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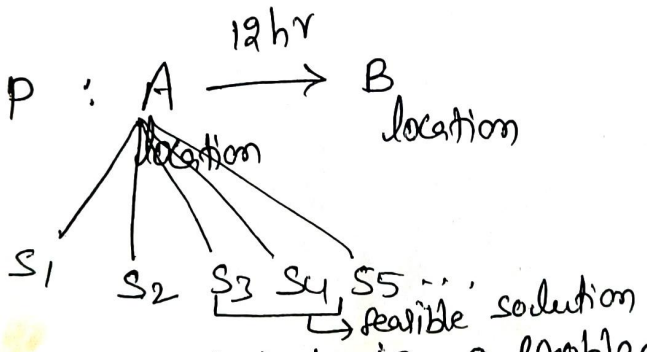
UNIT-IV

Greedy method Introduction :-

General Method :-

This method is used for solving optimization problems. the optimization problem means, the problem which requires either minimum result or maximum result.

definition :- Any subset that satisfies these constraints is called a feasible solutions. we need to find a feasible solution that either maximizes or minimizes a given objective function. A feasible solution that does this is called an ~~optical~~ optimal solution.



There is a constraint in a problem is 12 hr.

→ A solution which is satisfying the condition given in the problem is "feasible solution"

→ If the result should be minimum then it is called minimization problem.

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→ A solution which is already feasible, & also giving me minimum cost i.e best cost, then that solution is called optimal solution.

→ optimization ~~solution~~ problem means it requires min or maximum ~~solution~~ result.

1. Greedy method
2. Dynamic Programming
3. Branch & Bound.

These are the stratagies are used for solving optimization problems.

Greedy method :- It says that a problem should be solved in stages, in each stage we will consider one input from a given problem, and if that input is feasible then we will included in the solution so by including all those feasible inputs, then we will get optimal solutions

Algorithm :- the General method

$n = 5$

a	a_1	a_2	a_3	a_4	a_5
	1	2	3	4	5

```
Algorithm Greedy(a, n)
    // a[1:n] contains the n inputs
{
    solution = 0; // initialize the solution
```

