UNIVERSITETET I BERGEN

KANDIDAT

118

PRØVE

MNF130 0 Diskrete strukturer

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Informasjon/ Information

Oppgave	Tittel	Oppgavetype
i	Egenerklæring/ Declaration	Informasjon eller ressurser
i	Kontaktinfo under eksamen	Informasjon eller ressurser
i	Generelt info om denne eksamen	Informasjon eller ressurser

Compulsory assignments

Oppgave	Tittel	Oppgavetype
1	Compulsory assignments	Langsvar

Propositional logic

Oppgave	Tittel	Oppgavetype
2	Propositional logic: truth tables	Sammensatt

Propositional equivalences

Oppgave	Tittel	Oppgavetype
3	Propositional equivalences	Utregning

Nested quantifiers

Oppgave	Tittel	Oppgavetype
4	Nested quantifiers	Sant/usant
5	Nested quantifiers	Sant/usant

Prove a theorem

Oppgave	Tittel	Oppgavetype
6	Prove a theorem	Langsvar

Prove or disprove

Oppgave	Tittel	Oppgavetype
7	Prove or disprove	Langsvar

Mark the subset

Oppgave	Tittel	Oppgavetype
8	Mark the subset	Feltvalg
9	Mark the subset	Feltvalg

Set identities

Oppgave	Tittel	Oppgavetype
10	Set identities	Utregning

Composition of functions

Oppgave	Tittel	Oppgavetype
11	Function composition	Flervalg

Sequences

Oppgave	Tittel	Oppgavetype
12	Sequences	Langsvar

Modular arithmetic

Oppgave	Tittel	Oppgavetype
13	Modular arithmetic	Fyll inn tall

Greatest common divisors

Oppgave	Tittel	Oppgavetype

14	Euclidean algorithm	Sammensatt

Cryptography

Oppgave	Tittel	Oppgavetype
15	RSA encryption	Sammensatt

Mathematical induction

Oppgave	Tittel	Oppgavetype
16	Mathematical induction	Langsvar

Recursion and structural induction

Oppgave	Tittel	Oppgavetype
17	Structural induction	Langsvar

Counting

Oppgave	Tittel	Oppgavetype
18	Bit strings	Flervalg

Probability

Oppgave	Tittel	Oppgavetype
19	Probability	Flervalg

Graph theory

Oppgave	Tittel	Oppgavetype
20	Dijkstra's algorithm	Sammensatt

Trees

Oppgave	Tittel	Oppgavetype

21	Tree traversal	Flervalg

File upload

Oppgave	Tittel	Oppgavetype
22	File upload	Filopplasting

1 Compulsory assignments

This is a placeholder to put the marks for the compulsory quizzes and written assignments:

- 1 point for every passed quiz, maximum 10 points.
- the sum of grades on the written assignments, maximum 30 points.

You don't need to fill anything here.

Maks poeng: 40

2 Propositional logic: truth tables

Fill the following truth table.
Note:

- Use **numeric 1 for True, and numeric 0 for False**!
- You must answer this question in **Inspera**.

p	q	r	$p \to q$	$q \to r$	$p \to (q \to r)$	$(p \to q) \to r$
1	1	1	1	1	1	1
1	1	0	1	0	0	0
1	0	1	0	1	1	1
1	0	0	0	1	1	1
0	1	1	1	1	1	1
0	1	0	1	0	1	0
0	0	1	1	1	1	1
0	0	0	1	1	1	0

Are the compound propositions $p \to (q \to r)$ and $(p \to q) \to r$ logically equivalent?
Select an alternative

- Yes
- No

Maks poeng: 5.1

3 Propositional equivalences

Use logical equivalences to prove that $(p \to q) \wedge (\neg p \to q) \equiv q$.
Note:

- You **may submit an answer in Inspera, or a handwritten solution.**
- Start with the compound proposition $(p \to q) \wedge (\neg p \to q)$ on the first row.
- Start each subsequent row with the logical equivalence symbol $\equiv$, and use a logical equivalence to transform the previous row into a new, logically equivalent compound proposition. **Apply only one logical equivalence per row, and name the logical equivalence being used**.
- If you answer in Inspera, the following TeX symbols may be useful in the equation editor (TeX code typed in the white equation box is converted automatically):

