

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344. This document was prepared as an account of work sponsored by an agency of the United States government. Neither the United States government nor Lawrence Livermore National Security, LLC, nor any of their employees makes any warranty, expressed or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States government or Lawrence Livermore National Security, LLC. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States government or Lawrence Livermore National Security, LLC, and shall not be used for advertising or product endorsement purposes. LLNL-SM-660398 Copyright (c) 2013, Lawrence Livermore National Security, LLC. Produced
at the Lawrence Livermore National Laboratory. Written by the XBraid team. LLNL-CODE-660355. All rights reserved.
This file is part of XBraid. Please see the COPYRIGHT and LICENSE file for the copyright notice, disclaimer, and the GNU Lesser General Public License. For support, post issues to the XBraid Github page.

XBraid is free software; you can redistribute it and/or modify it under the terms of the GNU General Public License (as published by the Free Software Foundation) version 2.1 dated February 1999.

XBraid is distributed in the hope that it will be useful, but WITHOUT ANY WARRANTY; without even the IMPLIED WARRANTY OF MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE. See the terms and conditions of the GNU General Public License for more details.

You should have received a copy of the GNU Lesser General Public License along with this program; if not, write to the Free Software Foundation, Inc., 59 Temple Place, Suite 330, Boston, MA 02111-1307 USA

## Contents

1 Abstract ..... 2
2 XBraid Quickstart, User Advice, and License ..... 2
2.1 What is XBraid? ..... 2
2.2 About XBraid ..... 2
2.3 Documentation ..... 3
2.4 Advice to Users ..... 3
2.5 Building XBraid ..... 3
2.6 Meaning of the name ..... 4
2.7 License ..... 4
3 Introduction ..... 4
3.1 Overview of the XBraid Algorithm ..... 4
3.1.1 Two-Grid Algorithm ..... 9
3.1.2 Summary ..... 9
3.2 Overview of the XBraid Code ..... 10
3.2.1 Parallel decomposition and memory ..... 11
3.2.2 Cycling and relaxation strategies ..... 12
3.2.3 Overlapping communication and computation ..... 13
3.2.4 Configuring the XBraid Hierarchy ..... 13
3.2.5 Halting tolerance ..... 14
3.2.6 Debugging XBraid ..... 15
3.3 Computing Derivatives with XBraid_Adjoint ..... 16
3.3.1 Short Introduction to Adjoint-based Sensitivity Computation ..... 17
3.3.2 Overview of the XBraid_Adjoint Algorithm ..... 18
3.3.3 Overview of the XBraid_Adjoint Code ..... 18
3.4 XBraid Delta Correction: Accelerating Convergence and Estimating Lyapunov Vectors ..... 21
3.4.1 The Lyapunov Spectrum ..... 21
3.4.2 Overview of the Low-Rank Delta Correction Algorithm ..... 22
3.4.3 Overview of the Delta Correction Code ..... 22
3.4.4 Getting Started ..... 23
3.5 Citing XBraid ..... 24
3.6 Summary ..... 24
4 Examples ..... 24
4.1 The Simplest Example ..... 25
4.1.1 User Defined Structures and Wrappers ..... 25
4.1.2 Running XBraid for the Simplest Example ..... 28
4.2 Some Advanced Features ..... 29
4.3 Simplest example expanded ..... 32
4.4 One-Dimensional Heat Equation ..... 33
4.5 Two-Dimensional Heat Equation ..... 33
4.5.1 User Defined Structures and Wrappers ..... 34
4.5.2 Running XBraid for this Example ..... 36
4.5.3 Scaling Study with this Example ..... 37
4.6 Simplest XBraid_Adjoint example ..... 39
4.6.1 User Defined Structures and Wrappers ..... 39
4.6.2 Running XBraid_Adjoint for this example ..... 41
4.7 Optimization with the Simplest Example ..... 42
4.7.1 User Defined Structures and Wrappers ..... 43
4.7.2 Running an Optimization Cycle with XBraid_Adjoint ..... 44
4.8 A Simple Optimal Control Problem ..... 44
4.9 Chaotic Lorenz System With Delta Correction and Lyapunov Estimation ..... 45
4.9.1 User Defined Structures and Wrappers ..... 45
4.9.2 Running XBraid with Delta correction and Lyapunov Estimation ..... 48
4.10 Running and Testing XBraid ..... 48
4.11 Fortan90 Interface, C++ Interface, Python Interface, and More Complicated Examples ..... 49
5 Examples: compiling and running ..... 49
6 Drivers: compiling and running ..... 51
7 Coding Style ..... 53
8 File naming conventions ..... 53
9 Using Doxygen ..... 54
9.0.1 Documentation Strategy ..... 54
9.0.2 XBraid Doxygen details ..... 55
10 Regression Testing ..... 56
10.0.1 Overview ..... 56
10.0.2 Lowest Level Test Scripts ..... 56
10.0.3 Level 2 Scripts ..... 58
10.0.4 Level 3 Script ..... 59
10.0.5 Cronfile ..... 60
11 Module Index ..... 60
11.1 Modules ..... 60
12 Data Structure Index ..... 61
12.1 Data Structures ..... 61
13 File Index ..... 62
13.1 File List ..... 62
14 Module Documentation ..... 63
14.1 Fortran 90 interface options ..... 63
14.1.1 Detailed Description ..... 63
14.1.2 Macro Definition Documentation ..... 63
14.2 Error Codes ..... 64
14.2.1 Detailed Description ..... 64
14.2.2 Macro Definition Documentation ..... 64
14.3 User-written routines ..... 64
14.3.1 Detailed Description ..... 65
14.3.2 Typedef Documentation ..... 65
14.4 User-written routines for XBraid_Adjoint ..... 73
14.4.1 Detailed Description ..... 74
14.4.2 Typedef Documentation ..... 74
14.5 User interface routines ..... 76
14.5.1 Detailed Description ..... 76
14.6 General Interface routines ..... 76
14.6.1 Detailed Description ..... 78
14.6.2 Macro Definition Documentation ..... 78
14.6.3 Typedef Documentation ..... 78
14.6.4 Function Documentation ..... 78
14.7 Interface routines for XBraid_Adjoint ..... 97
14.7.1 Detailed Description ..... 98
14.7.2 Function Documentation ..... 98
14.8 XBraid status structures ..... 101
14.9 XBraid status routines ..... 101
14.9.1 Detailed Description ..... 102
14.9.2 Function Documentation ..... 102
14.10Inherited XBraid status routines ..... 117
14.10.1 Detailed Description ..... 119
14.10.2 Function Documentation ..... 119
14.11 XBraid status macros ..... 129
14.11.1 Detailed Description ..... 129
14.11.2 Macro Definition Documentation ..... 129
14.12XBraid test routines ..... 131
14.12.1 Detailed Description ..... 132
14.12.2 Function Documentation ..... 132
15 Data Structure Documentation ..... 140
15.1 _braid_Action Struct Reference ..... 140
15.1.1 Detailed Description ..... 140
15.1.2 Field Documentation ..... 140
15.2 _braid_BaseVector Struct Reference ..... 142
15.2.1 Detailed Description ..... 142
15.2.2 Field Documentation ..... 142
15.3 _braid_Basis Struct Reference ..... 143
15.3.1 Detailed Description ..... 143
15.3.2 Field Documentation ..... 143
15.4 _braid_CommHandle Struct Reference ..... 144
15.4.1 Detailed Description ..... 144
15.4.2 Field Documentation ..... 144
15.5 _braid_Core Struct Reference ..... 145
15.5.1 Detailed Description ..... 148
15.5.2 Field Documentation ..... 148
15.6 _braid_Grid Struct Reference ..... 163
15.6.1 Detailed Description ..... 164
15.6.2 Field Documentation ..... 164
15.7 _braid_Status Struct Reference ..... 166
15.7.1 Detailed Description ..... 167
15.7.2 Field Documentation ..... 167
15.8 _braid_Tape Struct Reference ..... 167
15.8.1 Detailed Description ..... 167
15.8.2 Field Documentation ..... 167
15.9 _braid_VectorBar Struct Reference ..... 168
15.9.1 Detailed Description ..... 168
15.9.2 Field Documentation ..... 168
15.10braid_AccessStatus Struct Reference ..... 169
15.10.1 Detailed Description ..... 169
15.10.2 Field Documentation ..... 169
15.11 braid_BufferStatus Struct Reference ..... 169
15.11.1 Detailed Description ..... 170
15.11.2 Field Documentation ..... 170
15.12braid_CoarsenRefStatus Struct Reference ..... 170
15.12.1 Detailed Description ..... 170
15.12.2 Field Documentation ..... 170
15.13braid_ObjectiveStatus Struct Reference ..... 171
15.13.1 Detailed Description ..... 171
15.13.2 Field Documentation ..... 171
15.14braid_Optim Struct Reference ..... 171
15.14.1 Detailed Description ..... 172
15.14.2 Field Documentation ..... 172
15.15braid_StepStatus Struct Reference ..... 173
15.15.1 Detailed Description ..... 174
15.15.2 Field Documentation ..... 174
15.16braid_SyncStatus Struct Reference ..... 174
15.16.1 Detailed Description ..... 174
15.16.2 Field Documentation ..... 174
16 File Documentation ..... 175
16.1 _braid.h File Reference ..... 175
16.1.1 Detailed Description ..... 177
16.1.2 Macro Definition Documentation ..... 177
16.1.3 Typedef Documentation ..... 180
16.1.4 Function Documentation ..... 180
16.1.5 Variable Documentation ..... 189
16.2 adjoint.h File Reference ..... 189
16.2.1 Detailed Description ..... 190
16.2.2 Function Documentation ..... 190
16.3 base.h File Reference ..... 191
16.3.1 Detailed Description ..... 192
16.3.2 Function Documentation ..... 192
16.4 braid.h File Reference ..... 205
16.4.1 Detailed Description ..... 207
16.5 braid_defs.h File Reference ..... 208
16.5.1 Detailed Description ..... 208
16.5.2 Macro Definition Documentation ..... 208
16.5.3 Typedef Documentation ..... 209
16.6 braid_status.h File Reference ..... 209
16.6.1 Detailed Description ..... 213
16.6.2 Macro Definition Documentation ..... 213
16.7 braid_test.h File Reference ..... 215
16.7.1 Detailed Description ..... 215
16.8 delta.h File Reference ..... 216
16.8.1 Detailed Description ..... 216
16.8.2 Macro Definition Documentation ..... 216
16.8.3 Function Documentation ..... 217
16.9 mpistubs.h File Reference ..... 217
16.9.1 Detailed Description ..... 217
16.10status.h File Reference ..... 218
16.10.1 Detailed Description ..... 218
16.10.2 Macro Definition Documentation ..... 218
16.10.3 Function Documentation ..... 219
16.11tape.h File Reference ..... 222
16.11.1 Detailed Description ..... 222
16.11.2 Enumeration Type Documentation ..... 222
16.11.3 Function Documentation ..... 223
16.12util.h File Reference ..... 224
16.12.1 Detailed Description ..... 225
16.12.2 Function Documentation ..... 225
Index ..... 227

## 1 Abstract

This package implements an optimal-scaling multigrid solver for the (non)linear systems that arise from the discretization of problems with evolutionary behavior. Typically, solution algorithms for evolution equations are based on a timemarching approach, solving sequentially for one time step after the other. Parallelism in these traditional time-integration techniques is limited to spatial parallelism. However, current trends in computer architectures are leading towards systems with more, but not faster, processors, i.e., clock speeds are stagnate. Therefore, faster overall runtimes must come from greater parallelism. One approach to achieve parallelism in time is with multigrid, but extending classical multigrid methods for elliptic operators to this setting is a significant achievement. In this software, we implement a nonintrusive, optimal-scaling time-parallel method based on multigrid reduction techniques. The examples in the package demonstrate optimality of our multigrid-reduction-in-time algorithm (MGRIT) for solving a variety of equations in two and three spatial dimensions. These examples can also be used to show that MGRIT can achieve significant speedup in comparison to sequential time marching on modern architectures.

It is strongly recommended that you also read Parallel Time Integration with Multigrid after reading the Overview of the XBraid Algorithm. It is a more in depth discussion of the algorithm and associated experiments.

## 2 XBraid Quickstart, User Advice, and License

### 2.1 What is XBraid?

XBraid is a parallel-in-time software package. It implements an optimal-scaling multigrid solver for the (non)linear systems that arise from the discretization of problems with evolutionary behavior.

This code and associated algorithms are developed at Lawrence Livermore National Laboratory, and at collaborating academic institutions, e.g., UNM.

For our publication list, please go here. There you will papers on XBraid and various application areas where XBraid has been applied, e.g., fluid dynamics, machine learning, parabolic equations, Burgers' equation, powergrid systems, etc.

### 2.2 About XBraid

Typically, solution algorithms for evolution equations are based on a time-marching approach, solving sequentially for one time step after the other. Parallelism in these traditional time-integration techniques is limited to spatial parallelism. However, current trends in computer architectures are leading towards systems with more, but not faster, processors, i.e., clock speeds are stagnate. Therefore, faster overall runtimes must come from greater parallelism. Our approach to achieve such parallelism in time is with multigrid.

In this software, we implement a non-intrusive, optimal-scaling time-parallel method based on multigrid reduction techniques (multigrid-reduction-in-time or MGRIT). A few important points about XBraid are as follows.

- The algorithm enables a scalable parallel-in-time approach by applying multigrid to the time dimension.
- It is designed to be nonintrusive. That is, users apply their existing sequential time-stepping code according to our interface, and then XBraid does the rest. Users have spent years, sometimes decades, developing the right time-stepping scheme for their problem. XBraid allows users to keep their schemes, but enjoy parallelism in the time dimension.
- XBraid solves exactly the same problem that the existing sequential time-stepping scheme does.
- XBraid is flexible, allowing for a variety of time stepping, relaxation, and temporal and spatial coarsening options.
- The full approximation scheme multigrid approach is used to accommodate nonlinear problems.
- XBraid written in MPI/C with C++, Fortran 90, and Python interfaces.
- XBraid is released under LGPL 2.1.


### 2.3 Documentation

- For examples of using XBraid, see the examples/ and drivers/ directories, and in particular examples/ex-01-*
- See the release page for links to precompiled documentation PDFs that go through, step-by-step, how to use XBraid.
- For tutorials, see the bottom of our publications page.
- For citing XBraid, see here.


### 2.4 Advice to Users

The field of parallel-in-time methods is in many ways under development, and success has been shown primarily for problems with some parabolic character. While there are ongoing projects (here and elsewhere) looking at varied applications such as hyperbolic problems, computational fluid dynamics, power grids, medical applications, and so on, expectations should take this fact into account. That being said, we strongly encourage new users to try our code for their application. Every new application has its own issues to address and this will help us to improve both the algorithm and the software. Please see our project publications website for our recent publications concerning some of these varied applications.

For bug reporting, please use the issue tracker here on Github. Please include as much relevant information as possible, including all the information in the "VERSION" file located in the bottom most XBraid directory. For compile and runtime problems, please also include the machine type, operating system, MPI implementation, compiler, and any error messages produced.

### 2.5 Building XBraid

- To specify the compilers, flags and options for your machine, edit makefile.inc. For now, we keep it simple and avoid using configure or cmake.
- To make the library, libbraid.a,

```
$ make
```

- To make the examples

```
$ make all
```

- The makefile lets you pass some parameters like debug with

```
$ make debug=yes
```

or
\$ make all debug=yes
It would also be easy to add additional parameters, e.g., to compile with insure.

- To set compilers and library locations, look in makefile.inc where you can set up an option for your machine to define simple stuff like

```
CC = mpicc
MPICC = mpicc
MPICXX = mpiCC
LFLAGS = -lm
```


### 2.6 Meaning of the name

We chose the package name XBraid to stand for Time-Braid, where $X$ is the first letter in the Greek word for time, Chronos. The algorithm braids together time-grids of different granularity in order to create a multigrid method and achieve parallelism in the time dimension.

### 2.7 License

This project is released under the LGPL v2.1 license. See files COPYRIGHT and LICENSE file for full details.
LLNL Release Number: LLNL-CODE-660355

## 3 Introduction

### 3.1 Overview of the XBraid Algorithm

The goal of XBraid is to solve a problem faster than a traditional time marching algorithm. Instead of sequential time marching, XBraid solves the problem iteratively by simultaneously updating a space-time solution guess over all time values. The initial solution guess can be anything, even a random function over space-time. The iterative updates to the solution guess are done by constructing a hierarchy of temporal grids, where the finest grid contains all of the time values for the simulation. Each subsequent grid is a coarser grid with fewer time values. The coarsest grid has a trivial number of time steps and can be quickly solved exactly. The effect is that solutions to the time marching problem on the coarser (i.e., cheaper) grids can be used to correct the original finest grid solution. Analogous to spatial multigrid, the coarse grid correction only corrects and accelerates convergence to the finest grid solution. The coarse grid does not need to represent an accurate time discretization in its own right. Thus, a problem with many time steps (thousands, tens of thousands or more) can be solved with 10 or 15 XBraid iterations, and the overall time to solution can be greatly sped up. However, this is achieved at the cost of more computational resources.

To understand how XBraid differs from traditional time marching, consider the simple linear advection equation, $u_{t}=$ $-c u_{x}$. The next figure depicts how one would typically evolve a solution here with sequential time stepping. The initial condition is
a wave, and this wave propagates sequentially across space as time increases.
XBraid instead begins with a solution guess over all of space-time, which for demonstration, we let be random. An XBraid iteration does


Figure 1 Sequential time stepping.

1. Relaxation on the fine grid, i.e., the grid that contains all of the desired time values. Relaxation is just a local application of the time stepping scheme, e.g., backward Euler.
2. Restriction to the first coarse grid, i.e., interpolate the problem to a grid that contains fewer time values, say every second or every third time value.
3. Relaxation on the first coarse grid
4. Restriction to the second coarse grid and so on...
5. When a coarse grid of trivial size (say 2 time steps) is reached, it is solved exactly.
6. The solution is then interpolated from the coarsest grid to the finest grid

One XBraid iteration is called a cycle and these cycles continue until the solution is accurate enough. This is depicted in the next figure, where only a few iterations are required for this simple problem.


Figure 2 XBraid iterations.

There are a few important points to make.

- The coarse time grids allow for global propagation of information across space-time with only one XBraid iteration. This is visible in the above figure by observing how the solution is updated from iteration 0 to iteration 1.
- Using coarser (cheaper) grids to correct the fine grid is analogous to spatial multigrid.
- Only a few XBraid iterations are required to find the solution over 1024 time steps. Therefore if enough processors are available to parallelize XBraid, we can see a speedup over traditional time stepping (more on this later).
- This is a simple example, with evenly space time steps. XBraid is structured to handle variable time step sizes and adaptive time step sizes.