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```
for (i=1 to n do
{
    x = select(a);
  If feasible(x, solution, n), then
  {solution = solution + x; // Solution = union (Solution, n);
  }}
} return solution;
```

eg:- To Buy a car, you have to choose the best car by using Greedy method.

Job sequencing with deadlines using greedy method:-

n=5

Jobs	J_1	J_2	J_3	J_4	J_5
Profit	20	15	10	5	1
deadline	2	2	1	3	3

Problem statement:- In Job sequencing problem, the objective is to find a sequence of jobs, which is completed within their deadlines and gives maximum profit.

Solution:- Let us consider, a set of n given Jobs which are associated with deadlines and profit is earned. If a Job is completed by its deadline. These Jobs need to be ordered in such a way that there is maximum profit.

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→ Initially, these jobs are ordered according to Profit.

i.e $P_1 \geq P_2 \geq P_3 > \cdots \geq P_n$.

eg:-

Problem:-

job	J1	J2	j3	J4	J5
deadline	2	1	3	2	1
Profit	60	100	20	40	20

Solution:- To solve this Problem, the given jobs are sorted according to their Profit in a descending order. Hence after sorting, the jobs are ordered as shown the following table

Job	j2	J1	J4	J3	J5
deadline	1	2	2	3	1
Profit	100	60	40	20	20

∴ Here the maximum deadline is 3.

→ and each job is completed within one hour only, but it can wait up to the deadline time.

$$0 \xrightarrow[J2]{100} 1 \xrightarrow[j1]{60} 2 \xrightarrow[J3]{20} 3$$

