Adding Supply/Demand Imbalance-Sensitivity to Simple Automated Trader-Agents*

Dave Cliff†
Zhen Zhang‡
Nathan Taylor§
Department of Computer Science,
University of Bristol, Bristol BS8 1UB, U.K.

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Abstract

In most major financial markets nowadays very many of the participants are "robot traders", autonomous adaptive software agents empowered to buy and sell quantities of tradeable assets, such as stocks and shares, currencies, and commodities; in the past two decades such robots have largely replaced human traders at the point of execution. This paper addresses the question of how to make minimally simple robot traders sensitive to any imbalance between supply and demand that may occur in the market in the course of a trading session. Such imbalances typically are transient, and their occurrence is unpredictable, but when they do arise any human traders would automatically shift the prices that they quote, to reflect their expectations (grounded in basic microeconomics) that a momentary excess supply is an indication that prices are about to fall, while an excess demand is an indication that prices are about to rise. This can result in prices moving against a trader that is attempting to buy or sell a large quantity of some asset. In this paper we describe our work on exploring the use of *multi-level order-flow imbalance* (MLOFI) as a usefully robust instantaneous measure of supply/demand imbalance, and we show how MLOFI can be used to give simple robot traders an *opinion* about where prices are heading in the immediate future, which mean that our imbalance-sensitive trading robots can serve as a platform for experimental study of issues arising in Nobel laureate Robert Shiller's recent work on *Narrative Economics*.

Keywords:
Market Impact, Adaptive Trader Agents, Financial Markets, Multi-Agent Systems.

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†OrcidID: 0000-0003-3822-9364; csdtc@bristol.ac.uk
‡OrcidID: 0000-0002-4618-6934; zhangzhen@enactusunnc.org
§OrcidID: 0000-0002-6813-2806; nathan.w.taylor@outlook.com

1 Introduction

In recent years Nobel prize-winner Robert Shiller has introduced [32] and popularised [33] the notion of *narrative economics* in which economic phenomena that defy explanation by conventional approaches can be better understood by reference to the *narratives* (i.e. the stories, the commentaries) told and believed by economic agents within the system of interest. As an example, in [33] Shiller argues that the stratospheric rise in price of cryptocurrencies like Bitcoin (which, by conventional economic analyses, should arguably be valueless) can be explained as a consequence of the stories that people tell each other, and themselves believe, about the future prospects of the currency as an investment. In the simplest case, a positive feedback loop of self-fulfilling prophecies can occur where everyone involved in the market convinces themselves that the price is set to rise, and so they seek to purchase more of the asset, and their enthusiasm draws in more buyers, and the consequent excess of demand over supply results in the price going up; this price increase then further confirms the beliefs of the market participants and draws additional in new buyers, and so the loop cycles around with the price rising on each iteration. Such economic "manias"' have long been documented (see e.g. [24]), but Shiller's analysis goes into greater depth, drawing analogies between the spread of narratives through the population of agents involved in some market, and the spread of infected individuals in a population of organisms experiencing an epidemic (or indeed a pandemic); and Shiller calls for new empirical research to track the spread of narratives (e.g. in online news and social media posts), and the changing nature of the narratives themselves (i.e., how the stories people tell each other change over time, creating new variants, in much the same was a mutations in a pathogen also create new variants).

In their 2021 prize-winning ICAART paper, Lomas & Cliff [25], showed how agent-based models (ABMs) could be used to study narrative economics, as an alternative to the empirical approach laid out by Shiller. Lomas & Cliff noted that a narrative, a story, is merely the external expression of an *opinion*, and there is a long-standing research theme in the social science simulation research literature that aims to understand how individuals in a population might influence others to change their opinion, or might themselves alter their own opinion as a result of interaction with other individuals: this field, known as *opinion dynamics* (OD) has seen the development of several notable models over the past 25 years, and Lomas & Cliff showed how these OD models could be integrated into agent-based models of contemporary automated financial markets in which traders buy and sell some asset by posting orders to an automated central exchange (such as a real-world stock-exchange, or a cryptocurrency trading website). The key innovation in Lomas & Cliff's work was that each trader-agent in their automated markets would determine the price that it was willing to buy or sell at by taking account of its opinion, and its opinion could be affected by 'social' interactions with other traders in the market, via one of the established OD models. Although there is a long tradition of agent-based models of financial markets (see e.g. [39, 11, 5, 23]), Lomas & Cliff were the first to integrate opinions into such market models.

And, although the OD literature is extensive, almost all of it explores models in which the opinions being held and altered by each agent are abstract and have no external reference that could establish whether a specific opinion is actually correct or not (e.g. the opinions could be about politics, or religion – topics on which people hold deeply-felt personal opinions, each believing their own opinion to be true, but for which there is no single objectively correct opinion); because of the abstract nature of the agents' opinions, many simulation experiments in OD research start by randomly assigning initial opinions to each agent, using draws from a distribution over the entire range of possible opinions. In much of the most widely referenced OD work, such as the Bounded Confidence (BC) model [22], the Relative Agreement (RA) model [16, 27], and the Relative Disagreement (RD) model [28], each agent in the ABM holds a single opinion that is represented as a real-valued number in the range $[-1,+1]$.

Lomas & Cliff's work is different from much of the existing OD literature because the trader-agents' opinions about the future price of an asset cannot sensibly be initialised by random assignments over the space of all possible prices. Furthermore, in Lomas & Cliff's work, an agent's opinion about whether the price of an asset will fall or rise in the immediate future can subsequently be established as either true or false, because either the price later really did go up, or it really did not. That is, the agents in Lomas & Cliff's model hold opinions about something that they can later shown to be wrong about: this also is unusual within the OD literature.

In this paper, we explore methods by which trader-agents in an automated market can reliable be given a means of forming an opinion about near-term movements in the price of an asset traded on the market. Technically, because the price is the only distinguishing variable about units of an asset, it is a *commodity*, but the model market used here is a close approximation of what happens in any major real-world financial market (in which, for example, individual share certificates are also, technically, commodity goods – because if I am selling one share of Apple Inc at $148, while you are selling one AAPL at $150, an interested buyer would distinguish between your share and my share purely on the basis of price).

Unsurprisingly, opinions about near-term movements in the price of an asset traded on an exchange are very important in real-world financial markets, which have seen an explosive prevalence of automated trading systems being installed and replacing human traders over the past 20 years. Financial markets populated by human traders often exhibit so-called *market impact*, where the prices quoted by traders shift in the direction of anticipated change, as a reaction to the arrival of a large (i.e., "block") buy or sell order for a particular asset: that is, mere knowledge of the presence of the block order is enough to trigger a change in the traders' quote-prices, before any transaction has actually taken place, because the traders know that a block buy order is likely to push the price of the asset up, and a block sell order is likely to push the price down, and so they adjust their quote-prices accordingly, in anticipation of the shift in price that they foresee coming as a consequence of the block-trade completing. This is bad news for the trader trying to buy or sell a block order: the moment she reveals her intention to buy a block, the market-price goes up; the moment she reveals her intention to sell, the price goes down. From the perspective of a block-trader, the market price *moves against her*, whether she is buying or selling, and this happens not because of the *price* she is quoting, but because of the *quantity* that she is attempting to transact.

Block-traders' collective desire to avoid market impact has long driven the introduction of automated trading techniques such as "VWAP engines" (which break block orders into a sequence of smaller sub-orders that are released into the market over a set period of time, with the intention of achieving a specific **v**olume-**w**eighted **a**verage **p**rice, hence VWAP), and has also driven the design of major new electronic exchanges such as London Stock Exchange's (LSE's) *Turquoise Plato* trading venue [26], in which block-traders are allowed to obscure the size of their blocks in a so-called *dark pool* market, with LSE's automated matching engine identifying one or more willing counterparties and only making full details of the block-trade known to all market participants after it has completed: see [30] for further discussion.

Many of the world's major financial markets now have very high levels of automated trading: in such markets most of the participants, the traders, are "robots" rather than humans: i.e., software systems for automated trading, empowered with the same legal sense of agency as a human trader, and hence "software agents" in the most literal sense of that phrase. Given that these software agents typically replace more than one human trader, and given that those human traders were widely regarded to have required a high degree of intelligence (and remuneration) to work well in a financial market, it is clear that the issue of designing well-performing robot traders presents challenges for research in agents *and* artificial intelligence, and hence is a research topic that is central to the themes of the ICAART conference.

