OBJECT ORIENTED ANALYSIS & DESIGN
DATA STRUCTURES & ALGORITHMS

Hashing - basic & advanced

Introduction of hash table

- Data structure that offers very fast insertion and searching, almost O(1).
- Relatively easy to program as compared to trees.
- Based on arrays, hence difficult to expand.
- No convenient way to visit the items in a hash table in any kind of order.

Hashing

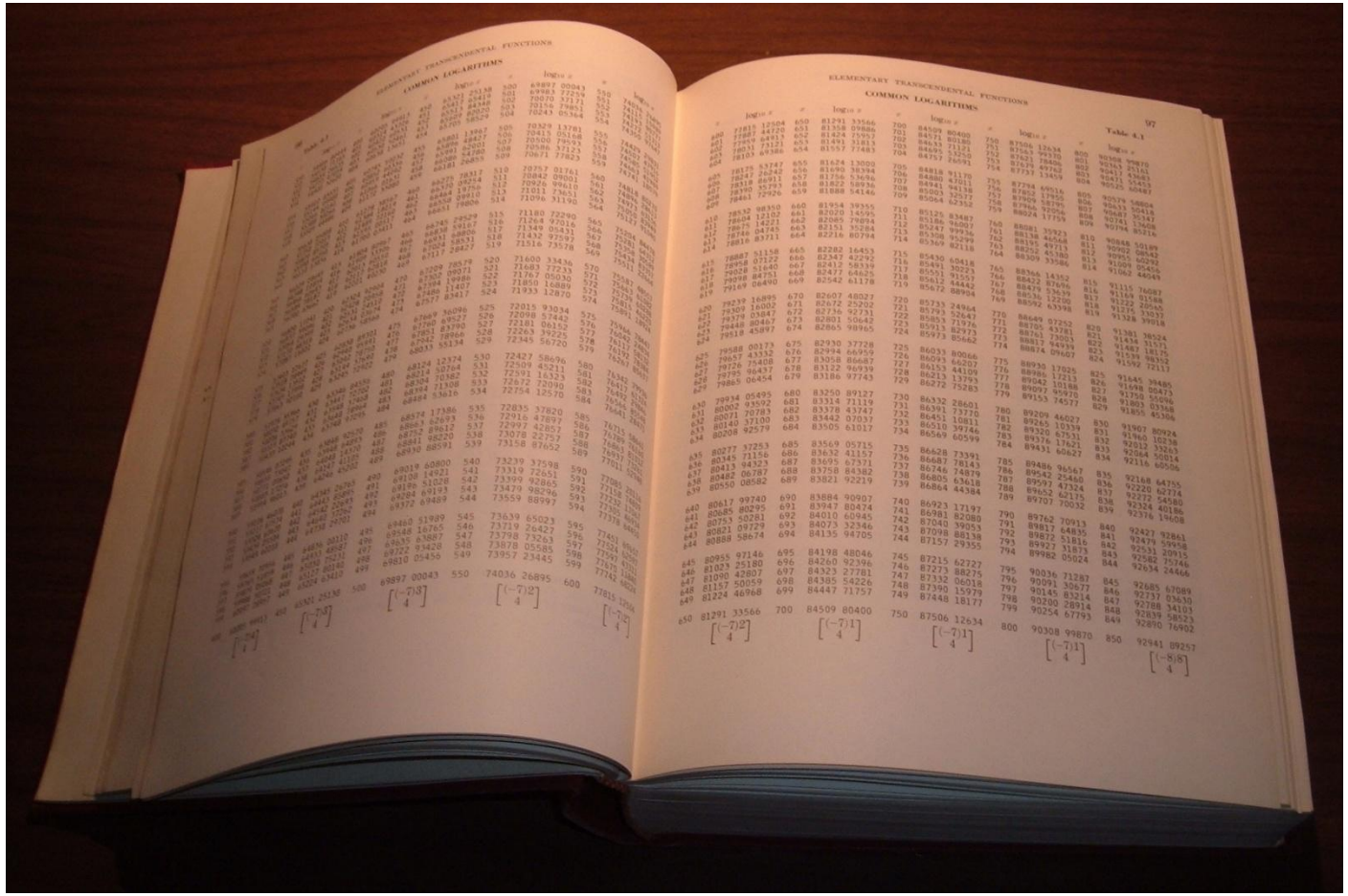
- A range of key values can be transformed into a range of array index values.
- A simple array can be used where each record occupies one cell of the array and the index number of the cell is the key value for that record.
- But keys may not be well arranged.
- In such a situation hash tables can be used.

