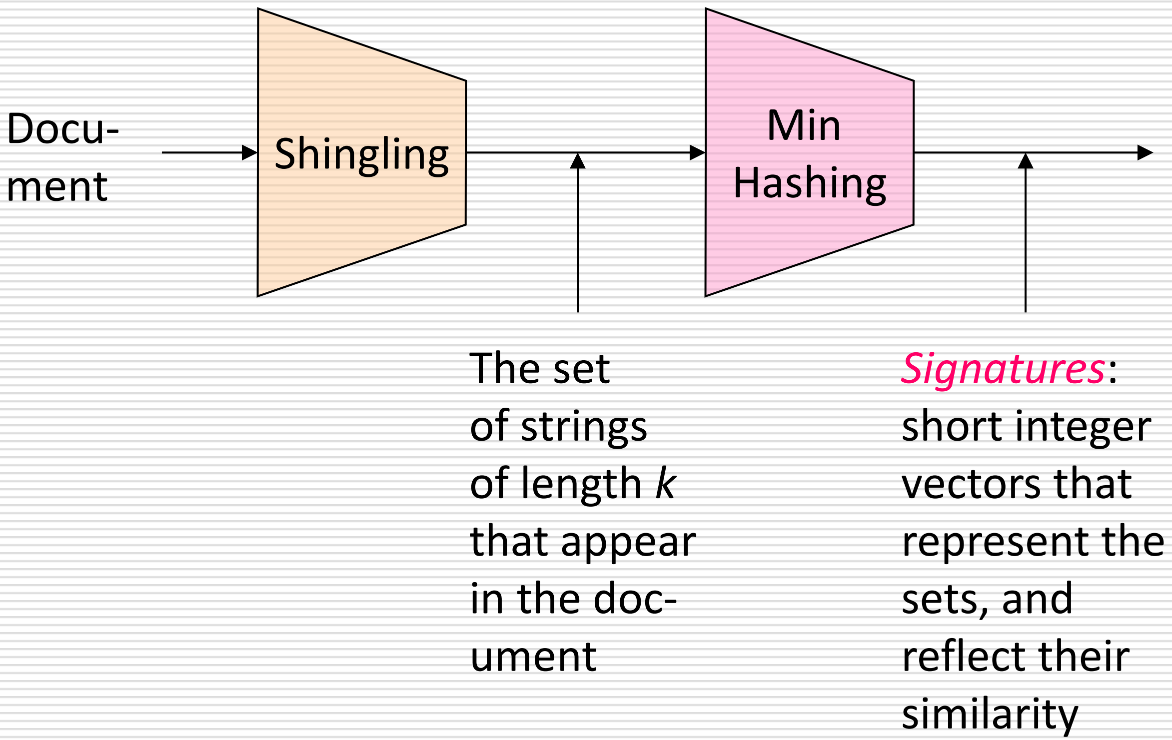
	doc1	*doc2*
go out	1	1
like travel	0	1
Every day	1	1
……	……	

Working Assumption

- Documents that have lots of shingles in common have similar text, even if the text appears in different order

- Caveat: You must pick *k* large enough, or most documents will have most shingles
  - *k* = 5 is OK for short documents
  - *k* = 10 is better for long documents

The Big Picture



- Step 2: *Minhashing:* Convert large sets to short signatures, while <u>preserving similarity</u>

Problem for Today's Lecture

precision? Shingling

large space requirement: MinHash

i

doc1	doc2	……	docN
1	1		1
0	1		1
1	1		1
0	1		1
0	0		0
1	1		1
0	1		1
1	1		1
0	0		0
……	……		……

From Sets to Boolean Matrices

- Rows = elements (shingles)
- Columns = sets (documents)
  - 1 in row *e* and column *s* if and only if *e* is a member of *s*
  - Column similarity is the Jaccard similarity of the corresponding sets (rows with value *1)*
  - Typical matrix is sparse!
- Each document is a column:
  - Example: sim(C_1 ,C_2) = ?
    - Size of intersection = 3; size of union = 6, Jaccard similarity (not distance) = 3/6
    - d(C_1,C_2) = 1 – (Jaccard similarity) = 3/6

Documents

Shingles

1	1	1	0
1	1	0	1
0	1	0	1
0	0	0	1
1	0	0	1
1	1	1	0
1	0	1	0
C_1	C_2		

Outline: Finding Similar Columns

- So far:
  - Documents → Sets of shingles
  - Represent sets as boolean vectors in a matrix
- Next goal: Find similar columns while computing small signatures
  - Similarity of columns == similarity of signatures

Outline: Finding Similar Columns

- Next Goal: Find similar columns, Small signatures
- Naïve approach:
  - 1) Signatures of columns: small summaries of columns
  - 2) Examine pairs of signatures to find similar columns
    - Essential: Similarities of signatures and columns are related
  - 3) Optional: Check that columns with similar signatures are really similar

Hashing Columns (Signatures)

- Key idea: "hash" each column C to a small *signature* $h(C)$, such that:
  - (1) $h(C)$ is small enough that the signature fits in RAM
  - (2) $sim(C_1, C_2)$ is the same as the "similarity" of signatures $h(C_1)$ and $h(C_2)$

- Goal: Find a hash function $h(\cdot)$ such that:
  - If $sim(C_1,C_2)$ is high, then with high prob. $h(C_1) = h(C_2)$
  - If $sim(C_1,C_2)$ is low, then with high prob. $h(C_1) \neq h(C_2)$

Min-Hashing

- Goal: Find a hash function $h(\cdot)$ such that:
  - if $sim(C_1,C_2)$ is high, then with high prob. $h(C_1) = h(C_2)$
  - if $sim(C_1,C_2)$ is low, then with high prob. $h(C_1) \neq h(C_2)$
- Clearly, the hash function depends on the similarity metric:
  - Not all similarity metrics have a suitable hash function
- There is a suitable hash function for the Jaccard similarity: It is called Min-Hashing