$\equiv$	\equiv
$\wedge$	\wedge
$\vee$	\vee
$\neg$	\neg
$\to$	\to

Fill in your answer here

$(p \to q) \wedge (\neg p \to q)$

$\equiv (\neg p \vee q) \wedge (p \to \neg q)$

$\equiv (\neg p \vee q) \wedge (\neg p \vee q)$

$\equiv p \vee (q \wedge \neg q)$

$\equiv p \vee (F)$

$\equiv p$

Maks poeng: 5

4 Nested quantifiers

What is the truth value of the statement $\forall n \exists m(m < n)$ if the universe of discourse is the set of nonnegative integers?

Note:

- You must **answer this question in Inspera**.
- To discourage guessing, you earn 1 point for a correct answer, but lose 0.5 point for a wrong answer.

Select one alternative:

- True
- False

Maks poeng: 1

5 Nested quantifiers

What is the truth value of the statement $\forall n \exists m(n < m)$ if the universe of discourse is the set of nonnegative integers?

Note:

- You must **answer this question in Inspera**.
- To discourage guessing, you earn 1 point for a correct answer, but lose 0.5 point for a wrong answer.

Select one alternative:

- True
- False

Maks poeng: 1

6 Prove a theorem

Suppose you are allowed to give either a direct proof or a proof by contraposition of the following: for all integers n, if $3n + 5$ is even, then n is odd. Which type of proof would be easier to give? Explain why and present your proof.

Note:

- You **may submit an answer in Inspera, or a handwritten solution**.

Fill in your answer here

In this example direct proof would be easier beacuse when using proof by contraposition we use a lot more hypothesis which is not nexesary in this example.

Proof by direct proof:

Definition:
n is odd integer, if ∃ integer k such that n = 2k+1

Assume n is odd
By defintion, ∃ integer k, such that n = 2k+1
3n+5 = 3(2k+1)+5=2(3k+4)
Let m = 3k+4
3n+5 = 2(m)
So, by definition, 3n+5 is even

Maks poeng: 5

7 Prove or disprove

Prove or disprove: For all real numbers x, $\lfloor x + \lfloor x \rfloor \rfloor = \lfloor 2x \rfloor$.
Note:

- You **may submit an answer in Inspera, or a handwritten solution**.

Fill in your answer here

The easiest way to prove this is to try to disproove it. All we need is one number which does not fit in the equation and the equation is disproven.

now if we plug in the number f.ex x = 1.6

On the left side we have □1.6+□1.6□□ = □1.6+1□=2 ,
and on the right side we have □2*1.6□ = □3.2□=3
thus □x+□x□□≠ □2x□, and the equation is false
This is becuase we have two flooring on the left side, if we only had the outer flooring the equation would be true