Concept of Hashing

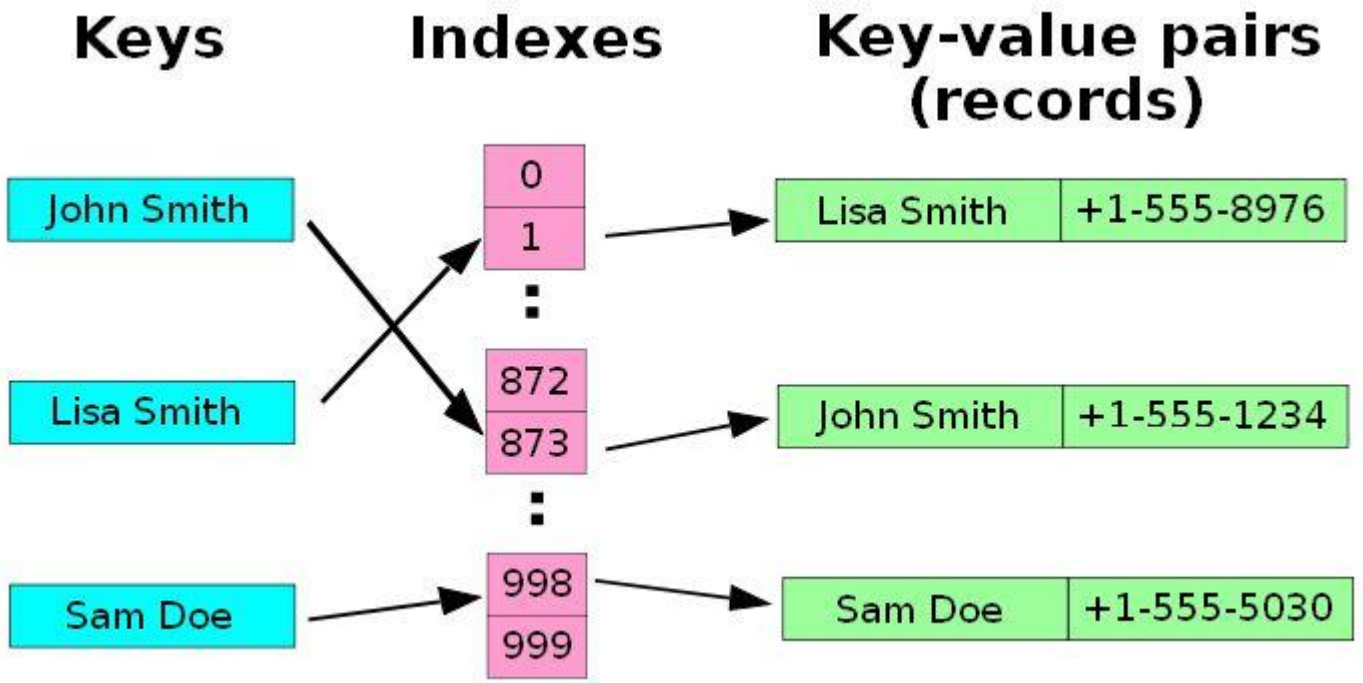
In CS, a **hash table**, or a **hash map**, is a data structure that associates keys (names) with values (attributes).

- ✓ Look-Up Table
- ✓ Dictionary
- ✓ Cache
- ✓ Extended Array

Tables of logarithms



Example



A small phone book as a hash table.

(Figure is from Wikipedia)

Dictionaries

Collection of pairs.

(key, value)

Each pair has a unique key.

Operations.

Get(theKey)

Delete(theKey)

Insert(theKey, theValue)

Just An Idea

Hash table :

- Collection of pairs,
- Lookup function (Hash function)

Hash tables are often used to implement associative arrays,

- Worst-case time for Get, Insert, and Delete is O(size).
- Expected time is O(1).

Origins of the Term

The term "hash" comes by way of analogy with its standard meaning in the physical world, to "chop and mix." **D. Knuth** notes that **Hans Peter Luhn** of IBM appears to have been the first to use the concept, in a memo dated January 1953; the term hash came into use some ten years later.

Search vs. Hashing

Search tree methods: key comparisons

- Time complexity: O(size) or O(log n)

Hashing methods: hash functions

- Expected time: O(1)

Types

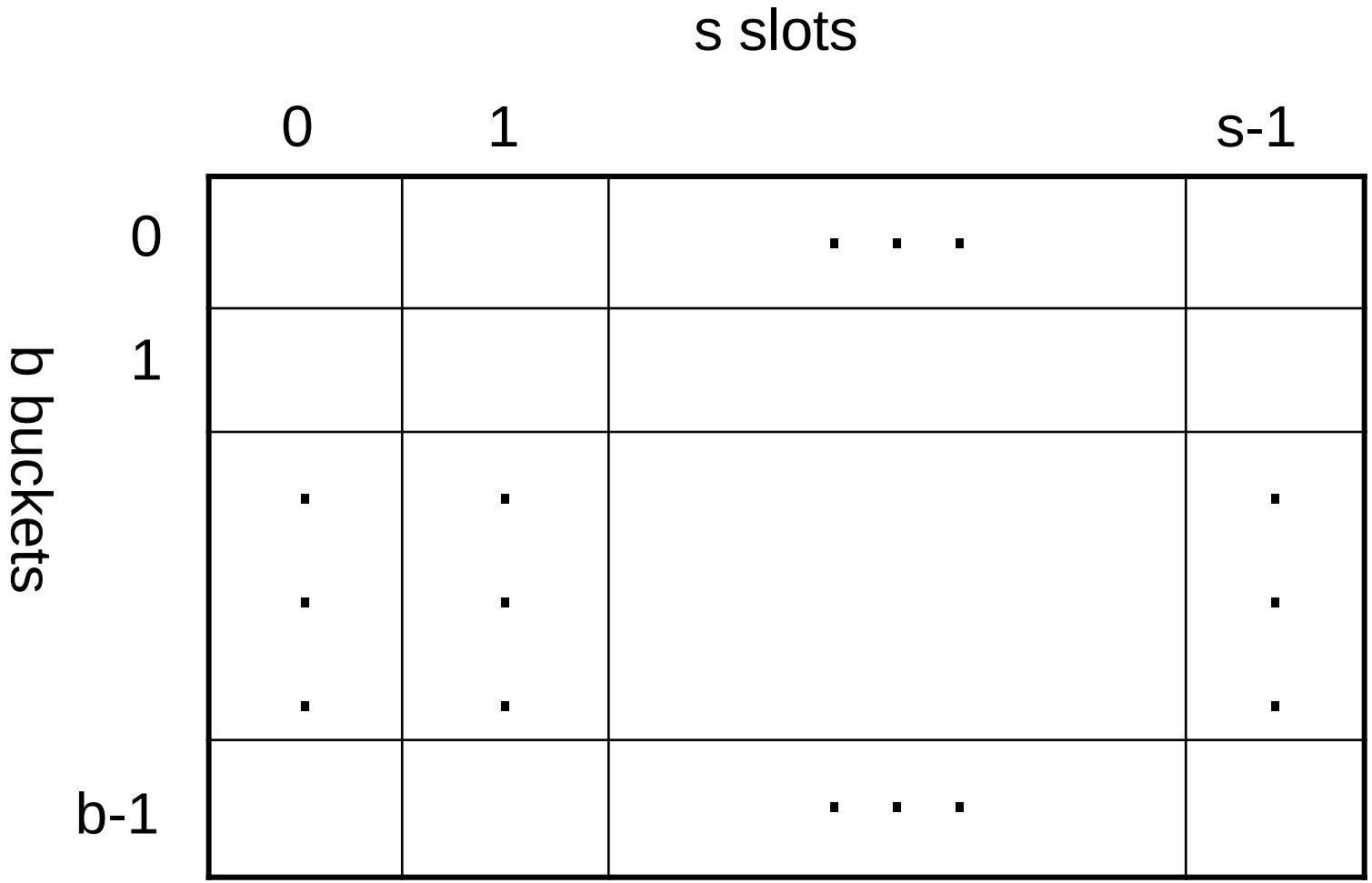
- Static hashing (section 8.2)
- Dynamic hashing (section 8.3)