Min-Hashing

- Imagine the rows of the boolean matrix permuted under random permutation π
- Define a "hash" function $h_\pi(C)$ = the index of the first (in the permuted order π) row in which column C has value 1:

$$h_\pi(C) = min_\pi\, \pi(C)$$

- Use several (e.g., 100) independent hash functions (that is, permutations) to create a signature of a column

Min-Hashing Example

Note: Another (equivalent) way is to store row indexes:

1	5	1	5
2	3	1	3
6	4	6	4

2nd element of the permutation is the first to map to a 1

Permutation π

2	4	3
3	2	4
7	1	7
6	3	2
1	6	6
5	7	1
4	5	5

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix M

2	1	2	1
2	1	4	1
1	2	1	2

4th element of the permutation is the first to map to a 1

Four Types of Rows

- Given cols C_1 and C_2, rows may be classified as:

	C_1	C_2
A	1	1
B	1	0
C	0	1
D	0	0

  - a = # rows of type A, etc.
- Note: $sim(C_1, C_2) = a/(a + b + c)$
- Then: $Pr[h(C_1) = h(C_2)] = Sim(C_1, C_2)$
  - Look down the cols C_1 and C_2 until we see a 1
  - If it's a type-*A* row, then $h(C_1) = h(C_2)$
    If a type-*B* or type-*C* row, then not

The Min-Hash Property

0	0
0	0
1	1
0	0
0	1
1	0

- Choose a random permutation π
- Claim: $\Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$
- Why?
  - Let X be a doc (set of shingles), $y \in X$ is a shingle
  - Then: $\Pr[\pi(y) = \min(\pi(X))] = 1/|X|$
    - It is equally likely that any $y \in X$ is mapped to the *min* element
  - Let y be s.t. $\pi(y) = \min(\pi(C_1 \cup C_2))$
  - Then either: $\pi(y) = \min(\pi(C_1))$ if $y \in C_1$, or

    $\pi(y) = \min(\pi(C_2))$ if $y \in C_2$
  - So the prob. that both are true is the prob. $y \in C_1 \cap C_2$
  - $\Pr[\min(\pi(C_1)) = \min(\pi(C_2))] = |C_1 \cap C_2| / |C_1 \cup C_2| = sim(C_1, C_2)$

One of the two cols had to have 1 at position *y*

Similarity for Signatures

- We know: $\Pr[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$
- Note: Because of the Min-Hash property, the similarity of columns is the same as the expected similarity of their signatures

Min-Hashing Example

Permutation π

2	4	3
3	2	4
7	1	7
6	3	2
1	6	6
5	7	1
4	5	5

Input matrix (Shingles x Documents)

1	0	1	0
1	0	0	1
0	1	0	1
0	1	0	1
0	1	0	1
1	0	1	0
1	0	1	0

Signature matrix *M*

2	1	2	1
2	1	4	1
1	2	1	2

Similarities:

	1-3	2-4	1-2	3-4
Col/Col	0.75	0.75	0	0
Sig/Sig	0.67	1.00	0	0

Min-Hash Signatures

- Pick K=100 random permutations of the rows
- Think of *sig*(C) as a column vector
- *sig*(C)[i] = according to the *i*-th permutation, the index of the first row that has a 1 in column *C*

  $$sig(C)[i] = \min(\pi_i(C))$$

- Note: The sketch (signature) of document *C* is small ~100 bytes!

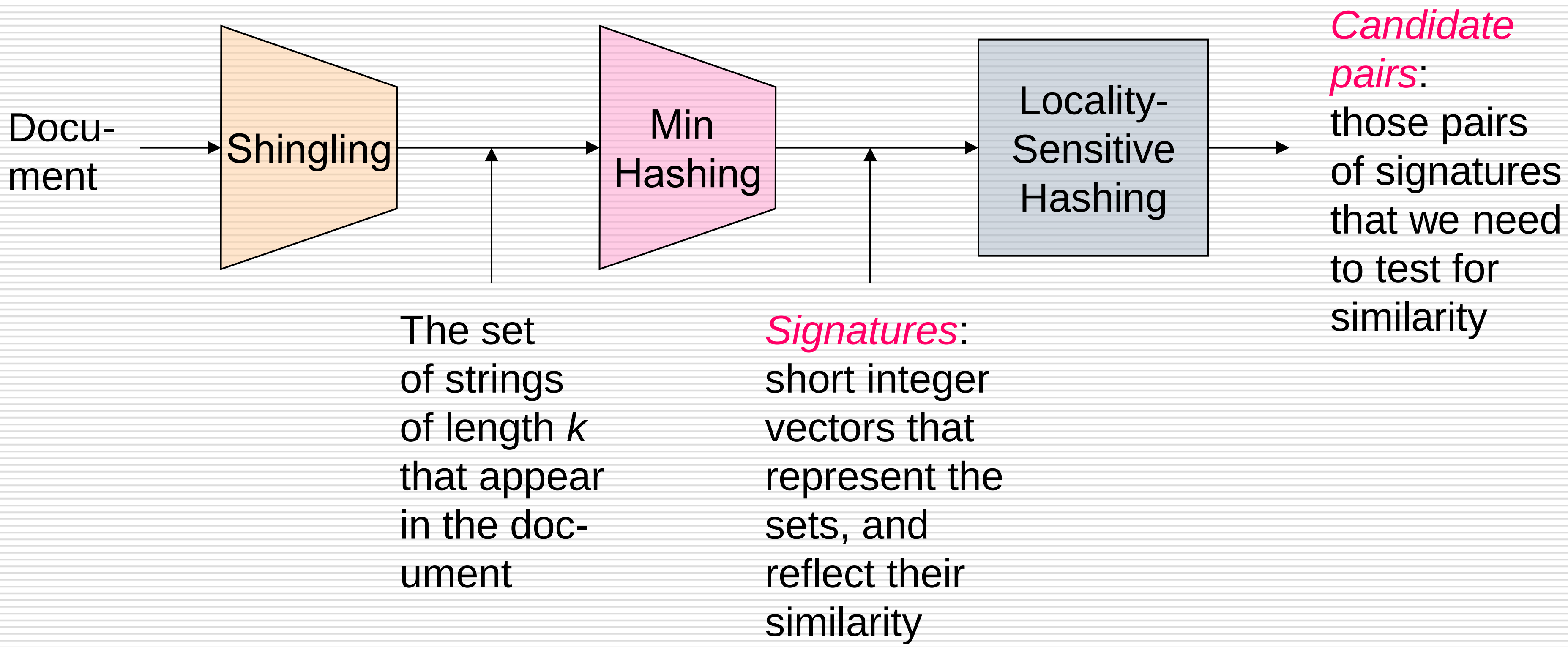
- We achieved our goal! We "compressed" long bit vectors into short signatures

