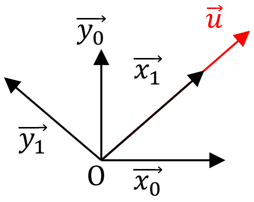
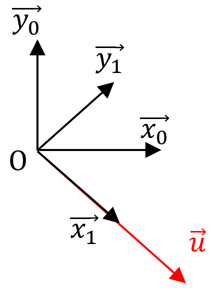
Exercice 1: Projections simples

Soit un vecteur u tel que :

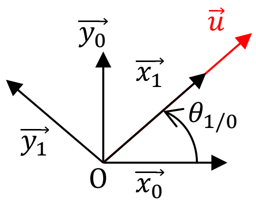
$$\vec{u} = u\overrightarrow{x_1}$$

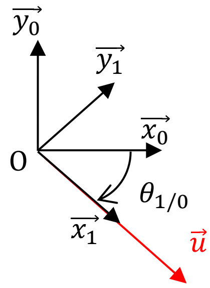




Pour chacun des deux cas proposés :

Question 1: Mettre en place le paramétrage angulaire θ_{10}





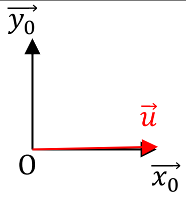
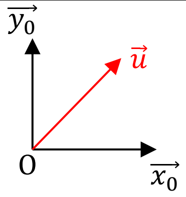
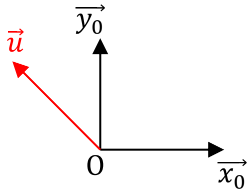
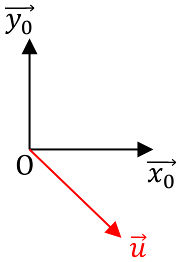
Question 2: Exprimer le vecteur $\boldsymbol{u}$ dans la base 0

$$\vec{u} = u\cos\theta_{1/0}\,\overrightarrow{x_0} + u\sin\theta_{1/0}\,\overrightarrow{y_0} = u\begin{pmatrix}\cos\theta_{1/0}\\ \sin\theta_{1/0}\\ 0\end{pmatrix}^{\mathcal{B}_0}$$

On prend $u = 1$ et on s'intéresse aux 4 valeurs suivantes : $\theta = \left(0; \frac{\pi}{4}; \frac{3\pi}{4}; -\frac{\pi}{4}\right)$

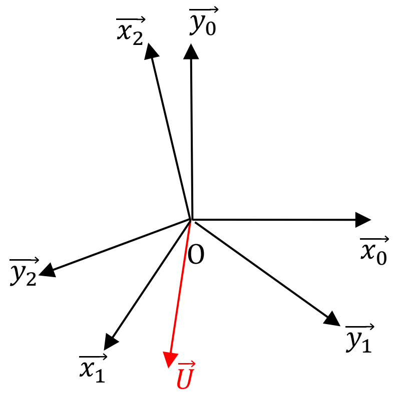
Question 3: Faire une représentation graphique pour chaque cas étudié et exprimer les composantes de $\vec{u}$ dans la base 0

$$\vec{u} = \begin{pmatrix} \cos\theta_{1/0} \\ \sin\theta_{1/0} \\ 0 \end{pmatrix}^{\mathcal{B}_0}$$

$\theta = 0$	$\theta = \frac{\pi}{4}$	$\theta = \frac{3\pi}{4}$	$\theta = -\frac{\pi}{4}$
			