To firm up our understanding, let's do a little math. Assume that you have a general system of ordinary differential equations (ODEs),

$$
\boldsymbol{u}^{\prime}(t)=\boldsymbol{f}(t, \boldsymbol{u}(t)), \quad \boldsymbol{u}(0)=\boldsymbol{u}_{0}, \quad t \in[0, T] .
$$

Next, let $t_{i}=i \delta t, i=0,1, \ldots, N$ be a temporal mesh with spacing $\delta t=T / N$, and $u_{i}$ be an approximation to $u\left(t_{i}\right)$. A general one-step time discretization is now given by

$$
\begin{aligned}
& \boldsymbol{u}_{0}=g_{0} \\
& \boldsymbol{u}_{i}=\Phi_{i}\left(\boldsymbol{u}_{i-1}\right)+\boldsymbol{g}_{i}, \quad i=1,2, \ldots, N
\end{aligned}
$$

Traditional time marching would first solve for $i=1$, then solve for $i=2$, and so on. For linear time propagators $\left\{\Phi_{i}\right\}$, this can also be expressed as applying a direct solver (a forward solve) to the following system:

$$
A \mathbf{u} \equiv\left(\begin{array}{cccc}
I & & & \\
-\Phi_{1} & I & & \\
& \ddots & \ddots & \\
& & -\Phi_{N} & I
\end{array}\right)\left(\begin{array}{c}
\boldsymbol{u}_{0} \\
\boldsymbol{u}_{1} \\
\vdots \\
\boldsymbol{u}_{N}
\end{array}\right)=\left(\begin{array}{c}
\boldsymbol{g}_{0} \\
\boldsymbol{g}_{1} \\
\vdots \\
\boldsymbol{g}_{N}
\end{array}\right) \equiv \mathbf{g}
$$

or

$$
A \mathbf{u}=\mathbf{g}
$$

This process is optimal and $O(N)$, but it is sequential. XBraid achieves parallelism in time by replacing this sequential solve with an optimal multigrid reduction iterative method ${ }^{1}$ applied to only the time dimension. This approach is

- nonintrusive, in that it coarsens only in time and the user defines $\Phi$. Thus, users can continue using existing time stepping codes by wrapping them into our framework.
- optimal and $\mathrm{O}(\mathrm{N})$, but $\mathrm{O}(\mathrm{N})$ with a higher constant than time stepping. Thus with enough computational resources, XBraid will outperform sequential time stepping.
- highly parallel

We now describe the two-grid process in more detail, with the multilevel analogue being a recursive application of the process. We also assume that $\Phi$ is constant for notational simplicity. XBraid coarsens in the time dimension with factor $m>1$ to yield a coarse time grid with $N_{\Delta}=N / m$ points and time step $\Delta T=m \delta t$.
The corresponding coarse grid problem,

$$
A_{\Delta}=\left(\begin{array}{cccc}
I & & & \\
-\Phi_{\Delta} & I & & \\
& \ddots & \ddots & \\
& & -\Phi_{\Delta} & I
\end{array}\right)
$$

is obtained by defining coarse grid propagators $\left\{\Phi_{\Delta}\right\}$ which are at least as cheap to apply as the fine scale propagators $\{\Phi\}$. The matrix $A_{\Delta}$ has fewer rows and columns than $A$, e.g., if we are coarsening in time by $2, A_{\Delta}$ has one half as many rows and columns.

This coarse time grid induces a partition of the fine grid into C-points (associated with coarse grid points) and F-points, as visualized next. C-points exist on both the fine and coarse time grid, but F-points exist only on the fine time scale.

[^0]

Every multigrid algorithm requires a relaxation method and an approach to transfer values between grids. Our relaxation scheme alternates between so-called F-relaxation and C-relaxation as illustrated next. F-relaxation updates the F-point values $\left\{u_{j}\right\}$ on interval $\left(T_{i}, T_{i+1}\right)$ by simply propagating the C-point value $u_{m i}$ across the interval using the time propagator $\{\Phi\}$. While this is a sequential process, each F-point interval update is independent from the others and can be computed in parallel. Similarly, C-relaxation updates the C-point value $u_{m i}$ based on the F-point value $u_{m i-1}$ and these updates can also be computed in parallel. This approach to relaxation can be thought of as line relaxation in space in that the residual is set to 0 for an entire time step.

The F updates are done simultaneously in parallel, as depicted next.


Figure 3 Update all F-point intervals in parallel, using the time propagator $\Phi$.

Following the F sweep, the C updates are also done simultaneously in parallel, as depicted next.


Figure 4 Update all C-points in parallel, using the time propagator $\Phi$.

In general, FCF- and F-relaxation will refer to the relaxation methods used in XBraid. We can say

- FCF- or F-relaxation is highly parallel.
- But, a sequential component exists equaling the number of F-points between two C-points.
- XBraid uses regular coarsening factors, i.e., the spacing of C-points happens every $m$ points.

After relaxation, comes forming the coarse grid error correction. To move quantities to the coarse grid, we use the restriction operator $R$ which simply injects values at C-points from the fine grid to the coarse grid,

$$
R=\left(\begin{array}{ccc}
I & & \\
0 & & \\
\vdots & & \\
0 & & \\
& I & \\
& 0 & \\
& \vdots & \\
& 0 & \\
& & \ddots
\end{array}\right)^{T} .
$$

The spacing between each $I$ is $m-1$ block rows. While injection is simple, XBraid always does an F-relaxation sweep before the application of $R$, which is equivalent to using the transpose of harmonic interpolation for restriction (see Parallel Time Integration with Multigrid). Another interpretation is that the F-relaxation compresses the residual into the C -points, i.e., the residual at all F -points after an F -relaxation is 0 . Thus, it makes sense for restriction to be injection.

To define the coarse grid equations, we apply the Full Approximation Scheme (FAS) method, which is a nonlinear version of multigrid. This is to accommodate the general case where $f$ is a nonlinear function. In FAS, the solution guess and residual (i.e., $\mathbf{u}, \mathbf{g}-A \mathbf{u}$ ) are restricted. This is in contrast to linear multigrid which typically restricts only the residual equation to the coarse grid. This algorithmic change allows for the solution of general nonlinear problems. For more details, see this PDF by Van Henson for a good introduction to FAS. However, FAS was originally invented by Achi Brandt.

A central question in applying FAS is how to form the coarse grid matrix $A_{\Delta}$, which in turn asks how to define the coarse grid time stepper $\Phi_{\Delta}$. One of the simplest choices (and one frequently used in practice) is to let $\Phi_{\Delta}$ simply be $\Phi$ but with the coarse time step size $\Delta T=m \delta t$. For example, if $\Phi=(I-\delta t A)^{-1}$ for some backward Euler scheme, then $\Phi_{\Delta}=(I-m \delta t A)^{-1}$ would be one choice.

With this $\Phi_{\Delta}$ and letting $\mathbf{u}_{\Delta}$ be the restricted fine grid solution and $\mathbf{r}_{\Delta}$ be the restricted fine grid residual, the coarse grid equation

$$
A_{\Delta}\left(\mathbf{v}_{\Delta}\right)=A_{\Delta}\left(\mathbf{u}_{\Delta}\right)+\mathbf{r}_{\Delta}
$$

is then solved. Finally, FAS defines a coarse grid error approximation $\mathbf{e}_{\Delta}=\mathbf{v}_{\Delta}-\mathbf{u}_{\Delta}$, which is interpolated with $P_{\Phi}$ back to the fine grid and added to the current solution guess. Interpolation is equivalent to injecting the coarse grid to the C-points on the fine grid, followed by an F-relaxation sweep (i.e., it is equivalent to harmonic interpolation, as mentioned above about restriction). That is,

$$
P_{\Phi}=\left(\begin{array}{ccc}
I & & \\
\Phi & & \\
\Phi^{2} & & \\
\vdots & & \\
\Phi^{m-1} & & \\
& I & \\
& \Phi & \\
& \Phi^{2} & \\
& \vdots & \\
& \Phi^{m-1} & \\
& & \ddots
\end{array}\right)
$$

where $m$ is the coarsening factor. See Two-Grid Algorithm for a concise description of the FAS algorithm for MGRIT.

### 3.1.1 Two-Grid Algorithm

The two-grid FAS process is captured with this algorithm. Using a recursive coarse grid solve (i.e., step 3 becomes a recursive call) makes the process multilevel. Halting is done based on a residual tolerance. If the operator is linear, this FAS cycle is equivalent to standard linear multigrid. Note that we represent $A$ as a function below, whereas the above notation was simplified for the linear case.

1. Relax on $A(\mathbf{u})=\mathbf{g}$ using FCF-relaxation
2. Restrict the fine grid approximation and its residual:

$$
\mathbf{u}_{\Delta} \leftarrow R \mathbf{u}, \quad \mathbf{r}_{\Delta} \leftarrow R(\mathbf{g}-A(\mathbf{u}),
$$

which is equivalent to updating each individual time step according to

$$
u_{\Delta, i} \leftarrow u_{m i}, \quad r_{\Delta, i} \leftarrow g_{m i}-A(\mathbf{u})_{m i} \quad \text { for } \quad i=0, \ldots, N_{\Delta} .
$$

3. Solve $A_{\Delta}\left(\mathbf{v}_{\Delta}\right)=A_{\Delta}\left(\mathbf{u}_{\Delta}\right)+\mathbf{r}_{\Delta}$
4. Compute the coarse grid error approximation: $\mathbf{e}_{\Delta}=\mathbf{v}_{\Delta}-\mathbf{u}_{\Delta}$
5. Correct: $\mathbf{u} \leftarrow \mathbf{u}+P \mathbf{e}_{\Delta}$

This is equivalent to updating each individual time step by adding the error to the values of $\mathbf{u}$ at the C -points:

$$
u_{m i}=u_{m i}+e_{\Delta, i},
$$

followed by an F-relaxation sweep applied to $\mathbf{u}$.

### 3.1.2 Summary

In summary, a few points are

- XBraid is an iterative solver for the global space-time problem.
- The user defines the time stepping routine $\Phi$ and can wrap existing code to accomplish this.
- XBraid convergence will depend heavily on how well $\Phi_{\Delta}$ approximates $\Phi^{m}$, that is how well a time step size of $m \delta t=\Delta T$ will approximate $m$ applications of the same time integrator for a time step size of $\delta t$. This is a subject of research, but this approximation need not capture fine scale behavior, which is instead captured by relaxation on the fine grid.
- The coarsest grid is solved exactly, i.e., sequentially, which can be a bottleneck for two-level methods like Parareal,
${ }^{2}$ but not for a multilevel scheme like XBraid where the coarsest grid is of trivial size.
- By forming the coarse grid to have the same sparsity structure and time stepper as the fine grid, the algorithm can recur easily and efficiently.
- Interpolation is ideal or exact, in that an application of interpolation leaves a zero residual at all F-points.
- The process is applied recursively until a trivially sized temporal grid is reached, e.g., 2 or 3 time points. Thus, the coarsening rate $m$ determines how many levels there are in the hierarchy. For instance in this figure, a 3 level hierarchy is shown. Three levels are chosen because there are six time points, $m=2$ and $m^{2}<6 \leq m^{3}$. If the coarsening rate had been $m=4$ then there would only be two levels because there would be no more points to coarsen!

> - $F$-point (fine grid only)
> - $C$-point (coarse grid)


By default, XBraid will subdivide the time domain into evenly sized time steps. XBraid is structured to handle variable time step sizes and adaptive time step sizes.

### 3.2 Overview of the XBraid Code

XBraid is designed to run in conjunction with an existing application code that can be wrapped per our interface. This application code will implement some time marching simulation like fluid flow. Essentially, the user has to take their application code and extract a stand-alone time-stepping function $\Phi$ that can evolve a solution from one time value to another, regardless of time step size. After this is done, the XBraid code takes care of the parallelism in the time dimension.

XBraid

- is written in C and can easily interface with Fortran, C++, and Python
- uses MPI for parallelism
- self documents through comments in the source code and through $*$.md files
- functions and structures are prefixed by braid
- User routines are prefixed by braid_
- Developer routines are prefixed by _braid_



### 3.2.1 Parallel decomposition and memory

- XBraid decomposes the problem in parallel as depicted next. As you can see, traditional time stepping only stores one time step at a time, but only enjoys a spatial data decomposition and spatial parallelism. On the other hand, XBraid stores multiple time steps simultaneously and each processor holds a space-time chunk reflecting both the spatial and temporal parallelism.
- XBraid only handles temporal parallelism and is agnostic to the spatial decomposition.

See braid_SplitCommworld.
Each processor owns a certain number of CF intervals of points. In the following figure, processor 1 and processor 2 each own 2 CF intervals. XBraid distributes intervals evenly on the finest grid.


- XBraid increases the parallelism significantly, but now several time steps need to be stored, requiring more memory. XBraid employs two strategies to address the increased memory costs.
- First, one need not solve the whole problem at once. Storing only one space-time slab is advisable. That is, solve for as many time steps (say $k$ time steps) as you have available memory for. Then move on to the next $k$ time steps.
- Second, XBraid provides support for storing only C-points. Whenever an F-point is needed, it is generated by F-relaxation. More precisely, only the red C-point time values in the previous figure are stored. Coarsening is usually aggressive with $m=8,16,32, \ldots$, so the storage requirements of XBraid are significantly reduced when compared to storing all of the time values.

Overall, the memory multiplier per processor when using XBraid is $O(1)$ if space-time coarsening (see The Simplest Example) is used and $O\left(\log _{m} N\right)$ for time-only coarsening. The time-only coarsening option is the default and requires no user-written spatial interpolation/restriction routines (which is the case for space-time coasrening). We note that the base of the logarithm is $m$, which can be quite large.

[^1]
### 3.2.2 Cycling and relaxation strategies

There are two main cycling strategies available in XBraid, F-and V-cycles. These two cycles differ in how often and the order in which coarse levels are visited. A V-cycle is depicted next, and is a simple recursive application of the Two-Grid Algorithm.


An F-cycle visits coarse grids more frequently and in a different order. Essentially, an F-cycle uses a V-cycle as the post-smoother, which is an expensive choice for relaxation. But, this extra work gives you a closer approximation to a two-grid cycle, and a faster convergence rate at the extra expense of more work. The effectiveness of a V-cycle as a relaxation scheme can be seen in Figure 2, where one V-cycle globally propagates and smoothes the error. The cycling strategy of an F-cycle is depicted next.


Next, we make a few points about F- versus V-cycles.

- One V-cycle iteration is cheaper than one F-cycle iteration.
- But, F-cycles often converge more quickly. For some test cases, this difference can be quite large. The cycle choice for the best time to solution will be problem dependent. See Scaling Study with this Example for a case study of cycling strategies.
- For exceptionally strong F-cycles, the option braid_SetNFMGVcyc can be set to use multiple V-cycles as relaxation. This has proven useful for some problems with a strongly advective nature.

The number of FC relaxation sweeps is another important algorithmic setting. Note that at least one F-relaxation sweep is always done on a level. A few summary points about relaxation are as follows.

- Using FCF, FCFCF, or FCFCFCF relaxation corresponds to passing braid_SetNRelax a value of 1,2 or 3 respectively, and will result in an XBraid cycle that converges more quickly as the number of relaxations grows.
- But as the number of relaxations grows, each XBraid cycle becomes more expensive. The optimal relaxation strategy for the best time to solution will be problem dependent.
- However, a good first step is to try FCF on all levels (i.e., braid_SetNRelax(core, -1, 1) ).
- A common optimization is to first set FCF on all levels (i.e., braid_setnrelax(core, -1, 1) ), but then overwrite the FCF option on level 0 so that only F-relaxation is done on level 0 , (i.e., braid_setnrelax(core, 0, 1) ). Another strategy is to use F-relaxation on all levels together with F-cycles.
- See Scaling Study with this Example for a case study of relaxation strategies.

There is also a weighted relaxation option, which applies weighted-Jacobi at the C-points during the C-relaxation. Experiments with the 1D heat equation and 1D advection showed iteration gains of $10-25 \%$ for V-cycles when the experimentally optimal weight was used.

- For the heat equation, a weight of around 1.3 was experimentally optimal
- For the advection equation, weights between 1.4 and 1.8 were experimentally optimal
- Set this option with braid_SetCRelaxWt, which allows you to set a global relaxation weight, or an individual weight for each level. In general, under-relaxation (weight $<1.0$ ) never improved performance, but over-relxation ( $1.0<$ weight $<2.0$ ) often offered some improvement.

Last, Parallel Time Integration with Multigrid has a more in depth case study of cycling and relaxation strategies

### 3.2.3 Overlapping communication and computation

XBraid effectively overlaps communication and computation. The main computational kernel of XBraid is one relaxation sweep touching all the CF intervals. At the start of a relaxation sweep, each process first posts a non-blocking receive at its left-most point. It then carries out F-relaxation in each interval, starting with the right-most interval to send the data to the neighboring process as soon as possible. If each process has multiple CF intervals at this XBraid level, the strategy allows for complete overlap.


### 3.2.4 Configuring the XBraid Hierarchy

Some of the more basic XBraid function calls allow you to control aspects discussed here.

- braid_SetFMG: switches between using F- and V-cycles.
- braid_SetMaxIter: sets the maximum number of XBraid iterations
- braid_SetCFactor: sets the coarsening factor for any (or all levels)
- braid_SetNRelax: sets the number of CF-relaxation sweeps for any (or all levels)
- braid_SetRelTol, braid_SetAbsTol: sets the stopping tolerance
- braid_SetMinCoarse: sets the minimum possible coarse grid size
- braid_SetMaxLevels: sets the maximum number of levels in the XBraid hierarchy


### 3.2.5 Halting tolerance

Another important configuration aspect regards setting a residual halting tolerance. Setting a tolerance involves these three XBraid options:

1. braid_PtFcnSpatialNorm

This user-defined function carries out a spatial norm by taking the norm of a braid_Vector. A common choice is the standard Eucliden norm (2-norm), but many other choices are possible, such as an L2-norm based on a finite element space.
2. braid_SetTemporalNorm

This option determines how to obtain a global space-time residual norm. That is, this decides how to combine the spatial norms returned by braid_PtFcnSpatialNorm at each time step to obtain a global norm over space and time. It is this global norm that then controls halting.
There are three tnorm options supported by braid_SetTemporalNorm. We let the summation index $i$ be over all C-point values on the fine time grid, $k$ refer to the current XBraid iteration, $r$ be residual values, space_time norms be a norm over the entire space-time domain and spatial_norm be the user-defined spatial norm from braid_PtFcnSpatialNorm. Thus, $r_{i}$ is the residual at the ith C-point, and $r^{(k)}$ is the residual at the kth XBraid iteration. The three options are then defined as,

- tnorm=1: One-norm summation of spatial norms

$$
\left\|r^{(k)}\right\|_{\text {space_time }}=\Sigma_{i}\left\|r_{i}^{(k)}\right\|_{\text {spatial_norm }}
$$

If braid_PtFcnSpatialNorm is the one-norm over space, then this is equivalent to the one-norm of the global space-time residual vector.

- tnorm=2: Two-norm summation of spatial norms

$$
\left\|r^{(k)}\right\|_{\text {space_time }}=\left(\Sigma_{i}\left\|r_{i}^{(k)}\right\|_{\text {Spatial_norm }}^{2}\right)^{1 / 2}
$$

If braid_PtFcnSpatialNorm is the Euclidean norm (two-norm) over space, then this is equivalent to the Euclidean-norm of the global space-time residual vector.

- tnorm=3: Infinity-norm combination of spatial norms

$$
\left\|r^{(k)}\right\|_{\text {space_time }}=\max _{i}\left\|r_{i}^{(k)}\right\|_{\text {spatial_norm }}
$$

If braid_PtFcnSpatialNorm is the infinity-norm over space, then this is equivalent to the infinity-norm of the global space-time residual vector.

## The default choice is tnorm=2

3. braid_SetAbsTol, braid_SetRelTol

- If an absolute tolerance is used, then

$$
\left\|r^{(k)}\right\|_{\text {space_time }}<\text { tol }
$$

defines when to halt.

