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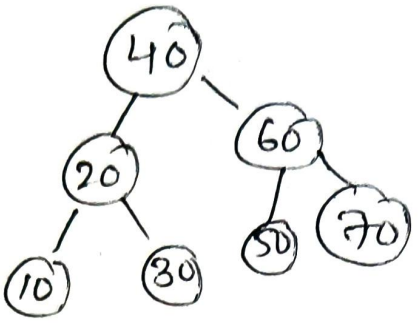
Optimal binary search tree :-

keys = 10, 20, 30, 40, 50, 60, 70



→ all the elements are it left side elements are smaller & right side elements are greater.

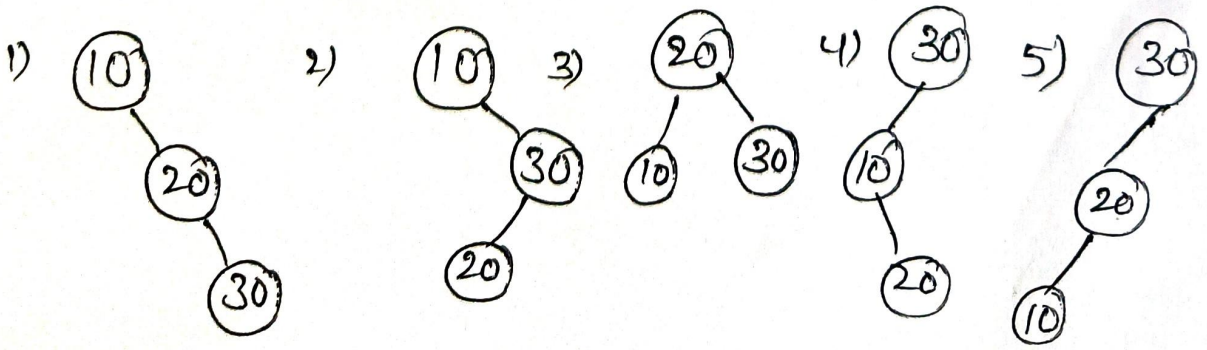
→ The maximum time will be if the element in the leaf

hieight of binary trees is $\boxed{\log n}$

Key = 45, then search in the binary tree.

so, then the element not found. then here some succesfull & some unsuccesfull.

eg:- key = 10, 20, 30



So, for 3 keys, then 5 binary trees are possible.

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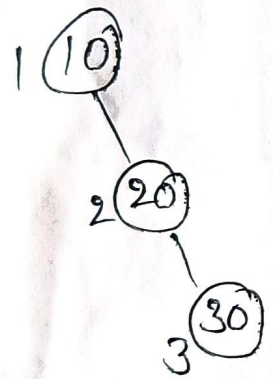
for n keys, $\frac{{}^{2n}C_n}{n+1} \Rightarrow \frac{{}^{2(3)}C_3}{3+1} = \frac{{}^{6}C_3}{4}$

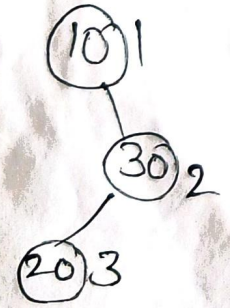
$${}^{6}C_3 = \frac{6!}{3!(6-3)!} = \frac{1\times\cancel{2}\times\cancel{3}\times4\times5\times6}{\cancel{3!}\,3!} = \frac{\overset{20}{\cancel{120}}}{\cancel{6}} = 20$$

$$= \frac{20}{4} = 5.$$

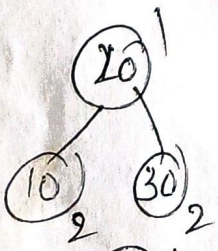
→ cost of searching in each tree. cost means dependent on the no. of comparisions required for searching for a key element