Static Hashing

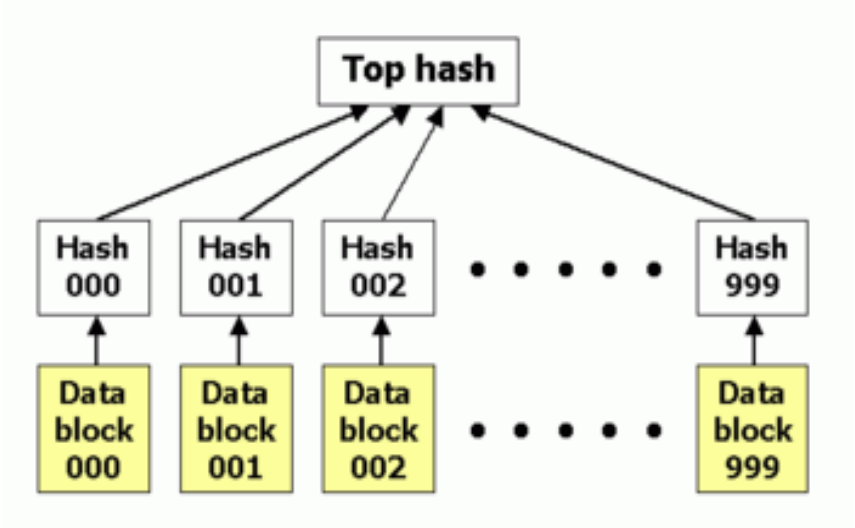
Key-value pairs are stored in a fixed size table called a *hash table*.

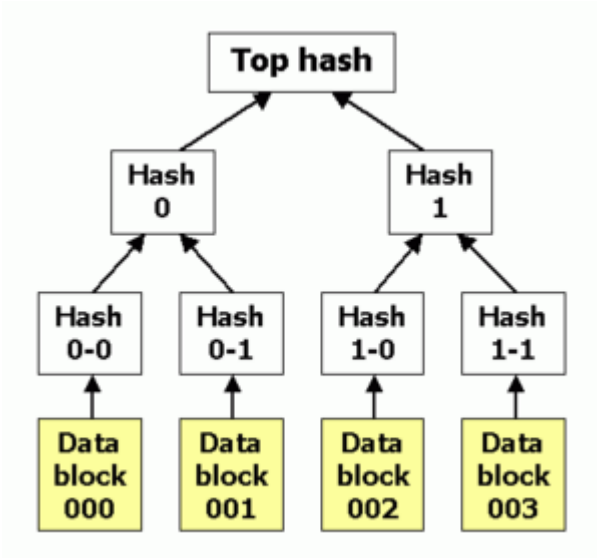
- A hash table is partitioned into many ***buckets***.
- Each bucket has many ***slots***.
- Each slot holds one record.
- A hash function f(x) transforms the identifier (key) into an address in the hash table

Hash table



Other Extensions





Hash List and Hash Tree

(Figure is from Wikipedia)

Ideal Hashing

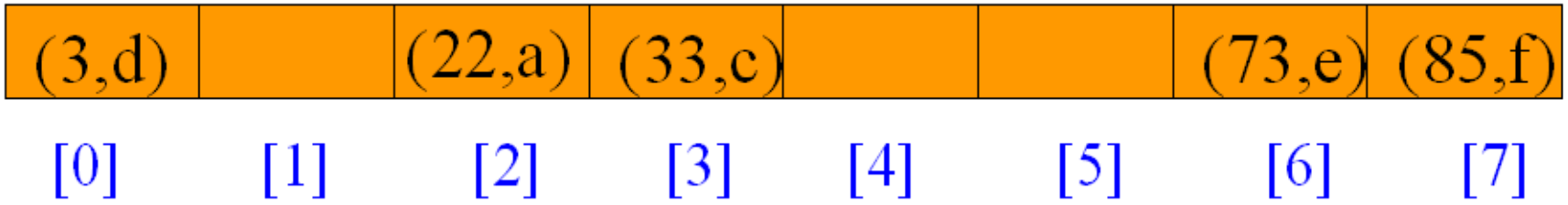
- Uses an array table[0:b-1].
  - ✓ Each position of this array is a bucket.
  - ✓ A bucket can normally hold only one dictionary pair.
- Uses a hash function f that converts each key k into an index in the range [0, b-1].
- Every dictionary pair (key, element) is stored in its home bucket table[f[key]].

