Probability

Definitions

Probability: A number between 0 and 1 representing the likelihood of an outcome happening.

Experiment: An observation of a process that can result in one of a range of defined outcomes.

Sample Space: The **set** of all possible outcomes of an experiment.

Set: A collection of well-defined **distinct** objects.

Event: A particular outcome or group of outcomes in the sample space. i.e. a subset of the sample space

For an **experiment**: tossing a coin, the **sample space** is the **set**: {H, T}. The **probability** of the **event**: "Heads" is:

$$P(Heads) = \frac{1}{2} = 0.5$$

$$P(event) = \frac{n(event\ outcomes)}{n(all\ outcomes)}$$

The **complement** of an **event** represents all of the other possible **outcomes**. i.e.

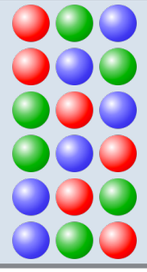
$$P(Heads) = 1 - P(Heads^C)$$
$$= P(Tails)$$

PERMUTATIONS

A **permutation** of a set is an arrangement of the objects, where the order of the arrangement **does** matter.

The number of **permutations** of N things, taken r things at a time:

$${}^NP_R = \frac{N!}{(N-r)!}$$



COMBINATIONS

A **combination** of a set is an arrangement of the objects, where the order of the arrangement **does not** matter.

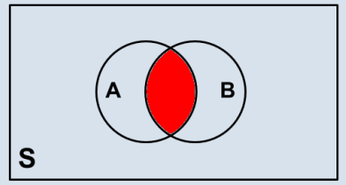
The number of **combinations** of N things, taken r things at a time:

$${}^NC_R = \binom{N}{r} = \frac{N!}{r! \times (N-r)!}$$

Combining Probability

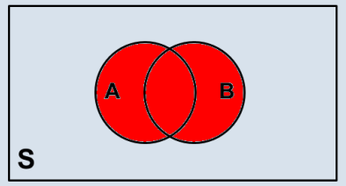
Compound Events: are the combination of at least two simple events.

INTERSECT (AND)



$P(A \cap B)$

UNION (OR)



$P(A \cup B)$

Addition Law

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Conditional Probability

The probability of event A occurring **given that** event B has already occurred is equal to the probability of events A and B occurring, scaled by the probability of just B occurring:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

The **conditioning event** 'filters' down space of outcomes to consider.

Example: Probability of the result of a dice roll begin a 5 given that the result is known to be odd:

$$P(5 \mid Odd) = \frac{P(5 \cap Odd)}{P(Odd)}$$
$$= \frac{1/6}{1/2}$$
$$= 1/3$$

Dependent and Independent Events

Independent: means the probability of an event A is unaffected by the whether an event B has happened.

Dependent: the occurrence of an event B, alters the probability of the occurrence of event A.

For **independent** events:

$$P(A \mid B) = P(A)$$

Multiplication Law

$$P(A \cap B) = P(A) \times P(B)$$

For **dependent** events:

$$P(A \mid B) \neq P(A)$$

Multiplication Law

$$P(A \cap B) = P(A \mid B) \times P(B)$$