$\vec{u} = \begin{pmatrix} \cos 0 \\ \sin 0 \\ 0 \end{pmatrix}^{\mathcal{B}_0}$ $\vec{u} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}^{\mathcal{B}_0}$	$\vec{u} = \begin{pmatrix} \cos\frac{\pi}{4} \\ \sin\frac{\pi}{4} \\ 0 \end{pmatrix}^{\mathcal{B}_0}$ $\vec{u} = \begin{pmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{pmatrix}^{\mathcal{B}_0}$	$\vec{u} = \begin{pmatrix} \cos\frac{3\pi}{4} \\ \sin\frac{3\pi}{4} \\ 0 \end{pmatrix}^{\mathcal{B}_0}$ $\vec{u} = \begin{pmatrix} -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \\ 0 \end{pmatrix}^{\mathcal{B}_0}$	$\vec{u} = \begin{pmatrix} \cos\left(-\frac{\pi}{4}\right) \\ \sin\left(-\frac{\pi}{4}\right) \\ 0 \end{pmatrix}^{\mathcal{B}_0}$ $\vec{u} = \begin{pmatrix} \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \\ 0 \end{pmatrix}^{\mathcal{B}_0}$

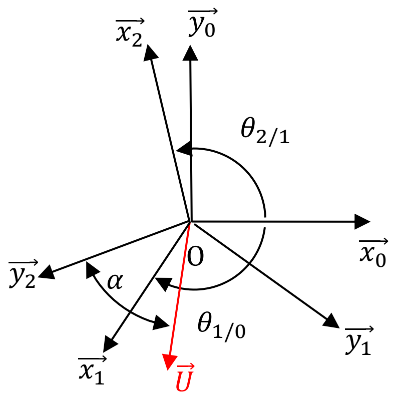
Exercice 2: Projection dans plusieurs bases

$$\vec{U} = U\vec{u}$$

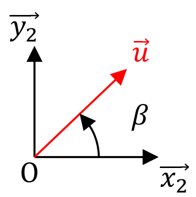
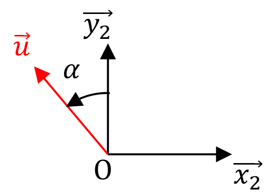


Soient deux bases 1 et 2 en rotation l'une par rapport à l'autre et une base 0.
Le vecteur $\vec{U}$ est fixe dans la base 2. On définit un angle non orienté α inférieur à 180° entre $\vec{U}$ et $\overrightarrow{y_2}$.

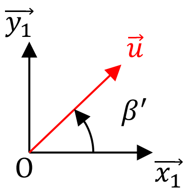
Question 1: Proposer le paramétrage angulaire $(\theta_{10}, \theta_{21}, \alpha)$



Question 2: Exprimer le vecteur $\vec{U}$ dans la base 2

 $\beta = (\widehat{\overrightarrow{x_2}, \vec{u}}) = (\widehat{\overrightarrow{x_2}, \overrightarrow{y_2}}) + (\widehat{\overrightarrow{y_2}, \vec{u}}) = \frac{\pi}{2} + \alpha$ $\vec{u} = U(\cos\beta\,\overrightarrow{x_2} + \sin\beta\,\overrightarrow{y_2})$ $\vec{u} = U\left(\cos\left(\frac{\pi}{2} + \alpha\right)\overrightarrow{x_2} + \sin\left(\frac{\pi}{2} + \alpha\right)\overrightarrow{y_2}\right)$ $\vec{u} = U(-\sin\alpha\,\overrightarrow{x_2} + \cos\alpha\,\overrightarrow{y_2})$	 $\vec{u} = U(-\sin\alpha\,\overrightarrow{x_2} + \cos\alpha\,\overrightarrow{y_2})$

