1(a) $\bar{X}_1 = 130$ $S_1 = 100$

$\bar{X}_2 = 150$ $S_2 = 40$

$\bar{X}_3 = 250$ $S_3 = 150$

2a) n observasjoner $t_1 \ldots t_n$ finn max $\hat{\theta}$ θ

t_i: 1, 1.4, 2, 0.5, 0.7, 2.0, 1.3, 1.1, 1.8, 0.2,

$f(t) = \theta e^{-\theta t}, \; t \geq 0$

Sansynligheten:

$$S(\theta \mid t_1, \ldots t_n) = \theta e^{-\theta x_1} \cdots \theta e^{-\theta x_n}$$

$$= \theta^n \left[e^{-\theta(x_1 + \ldots + x_n)} \right]$$

Så deriver vi den slik at vi kan sette funksjonen lik 0 for å så finne $\hat{\theta}$. Og siden funksjonen har e vil det være lettere og finne svaret om vi bruker log til å derivere.

$$\frac{d}{d\theta} \log\left(\theta^n \left[e^{-\theta(x_1 + \ldots + x_n)} \right]\right)$$

$$d/d\theta \; \log(\theta^n) + \log\left[e^{-\theta(x_1 + \ldots + x_n)} \right]$$

$$d/d\theta \; n \log(\theta) - \theta(x_1 + \ldots + x_n)$$

$$n \frac{1}{\theta} - (x_1 + \ldots + x_n)$$

$$0 = n\frac{1}{\theta} - (x_1 + \ldots + x_n)$$

$$\hat{\theta} = \frac{n}{(x_1 + \ldots + x_n)}$$

$$\hat{\theta} = 10 : (1 + 1{,}4 + 2 + 0{,}5 + 0{,}7 + 2 + 1{,}3 + 1{,}1 + 1{,}8 + 0{,}2)$$

$$\hat{\theta} = 0{,}8\overline{33}$$

2b) Nei, fordi

3a) $L(\theta) = \left(\frac{\theta^3}{3!} e^{-\theta} \cdot \frac{(2\theta)^5}{5!} e^{-2\theta}\right)$

$$= \frac{\theta^3 \cdot e^{-\theta} \cdot 2^5 \cdot \theta^5 \cdot e^{-2\theta}}{3!\,5!} = \frac{2^5 \theta^8 e^{-3\theta}}{3!\,5!}$$

$l(\theta) = \ln(L(\theta)) = \ln\left(\frac{2^5 \theta^8 e^{-3\theta}}{3!\,5!}\right)$

$$= \ln(2^5 \cdot \theta^8 \cdot e^{-3\theta}) - \ln(3! \cdot 5!)$$

$$= \ln 2^5 + \ln \theta^8 + \ln e^{-3\theta} - \ln 3! - \ln 5!$$

$$= \underline{\underline{5\ln 2 - \ln(5!) - \ln(3!) + 8\ln\theta - 3\theta}}$$

Se figur 2 for å se $l(\theta)$ her kan vi lese av at maksimal punktet til $l(\theta)$ er ca. $\underline{\underline{2,5}}$

3b) $\frac{d}{d\theta} l(\theta) = \frac{d}{d\theta}\left(5\ln 2 - \ln(5!) - \ln(3!) + 8\ln\theta - 3\theta\right)$

$$= 8/\theta - 3$$

$$0 = 8/\theta - 3$$

$$\underline{\underline{\hat{\theta} = 8/3}}$$

4a) X_1, X_2, X_3 er alle binomisk fordelt med $n=1$ og henholdsvis $p_1 = \theta$, $p_2 = 4\theta$ og $p_3 = 5\theta$.