Implementation Trick

- Permuting rows even once is prohibitive
- Row hashing!
  - Pick K = 100 hash functions k_i
  - Ordering under k_i gives a random row permutation!
- One-pass implementation
  - For each column C and hash-func. k_i keep a "slot" for the min-hash value
  - Initialize all $sig(C)[i] = \infty$
  - Scan rows looking for 1s
    - Suppose row j has 1 in column C
    - Then for each k_i :
      - If $k_i(j) < sig(C)[i]$, then $sig(C)[i] \leftarrow k_i(j)$

How to pick a random hash function h(x)?
Universal hashing:
$h_{a,b}(x)=((a \cdot x+b) \bmod p) \bmod N$
where:
a,b … random integers
p … prime number (p > N)

The Big Picture



- Step 3: *Locality-Sensitive Hashing:*
  Focus on pairs of signatures likely to be from similar documents

Motivation for LSH

- Suppose we need to find near-duplicate documents among $N = 1$ million documents

- Naïvely, we would have to compute pairwise Jaccard similarities for every pair of docs
  - $N(N-1)/2 \approx 5*10^{11}$ comparisons
  - At 10^5 secs/day and 10^6 comparisons/sec, it would take 5 days

- For $N = 10$ million, it takes more than a year…

Problem for Today's Lecture

precision? Shingling

n

high time complexity: O(n^2) ⇨ O(n)

LSH

doc1	doc2		docN
1	1		1
0	1		1
1	1		1
0	1		1
0	0	......	0
1	1		1
0	1		1
1	1		1
0	0		0
......	......		......

large space requirement: MinHash

i

LSH: First Cut

2	1	4	1
1	2	1	2
2	1	2	1

- Goal: Find documents with Jaccard similarity at least *s* (for some similarity threshold, e.g., *s*=0.8)

- LSH – General idea: Use a function *f(x,y)* that tells whether *x* and *y* is a *candidate pair:* a pair of elements whose similarity must be evaluated

- For Min-Hash matrices:
  - Hash columns of signature matrix *M* to many buckets
  - Each pair of documents that hashes into the same bucket is a candidate pair

Candidates from Min-Hash

2	1	4	1
1	2	1	2
2	1	2	1

- Pick a similarity threshold *s* (0 < s < 1)

- Columns *x* and *y* of *M* are a candidate pair if their signatures agree on at least fraction *s* of their rows:
  $M(i, x) = M(i, y)$ for at least frac. *s* values of *i*
  - We expect documents *x* and *y* to have the same (Jaccard) similarity as their signatures