- If a relative tolerance is used, then

$$
\frac{\left\|r^{(k)}\right\|_{\text {space_time }}}{\left\|r^{(0)}\right\|_{\text {space_time }}}<\text { tol }
$$

defines when to halt. That is, the current kth residual is scaled by the initial residual before comparison to the halting tolerance. This is similar to typical relative residual halting tolerances used in spatial multigrid, but can be a dangerous choice in this setting.

Care should be practiced when choosing a halting tolerance. For instance, if a relative tolerance is used, then issues can arise when the initial guess is zero for large numbers of time steps. Taking the case where the initial guess (defined by braid_PtFcnInit) is 0 for all time values $t>0$, the initial residual norm will essentially only be nonzero at the first time value,

$$
\left\|r^{(0)}\right\|_{\text {space_time }} \approx\left\|r_{1}^{(k)}\right\|_{\text {spatial_norm }}
$$

This will skew the relative halting tolerance, especially if the number of time steps increases, but the initial residual norm does not.

A better strategy is to choose an absolute tolerance that takes your space-time domain size into account, as in Section Scaling Study with this Example, or to use an infinity-norm temporal norm option.

### 3.2.6 Debugging XBraid

Wrapping and debugging a code with XBraid typically follows a few steps.

- Test your wrapped functions with XBraid test functions, e.g., braid_TestClone or braid_TestSum.
- Set max levels to 1 (braid_SetMaxLevels) and run an XBraid simulation. You should get the exact same answer as that achieved with sequential time stepping. If you make sure that the time-grids used by XBraid and by sequential time stepping are bit-wise the same (by using the user-defined time grid option braid_SetTimeGrid ), then the agreement of their solutions should be bit-wise the same.
- Continue with max levels equal to 1 , but switch to two processors in time. Check that the answer again exactly matches sequential time stepping. This test checks that the information in braid_Vector is sufficient to correctly start the simulation on the second processor in time.
- Set max levels to 2, halting tolerance to 0.0 (braid_SetAbsTol), max iterations to 3 (braid_SetMaxlter) and turn on the option braid_SetSeqSoln.
This will use the solution from sequential time-stepping as the initial guess for XBraid and then run 3 iterations. The residual should be exactly 0 each iteration, verifying the fixed-point nature of XBraid and a (hopefully!) correct implementation. The residual may be on the order of machine epsilon (or smaller). Repeat this test for multiple processors in time (and space if possible).
- A similar test turns on debug level printing by passing a print level of 3 to braid_SetPrintLevel. This will print out the residual norm at each C-point. XBraid with FCF-relaxation has the property that the exact solution is propagated forward two C-points each iteration. Thus, this should be reflected by numerically zero residual values for the first so many time points. Repeat this test for multiple processors in time (and space if possible).
- Finally, run some multilevel tests, making sure that the XBraid results are within the halting tolerance of the solutions generated by sequential time-stepping. Repeat this test for multiple processors in time (and space if possible).
- Congratulations! Your code is now verified.

One detail that can rarely affect the fixed-point test (and other tests) concerns the time-step size computation in XBraid. XBraid computes the time-step value with the formula

$$
t_{i}=t_{0}+(i / N) *\left(T-t_{0}\right), \quad i=1,2 \ldots, N
$$

where $N$ is the number of time-steps (not counting $t_{0}$ ), the integer division with $N$ is cast as a float, $t_{0}$ is the global start time, and $T$ is the global end time. This formula guarantees that the last time-value $t_{N}=T$ and that the $t_{i}$ are evenly spaced (to within floating point accuracy). But, this formula also means that in some cases the time-step size can vary when not expected. For example, the time-step size can be uniform in exact arithmetic, but vary by a small amount (in the least significant bit) in floating-point arithmetic. For instance, a time-interval of $[0,1]$ and $N=5$ can yield this phenomenon.

This phenomenon can cause fixed-point issues, for example, if you precompute values based on the time-step size, or use the time-step size as a dictionary key. If you suspect this is an issue, it is recommended to use for your debugging tests, $t_{0}, T$, and $N$ that do not produce this phenomenon, or to use a user-specified time-grid with braid_SetTimeGrid .

### 3.3 Computing Derivatives with XBraid_Adjoint

XBraid_Adjoint has been developed in collaboration with the Scientific Computing group at TU Kaiserslautern, Germany, and in particular with Dr. Stefanie Guenther and Prof. Nicolas Gauger.

In many application scenarios, the ODE system is driven by some independent design parameters $\rho$. These can be any time-dependent or time-independent parameters that uniquely determine the solution of the ODE (e.g. a boundary condition, material coefficients, etc.). In a discretized ODE setting, the user's time-stepping routine might then be written as

$$
u_{i}=\Phi_{i}\left(u_{i-1}, \rho\right), \quad \forall i=1, \ldots N
$$

where the time-stepper $\Phi_{i}$, which propagates a state $u_{i-1}$ at a time $t_{i-1}$ to the next time step at $t_{i}$, now also depends on the design parameters $\rho$. In order to quantify the simulation output for the given design, a real-valued objective function can then be set up that measures the quality of the ODE solution:

$$
J(\mathbf{u}, \rho) \in \mathrm{R}
$$

Here, $\mathbf{u}=\left(u_{0}, \ldots, u_{N}\right)$ denotes the space-time state solution for a given design.
XBraid_Adjoint is a consistent discrete time-parallel adjoint solver for XBraid which provides sensitivity information of the output quantity $J$ with respect to the user-defined design parameters $\rho$. The ability to compute sensitivities can greatly improve and enhance the simulation tool, for example for solving

- Design optimization problems,
- Optimal control problems,
- Parameter estimation for validation and verification purposes,
- Error estimation,
- Uncertainty quantification techniques.

XBraid_Adjoint is non-intrusive with respect to the adjoint time-stepping scheme so that existing time-serial adjoint codes can be integrated easily though an extended user-interface.

### 3.3.1 Short Introduction to Adjoint-based Sensitivity Computation

Adjoint-based sensitivities compute the total derivative of $J$ with respect to changes in the design parameters $\rho$ by solving additional so-called adjoint equations. We will briefly introduce the idea in the following. You can skip this section, if you are familiar with adjoint sensitivity computation in general and move to Overview of the XBraid_Adjoint Algorithm immedately. Information on the adjoint method can be found in [Giles, Pierce, 2000] ${ }^{3}$ amongst many others.

Consider an augmented (so-called Lagrange) funtion

$$
L(\mathbf{u}, \rho)=J(\mathbf{u}, \rho)+\overline{\mathbf{u}}^{T} A(\mathbf{u}, \rho)
$$

where the discretized time-stepping ODE equations in

$$
A(\mathbf{u}, \rho):=\left(\begin{array}{c}
\Phi_{1}\left(u_{0}, \rho\right)-u_{1} \\
\vdots \\
\Phi_{N}\left(u_{N-1}, \rho\right)-u_{N}
\end{array}\right)
$$

have been added to the objective function, and multiplied with so-called adjoint variables $\overline{\mathbf{u}}=\left(\bar{u}_{1}, \ldots, \bar{u}_{N}\right)$. Since the added term is zero for all design and state variables that satisfy the discrete ODE equations, the total derivative of $J$ and $L$ with respect to the design match. Using the chain rule of differentiation, this derivative can be expressed as

$$
\frac{\mathrm{d} J}{\mathrm{~d} \rho}=\frac{\mathrm{d} L}{\mathrm{~d} \rho}=\frac{\partial J}{\partial \mathbf{u}} \frac{\mathrm{~d} \mathbf{u}}{\mathrm{~d} \rho}+\frac{\partial J}{\partial \rho}+\overline{\mathbf{u}}^{T}\left(\frac{\partial A}{\partial \mathbf{u}} \frac{\mathrm{~d} \mathbf{u}}{\mathrm{~d} \rho}+\frac{\partial A}{\partial \rho}\right)
$$

where $\partial$ denotes partial derivatives - in contrast to the total derivative (i.e. the sensitivity) denoted by d .
When computing this derivative, the terms in red are the ones that are computationally most expensive. In fact, the cost for computing these sensitivities scale linearly with the number of design parameters, i.e. the dimension of $\rho$. These costs can grow quickly. For example, consider a finite differencing setting, where a re-computation of the entire spacetime state would be necessary for each design variable, because a perturbation of the design must be computed in all the unit directions of the design space. In order to avoid these costs, the adjoint method aims to set the adjoint variable $\overline{\mathbf{u}}$ such that these red terms add up to zero in the above expression. Hence, if we solve first for

$$
\left(\frac{\partial J}{\partial \mathbf{u}}\right)^{T}+\left(\frac{\partial A}{\partial \mathbf{u}}\right)^{T} \overline{\mathbf{u}}=0
$$

for the adjoint variable $\overline{\mathbf{u}}$, then the so-called reduced gradient of $J$, which is the transpose of the total derivative of $J$ with respect to the design, is given by

$$
\left(\frac{\mathrm{d} J}{\mathrm{~d} \rho}\right)^{T}=\left(\frac{\partial J}{\partial \rho}\right)^{T}+\left(\frac{\partial A}{\partial \rho}\right)^{T} \overline{\mathbf{u}}
$$

The advantage of this strategy is, that in order to compute the sensitivity of $J$ with respect to $\rho$, only one additional space-time equation (adjoint) for $\overline{\mathbf{u}}$ has to be solved, in addition to evaluating the partial derivatives. The computational cost for computing $\mathrm{d} J / \mathrm{d} \rho$ therefore does not scale in this setting with the number of design parameters.

For the time-dependent discrete ODE problem, the adjoint equation from above reads

$$
\text { unsteady adjoint: } \quad \bar{u}_{i}=\partial_{u_{i}} J(\mathbf{u}, \rho)^{T}+\left(\partial_{u_{i}} \Phi_{i+1}\left(u_{i}, \rho\right)\right)^{T} \bar{u}_{i+1} \quad \forall i=N \ldots, 1
$$

using the terminal condition $u_{N+1}:=0$. The reduced gradient is given by
reduced gradient:

$$
\left(\frac{\partial J}{\partial \rho}\right)^{T}=\partial_{\rho} J(\mathbf{u}, \rho)^{T}+\sum_{i=1}^{N}\left(\partial_{\rho} \Phi_{i}\left(u_{i-1}, \rho\right)\right)^{T} \bar{u}_{i}
$$

[^2]
### 3.3.2 Overview of the XBraid_Adjoint Algorithm

The unsteady adjoint equations can in principle be solved 'backwards in time" in a time-serial manner, starting from the terminal condition $\bar{u}_{N+1}=0$. However, the parallel-in-time XBraid_Adjoint solver offers speedup by distributing the backwards-in-time phase onto multiple processors along the time domain. Its implementation is based on techniques of the reverse-mode of Automatic Differentiation applied to one primal XBraid iteration. To that end, each primal iteration is augmented by an objective function evaluation, followed by updates for the space-time adjoint variable $\overline{\mathbf{u}}$, as well as evaluation of the reduced gradient denoted by $\bar{\rho}$. In particular, the following so-called piggy-back iteration is performed:

1. XBraid: update the state and evaluate the objective function

$$
\mathbf{u}^{(k+1)} \leftarrow \operatorname{XBraid}\left(\mathbf{u}^{(k)}, \rho\right), \quad J \leftarrow J\left(\mathbf{u}^{(k)}, \rho\right)
$$

2. XBraid_Adjoint: update the adjoint and evaluate the reduced gradient

$$
\overline{\mathbf{u}}^{(k+1)} \leftarrow \text { XBraid_Adjoint }\left(\mathbf{u}^{(k)}, \overline{\mathbf{u}}^{(k)}, \rho\right), \quad \bar{\rho} \leftarrow\left(\frac{\mathrm{d} J\left(\mathbf{u}^{(k)}, \rho\right)}{\mathrm{d} \rho}\right)^{T}
$$

Each XBraid_Adjoint iteration moves backwards though the primal XBraid multigrid cycle. It collects local partial derivatives of the elemental XBraid operations in reverse order and concatenates them using the chain rule of differentiation. This is the basic idea of the reverse mode of Automatic Differentiation (AD). This yields a consistent discrete time-parallel adjoint solver that inherits the parallel scaling properties of the primal XBraid solver.

Further, XBraid_Adjoint is non-intrusive for existing adjoint methods based on sequential time marching schemes. It adds additional user-defined routines to the primal XBraid interface, in order to define the propagation of sensitivities of the forward time stepper backwards-in-time and the evaluation of partial derivatives of the local objective function at each time step. In cases where a time-serial unsteady adjoint solver is already available, this backwards time stepping capability can be easily wrapped according to the adjoint user interface with little extra coding.

The adjoint solve in the above piggy-back iteration converges at the same convergence rate as the primal state variables. However since the adjoint equations depend on the state solution, the adjoint convergence will slightly lag behind the convergence of the state. More information on convergence results and implementational details for XBraid_Adjoint can be found in [Gunther, Gauger, Schroder, 2017]. ${ }^{4}$

### 3.3.3 Overview of the XBraid_Adjoint Code

XBraid_Adjoint offers a non-intrusive approach for time-parallelization of existing time-serial adjoint codes. To that end, an extended user-interface allows the user to wrap their existing code for evaluating the objective function and performing a backwards-in-time adjoint step into routines according to the XBraid_Adjoint interface.

[^3]3.3.3.1 Objective function evaluation The user-interface for XBraid_Adjoint allows for objective functions of the following type:
$$
J=F\left(\int_{t_{0}}^{t^{1}} f(u(t), \rho) \mathrm{d} t\right)
$$

This involves a time-integral part of some time-dependent quantity of interest $f$ as well as a postprocessing function $F$. The time-interval boundaries $t_{0}, t_{1}$ can be set using the options braid_SetTStartObjective and braid_SetTStopObjective, otherwise the entire time domain will be considered. Note that these options can be used for objective functions that are only evaluated at one specific time instance by setting $t_{0}=t_{1}$ (e.g. in cases where only the last time step is of interest). The postprocessing function $F$ offers the possibility to further modify the time-integral, e.g. for setting up a tracking-type objective function (substract a target value and square), or for adding relaxation or penalty terms. While defining $f$ is mandatory for XBraid_Adjoint, the postprocessing routine $F$ is optional and is passed to XBraid_Adjoint though the optional braid_SetPostprocessObjective and braid_SetPostprocessObjective_diff routines. XBraid_Adjoint will perform the time-integration by summing up the $f$ evaluations in the given time-domain

$$
I \leftarrow \sum_{i=i_{0}}^{i_{1}} f\left(u_{i}, \rho\right)
$$

followed by a call to the postprocessing function $F$, if set:

$$
J \leftarrow F(I, \rho)
$$

Note that any integration rule for computing $I$, e.g. for scaling contributions from $f()$, must be done by the user.
3.3.3.2 Partial derivatives of user-routines The user needs to provide the derivatives of the time-stepper $\Phi$ and function evaluation $f$ (and potentially $F$ ) for XBraid_Adjoint. Those are provided in terms of transposed matrix-vector products in the following way:

## 1. Derivatives of the objective function $J$ :

- Time-dependent part $f$ : The user provides a routine that evaluates the following transposed partial derivatives of $f$ multiplied with the scalar input $\bar{F}$ :

$$
\begin{gathered}
\bar{u}_{i} \leftarrow\left(\frac{\partial f\left(u_{i}, \rho\right)}{\partial u_{i}}\right)^{T} \bar{F} \\
\bar{\rho} \leftarrow \bar{\rho}+\left(\frac{\partial f\left(u_{i}, \rho\right)}{\partial \rho}\right)^{T} \bar{F}
\end{gathered}
$$

The scalar input $\bar{F}$ equals 1.0 , if no postpocessing function $F$ has been set.

- Postprocessing $F$ : If the postprocessing routine has been set, the user needs to provide it's transposed partial derivatives in the following way:

$$
\begin{gathered}
\bar{F} \leftarrow \frac{\partial F(I, \rho)}{\partial I} \\
\bar{\rho} \leftarrow \rho+\frac{\partial F(I, \rho)}{\partial \rho}
\end{gathered}
$$

2. Derivatives of the time-stepper $\Phi_{i}$ : The user provides a routine that computes the following transposed partial derivatives of $\Phi_{i}$ multiplied with the adjoint input vector $\bar{u}_{i}$ :

$$
\begin{gathered}
\bar{u}_{i} \leftarrow\left(\frac{\partial \Phi\left(u_{i}, \rho\right)}{\partial u_{i}}\right)^{T} \bar{u}_{i} \\
\bar{\rho} \leftarrow \bar{\rho}+\left(\frac{\partial \Phi\left(u_{i}, \rho\right)}{\partial \rho}\right)^{T} \bar{u}_{i}
\end{gathered}
$$

Note that the partial derivatives with respect to $\rho$ always update the reduced gradient $\bar{\rho}$ instead of overwriting it (i.e. they are a plus-equal operation, $+=$ ). Therefore, the gradient needs to be reset to zero before each iteration of XBraid_ $\hookleftarrow$ Adjoint, which is taken care of by XBraid_Adjoint calling an additional user-defined routine braid_PtFcnResetGradient.

Depending on the nature of the design variables, it is neccessary to gather gradient information in $\bar{\rho}$ from all timeprocessors after XBraid_Adjoint has finished. It is the user's responsibility to do that, if needed, e.g. through a call to MPI_Allreduce.
3.3.3.3 Halting tolerance Similar to the primal XBraid algorithm, the user can choose a halting tolerance for XBraid $\hookleftarrow$ _Adjoint which is based on the adjoint residual norm. An absolute tolerance (braid_SetAbsTolAdjoint)

$$
\left\|\overline{\mathbf{u}}^{(k)}-\overline{\mathbf{u}}^{(k-1)}\right\|_{\text {space_time }}<\text { tol_adjoint }
$$

or a relative tolerance (braid_SetRelTolAdjoint)

$$
\frac{\left\|\overline{\mathbf{u}}^{(k)}-\overline{\mathbf{u}}^{(k-1)}\right\|_{\text {space_time }}}{\left\|\overline{\mathbf{u}}^{(1)}-\overline{\mathbf{u}}^{(0)}\right\|_{\text {space_time }}}<\text { tol_adjoint }
$$

can be chosen.
3.3.3.4 Finite Difference Testing You can verify the gradient computed from XBraid_Adjoint using Finite Differences. Let $e_{i}$ denote the $i$-th unit vector in the design space, then the i-th entry of the gradient should match with

$$
i \text {-th Finite Difference: } \frac{J\left(\mathbf{u}_{\rho+h e_{i}}, \rho+h e_{i}\right)-J(\mathbf{u}, \rho)}{h}
$$

for a small perturbation $h>0$. Here, $\mathbf{u}_{\rho+h e_{i}}$ denotes the new state solution for the perturbed design variable. Keep in mind, that round-off errors have to be considered when computing the Finite Differences for very small perturbations $h \rightarrow 0$. Hence, you should vary the parameter to find the best fit.

In order to save some computational work while computing the perturbed objective function value, XBraid_Adjoint can run in ObjectiveOnly mode, see braid_SetObjectiveOnly. When in this mode, XBraid_Adjoint will only solve the ODE system and evaluate the objective function, without actually computing its derivative. This option might also be useful within an optimization framework e.g. for implementing a line-search procedure.

### 3.3.3.5 Getting started

- Look at the simple example Simplest XBraid_Adjoint example in order to get started. This example is in examples/ex-01-adjoint.c, which implements XBraid_Adjoint sensitivity computation for a scalar ODE.