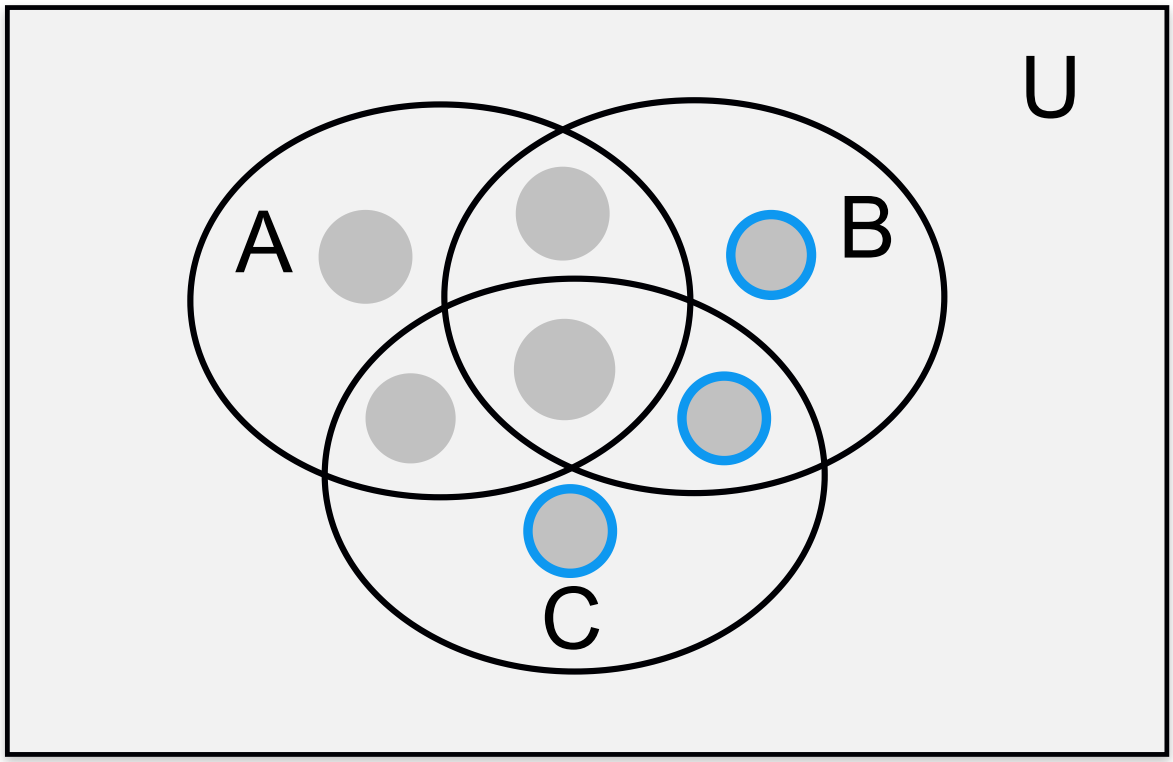
Maks poeng: 3

8 Mark the subset

Mark all the hotspots in areas on the Venn diagram that make up $(B \cup C) - A$.
Note:

- You must **answer this question in Inspera**.
- You earn 1.4 point if you mark exactly the right combination of hotspots, 0 otherwise.