⇒ average no. of comparisions are $\frac{1+2+3}{3} = 2$



$\Rightarrow \frac{1+2+3}{3} = \frac{6}{3} = 2$



$\Rightarrow \frac{1+2+2}{3} = \frac{5}{3} = 1.6$

$= 6/3 = 2$

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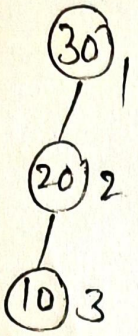
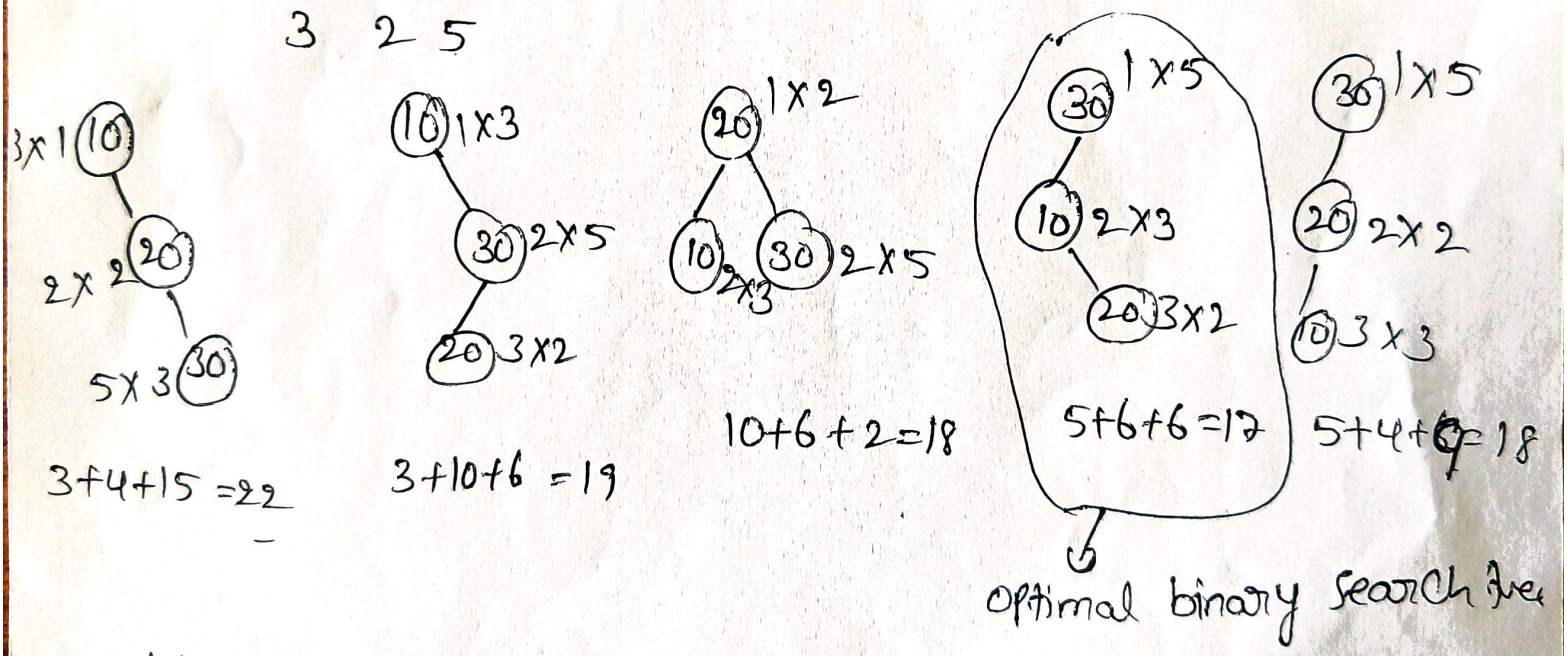


$\Rightarrow \frac{1+2+3}{3} = 2$

⇒ one of the tree is average comparision is less. this is balanced binary search tree.

⇒ Optimal binary search tree : In this problem, we consider the frequency of searching of those keys.

key ⇒ 10, 20, 30
3 2 5



⇒ which tree is the smallest number then that is optimal binary search tree

⇒ based on the frequency is that cost of that binary search tree is minimum

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	1	2	3	4
Keys	10	20	30	40
Frequency	4	2	6	3

i \ j	0	1	2	3	4
0	0	4	8^1	20^2	26^3
1		0	2	10^3	16^3
2			0	6	12^3
3				0	3
4					0