→ From these set of jobs, first we select J2, as it can be completed within its deadline & contributes maximum Profit

→ Next, J1 is selected as it gives more Profit compared to J4.

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→ Next, J4 cannot be selected as its deadline is over, hence J5 is selected as it executes within its deadline.

→ Job J3 is discarded as it cannot be executed with in its deadline.

→ The solution is the sequence of jobs (J2, J1, J3) which are being executed within their deadline and gives maximum profit

The profit of this sequence is 100+60+20 = 180

Job Consider	Slot asign	Solution	Profit
0	–	0	0
J1	[1,2]	J1	60
J2	[0,1] [1,2]	J1, J2	100+60=160
J3	[0,1] [1,2] [2,3]	J1, J2, J3	100+60+20 = 180
J4	[0,1] [1,2] [2,3]	J1, J2, J3	180
J5	[0,1] [1,2] [2,3]	J1, J2, J3	180

∴ Here, Job 4 contains deadline 2, then it already over, it not executed.

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$n = 7$

eg:-

Jobs	J1	J2	J3	J4	J5	J6	J7
Profit	35	30	25	20	15	12	5
deadline	3	4	4	2	3	1	2

Solution:- Here, the maximum deadline is 4.,

$$0 \xrightarrow[20]{J4} 1 \xrightarrow[25]{J3} 2 \xrightarrow[35]{J1} 3 \xrightarrow[30]{J2} 4$$

→ In this each device is completed within one hour but it can wait until the deadline time is completed

→ $j_5, J6, J7$ jobs are discarded because all the slots are completed.

→ Final Profit is $\{J_1, J_2, J_3, J_4\}$

$$\Rightarrow 35+30+25+20 = \underline{110}$$

$\boxed{\text{In this, the maximum profit is 110}}$

Knapsack problem using greedy method:-

$n = 7$ objects

$m = 15$

objects	0	1	2	3	4	5	6	7
Profit	P	10	5	15	7	6	18	3