This paper addresses the question of best how to give robot traders an appropriate anticipatory sensitivity to large orders, such that markets populated entirely by robot traders also show market-impact effects. This is desirable because the impact-sensitive robot traders will get a better price for their transactions when block orders do arrive, and also because simulated markets populated by impact-sensitive automated traders can be studied to explore the pros and cons of various impact-mitigation or avoidance techniques. We show here that well-known and long-established trader-agent strategies can be extended by giving them appropriately robust sensitivity to the imbalance between buy and sell orders issued by traders on the exchange, orders that are aggregated on the market's limit order-book (LOB), the data-structure at the heart of most electronic exchanges.

To the best of our knowledge, the first paper to report on the use of an imbalance metric to give ABM automated traders impact-sensitivity was the 2019 publication by Church & Cliff [6], in which they demonstrated how a minimal nonadaptive trader-agent called *Shaver* (which, following the convention of practice in this field, is routinely referred to in abbreviated form via a psuedo-ticker-symbol: "SHVR") could be extended to show impact effects by addition of an imbalance metric, and Church & Cliff gave the name *ISHV* to their Imbalance-SHVR trader-agent [6]. SHVR is a trader-agent strategy built-in to the popular open-source financial exchange simulator called BSE [2], which Church & Cliff used as the platform for their research. Although Church & Cliff deserve some credit for the proof-of-concept that ISHV provides, we argue here that the imbalance-metric they employed is too fragile for practical purposes because very minor changes in the supply and demand can cause their metric to swing wildly between the extremes of its range. One of the major contributions of our paper here (which is revised and extended from [44]) is the demonstration that a much better, more robust, metric known as *multi-level order-flow imbalance* (MLOFI) can be used instead of the comparatively very fragile metric proposed by Church & Cliff. Another major contribution of this paper is our demonstration of the addition of MLOFI-based impact-sensitivity to the very well-known and widely cited public-domain adaptive trader-agent strategies ZIP [8] and AA [41, 40]. Although our primary aim was to add impact sensitivity to these two machine-learning-based trader-agent strategies, we also demonstrate in this paper that ISHV can be altered/extended to use MLOFI, and our improvement of Church & Cliff's work in that regard is an additional contribution of this paper.

The extended versions of the AA, ZIP, and ISHV trader-agent strategies that we introduce here are named ZZIAA, ZZIZIP, and ZZISHV respectively. In this paper, after our criticism of Church & Cliff's methods, we describe a more mathematically sophisticated approach to measuring imbalance, which is more robust, and which we incorporate into our agent extensions. We then present results from testing our extended trader-agents on BSE, the same platform that was used in Church & Cliff's work. Full details of the work reported here are available in [46], and all the relevant source-code has been made freely available on *GitHub* [45].

Section 2 of this paper presents an overview of relevant background material: readers already familiar with automated trading systems, contemporary electronic financial exchanges, and the mathematics of order-flow imbalance, can safely skip ahead straight to Section 3. In Section 3 we describe the steps taken to add MLOFI-based impact-sensitivity to ZIP, AA, ISHV, and PRZI; and illustrative results from ABMs populated by those extended trading algorithms are presented.

2 Prior Work

2.1 Automated Traders

Since the mid-1990s researchers in universities and in the research labs of major corporations such as IBM and Hewlett-Packard have published details of various strategies for autonomous trader-agents, often incorporating AI and/or machine learning (ML) methods so that the automated trader can adapt its behaviors to prevailing market conditions. Notable trading strategies in this body of literature include: Kaplan's "Sniper" strategy [31]; Gode & Sunder's ZIC [20]; the ZIP strategy developed at Hewlett-Packard [8]; the GD strategy reported by Gjerstad & Dickhaut [19] the MGD and GDX strategies developed by IBM researchers [37, 38]; Gjerstad's HBL [18]; Duffy & Unver's NZI [17];Vytelingum's AA [41, 40]; the Roth-Erev approach (see e.g. [29]); Arifovic & Ledyard's IEL [1]; and the recently-introduced PRZI [7], described further in Section 3.2. However, for reasons discussed at length in a recent review of key papers in the field [34] this sequence of publications concentrated on the issue of developing trading strategies for orders where the quantities were all in the same order of magnitude (and often, where the quantity was fixed at one, a single unit per order). That is, *none* of the key papers listed here deal with trading strategies for outsize block orders, and none of them directly explore the issue of how an automated trader can best deal with, or avoid, market impact.

Trader-agent strategies such as Sniper, ZIC, ZIP, GD and MGD were all developed to operate in electronic markets that were based on old-school open-outcry trading pits, as were common on major financial exchanges until face-to-face human-to-human bargaining was replaced by negotiation of trades via electronic communication media; but more recent work has concentrated on developing trading agents that issue bids and asks (i.e. quotations for orders to buy or to sell) to a centralised electronic exchange (such as a major stock-market like NYSE or NASDAQ or LSE) where the exchange's *matching engine* then either matches the trader's quote with a willing counterparty (in which case a transaction is recorded between the two counterparties, the buyer and the seller) or the quote is added to a data-structure called the *Limit Order Book* (LOB) that is maintained by the exchange and published to all traders whenever it changes. The LOB aggregates and anonymises all outstanding orders: it has two sides or halves: the bid-side and the ask-side. Each side of the book shows a summary of all outstanding orders, arranged from best to worst: this means that the bid-side is arranged in descending price-order, and the ask side is arranged in ascending price-order, such that at the "top of the book" on the two sides the best bid and ask are visible. For all orders currently sat on the LOB, if there are multiple orders at the same price then the quantities of those orders are aggregated together, and often multiple orders at the same price will be later matched with a counterparty in a sequence given by the orders' arrival times, in a first-in-first-out fashion. The public LOB shows only, for each side of the book, the prices at which orders have been lodged with the exchange, and the total quantity available at each of those prices: if no orders are resting at the exchange for a particular price, then that price is usually omitted from the LOB rather than being shown with a corresponding quantity of zero. Illustrations of LOBs appear later in this paper.

The difference between the price of the best bid on the LOB at time t and the price of the best ask at t is known as the *spread*. The mid-point of the spread (i.e. the arithmetic mean of the best bid and the best ask) is known as the *mid-price* which is denoted here by P_{mid}. The mid-price is very commonly used as a single-valued statistic to summarise the current state of the market, and as an estimate of what the next transaction price would be. However, the midprice pays no attention to the quantities that are bid and offered. If the current best bid is for a quantity of one at a price of \$10 and the current best ask is for a quantity of 200 at a price of \$20 then the mid-price is \$15 but that fails to capture that there is a much larger quantity being offered than being bid: basic microeconomics, the theory of supply and demand, would tell even the most casual observer that with such heavy selling pressure then actual transaction prices are likely to trend down – in which case the mid-price of \$15 is likely to be an overestimate of the next transaction price. Similarly, if the bid and ask prices remain the same but the *imbalance* between supply and demand is instead reversed, then the fact that there is a revealed desire for 200 units to be purchased but only one unit on sale at the current best ask would surely be a reasonable indication that transaction prices are likely to be pushed up by buying pressure, in which case the mid-price of \$15 will turn out to be an underestimate. This lack of quantity-sensitivity in the mid-price calculation leads many market practitioners to instead monitor the *micro-price*, denoted here by P_{micro}, which is a quantity-weighted average of the best bid and best ask prices, and which does move in the direction indicated by imbalances between supply and demand at the top of the LOB: see, e.g., [4].

To the best of our knowledge the first impact-sensitive ABM trader-agent was *ISHV* [6]. ISHV is based on the *SHVR* trader built into the popular *BSE* public-domain financial-market simulator [2, 9]. A SHVR trader simply posts the buy/sell order with its price set one penny higher/lower than the current best bid/ask. This single instruction gives it a parasitic nature, in the sense that it can mimic the price-convergence behaviour of other strategies being used by other traders in the market.