Case i) $j - i = 0$

$0-0=0$ $\quad C[0,0] =$

$1-1=0$ $\quad C(1,1)$

$2-2=0$ $\quad C(2,2)$

$3-3=0$ $\quad C(3,3)$

$4-4=0$ $\quad C(4,4)$

Case ii) $j-i=1$

$1-0=1$ $\quad C(0,1)$

$2-1=1$ $\quad C(1,2)$

$3-2=1$ $\quad C(2,3)$

$4-3=1$ $\quad C(3,4)$

C(0,1)	C(1,2) ignore	C(2,3)	C(3,4)
10	20	30	40
4	2	6	3

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$j - i = 2$

$2 - 0 = 2 \quad (0, 2)$

$3 - 1 = 2 \quad (1, 3)$

$4 - 2 = 2 \quad (2, 4)$

$C(\overset{i}{0}, \overset{j}{2})$

ignore zero

10 20

4 2

10 1x4 20 1x2

20 2x2 10 2x4

4+4=8 2+8=10

$c(i,j) = \min_{i<k\le j} \{c(i,k-1) + c(k,j)\} + w(i,j)$

$c(1,2) = \min\{c(1,1) + c(2,2)$

⇒ minimum cost is 8 and which key is forming a root one.

by using formula

$C(0,0) + c(1,2) + w(0,2)$

$0 + 2 + 6 = 8$

$c(0,1) + c(2,2) + c(0,2)$

$4 + 0 + 6 = 10$

$c(1,3)$ ⇒ just ignore 1.

20 30

2 6

2x1 20 30 1x6

6x2 30 20 2x2

6+4=10

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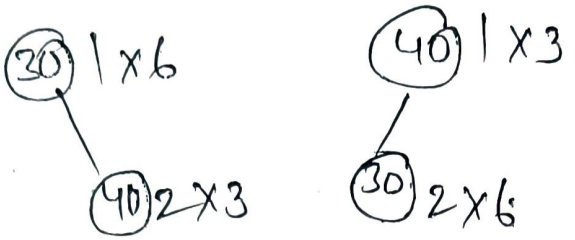
⇒ The root is the 3rd key

c(2,4) ⇒ ignore 2

3, 4

30 40

6 3



6+6 = 12 3+12 = 15

⇒ Minimum cost is the 12. Key is 3rd.

j − i = 3

3 − 0 = 3 (0,3)

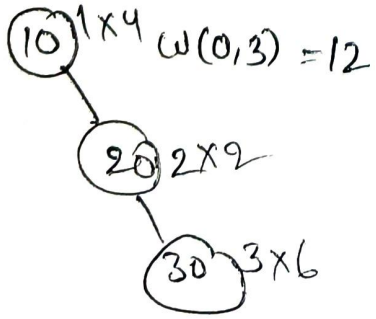
4 − 1 = 3 (1,4)

c(0,3) ⇒ ignore zero

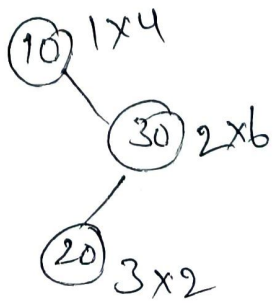
1 2 3

10 20 30

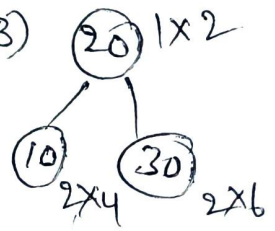
4 2 6



2)



3)



4+4+18 = 26 4+12+6 = 22 2+8+12 = 20

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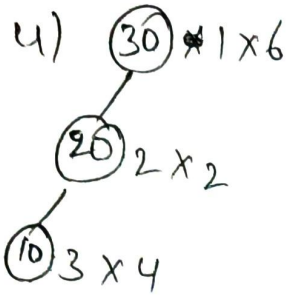
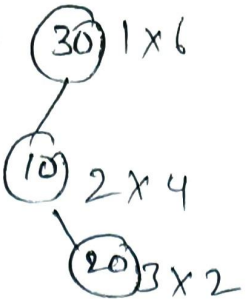
4) 

$6+4+12=22$

5) 