### 3.4 XBraid Delta Correction: Accelerating Convergence and Estimating Lyapunov Vectors

Certain systems, especially chaotic systems, exhibit sensitivity to perturbations along a trajectory, where such perturbations can grow exponentially fast in time. While this sensitivity may go unnoticed in a serial time-marching simulation, it can seriously degrade the convergence rate of XBraid. The propagation of small perturbations along such an unstable trajectory is governed by the linear tangent propagator, $F_{i}$, which for a discrete time system, corresponds with the Jacobian of the time-stepping operator, $\frac{d \Phi}{d u_{i}}$. i.e. if $v_{i}$ is a small perturbation to the solution $u_{i}$ at time $i$, then

$$
\Phi\left(u_{i}+v_{i}\right) \approx \Phi\left(u_{i}\right)+\frac{d \Phi}{d u} \cdot v_{i}=u_{i+1}+v_{i+1}
$$

and we see that the propagation of $v$ along a fixed trajectory $u$ is determined by the linear recurrance $v_{i+1}=F_{i} v_{i}$. Since a different propagator is used on the coarse grid, $\Phi_{\Delta}$, the coarse grid equation will have a different linear tangent propagator, and the propagation of small perturbations could be catastrophically wrong. Thus, to correct this, XBraid Delta correction uses Jacobians of $\Phi$, computed on the fine grid, to correct the coarse grid operator, i.e. the Delta correction is given by

$$
\Delta_{i}=\frac{d \Phi^{m}}{d u_{i-m}}-\frac{d \Phi_{\Delta}}{d u_{i-m}},
$$

and it is used to correct the coarse grid time-stepping operator $\Phi_{\Delta}$ like

$$
u_{i}=\Phi_{\Delta}\left(u_{i-m}\right)+\Delta_{i} u_{i-m}+\tau_{i},
$$

where $\tau_{i}$ is the FAS tau-correction term. This ensures that, as the solution $u$ converges, the linear tangent propagator on the coarse grid will approach that of the fine-grid.

The Xbraid Delta correction option can potentially accelerate convergence, (converging quadratically in special cases) at the cost of each iteration being more costly. It is intended to be used for chaotic, unsteady, or otherwise challenging systems, but it is very unlikely to provide convergence when the basic XBraid iteration is unstable. Care should be exercised when using this option, see the paper https://arxiv.org/abs/2208.12629. The option also provides estimates for the Lyapunov vectors and exponents of the system, which are explained in more detail below.

### 3.4.1 The Lyapunov Spectrum

The Lyapunov exponents (LEs) of a system characterize the average growth rate of these perturbations, and the associated Lyapunov vectors (LVs) give the directions along which these perturbations grow with that particular rate. A system has as many LEs and associated LVs as spatial degrees of freedom. A positive LE, $\lambda_{j}>0$, indicates that a perturbation in the direction of the associated $\mathrm{LV}, \psi_{j}$ will grow exponentially fast, with average rate $\lambda_{j}$. Likewise, a negative LE indicates exponential decay of perturbations in the direction of the associated LV, and a vanishing LE indicates that, on average, a perturbation along in the associated direction does not grow or decay. The full Lyapunov spectrum of a system qualitatively describes the nonlinear system, and a chaotic system will have at least one LE which is positive. The subsets of LVs having positive, vanishing, and negative exponents are called the unstable, neutral, and stable manifolds, respectively. Note, the $\psi_{j}$ are functions of time.

In many cases, the Lyapunov spectrum on the coarse grid, induced by $\Phi_{\Delta}$, will not match that of the fine grid, since they will have different linear tangent propagators. The result of this is that, for a chaotic system, a small error may grow very large during the coarse grid solve, where it will grow along the unstable LVs which don't match those of the fine grid, causing degradation of convergence and stalling.

### 3.4.2 Overview of the Low-Rank Delta Correction Algorithm

While using the full Jacobian of the time-stepping operators yields quadratic convergence, the computation of the Jacobian is too expensive for systems with many spatial dimensions, since computing the Jacobian for a system having $n_{x}$ spatial degrees of freedom will require $\mathcal{O}\left(n_{x}^{2}\right)$ work. For this reason, XBraid instead computes the action of the Jacobian on a small number $k$, of basis vectors, $\Psi_{i}$ which are initialized by the user. Then a low rank approximation (of rank $k$ ) of $\Delta_{i}$ is used in place of the full matrix, i.e. the correction on the coarse grid becomes

$$
u_{i}=\Phi_{\Delta}\left(u_{i-m}\right)+\Delta_{i} \Psi_{i} \Psi_{i}^{T} u_{i-m}+\tau_{i}
$$

where the $k \times n_{x}$ matrices $\left(\Delta_{i} \Psi_{i}\right)$ and $\Psi_{i}$ are stored as seperate factors. This reduces the overall work of computing the Delta correction to $\mathcal{O}\left(k n_{x}\right)$.

By default, Delta correction will use the user initialized basis, but the Lyapunov estimation option allows Braid to compute estimates to the first $k$ backward Lyapunov vectors of the system, using the initialized basis as an initial guess, and the Delta correction will be computed on the computed Lyapunov basis, meaning that the corrections will target the unstable manifold of the system first. This is especially useful for chaotic systems, where the dimension of the unstable manifold is often much smaller than the total number of spatial dimensions. The Lyapunov vectors are orthonormalized at the C points using modified Gram-Schmidt, according to the recurrance

$$
\Psi_{i} R_{i}=\left(\frac{d \Phi}{d u_{i-1}}\right) \Psi_{i-1}
$$

where $R_{i}$ is an upper triangular matrix. Repeated iteration of this, as $i \rightarrow \infty$ will cause the $k$ columns of $\Psi_{i}$ to converge to the first $k$ backward LVs, while the diagonal entries of each $R_{i}$ will contain the local Lyapunov exponents, whose average over time yields the true LEs. Lyapunov estimation in XBraid essentially applies the MGRIT algorithm to the above recurrance relationship, solving for the LVs and LEs parallel-in-time, simultaneously with the state solution. These estimated LVs then provide a basis for Delta correction, which targets the slowest converging modes of error, which are along the unstable and neutral manifolds.

### 3.4.3 Overview of the Delta Correction Code

The Delta correction maintains the non-intrusive philosophy used by the rest of the XBraid code, and thus the user must provide a couple of new wrapper functions in order to enable the feature, including the added requirement that the user's step function be able to compute the Jacobian vector product for the $k$ basis vectors of $\Psi$. Delta correction is enabled by calling braid_SetDeltaCorrection which requires the number (rank) of basis vectors, a pointer to a function which initializes basis vectors, and a pointer to a function which computes the inner product between two user vectors. These are described in more detail below.

Lyapunov vector estimation is enabled by calling the function braid_SetLyapunovEstimation which controls whether LVs are estimated on the coarse grid (more serial work, much more accurate) and whether LVs are computed during FCF relaxation (more parallel work, less accurate). The LV and LE estimates are available through the AccessStatus structure.

To mitigate some of the extra cost of Delta correction, while still maintaining some accelerated convergence, Delta correction may be deferred to a coarse grid, meaning that Delta corrections will not be computed on the fine grid, but will be computed on all coarser grids after the specfied level. Delta correction may also be deferred to a later iteration, meaning that XBraid will proceed without Delta correction until the given iteration. These options are controlled via the function braid_SetDeferDelta.
3.4.3.1 Step Function Jacobian Vector Product The user's step function can access references to the $k$ Lyapunov basis vectors from the StepStatus structure (see also examples/ex-07), and for each basis vector $\psi_{j}$, the step function should be able to compute the Jacobian-vector product

$$
\psi_{j} \leftarrow\left(\frac{d \Phi}{d u}\right) \psi_{j}
$$

While some innacuracy here is acceptable, (so e.g. finite difference approximations may be used), if the Jacobian product is too innacurate, there may be no benefit from using Delta correction, since the correction will be innacurate. It is very important that the set of vectors $\psi_{j}$ remain linearly independentt after being propagated by the user's step function, so it is advised not to use an approximation of rank lower than the number of basis vectors used, e.g. a Krylov subspace approximation of the Jacobian of dimension less than $k$ should not be used for this purpose.
3.4.3.2 Inner Product Function The user must provide a function braid_PtFcnInnerProd which computes an inner product between two user vectors and returns a scalar result. The Euclidean dot product between two vectors is an example. This function is used to project the state vector onto the basis vectors and for Gram-Schmidt orthonormalization of basis vectors.
3.4.3.3 Basis Vector Initialization Function The user must provide a function braid_PtFcnInitBasis which initializes a single basis vector, at a given time with a given spatial index. The spatial index is simply used to distinguish between the different basis vectors at a given time point. The basis vectors may be the columns of the identity matrix, a Fourier basis, or any other linearly independent basis of physical relevance to the system. While the vectors need not be orthonormal, they must be linearly independent, since they will be orthonormalized using modified Gram-Schmidt.
3.4.3.4 Buffer Size Function The user buffer size function is reused by Delta correction to allocate a buffer to pack the basis vectors, althought the user may specify a different size for the state vector and the basis vectors. The size of the state vector should be set as normal, but the user may set an optional size for the basis vectors through the Buffer $\hookleftarrow$ Status structure. This is useful in case the state vector contains time-dependent information which is not propagated by $\Phi$, e.g. a time-dependent forcing term, and which does not need to be duplicated in every single basis vector. The user provided buffer packing and unpacking functions do not need to be changed for Delta correction, but they should be aware of any differences between state vectors and basis vectors.
3.4.3.5 Testing Delta Correction Wrapper Functions A routine for testing the user provided inner product function is provided in braid_TestlnnerProd. A routine for testing the users basis initialization, step, buffer size, buffer packing, and buffer unpacking functions for use with Delta correction is provided in braid_TestDelta. These functions can be accessed by including the braid_test header file.

### 3.4.4 Getting Started

To familiarize yourself with XBraid Delta correction, please see the example Lorenz System with Delta Correction, located in examples/ex-07.c, which demonstrates solving the chaotic Lorenz system using Delta correction and Lyapunov estimation.

### 3.5 Citing XBraid

To cite XBraid, please state in your text the version number from the VERSION file, and please cite the project website in your bibliography as
[1] XBraid: Parallel multigrid in time. http://llnl.gov/casc/xbraid.

The corresponding BibTex entry is

```
@misc{xbraid-package,
    title = {{XB}raid: Parallel multigrid in time},
    howpublished = {\url{http://llnl.gov/casc/xbraid}}
    }
```


### 3.6 Summary

- XBraid applies multigrid to the time dimension.
- This exposes concurrency in the time dimension.
- The potential for speedup is large, 10x, 100x, ...
- This is a non-intrusive approach, with an unchanged time discretization defined by user.
- Parallel time integration is only useful beyond some scale.

This is evidenced by the experimental results below. For smaller numbers of cores sequential time stepping is faster, but at larger core counts XBraid is much faster.

- The more time steps that you can parallelize over, the better your speedup will be.
- XBraid is optimal for a variety of parabolic problems (see the examples directory).
- XBraid_Adjoint provides time-parallel adjoint-based sensitivities of output quantities with respect to user-defined design variables
- It is non-intrusive with respect to existing adjoint time-marching schemes
- It inherits parallel scaling properties from XBraid


## 4 Examples

This section is the chief tutorial of XBraid, illustrating how to use it through a sequence of progressively more sophisticated examples.

### 4.1 The Simplest Example

### 4.1.1 User Defined Structures and Wrappers

The user must wrap their existing time stepping routine per the XBraid interface. To do this, the user must define two data structures and some wrapper routines. To make the idea more concrete, we now give these function definitions from examples/ex-01, which implements a scalar ODE,

$$
u_{t}=\lambda u
$$

The two data structures are:

1. App: This holds a wide variety of information and is global in that it is passed to every function. This structure holds everything that the user will need to carry out a simulation. Here for illustration, this is just an integer storing a processor's rank.
```
typedef struct _braid_App_struct
{
    int rank;
} my_App;
```

2. Vector: this defines (roughly) a state vector at a certain time value. It could also contain any other information related to this vector which is needed to evolve the vector to the next time value, like mesh information. Here, the vector is just a scalar double.
```
typedef struct _braid_Vector_struct
{
    double value;
} my_Vector;
```

The user must also define a few wrapper routines. Note, that the app structure is the first argument to every function.

1. Step: This function tells XBraid how to take a time step, and is the core user routine. The user must advance the vector $u$ from time tstart to time tstop. Note how the time values are given to the user through the status structure and associated Get routine. Important note: the $g_{i}$ function from Overview of the XBraid Algorithm must be incorporated into Step, so that the following equation is solved by default.

$$
\Phi\left(u_{i}\right)=0
$$

The ustop parameter serves as an approximation to the solution at time tstop and is not needed here. It can be useful for implicit schemes that require an initial guess for a linear or nonlinear solver. The use of fstop is an advanced parameter (not required) and forms the the right-hand side of the nonlinear problem on the given time grid. This value is only nonzero when providing a residual with braid_SetResidual. More information on how to use this optional feature is given below.

Here advancing the solution just involves the scalar $\lambda$.

```
int
my_Step(braid_App app,
    braid_Vector ustop,
    braid_Vector fstop,
    braid_Vector u,
    braid_StepStatus status)
{
```

```
    double tstart; /* current time */
    double tstop; /* evolve to this time*/
    braid_StepStatusGetTstartTstop(status, &tstart, &tstop);
    /* Use backward Euler to propagate solution */
    (u->value) = 1./(1. + tstop-tstart)*(u->value);
    return 0;
}
```

2. Init: This function tells XBraid how to initialize a vector at time $t$. Here that is just allocating and setting a scalar on the heap.
```
int
my_Init(braid_App app,
        double t,
        braid_Vector *u_ptr)
{
    my_Vector *u;
    u = (my_Vector *) malloc(sizeof(my_Vector));
    if (t == 0.0) /* Initial condition */
    {
        (u->value) = 1.0;
    }
    else /* All other time points set to arbitrary value */
    {
        (u->value) = 0.456;
    }
    *u_ptr = u;
    return 0;
}
```

3. Clone: This function tells XBraid how to clone a vector into a new vector.
```
int
my_Clone(braid_App app,
    braid_Vector u,
    braid_Vector *v_ptr)
{
    my_Vector *v;
    v = (my_Vector *) malloc(sizeof(my_Vector));
    (v->value) = (u->value);
    *v_ptr = v;
    return 0;
}
```

4. Free: This function tells XBraid how to free a vector.
```
int
my_Free(braid_App app,
        braid_Vector u)
{
    free(u);
    return 0;
}
```

5. Sum: This function tells XBraid how to sum two vectors (AXPY operation).
```
int
my_Sum(braid_App app,
    double alpha,
    braid_Vector x,
    double beta,
```

```
        braid_Vector y)
{
    (y->value) = alpha*(x->value) + beta*(y->value);
    return 0;
}
```

6. SpatialNorm: This function tells XBraid how to take the norm of a braid_Vector and is used for halting. This norm is only over space. A common norm choice is the standard Euclidean norm, but many other choices are possible, such as an L2-norm based on a finite element space. The norm choice should be based on what makes sense for your problem. How to accumulate spatial norm values to obtain a global space-time residual norm for halting decisions is controlled by braid_SetTemporalNorm.
```
int
my_SpatialNorm(braid_App app,
    braid_Vector u,
    double *norm_ptr)
{
    double dot;
    dot = (u->value)*(u->value);
    *norm_ptr = sqrt(dot);
    return 0;
}
```

7. Access: This function allows the user access to XBraid and the current solution vector at time $t$. This is most commonly used to print solution(s) to screen, file, etc... The user defines what is appropriate output. Notice how you are told the time value $t$ of the vector $u$ and even more information in astatus. This lets you tailor the output to only certain time values at certain XBraid iterations. Querying astatus for such information is done through braid_AccessStatusGet**(..) routines.

The frequency of the calls to access is controlled through braid_SetAccessLevel. For instance, if access_level is set to 2 , then access is called every XBraid iteration and on every XBraid level. In this case, querying astatus to determine the current XBraid level and iteration will be useful. This scenario allows for even more detailed tracking of the simulation. The default access_level is 1 and gives the user access only after the simulation ends and only on the finest time-grid.

Eventually, this routine will allow for broader access to XBraid and computational steering.

See examples/ex-03 and drivers/drive-diffusion for more advanced uses of the access function. In drive-diffusion, access is used to write solution vectors to a GLVIS visualization port, and ex-03 uses access to write to .vtu files.

```
int
my_Access(braid_App app,
            braid_Vector u,
            braid_AccessStatus astatus)
{
    int index;
    char filename[255];
    FILE *file;
    braid_AccessStatusGetTIndex(astatus, &index);
    sprintf(filename, "%s.%04d.%03d", "ex-01.out", index, app->rank);
    file = fopen(filename, "w");
    fprintf(file, "%.14e\n", (u->value));
    fflush(file);
    fclose(file);
    return 0;
}
```

8. BufSize, BufPack, BufUnpack: These three routines tell XBraid how to communicate vectors between processors. BufPack packs a vector into a void $*$ buffer for MPI and then BufUnPack unpacks the void $*$ buffer into a vector. Here doing that for a scalar is trivial. BufSize computes the upper bound for the size of an arbitrary vector.

Note how BufPack also sets the size in bstatus. This value is optional, but if set it should be the exact number of bytes packed, while BufSize should provide only an upper-bound on a possible buffer size. This flexibility allows for the buffer to be allocated the fewest possible times, but smaller messages to be sent when needed. For instance, this occurs when using variable spatial grid sizes. To avoid MPI issues, it is very important that BufSize be pessimistic, provide an upper bound, and return the same value across processors.

In general, the buffer should be self-contained. The receiving processor should be able to pull all necessary information from the buffer in order to properly interpret and unpack the buffer.

```
int
my_BufSize(braid_App app,
    int *size_ptr,
    braid_BufferStatus bstatus)
{
    *size_ptr = sizeof(double);
    return 0;
}
int
my_BufPack(braid_App app,
            braid_Vector u,
            void *buffer,
            braid_BufferStatus bstatus)
{
    double *dbuffer = buffer;
    dbuffer[0] = (u->value);
    braid_BufferStatusSetSize( bstatus, sizeof(double) );
    return 0;
}
int
my_BufUnpack(braid_App app,
                        void *buffer,
            braid_Vector *u_ptr,
            braid_BufferStatus bstatus)
{
    double *dbuffer = buffer;
    my_Vector *u;
    u = (my_Vector *) malloc(sizeof(my_Vector));
    (u->value) = dbuffer[0];
    *u_ptr = u;
    return 0;
}
```


### 4.1.2 Running XBraid for the Simplest Example

A typical flow of events in the main function is to first initialize the app structure.

```
/* set up app structure */
app = (my_App *) malloc(sizeof(my_App));
(app->rank) = rank;
```

Then, the data structure definitions and wrapper routines are passed to XBraid. The core structure is used by XBraid for internal data structures.

```
braid_Core core;
braid_Init(MPI_COMM_WORLD, comm, tstart, tstop, ntime, app,
    my_Step, my_Init, my_Clone, my_Free, my_Sum, my_SpatialNorm,
    my_Access, my_BufSize, my_BufPack, my_BufUnpack, &core);
```

Then, XBraid options are set.

```
braid_SetPrintLevel( core, 1);
braid_SetMaxLevels(core, max_levels);
braid_SetAbsTol(core, tol);
braid_SetCFactor(core, -1, cfactor);
```

Then, the simulation is run.
braid_Drive(core);

Then, we clean up.
braid_Destroy(core);

Finally, to run ex-01, type
ex-01

### 4.2 Some Advanced Features

We now give an overview of some optional advanced features that will be implemented in some of the following examples.

