Lecture 3

Supervised Learning II

Probability theory

A random variable represents all the values that could be selected from  
a set of possible values.

Probability theory

Example

If 𝑋 is a random variable over the days of the week, then 𝑋 could be any of the  
values in the following set:

{𝑀𝑜𝑛𝑑𝑎𝑦, 𝑇𝑢𝑒𝑠𝑑𝑎𝑦, 𝑊𝑒𝑑𝑛𝑒𝑠𝑑𝑎𝑦, 𝑇ℎ𝑢𝑟𝑠𝑑𝑎𝑦, 𝐹𝑟𝑖𝑑𝑎𝑦, 𝑆𝑎𝑡𝑢𝑟𝑑𝑎𝑦, 𝑆𝑢𝑛𝑑𝑎𝑦}

Probability theory

A discrete random variable is drawn from a countable set of possible  
outcomes—both finite and infinite. F. ex. the set over the people in  
class or all the stars in the universe.

A continuous random variable is drawn from an uncountable set of  
outcomes: F. ex. the height of a person.

Probability theory

A random n-dimensional vector is a vector that consists of n random  
variables.

Probability theory.

Example

If X is a random vector over voluntary deliveries and final grades, then X  
could be any of the values in the following set:  
{[0, 𝐸]  
[3, 𝐴]  
[2, 𝐵]  
[1, 𝐶]  
⋮}  
(all the combinations)

Probability distribution  
A probability distribution is a description of the likelihood of all possible  
states of a random variable.  
E.g. 𝑃(𝑥 = 𝑚𝑜𝑛𝑑𝑎𝑦)

Probability distribution  
We can also describe a probability distribution as a function over a  
random variable and outputs a continuous value between 0 and 1.  
Example: Throwing a dice  
P(throw=3) = 1/6

Probability distribution  
A probability distribution over many variables is a  
“joint probability distribution”  
𝑃(𝑥 = 𝑚𝑜𝑛𝑑𝑎𝑦, 𝑦 = 𝑟𝑎𝑖𝑛)

Probability distribution  
A distribution where all the possibilities are equally likely is called a  
”uniform distribution”  
E.g. the probability of landing on any side of a fair dice

Probability density  
The probability of a continuous random variable is represented by a  
probability density function. The area under the function is equal to 1.

Conditional probability  
Conditional probability is the probability of some event y, given that  
some other event x has happened.