1a) Sannsynlighet for at det er nøyaktig 2 defekte deler:

$$P(X=k) = \frac{\binom{r}{k}\binom{n-r}{m-k}}{\binom{n}{m}} = \frac{\binom{3}{2}\binom{18}{5}}{\binom{21}{7}} = \underline{\underline{0,221}}$$

b) forventning: $E(x) = m \cdot \frac{r}{n}$

Varians: $V(x) = m\frac{r}{n}\left(1-\frac{r}{n}\right)\frac{n-m}{n-1}$

$$E(x) = 7 \cdot \frac{3}{21} = \underline{\underline{1}}$$

$$V(x) = 7\,\frac{3}{21}\left(1-\frac{3}{21}\right)\frac{21-7}{21-1} = 3/5 = \underline{\underline{0,6}}$$

2 Geometrisk fordeling forventning: $E(X) = 1/p$

$E(Q) = 1/0{,}05 = 20$

Siden algorithmen broker en halvtime på å kjøre betyr dette at den broker 20 halvtimer, eller 10 timer
Så programmereren burde bruke algoritme B

3a) $P(x;\lambda) = \dfrac{e^{-\lambda}\lambda^{x}}{x!}$ $\lambda_1 = 3$ $\lambda_2 = 5$

$P(2;3) = \dfrac{e^{-3}\cdot 3^{2}}{2!} = 0{,}224$

$P(1;3) = \dfrac{e^{-3}\cdot 3^{1}}{1!} = 0{,}149$

$p(0;3) = \dfrac{e^{-3}\cdot 3^{0}}{0!} = 0{,}05$

$p(x \leq 2) = 0{,}224 + 0{,}149 + 0{,}05 = 0{,}423$

$p(x \geq 3) = 1 - P(x \leq 2) = \underline{\underline{0{,}577}}$

3b $P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$

$n=5$, $k=3$, $p=0{,}2$

$P(X=3) = \binom{5}{3} 0{,}2^3 (1-0{,}2)^{5-3} = \underline{\underline{0{,}0512}}$

Sansynligheten for at 3av de 5 hjortene blir observert e på rundt 5%.

Forventet antall hjort som vil bli observert er:

$E(x) = np = 5 \cdot 0{,}2 = \underline{\underline{1}}$

4 $E(X)=0{,}6 \quad E(Y)=0{,}5 \quad V(X)=1 \quad V(Y)=2$

$\rho(X,Y)=0{,}5$

$E(U)=E(X)+E(Y)=0{,}6+0{,}5=\underline{1{,}1}$

$E(V)=2E(Y)=2\cdot 0{,}5=\underline{1}$

$\rho=\mathrm{Corr}(X,Y)=\dfrac{\mathrm{Cov}(X,Y)}{\sigma_x\sigma_y} \qquad \mathrm{Cov}(X,Y)=\rho\,\sigma_x\sigma_y$

$\sigma_x^2=V(X)$ & $\sigma_y^2=V(Y)$

$\mathrm{Cov}(X,Y)=0{,}5\cdot\sqrt{1}\cdot\sqrt{2}=\dfrac{\sqrt{2}}{2}\approx\underline{0{,}71}$

$V(U)=V(X+Y)=V(X)+V(Y)+2\mathrm{Cov}(X,Y)$

$=1+2+2\cdot\dfrac{\sqrt{2}}{2}=3+\sqrt{2}=\underline{4{,}41}$

$V(W)=V(2Y)=2^2\cdot 2=\underline{8}$

$V(X)+V(Y)+2\mathrm{Cov}(X,Y) \Leftrightarrow \mathrm{Cov}(X,Y)=\frac{1}{2}\left(V(X,Y)-V(X)-V(Y)\right)$

$V(U+W)=V(X+3Y)=V(X)+9\cdot V(Y)+6\mathrm{Cov}(X,Y)$

$=1+18+6\cdot\dfrac{\sqrt{2}}{2}=19+3\sqrt{2}=\underline{23{,}24}$

body sha

$$Cov(U,W) = 1/2\left(V(U+W) - V(U) - V(W)\right)$$

$$= 1/2\left((19+3\sqrt{2}) - (3+\sqrt{2}) - 8\right) = 4+\sqrt{2} = \underline{\underline{5{,}414}}$$

$$Corr(U,W) = \frac{Cov(U,W)}{\sigma_U \cdot \sigma_W} = \frac{4+\sqrt{2}}{\sqrt{3+\sqrt{2}} \cdot \sqrt{8}} = \underline{\underline{0{,}911}}$$

5a) $f(x)\begin{cases} k(x-x^4) & \text{if } 0 \leq x \leq 1 \\ 0 & \text{ellers} \end{cases}$

for at denne sannsynlighetstettheten skal bli gyldig må området under grafen være lik 1.

$$\int_0^1 k(x-x^4)\,dx = k\left[\frac{1}{2}x^2 - \frac{1}{5}x^5\right]_0^1 = k\left(\frac{1}{2}-\frac{1}{5}\right) = k\frac{3}{10}$$

$$k\frac{3}{10} = 1 \qquad \underline{\underline{k = \frac{10}{3}}}$$

b) $E(x) = \mu = \int_{-\infty}^{\infty} x \cdot f(x)\,dx \qquad V(x) = E(x^2) - E(x)^2$

Siden $f(x) = 0$ når den er utenfor 0 og 1 bryr vi oss bare om alt mellom 0 og 1

$$\int_0^1 x k(x-x^4)\,dx = \int_0^1 x\frac{10}{3}(x-x^4)\,dx$$

$$= \frac{10}{3}\left[\frac{1}{3}x^3 - \frac{1}{6}x^6\right]_0^1 = \frac{10}{3}\left(\frac{1}{3}-\frac{1}{6}\right) = \frac{10}{9} - \frac{10}{18} = \underline{\underline{\frac{5}{9}}}$$

$$E(x^2) = \int_0^1 \frac{10}{3}x(x-x^4)\,dx = \underline{\frac{5}{14}}$$

$$V(x) = E(x^2) - E(x)^2 = 5/14 - (5/9)^2 = \frac{55}{1134} = \underline{\underline{0{,}0485}}$$

5c) $E(4X+3)$ $V(4X+3)$

$$E(4X+3) = 4E(X)+3 = 4\cdot\frac{5}{9}+3 = \underline{\underline{5{,}\overline{22}}}$$

$$V(4X+3) = 4^2 V(X) = 16\cdot {}^{55}\!/_{1134} = \underline{\underline{0{,}776}}$$

6a)

$X \backslash Y$	0	1	2	$p_X(x)$
0	0,05	0,25	0,1	0,4
1	0,1	0,05	0,1	0,25
2	0,25	0	0,1	0,35
$p_Y(y)$	0,4	0,3	0,3	

Bruker $p_X(x) = \sum_y p(x,y)$ og $p_Y(y) = \sum_x p(x,y)$

Betinget sannsynlighet: $P(X=2|Y=2) = \frac{P(2,2)}{p_Y(2)} = \frac{0,1}{0,3} = 0,\overline{33}$

Nei, X og Y er ikke uavhengige i dette tilfelle fordi: $P(X=2|Y=2) \neq P(X=2) = p_X(2) = 0,35$