Click the image



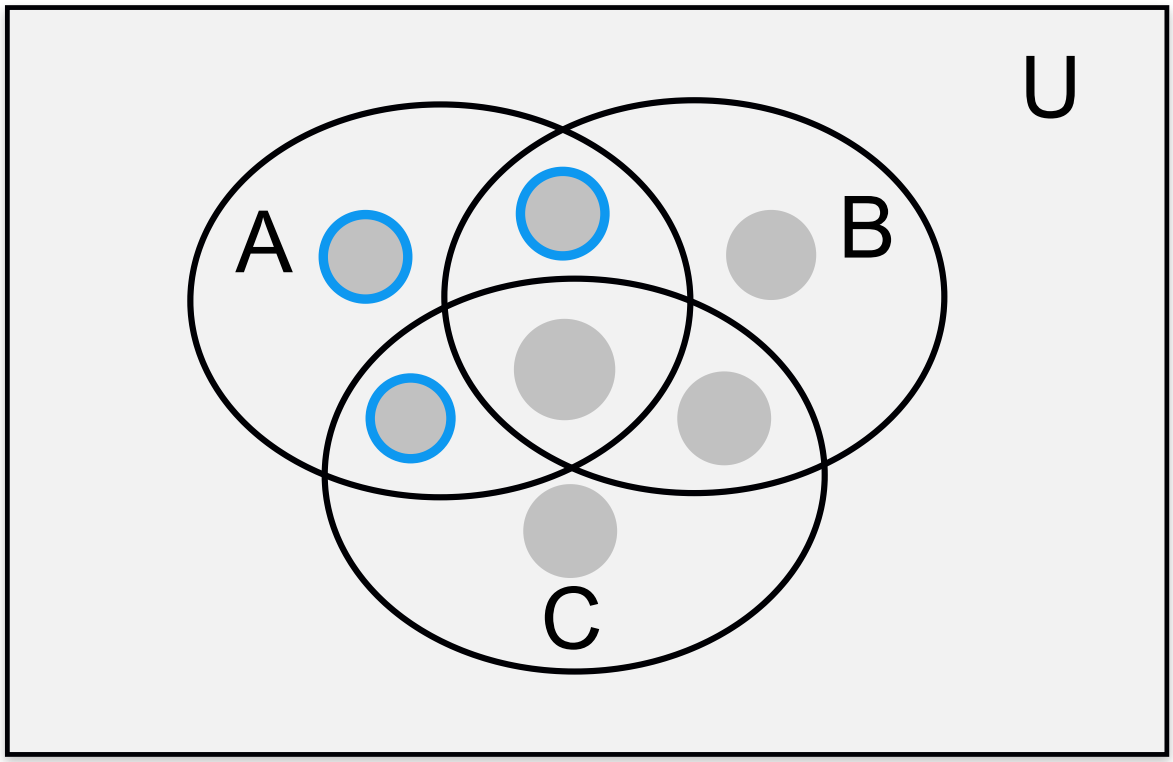
Maks poeng: 1.4

9 Mark the subset

Mark all the hotspots in areas on the Venn diagram that make up $A - (B \cap C)$.
Note:

- You must **answer this question in Inspera**.
- You earn 1.4 point if you mark exactly the right combination of hotspots, 0 otherwise.

Click the image



Maks poeng: 1.4

10 Set identities

Let A, B, and C be sets within the same universal set U. Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ by giving a containment proof (that is, prove that the left side is a subset of the right side and that the right side is a subset of the left side).

Note:

- You **may submit an answer in Inspera, or a handwritten solution**.
- If you type your answer in Inspera, the following TeX symbols may be useful in the equation editor (TeX code typed in the white equation box is converted automatically):

$\neg$	\neg
$\vee$	\vee
$\wedge$	\wedge
$\in, \notin$	\in, \not\in
$\overline{A}$	\overline{A}
$\cup$	\cup
$\cap$	\cap

Fill in your answer here

$B \cap C \subseteq B$ *and* $B \cap C \subseteq C$

$A \cup (B \cap C) \subseteq A \cup B$

$A \cup (B \cap C) \subseteq A \cup C$

This shows that $A \cup (B \cap C)$ *is contained in both* $A \cup B$ *and* $A \cup C$

$A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$. *This proves containment in one direction*

For the opposite direction, $\sup$ *pose that* $x \in (A \cup B) \cap (A \cup C)$

There are two possibilities: *either* $x \in A$ *or* $x \notin A$

If $x \in A$ *then certainly* $x \in A \cup (B \cap C)$

On the other hand, *if* $x \notin A$, *then x must be in both B and C*, $\sin$ *ce* $x \in (A \cup B) \cap (A \cup C)$

In both cases we have $x \in A \cup (B \cap C)$

This proves the containment $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$

Maks poeng: 5

11 Function composition

Suppose $g : A \to A$ and $f : A \to A$, where $A = \{1, 2, 3, 4\}$, and the functions f and g are defined by $f = \{(1, 3), (2, 2), (3, 4), (4, 2)\}$ and $g = \{(1, 4), (2, 1), (3, 1), (4, 2)\}$. Find $f \circ g$.

Note:

- You must **answer this question in Inspera**.
- To discourage guessing, you earn 2 points for a correct answer, but lose 1 point for a wrong answer.

Select one alternative

- $f \circ g = \{(1, 1), (2, 1), (3, 2), (4, 1)\}$
- $f \circ g = \{(1, 2), (2, 3), (3, 3), (4, 2)\}$
- $f \circ g = \{(1, 4), (2, 2), (3, 2), (4, 2)\}$
- $f \circ g = \{(1, 2), (2, 4), (3, 4), (4, 1)\}$

Maks poeng: 2

12 Sequences

Let $a_n = 3^{n+4}$ for $n \geq 0$. Find the values of a_0 and a_1, and verify that a_n is a solution to the recurrence relation $a_n = 4a_{n-1} - 3a_{n-2}$ for $n \geq 2$.