Instead of shaving the best bid or offer by one penny, Church & Cliff's ISHV trader instead chooses to shave by an amount Δ_s which varies with Δ_m defined in Equation 1:

$$\Delta_m = P_{\text{micro}} - P_{\text{mid}} \tag{1}$$

The difference of the micro-price and the mid-price can identify the degree of supply/demand imbalance to a useful extent. If $\Delta_m \approx 0$, there is no obvious imbalance in the market. If $\Delta_m < 0$, then the quantities of the best bid and the best offer on the LOB indicate that supply exceeds demand and the subsequent transactions prices are likely to decrease; whereas $\Delta_m > 0$ indicates that demand outweighs supply and subsequent transaction prices will have an upward tendency.

Church & Cliff give pseudocode for ISHV in [6] and source-code for a Python implementation is freely available at [2]. ISHV implements a function that maps from Δ_m to Δ_s to determine how much it will shave off its price. For an ISHV buyer, if $\Delta_m < 0$, SHVR 'predicts' the price will shift in its favour and shaves its price as little as possible (the exchange's minimum tick-size Δ_p – often one penny or one cent – is chosen as the value for Δ_s): we might say that in this situation SHVR is *relaxed* However, if $\Delta_m > 0$, ISHV 'predicts' that later prices will be worse and so it attempts to shave a large amount off, using $\Delta_s = C\Delta_p + M\Delta_m\Delta_p$, where C and M are two constants that determine the SHVR's response to the imbalance (they are the y-intersect and gradient for a linear response function; nonlinear response functions could be used instead): in such a situation we might say that SHRV is *urgent*. The algorithm for an ISHV seller is the same *mutatis mutandis*. Church & Cliff showed that ISHV can identify and respond appropriately to the presence of a block order signal at the top of the LOB.

2.2 Critique of Church & Cliff

Church & Cliff were careful to flag their ISHV trader as only a proof-of-concept (PoC): ISHV was developed to enable the study of coupled lit/dark trading polls such as LSE *Turquoise Plato* system in commercial operation in London, as mentioned in the Introduction to this paper. Without impact-sensitive trader-agents, it is not possible to build agent-based models of contemporary real-world trading venues such as LSE Turquoise Plato. Having experimented further with Church & Cliff's PoC system, we came to realise that there are severe limitations in ISHV: these limitations stem from the fact that Equation 1, which is at the heart of ISHV, uses values *only found at the top of the LOB*: Equation 1 involves only the price and quantity of the best bid and the best ask. As we will demonstrate in the next section, this makes the method introduced by Church & Cliff so fragile that it is unlikely to be usable in anything but the simplest of simulation studies; as we show in the next section, for real-world markets it is necessary to look deeper into the LOB, to delve below the top of the LOB.

For brevity, we will limit ourselves here to presenting a qualitative illustrative example which demonstrates how wildly fragile the Church & Cliff method is. For a longer and more detailed discussion, see Chapter 3 of [46].

Consider a situation in which the top of the LOB has a best bid price of $10 and a best ask price of $20, as before, and where the quantity at the best bid is 200 and at the best ask is 1. As we explained in the previous section, this huge imbalance between supply and demand at the top of the book indicate that the excess demand is likely to push transaction prices up in the immediate future. Church & Cliff's ISHV does the right thing in this situation.

Now consider what happens if the next order to arrive at the exchange is a bid for $11 at a quantity of 1. Because this fresh bid is at a higher price than the current best bid, it is inserted at the top of the bid-side of the LOB. The previous best-bid, for 200 at $10, gets shuffled down to the second layer of the LOB. At that point, the best bid and the best ask each show a quantity of one, and so ISHV acts as if there is no imbalance in the market, despite the fact when viewing *the whole LOB* it is clear that the quantity bid is now 201 (i.e. 1 at $11 and 200 at $10) while the ask quantity is still only 1: if anything, the imbalance has *increased* but ISHV reacts as if it had *disappeared* because ISHV looks only at the top of the LOB.

There is more that could be said, but this should be enough to convince the reader that any impact-sensitive trader-agent algorithm that looks only at the data at the top of the LOB is surely going to get it wrong very often, because it is ignoring the supply and demand information, the quantities and the prices, which lie deeper in the LOB. What we introduce in the rest of this paper addresses this problem.

2.3 Measuring Imbalance

A reliable metric is needed to capture the quantity imbalance between the supply side and the demand side, at multiple levels in the LOB (i.e., not just the top) and which can quantitatively indicate how much the imbalance will affect the market. We first discuss the Order-Flow Imbalance (OFI) metric introduced by [10] and then describe the extension of this into a reliable Multi-Level OFI (MLOFI) metric recently reported by [43]. After that, we show how MLOFI can be used to give robust impact-sensitivity to ISHV [6], AA [41, 40], and ZIP [8]. AA and ZIP are of particular interest because in previous papers published at IJCAI and at ICAART it was demonstrated that these two strategies can each reliably outperform human traders [12, 14, 13, 15].

2.3.1 Order Flow Imbalance (OFI)

Cont *et al.* argued that previous studies modelling impact are extremely complex, and that instead a single factor, the order flow imbalance (OFI), can adequately explain the impact ($R^2 = 67\%$ in their research) [10]. They indicated that OFI has a positive linear relation with mid-price changes, and that the market depth D is inversely proportional to the scope of the relationship. OFI means the net order flow at the bid-side and the ask-side, and the market depth, D, represents the size at each bid/ask quote price.

To calculate the OFI they focused on the "Level 1 order book", i.e. the best bid and ask at the top of the LOB. Between any two events ($event_n$ and $event_{n-1}$), only one change happens in the LOB (check the condition from top to bottom, and from left to right; in other words, we should compare the change of price first and if the price does not change, then compare the change of quantity). Using $D\uparrow$ and $D\downarrow$ to respectively denote an increase and a reduction in demand; and $S\uparrow$ and $S\downarrow$ to denote an increase/decrease in supply, Cont et al. had:

$$\begin{array}{lcl} p_n^b > p_{n-1}^b & \vee & q_n^b > q_{n-1}^b \Longrightarrow D\uparrow \\ p_n^b < p_{n-1}^b & \vee & q_n^b < q_{n-1}^b \Longrightarrow D\downarrow \\ p_n^a < p_{n-1}^a & \vee & q_n^a > q_{n-1}^a \Longrightarrow S\uparrow \\ p_n^a > p_{n-1}^a & \vee & q_n^a < q_{n-1}^a \Longrightarrow S\downarrow \end{array}$$

Where p^b is the best bid price; q^b the size of the best bid price; p^a the best ask price; and q^a the size of the best ask price. The variable e_n is defined to measure this tick change between two events, ($event_n$ and $event_{n-1}$), shown in Equation 2, where I can be regarded as a Boolean variable.

$$\begin{aligned} e_n &= I_{\{p_n^b > p_{n-1}^b\}} q_n^b - I_{\{p_n^b \leq p_{n-1}^b\}} q_{n-1}^b \\ &\quad - I_{\{p_n^a < p_{n-1}^a\}} q_n^a + I_{\{p_n^a \geq p_{n-1}^a\}} q_{n-1}^a \end{aligned} \tag{2}$$

The rules for I are as follows, and only one of them will happen between any two consecutive events:

1. if p^b increases, $e_n = q_n^b$
2. if p^b decreases, $e_n = -q_{n-1}^b$
3. if p^a increases, $e_n = q_{n-1}^a$
4. if p^a decreases, $e_n = -q_n^a$
5. if p^b remains same and $q_n^b \neq q_{n-1}^b$, $e_n = q_n^b - q_{n-1}^b$
6. if p^a remains same and $q_n^a \neq q_{n-1}^a$, $e_n = q_{n-1}^a - q_n^a$

If $N(t_k)$ is the number of events during [0, t_k], then OFI_k refers to the cumulative effect of e_n that has occurred over the time interval [$t_k - 1$, t_k], as shown in Equation 3.