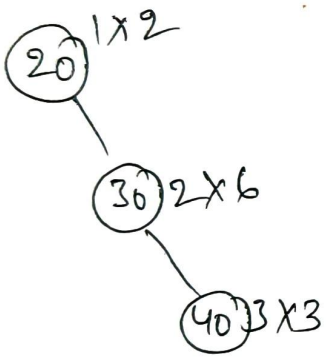
$6+8+6=20$

~~1) C(0,0) + C(1,3) + w(0,3)~~

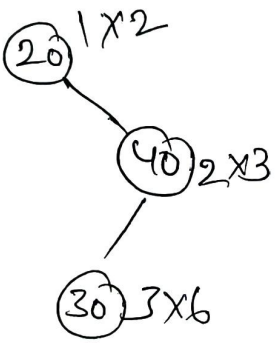
minimum cost is 20

C(1,4) ignore one

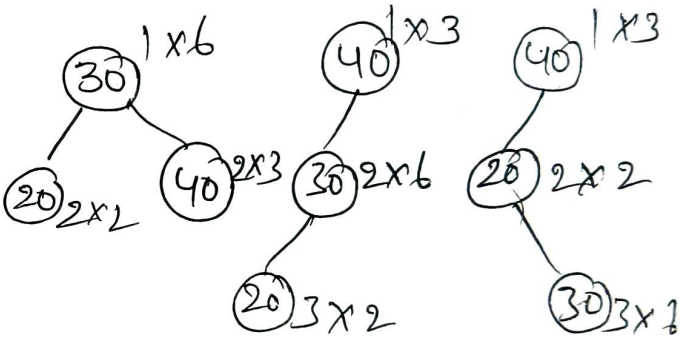
2	3	4
20	30	40
2	6	3



$2+12+9=23$



$2+6+18=26$



$6+4+6=16$ $\quad =3+12+6=21$ $\quad 3+4+18=25$

minimume cost is 16 the root is 3

Now we will calculate the values $j-i=4$

when $j=4$ & $i=0$ then $j-i=4$

In this case we will consider four keys 10, 20, 30, 40. The frequencies of 10, 20, 30 & 40 are 4, 2, 6, 3 respectively

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weight of $w[0,4] = 4+2+6+3 = 15$.

→ If we consider 10 as the root node then

$$c[0,4] = \min\{c(0,0) + c(1,4) + w[0,4]\}$$
$$= \min\{0+16\} + 15 = 31$$

→ if we consider 20 as the root node then

$$c[0,4] = \min\{c(0,1) + c(2,4)\} + w[0,4]$$
$$= \min\{4+12\} + 15$$
$$= 16+15 = 31$$

→ if we consider 30 as the root node then

$$c[0,4] = \min\{c(0,2) + c(3,4)\} + w[0,4]$$
$$= \min\{8+3\} + 15$$
$$= 26$$

→ if we consider consider 40 as the root node

$$c[0,4] = \min\{c(0,3) + c(4,4)\} + w[0,4]$$
$$= \min\{20+0\} + 15$$
$$= 35$$

In the above case, we have observed that 26 is the minimum cost. therefore, $c[0,4]$ is equal to 26. then the root is 3.

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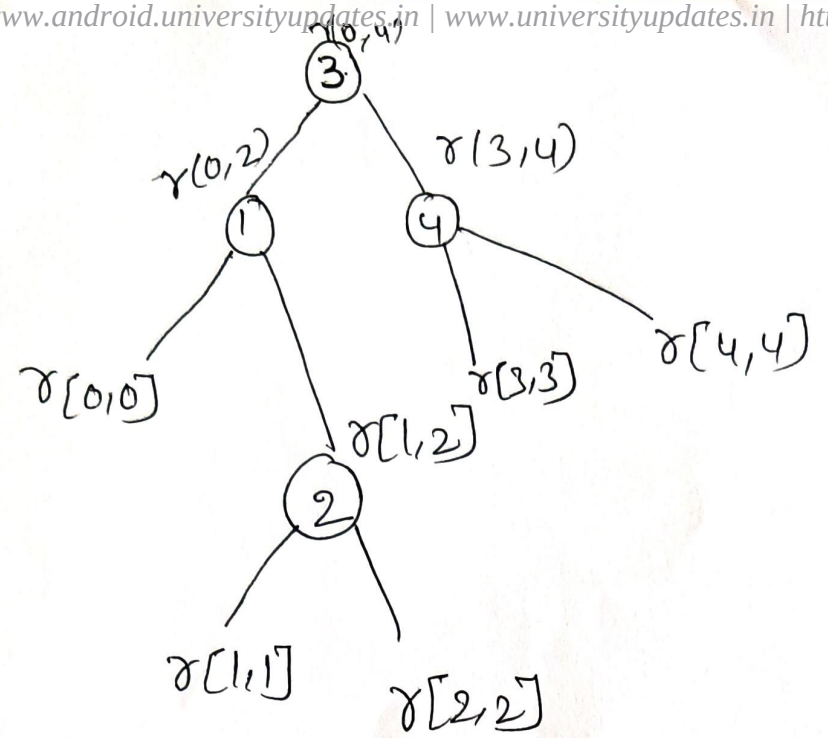
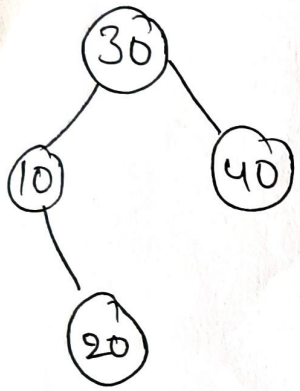