Example

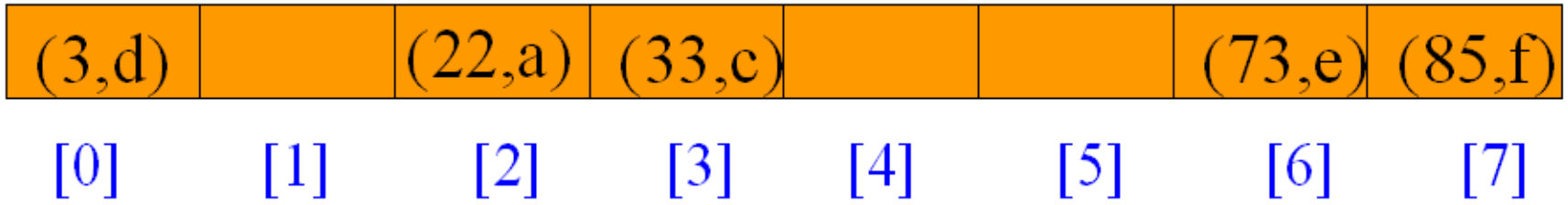
Pairs are: (22,a), (33,c), (3,d), (73,e), (85,f).

Hash table is table[0:7], b = 8.

Hash function is key (mod 11).



What Can Go Wrong?



Where does (26,g) go?

Keys that have the same home bucket are synonyms.

22 and 26 are synonyms with respect to the hash function that is in use.

The bucket for (26,g) is already occupied.

Some Issues

Choice of hash function.

Really tricky!

To avoid collision (two different pairs are in the same the same bucket.)

Size (number of buckets) of hash table.

Overflow handling method.

Overflow: there is no space in the bucket for the new pair.

Example (fig 8.1)

synonyms:
char, ceil,
clock, ctime

↑

overflow

	Slot 0	Slot 1
0	acos	atan
1		
2	char	ceil
3	define	
4	exp	
5	float	floor
6		
…		
25		

synonyms

synonyms

Choice of Hash Function

- Requirements
  - ✓ easy to compute
  - ✓ minimal number of collisions

- If a hashing function groups key values together, this is called clustering of the keys.

- A good hashing function distributes the key values uniformly throughout the range.

Some hash functions

- Middle of square

  H(x):= return middle digits of x^2

- Division

  H(x):= return x % k

- **Multiplicative:**

  H(x):= return the first few digits of the fractional part of x*k, where k is a fraction.

  advocated by D. Knuth in TAOCP vol. III.

Some hash functions II

Folding:

Partition the identifier x into several parts, and add the parts together to obtain the hash address

e.g. x=12320324111220; partition x into 123,203,241,112,20; then return the address 123+203+241+112+20=699

Shift folding vs. folding at the boundaries

Digit analysis:

If all the keys have been known in advance, then we could delete the digits of keys having the most skewed distributions, and use the rest digits as hash address.

Hashing By Division

- Domain is all integers.
- For a hash table of size *b*, the number of integers that get hashed into bucket *i* is approximately $2^{32}/b$.
- The division method results in a uniform hash function that maps approximately the same number of keys into each bucket.

Hashing By Division II

In practice, keys tend to be correlated.

If divisor is an even number, odd integers hash into odd home buckets and even integers into even home buckets.

20%14 = 6, 30%14 = 2, 8%14 = 8

15%14 = 1, 3%14 = 3, 23%14 = 9

divisor is an odd number, odd (even) integers may hash into any home.

20%15 = 5, 30%15 = 0, 8%15 = 8

15%15 = 0, 3%15 = 3, 23%15 = 8

Hashing By Division III

- Similar biased distribution of home buckets is seen in practice, when the divisor is a multiple of prime numbers such as 3, 5, 7, …
- The effect of each prime divisor p of b decreases as p gets larger.
- Ideally, choose large prime number b.
- Alternatively, choose b so that it has no prime factors smaller than 20.

Criterion of Hash Table

- The key density (or identifier density) of a hash table is the ratio n/T
  n is the number of keys in the table
  T is the number of distinct possible keys

- The loading density or loading factor of a hash table is □ = n/(sb)
  s is the number of slots
  b is the number of buckets

Example

synonyms:
char, ceil,
clock, ctime

↑

overflow

	Slot 0	Slot 1
0	acos	atan
1		
2	char	ceil
3	define	
4	exp	
5	float	floor
6		
…		
25		

synonyms

synonyms

b=26, s=2, n=10, □=10/52=0.19, f(x)=the first char of x

Overflow Handling

An overflow occurs when the home bucket for a new pair (key, element) is full.

We may handle overflows by:

- Search the hash table in some systematic fashion for a bucket that is not full.
  - Linear probing (linear open addressing).
  - Quadratic probing.
  - Random probing.

- Eliminate overflows by permitting each bucket to keep a list of all pairs for which it is the home bucket.
  - Array linear list.
  - Chain.

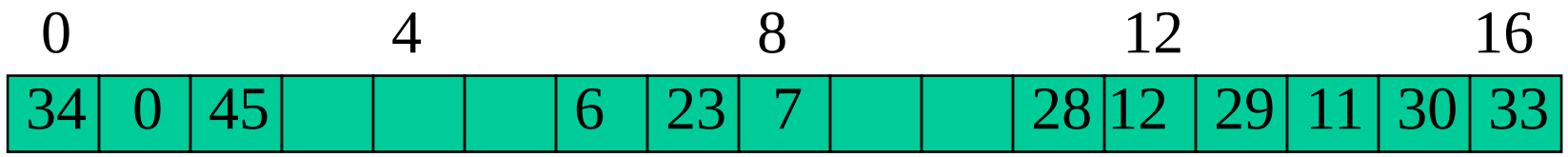
Linear probing (linear open addressing)

Open addressing ensures that all elements are stored directly into the hash table, thus it attempts to resolve collisions using various methods.

Linear Probing resolves collisions by placing the data into the next open slot in the table.