$E(X) = p$ $\qquad V(X) = p(1-p)$

$E(X_1) = \theta$ $\qquad V(X_1) = \theta(1-\theta)$

$E(X_2) = 4\theta$ $\qquad V(X_2) = 4\theta(1-4\theta)$

$E(X_3) = 5\theta$ $\qquad V(X_3) = 5\theta(1-5\theta)$

4b) $E(\hat{\theta}_1) = E\left(\frac{1}{10}(X_1 + X_2 + X_3)\right) = \frac{1}{10}\left(E(X_1) + E(X_2) + E(X_3)\right)$

$= \frac{1}{10}(\theta + 4\theta + 5\theta) = \underline{\underline{\theta}}$

$E(\hat{\theta}_2) = E\left(\frac{1}{3}\left(X_1 + \frac{X_2}{4} + \frac{X_3}{5}\right)\right) = \frac{1}{3}\left(E(X_1) + \frac{E(X_2)}{4} + \frac{E(X_3)}{5}\right)$

$= \frac{1}{3}\left(\theta + \frac{4\theta}{4} + \frac{5\theta}{5}\right) = \underline{\underline{\theta}}$

$V(\hat{\theta}_1) = V\left(\frac{1}{10}(X_1 + X_2 + X_3)\right) = \frac{1}{10^2}\left(V(X_1) + V(X_2) + V(X_3)\right)$

$= \frac{1}{10^2}\left(\theta(1-\theta) + 4\theta(1-4\theta) + 5\theta(1-5\theta)\right)$

$= \frac{1}{10^2}\left(\theta - \theta^2 + 4\theta - 16\theta^2 + 5\theta - 25\theta^2\right)$

$= \frac{1}{10^2}\left(10\theta - 42\theta^2\right) = \frac{1}{10}\left(\theta - \frac{42}{10}\theta^2\right)$

$V(\hat{\theta}_2) = V\left(\frac{1}{10}(X_1 + X_2/4 + X_3/5)\right) = 1/3^2\left(V(X_1) + 1/4^2\, V(X_2) + 1/5^2\, V(X_3)\right)$

$= 1/3^2\left(\theta(1-\theta) + 1/4^2\, 4\theta(1-4\theta) + 1/5^2\, 5\theta(1-5\theta)\right)$

$= 1/3^2\left(\theta - \theta^2 + \theta/4 - \theta^2 + \theta/5 - \theta^2\right)$

$= 1/3^2\left(\left(1 + \frac{1}{4} + \frac{1}{5}\right)\theta - 3\theta^2\right)$

$V(\hat{\theta}_1 = 0{,}2) = 1/10\left(0{,}2 - 42/10 \cdot (0{,}2)^2\right) = 0{,}0032$

$V(\hat{\theta}_2 = 0{,}2) = 1/3^2\left((1 + 1/4 + 1/5)\,0{,}2 - 3 \cdot (0{,}2)^2\right) = 0{,}01\overline{88}$

Siden $\hat{\theta}_1$ har minst varians er dette den beste estimatet

5a) i $P(X<1,5) = P\left(\frac{X-\mu}{\sigma} < \frac{1,5-\mu}{\sigma}\right) = P\left(Z < \frac{1,5-2,2}{0,8}\right)$

$= P(Z < -0,88) = \underline{\underline{0,1894}}$

Broker tabell A.3 i boka

ii $P(2<X<2,5) = P(X<2,5) - P(X<2)$

$= P\left(Z < \frac{2,5-\mu}{\sigma}\right) - P\left(Z < \frac{2-\mu}{\sigma}\right)$

$= P(Z<0,38) - P(Z<-0,25) = 0,648 - 0,4052 = \underline{\underline{0,24}}$

5b) $P(2<X<2,5 \mid X>1,5) = \frac{P(X<2,5) - P(X<2)}{P(X>1,5)}$

$= P\left(Z < \frac{2,5-\mu}{\sigma}\right) - P\left(Z - \frac{2-\mu}{\sigma}\right)$

$= \frac{0,24}{1-0,1894} = \underline{\underline{0,2961}}$

5c $\bar{y} - z_{0,05}\,\sigma_Y/\sqrt{n},\ \bar{y} + z_{0,05},\ \sigma_y/\sqrt{n}$

/

6 $\bar{y} = \frac{1}{5}\sum_{i=1}^{5} y_i = 284$

$S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n} (y_i - \bar{y})^2} = \sqrt{2342/4} = 24{,}2$

bruker tabell A5

$t_{0,025,n-1} = \underline{\underline{2,776}}$