$r[\overset{i}{0},\overset{j}{4}]$, $k=3$

$r(\overset{i}{0},\overset{k-1}{2})$, $r[\overset{k,}{3},\overset{j}{4}]$



∴ optimal binary search tree.

the formula is $\boxed{C[i,j] = \min\{C[i,k-1] + C[k,j]\} + \omega(i,j)}$

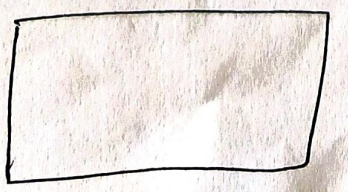
0/1 Knapsack Problem:-

$m=8$ $\quad P = \{1,2,5,6\}$

$n=4$ $\quad \omega = \{2,3,4,5\}$

$x_i = \{1,0,0 \ldots\}$

$x_i = 0/1$



$m=8$

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$x_i = 0/1$ → it means that object is carried or not.

$$\max \sum p_i x_i$$

$$\sum w_i x_i \le m.$$

→ Dynamic Programming is Usefull for solving optimization problems.

→ Dynamic Programming says that the problem should be solved in sequence of decisions.

→ Dynamic Programming says that we should try all possible solutions & pick up the best one.

0 0 0 0 → No objects
1 1 1 1 → all objects taken
0 0 0 1 → last " "
1 0 0 0 → first " "
1 1 0 0 → first two objects taken

2^4 solutions, 2^n, $\boxed{O(2^n)}$ → Time complexity

p	w
1	2
2	3
5	4
6	5

Capacity ↓ \ w →	0	1	2	3	4	5	6	7	8
0	0	0	0	0	0	0	0	0	0
1	0	0	1	1	1	1	1	1	1
2	0	0	1	2	2	3	3	3	3
3	0	0	1	2	5	5	6	7	7
4	0	0	1	2	5	6	6	7	8

$$V[i,w] = \max\{V[i-1,w], V[i-1, w-w[i]] + P[i]\}$$

$$V[4,1] = \max[V[3,1], V[3,1-5] + 6]$$

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$V[4,1] = \max\{V[3,1], V[3,1-5]+6\}$

$3, -4$

$V[4,5] = \max[V[3,5], V[3,5-5]+6$

$5 \qquad 3,0$

$\underline{\underline{0+6}}$

$V[4,6] = \max[V[3,6], V[3,6-5]+6)$

$3,1$

$= 6, 0+6$

$V[4,7] = \max[V[3,7], V[3,7-5]+6]$

$3, 2$

$= 7, \underline{1+6}$

$V[4,8] = \max[V[3,8], V[3,8-5]+6)$

$3,3$

$7, \underline{2+6}$

x_1	x_2	x_3	x_4
0	1	0	1

$8 - 6 = 2$ (profit)

$2 - 2 = 0$ (profit)

<u>Sets method :-</u>

$m = 8 \qquad P = \{1, 2, 5, 6\}$

$n = 4 \qquad w = \{2, 3, 4, 5\}$

$(P, w) \quad S^0 \{(0,0)\}$ (no profit, no weight) $\qquad S_1^0 = \{(1,2)\}$

$S^1 = \{(0,0)(1,2)\}$ → we have consider first object

$S_1^1 = \{(2,3)(3,5)\}$

$S^2 = \{(0,0)(1,2)(2,3)(3,5)\}$

$S_1^2 = \{(5,4), (6,6), (7,7), \cancel{(8,9)}\}$

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$S^3 = \{(0,0)(1,2)(2,3)(\cancel{3,5})(5,4)(6,6)(7,7)\}$