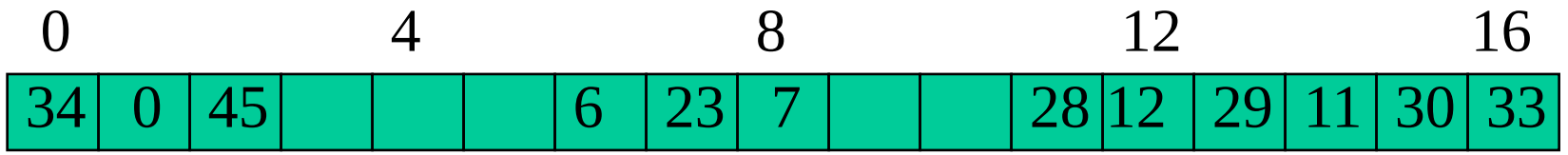
Linear Probing – Get And Insert

divisor = b (number of buckets) = 17.
Home bucket = key % 17.

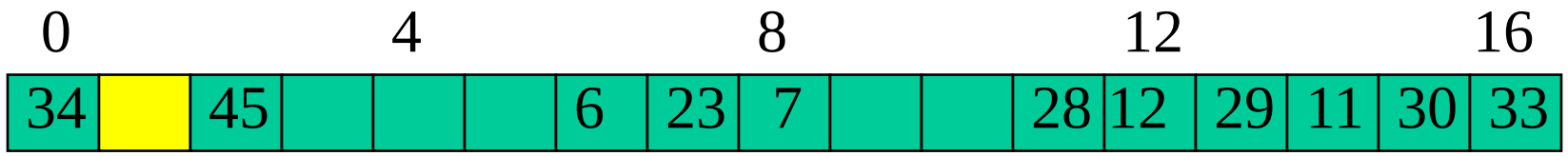


- Insert pairs whose keys are 6, 12, 34, 29, 28, 11, 23, 7, 0, 33, 30, 45

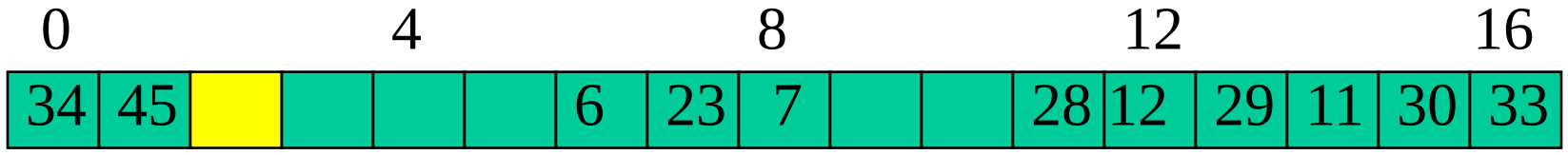
Linear Probing – Delete



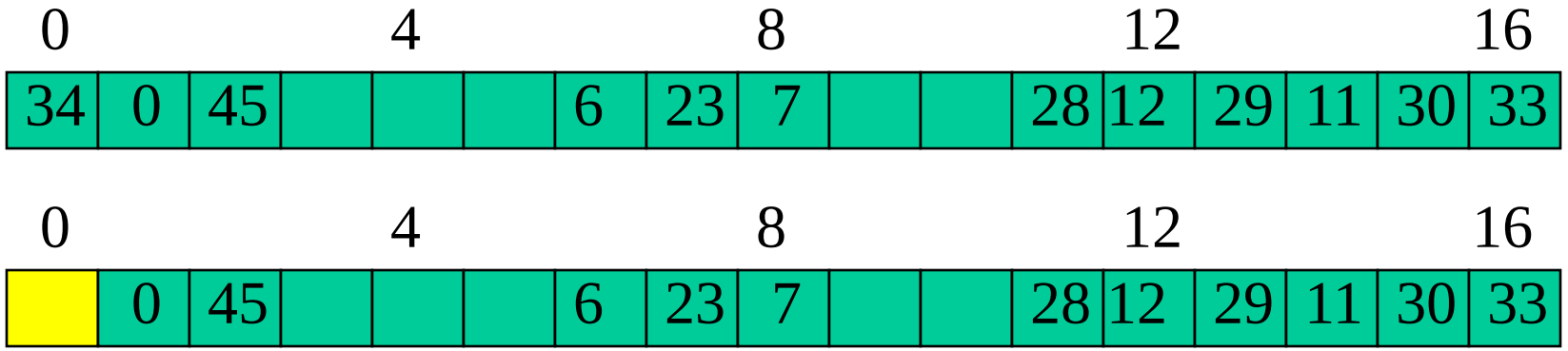
Delete(0)



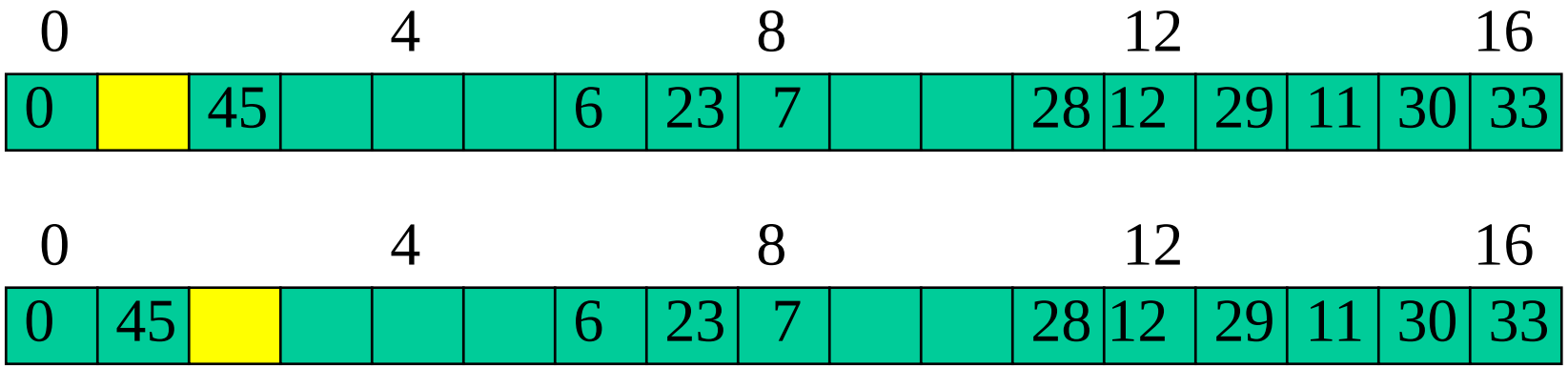
- Search cluster for pair (if any) to fill vacated bucket.



