$$Pat\_Dist(P_1, P_2) = 1 - \frac{|T(P_1) \cap T(P_2)|}{|T(P_1) \cup T(P_2)|}$$

We assume that $T(P_1) = (t_1, t_2, t_3, t_4, t_5)$. $T(P_2) = (t_1, t_2, t_3, t_4, t_5)$ where t is the transaction dataset distance. T_1 is $D(P_1, P_2)$

According to the Pat_Dist, which is a valid distance metric, it has the following Properities:

(1) $Pat\_Dist(P_1, P_2) > 0, \forall P_1 \neq \forall P_2$

(2) $Pat\_Dist(P_1, P_2) = 0, \quad \forall P_1 = \forall P_2$

(3) $Pat\_Dist(P_1, P_2) = Pat\_Dist(P_2, P_1)$

(4) $Pat\_Dist(P_1 + P_2) + Pat\_Dist(P_2, P_3) \geq Pat\_Dist(P_1, P_3) \quad \forall P_1, \forall P_2, \forall P_3$

Then we assume the variables:

$|T(P_1)| = a$, $|T(P_2)| = b$, $|T(P_3)| = c$, $|T(P_1) \cap T(P_2)| = b_1$, $|T(P_2) - T(P_1) \cap T(P_2)| = b_2$

$|T(P_1) \cap T(P_3)| = C_1$ $|T(P_3) - T(P_1)| = C_2$, $|T(P_1) \cap T(P_2) \cap T(P_3)| = d$

$T(P_2) \cap T(P_3) - |T(P_1) \cap T(P_2) \cap T(P_3)| = d_2$

Since $(T(P_1) \cap T(P_2)) \cup T(P_1) \cap T(P_3)) \subseteq T(P_1)$

$\Rightarrow |T(P_1) \cap T(P_2)| + |T(P_1) \cap T(P_3)| - |T(P_1) \cap T(P_2) \cap T(P_3)| \leq |T(P_1)|$

$\Rightarrow b_1 + C_1 - d_1 \leq a$ ①

$Pat\_Dist(P_1, P_2) + Pat\_Dist(P_2, P_3) \geq Pat\_Dist(P_1, P_3)$

$$\Rightarrow \frac{b_1}{a + b_2} \underset{d_1}{+} \frac{C_1}{a + C_2} \leq 1 + \frac{d_1 + d_2}{b_1 + b_2 + C_1 + C_2 - d_1 - d_2}$$

$$\begin{aligned}
\Rightarrow 1 + \frac{d_1 + d_2}{b_1 + b_2 + C_1 + C_2 - d_1 - d_2} &\geq 1 + \frac{d_1}{b_1 + b_2 + C_1 + C_2 - d_1} \quad (d_2 \geq 0) \\
&\geq 1 + \frac{d_1}{a + b_2 + C_2} = \frac{a + b_2 + C_2 + d_1}{a + b_2 + C_2} \quad ① \\
&\geq \frac{b_1 + C_1 + b_2 + C_2}{a + b_2 + C_2} = \frac{b_1 + C_2}{a + b_2 + C_2} + \frac{C_1 + b_2}{a + b_2 + C_2} \\
&\geq \frac{b_1}{a + b_2} + \frac{C_1}{a + C_2}
\end{aligned}$$

$$(a+b_2 \geq b_1,\ c_2 \geq 0)$$

$$(a+c_2 \geq c_1,\ b_2 \geq 0)$$

Thus, the distance between equation is correct.