Compulsory Assignment 1

Question 1.

a)

p	q	r	$p\to q$	$q\to r$	$(p\to q)\wedge(q\to r)$	$p\to r$	$[(p\to q)\wedge(q\to r)]\to(p\to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

b)

We can see by the truth table above, that the answer to the compound proposition is true regardless of the values of p, q, and r. Therefore, it is a tautology.

c)

For this exercise we will prove the logical equivalence between $(p\wedge\neg q)\to r$ and $p\to(q\vee r)$, (marked in grey), using truth tables.

p	q	r	$p\wedge\neg q$	$(p\wedge\neg q)\to r$		$q\vee r$	$p\to(q\vee r)$
T	T	T	F	T		T	T
T	T	F	F	T		T	T
T	F	T	T	T		T	T
T	F	F	T	F		F	F
F	T	T	F	T		T	T
F	T	F	F	T		T	T
F	F	T	F	T		T	T
F	F	F	F	T		F	T

d)

The truth value for $(\forall n\exists m(n+m=0))$ statement is true. The reason why is if you have a number n you can find the opposite m to make $n + m = 0$. To give an example if you have $n = -1$ then your m will be $-(-1)$, and $-1 + (-(-1)) = 0$.

For the other statement, $(\exists n\forall m(n<m^2))$, despite what we choose m to be, it will always be positive because we square it. That means we can choose n to be -1 and the statement will always be true.

Question 2.

Prove that if *m* and *n* are integers and *mn* is even, the *nm* is even, or *n* is even.

First, we will look at what happens when both m and n is an odd number. Assume that *m = 2k+1* and n = 2i+1, where k and i are in the domain of **Z**.
mn = (2k+1)*(2i+1)
= 4ki+2(k+i)+1
= 2(2ki+k+i)+1

Since both k and i are just integers we can write (2ki+k+i) as r. Which makes mn, = (2r+1), an odd number when both m and n are odd.

When we look at an instance where one of them is even, we will get the following:

Assume that m = 2k+1 and n = 2i, where both k and i are in the domain of **Z**.
mn = (2k+1)*(2i)
= 2(2ki+i)

Since both k and i are just integers we can write 2(2ki+i) as r. Which makes mn, = 2r, an even number when either m or n is an even number.

Finally, when we have two even numbers, we will get the following equations:

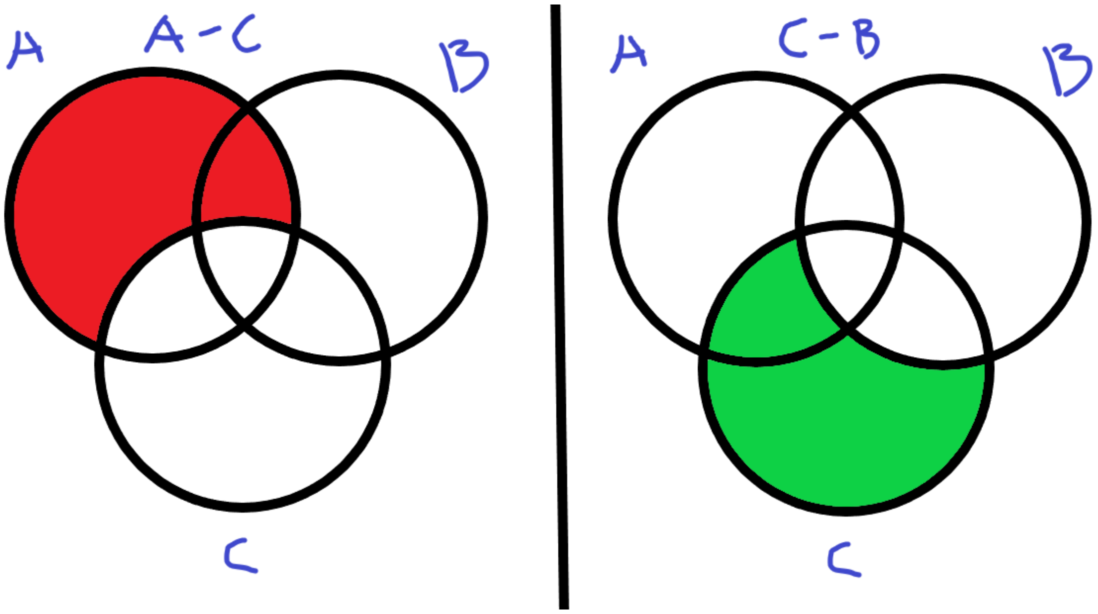
Assume that m = 2k and n = 2i, where both k and i are in the domain of **Z**.
mn = (2k)*(2i)
= 4ki

Since both k and i are just integers we can write (ki) as r. Which makes mn, = 2r, an even number when both m and n are even numbers.