1. SCoarsen, SRestrict: These are advanced options that allow for coarsening in space while you coarsen in time. This is useful for maintaining stable explicit schemes on coarse time scales and is not needed here. See examples/ex-02 for a simple example of this feature, and then drivers/drive-diffusion and drivers/drive-diffusion-2D for more advanced examples of this feature.

These functions allow you to vary the spatial mesh size on XBraid levels as depicted here where the spatial and temporal grid sizes are halved every level.

2. Residual: A user-defined residual can be provided with the function braid_SetResidual and can result in substantial computational savings, as explained below.
However to use this advanced feature, one must first understand how XBraid measures the residual. XBraid computes residuals of this equation,

$$
A_{i}\left(u_{i}, u_{i-1}\right)=f_{i}
$$

where $A_{i}($,$) evaluates one block-row of the the global space-time operator A$. The forcing $f_{i}$ is the XBraid forcing, which is the FAS right-hand-side term on coarse grids and 0 on the finest grid. The PDE forcing goes inside of $A_{i}$.
Since XBraid assumes one-step methods, $A_{i}()$ is defined to be

$$
A_{i}\left(u_{i}, u_{i-1}\right)=-\Phi\left(u_{i-1}\right)+\Psi\left(u_{i}\right)
$$

i.e., the subdiagonal and diagonal blocks of $A$.

Default setting: In the default XBraid setting (no residual option used), the user only implements Step() and Step() will simply apply $\Phi()$, because $\Psi()$ is assumed to be the identity. Thus, XBraid can compute the residual using only the user-defined Step() function by combining Step() with the Sum() function, i.e.

$$
r_{i}=f_{i}+\Phi\left(u_{i-1}\right)-u_{i} .
$$

The fstop parameter in Step() corresponds to $f_{i}$, but is always passed in as NULL to the user in this setting and should be ignored. This is because XBraid can compute the contribution of $f_{i}$ to the residual on its own using the Sum() function.
An implication of this is that the evaluation of $\Phi()$ on the finest grid must be very accurate, or the residual will not be accurate. This leads to a nonintrusive, but expensive algorithm. The accuracy of $\Phi()$ can be relaxed on coarser grids to save computations.

Residual setting: The alternative to the above default least-intrusive strategy is to have the user define

$$
A_{i}\left(u_{i}, u_{i-1}\right)=-\Phi\left(u_{i-1}\right)+\Psi\left(u_{i}\right)
$$

directly, which is what the Residual function implements (set with braid_PtFcnResidual). In other words, the user now defines each block-row of the space-time operator, rather than only defining $\Phi()$. The user Residual() function computes $A_{i}\left(u_{i}, u_{i-1}\right)$ and XBraid then subtracts this from $f_{i}$ to compute $r_{i}$.
However, more care must now be taken when defining the Step() function. In particular, the fstop value (i.e., the $f_{i}$ value) must be taken into account. Essentially, the definition of Step() changes so that it no longer defines $\Phi()$, but instead defines a (possibly inexact) solve of the equation defined by

$$
A_{i}\left(u_{i}, u_{i-1}\right)=f_{i}
$$

Thus, Step() must be compatible with Residual(). Expanding the previous equation, we say that Step() must now compute

$$
u_{i}=\Psi^{-1}\left(f_{i}+\Phi\left(u_{i-1}\right)\right)
$$

It is clear that the fstop value (i.e., the $f_{i}$ value) must now be given to the $\operatorname{Step}()$ function so that this equation can be solved by the user. In other words, fstop is now no longer NULL.

Essentially, one can think of Residual() as defining the equation, and Step() defining a preconditioner for that row of the equation, or an inexact solve for $u_{i}$.
As an example, let $\Psi=(I+\Delta t L)$, where $L$ is a Laplacian and $\Phi=I$. The application of the residual function will only be a sparse matrix-vector multiply, as opposed to the default case where an inversion is required for $\Phi=(I+\Delta t L)^{-1}$ and $\Psi=I$. This results in considerable computational savings. Moreover, the application of Step() now involves an inexact inversion of $\Psi$, e.g., by using just one spatial multigrid V-cycle. This again results in substantial computation savings when compared with the naive approach of a full matrix inversion.

Another way to think about the compatibility between $\Psi$ and $\Phi$ is that

$$
f_{i}-A_{i}\left(u_{i}, u_{i-1}\right)=0
$$

must hold exactly if $u_{i}$ is an exact propagation of $u_{i-1}$, that is,

$$
f_{i}-A_{i}\left(S t e p\left(u_{i-1}, f_{i}\right), u_{i-1}\right)=0
$$

must hold. When the accuracy of the Step() function is reduced (as mentioned above), this exact equality with 0 is lost, but this should evaluate to something small. There is an XBraid test function braid_TestResidual that tests for this compatibility.

The residual feature is implemented in the examples examples/ex-01-expanded.c, examples/ex-02. $\leftarrow$ c, and examples/ex-03.c.
3. Adaptive and variable time stepping: This feature is available by first calling the function braid_SetRefine in the main driver and then using braid_StepStatusSetRFactor in the Step routine to set a refinement factor for interval [tstart, tstop]. In this way, user-defined criteria can subdivide intervals on the fly and adaptively refine in time. For instance, returning a refinement factor of 4 in Step will tell XBraid to subdivide that interval into 4 evenly spaced smaller intervals for the next iteration. Refinement can only be done on the finest XBraid level.

The final time grid is constructed adaptively in an FMG-like cycle by refining the initial grid according to the requested refinement factors. Refinement stops when the requested factors are all one or when various upper bounds are reached such as the max number of time points or max number of time grid refinement levels allowed. No restriction on the refinement factors is applied within XBraid, so the user may want to apply his own upper bound on the refinement factors to avoid over-refinement. See examples/ex-01-refinement.c and examples/ex-03.c for an implementation of this.
4. Richardson-based Error Estimation and Extrapolation: This feature allows the user to access built-in Richardson-based error estimates and accuracy improving extrapolation. The error estimates and/or extrapolation can be turned on by using braid_SetRichardsonEstimation. Moreover, this feature can be used in conjunction with the above discussed function, braid_StepStatusSetRFactor, to achieve easy-to-use adaptive refinement in time.

Essentially, Richardson extrapolation (RE) is used to improve the accuracy of the solution at the C-points on the finest level. When the built-in error estimate option is turned on, RE is used to estimate the local truncation error at each point. These estimates can be accessed through StepStatus and AccessStatus functions.
The Richardson-based error estimates and extrapolation are only available after the first Braid iteration, in that the coarse level solution must be available to compute the error estimate and/or extrapolation. Thus, after an adaptive refinement (and new hierarchy is constructed), another iteration is again required for the error estimates to be available. If the error estimate isn't available, Braid returns a value of -1. See this example for more details
examples/ex-06.c
5. Shell-vector: This feature supports the use of multi-step methods. The strategy for BDF-K methods is to allow for the lumping of $k$ time points into a single XBraid vector. So, if the problem had 100 time points and the time-stepper was BDF-2, then XBraid would only see 50 time points but each XBraid vector would contain two separate time points. By lumping 2 time points into one vector, the BDF-2 scheme remains one-step and compatible with XBraid.

However, the time-point spacing between the two points internal to the vector stays the same on all time grids, while the spacing between vectors grows on coarse time grids. This creates an irregular spacing which is problematic for BDF-k methods. Thus the shell-vector strategy lets meta-data be stored at all time points, even for F-points which are usually not stored, so that the irregular spacings can be tracked and accounted for with the BDF method. (Note, there are other possible uses for shell-vectors.)
There are many strategies for handling the coarse time-grids with BDF methods (dropping the BDF order, adjusting time-point spacings inside the lumped vectors, etc...). Prospective users are encouraged to contact the devlopers through the XBraid Github page and issue tracker. This area is active research.
See examples/ex-01-expanded-bdf2.c.
6. Storage: This option (see braid_SetStorage) allows the user to specify storage at all time points ( $C$ and $F$ ) or only at C-points. This extra storage is useful for implicit methods, where the solution value from the previous XBraid iteration for time step $i$ can be used as the initial guess when computing step $i$ with the implicit solver. This is often a better initial guess than using the solution value from the previous time step $i-1$. The default is to store only C-point values, thus the better initial guess is only available at C-points in the default setting. When storage is turned on at F-points, the better initial guess becomes available everywhere.
In general, the user should always use the ustop parameter in Step() as the initial guess for an implicit solve. If storage is turned on (i.e., set to 0 ), then this value will always be the improved initial guess for C - and F -points. If storage is not turned on, then this will be the improved guess only for C -points. For F -points, it will equal the solution from the previous time step.
See examples/ex-03 for an example which uses this feature.
7. Delta Correction and Lyapunov Vector Estimation: These options (see braid_SetDeltaCorrection and braid_SetLyapunovEstimation) allow XBraid to accelerate convergence by using Delta correction, which was originally designed for use with chaotic systems. The feature works by using low rank approximations to the Jacobian of the fine grid time-stepper as a linear correction to the coarse grid time-stepper. This can converge quadratically in some cases. LyapunovEstimation is not required for Delta correction, but for chaotic systems, the unstable modes of error, corresponding with the first few Lyapunov vectors, are often the slowest to converge. Thus, Lyapunov estimation targets these modes by computing estimates to the backward Lyapunov vectors of the system, then computing the Delta correction using these vectors as a basis.

See examples/ex-07 for an example which uses these features.

### 4.3 Simplest example expanded

These examples build on The Simplest Example, but still solve the scalar ODE,

$$
u_{t}=\lambda u
$$

The goal here is to show more advanced features of XBraid.

- examples/ex-01-expanded.c: same as ex-01.c but adds more XBraid features such as the residual feature, the user defined initial time-grid and full multigrid cycling.
- examples/ex-01-expanded-bdf2.c: same as ex-01-expanded.c, but uses BDF2 instead of backward Euler. This example makes use of the advanced shell-vector feature in order to implement BDF2.
- examples/ex-01-expanded-f.f90: same as ex-01-expanded.c, but implemented in f90.
- examples/ex-01-refinement.c: same as ex-01.c, but adds the refinement feature of XBraid. The refinement can be arbitrary or based on error estimate.


### 4.4 One-Dimensional Heat Equation

In this example, we assume familiarity with The Simplest Example. This example is a time-only parallel example that implements the 1D heat equation,

$$
\delta / \delta_{t} u(x, t)=\Delta u(x, t)+g(x, t)
$$

as opposed to The Simplest Example, which implements only a scalar ODE for one degree-of-freedom in space. There is no spatial parallelism, as a serial cyclic reduction algorithm is used to invert the tri-diagonal spatial operators. The space-time discretization is the standard 3-point finite difference stencil ( $[-1,2,-1]$ ), scaled by mesh widths. Backward Euler is used in time.

This example consists of three files and two executables.

- examples/ex-02-serial.c: This file compiles into its own executable ex-02-serial and represents a simple example user application that does sequential time-stepping. This file represents where a new XBraid user would start, in terms of converting a sequential time-stepping code to XBraid.
- examples/ex-02.c: This file compiles into its own executable ex-02 and represents a time-parallel XBraid wrapping of the user application ex-02-serial.
- ex-02-lib.c: This file contains shared functions used by the time-serial version and the time-parallel version. This file provides the basic functionality of this problem. For instance, take_step(u, tstart, tstop, ...) carries out a step, moving the vector $u$ from time tstart to time tstop.


### 4.5 Two-Dimensional Heat Equation

In this example, we assume familiarity with The Simplest Example and describe the major ways in which this example differs. This example is a full space-time parallel example, as opposed to The Simplest Example, which implements only a scalar ODE for one degree-of-freedom in space. We solve the heat equation in 2D,

$$
\delta / \delta_{t} u(x, y, t)=\Delta u(x, y, t)+g(x, y, t)
$$

For spatial parallelism, we rely on the hypre package where the SemiStruct interface is used to define our spatial discretization stencil and form our time stepping scheme, the backward Euler method. The spatial discretization is just the standard 5 -point finite difference stencil ( $[-1 ;-1,4,-1 ;-1]$ ), scaled by mesh widths, and the PFMG solver is used for the solves required by backward Euler. Please see the hypre manual and examples for more information on the SemiStruct interface and PFMG. Although, the hypre specific calls have mostly been abstracted away for this example, and so it is not necessary to be familiar with the SemiStruct interface for this example.

This example consists of three files and two executables.

- examples/ex-03-serial.c: This file compiles into its own executable ex-03-serial and represents a simple example user application. This file supports only parallelism in space and represents a basic approach to doing efficient sequential time stepping with the backward Euler scheme. Note that the hypre solver used (PFMG) to carry out the time stepping is highly efficient.
- examples/ex-03.c: This file compiles into its own executable ex-03 and represents a basic example of wrapping the user application ex-03-serial. We will go over the wrappers below.
- ex-03-lib.c: This file contains shared functions used by the time-serial version and the time-parallel version. This is where most of the hypre specific calls reside. This file provides the basic functionality of this problem. For instance, take_step( $u$, tstart, tstop, ...) carries out a step, moving the vector $u$ from time tstart to time tstop and setUpImplicitMatrix(...) constructs the matrix to be inverted by PFMG for the backward Euler method.


### 4.5.1 User Defined Structures and Wrappers

We now discuss in more detail the important data structures and wrapper routines in examples/ex-03.c. The actual code for this example is quite simple and it is recommended to read through it after this overview.

The two data structures are:

1. App: This holds a wide variety of information and is global in that it is passed to every user function. This structure holds everything that the user will need to carry out a simulation. One important structure contained in the app is the simulation_manager. This is a structure native to the user code ex-03-lib.c. This structure conveniently holds the information needed by the user code to carry out a time step. For instance,
```
app->man->A
```

is the time stepping matrix,

```
app->man->solver
```

is the hypre PFMG solver object,

```
app->man->dt
```

is the current time step size. The app is defined as

```
typedef struct _braid_App_struct {
    MPI_Comm comm; /* global communicator */
    MPI_Comm comm_t; /* communicator for parallelizing in time */
    MPI_Comm comm_x; /* communicator for parallelizing in space */
    int pt; /* number of processors in time */
    simulation_manager *man; /* user's simulation manager structure */
    HYPRE_SStructVector e; /* temporary vector used for error computations */
    int nA; /* number of spatial matrices created */
    HYPRE_SStructMatrix *A; /* array of spatial matrices, size nA, one per level*/
    double *dt_A; /* array of time step sizes, size nA, one per level*/
    HYPRE_StructSolver *solver; /* array of PFMG solvers, size nA, one per level*/
    int use_rand; /* binary value, use random or zero initial guess */
    int *runtime_max_iter; /* runtime info for number of PFMG iterations*/
    int *max_iter_x; /* maximum iteration limits for PFMG */
} my_App;
```

The app contains all the information needed to take a time step with the user code for an arbitrary time step size. See the Step function below for more detail.
2. Vector: this defines a state vector at a certain time value.

Here, the vector is a structure containing a native hypre data-type, the SStructVector, which describes a vector over the spatial grid. Note that my_Vector is used to define braid_Vector.

```
typedef struct _braid_Vector_struct {
    HYPRE_SStructVector x;
} my_Vector;
```

The user must also define a few wrapper routines. Note, that the app structure is the first argument to every function.

1. Step: This function tells XBraid how to take a time step, and is the core user routine. This function advances the vector $u$ from time tstart to time tstop. A few important things to note are as follows.

- The time values are given to the user through the status structure and associated Get routines.
- The basic strategy is to see if a matrix and solver already exist for this $d t$ value. If not, generate a new matrix and solver and store them in the app structure. If they do already exist, then re-use the data.
- To carry out a step, the user routines from ex-03-lib. c rely on a few crucial data members man->dt, man->A and man-solver. We overwrite these members with the correct information for the time step size in question. Then, we pass man and $u$ to the user function take_step(...) which evolves $u$.
- The forcing term $g_{i}$ is wrapped into the take_step(...) function. Thus, $\Phi\left(u_{i}\right) \rightarrow u_{i+1}$.

```
int my_Step(braid_App app,
    braid_Vector u,
    braid_StepStatus status)
{
    double tstart; /* current time */
    double tstop; /* evolve u to this time*/
    int i, A_idx;
    int iters_taken = -1;
    /* Grab status of current time step */
    braid_StepStatusGetTstartTstop(status, &tstart, &tstop);
    /* Check matrix lookup table to see if this matrix already exists*/
    A_idx = -1.0;
    for( i = 0; i < app->nA; i++ ) {
        if( fabs( app->dt_A[i] - (tstop-tstart) )/(tstop-tstart) < 1e-10) {
            A_idx = i;
            break;
        }
    }
    /* We need to "trick" the user's manager with the new dt */
    app->man->dt = tstop - tstart;
    /* Set up a new matrix and solver and store in app */
    if( A_idx == -1.0 ) {
        A_idx = i;
        app->nA++;
        app->dt_A[A_idx] = tstop-tstart;
        setUpImplicitMatrix( app->man );
        app->A[A_idx] = app->man->A;
        setUpStructSolver( app->man, u->x, u->x );
        app->solver[A_idx] = app->man->solver;
    }
    /* Time integration to next time point: Solve the system Ax = b.
    * First, "trick" the user's manager with the right matrix and solver */
    app->man->A = app->A[A_idx];
    app->man->solver = app->solver[A_idx];
    ...
```

```
    /* Take step */
    take_step(app->man, u->x, tstart, tstop);
    ..
    return 0;
}
```

2. There are other functions, Init, Clone, Free, Sum, SpatialNorm, Access, BufSize, BufPack and Buf $\hookleftarrow$ Unpack, which also must be written. These functions are all simple for this example, as for the case of The Simplest Example. All we do here is standard operations on a spatial vector such as initialize, clone, take an inner-product, pack, etc... We refer the reader to ex-03. c.

### 4.5.2 Running XBraid for this Example

To initialize and run XBraid, the procedure is similar to The Simplest Example. Only here, we have to both initialize the user code and XBraid. The code that is specific to the user's application comes directly from the existing serial simulation code. If you compare ex-03-serial. c and ex-03.c, you will see that most of the code setting up the user's data structures and defining the wrapper functions are simply lifted from the serial simulation.

Taking excerpts from the function main() in ex-03.c, we first initialize the user's simulation manager with code like

```
app->man->px = 1; /* my processor number in the x-direction */
app->man->py = 1; /* my processor number in the y-direction */
    /* px*py=num procs in space */
app->man->nx = 17; /* number of points in the x-dim */
app->man->ny = 17; /* number of points in the y-dim */
app->man->nt = 32; /* number of time steps */
```

We also define default XBraid parameters with code like

```
...
max_levels = 15; /* Max levels for XBraid solver */
min_coarse = 3; /* Minimum possible coarse grid size */
nrelax = 1; /* Number of CF relaxation sweeps on all levels */
```

The XBraid app must also be initialized with code like

```
app->comm = comm;
app->tstart = tstart;
app->tstop = tstop;
app->ntime = ntime;
```

Then, the data structure definitions and wrapper routines are passed to XBraid.

```
braid_Core core;
braid_Init(MPI_COMM_WORLD, comm, tstart, tstop, ntime, app,
    my_Step, my_Init, my_Clone, my_Free, my_Sum, my_SpatialNorm,
    my_Access, my_BufSize, my_BufPack, my_BufUnpack, &core);
```

Then, XBraid options are set with calls like

```
braid_SetPrintLevel( core, 1);
braid_SetMaxLevels(core, max_levels);
braid_SetNRelax(core, -1, nrelax);
...
```

Then, the simulation is run.

```
braid_Drive(core);
```

Then, we clean up.

```
braid_Destroy(core);
```

Finally, to run ex-03, type
ex-03 -help

As a simple example, try the following.

```
mpirun -np 8 ex-03 -pgrid 2 2 2 -nt 256
```


### 4.5.3 Scaling Study with this Example

Here, we carry out a simple strong scaling study for this example. The "time stepping" data set represents sequential time stepping and was generated using examples/ex-03-serial. The time-parallel data set was generated using examples/ex-03. The problem setup is as follows.