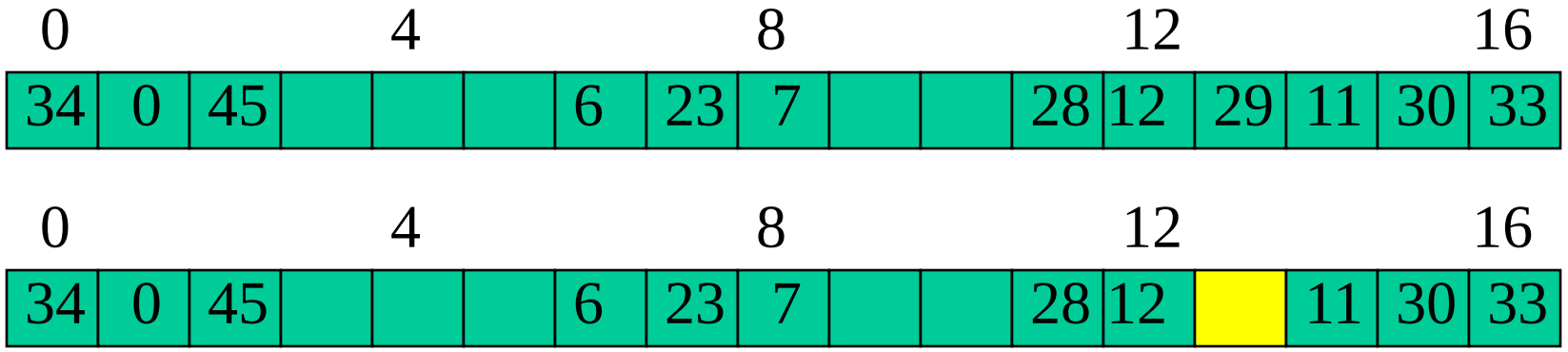
Linear Probing – Delete(34)



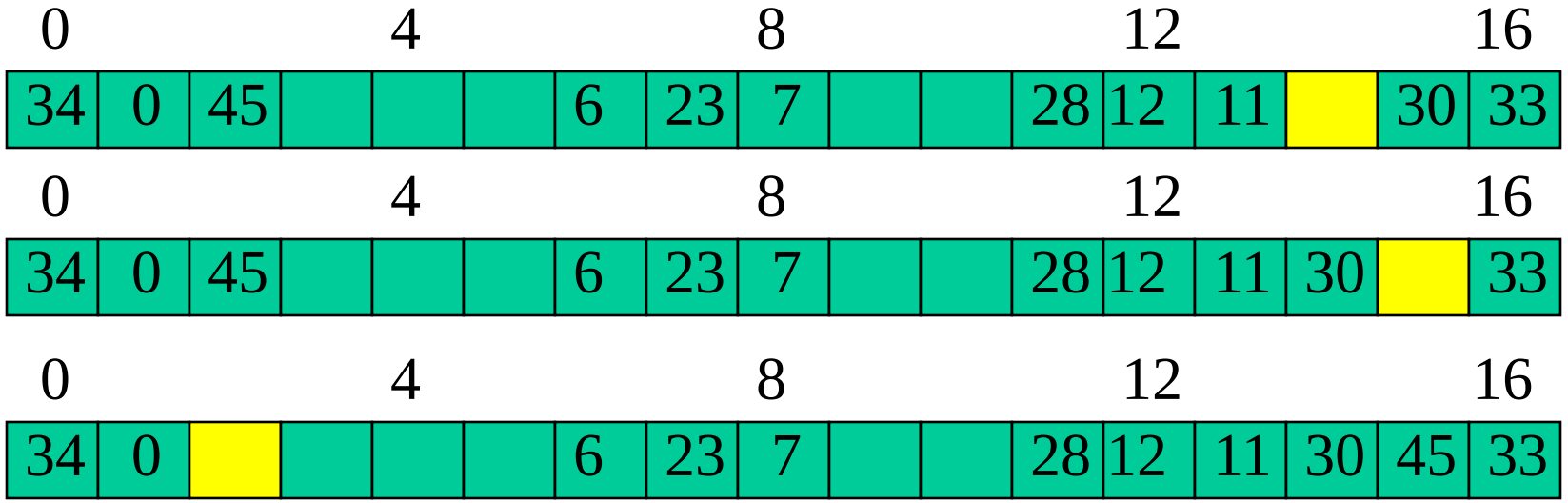
Search cluster for pair (if any) to fill vacated bucket.



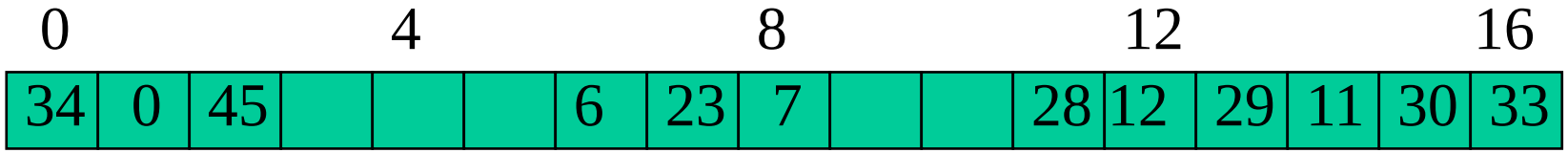
Linear Probing – Delete(29)



Search cluster for pair (if any) to fill vacated bucket.