$$OFI_k = \sum_{n=N(t_{k-1})+1}^{N(t_k)} e_n \tag{3}$$

After this, a linear regression equation can be built, per Equation 4, where $\Delta P_k = (P_k - P_{k-1})/\delta$ and δ is the tick size (1 cent in Cont et al.'s experiments), β is the price impact coefficient, and ε_k is the noise term mainly caused by contributions from lower levels of the LOB:

$$\Delta P_k = \beta OFI_k + \varepsilon_k \tag{4}$$

Moreover, Cont et al. stated that the market depth, D, is an important contributing factor to the fluctuations, and is inversely proportional to mid-price changes. They defined the average market depth, AD_k, in the "Level 1 order book" as shown in Equation 5; and β can be measured by AD_k, shown in Equation 6, where λ and c are constants and

v_k is a noise term.

$$AD_k = \frac{1}{2(N(T_k) - N(T_k) - 1)} \sum_{N(T_{k-1})+1}^{N(T_k)} (q_n^B + q_n^A) \tag{5}$$

$$\beta_k = \frac{c}{AD_k^{\lambda}} + v_k \tag{6}$$

Given equations 4 and 6, the relationship between ΔP and OFI and AD is constructed as seen in Equation 7, according to which, Cont et al. ran the linear regression by using the 21-trading-day data from 50 randomly chosen US stocks, and the average $R^2 = 67\%$. They demonstrated that OFI is positive in relation to the change of mid-price. If $OFI > 0$, meaning a net inflow on the bid side or a net outflow on the ask side, the mid-price has a significantly increasing momentum, and the higher OFI is, the more the mid-price will increase. Conversely, if $OFI < 0$, meaning a net outflow on the bid side or a net inflow on the ask side, the mid-price has a significantly decreasing momentum, and the lower OFI is, the more the mid-price will decrease.

$$\Delta P_k = \frac{c}{AD_k^{\lambda}} OFI_k + \varepsilon_k \tag{7}$$

OFI is clearly a useful metric, but it operates only on values found at the top of the LOB, i.e. the best bid and ask. In that sense, it is as sensitive to changes at the top of the book as is the Church & Cliff Δ_m metric. Next we describe how OFI can be extended to be sensitive to values at multiple levels in the LOB, which gives us Multi-Level OFI, or MLOFI.

2.3.2 Multi-Level Order Flow Imbalance

Fortunately, [43] demonstrated how to measure multi-level order flow imbalance (MLOFI). A quantity vector, v, is used to record the OFI at each discrete level in the LOB: see Equation 8, where m denotes the depth of price level in the LOB. The level-m bid-price refers to the m-highest prices among bids in the LOB, and the level-m ask-price refers to the LOB's m-lowest priced asks.

$$v = \begin{pmatrix} MLOFI_1 \\ MLOFI_2 \\ \cdots \\ MLOFI_m \end{pmatrix} \tag{8}$$

The time when an n_{th} event occurs is denoted by τ_n; $p_b^m(\tau_n)$ signifies the level-m bid-price; $p_a^m(\tau_n)$ denotes the level-m ask-price; $q_b^m(\tau_n)$ refers to the total quantity at the level-m bid-price, and $q_a^m(\tau_n)$ refers to the total quantity at the level-m ask-price.

Similar to the OFI defined in Section 2.3.1, the level-m OFI between two consecutive events occurring at times τ_s and τ_n ($s = n-1$) can be calculated as follows:

$$\Delta W^m(\tau_n) = \begin{cases} q_b^m(\tau_n), & \text{if } p_b^m(\tau_n) > p_b^m(\tau_s) \\ q_b^m(\tau_n) - q_b^m(\tau_s), & \text{if } p_b^m(\tau_n) = p_b^m(\tau_s) \\ -q_b^m(\tau_m), & \text{if } p_b^m(\tau_n) < p_b^m(\tau_s) \end{cases} \tag{9}$$

and

$$\Delta V^m(\tau_n) = \begin{cases} -q_a^m(\tau_m), & \text{if } p_a^m(\tau_n) > p_a^m(\tau_s) \\ q_a^m(\tau_n) - q_a^m(\tau_s), & \text{if } p_a^m(\tau_n) = p_a^m(\tau_s) \\ q_a^m(\tau_n), & \text{if } p_a^m(\tau_n) < p_a^m(\tau_s) \end{cases} \tag{10}$$

where $\Delta W^m(\tau_n)$ measures the order flow imbalance of the bid side in the level-m and $\Delta V^m(\tau_n)$ measures the order flow imbalance of the ask side in the level-m.

From equations 9 and 10, we can get the MLOFI in the level-m over the time interval $[t_k - 1, t_k]$:

$$MLOFI_k^m = \sum_{\{n|t_{k-1}<\tau_n<t_k\}} e^m(\tau_n) \tag{11}$$

where

$$e^m(\tau_n) = \Delta W^m(\tau_n) - \Delta V^m(\tau_n) \tag{12}$$

We now give four illustrative examples of the MLOFI mechanism in action: for ease of explanation, we'll only consider the 3-level OFI, and we'll assume that there is only evert one event that occurs during the time interval $[t_{k-1}, t_k]$.

2.3.3 Case 1: New order at level-1 of the LOB

Figure 1 shows the situation of the LOB at time t_{k-1}, and then the updated LOB at time t_k after a new buy order has arrived.

Limit Order Book time $= t_{k-1}$			
Bid Q	Bid P	Ask P	Ask Q
7	90	95	3
2	87	98	5
10	82	100	1

Limit Order Book time $= t_k$			
Bid Q	Bid P	Ask P	Ask Q
5	**93**	95	3
7	90	98	5
2	87	100	1

Figure 1: Left: the initial LOB at time t_{k-1}; Right: the updated LOB at time t_k after a new buy order has been added: details of the LOB changed as a result are highlighted in bold font. In the column headings, P stands for Price and Q stands for Quantity.

- Level-1: since $p_b^1(t_k) > p_b^1(t_{k-1})$ (i.e. $93 > 90$), $MLOFI_k^1 = q_b^1(t_k) = 5$;
- Level-2: since $p_b^2(t_k) > p_b^2(t_{k-1})$ (i.e. $90 > 87$), $MLOFI_k^2 = q_b^2(t_k) = 7$;
- Level-3: since $p_b^3(t_k) > p_b^3(t_{k-1})$ (i.e. $87 > 82$), $MLOFI_k^3 = q_b^3(t_k) = 2$;

So, the quantity vector v_k is:

$$v_k = \begin{pmatrix} 5 \\ 7 \\ 2 \end{pmatrix} \tag{13}$$

All three numbers in v_k are positive, which indicates the upward trend of the price.

2.3.4 Case 2: Partial fulfillment or cancellation

A new sell limit order crosses the spread, or a buy limit order at the best-bid position cancels. Figure 2 shows the resultant LOB.

Limit Order Book time $= t_{k-1}$			
Bid Q	Bid P	Ask P	Ask Q
7	90	95	3
2	87	98	5
10	82	100	1

Limit Order Book time $= t_k$			
Bid Q	Bid P	Ask P	Ask Q
2	**90**	95	3
2	87	98	5
10	82	100	1

Figure 2: Left: the initial LOB at time t_{k-1}; Right: the updated LOB at time t_k after the quantity available at the best bid price is altered, either as a result of one or more sellers hitting that bid price with a total quantity of five, or five items are deleted from the LOB as a result of one or more order cancellations. Format as for Figure 1.

Level-1: as $p_b^1(t_k) = p_b^1(t_{k-1})$ (i.e. $90 = 90$), $MLOFI_k^1 = q_b^1(t_k) - q_b^1(t_{k-1}) = 2 - 5 = -3$;
Level-2: as $p_b^2(t_k) = p_b^2(t_{k-1})$ (i.e. $87 = 87$), $MLOFI_k^2 = q_b^2(t_k) - q_b^2(t_{k-1})\ 2 - 2 = 0$;
Level-3: as $p_b^3(t_k) = p_b^3(t_{k-1})$ (i.e. $82 = 82$), $MLOFI_k^2 = q_b^3(t_k) - q_b^3(t_{k-1}) = 0$;

So the quantity vector v_k is:

$$v_k = \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix} \tag{14}$$

Where -3 at Level 1 indicates a potential downward trend for the price, because the total demand on the bid side decreases.