Note:

- You **may submit an answer in Inspera, or a handwritten solution**.

Fill in your answer here

a_n = 3^(n+4) for n >=0

a_0 = 3^(0+4) = 81
a_1 = 3^(1+4) = 243
a_2 = 3^(2+4) = 729

a_n = 4a_(n-1)-3a_(n-2) for n>=2.

a_2 = 4*243-3*81 = 729

We can see that a_2 has the same value for both relations.

Maks poeng: 4

13 Modular arithmetic

Find the smallest integer $a > 1$ such that $a - 1 \equiv 2a \pmod{11}$.
Note:

- You must **answer this question in Inspera**.

$a =$ 10 .

Maks poeng: 1

14 Euclidean algorithm

Use the Euclidean algorithm to find $\gcd(662, 414)$, and write the result as a linear combination of 662 and 414.

Note:

- You must **answer this question in Inspera**.

$\gcd(662, 414) =$ 2 $=$ -5 $\cdot 662 +$ 8 $\cdot 414$

Maks poeng: 3

15 RSA encryption

Encrypt the message KING using the RSA system with $n = 43 \cdot 61$ and $e = 13$, translating each letter into integers ($A = 00, B = 01, \ldots$) and grouping together pairs of integers. Write your answer as two groups of pairs of integers in the boxes specified below.

Note:

- You must answer this question in Inspera

1740 2314

Maks poeng: 2

16 Mathematical induction

Use mathematical induction to prove that $1-2+2^2-2^3+\cdots+(-1)^n2^n=\frac{1}{3}\left(2^{n+1}(-1)^n+1\right)$ for all integers $n \geq 0$. Structure your answer as follows:

1. Show that the basis step is true.
2. State the inductive hypothesis.
3. Prove the inductive step, identifying where you use the inductive hypothesis.

Note:

- You **may submit an answer in Inspera, or a handwritten solution**.

Fill in your answer here

Define P(n):
P(n) = 1-2+2^2-2^3+...+(-1)^n*2^n=1/3*(2^(n+1)*(-1)^n+1)

Basis step - show P(0):
1-2+2^2-2^3+...+(-1)^n*2^n=1/3*(2^(n+1)*(-1)^n+1)
=(-1)^0*2^0=1/3*(2^1*1+1)
=1=1.
So we have prove the basis step and P(n) is true for n = 0

Induction hypothesis:
Assume P(m) is true for some arbitrary m>=0
$\sum_{k=0}^{m}(-1)^m(2)^m=\frac{(2^{m+1}(-1)^m+1)}{3}$

inductive step - verify P(m+1):
$\sum_{k=0}^{m+1}(-1)^m(2)^m=\sum_{k=0}^{m+1}(-1)^m(2)^m+(-1)^{m+1}(2)^{m+1}$
plug in the unductive hypotesis on the right side:
$=\frac{(2^{m+1}(-1)^m+1)}{3}+(-1)^{m+1}(2)^{m+1}$
$=\frac{(2^{m+1}(-1)^m+1)+3(-1)^{m+1}(2)^{m+1}}{3}$
$=\frac{(-1)^m+3(-1)^{m+1}2(2)^{m+1}+1}{3}$

Maks poeng: 6

17 Structural induction

Let S be the subset of the ordered pairs of integers defined recursively by
Basis step: $(0,0) \in S$
Recursive step: If $(a,b) \in S$, then $(a,b+1) \in S$ and $(a+1,b+1) \in S$.
a) List the elements of S produced by the first three applications of the recursive definition.
b) Use structural induction to show that $a \leq b$ whenever $(a,b) \in S$. Structure your answer as follows:

1. Show that the basis step is true.
2. State the inductive hypothesis.
3. Prove the inductive step, identifying where you use the inductive hypothesis.