$S^3_1 = \{(6,5),(7,7)(8,8),\cancel{(11,9),(12,11)}$
$\cancel{(13,12)}\}$

$S^4 = \{(0,0)(1,2)(2,3)(5,4)(\cancel{6,6})\cancel{(2,3)},\cancel{6,5}(6,5),(7,7),(8,8)\}$

belong to

(1) $(8,8) \in S^4$

but $(8,8) \notin S^3$ $\therefore x_4 = 1$

$\cancel{(8,8)}$ $\cancel{(8-6)}$ $(8-6, 8-5) = (2,3)$

(2) $(2,3) \in S^3$

but $(2,3) \in S^2$ $\therefore x_3 = 0$

(3) $\cancel{(2,3)}$ $(2,3) \in S^2$

but $(2,3) \notin S^1$ $\therefore x_2 = 1$

$(2-2, 3-1) = (0,0)$

(4) $(0,0) \in S^1$ & $(0,0) \in S^0$ $x_1 = 0$

Solution is $\{0 \quad 1 \quad 0 \quad 1\}$

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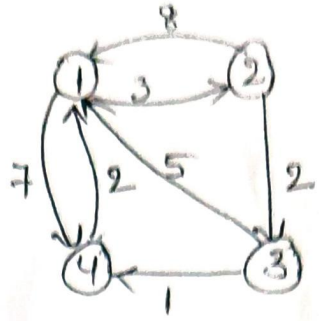
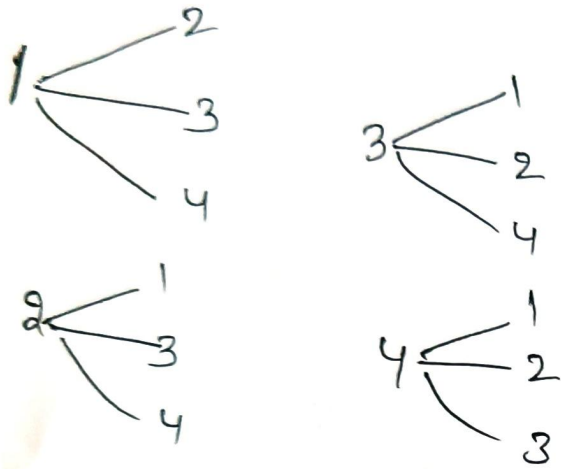
All Pairs Shortest Path:-

→ find the shortest path in every pair of vertices

$$A^0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & \infty \\ 3 & 5 & \infty & 0 & 1 \\ 4 & 2 & \infty & \infty & 0 \end{array}$$

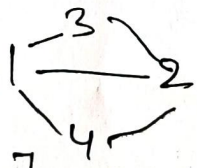




$n^2 \times n = O(n^3)$.

Dynamic Programming says that the problem should by solved by taking sequence of decisions in each stage we will take the decisions.

vertices 1, 2 →



$A^k[i,j] = \min\{A^{k-1}[i,j], A^{k-1}[i,k] + A^{k-1}[k,j]\}$

$$A^1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & 15 \\ 3 & 5 & 8 & 0 & 1 \\ 4 & 2 & 5 & \infty & 0 \end{array}$$

vertex 1

$A^0[2,3]$ $\quad$ $A^0[2,1] + A^0[1,3]$

$2 \quad < \quad 8 + \infty$

This is less so there is no better path via vertex 1.

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$A^0[2,4]$ $\quad A^0[2,1]+A^0[1,4]$

$\infty \quad > \quad 8+7$

$=15$

so update this path $= 15$

$A^0[3,2] \quad A^0[3,1]+A^0[1,2]$

$\infty \quad > \quad 5+3 = 8$

$A^0[3,4] \quad A^0[3,1]+A^0[1,4]$

$1 \quad < \quad 5+7 = 12$

$A^0[4,2] \quad A^0[4,1]+A^0[1,2]$

$\infty \quad > \quad 2+3 = 5$

$A^0[4,3] \quad A^0[4,1]+A^0[1,3]$

$\infty \quad 2+\infty$

$$A^1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & \infty & 7 \\ 2 & 8 & 0 & 2 & 15 \\ 3 & 5 & 8 & 0 & 1 \\ 4 & 2 & 5 & \infty & 0 \end{array}$$

Now I will prepare a matrix which going via vertex 2

$$A^2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 5 & 7 \\ 2 & 8 & 0 & 2 & 15 \\ 3 & 5 & 8 & 0 & 1 \\ 4 & 2 & 5 & 7 & 0 \end{array}$$