2.3.5 Case 3: full fulfillment or cancellation

This is similar to Case 2, but (as illustrated in Figure 3) assumes that all orders at Level 1 in the ask book (A_1) are transacted by an incoming buy order, or that the orders making up A_1 are cancelled. In this case, we need to consider the change on the ask side.

Here we have:
A_1: $p_a^1(t_k) > p_a^1(t_{k-1}) \implies \Delta V^1(t_k) = -q_a^1(t_{k-1}) = -3$; $MLOFI_k^1 = -\Delta V^1(t_k) = 3$;
A_2: $p_a^2(t_k) > p_a^2(t_{k-1}) \implies \Delta V^1(t_k) = -q_a^2(t_{k-1}) = -5$; $MLOFI_k^2 = -\Delta V^1(t_k) = 5$;
A_3: $p_a^3(t_k) > p_a^3(t_{k-1}) \implies \Delta V^1(t_k) = -q_b^2(t_{k-1}) = -1$; $MLOFI_k^3 = -\Delta V^1(t_k) = 1$;

So the quantity vector v_k shown in Equation 15 demonstrates that if the supply reduces or a buy has sufficient interest to transact, the price tends to go up.

$$v_k = \begin{pmatrix} 3 \\ 5 \\ 1 \end{pmatrix} \tag{15}$$

Limit Order Book time $= t_{k-1}$			
Bid Q	Bid P	Ask P	Ask Q
7	90	95	3
2	87	98	5
10	82	100	1

Limit Order Book time $= t_k$			
Bid Q	Bid P	Ask P	Ask Q
7	90	98	5
2	87	100	1
10	82	**105**	**7**

Figure 3: Left: the initial LOB at time t_{k-1}; Right: the updated LOB at time t_k after the quantity available at the best ask price is entirely consumed, either as a result of one or more buyers lifting that ask price with a total quantity of three, or three items being deleted from the ask side of the LOB as a result of one or more order cancellations. The whole ask side of the LOB shifts up, revealing previously hidden sell orders at a price of 105. Format as for Figure 1.

Limit Order Book time $= t_{k-1}$			
Bid Q	Bid P	Ask P	Ask Q
7	90	95	3
2	87	98	5
10	82	100	1

Limit Order Book time $= t_k$			
Bid Q	Bid P	Ask P	Ask Q
7	90	95	3
100	**89**	98	5
2	87	100	1

Figure 4: Left: the initial LOB at time t_{k-1}; Right: the updated LOB at time t_k after a block bid order arrives, at a price that is below the current best. Format as for Figure 1.

2.3.6 Case 4: New order at level-*m* of the LOB

Assuming now that a new large-sized order comes to the level-2 ask, if we only consider order flow imbalance in the top level of the LOB, we cannot detect this new block order. This is the reason why we choose to use MLOFI.

As there is no change in the level-1 bid, $MLOFI_k^1 = 0$. Because a new order comes to the second-level bid, $p_b^2(t_k) > p_b^2(t_{k-1})$ (i.e. $89 > 87$) and $MLOFI_k^2 = q_b^2(t_k) = 100$. Based on the same rule, $MLOFI_k^3 = q_b^3(t_k) = 2$. So, the quantity vector v_k is:

$$v_k = \begin{pmatrix} 0 \\ 100 \\ 2 \end{pmatrix} \tag{16}$$

If we only care about first-level order flow imbalance, we get $OFI = 0$. However, if we consider second and third levels, we get $MLOFI_k^2 = 100$ and $MLOFI_k^3 = 2$, which indicate a huge surplus on the demand side. If a trader can obtain this information and take action accordingly, it may result in larger profits or smaller losses.

3 Adding MLOFI-impact to Robot Traders

In our work thus far, we have explored adding MLOFI-based impact-sensitivity to Vytelingum's [40] AA trading strategy, creating an extended AA that we refer to as ZZIAA. The impact-sensitivity source-code that we developed for ZZIAA was then added to Cliff's [8] ZIP strategy, giving ZZIZIP, and to the ISHV strategy introduced by Church & Cliff [6], giving ZZISHV: how we did this, and the results we got, are described in Section 3.1, which is adapted and abridged from [46]. Then Section 3.2 describes our work on adding MLOFI-based impact sensitivity to the recently-developed PRZI strategy [7], which is adapted and abridged from [36]

3.1 Simple robot traders with impact: AA, ZIP, and ISHV

3.1.1 Implementation

In this section we describe how ZZIAA is created, by the addition of MLOFI-style imbalance-sensitivity to the original AA trader strategy. Our intention for ZZIAA was to develop an "impact-sensitivity" module of code that is not deeply embedded into the original AA [40] so that, if successful, this relatively independent module could also easily be applied to other trading algorithms. For this reason we chose the Widrow-Hoff delta rule to update the quote of the ZZIAA towards an impact-sensitive quote, as shown in Equation 17. The $p_{\mathrm{AA}}(t+1)$ is derived from the long-term and short-term factors using the information at time t (see [40]), and $\tau(t)$ is the target price computed with consideration of MLOFI:

$$p_{\mathrm{IAA}}(t+1) = p_{\mathrm{AA}}(t+1) + \Delta(t) \tag{17}$$

where

$$\Delta(t) = \beta(\tau(t) - p_{\mathrm{AA}}(t+1)) \tag{18}$$

and

$$\tau(t) = p_{\mathrm{benchmark}}(t) + o_{\mathrm{offset}}(t) \tag{19}$$

The core of the IAA derivation is how to find $\tau(t)$, which consists of two parts, the benchmark price $p_{\text{benchmark}}(t)$ and $o_{\text{offset}}(t)$. The $p_{\text{benchmark}}(t)$ depends on whether the mid-price exists. As Equation 20 shows, if the mid-pice is available, we can set $p_{\text{benchmark}}(t)$ as the mid-price, but if it is not, we set $p_{\text{benchmark}}(t)$ as $p_{AA}(t+1)$, which can be obtained at time t.

$$p_{\text{benchmark}}(t) = \begin{cases} p_{\text{mid}}(t), & \text{if } \exists p_{\text{mid}} \\ p_{\text{AA}}(t+1), & \text{if } \nexists p_{\text{mid}} \end{cases} \tag{20}$$

The $o_{\text{offset}}(t)$ is derived from the MLOFI and the average depth. Assume that we consider M numbers of levels MLOFI in the LOB, shown in Equation 21, and that each MLOFI captures the last N events shown in Equation 22.

$$a(t) = \begin{pmatrix} \text{MLOFI}_1(t) \\ \text{MLOFI}_2(t) \\ \dots \\ \dots \\ \text{MLOFI}_M(t) \end{pmatrix} \tag{21}$$

where

$$\text{MLOFI}_M(t) = \sum_{n=1}^{N} e_n^m \tag{22}$$

We can define the average market depth for m levels in a similar way:

$$d(t) = \begin{pmatrix} \text{AD}_1(t) \\ \text{AD}_2(t) \\ \dots \\ \dots \\ \text{AD}_M(t) \end{pmatrix} \tag{23}$$

where:

$$\text{AD}_M(t) = \frac{1}{N} \sum_{n=1}^{N} \frac{q_{M_n}^a + q_{M_n}^b}{2} \tag{24}$$

Knowing the quantity vector $a(t)$, we need a mechanism to switch this vector to a scalar. Similar to Equation 7, we define the offset as Equation 25.

$$v_{\text{offset}} = \sum_{i=0}^{i=m-1} \alpha^i \frac{c * \text{MLOFI}_{(i+1)}(t)}{\text{AD}_{(i+1)}(t)} \tag{25}$$

where α is the decay factor (initialized as 0.8) and c is a constant (we use $c = 5$). Note: if $AD_m(t) = 0$, the item $\alpha^{m-1} \frac{c*\text{MLOFI}_m(t)}{\text{AD}_m(t)}$ will not be counted.

To summarise, our work extends AA by the novel introduction of prior contributions to the econometrics of LOB imbalance from Cont et al. and of Xu et al. in the following ways:

- Cont et al. and Xu et al. run linear regressions to build their model and use statistical methods to test the significance of factors. The constants such as c come from modelling real-world data. However our version does not run a linear regression and the constants such as c and α are determined based on previous studies [10, 43]. We can check the model's performance by exploring different values of constants.