Question 3: Exprimer le vecteur $\vec{U}$ dans la base 1

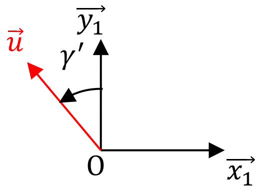


$$\beta' = \left(\widehat{\overrightarrow{x_1},\vec{u}}\right) = \left(\widehat{\overrightarrow{x_1},\overrightarrow{x_2}}\right) + \left(\widehat{\overrightarrow{x_2},\overrightarrow{y_2}}\right) + \left(\widehat{\overrightarrow{y_2},\vec{u}}\right) = \theta_{12} + \frac{\pi}{2} + \alpha$$

$$\vec{u} = U(\cos\beta\,\overrightarrow{x_1} + \sin\beta\,\overrightarrow{y_1})$$

$$\vec{u} = U\left(\cos\left(\theta_{12} + \frac{\pi}{2} + \alpha\right)\overrightarrow{x_1} + \sin\left(\theta_{12} + \frac{\pi}{2} + \alpha\right)\overrightarrow{y_1}\right)$$

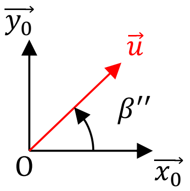
$$\vec{u} = U(-\sin(\theta_{12} + \alpha)\,\overrightarrow{x_1} + \cos(\theta_{12} + \alpha)\,\overrightarrow{y_1})$$



$$\gamma' = \left(\widehat{\overrightarrow{y_1},\vec{u}}\right) = \left(\widehat{\overrightarrow{y_1},\overrightarrow{y_2}}\right) + \left(\widehat{\overrightarrow{y_2},\vec{u}}\right) = \theta_{12} + \alpha$$

$$\vec{u} = U(-\sin(\theta_{12} + \alpha)\,\overrightarrow{x_1} + \cos(\theta_{12} + \alpha)\,\overrightarrow{y_1})$$

Question 4: Exprimer le vecteur $\vec{U}$ dans la base 0

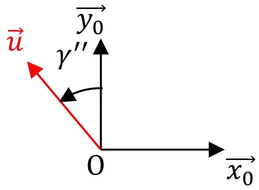


$$\beta'' = \left(\widehat{\overrightarrow{x_0},\vec{u}}\right) = \left(\widehat{\overrightarrow{x_0},\overrightarrow{x_2}}\right) + \left(\widehat{\overrightarrow{x_2},\overrightarrow{y_2}}\right) + \left(\widehat{\overrightarrow{y_2},\vec{u}}\right) = \theta_{02} + \frac{\pi}{2} + \alpha$$

$$\vec{u} = U(\cos\beta\,\overrightarrow{x_0} + \sin\beta\,\overrightarrow{y_0})$$

$$\vec{u} = U\left(\cos\left(\theta_{02} + \frac{\pi}{2} + \alpha\right)\overrightarrow{x_0} + \sin\left(\theta_{02} + \frac{\pi}{2} + \alpha\right)\overrightarrow{y_0}\right)$$

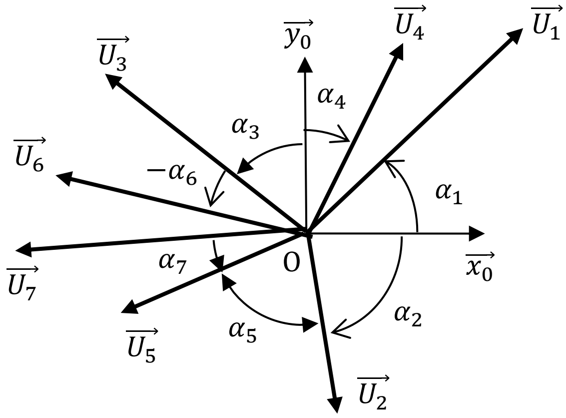
$$\vec{u} = U(-\sin(\theta_{02} + \alpha)\,\overrightarrow{x_0} + \cos(\theta_{02} + \alpha)\,\overrightarrow{y_0})$$



$$\gamma'' = \left(\widehat{\overrightarrow{y_0},\vec{u}}\right) = \left(\widehat{\overrightarrow{y_0},\overrightarrow{y_2}}\right) + \left(\widehat{\overrightarrow{y_2},\vec{u}}\right) = \theta_{02} + \alpha$$

$$\vec{u} = U(-\sin(\theta_{02} + \alpha)\,\overrightarrow{x_0} + \cos(\theta_{02} + \alpha)\,\overrightarrow{y_0})$$

