I -

- 1.a $\mathbb{N}^*$
- 1.b $\mathbb{P}(X \geq x) = (1-p)^{x-1}$
- 1.c $\mathbb{P}(T \geq t) = \mathbb{P}(X \geq t)\mathbb{P}(Y \geq t) = (1-p)^{2t-2}$
- 1.d $\mathbb{P}(T = t) = (1-p)^{2t-2}(2p - p^2)$

II -

- 2.a $\mathbb{N}$
- 2.b $\frac{p}{p-1}\ln p$
- 2.c

$$\begin{aligned}(T = t) &= (X = t, Y > t) \cup (X > t, Y = t) \cup (X = t, Y = t)\\ &= \bigcup_{i=t+1}^{\infty}(X = t, Y = i) \cup \bigcup_{j=t+1}^{\infty}(X = j, Y = t) \cup (X = t, Y = t)\end{aligned}$$

$$\begin{aligned}\mathbb{P}(T = t) &= \sum_{i=t+1}^{+\infty}\mathbb{P}(X = t, Y = i) + \sum_{j=t+1}^{+\infty}\mathbb{P}(X = j, Y = t) + \mathbb{P}(X = t, Y = t)\\ &= \sum_{i=t+1}^{+\infty}\mathbb{P}(X = t)\mathbb{P}(Y = i) + \sum_{j=t+1}^{+\infty}\mathbb{P}(X = j)\mathbb{P}(Y = t) + \mathbb{P}(X = t)\mathbb{P}(Y = t)\\ &= p(1-p)^{t-1}\sum_{i=t+1}^{+\infty}p(1-p)^{i-1} + p(1-p)^{t-1}\sum_{j=t+1}^{+\infty}p(1-p)^{j-1} + p(1-p)^{t-1} * p(1-p)^{t-1}\\ &= 2p(1-p)^{t-1}(1-p)^t + p^2(1-p)^{2t-2}\\ &= p(2-p)(1-p)^{2t-2}\end{aligned}$$

$$\begin{aligned}(T = t, Z = 0) &= (\min(X, Y) = t, X = Y)\\ \mathbb{P}(T = t, Z = 0) &= \mathbb{P}(X = t, Y = t) = \mathbb{P}(X = t)\mathbb{P}(Y = t)\\ &= p(1-p)^{t-1} * p(1-p)^{t-1} = p(2-p)(1-p)^{2t-2}\\ &= \left(p(2-p)(1-p)^{2t-2}\right)\left(\frac{p}{2-p}\right)\\ &= \mathbb{P}(T = t)\mathbb{P}(Z = 0)\end{aligned}$$

$$\begin{aligned}
(T=t, Z=z) &= (\min(X,Y)=t, |X-Y|=z) = (X=t, Y=z+t) \cup (X=z+t, Y=t) \\
\mathbb{P}(T=t, Z=z) &= \mathbb{P}(X=t, Y=t+z) + \mathbb{P}(X=t+z, Y=t) \\
&= \mathbb{P}(X=t)\mathbb{P}(Y=t+z) + \mathbb{P}(X=t+z)\mathbb{P}(Y=t) \\
&= 2p(1-p)^{t-1} * p(1-p)^{t+z-1} \\
&= \left(p(2-p)(1-p)^{2t-2}\right)\left((1-p)^z \frac{2p}{2-p}\right) \\
&= \mathbb{P}(T=t)\mathbb{P}(Z=z)
\end{aligned}$$

$1-p \in]0,1[$