- In the prior work, $MLOFI_M(t)$ and $AD_M(t)$ are influenced by the events within a specified time interval. In contrast, in our work, $MLOFI_M(t)$ and $AD_M(t)$ are calculated based on the last N events that occurred in the LOB, regardless of length of the time interval between successive events.

3.1.2 Results

Because our MOLFI-based "impact sensitive" module added to AA was deliberately developed in a non-intrusive way, it can easily be replicated into any other algorithm. In this section we first show results from ZZIAA and then we follow those with results from adding the MLOFI module to ISHV (giving ZZISHV), and to ZIP (giving ZZIZIP). Because of space limitations, the performance comparisons shown here focus on situations where the imbalance would cause a problem for the non-imbalance sensitive versions of the trader agents – and we demonstrate that our extended trader agents are indeed superior. Extensive sets of further results are presented in [46], which demonstrate that the extended trader-agents perform the same as the unextended versions in situations where there is no imbalance to be concerned about in the LOB.

For each A:B comparison we ran 100 trials in BSE [2], the same open-source simulator of a financial exchange that was used by Church & Cliff. Each trial involved creating a market where there were N traders of type A (e.g., ZIP) and N traders of type B (e.g., ZZIZIP) who were allocated the role of buyers, and similarly N of type A and N of type B who were allocated the role of sellers. Thus one market trial involved a total of $4N$ trader-agents: for the results presented here we used $N = 10$. As is entirely commonplace in all such experimental work, buyers were issued with assignments of cash, and sellers with assignments of items to sell, and each trader was given a private *limit price*: the price below which a seller could not sell and above which a buyer could not buy. The distribution of limit prices in the market determines that market's supply and demand curves, and the intersection of those two curves indicates the *competitive equilibrium price* that transaction prices are expected to converge to.

Although very many of the previous trader-agent papers that we have cited here have monitored the *efficiency* of the traders' activity in the market, we instead monitored *profitability* (which only differs from efficiency by some constant coefficient). Each individual market trial would allow the traders to interact via the LOB-based exchange in BSE for a fixed period of time, and at the end of the session the average profit of the Type A traders would be recorded, along with the average profit of the Type B traders. In the results presented here we conducted 100 independent and identically distributed market trials for each A:B comparison, giving us 100 pairs of profitability figures. To summarise those results we plot as box-and-whisker charts the distribution of profitability values for traders of Type A, the distribution of profitability values for traders of Type B, and the distribution of profitability-difference values (i.e., for each of the 100 trials, for trial t compute the difference between the profitability of Type A traders and the profitability of Type B traders in trial i). We used the Wilcoxon-Mann-

Whitney U Test to determine whether the differences we observed were significant.

3.1.3 ZZIAA

Figure 5 summarises the comparison data generated between AA and ZZIAA. In the U test, when comparing the ZZIAA with AA, $p = 0.007$ which meant that the profit difference between ZZIAA and AA was statistically significant.

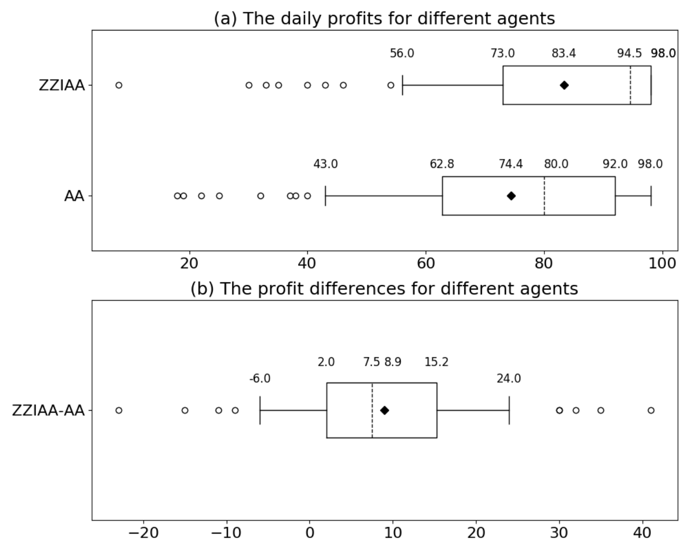

Figure 5: Profit distributions from original AA tested against ZZIAA: upper figure shows profit data for ZZIAA and AA; lower figure shows the difference in profits calculated across paired data for the two types of agent.

3.1.4 Comparison of ZZISHV and ISHV

We can see from Figure 6 that the profit generated by ZZISHV was much greater than ISHV. However, this only means that ZZISHV is better than ISHV under this particular market condition, and this might not be the case under other market conditions. In the test, the outperformance of ZZISHV can easily be explained: as a seller, when ISHV met favourable imbalances, it worked like SHVR and posted a price one penny lower than the current best ask; in contrast, under the same condition, ZZISHV chose to set price Δ_p higher than the current best ask and seek for transaction opportunities some time later. For example, assume that the current best ask is 70 and ISHV will post an order with the price equal to 69. Assume that ZZISHV gets the offset value equal to 20 from the "impact-sensitive" module, and the quoted price will be 90.

The aim of both ISHV and ZZISHV is the same: to be sensitive to imbalances in the market. The former uses a function that maps from Δ_m to Δ_s to achieve this objective and Δ_m is generated based on the mid- and micro-prices in the market. In

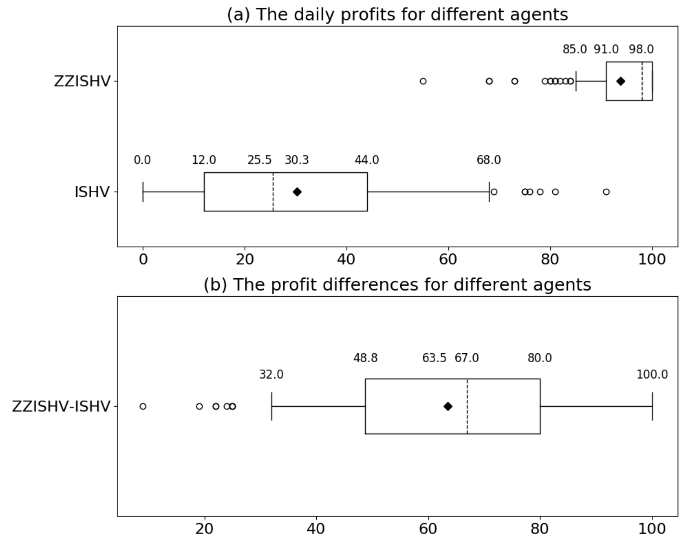

Figure 6: Performance of ZZISHV and ISHV when facing large-sized orders from the bid side. Format as for Figure 5.

contrast, the latter uses MLOFI to achieve the goal. The biggest difference between ISHV and ZZISHV is that ISHV can only be sensitive to imbalances at the top of the LOB and the MLOFI mechanism helps ZZISHV to be sensitive to m-level imbalances on the LOB and thus detect them earlier than ISHV in some cases. The drawback comes in determination of appropriate parameter values for both ISHV and ZZISHV, where trial-and-error is the best current option. In the map function of ISHV ($\Delta_s = C\Delta_p \pm M\Delta_m\Delta_p$ if the imbalance is significant), the parameters C and M were somewhat arbitrarily set in [6] to C=2 and M=1. For ZZISHV, when quantifying MLOFI, we use Equation 25, and the key parameter c and decay factor α are artificially determined. We set $m = 5$ (consistent with the result from [10]) and $\alpha = 0.8$. The optimal values of these parameters are not known; poor choices of these constants may cause agents to perform badly.

3.1.5 Comparison of ZZIZIP and ZIP

ZZIZIP is ZIP with the addition of the MLOFI module. In the example we present here, sellers will face an excess imbalance from the demand side. The box plots in Figure 7 illustrate the results: ZZIZIP has less variance than ZIP and their median profitability was slightly higher than that of ZIP; in the second figure, we can see that although there were some outliers on both the top and bottom, and the bottom whisker was located below zero, the whole box was distributed beyond zero. Employing the U Test, we got $p = 0.002$ and can therefore conclude that the profit generated by ZZIZIP was statistically significantly greater than ZIP. Despite this, it is worth noting that the average difference in profitability is less than half of the difference between AA and

IAA, given that other conditions remain unchanged. So, our next question is: what causes the smaller difference in profits between ZZIZIP and ZIP?