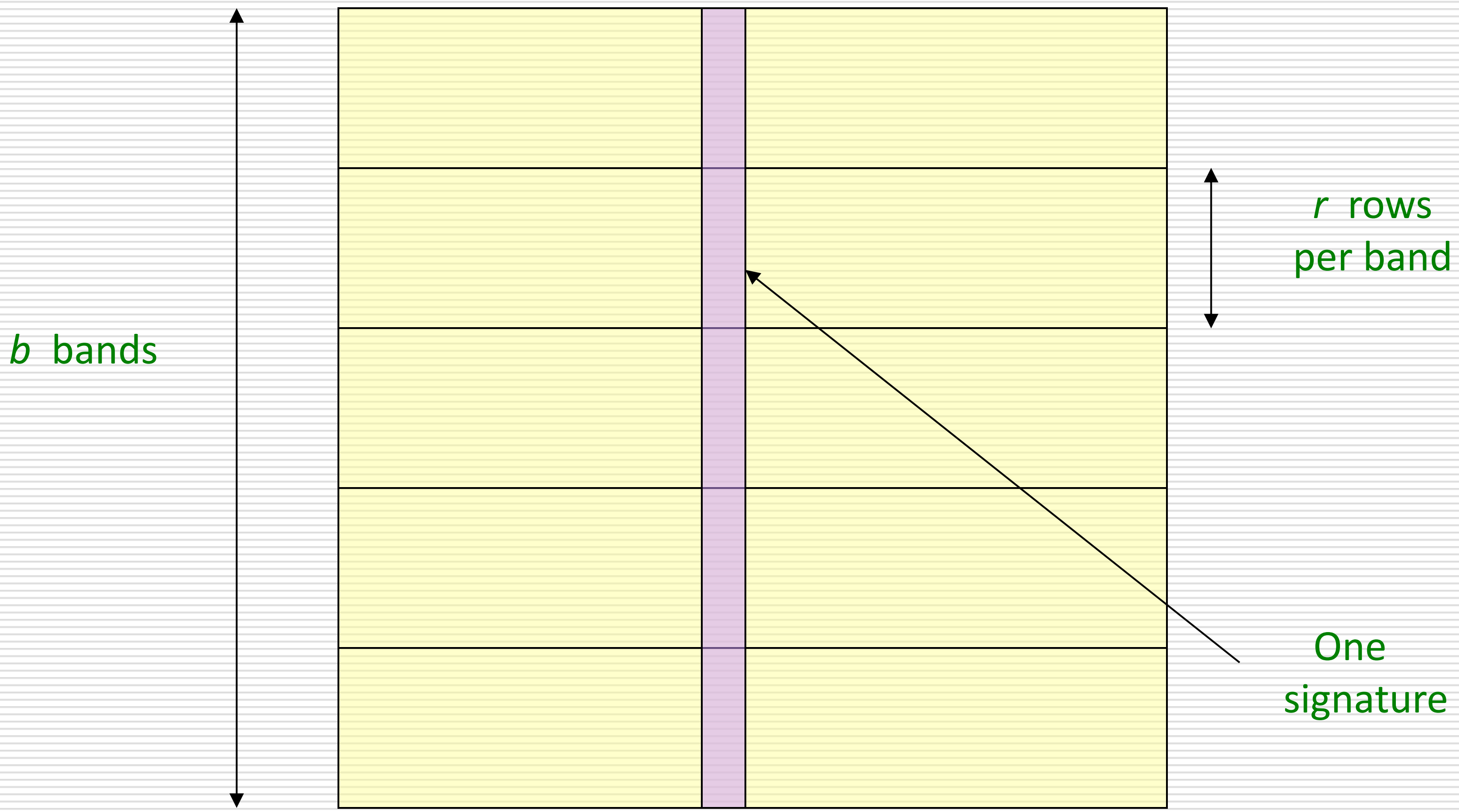
LSH for Min-Hash

2	1	4	1
1	2	1	2
2	1	2	1

- Big idea: Hash columns of signature matrix *M* several times
- Arrange that (only) similar columns are likely to hash to the same bucket, with high probability
- Candidate pairs are those that hash to the same bucket

Partition M into b Bands

2	1	4	1
1	2	1	2
2	1	2	1



Signature matrix M

Partition M into Bands

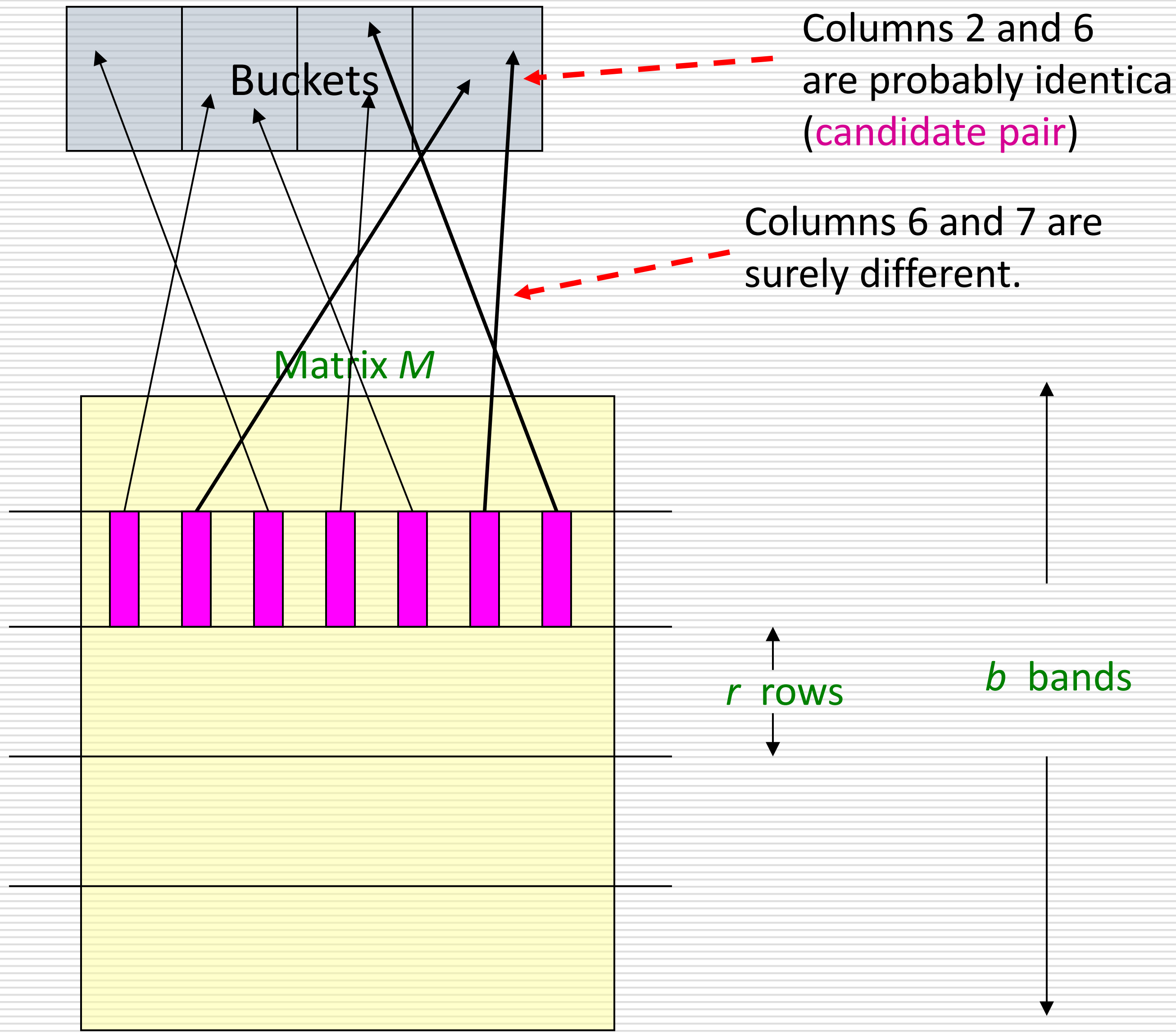
- Divide matrix ***M*** into ***b*** bands of ***r*** rows