weight	w	2	3	5	7	1	4	1

In this, $n = 7$ no. of objects $= 7$ & capacity of the bag is 15

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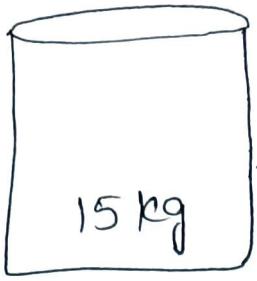

→ It is a bag or container

→ It is also called container loading problem (or) knapsack problem.

→ we are given n objects & a knapsack (or) bag. objects i has a weight w_i & the knapsack as capacity w. The object is obtain a filling of the knapsack that maximizes the total profit earned.

→ here the constraint is maximum capacity 15.

→ $0 \le x \le 1$, starting from 0 to 1. , It can take fractions also

$$x \left(\begin{matrix} 1 & 2/3 & 1 & 0 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \end{matrix} \right)$$

m=15

m = 15 − 1 = 14

14 − 2 = 12

12 − 4 = 8

8 − 5 = 3

3 − 1 = 2

2 − 2 = 0 ✗

m = 7

m = 15

Objects	1	2	3	4	5	6	7
Profit	10	5	15	7	6	18	3
weight	2	3	5	7	1	4	1
P/w	5	1.3	3	1	6	4.5	3

→ we should select the objects, who are having highest profits, per kg who are heighest we will take that.

we calculate $\frac{P}{w} \Rightarrow \frac{10}{2}\ \frac{5}{3}\ \frac{15}{5}\ \frac{7}{7}\ \frac{6}{1}\ \frac{18}{4}\ \frac{3}{1}$

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$$x\begin{pmatrix} 1 & 2/3 & 1 & 0 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 \end{pmatrix}$$

$\Rightarrow$ we will calculate $x \times$ weight

$= 1\times2 + \frac{2}{\cancel{3}}\times\cancel{3} + 1\times5 + 0\times7 + 1\times1 + 1\times4 + 1\times1$

$= 2+2+5+0+1+4+1 = 15$

$\Sigma x_i w_i = 15$