- Backwards Euler is used as the time stepper. This is the only time stepper supported by ex-03.
- We used a Linux cluster with 4 cores per node, a Sandybridge Intel chipset, and a fast Infiniband interconnect.
- The space-time problem size was $129^{2} \times 16,192$ over the unit cube $[0,1] \times[0,1] \times[0,1]$.
- The coarsening factor was $m=16$ on the finest level and $m=2$ on coarser levels.
- Since 16 processors optimized the serial time stepping approach, 16 processors in space are also used for the XBraid experiments. So for instance 512 processrs in the plot corresponds to 16 processors in space and 32 processors in time, $16 * 32=512$. Thus, each processor owns a space-time hypercube of $\left(129^{2} / 16\right) \times$ $(16,192 / 32)$. See Parallel decomposition and memory for a depiction of how XBraid breaks the problem up.
- Various relaxation and V and F cycling strategies are experimented with.
- V-cycle, FCF denotes V-cycles and FCF-relaxation on each level.
- V-cycle, F-FCF denotes V-cycles and F-relaxation on the finest level and FCF-relaxation on all coarser levels.
- F-cycle, F denotes F-cycles and F-relaxation on each level.
- The initial guess at time values for $t>0$ is zero, which is typical.
- The halting tolerance corresponds to a discrete L2-norm and was

$$
\text { tol }=\frac{10^{-8}}{\sqrt{\left(h_{x}\right)^{2} h_{t}}}
$$

where $h_{x}$ and $h_{t}$ are the spatial and temporal grid spacings, respectively.
This corresponds to passing tol to braid_SetAbsTol, passing 2 to braid_SetTemporalNorm and defining braid_PtFcnSpatialNorm to be the standard Euclidean 2-norm. All together, this appropriately scales the space-time residual in way that is relative to the number of space-time grid points (i.e., it approximates the L2-norm).

To re-run this scaling study, a sample run string for ex-03 is

```
mpirun -np 64 ex-03 -pgrid 4 4 4 -nx 129 129 -nt 16129 -cf0 16 -cf 2 -nu 1 -use_rand 0
```

To re-run the baseline sequential time stepper, ex-03-serial, try

```
mpirun -np 64 ex-03-serial -pgrid 8 8 -nx 129 129 -nt 16129
```

For explanations of the command line parameters, type

$$
\begin{aligned}
& \text { ex-03-serial -help } \\
& \text { ex-03 -help }
\end{aligned}
$$

Regarding the performance, we can say

- The best speedup is $10 x$ and this would grow if more processors were available.
- Although not shown, the iteration counts here are about 10-15 XBraid iterations. See Parallel Time Integration with Multigrid for the exact iteration counts.
- At smaller core counts, serial time stepping is faster. But at about 256 processors, there is a crossover and XBraid is faster.
- You can see the impact of the cycling and relaxation strategies discussed in Cycling and relaxation strategies. For instance, even though V-cycle, F-FCF is a weaker relaxation strategy than V-cycle, FCF (i.e., the XBraid convergence is slower), V-cycle, F-FCF has a faster time to solution than $V$-cycle, $F C F$ because each cycle is cheaper.
- In general, one level of aggressive coarsening (here by a factor 16) followed by slower coarsening was found to be best on this machine.

Achieving the best speedup can require some tuning, and it is recommended to read Parallel Time Integration with Multigrid where this 2D heat equation example is explored in much more detail.

### 4.6 Simplest XBraid_Adjoint example

The file examples/ex-01-adjoint.c extends the simple scalar ODE example in ex-01.c for computing adjoint-based sensitivities. See The Simplest Example. The scalar ODE is

$$
u_{t}(t)=\lambda u(t) \quad \forall t \in(0, T),
$$

where $\lambda$ is considered the design variable. We consider an objective function of the form

$$
J(u, \lambda)=\int_{0}^{T} \frac{1}{T}\|u(t)\|^{2} d t
$$

### 4.6.1 User Defined Structures and Wrappers

The two user-defined data structures are:

1. Vector: This structure is unchanged from The Simplest Example, and contains a single scalar representing the state at a given time.
```
typedef struct _braid_Vector_struct
{
    double value;
} my_Vector;
```

2. App: This structure holds two additional elements when compared to The Simplest Example : the design and the reduced gradient. This ensures that both are accessible in all user routines.
```
typedef struct _braid_App_struct
{
    int rank;
    double design;
    double gradient;
} my_App;
```

The user must also define a few additional wrapper routines. Note, that the app structure continues to be the first argument to every function.

1. All user-defined routines from examples/ex-01.c stay the same, except Step (), which must be changed to account for the new design parameter in app.
2. The user's Step routine queries the app to get the design and propagates the braid_Vector $u$ forward in time for one time step:
```
int
my_Step(braid_App app,
        braid_Vector ustop,
        braid_Vector fstop,
        braid_Vector u,
        braid_StepStatus status)
{
    double tstart; /* current time */
    double tstop; /* evolve to this time*/
    braid_StepStatusGetTstartTstop(status, &tstart, &tstop);
    /* Get the design variable from the app */
    double lambda = app->design;
    /* Use backward Euler to propagate the solution */
    (u->value) = 1./(1. - lambda * (tstop-tstart))*(u->value);
    return 0;
}
```

3. ObjectiveT: This new routine evaluates the time-dependent part of the objective function at a local time $t_{i}$, i.e. it returns the integrand $f\left(u_{i}, \lambda\right)=\frac{1}{T}\left\|u_{i}\right\|_{2}^{2}$.
```
int
my_ObjectiveT(braid_App app,
    braid_Vector u,
    braid_ObjectiveStatus ostatus,
    double *objectiveT_ptr)
{
    /* Get the total number of time steps */
    braid_ObjectiveStatusGetNTPoints(ostatus, &ntime);
    /* Evaluate the local objective: 1/N u(t)^2 */
    objT = 1. / ntime * (u->value) * (u->value);
    *objectiveT_ptr = objT;
    return 0;
}
```

The ObjectiveStatus can be queried for information about the current status of XBraid (e.g., what is the current time value, time-index, number of time steps, current iteration number, etc...).
XBraid_Adjoint calls the Ob jectiveT function on the finest time-grid level during the down-cycle of the multigrid algorithm and adds the value to a global objective function value with a simple summation. Thus, any user-specific integration formula of the objective function must be here.
4. ObjectiveT_diff: This new routine updates the adjoint variable u_bar and the reduced gradient with the transposed partial derivatives of Ob jectiveT multiplied by the scalar input $\bar{F}$, i.e.,

$$
\bar{u}_{i}={\frac{\partial f\left(u_{i}, \lambda\right)^{T}}{\partial u_{i}}} \bar{F} \quad \text { and } \quad \bar{\rho}+=\frac{\partial f\left(u_{i}, \lambda\right)^{T}}{\partial \rho} \bar{F} .
$$

Note that $\bar{u}_{i}$ gets overwritten (" $=$ "), whereas $\rho$ is updated (" $+=$ ").

```
int
my_ObjectiveT_diff(braid_App app,
    braid_Vector u,
    braid_Vector u_bar,
    braid_Real F_bar,
    braid_ObjectiveStatus ostatus)
{
    int ntime;
    double ddu; /* Derivative wrt u */
    double ddesign; /* Derivative wrt design */
    /* Get the total number of time steps */
    braid_ObjectiveStatusGetNTPoints(ostatus, &ntime);
    /* Partial derivative with respect to u times F_bar */
    ddu = 2. / ntime * u->value * F_bar;
    /* Partial derivative with respect to design times F_bar*/
    ddesign = 0.0 * F_bar;
    /* Update u_bar and gradient */
    u_bar->value = ddu;
    app->gradient += ddesign;
    return 0;
}
```

5. Step_diff: This new routine computes transposed partial derivatives of the Step routine multiplied with the adjoint vector u_bar ( $\bar{u}_{i}$ ), i.e.,

$$
\bar{u}_{i}=\left(\frac{\partial \Phi_{i+1}\left(u_{i}, \rho\right)}{\partial u_{i}}\right)^{T} \bar{u}_{i} \quad \text { and } \quad \bar{\rho}+=\left(\frac{\partial \Phi_{i+1}\left(u_{i}, \rho\right)}{\partial \rho}\right)^{T} \bar{u}_{i} .
$$

```
int
my_Step_diff(braid_App app,
braid_Vector ustop,
braid_Vector u,
braid_Vector ustop_bar,
braid_Vector u_bar,
braid_StepStatus status)
{
    double ddu; /* Derivative wrt u */
    double ddesign; /* Derivative wrt design */
    /* Get the time step size */
    double tstop, tstart, deltat;
    braid_StepStatusGetTstartTstop(status, &tstart, &tstop);
    deltat = tstop - tstart;
    /* Get the design from the app */
    double lambda = app->design;
    /* Transposed derivative of step wrt u times u_bar */
    ddu = 1./(1. - lambda * deltat) * (u_bar->value);
    /* Transposed derivative of step wrt design times u_bar */
    ddesign = (deltat * (u->value)) / pow(1. - deltat*lambda,2) * (u_bar->value);
    /* Update u_bar and gradient */
    u_bar->value = ddu;
    app->gradient += ddesign;
    return 0;
}
```

Important note on the usage of ustop: If the Step routine uses the input vector ustop instead of $u$ (typically for initializing a (non-)linear solve within $\Phi$ ), then Step_diff must update ustop_bar instead of u_bar and set u_bar to zero:

$$
\overline{u s t o p}+=\left(\frac{\partial \Phi_{i+1}(\text { ustop }, \rho)}{\partial \text { ustop }}\right)^{T} \bar{u}_{i} \quad \text { and } \quad \bar{u}_{i}=0.0 .
$$

6. ResetGradient: This new routine sets the gradient to zero.
```
int
my_ResetGradient(braid_App app)
{
    app->gradient = 0.0;
    return 0;
}
```

XBraid_Adjoint calls this routine before each iteration such that old gradient information is removed properly.

### 4.6.2 Running XBraid_Adjoint for this example

The workflow for computing adjoint sensitivities with XBraid_Adjoint alongside the primal state computation closely follows XBraid's workflow. The user's main file will first set up the app structure, holding the additional information on an initial design and zero gradient. Then, all the setup calls done in Running XBraid for the Simplest Example will also be done.

The XBraid_Adjoint specific calls are as follows. After braid_Init ( . . ) is called, the user initializes XBraid_Adjoint by calling

```
/* Initialize XBraid_Adjoint */
braid_InitAdjoint( my_ObjectiveT, my_ObjectiveT_diff, my_Step_diff, my_ResetGradient, &core);
```

Next, in addition to the usual XBraid options for controlling the multigrid iterations, the adjoint solver's accuracy is set by calling

```
braid_SetAbsTolAdjoint(core, 1e-6);
```

After that, one call to

```
/* Run simulation and adjoint-based gradient computation */
braid_Drive(core);
```

runs the multigrid iterations with additional adjoint sensitivity computations (i.e. the piggy-back iterations). After it finishes, the objective function value can be accessed by calling

```
/* Get the objective function value from XBraid */
braid_GetObjective(core, &objective);
```

Further, the reduced gradient, which is stored in the user's App structure, holds the sensitivity information $d J / d \rho$. As this information is local to all the time-processors, the user is responsible for summing up the gradients from all time-processors, if necessary. This usually involves an MP I_Allreduce call as in

```
/* Collect sensitivities from all processors */
double mygradient = app->gradient;
MPI_Allreduce(&mygradient, &(app->gradient), 1, MPI_DOUBLE, MPI_SUM, MPI_COMM_WORLD);
```

Lastly, the gradient computed with XBraid_Adjoint is verified using Finite Differences. See the source code examples/ex-01-adjoint.c for details.

### 4.7 Optimization with the Simplest Example

The file examples/ex-01-optimization.c implements a simple optimization iteration by extending examples/ex-01-adjoint.c, described in Simplest XBraid_Adjoint example. This example solves an inverse design problem for the simple scalar ODE example:

$$
\begin{gathered}
\min \quad \frac{1}{2}\left(\int_{0}^{T} \frac{1}{T}\|u(t)\|^{2} d t-J_{\text {Target }}\right)^{2}+\frac{\gamma}{2}\|\lambda\|^{2} \\
\text { s.t. } \quad \frac{\partial}{\partial t} u(t)=\lambda u(t) \quad \forall t \in(0, T)
\end{gathered}
$$

where $J_{\text {Target }}$ is a fixed and precomputed target value and $\gamma>0$ is a fixed relaxation parameter. Those fixed values are stored within the App.

### 4.7.1 User Defined Structures and Wrappers

In order to evaluate the time-independent part of the objective function (e.g. the postprocessing function $F$ ) and its derivative, two additional user routines are necessary. There are no new user-defined data structures.

1. PostprocessObjective: This function evaluates the tracking-type objective function and the regularization term. The input variable integral contains the integral-part of the objective and returns the objective that is to be minimized $F(I)$ :
```
/* Evaluate the time-independent part of the objective function */
int
my_PostprocessObjective(braid_App app,
    double integral,
    double *postprocess
    )
{
    double F;
    /* Tracking-type functional */
    F = 1./2. * pow(integral - app->target,2);
    /* Regularization term */
    F += (app->gamma) / 2. * pow(app->design,2);
    *postprocess = F;
        return 0;
}
```

1. PostprocessObjective_diff: This provides XBraid_Adjoint with the partial derivatives of the Postprocess $\leftarrow$ Objective routine, i.e.
```
            \overline{F}=\frac{\partialF(I,\lambda)}{\partialI}\quad\mathrm{ and }\quad\overline{\rho}+=\frac{\partialF(I,\lambda)}{\partial\lambda}
    int
    my_PostprocessObjective_diff(braid_App app,
    double integral,
    double *F_bar
    )
    {
    /* Derivative of tracking type function */
    *F_bar = integral - app->target;
    /* Derivative of regularization term */
    app->gradient += (app->gamma) * (app->design);
    return 0;
    }
```

These routines are optional for XBraid_Adjoint. Therefore, they need to be passed to XBraid_Adjoint after the initialization with braid_Init (. . .) and braid_InitAdjoint (. . .) in the user's main file:

```
/* Optional: Set the tracking type objective function and derivative */
braid_SetPostprocessObjective(core, my_PostprocessObjective);
braid_SetPostprocessObjective_diff(core, my_PostprocessObjective_diff);
```


### 4.7.2 Running an Optimization Cycle with XBraid_Adjoint

XBraid_Adjoint does not natively implement any optimization algorithms. Instead, we provide examples showing how one can easily use XBraid_Adjoint inside an optimization cycle. Here, one iteration of the optimization cycle consists of the following steps:

1. First, we run XBraid_Adjoint to solve the primal and adjoint dynamics:
```
braid_Drive(core);
```

2. Get the value of the objective function with
```
braid_GetObjective(core, &objective);
```

3. Gradient information is stored in the app structure. Since it is local to all temporal processors, we need to invoke an MPI_Allreduce call which sums up the local sensitivities:
```
mygradient = app->gradient;
MPI_Allreduce(&mygradient, &app->gradient, 1, MPI_DOUBLE, MPI_SUM, MPI_COMM_WORLD);
```

Note: For time-dependent design variables, summing over all processors might not be necessary, since information is needed only locally in time. See examples/ex-04.c for a time-dependent design example.
4. Update the design variable using the gradient information. Here, we implement a simple steepest descent update into the direction of the negative gradient:

```
app->design -= stepsize * app->gradient;
```

Here, a fixed step size is used to update the design variable. Usually, a line-search procedure should be implemented in order to find a suitable step length that minimizes the objective function along the update direction. However to carry out a line search, we must re-evaluate the objective function for different design value(s). Thus, the option braid_SetObjectiveOnly(core, 1) can be used. After this option has been set, any further call to braid_Drive (core) will then only run a primal XBraid simulation and carry out an objective function evaluation. No gradients will be computed, which saves computational time. After the line search, make sure to reset XBraid_Adjoint for gradient computation with braid_SetobjectiveOnly (core, 0).
5. The optimization iterations are stopped when the norm of the gradient is below a prescribed tolerance.

### 4.8 A Simple Optimal Control Problem

This example demonstrates the use of XBraid_Adjoint for solving an optimal control problem with time-dependent design variables:

$$
\begin{array}{rlr}
\min \int_{0}^{1} u_{1}(t)^{2}+u_{2}(t)^{2}+\gamma c(t)^{2} d t & \\
& & \\
\text { s.t. } & \frac{\partial}{\partial t} u_{1}(t)=u_{2}(t) & \forall t \in(0,1) \\
& \frac{\partial}{\partial t} u_{2}(t)=-u_{2}(t)+c(t) & \forall t \in(0,1)
\end{array}
$$

with initial condition $u_{1}(0)=0, u_{2}(0)=-1$ and piecewise constant control (design) variable $c(t)$.

The example consists of three files, meant to indicate how one can take a time-serial implementation for an optimal control problem and create a corresponding XBraid_Adjoint implementation.

- examples/ex-04-serial.c: Compiles into its own executable examples/ex-04-serial, which solves the optimal control problem using time-serial forward-propagation of state variables and time-serial backward-propagation of the adjoint variables in each iteration of an outer optimization cycle.
- examples/ex-04.c: Compiles into ex-04. This solves the same optimization problem in time-parallel by replacing the forward- and backward-propagation of state and adjoint by the time-parallel XBraid and XBraid_↔ Adjoint solvers.
- examples/ex-04-lib.c: Contains the routines that are shared by both the serial and the time-parallel implementation. Study this file, and discover that most of the important code setting up the user-defined data structures and wrapper routines are simply lifted from the serial simulation.


### 4.9 Chaotic Lorenz System With Delta Correction and Lyapunov Estimation

This example demonstrates acceleration of XBraid convergence and Lyapunov analysis of a system with Delta correction. Familiarity with The Simplest Example is assumed. This example solves the chaotic Lorenz system in three dimensions, defined by the system

$$
\left\{\begin{array}{l}
x^{\prime}=\sigma(y-x), \\
y^{\prime}=x(\rho-z)-y, \\
z^{\prime}=x y-\beta z,
\end{array}\right.
$$

where $\sigma=10, \rho=28$, and $\beta=8 / 3$. This system is chaotic, with the greatest Lyapunov exponent being $\approx 0.9$. Here, Delta correction is used to accelerate convergence to the solution, while Lyapunov estimation is used to simultaneously compute the Lyapunov vectors and Lyapunov exponents along the trajectory.

### 4.9.1 User Defined Structures and Wrappers

Most of the user defined structures and wrappers are defined exactly as in previous examples, with the exception of Step(), BufSize(), and Access(), which are modified to accomodate the Lyapunov vectors, and InnerProd() and Init $\hookleftarrow$ Basis(), which are new functions required by Delta correction.