- For each band, hash its portion of each column to a hash table with ***k*** buckets
  - Make ***k*** as large as possible

- ***Candidate*** column pairs are those that hash to the same bucket for **≥ 1** band

- Tune ***b*** and ***r*** to catch most similar pairs, but few non-similar pairs

Hashing Bands



Simplifying Assumption

- There are enough buckets that columns are unlikely to hash to the same bucket unless they are identical in a particular band
- Hereafter, we assume that "same bucket" means "identical in that band"
- Assumption needed only to simplify analysis, not for correctness of algorithm

Example of Bands

2	1	4	1
1	2	1	2
2	1	2	1

Assume the following case:

- Suppose 100,000 columns of *M* (100k docs)
- Signatures of 100 integers (rows)
- Therefore, signatures take 40Mb
- Choose *b* = 20 bands of *r* = 5 integers/band

- Goal: Find pairs of documents that are at least *s = 0.8* similar

C_1, C_2 are 80% Similar

2	1	4	1
1	2	1	2
2	1	2	1

- Find pairs of $\geq s$=0.8 similarity, set b=20, r=5
- Assume: sim(C_1, C_2) = 0.8
  - Since sim(C_1, C_2) $\geq$ s, we want C_1, C_2 to be a candidate pair: We want them to hash to at least 1 common bucket (at least one band is identical)
- Probability C_1, C_2 identical in one particular band: $(0.8)^5 = 0.328$
- Probability C_1, C_2 are *not* similar in all of the 20 bands: $(1-0.328)^{20} = 0.00035$
  - i.e., about 1/3000th of the 80%-similar column pairs are false negatives (we miss them)
  - We would find 99.965% pairs of truly similar documents

C_1, C_2 are 30% Similar

2	1	4	1
1	2	1	2
2	1	2	1

- Find pairs of $\geq s$=0.8 similarity, set b=20, r=5
- Assume: sim(C_1, C_2) = 0.3
  - Since sim(C_1, C_2) < s we want C_1, C_2 to hash to NO common buckets (all bands should be different)
- Probability C_1, C_2 identical in one particular band: $(0.3)^5 = 0.00243$
- Probability C_1, C_2 identical in at least 1 of 20 bands: $1 - (1 - 0.00243)^{20} = 0.0474$
  - In other words, approximately 4.74% pairs of docs with similarity 0.3% end up becoming candidate pairs
    - They are false positives since we will have to examine them (they are candidate pairs) but then it will turn out their similarity is below threshold s