Note:

- You **may submit an answer in Inspera, or a handwritten solution.**

Fill in your answer here

Lmao imagine having two induction questions in one exam xD □□

Maks poeng: 5

18 Bit strings

How many bit strings of length 10 have exactly six 0s?
Note:

- You must **answer this question in Inspera.**
- To discourage guessing, you earn 1 point for the correct answer, but lose 0.5 point for a wrong answer

Select one alternative:

- $\binom{10}{4}$
- $\binom{10}{6}$
- 2^4
- 2^6

Maks poeng: 1

19 Probability

What is the probability that a fair coin lands heads four times out of five flips?

Note:

- You must **answer this question in Inspera.**
- To discourage guessing, you earn 1 point for the correct answer, but lose 0.5 point for a wrong answer

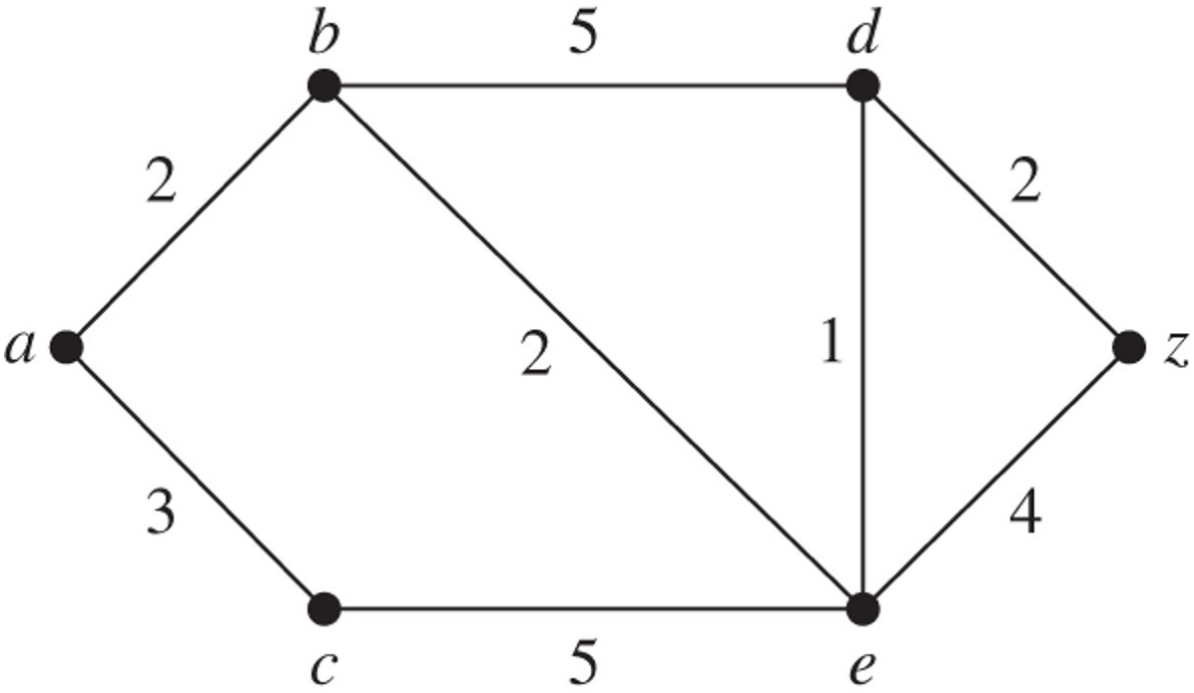
Select one alternative:

- $\frac{5}{2^5}$
- $4!/5!$
- $4/5$
- $\left(\frac{1}{2}\right)^4$

Maks poeng: 1

20 Dijkstra's algorithm

For the undirected weighted graph shown in the figure, use Dijkstra's algorithm (the version in the lecture slides) to find the length of a shortest path from vertex *a* to all other vertices. Write down explicitly the steps of the algorithm by completing the tables below.



Note:

- You must **answer this question in Inspera.**

Table 1. The current set S of processed vertices and its neighbourhoud N(S) at each iteration of the algorithm

In the table below, fill each entry with **a list of vertices, ordered alphabetically, separated by commas, and do not include surrounding { }** (see the example entries). Failure to follow this notation will result in your answer being wrongly parsed by the system.