1. Step: Here the Step function is required to do two things:

- Propagate the state vector (as in regular XBraid)

$$
u \leftarrow \Phi(u)
$$

- Propagate a number of basis vectors using the Jacobian vector product (new functionality required by Delta correction)

$$
\psi_{j} \leftarrow\left(\frac{d \Phi}{d u}\right) \psi_{j}
$$

The number of basis vectors to be propagated is accessed via braid_StepStatusGetDeltaRank, and references to the vectors themselves are accessed via braid_StepStatusGetBasisVec. In this example, the full Jacobian of Step is used to propagate the basis vectors, but finite differencing or even forward-mode automatic differentiation are other ways of propagating the basis vectors.

```
int my_Step(braid_App app,
            braid_Vector ustop,
            braid_Vector fstop,
            braid_Vector u,
            braid_StepStatus status)
{
/* for Delta correction, the user must propagate the solution vector
* (as in a traditional Braid code) as well as the Lyapunov vectors.
* The Lyapunov vectors are available through the StepStatus structure,
    * and are propagated by the Jacobian of the time-step function. (see below)
```

```
    */
    double tstart; /* current time */
    double tstop; /* evolve to this time */
    braid_StepStatusGetTstartTstop(status, &tstart, &tstop);
    double h; /* dt value */
    h = tstop - tstart;
    // get the number of Lyapunov vectors we need to propagate
    int rank; /* rank of Delta correction */
    braid_StepStatusGetDeltaRank(status, &rank);
    MAT Jacobian = {{0., 0., 0.}, {0., 0., 0.}, {0., 0., 0.}};
    if (rank > 0) // we are propagating Lyapunov vectors
    {
        Euler((u->values), h, &Jacobian);
    }
    else
    {
        Euler((u->values), h, NULL);
    }
    for (int i = 0; i < rank; i++)
    {
        // get a reference to the ith Lyapunov vector
        my_Vector *psi;
        braid_StepStatusGetBasisVec(status, &psi, i);
        // propagate the vector from tstart to tstop
    if (psi)
        {
        MatVec(Jacobian, psi->values);
        }
    }
    /* no refinement */
    braid_StepStatusSetRFactor(status, 1);
    return 0;
}
```

2. BufSize(): There is an additional option to set the size of a single basis vector here, via braid_BufferStatusSetBasisSize.
```
int my_BufSize(braid_App app, int *size_ptr, braid_BufferStatus bstatus)
{
    /* Tell Braid the size of a state vector */
    *size_ptr = VecSize * sizeof(double);
    /*
    * In contrast with traditional Braid, you may also specify the size of a single
    * Lyapunov basis vector,in case it is different from the size of a state vector.
    * Note: this isn't necessary here, but for more complicated applications this
    * size may be different.
    */
    braid_BufferStatusSetBasisSize(bstatus, VecSize * sizeof(double));
    return 0;
}
```

3. Access: Here, the Access function is used to access the Lyapunov vector estimates via the same API as for Step. Also, the local Lyapunov exponents are accessed via braid_AccessStatusGetLocalLyapExponents.
```
int my_Access(braid_App app, braid_Vector u, braid_AccessStatus astatus)
{
    FILE *file = (app->file);
    int index, i;
    double t;
```

```
    braid_AccessStatusGetT(astatus, &t);
    braid_AccessStatusGetTIndex(astatus, &index);
    fprintf(file, "%d", index);
    for (i = 0; i < VecSize; i++)
    {
        fprintf(file, " %.14e", (u->values[i]));
    }
    fprintf(file, "\n");
    fflush(file);
    /* write the lyapunov vectors to file */
    file = app->file_lv;
    int local_rank, num_exp;
    braid_AccessStatusGetDeltaRank(astatus, &local_rank);
    num_exp = local_rank;
    double *exponents = malloc(local_rank * sizeof(double));
    if (num_exp > 0)
    {
        braid_AccessStatusGetLocalLyapExponents(astatus, exponents, &num_exp);
    }
fprintf(file, "%d", index);
for (int j = 0; j < local_rank; j++)
{
    my_Vector *psi;
    braid_AccessStatusGetBasisVec(astatus, &psi, j);
    if (psi)
    {
                if (j < num_exp)
            {
                (app->lyap_exps)[j] += exponents[j];
                    fprintf(file, " %.14e", exponents[j]);
            }
            else
            {
                    fprintf(file, " %.14e", 0.);
            }
            for (i = 0; i < VecSize; i++)
            {
                    fprintf(file, " %.14e", (psi->values[i]));
        }
    }
    }
    fprintf(file, "\n ");
    fflush(file);
    free(exponents);
    return 0;
```

$\}$
4. InnerProd: This function tells XBraid how to compute the inner product between two Vector structures. This is required by Delta correction in order to project user vectors onto the basis vectors, and for orthonormalization of the basis vectors. Here, the standard dot product is used.

```
int my_InnerProd(braid_App app, braid_Vector u, braid_Vector v, double *prod_ptr)
{
    /*
    * For Delta correction, braid needs to be able to compute an inner product
    * between two user vectors, which is used to project the user's vector onto
    * the Lyapunov basis for low-rank Delta correction. This function should
    * define a valid inner product between the vectors *u* and *v*.
    */
    double dot = 0.;
    for (int i = 0; i < VecSize; i++)
    {
        dot += (u->values[i]) * (v->values[i]);
```

```
    }
    *prod_ptr = dot;
    return 0;
}
```

5. InitBasis: This function tells XBraid how to initialize a single basis vector, with spatial index $j$ at time $t$. This initializes the column $j$ of the matrix $\Psi$ whose columns are the basis vectors used for Delta correction. Here, we simply use column $j$ of the identity matrix. It is important that the vectors initialized by this function are linearly independent, or Lyapunov estimation will not work.
```
int my_InitBasis(braid_App app, double t, int index, braid_Vector *u_ptr)
{
    /*
    * For Delta correction, an initial guess is needed for the Lyapunov basis vectors.
    * This function initializes the basis vector with spatial index *index* at time *t*.
    * Note that the vectors at each index *index* must be linearly independent.
    */
    my_Vector *u;
    u = (my_Vector *)malloc(sizeof(my_Vector));
    // initialize with the columns of the identity matrix
    VecSet(u->values, 0.);
    u->values[index] = 1.;
    *u_ptr = u;
    return 0;
}
```


### 4.9.2 Running XBraid with Delta correction and Lyapunov Estimation

XBraid is initialized as before, and most XBraid features are compatible, however, this does not include Richardson extrapolation, the XBraid_Adjoint feature, the Residual option, and spatial coarsening. Delta correction and Lyapunov estimation are turned on by calls to braid_SetDeltaCorrection and braid_SetLyapunovEstimation, respectively, where the number of basis vectors desired (rank of low-rank Delta correction) and additional wrapper functions InnerProd and InitBasis are passed to XBraid and options regarding the estimation of Lyapunov vectors and exponents are set. Further, the function braid_SetDeferDelta gives more options allowing Delta correction to be deferred to a later iteration, or a coarser grid. This is illustrated in the folowing exerpt from this example's main() function:

```
if (delta_rank > 0)
{
    braid_SetDeltaCorrection(core, delta_rank, my_InitBasis, my_InnerProd);
    braid_SetDeferDelta(core, defer_lvl, defer_iter);
    braid_SetLyapunovEstimation(core, relax_lyap, lyap, relax_lyap || lyap);
}
```


### 4.10 Running and Testing XBraid

The best overall test for XBraid, is to set the maximum number of levels to 1 (see braid_SetMaxLevels) which will carry out a sequential time stepping test. Take the output given to you by your Access function and compare it to output from a non-XBraid run. Is everything OK? Once this is complete, repeat for multilevel XBraid, and check that the solution is correct (that is, it matches a serial run to within tolerance).

At a lower level, to do sanity checks of your data structures and wrapper routines, there are also XBraid test functions, which can be easily run. The test routines also take as arguments the app structure, spatial communicator comm_x, a stream like stdout for test output and a time step size $d t$ to test. After these arguments, function pointers to wrapper routines are the rest of the arguments. Some of the tests can return a boolean variable to indicate correctness.

```
/* Test init(), access(), free() */
braid_TestInitAccess( app, comm_x, stdout, dt, my_Init, my_Access, my_Free);
/* Test clone() */
braid_TestClone( app, comm_x, stdout, dt, my_Init, my_Access, my_Free, my_Clone);
/* Test sum() */
braid_TestSum( app, comm_x, stdout, dt, my_Init, my_Access, my_Free, my_clone, my_Sum);
/* Test spatialnorm() */
correct = braid_TestSpatialNorm( app, comm_x, stdout, dt, my_Init, my_Free, my_Clone,
                                    my_Sum, my_SpatialNorm);
/* Test bufsize(), bufpack(), bufunpack() */
correct = braid_TestBuf( app, comm_x, stdout, dt, my_Init, my_Free, my_Sum, my_SpatialNorm,
    my_BufSize, my_BufPack, my_BufUnpack);
/* Test coarsen and refine */
correct = braid_TestCoarsenRefine(app, comm_x, stdout, 0.0, dt, 2*dt, my_Init,
    my_Access, my_Free, my_Clone, my_Sum, my_SpatialNorm,
    my_CoarsenInjection, my_Refine);
correct = braid_TestCoarsenRefine(app, comm_x, stdout, 0.0, dt, 2*dt, my_Init,
    my_Access, my_Free, my_Clone, my_Sum, my_SpatialNorm,
    my_CoarsenBilinear, my_Refine);
/**
    * Test innerprod(), initbasis(), step(), bufsize(), bufpack(), bufunpack()
    * for use with Delta correction
*/
correct = braid_TestInnerProd(app, comm_x, stdout, 0.0, 1.0,
                                    my_Init, my_Free, my_Sum, my_InnerProd);
correct = braid_TestDelta(app, comm_x, stdout, 0.0, dt, delta_rank, my_Init,
    my_InitBasis, my_Access, my_Free, my_Sum, my_BufSize,
    my_BufPack, my_BufUnpack, my_InnerProd, my_Step);
```


### 4.11 Fortan90 Interface, C++ Interface, Python Interface, and More Complicated Examples

We have Fortran90, C++, and Python interfaces. For Fortran 90, see examples/ex-01f.f90. For C++ see braid.hpp and examples/ex-01-pp.cpp For more complicated C++ examples, see the various C++ examples in drivers/drive-**.cpp. For Python, see the directories examples/ex-01-cython and examples/ex-01-cython-alt.

For a discussion of more complex problems please see our project publications website for our recent publications concerning some of these varied applications.

## 5 Examples: compiling and running

For C/C++/Fortran examples, type

```
ex-* -help
```

for instructions on how to run. To run the $\mathrm{C} / \mathrm{C}++/$ Fortran examples, type

```
mpirun -np 4 ex-* [args]
```

For the Cython examples, see the corresponding $*$.pyx file.

1. ex-01 is the simplest example. It implements a scalar ODE and can be compiled and run with no outside dependencies. See Section (The Simplest Example) for more discussion of this example. There are seven versions of this example,

- ex-01.c: simplest possible implementation, start reading this example first
- ex-01-expanded.c: same as ex-01.c but adds more XBraid features
- ex-01-expanded-bdf2.c: same as ex-01-expanded.c, but uses BDF2 instead of backward Euler
- ex-01-expanded-f.f90: same as ex-01-expanded.c, but implemented in $f 90$
- ex-01-refinement.c: same as ex-01.c, but adds the refinement feature
- ex-01-adjoint.c: adds adjoint-based gradient computation to ex-01.c
- ex-01-optimization.c: gradient-based optimization cycle for ex-01-c
- ex-01-cython/: is a directory containing an example using the Braid-Cython interface defined in braid. $\hookleftarrow$ pyx ( braid/braid.pyx ). It solves the same scalar ODE equation as the ex-01 series described above. This example uses a Python-like syntax, in contrast to the ex-01-cython-alt example, which uses a C-style syntax. For instructions on running and compiling, see
examples/ex-01-cython/ex_01.pyx
and
examples/ex-01-cython/ex_01-setup.py
- ex-01-cython-alt: is a directory containing another example using the Braid-Cython interface defined in braid.pyx ( braid/braid.pyx ). It solves the same scalar ODE equation as the ex-01 series described above. This example uses a lower-level C-like syntax for most of it's code, in contrast to the ex-01-cython example, which uses a Python-style syntax.

For instructions on running and compiling, see
examples/ex-01-cython-alt/ex_01_alt.pyx
and
examples/ex-01-cython-alt/ex_01_alt-setup.py
2. ex-02 implements the 1D heat equation on a regular grid, using a very simple implementation. This is the next example to read after the various ex-01 cases.
3. ex-03 implements the 2D heat equation on a regular grid. You must have hypre installed and these variables in examples/Makefile set correctly

```
HYPRE_DIR = ../../linear_solvers/hypre
HYPRE_FLAGS = -I$(HYPRE_DIR)/include
HYPRE_LIB = -L$(HYPRE_DIR)/lib -lHYPRE
```

Only implicit time stepping (backward Euler) is supported. See Section (Two-Dimensional Heat Equation) for more discussion of this example. The driver

```
drivers/drive-diffusion
```

is a more sophisticated version of this simple example that supports explicit time stepping and spatial coarsening.
4. ex-04 solves a simple optimal control problem with time-dependent design variable using a simple steepestdescent optimization iteration.
5. Directory ex-05-cython/ solves a simple 1D heat equation using the Cython interface
examples/ex-05-cython/ex_05.pyx
and
examples/ex-05-cython/ex_05-setup.py
6. ex-06 solves a simple scalar ODE, but allows for use of the built-in Richardson-based error estimator and accuracy improving extrapolation. With the "-refinet" option, the error estimator allows for adaptive refinement in time, and with the "-richardson" option, Richardson extrapolation is used improve the solution at fine-level C-points.
The viz script,
examples/viz-ex-06.py
allows you to visualize the solution, error, and error estimate. The use of "-richardson" notably improves the accuracy of the solution.

The Richardson-based error estimates and/or extrapolation are only available after the first Braid iteration, in that the coarse level solution must be available to compute the error estimate and extrapolation. Thus, after an adaptive refinement (and new hierarchy is constructed), another iteration is again required for the error estimate to be available. If the error estimate isn't available, Braid returns a value of -1 . See this example and the comments therein for more details.
7. ex-07 solves the chaotic Lorenz system, utilizing the Delta correction feature to accelerate Braid convergence while estimating the Lyapunov vectors and Lyapunov exponents.
The viz script,
examples/viz-ex-07.py
Plots the solution trajectory in 3D along with the estimated Lyapunov basis vectors computed by Braid. The Lyapunov vectors define a basis for the stable, neutral, and unstable manifolds of the system, and the Lyapunov exponents give qualitative information about the dynamics of the system.
The command line argument "-rank" controls the number of Lyapunov vectors which are tracked, with "-rank 0 " turning Delta correction off, and "-rank 3" giving a full-rank Delta correction, since the Lorenz system is 3dimensional. The "-defer-lvl" and "-defer-iter" arguments control whether the Delta correction is deferred to a coarse level, or later iteration, respectively. For more information about these options, use "\$ examples/ex-07 -help".

## 6 Drivers: compiling and running

Type
drive-* -help
for instructions on how to run any driver.
To run the examples, type

```
mpirun -np 4 drive-* [args]
```

1. drive-diffusion-2D implements the 2D heat equation on a regular grid. You must have hypre installed and these variables in examples/Makefile set correctly
```
HYPRE_DIR = ../../linear_solvers/hypre
HYPRE_FLAGS = -I$(HYPRE_DIR)/include
HYPRE_LIB = -L$(HYPRE_DIR)/lib -lHYPRE
```

This driver also support spatial coarsening and explicit time stepping. This allows you to use explicit time stepping on each Braid level, regardless of time step size.
2. drive-burgers-1D implements Burger's equation (and also linear advection) in 1D using forward or backward Euler in time and Lax-Friedrichs in space. Spatial coarsening is supported, allowing for stable time stepping on coarse time-grids.
See also viz-burgers.py for visualizing the output.
3. drive-diffusion is a sophisticated test bed for finite element discretizations of the heat equation. It relies on the mfem package to create general finite element discretizations for the spatial problem. Other packages must be installed in this order.

- Unpack and install Metis
- Unpack and install hypre
- Unpack mfem. Then make sure to set these variables correctly in the mfem Makefile:

```
USE_METIS_5 = YES
HYPRE_DIR = where_ever_linear_solvers_is/hypre
```

- Make the parallel version of mfem first by typing

```
make parallel
```

- Make GLVIS. Set these variables in the glvis makefile

```
MFEM_DIR = mfem_location
MFEM_LIB = -L$ (MFEM_DIR) -lmfem
```

- Go to braid/examples and set these Makefile variables,

```
METIS_DIR = ../../metis-5.1.0/lib
MFEM_DIR = ../../mfem
MFEM_FLAGS = -I$(MFEM_DIR)
MFEM_LIB = -L$(MFEM_DIR) -lmfem -L$(METIS_DIR) -lmetis
```

then type
make drive-diffusion

- To run drive-diffusion and glvis, open two windows. In one, start a glvis session
./glvis

Then, in the other window, run drive-diffusion
mpirun -np ... drive-diffusion [args]

Glvis will listen on a port to which drive-diffusion will dump visualization information.
4. The other drive-.cpp files use MFEM to implement other PDEs

- drive-adv-diff-DG: implements advection(-diffusion) with a discontinuous Galerkin discretization. This driver is under developement.
- drive-diffusion-1D-moving-mesh: implements the 1D heat equation, but with a moving mesh that adapts to the forcing function so that the mesh equidistributes the arc-length of the solution.
- drive-diffusion-1D-moving-mesh-serial: implements a serial time-stepping version of the above problem.
- drive-pLaplacian: implements the 2D the $p$-Laplacian (nonlinear diffusion).
- drive-diffusion-ben: implements the 2D/3D diffusion equation with time-dependent coefficients. This is essentially equivalent to drive-diffusion, and could be removed, but we're keeping it around because it implements linear diffusion in the same way that the p-Laplacian driver implemented nonlinear diffusion. This makes it suitable for head-to-head timings.
- drive-lin-elasticity: implements time-dependent linearized elasticity and is under development.
- drive-nonlin-elasticity: implements time-dependent nonlinear elasticity and is under development.

5. Directory drive-adv-diff-1D-Cython/ solves a simple 1D advection-diffussion equation using the Cython interface and numerous spatial and temporal discretizations

> drivers/drive-adv-diff-1D-Cython/drive_adv_diff_1D.pyx
and

> drivers/drive-adv-diff-1D-Cython/drive_adv_diff_1D-setup.py
6. Directory drive-Lorenz-Delta/ implements the chaotic Lorenz system, with its trademark butterfly shaped attractor. The driver uses the Delta correction feature and Lyapunov estimation to solve for the backward Lyapunov vectors of the system and to accelerate XBraid convergence. Visualize the solution and the Lyapunov vectors with vis $\hookleftarrow$ _lorenz_LRDelta.py Also see example 7 (examples/ex-07.c). This driver is in a broken state, and needs updating for compatibility with new Delta correction implementation.
7. Directory drive-KS-Delta/ solves the chaotic Kuramoto-Sivashinsky equation in 1D, using fourth order finite differencing in space and the Lobatto IIIC fully implicit RK method in time. The driver also uses Delta correction and Lyapunov estimation to accelerate convergence and to generate estimates to the unstable Lyapunov vectors for the system.

## 7 Coding Style

Code should follow the ellemtel style. See braid/misc/sample_c_code.c, and for emacs and vim style files, see braid $/ \mathrm{misc} /$ sample.vimrc, and braid $/ \mathrm{misc} /$ sample.emacs.