LSH Involves a Tradeoff

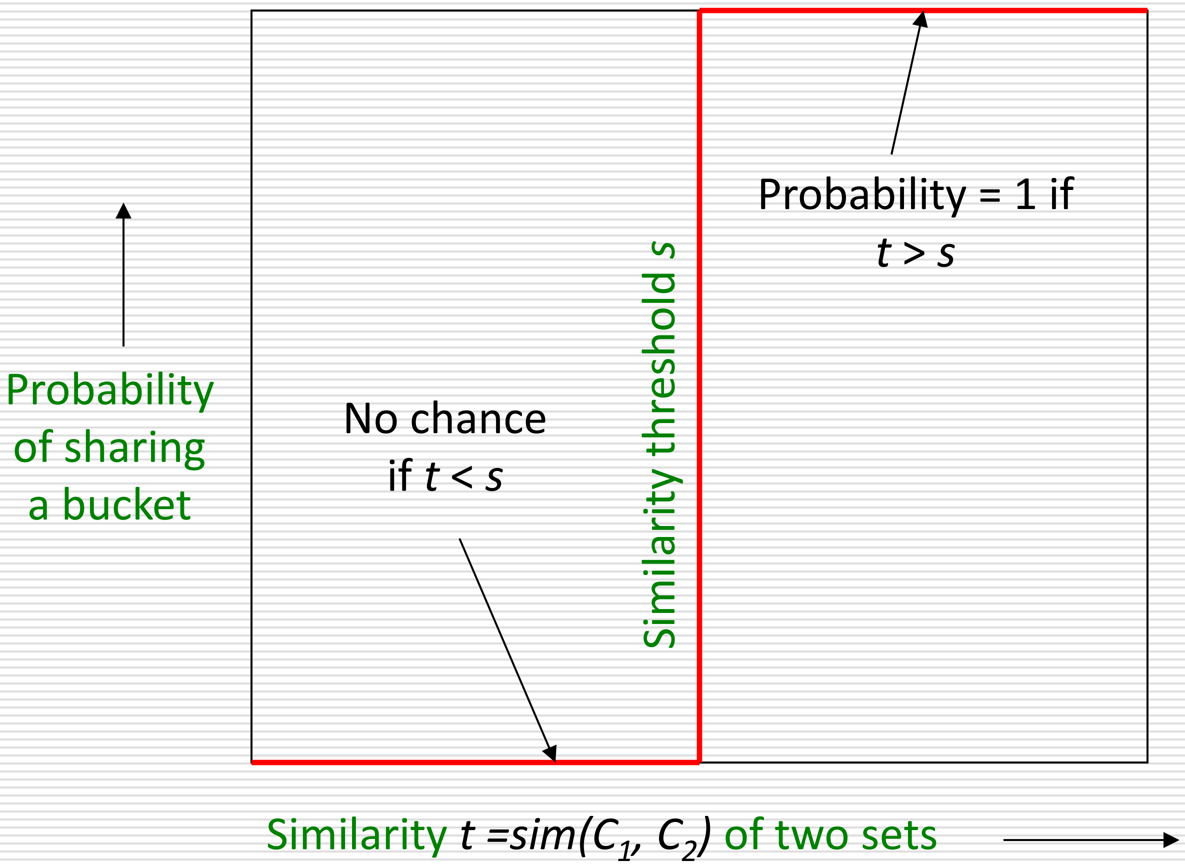
2	1	4	1
1	2	1	2
2	1	2	1

- Pick:
  - The number of Min-Hashes (rows of *M*)
  - The number of bands *b*, and
  - The number of rows *r* per band

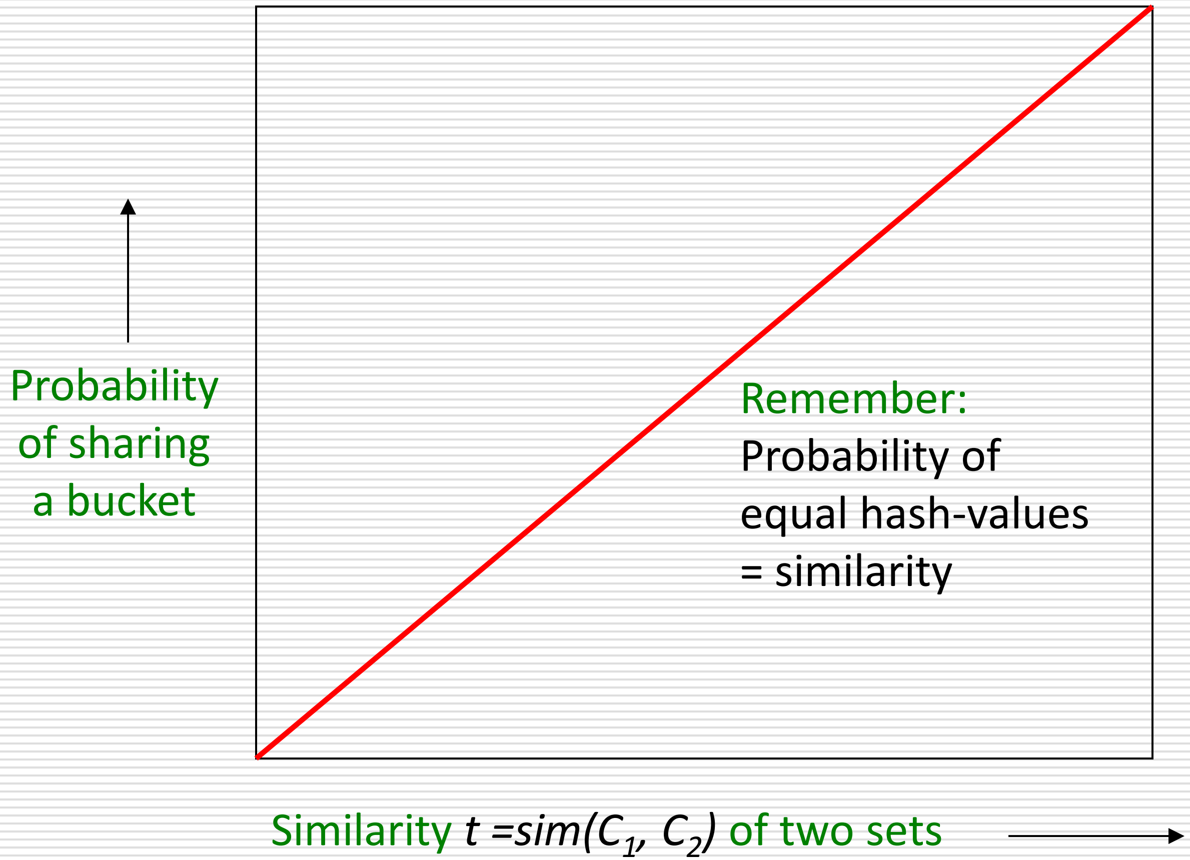
  to balance false positives/negatives

- Example: If we had only 15 bands of 5 rows, the number of false positives would go down, but the number of false negatives would go up