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$A^1[1,3]$ $\quad$ $A^1[1,2] + A^1[2,3]$

$\infty \quad > \quad 3+2=5$

$A^1[1,4]$ $\quad$ $A^1[1,2] + A^1[2,4]$

$7 \quad < \quad 3+15=18$

$A^1[3,1]$ $\quad$ $A^1[3,2] + A^1[2,1]$

$5 \quad < \quad 8+8=16$

$A^1[3,4]$ $\quad$ $A^1[3,2] + A^1[2,4]$

$1 \quad < \quad 8+15=23$

$A^1[4,1]$ $\quad$ $A^1[4,2] + A^1[2,1]$

$2 \quad < \quad 5+8=13$

$A^1[4,3]$ $\quad$ $A^1[4,2] + A^1[2,3]$

$\infty \quad 5+2=7$

$$A^3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 5 & 6 \\ 2 & 7 & 0 & 2 & 3 \\ 3 & 5 & 8 & 0 & 1 \\ 4 & 2 & 5 & 7 & 0 \end{array}$$

vertex 3

$A^2[1,2]$ $\quad$ $A^2[1,3] + A^2[3,2]$

$3 \quad < \quad 5+8=13$

$A^2[1,4]$ $\quad$ $A^2[1,3] + A^2[3,4]$

$7 \quad 5+1=6$

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$$A^4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 3 & 5 & 6 \\ 2 & 5 & 0 & 2 & 3 \\ 3 & 3 & 6 & 0 & 1 \\ 4 & 2 & 5 & 7 & 0 \end{array}$$

$$A^k[i,j] = \min\left[A^{k-1}[i,j] ; A^{k-1}[i,k] + A^{k-1}[k,j]\right]$$

```
for(k=1; k<=n; k++)
{
  for(i=1; i<=n; i++)
  {
    for(j=1; j<=n; j++)
    {
      A[i,j] = min(A[i,j], A[i,k]+A[k,j]);
    }
  }
}
```

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Reliability design :-

In this section, we will see how to use dynamic programming to solve a problem with a multiplicative optimization function. The problem is to design a system that is composed of several devices connected in series. Let r_i be the probability of device D_i. Then the reliability of the entire system is product r_i.

fig: n devices D_i $(1 \le i \le n)$ connected in series.

D_i	D_2	D_3	D_4
C_1	C_2	C_3	C_4
r_1	r_2	r_3	r_4
.9	.9	.9	.9

$\prod r_i = 0.9^4 = 0.6561.$

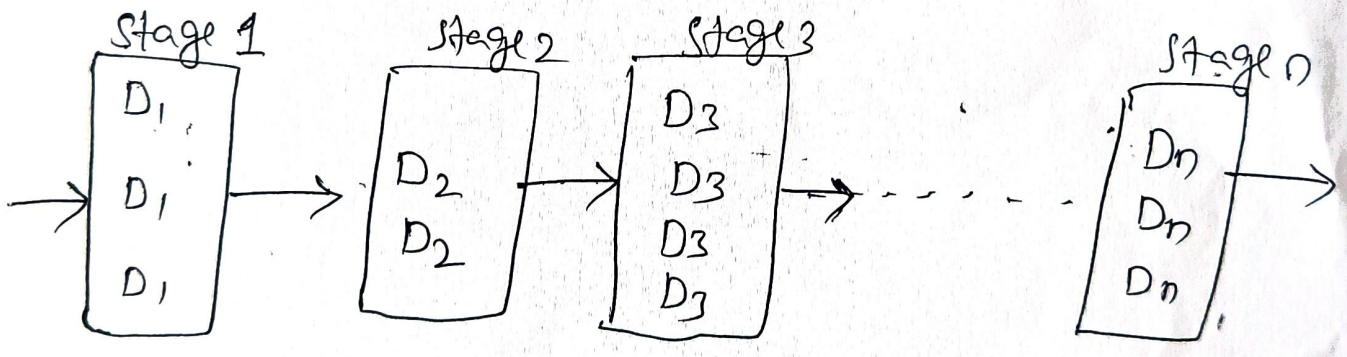

fig: multiple devices connected in parallel in each stage

→ if stage i contains m_i copies of device D_i, then the probability that all m_i have a malfunction is $(1-r_i)^{m_i}$ (m_i copies)