Iteration	S	N(S)
0	a	b,c
1	a,b	c,d,e
2	a,b,e	c,d,z
3	a,b,e,d	c,z
4	a,b,d,e,z	c
5	a,b,c,d,e,z	∅

Table 2. The current shortest path length from vertex *a* to every other vertex at each iteration of the algorithm

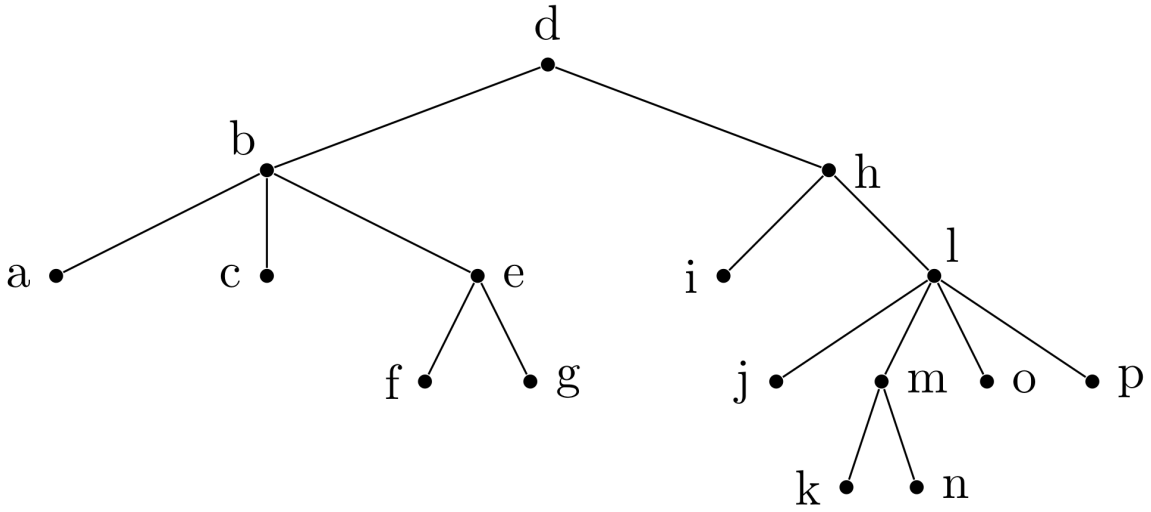
In the table below, fill each entry with **an integer number, or a - (minus sign) if a vertex has not yet been considered** (see the example entries). Failure to follow this notation will result in your answer being wrongly parsed by the system.
The column headers correspond to the iteration count in the previous table.

Vertex	0	1	2	3	4	5
a	0	0	0	0	0	0
b	-	2	2	2	2	2
c	-	3	3	3	3	3
d	-	-	7	5	5	5
e	-	-	4	4	4	4
z	-	-	-	8	7	7

Maks poeng: 5.1

21 Tree traversal

Which ordering of vertices is the **postorder** traversal of the tree shown below:



Note:

- You must **answer this question in Inspera.**
- To discourage guessing, you earn 2 points for the correct answer, but lose 1 point for a wrong answer.

Select one alternative:

- a,b,c,f,e,g,d,i,h,j,l,k,m,n,o,p
- d,b,h,a,c,e,i,l,f,g,,j,m,o,p,k,n
- d,b,a,c,e,f,g,h,i,l,j,m,k,n,o,p
- a,c,f,g,e,b,i,j,k,n,m,o,p,l,h,d

Maks poeng: 2

22 File upload

If you have handwritten answers to some or all of the manually graded questions:

- Make a scan or take a photo of every page, and merge into a **single PDF file**.
- Clearly write **your candidate number, question number and page number on each page** of the uploaded file.

If you answered all questions in Inspera, you don't have to do anything here.

Upload your file here. Maximum one file. Maximum file size is 2 GB.

Følgende filtyper er tillatt: **.pdf** Maksimal filstørrelse er **2 GB**.

Velg fil for opplasting

NB 1: you can only upload ONE single file
NB 2: only PDF is allowed
NB 3: maximum file size is 2 GB.

Maks poeng: 0