Analysis of LSH – What We Want



What 1 Band of 1 Row Gives You

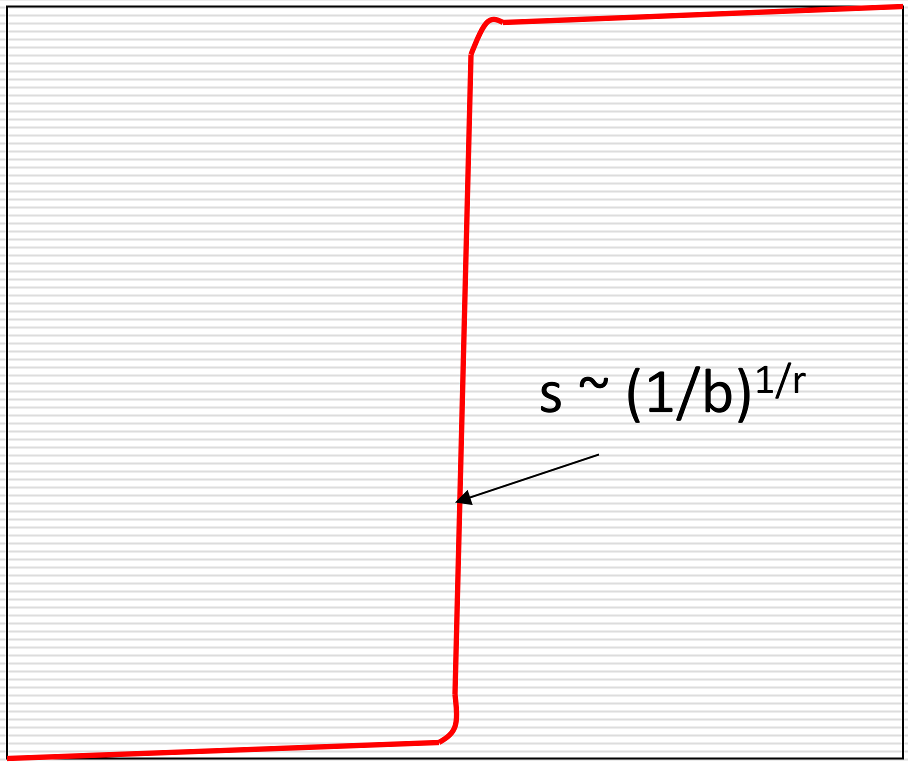


b bands, r rows/band

- Columns C_1 and C_2 have similarity $\boldsymbol{t}$
- Pick any band ($\boldsymbol{r}$ rows)
  - Prob. that all rows in band equal = $\boldsymbol{t^r}$
  - Prob. that some row in band unequal = $\boldsymbol{1 - t^r}$
- Prob. that no band identical = $\boldsymbol{(1 - t^r)^b}$
- Prob. that at least 1 band identical = $\boldsymbol{1 - (1 - t^r)^b}$

What b Bands of r Rows Gives You

Probability of sharing a bucket



Similarity t=$sim(C_1, C_2)$ of two sets

At least one band identical

No bands identical

$$1 - (1 - t^{\,r})^{b}$$

Some row of a band unequal

All rows of a band are equal

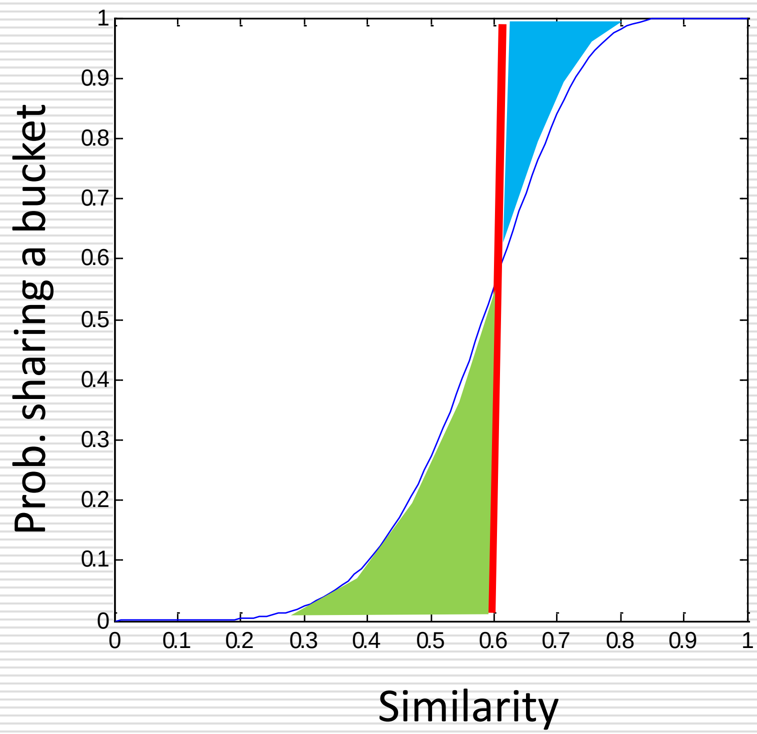
Example: $b = 20$; $r = 5$

- **Similarity threshold s**
- **Prob. that at least 1 band is identical:**

s	$1-(1-s^r)^b$
.2	.006
.3	.047
.4	.186
.5	.470
.6	.802
.7	.975
.8	.9996

Picking *r* and *b*: The S-curve

- Picking *r* and *b* to get the best S-curve
  - 50 hash-functions (r=5, b=10)



Blue area: False Negative rate
Green area: False Positive rate

LSH Summary

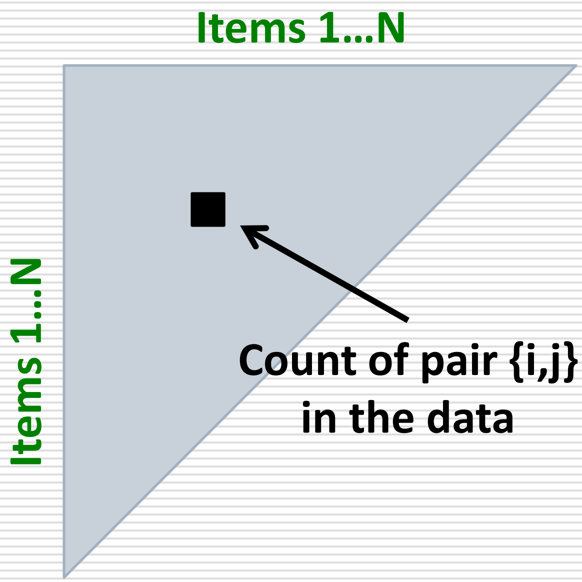
- Tune ***M*, *b*, *r*** to get almost all pairs with similar signatures, but eliminate most pairs that do not have similar signatures
- Check in main memory that **candidate pairs** really do have **similar signatures**
- **Optional:** In another pass through data, check that the remaining candidate pairs really represent similar documents