Profit $= 1\times10 + \frac{2}{3}\times5 + 1\times15 + 0\times7 + 1\times6 + 1\times18 + 1\times3$

$\Sigma x_i P_i = 10 + 2\times1.3 + 15 + 6 + 18 + 3$

$= 54.6$

$\Rightarrow$ constraint is $\boxed{\Sigma x_i w_i \leq m.}$

$\Rightarrow$ objective is $\boxed{\max \Sigma x_i P_i}$

Minimum cost spanning trees:-

Let $G = (V, E)$

here V is a set of vertices $V = \{1, 2, 3, 4, 5, 6\}$

E is a set Edges $E = \{(1,2)(2,3)(3,4)\ldots\}$

Definition:- A spanning tree is a subset of graph, (or) subgraph of a graph. having all vertices but only $n-1$ edges. It is called as spanning tree

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→

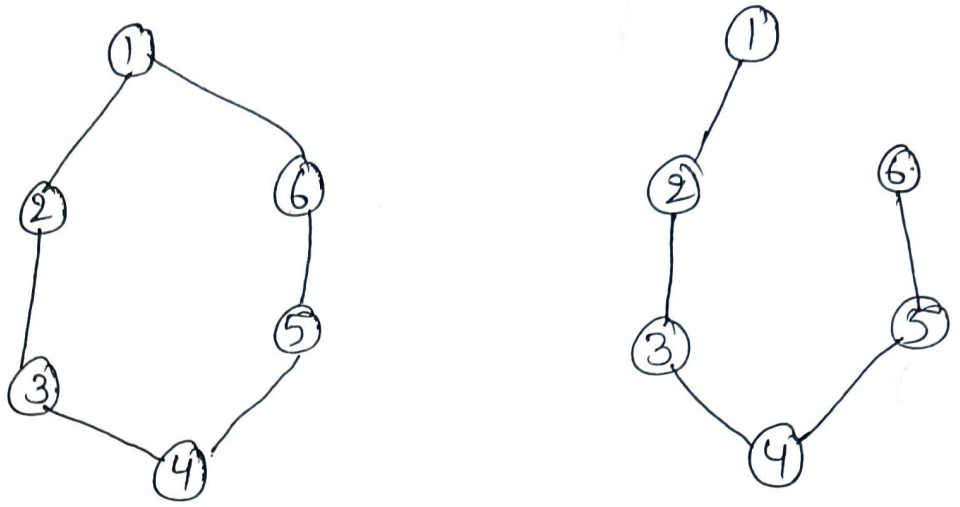


fig: A graph and it's minimum cost spanning tree

→ spanning will not having a cycle

$|V| = n = 6$ vertices (vertices, no of.)

$E = N-1 = 5$ edges (edges)

→ $S \subseteq G$ [∵ S is a subgraph of a graph]

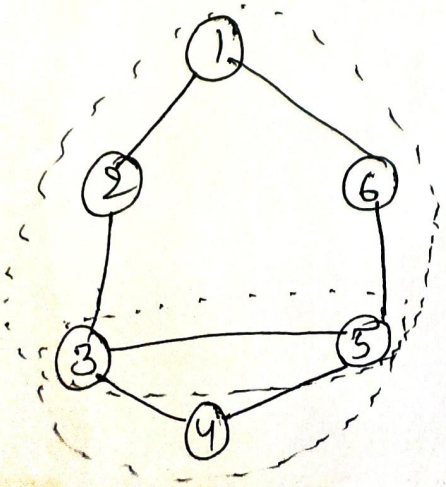
$S = (V, E')$

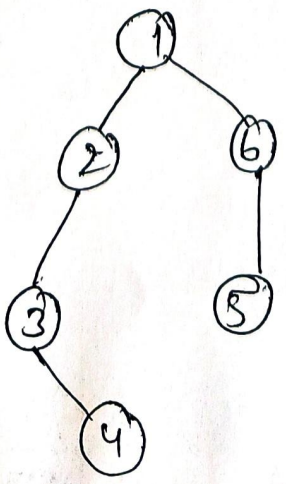
$V' = V'$

$|E| = |V| - 1$

→ $|E| = 6$ $\quad {}^6C_5 \Rightarrow 6$

I can draw 6 different spanning trees





∴ ${}^7C_5 = 21$

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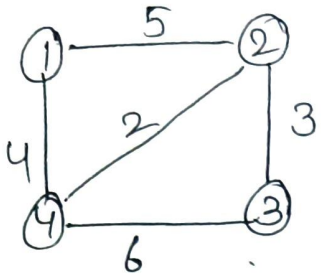
$|E| C_{|V|-1}$ — no. of cycles

eg:-

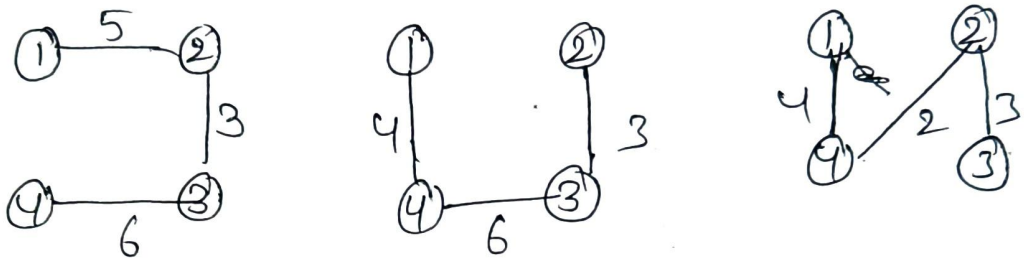


$V = \{1, 2, 3, 4\}$ $V = 4$

Edges $= V-1 = 3$.

fig: weighted graph.

solution:-



cost = 14 cost = 13 cost = 9

→ If you have a weighted graph, and you get different spanning trees, the cost of the spanning tree is may be varient. we can get different cost of spanning trees.