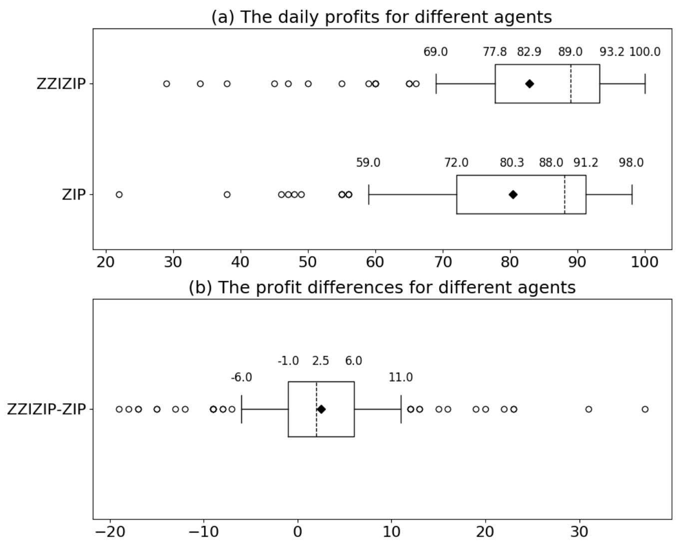

Figure 7: Performance of ZIP and ZZIZIP when facing large-sized orders from the bid side. Format as for Figure 5.

To answer this, we need to examine how ZIP works. ZIP uses the Widrow-Hoff Delta rule to update its next quote-price towards its current target price. The current target price is based on the last quote price in the market. Due to this, the last quote price affects the bidding behaviour of ZIP considerably. In this test, on the ask side, the 10 ZIP sellers were not impact-sensitive and the 10 ZZIZIP sellers were. But, although the ZIP traders were not themselves impact-sensitive, they were affected by the quote prices coming from the ZZIZIP active in the same market, and so the ZIPs' quote prices approached the ZZIZIPs' to some extent. In other words, this adaptive mechanisms within the non-impact-sensitive ZIP gave it a degree of impact-sensitivity, because it was influenced by the activities of the impact-sensitive traders in the market. In the test, if we treat ZZIZIP and ZIP as a group, the average profit generated is 84.82 (95% CI: [82.16, 87.48]). If we replace 10 ZZIZIPs with 10 ZIPs (total 20 ZIP sellers), the average profit of ZIP is 79.21 (95% CI: [77.11, 81.31]). With the presence of ZZIZIP, all sellers tend to make more profit.

3.2 MLOFI Opinionated PRZI traders for Narrative Economics

Having established in the previous section that MLOFI works well for giving autonomous trader-agents a robust sensitivity to supply/demand imbalance, we turn now to use of MLOFI in an agent-based model of a financial market in which the agents' trading activities are driven at least in part by their opinion of future events in the market, and in which agents interact with one another via two channels: posting buy/sell orders at the central financial exchange, which either result in an immediate transaction or are added to the LOB; and 'social' interactions in which agents active in the market engage in some number of pairwise interactions with one or more other agents which can result in the opinions of the interacting agents being altered via some opinion dynamics (OD) model. That is, this is where we bring things back to the narrative economics of Nobel laureate Robert Shiller [32, 33], and the agent-based modelling work of Lomas & Cliff [25], all of which was discussed in the Introduction section of this paper.

In a paper released in early 2021, Cliff [7] introduced a new minimal-intelligence automated trading algorithm named *Parameterized-Response Zero Intelligence*, or PRZI. PRZI's response is parameterized in the sense that its behavior at any one time is governed by the current value of its strategy parameter, denoted by $s \in [-1,+1] \in \mathbb{R}$. In common with many of the trading strategies reviewed in Section 2, a PRZI trader stochastically generates the prices for its bid or offer orders as iid draws from some random distribution. For example, Gode & Sunder's [20] ZIC strategy uses a uniform distribution bounded on one side by trader i's limit price λ_i (the price that i may not pay above when buying, or sell below when selling) and bounded on the other side by a system constant. The space of possible trading strategies that PRZI can exhibit does include ZIC, as the case when $s = 0$; but as s is moved further from zero, towards $s = \pm 1$, the shape of the PRZI trader's distribution function smoothly deforms to be skewed to the left or the right, giving rise to stochastically-generated quote-prices that are reasonable models of the trader either becoming more *urgent* in the pricing of its quotes (i.e., more likely to generate a quote-price that is attractive to potential counterparties, so more likely to lead to a transaction, but at the cost of making less profit on the transaction when it does happen) or more *relaxed* in the pricing of its quotes (i.e., more likely to generate profit, but less attractive to the counterparty side and hence likely to result in a longer wait before a willing counterparty is found). At the absolute extremes of $s = -1$ and $s = +1$, the most urgent PRZI trading strategy is identical to the GVWY strategy introduced in [2, 9] which simply generates quote price of λ_i with probability 1, maximising the chances of a transaction but offering the strong likelihood of making no profit at all; while the least urgent PRZI strategy is identical to the SHVR strategy introduced in [2, 9] which simply adds one penny to the price of the best bid, or shaves one penny of the price of the best ask.

Just as Lomas & Cliff added opinions-dynamics to the pre-established ZIC [20] and NZI [17] strategies to give new *opinionated* extensions referred to as OZIC and ONZI, we add opinions to PRZI to give OPRZI. If an OPRZI trader i at time t has opinion-value $o_i(t) \in [-1,+1] \in \mathbb{R}$ then what is required is to map this onto i's strategy at time t denoted by $s_i(t)$, i.e. $s_i(t) = F(o_i(t))$ s.t. $s_i(t) \in [-1,+1] \in \mathbb{R}$. However, a moment's thought reveals that in our ABM a trader's opinion can be affected by more than one factor: specifically there is the *local* influence of the trader's social interactions with other traders (i.e., the opinion dynamics aspect, the narrative economics factor); and then there is the *global* factor that all traders in the market can see the exchange's LOB, and any imbalance on the LOB (as measured by MLOFI) will also affect a trader's opinion of where prices are heading in the near-term.

Let the market be populated by a total of N_P trader-agents each running OPRZI, and let $o_i(t)_l \in [-1,+1] \in \mathbb{R}$ denote trader i's *local* opinion, which is maintained via an opinion dynamics model such as BC [22], RA [16, 27] ,or RD [28]; and let $o_i(t)_g \in [-1,+1] \in \mathbb{R}$ denote the *global* opinion, which i derives from data published to all by the market's central exchange – in principle, different traders could have ways of computing $o_i(t)_g$, but in the initial studies reported here we give the same method to all traders, and they all use $o_i(t)_g = \sigma(MLOFI(t))$ where $\sigma(.)$ is a sigmoidal function with asymptotes at ± 1 s.t. $o_i(t)_g \in [-1,1]$.

For completeness we need also to define the global opinion factor $o_i(t)_g$ in the situations where one or both sides of the LOB are empty, in which case MLOFI is undefined. In a real market there are two plausible commentaries on a situation in which one or both sides of the LOB are empty: one would involve words to the effect of *"...well, the market's only just opened, and there's clearly not enough information in the LOB, not enough market activity yet, to form a judgement, so we'll stick with global opinion of zero, i.e. we'll sit on the fence for the time being'*; while the other could be along the lines of *"...there's a major supply/demand imbalance because one side of the LOB is empty while the other side is not, and so there must be very heavy excess demand or supply and so the global opinion should go fully to +1 or -1 depending on which side is empty"*. In a real market, which of those two narratives you'd settle upon would depend on wider context, such as how long the market has been open and what the total order volume is on the nonempty side of the LOB: if the market has only just opened, and there are just the first few orders populating the LOB, then $o_i(t)_g = 0$ is a reasonable assumption (the alternative, assuming +1 or -1, might introduce some fairly large opinion-swings at the start of the market) but if you're mid-way through a trading session and the LOB has previously been well populated with orders on both sides of the book but you then find one side of the LOB emptying, that could be a sign that there has been a major reduction in supply/demand, and then assuming $o_i(t)_g = \pm 1$ would make much more sense. In the interests of minimality, in the work reported here $o_i(t)_g = 0$ has been used if either/both sides of the LOB are empty, because that's less likely to introduce wild swings; exploring alternative approaches remains a topic for further work.