Summary: 3 Steps

- **Shingling:** Convert documents to sets
  - We used hashing to assign each shingle an ID
- **Min-Hashing:** Convert large sets to short signatures, while preserving similarity
  - We used **similarity preserving hashing** to generate signatures with property $\mathbf{Pr}[h_\pi(C_1) = h_\pi(C_2)] = sim(C_1, C_2)$
  - We used hashing to get around generating random permutations
- **Locality-Sensitive Hashing:** Focus on pairs of signatures likely to be from similar documents
  - We used hashing to find **candidate pairs** of similarity $\geq$ **s**

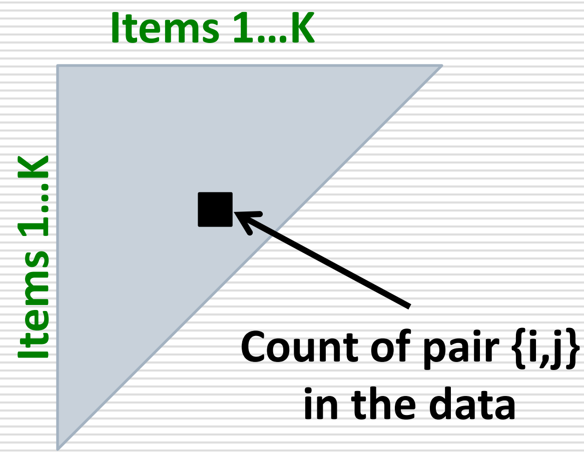
Relation to Previous Lecture

- Last time: Finding frequent pairs



Naïve solution:
Single pass but requires space quadratic in the number of items

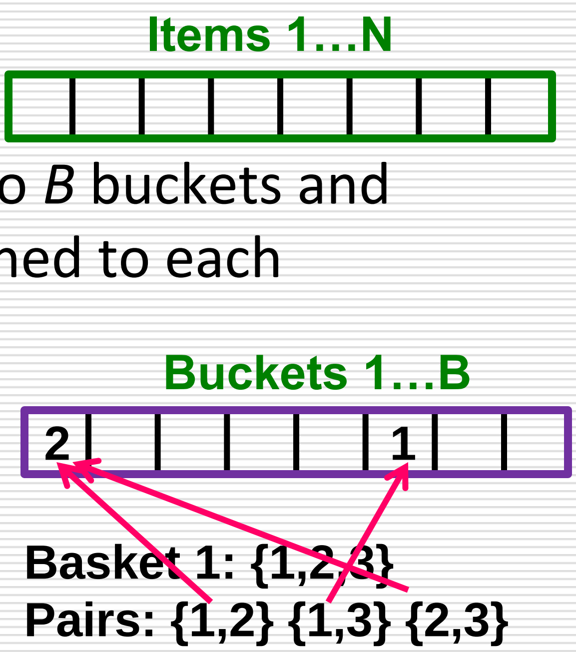
N … number of distinct items
K … number of items with support ≥ *s*



A-Priori:
First pass: Find frequent singletons
For a pair to be a frequent pair candidate, its singletons have to be frequent!
Second pass:
Count only candidate pairs!

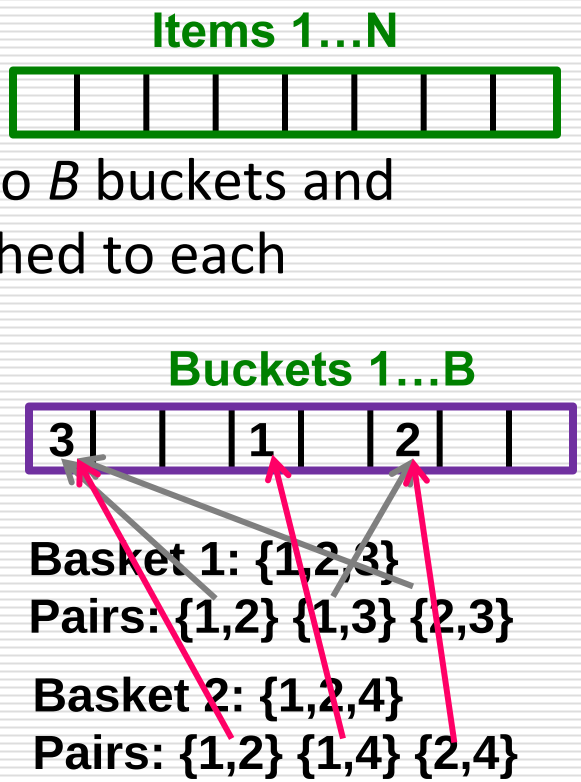
Relation to Previous Lecture

- Last time: Finding frequent pairs
- Further improvement: PCY
  - Pass 1:
    - Count exact frequency of each item:
    - Take pairs of items {i,j}, hash them into *B* buckets and count of the number of pairs that hashed to each bucket:



Relation to Previous Lecture

- Last time: Finding frequent pairs
- Further improvement: PCY
  - Pass 1:
    - Count exact frequency of each item:
    - Take pairs of items {i,j}, hash them into *B* buckets and count of the number of pairs that hashed to each bucket:
  - Pass 2:
    - For a pair {i,j} to be a candidate for a frequent pair, its singletons {i}, {j} have to be frequent and the pair has to hash to a frequent bucket!



Relation to Previous Lecture

- Last time: Finding frequent pairs
- Fu

Previous lecture: A-Priori

Main idea: Candidates

Instead of keeping a count of each pair, only keep a count of candidate pairs!

Today's lecture: Find pairs of similar docs

Main idea: Candidates

-- Pass 1: Take documents and hash them to buckets such that <u>documents that are similar hash to the same bucket</u>

-- Pass 2: Only compare documents that are candidates (i.e., they hashed to a same bucket)

Benefits: Instead of $O(N^2)$ comparisons, we need $O(N)$ comparisons to find similar documents

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