This proves the original question that mn is even when both of them or one of them is even.

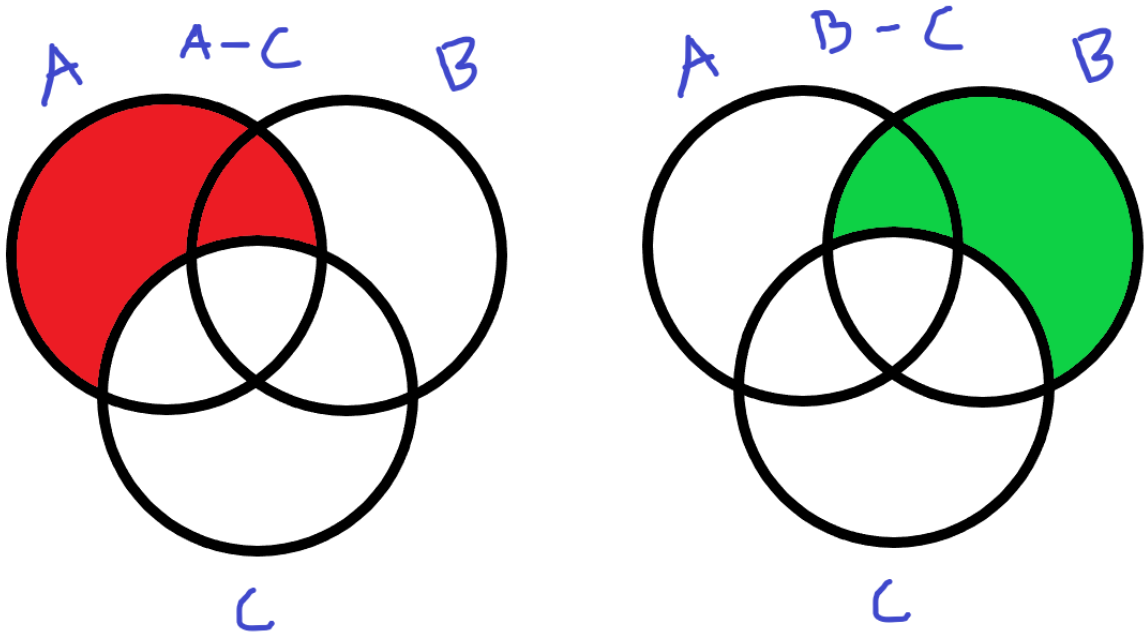
Question 3.

a)



$$(A - C) = \{x | x \in A \land x \notin C\}$$
$$(C - B) = \{x | x \in C \land x \notin B\}$$
$$(A - C) \cap (C - B)$$
$$(x \in A \land x \notin C) \land (x \in C \land x \notin B)$$
$$\left(x \in A \land x \in \overline{C}\right) \land \left(x \in C \land x \in \overline{B}\right)$$
$$\left(x \in A \land x \in \overline{B}\right) \land \left(x \in C \land x \in \overline{C}\right)$$
$$\left(x \in A \land x \in \overline{B}\right) \land (\emptyset)$$
$$(\emptyset)$$

This proves that $(A - C) \cap (C - B)$, is an empty set b)



$$(A - C) = \{x | x \in A \land x \notin C\}$$
$$(B - C) = \{x | x \in B \land x \notin C\}$$
$$(A - C) \cap (B - C)$$
$$(x \in A \land x \notin C) \land (x \in B \land x \notin C)$$
$$\left(x \in A \land x \in \overline{C}\right) \land \left(x \in B \land x \in \overline{C}\right)$$
$$(x \in A \land x \in B) \land (x \notin C \land x \notin C)$$
$$(x \in A \land x \in B) \land (T)$$
$$(T)$$

This means that the statement, $(A - C) \cap (B - C)$, is a set that contains something.

c)

$$f(x) = x^2$$
$$f: \mathbb{R} \to \mathbb{R}$$

As stated by the book this is the definition of an injective function:

A function f is said to be one-to-one, or an injection, if and only if $f(a) = f(b)$ implies that $a = b$ for all a and b in the domain of f.

$$a = 2$$
$$b = -2$$
$$f(a) = 4$$
$$f(b) = 4$$

In this instance $f(a) = f(b)$, but $a \neq b$. Which means f is not one-to-one (injective). This also implies f can't be a one-to-one correspondence (bijective).

Furthermore, the book states:

A function f from A to B is called onto, or a surjection, if and only if for every element $b \in B$ there is an element $a \in A$ with $f(a) = b$.

And since there doesn't exist a single a in $\mathbb{R}$ that makes $f(a) = -1$, f isn't an onto (surjective) function ether.