Exercice 3: Somme de vecteurs



Question 1: Donner l'expression de $\vec{U}$ dans la base 0.

$$i = 1:2: \overrightarrow{u_i} = \cos\alpha_i\,\overrightarrow{x_0} + \sin\alpha_i\,\overrightarrow{y_0} = \begin{pmatrix}\cos\alpha_i\\ \sin\alpha_i\\ 0\end{pmatrix}^{\mathcal{B}_0}$$

$$i = 3:4: \overrightarrow{u_i} = -\sin\alpha_i\,\overrightarrow{x_0} + \cos\alpha_i\,\overrightarrow{y_0} = \begin{pmatrix}-\sin\alpha_i\\ \cos\alpha_i\\ 0\end{pmatrix}^{\mathcal{B}_0}$$

$$\overrightarrow{u_5} = \cos(\alpha_2-\alpha_5)\,\overrightarrow{x_0} + \sin(\alpha_2-\alpha_5)\,\overrightarrow{y_0} = \begin{pmatrix}\cos(\alpha_2-\alpha_5)\\ \sin(\alpha_2-\alpha_5)\\ 0\end{pmatrix}^{\mathcal{B}_0}$$

$$\overrightarrow{u_6} = -\sin(\alpha_3-\alpha_6)\,\overrightarrow{x_0} + \cos(\alpha_3-\alpha_6)\,\overrightarrow{y_0} = \begin{pmatrix}-\sin(\alpha_3-\alpha_6)\\ \cos(\alpha_3-\alpha_6)\\ 0\end{pmatrix}^{\mathcal{B}_0}$$

$$\overrightarrow{u_7} = \cos(\alpha_2-\alpha_5-\alpha_7)\,\overrightarrow{x_0} + \sin(\alpha_2-\alpha_5)\,\overrightarrow{y_0} = \begin{pmatrix}\cos(\alpha_2-\alpha_5-\alpha_7)\\ \sin(\alpha_2-\alpha_5-\alpha_7)\\ 0\end{pmatrix}^{\mathcal{B}_0}$$

$$\begin{aligned}\vec{U} = \sum_{i=1}^{4}\overrightarrow{U_i} = \sum_{i=1}^{4}U_i\overrightarrow{u_i}\\ &= \sum_{i=1}^{2}\left[U_i\begin{pmatrix}\cos\alpha_i\\ \sin\alpha_i\\ 0\end{pmatrix}^{\mathcal{B}_0}\right] + \sum_{i=3}^{4}\left[U_i\begin{pmatrix}-\sin\alpha_i\\ \cos\alpha_i\\ 0\end{pmatrix}^{\mathcal{B}_0}\right] + U_5\begin{pmatrix}\cos(\alpha_2-\alpha_5)\\ \sin(\alpha_2-\alpha_5)\\ 0\end{pmatrix}^{\mathcal{B}_0}\\ &+ U_6\begin{pmatrix}-\sin(\alpha_3-\alpha_6)\\ \cos(\alpha_3-\alpha_6)\\ 0\end{pmatrix}^{\mathcal{B}_0}\end{aligned}$$

$$\vec{U} = \begin{pmatrix}U_1\cos\alpha_1 + U_2\cos\alpha_2 - U_3\sin\alpha_3 - U_4\sin\alpha_4 + U_5\cos(\alpha_2-\alpha_5) - U_6\sin(\alpha_3-\alpha_6)\\ U_1\sin\alpha_1 + U_2\sin\alpha_2 + U_3\cos\alpha_3 - U_4\cos\alpha_4 + U_5\sin(\alpha_2-\alpha_5) + U_6\cos(\alpha_3-\alpha_6)\\ 0\end{pmatrix}^{\mathcal{B}_0}$$