→ here, we find out Minimum cost spanning tree, by using, you can try all possible spanning trees and sele minimum cost spanning tree. but It is a too lengthy meth

→ By using greedy method we can draw spanning trees.

The methods are 1) prim's algorithm

2) kruskals algorithm

There are two algorithms for finding minimum cost spannin trees.

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eg:- Prim's algorithm :-

A Prim's algorithm is a greedy method to obtain a minimum cost spanning tree builds this tree edge by edge. The next edge to include is choosen according to the minimum cost, that vertices must be connected to already connected vertices.

eg:-

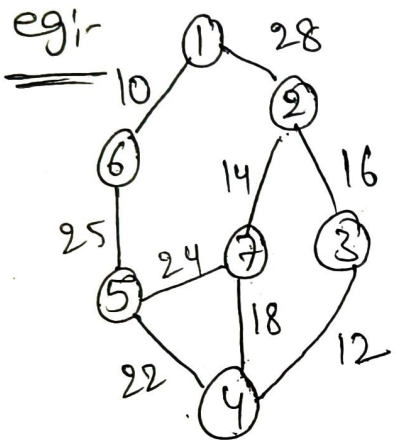
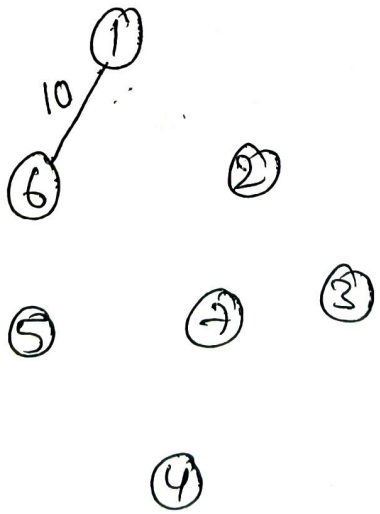
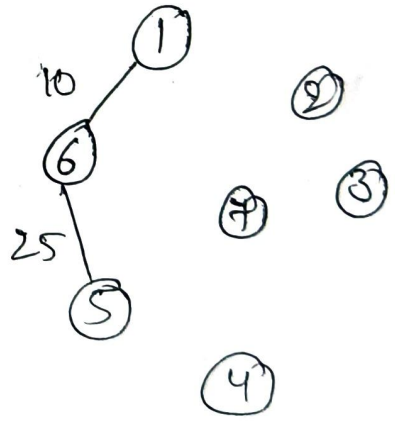


fig: weighted graph.

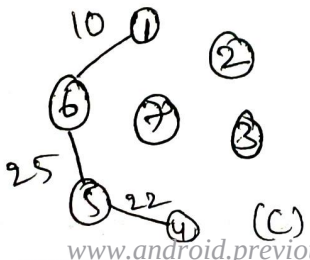
Solution :-



(a)



(b)



(c)

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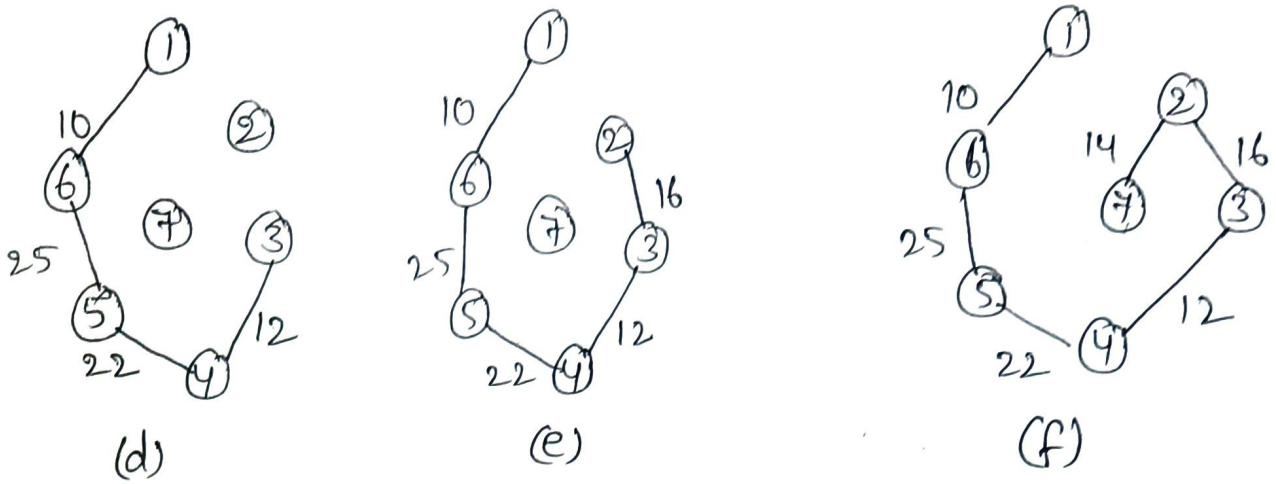

fig:- Stages in Prim's algorithms.

→ The algorithm will start with a tree that includes only a minimum-cost edge of G. Then, edges are added to this tree one by one.

→ The ~~next~~ next edge (i, j) to be added is such that i is a vertex already included in the tree, j is a vertex not yet included.

→ The cost of tree is cost = 99.

<u>Kruskal's algorithm</u>:- It is also follows greedy approach.

In this algorithm, always select minimum cost.

eg:-

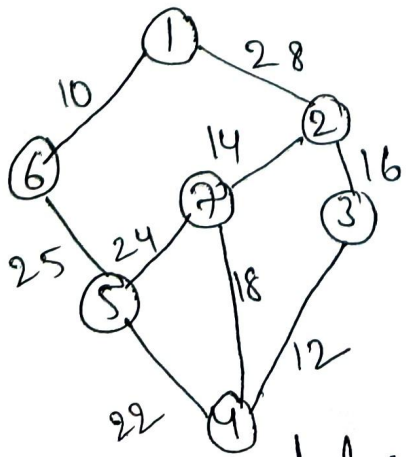

fig:- weighted graph