The local and global sources of opinion are combined as a simple linear weighted combination:

$$o_i(t) = \omega_i(t) o_i(t)_l + (1 - \omega_i(t)) o_i(t)_g \qquad (26)$$

Where $\omega_i(t) \in [0,1] \in \mathbb{R}$ is trader i's opinion-weighting: if $\omega_i = 0$, the trader ignores local opinion and pays attention only to global opinion; if $\omega_i = 1$ then it ignores global information and is influenced only by the opinions of other traders that i interacts with. Such a simple linear combination of global and local opinions has also recently been shown in [21] to be useful in an agent-based model of opinion dynamics among a population of bettors gambling on the outcome of horse-racing events.

We first established a set of baseline results from a market in which nothing happens, and in which all traders hold moderate (near-zero) opinions: these results were then used for comparison in analysis of results from subsequent experiments where controlled interventions were made, and results observed. All our results from markets populated by OPRZI traders are visualized and analyzed in [36]. In this paper, we limit ourselves to presenting one set of key illustrative results, in which the market is first allowed to run for a period of time to stabilise to a steady state, and then at a predetermined time denoted t_I, we inject extreme opinions into some number N_I of the traders in the market, and observe the subsequent spread of extreme opinions in the market, and the effect those extreme opinions have on the dynamics of transaction prices on the exchange.

The manner in which extreme opinions are injected into the market requires some care when (as with the RA [16, 27] and RD [28] opinion-dynamics models) the extent to which one agent's opinion can be affected by another can be dependent on how close those two agents' opinions are before they interact: intuitively, this captures the everyday observation that someone who (for example) holds right-of-centre political view is more likely to be influenced to a slightly further right-wing view by interactions with a fellow right-winger, than they are to be convinced to move toward the left by interacting with an individual who holds extreme left-wing views; once the difference in opinions between two agents is too great, they lose the ability to influence one another. As is explained at length in [36], this required that the injection of extremist into the market is done with some sophistication: the agents selected to be made to hold extreme views are set on a trajectory where their opinion is made slightly more extreme on each timestep, so that they can influence some number of moderates to start moving to a more extreme opinion (who will then in turn influence other moderates, and so on). This is in contrast to making the injection of extremists as a sudden step-change in the selected agents' opinions, which would immediately render them all as absolute extremists but would leave them without any influence over the remaining moderates.

Figure 8 shows a scatter-plot of transaction prices in a single ABM experiment where the market is populated by OPRZI traders, all of whom hold initially moderate opinions, and where at $T_I = 300$ some number of extremists are injected, whose opinions steadily ramp up to be ever more positive, eventually stopping at the system limit of $o_i(t) = +1.0$. As time progresses, their influence raises the opinion of previously moderate or neutral traders to also become more positive. As more traders' opinions are increasingly positive, indicating that a price rise is imminent, the OPRZI buyers bid more urgently and the OPRZI sellers offer at relaxed prices, and so the distribution of transaction prices rises in the period after T_I. The experiment is terminated when all traders hold extreme-positive views with opinion-values greater than some pre-set threshold (e.g. $o_i(t) \geq 0.95; \forall i$).

4 Discussion & Conclusion

We know of no agent-based model (ABM) prior to Church & Cliff's [6] in which trader-agents are given a sensitivity to quantity imbalances between the bid and ask sides of the LOB. Such imbalances are often (but not always) caused by the arrival of one or more block orders on one side of the LOB. In this paper we have provided a constructive critique of Church & Cliff's method, pointing out the extreme fragility of imbalance-sensitivity metrics like theirs that monitor only the top of the LOB. We then explained the OFI and MLOFI metrics of [10] and [43] respectively, and demonstrated how MLOFI could be integrated within Vytelingum's AA trading-agent strategy to give ZZIAA. We demonstrated that ZZIAA performs extremely well: it performs the same as AA when there is no imbalance, and significantly outperforms AA in the presence

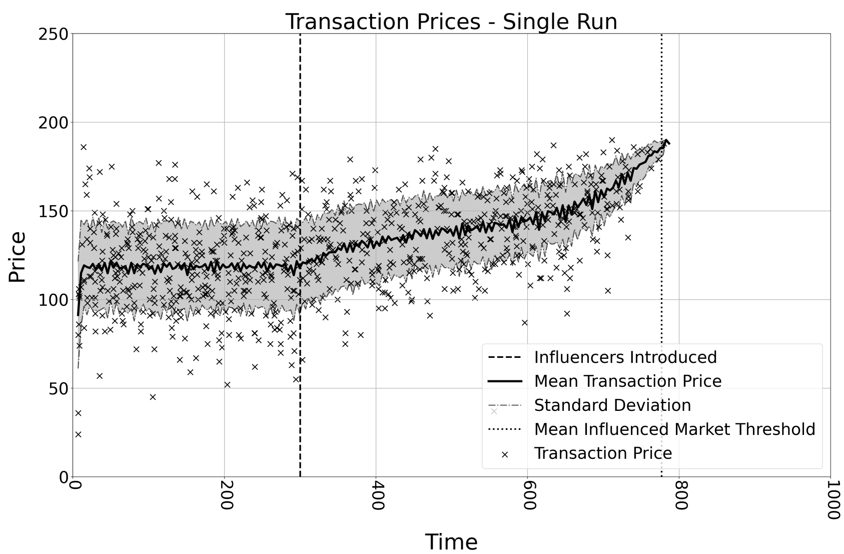

Figure 8: Transaction-price time-series: data-points marked by crosses come from a single experiment in which at time $T_I = 300$ (indicated by the vertical dashed line) a number of the OPRZI trader-agents are injected into the market, to become progressively more extreme, holding ever more positive opinions, which influence other moderate or neutral traders to also become more positive. As more traders' opinions are increasingly positive, indicating that a price rise is imminent, the OPRZI buyers bid more urgently and the OPRZI sellers offer at relaxed prices, and so the distribution of transaction prices rises in the period after T_I.b The vertical dotted line at time of approx. 780 marks the point at which all traders have become extremists, at which point the session is halted. The solid dark line shows the mean transaction price, and the gray-shaded envelope indicates plus and minus one standard deviation, for 200 i.i.d. repetitions of this experiment.

of major LOB imbalance. We then showed how the imbalance-sensitivity mechanisms that we developed for ZZIAA can readily be incorporated into other trading-agent algorithms such as ZIP [8] and SHVR [9]. Results from ZZIZIP and ZZISHV are similarly very good and further demonstrate that the mechanisms developed here have given robust imbalance-sensitivity to a range of trader-agent strategies.

Having demonstrated the robustness of MLOFI in long-established trader-agent strategies, we then explained how MLOFI can be used to give trader-agents within an ABM an *opinion* about near-term price movements that is grounded in the facts of the current state of the market's limit order book (LOB), as a *global* opinion-factor affecting all traders in the market, and how this global factor can be combined with a *local* opinion-factor where a trader's own opinion can be influenced by, and can have influence over, the opinions of other traders that it interacts with 'socially' (via an established opinion-dynamics model such as BC [22], RA [16, 27], or RD [28]) without reference to the realities of the market's LOB. Aiming for Occam's-razor minimalism, we have established that a simple linear combination of these two opinion factors is sufficient to give an ABM that seems well set to serve as a platform for examining various aspects in Shiller's thinking on narrative economics.

In future work we intend to explore the addition of MLOFI-based impact-sensitivity to contemporary adaptive trader-agents based on deep learning neural networks [3, 42], and we are currently exploring mechanisms that will allow opinionated traders to *introspect*, such that when their opinions about future events turn out to be wrong, they change the way that they form subsequent opinions. Complete details of the work described here are given in [46, 36] and all of our relevant source-code for the ABMs described here has been made freely available as open-source code on GitHub [45, 35], enabling other researchers to examine, replicate, and extend our work.

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