$r_1 = 0.9$

$1 - r_1 = 1 - 0.9 = 0.1.$

$(1-r_1)^3 = (0.1)^3 = 0.001$

$1-(1-r_1)^3 = 0.999$

∴ The reliability of stage i becomes $\boxed{1-(1-r_i)^{m_i}}$

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$C = 105$

eg:-

D_i	c_i	r_i	u_i
D_1	30$	0.9	2
D_2	15$	0.8	3
D_3	20$	0.5	3

$$\Sigma c_i = C_1 + C_2 + C_3$$

$$= 30 + 15 + 20 = 65\$$$

remaining $C - \Sigma c_i = 105 - 65 = 40\$$.

$$\frac{40}{30} = 1$$

$$\left[\frac{C - \Sigma c_i}{c_i}\right] + 1 = \frac{40}{30} = 1 + 1 = 2$$

$$\frac{40}{15} = 2 + 1 = 3$$

$$\frac{40}{20} = 2 + 1 = 3$$

upper bound of u_i

$$\boxed{u_i = \left[\frac{C - \Sigma c_i}{c_i}\right] + 1}$$

→ We have to find out how many copies of each device we should take such that the reliability of the system is maximized.

→ we will use set method to solve this problem, 0/1 of knapsack problem.

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set of order pair = (R, C)
R → Reliability, C → cost

D_i	C_i	r_i	u_i
D_1	30	0.9	2
D_2	15	0.8	3
D_3	20	0.5	3

$$\boxed{S^0 = \{(1, 0)\}}$$

Consider D_1

S_1^1 (device) $= \{(0.9, 30)\}$

S_2^1 (copy) $= \{(0.99, 60)\}$

$$\boxed{S^1 = \{(0.9, 30), (0.99, 60)\}}$$

$$1-(1-r_1)^2 = 1-(1-0.9)^2$$
$$= 1-(0.1)^2$$
$$= 1-0.01$$
$$= (0.99, 60)$$

Consider D_2

S_1^2 (device) $= \{(0.72, 45), (0.792, 75)\}$ (0.9×0.8, 0.8×0.99)

S_2^2 (copy) $= \{(0.864, 60), (0.9504, 90)\}$ (0.96×0.9)

$S_3^2 = \{(0.8928, 75), (\text{———}, 105)\}$

$$\boxed{S^2 = \{(0.72, 45), (0.792, 75), (0.864, 60), (0.8928, 75)\}}$$

S_2^2:
$= 1-(1-r_2)^2$
$= 1-(1-0.8)^2$
$= 1-(0.2)^2$
$= 1-0.04$
$= 0.96, 30$

S_3:
$= 1-(1-r_3)^3$
$= 1-(1-0.8)^3$
$= 1-(0.2)^3$
$= 1-0.008$
$= 0.992$ 45

→ here, the reliability is increased & cost is increased. In second set, the reliability is increased & ~~that~~ the cost is increased. but, third set, the reliability is increased but cost is decreased. so you can apply the dominance rule to cut the higher cost of set.

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$S^2 = \{(0.72, 45), (0.792, 75), \boxed{(0.864, 60)}, (0.8928, 75)$

$\therefore$ cut the ~~high~~ maximum of cost of set.

Consider D_3.

$S_1^3 = \{(0.36, 65), (0.432, 80), (0.4464, 95)\}$

$S_2^3 = \{(0.54, 85), \boxed{(0.648, 100)}, (_, 115)\}$

$S_3^3 = \{(0.63, 105), (_, 120), (_, 135)\}$

$S^3 = \{(0.36, 65), (0.432, 80), (0.4464, 95), (0.54, 85), (0.648, 100), (0.63, 105)\}$

$$\boxed{S^3 = \{(0.36, 65)(0.432, 80), (0.54, 85), \boxed{(0.648, 100)}\}}$$

$r_3 = 0.5$

$1-(1-r_3)^2$

$1-(1-0.5)^2$

$1-(0.5)^2$

$1-0.25$

$= 0.75, 40$

$1-(1-r_3)^3$

$1-(1-0.5)^3$

$= 1-(0.5)^3$

$= 1-0.125$

$= 0.875, 60$

The maximum reliability is $(0.648, 100)$

reliability $= 0.648$

cost $= 100$

$D_1 = 1$, $D_2 = 2$, $D_3 = 2$

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