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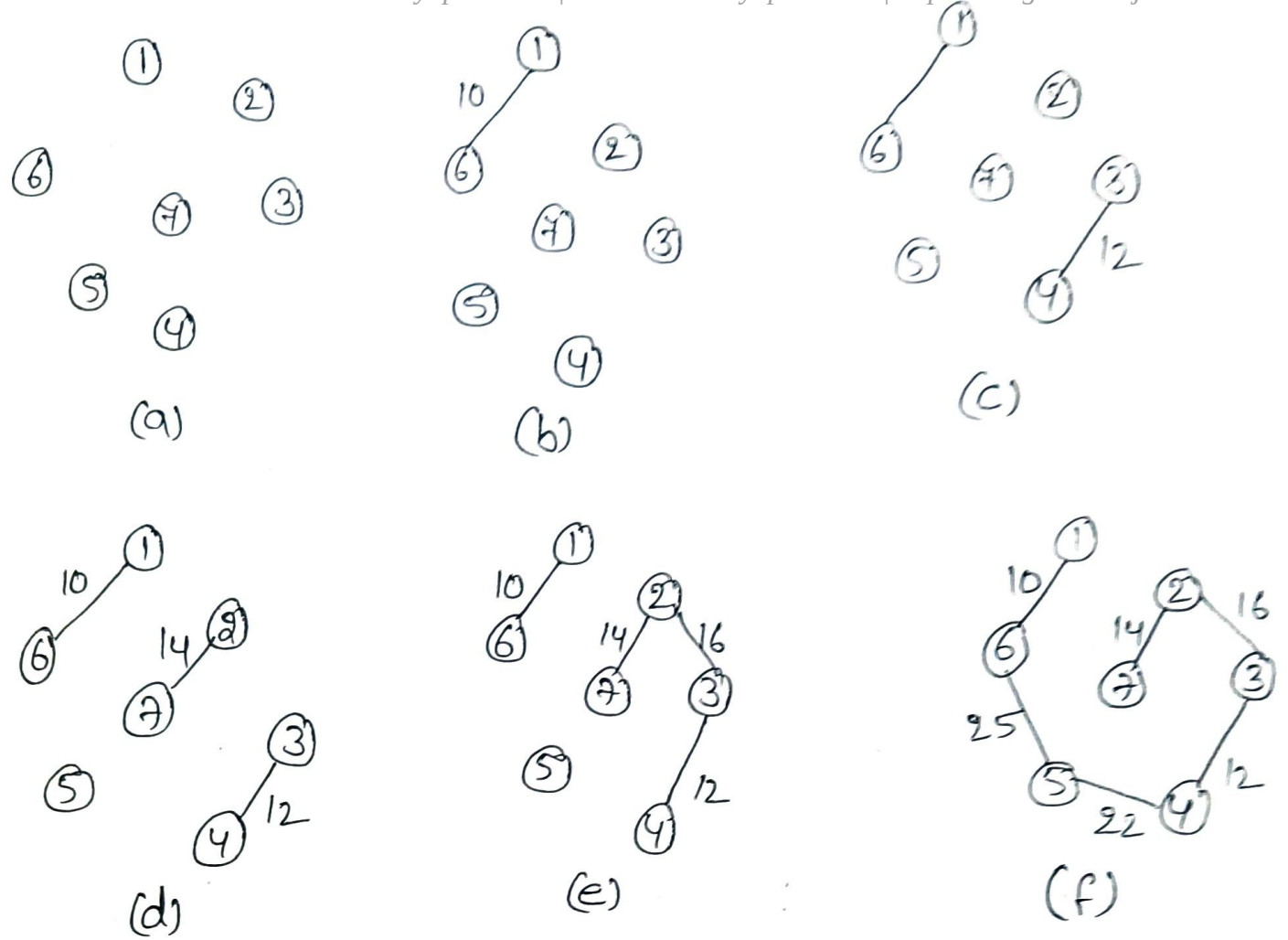


fig: stages in Kruskal's algorithm.

→ total cost is cost = 99.

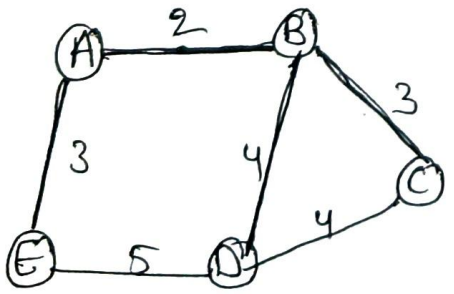
$|E|$

$|V| - 1$

$O(|V| \log |E|)$

Time taken for this algorithm $O(n, e) = O(n^2)$

eg:



Cost = 2+3+3+4 = 12, Cost = 2+3+3+4 = 12.

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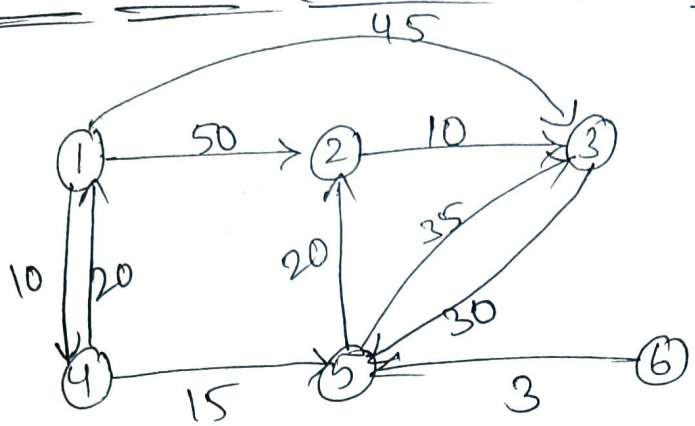
Single-source shortest path problem :-



(a) graph

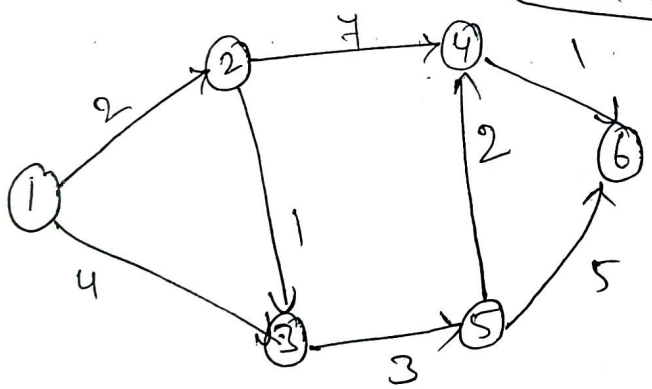
Path	Length
1) 1,4	10
2) 1,4,5	25
3) 1,4,5,2	45
4) 1,3	45

(b) Shortest paths from 1.

Fig: Graph & Shortest Paths from vertex 1 to all destinations.

Def:- Single source shortest path problem consist of finding the shortest path b/n a given vertex v & all other vertices in the graph. By using Dijkstra algorithm we can solve this problem

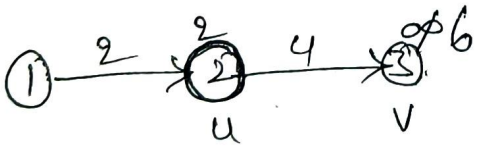
Dijkstra Algorithm :-

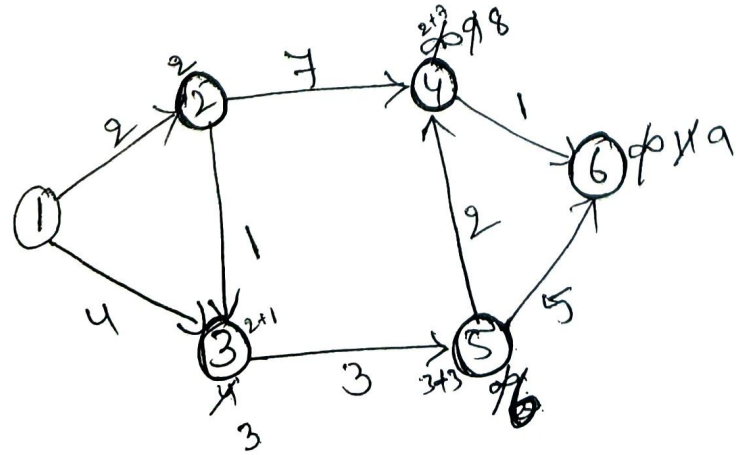


Relaxation

if $(d[u] + c[u,v] < d[v])$

$d[v] = d[u] + c(u,v)$





v	d[v]
2	2
3	3
4	8
5	6
6	9

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$n = |V|$ $|V|$ [$\because$ no. of vertices $= n$]

$n \times n = n^2$

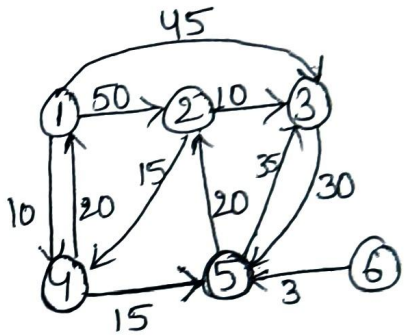
n ↓ no. of vertices, n ↳ Relaxed vertices

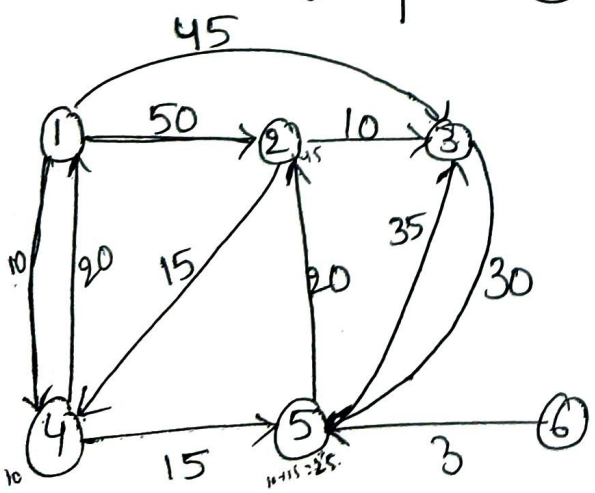
$\theta(|V|)^2$

$\boxed{\text{Time complexity} = \theta(n^2)}$

Eg:- Starting vertex 1

Selected vertex	2	3	4	5	6
4	50	45	10	∞	∞
5	50	45	10	25	∞
2	45	45	10	25	∞
3	45	45	10	25	∞
6	45	45	10	25	∞





(a) Graph

	Path	Length
1)	1, 4	10
2)	1, 4, 5	25
3)	1, 4, 5, 2	45