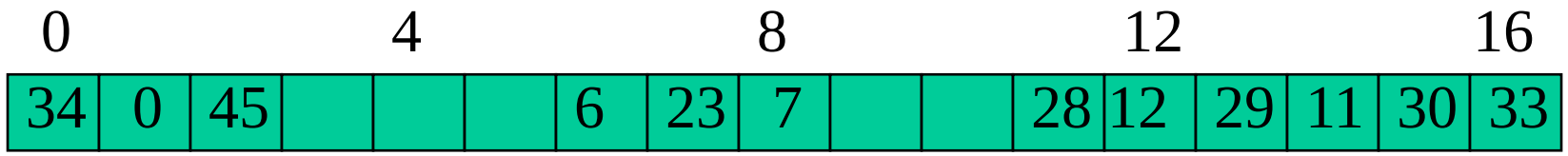


Performance Of Linear Probing



Worst-case find/insert/erase time is □(n), where n is the number of pairs in the table. This happens when all pairs are in the same cluster.

Expected Performance



α = loading density = (number of pairs)/b.

α = 12/17.

S_n = expected number of buckets examined in a successful search when n is large

U_n = expected number of buckets examined in a unsuccessful search when n is large

Time to put and remove is governed by U_n.

Problem of Linear Probing

- Identifiers tend to cluster together
- Adjacent cluster tend to coalesce
- Increase the search time

Quadratic Probing

- Linear probing searches buckets (H(x)+i2)%b
- Quadratic probing uses a quadratic function of i as the increment
- Examine buckets H(x), (H(x)+i2)%b, (H(x)-i2)%b, for 1<=i<=(b-1)/2
- b is a prime number of the form 4j+3, j is an integer

Random Probing

- Random Probing works incorporating with random numbers.

  H(x):= (H’(x) + S[i]) % b

  S[i] is a table with size b-1

  S[i] is a random permuation of integers [1,b-1].

Rehashing

Rehashing: Try H_1, H_2, …, H_m in sequence if collision occurs. Here H_i is a hash function.

Double hashing is one of the best methods for dealing with collisions.

If the slot is full, then a second hash function is calculated and combined with the first hash function.

$H(k, i) = (H_1(k) + i\ H_2(k)\ )\ \%\ m$

Summary:
Hash Table Design

Performance requirements are given, determine maximum permissible loading density. Hash functions must usually be custom-designed for the kind of keys used for accessing the hash table.

We want a successful search to make no more than 10 comparisons (expected).

$S_n \sim \frac{1}{2}(1 + 1/(1 - \square))$

$\square <= 18/19$

Summary:
Hash Table Design II

We want an unsuccessful search to make no more than 13 comparisons (expected).

Un ~ ½(1 + 1/(1 – □)2)

□ <= 4/5

So □ <= min{18/19, 4/5} = 4/5.

Summary:
Hash Table Design III

Dynamic resizing of table.

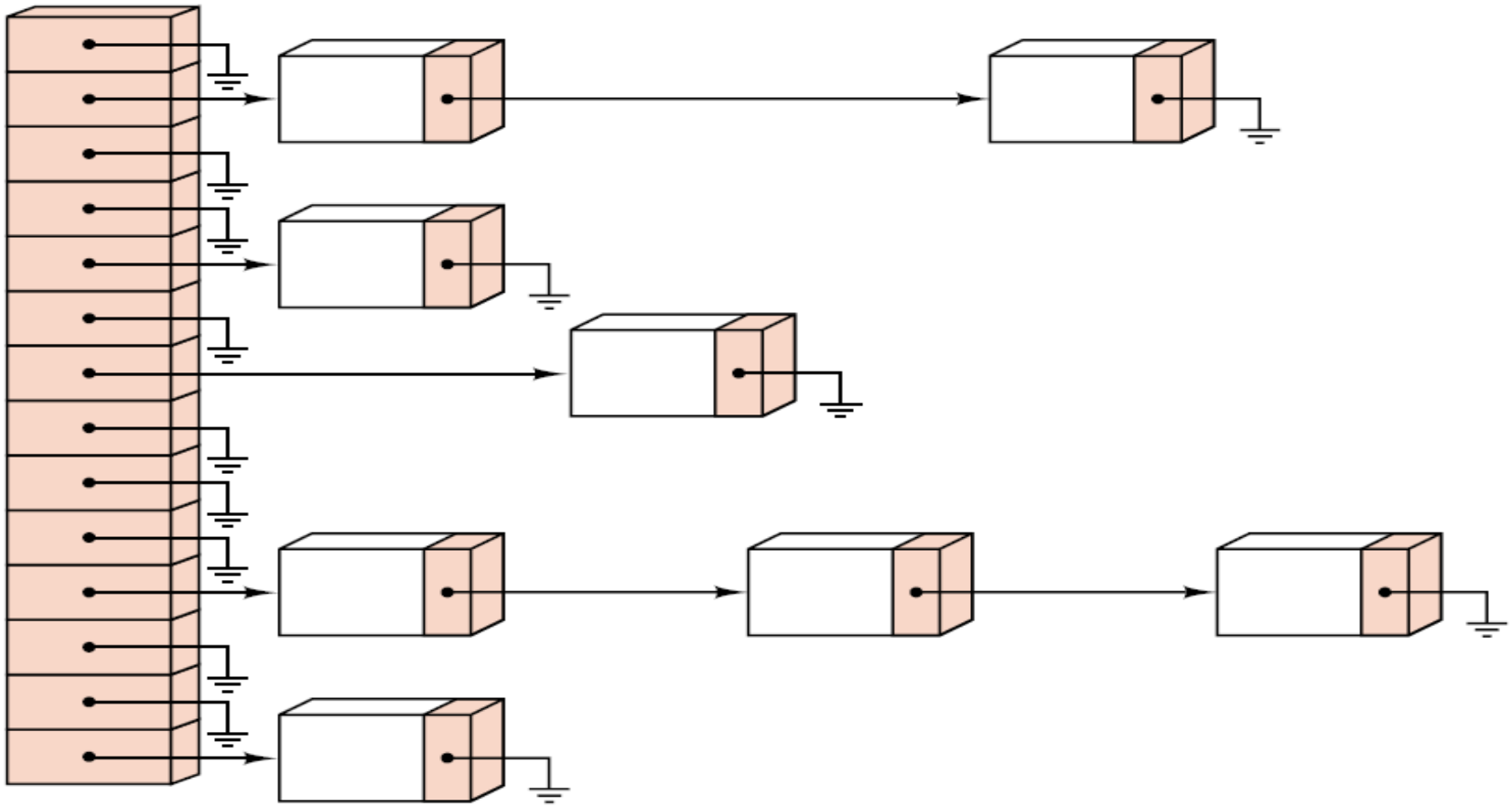
Whenever loading density exceeds threshold (4/5 in our example), rehash into a table of approximately twice the current size.'