4)	1, 3, ~~4, 5, 2~~	45

(b) Shortest Paths from 1

Fig: Graph & Shortest paths from vertex 1 to all destinations.

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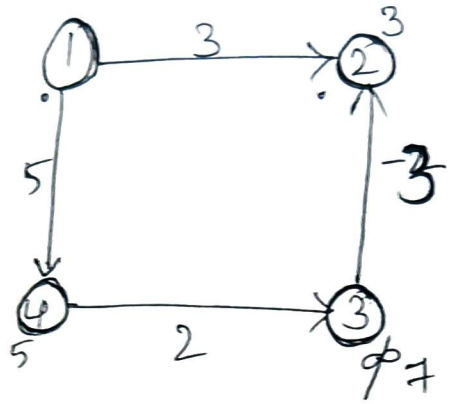
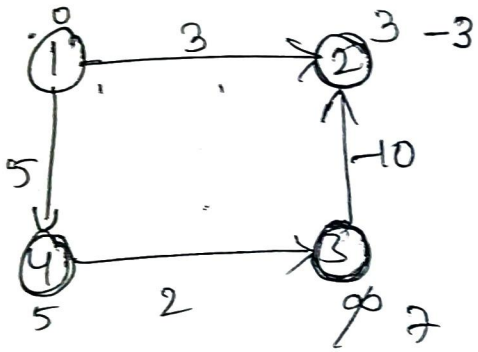
disadvantage of Dijkstra algorithm:-



→ Dijkstra algorithm works on negative weights also.



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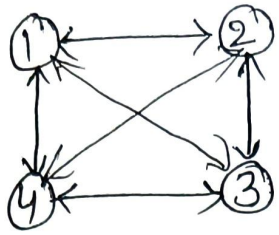
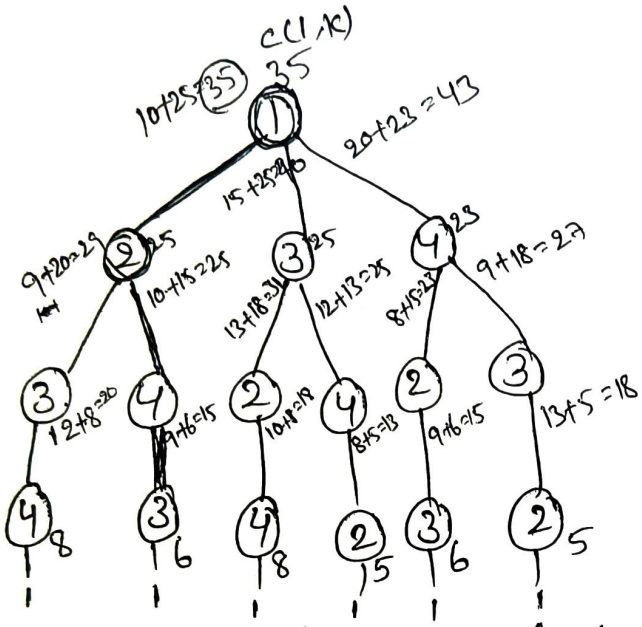
Traveling sales Person Problem :-

$$A = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 10 & 15 & 20 \\ 2 & 5 & 0 & 9 & 10 \\ 3 & 6 & 13 & 0 & 12 \\ 4 & 8 & 8 & 9 & 0 \end{array}$$





→ first, we have to select any vertex as the starting vertex, and find out the path, ie. going to all the vertices and returning back to the starting vertex it means covering all vertices.

→ The cost of shortest route is 35

$$g(1,\{2,3,4\}) = \min_{K \in 2,3,4} \{ c_{1K} + g(K, \{2,3,4\} - \{K\}) \}$$

$$\boxed{g(i,S) = \min_{K \in S} \{ c_{iK} + g(K, S - \{K\}) \}}$$

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$g(2,\phi) = c_{21} = 5$

$g(3,\phi) = 6$

$g(4,\phi) = 8$

$g(2,\{3\}) = 15$

$g(2,\{4\}) = 8$

$g(3,\{2\}) = 5$

$g(3,\{4\}) = 8$

$g(4,\{2\}) = 5$

$g(4,\{3\}) = 6$

$g(2,\{3,4\}) = 25$

$g(3,\{2,4\}) = 25$

$g(4,\{2,3\}) = 23$

$g(1,\{2,3,4\}) = \min\{c_{12} + g(2,\{3,4\}), c_{13} + g(3,\{2,4\}), c_{14} + g(4,\{2,3\})\}$

$10 + 25 = 35$

$15 + 25 = 40$

$20 + 23 = 43$

$\min\{35\}$

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