Fixed table size.

Loading density <= 4/5 => b >= 5/4*1000 = 1250.

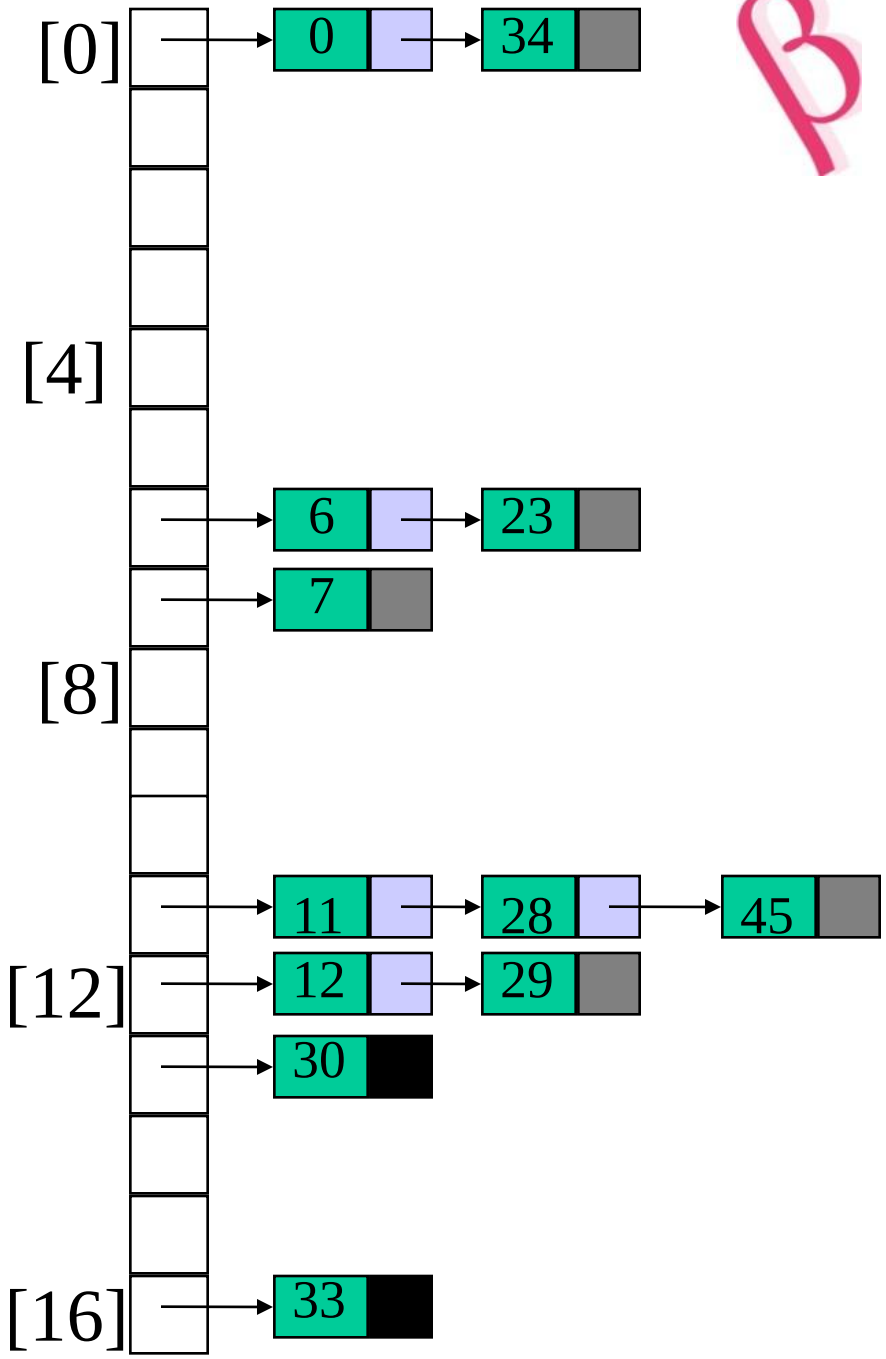
Pick b (equal to divisor) to be a prime number or an odd number with no prime divisors smaller than 20.

Figure of Chaining



Sorted Chains

- Put in pairs whose keys are 6, 12, 34, 29, 28, 11, 23, 7, 0, 33, 30, 45
- Bucket = key % 17.



Expected Performance

Note that □ >= 0.

Expected chain length is □.

S_n ~ 1 + □/2.

U_n ~ □

Refer to the theorem 8.1 of textbook, and refer to D. Knuth's TAOCP vol. III for the proof.

Comparison : Load Factor

If **open addressing** is used, then each table slot holds at most one element, therefore, the loading factor can **never** be greater than 1.

If **external chaining** is used, then each table slot can hold many elements, therefore, the loading factor **may be** greater than 1.

Conclusion

The main **tradeoffs** between these methods are that **linear probing** has the best cache performance but is most sensitive to clustering, while **double hashing** has poorer cache performance but exhibits virtually no clustering; **quadratic probing** falls in between the previous two methods.

Thank You!