## 8 File naming conventions

These are the general filenaming conventions for Braid

User interface routines in braid begin with braid_ and all other internal non-user routines begin with _braid_. This helps to prevent name clashes when working with other libraries and helps to clearly distinguish user routines that are supported and maintained.

To keep things somewhat organized, all user header files and implementation files should have names that begin with braid, for example, braid.h, braid.c, braid_status.c, ... There should be no user interface prototypes or implementations that appear elsewhere.

Note that it is okay to include internal prototypes and implementations in these user interface files when it makes sense (say, as supporting routines), but this should generally be avoided.

An attempt has been made to simplify header file usage as much as possible by requiring only one header file for users, braid.h, and one header file for developers, _braid.h.

## 9 Using Doxygen

To build the documentation, doxygen must be version 1.8 or greater. XBraid documentation uses a markdown syntax both in source file comments and in $*$.md files.

To make the documentation,

```
$ make user_manual
$ acroread user_manual.pdf
```

or to make a more extensive reference manual for developers,

```
$ make developer_manual
$ acroread developer_manual.pdf
```

Developers can run doxygen from a precompiled binary, which may or may not work for your machine,

```
/usr/casc/hypre/braid/share/doxygen/bin/doxygen
```

or build doxygen from

```
/usr/casc/hypre/braid/share/doxygen.tgz
```

- Compiling doxygen requires a number of dependencies like Bison, GraphViz and Flex. Configure will tell you what you're missing
- Unpack doxygen.tgz, then from the doxygen directory

```
./configure --prefix some_dir_in_your_path
make
make install
```


### 9.0.1 Documentation Strategy

- The doxygen comments are to be placed in the header files.
- A sample function declaration using the documenation approach using markdown (including typesetting equations) is in braid.h for the function braid_Init()
- A sample structure documentation is in _braid.h for _braid_Core_struct
- Descriptors for files can also be added, as at the top of braid.h
- The Doxygen manual is at http://www.stack.nl/~dimitri/doxygen/manual/index.html


### 9.0.2 XBraid Doxygen details

The user and developer manuals are ultimately produced by Latex. The formatting of the manuals is configured according to the following.

- docs/local_doxygen.sty
- Latex style file used
- docs/user_manual_header.tex
- User manual title page and header info
- docs/developer_manual_header.tex
- Developer manual title page and header info
- *.md
- Any file ending in .md is extra documentation in markdown format, like Introduction.md or the various Readme.md files in each directory.
This material can be read in plain-text or when it's compiled by Doxygen and Latex.
- docs/user_manual.conf
- Doxygen configure file for the user manual
- The FILE_NAMES tag is a filter to only include the user interface routines in braid.h
- The INPUT tag orders the processing of the files and hence the section ordering
- docs/reference_manual.conf
- Same as user_manual.conf, but the FILE_NAMES tag does not exclude any file from processing.
- docs/img
- Contains the images
- To regenerate generic doxygen latex files, type
\$ doxygen -w latex header.tex footer.tex doxygen. sty doxy.conf
If this is done, then the .conf file must be changed to use the new header file and to copy the local_doxygen.sty file to the latex directory.


## 10 Regression Testing

### 10.0.1 Overview

- There are three levels in the testing framework. At each level, the fine-grain output from a testscript. sh is dumped into a directory testscript. dir, with the standard out and error stored in testscript. out and testscript.err. The test testscript.sh passes if testscript.err is empty (nothing is written to standard error).
- Basic instructions: run a test with a command like

```
$ ./test.sh diffusion2D.sh
```

Then, see if diffusion2D. err is of size 0 . If it is not, look at it's contents to see which test failed.

- To add a new regression test, create a new lowest level script like diffusion2D.sh and then call it from a machine script at level 2.
- Regression tests should be run before pushing code. It is recommended to run the basic (lowest level) tests like diffusion2d.sh or machine test like machine-tux.sh


### 10.0.2 Lowest Level Test Scripts

As an example, here we look at one of the lowest level tests, the diffusion2d test.
Files used:

- test.sh
- diffusion2D.sh
- diffusion2D.saved

Output:

- diffusion2D.dir
- diffusion2D.err
- diffusion2D.out

At this level, we execute
\$ ./test.sh diffusion2D.sh
or just
\$ ./diffusion2D.sh

The script diffusion2D.sh must create diffusion2D.dir and place all fine-grain test output in this directory. test.sh captures the standard out and error in diffusion2D.out and diffusion2D.err. The test diffusion2D.sh passes if diffusion2D.err is empty (nothing is written to standard error).

The strategy for low level scripts like diffusion2D.sh is to run a sequence of tests such as

```
$ mpirun -np 1 ../examples/ex-02 -pgrid 1 1 1 -nt 256
$ mpirun -np 4 ../examples/ex-02 -pgrid 1 1 4 -nt 256
```

The output from the first mpirun test must then be written to files named

```
diffusion2D.dir/unfiltered.std.out.0
diffusion2D.dir/std.out.0
diffusion2D.dir/std.err.0
```

and the second mpirun test similarly writes the files

```
diffusion2D.dir/unfiltered.std.out.1
diffusion2D.dir/std.out.1
diffusion2D.dir/std.err.1
```

Subsequent tests are written to higher numbered files. The unfiltered.std.out. num file contains all of the standard out for the test, while std. out. num contains filtered output (usually from a grep command) and could contain the output lines such as iteration numbers and number of levels. The file std.err. num contains the standard error output.

To see if a test ran correctly, std.out. num is compared to saved output in diffusion2D.saved. The file diffusion2D.saved contains the concatenated output from all the tests that diffusion2D.sh will run. For the above example, this file could look like

```
# Begin Test 1
number of levels = 6
iterations = 16
# Begin Test 2
number of levels = 4
iterations = 8
```

This saved output is split into an individual file for each test (using \# Begin Test as a delimiter) and these new files are placed in diffusion2D.dir. So, after running these two regression tests, diffusion2D. dir will contain

```
diffusion2D.saved.0
diffusion2D.saved.1
unfiltered.std.out.0
std.out.0
std.err.0
unfiltered.std.out.1
std.out.1
std.err.1
```

An individual test has passed if std.err.num is empty. The file std.err.num contains a diff between diffusion2D.save. num and std. out. num (the diff ignores whitespace and the delimiter \# Begin Test).

Last in the directy where you ran ./test.sh diffusion2d.sh, the files

```
diffusion2D.err
diffusion2D.out
```

will be created. If all the tests passed then diffusion2D. err will be empty. Otherwise, it will contain the filenames of the std.err. num files that are non-empty, representing failed tests.

### 10.0.3 Level 2 Scripts

As an example, here we look at one of the Level 2 tests, the machine-tux test that Jacob runs. Files used:

- machine-tux.sh

Output:

- machine-tux.dir
- machine-tux.err (only generated if autotest.sh is used to run machine-tux.sh)
- machine-tux. out (only generated if autotest.sh is used to run machine-tux.sh)

At this level, we execute

The autotest framework (autotest.sh) calls machine scripts in this way. Each machine script should be short and call lower-level scripts like diffusion2D.sh. The output from lower-level scripts must be moved to machine-tux.dir like this:

```
$ ./test.sh diffusion2D.sh
$ mv -f diffusion2D.dir machine-tux.dir
$ mv -f diffusion2D.out machine-tux.dir
$ mv -f diffusion2D.err machine-tux.dir
```

All error files from diffusion2D.sh will be placed in machine-tux. dir, so if machine-tux. dir has all zero $*$. err files, then the machine-tux test has passed.

To begin testing on a new machine, like vulcan, add a new machine script similar to machine-tux. sh and change autotest. sh to recognize and run the new machine test. To then use autotest. sh with the machine script, you'll have to set up a passwordless connection from the new machine to

### 10.0.4 Level 3 Script

Here we look at the highest level, where autotest. sh runs all of the level 2 machine tests and emails out the results.

Files used:

- autotest.sh

Output:

- test/autotest_finished
- /usr/casc/hypre/braid/testing/AUTOTEST-20**.**.**-Day
- Email to recipients listed in autotest.sh

At the highest level sits autotest. sh and is called automatically as a cronjob. If you just want to check to see if you've broken anything with a commit, just use lower level scripts.

There are four steps to running autotest.

- Step 1
\$ ./autotesh.sh -init
will do a pull from master for the current working repository and recompile Braid. The autotest output files (autotest.err and autotest. out) and the output directory (autotest_finished) are initialized.
- Step 2
\$ ./autotest.sh -tux343
will run the autotests on tux 343 . This command will look for a machine-tux. sh, and execute it, moving the resulting

$$
\begin{aligned}
& \text { machine-tux.dir } \\
& \text { machine-tux.err } \\
& \text { machine-tux.out }
\end{aligned}
$$

into test/autotest_finished.

- Step 3
\$ ./autotest.sh -remote-copy
will copy /test/autotest_finished/* to a time-stamped directory such as
/usr/casc/hypre/braid/testing/AUTOTEST-2013.11.18-Mon
Alternatively,
\$ ./autotesh.sh -remote-copy tux343
will ssh through tux343 to copy to /usr/casc. Multiple machines may independently be running regression tests and then copy to AUTOTEST-2013.11.18-Mon.
- Step 4

```
$ ./autotest.sh -summary-email
```

will email everyone listed in the \$email_list (an autotest.sh variable)

### 10.0.5 Cronfile

To add entries to your crontab, First, put your new cronjob lines into cronfile. Then see what you already have in your crontab file with

```
$ crontab -1
```

Next, append to cronfile whatever you already have

```
$ crontab -l >> cronfile
```

Finally, tell crontab to use your cronfile

```
$ crontab cronfile
```

Then make sure it took affect with
\$ crontab -l

Crontab entry format uses ' $*$ ' to mean "every" and ' $* / \mathrm{m}$ ' to mean "every m-th". The first five entries on each line correspond respectively to:

- minute (0-56)
- hour (0-23)
- day of month (1-31)
- month (1-12)
- day of week (0-6)(0=Sunday)

Jacob's crontab (on tux343):

```
00 01 * * * source /etc/profile; source $HOME/.bashrc; cd $HOME/joint_repos/braid/test; ./autotest.sh -init
10 01 * * * source /etc/profile; source $HOME/.bashrc; cd $HOME/joint_repos/braid/test; ./autotest.sh -tux343
40 01 * * * source /etc/profile; source $HOME/.bashrc; cd $HOME/joint_repos/braid/test; ./autotest.sh -remote-c
50 01 * * * source /etc/profile; source $HOME/.bashrc; cd $HOME/joint_repos/braid/test; ./autotest.sh -summary-
OO 02 * * * source /etc/profile; source $HOME/.bashrc; cd $HOME/joint_repos/braid/test; ./autotest.sh -create-t
```


## 11 Module Index

### 11.1 Modules

Here is a list of all modules:
Fortran 90 interface options ..... 63
Error Codes ..... 64
User-written routines ..... 64
User-written routines for XBraid_Adjoint ..... 73
User interface routines ..... 76
General Interface routines ..... 76
Interface routines for XBraid_Adjoint ..... 97
XBraid status structures ..... 101
XBraid status routines ..... 101
Inherited XBraid status routines ..... 117
XBraid status macros ..... 129
XBraid test routines ..... 131

## 12 Data Structure Index

### 12.1 Data Structures

Here are the data structures with brief descriptions:
_braid_Action 140
_braid_BaseVector 142
_braid_Basis 143
_braid_CommHandle 144
_braid_Core 145
_braid_Grid 163
_braid_Status 166
_braid_Tape 167
_braid_VectorBar 168
braid_AccessStatus 169
braid_BufferStatus 169
braid_CoarsenRefStatus 170
braid_ObjectiveStatus 171

```
braid_Optim 171
braid_StepStatus 173
braid_SyncStatus 174
```


## 13 File Index

### 13.1 File List

Here is a list of all files with brief descriptions:

```
_braid.h
    Define headers for XBraid internal (developer) routines and XBraid internal structure declarations175
adjoint.h
    Define internal XBraid headers for the adjoint feature189
```

base.h
Define XBraid internal headers for wrapper routines of user-defined functions ..... 191
braid.h
Define headers for user-interface routines ..... 205
braid defs.h
Definitions of braid types, error flags, etc.. ..... 208
braid_status.hDefine headers for the user-interface with the XBraid status structures, allowing the user to get/setstatus structure values209
braid_test.h
Define headers for XBraid user-test routines ..... 215
delta.h
Define internal XBraid headers for Delta correction ..... 216
mpistubs.h
XBraid internal headers to define fake MPI stubs. This ultimately allows the user to generate purelyserial codes without MPI217
status.hDefine the XBraid internal headers for the XBraid status structure routines, and define the statusstructures themselves218
tape.h
Define the XBraid internal headers for the action-tape routines (linked list for AD) ..... 222
util.h
Define XBraid internal headers for utility routines ..... 224

## 14 Module Documentation

### 14.1 Fortran 90 interface options

## Macros

- \#define braid_FMANGLE 1
- \#define braid_Fortran_SpatialCoarsen 0
- \#define braid_Fortran_Residual 1
- \#define braid_Fortran_TimeGrid 1
- \#define braid_Fortran_Sync 1


### 14.1.1 Detailed Description

Allows user to manually, at compile-time, turn on Fortran 90 interface options

### 14.1.2 Macro Definition Documentation

14.1.2.1 braid_FMANGLE \#define braid_FMANGLE 1

Define Fortran name-mangling schema, there are four supported options, see braid_F90_iface.c
14.1.2.2 braid_Fortran_Residual \#define braid_Fortran_Residual 1

Turn on the optional user-defined residual function
14.1.2.3 braid_Fortran_SpatialCoarsen \#define braid_Fortran_SpatialCoarsen 0

Turn on the optional user-defined spatial coarsening and refinement functions
14.1.2.4 braid_Fortran_Sync \#define braid_Fortran_Sync 1

Turn on the optional user-defined sync function
14.1.2.5 braid_Fortran_TimeGrid \#define braid_Fortran_TimeGrid 1

Turn on the optional user-defined time-grid function

### 14.2 Error Codes

## Macros

- \#define braid_INVALID_RNORM -1
- \#define braid_ERROR_GENERIC 1 /* generic error */
- \#define braid_ERROR_MEMORY $2 / *$ unable to allocate memory $* /$
- \#define braid_ERROR_ARG 4 /* argument error */


### 14.2.1 Detailed Description

### 14.2.2 Macro Definition Documentation

14.2.2.1 braid_ERROR_ARG \#define braid_ERROR_ARG 4 /* argument error */
14.2.2.2 braid_ERROR_GENERIC \#define braid_ERROR_GENERIC 1 /* generic error */
14.2.2.3 braid_ERROR_MEMORY \#define braid_ERROR_MEMORY 2 /* unable to allocate memory */
14.2.2.4 braid_INVALID_RNORM \#define braid_INVALID_RNORM -1

Value used to represent an invalid residual norm

### 14.3 User-written routines

## Modules

- User-written routines for XBraid_Adjoint


## Typedefs

- typedef struct _braid_App_struct * braid_App
- typedef struct _braid_Vector_struct * braid_Vector
- typedef braid_Int(* braid_PtFcnStep) (braid_App app, braid_Vector ustop, braid_Vector fstop, braid_Vector u, braid_StepStatus status)
- typedef braid_Int(* braid_PtFcnInit) (braid_App app, braid_Real t, braid_Vector *u_ptr)
- typedef braid_Int(* braid_PtFcnInitBasis) (braid_App app, braid_Real t, braid_Int index, braid_Vector *u_ptr)
- typedef braid_Int(* braid_PtFcnClone) (braid_App app, braid_Vector u, braid_Vector *v_ptr)
- typedef braid_Int(* braid_PtFcnFree) (braid_App app, braid_Vector u)
- typedef braid_Int(* braid_PtFcnSum) (braid_App app, braid_Real alpha, braid_Vector x, braid_Real beta, braid_Vector y)
- typedef braid_Int(* braid_PtFcnSpatialNorm) (braid_App app, braid_Vector u, braid_Real *norm_ptr)
- typedef braid_Int(* braid_PtFcnInnerProd) (braid_App app, braid_Vector u, braid_Vector v, braid_Real *prod_ptr)
- typedef braid_Int(* braid_PtFcnAccess) (braid_App app, braid_Vector u, braid_AccessStatus status)
- typedef braid_Int(* braid_PtFcnSync) (braid_App app, braid_SyncStatus status)
- typedef braid_Int(* braid_PtFcnBufSize) (braid_App app, braid_Int *size_ptr, braid_BufferStatus status)
- typedef braid_Int(* braid_PtFcnBufPack) (braid_App app, braid_Vector u, void *buffer, braid_BufferStatus status)
- typedef braid_Int(* braid_PtFcnBufUnpack) (braid_App app, void *buffer, braid_Vector *u_ptr, braid_BufferStatus status)
- typedef braid_Int(* braid_PtFcnBufAlloc) (braid_App app, void **buffer, braid_Int nbytes, braid_BufferStatus status)
- typedef braid_Int(* braid_PtFcnBufFree) (braid_App app, void $* *$ buffer)
- typedef braid_Int(* braid_PtFcnResidual) (braid_App app, braid_Vector ustop, braid_Vector r, braid_StepStatus status)
- typedef braid_Int(* braid_PtFcnSCoarsen) (braid_App app, braid_Vector fu, braid_Vector $*$ cu_ptr, braid_↔ CoarsenRefStatus status)
- typedef braid_Int(* braid_PtFcnSRefine) (braid_App app, braid_Vector cu, braid_Vector *fu_ptr, braid_Coarsen $\hookleftarrow$ RefStatus status)
- typedef braid_Int(* braid_PtFcnSInit) (braid_App app, braid_Real t, braid_Vector $* u \_p t r$ )
- typedef braid_Int(* braid_PtFcnSClone) (braid_App app, braid_Vector u, braid_Vector *v_ptr)
- typedef braid_Int(* braid_PtFcnSFree) (braid_App app, braid_Vector u)
- typedef braid_Int(* braid_PtFcnTimeGrid) (braid_App app, braid_Real *ta, braid_Int *ilower, braid_Int *iupper)


### 14.3.1 Detailed Description

These are all the user-written data structures and routines. There are two data structures (braid_App and braid_Vector) for the user to define. And, there are a variety of function interfaces (defined through function pointer declarations) that the user must implement.

### 14.3.2 Typedef Documentation

### 14.3.2.1 braid_App typedef struct _braid_App_struct* braid_App

This holds a wide variety of information and is global in that it is passed to every function. This structure holds everything that the user will need to carry out a simulation. For a simple example, this could just hold the global MPI communicator and a few values describing the temporal domain.
14.3.2.2 braid_PtFcnAccess typedef braid_Int (* braid_PtFcnAccess) (braid_App app, braid_Vector u,
braid_AccessStatus status)

Gives user access to XBraid and to the current vector $u$ at time $t$. Most commonly, this lets the user write the vector to screen, file, etc... The user decides what is appropriate. Note how you are told the time value $t$ of the vector $u$ and other information in status. This lets you tailor the output, e.g., for only certain time values at certain XBraid iterations. Querying status for such information is done through braid_AccessStatusGet**(..) routines.

The frequency of XBraid's calls to access is controlled through braid_SetAccessLevel. For instance, if access_level is set to 3 , then access is called every XBraid iteration and on every XBraid level. In this case, querying status to determine the current XBraid level and iteration will be useful. This scenario allows for even more detailed tracking of the simulation.

Eventually, access will be broadened to allow the user to steer XBraid.
Parameters

| $a p p$ | user-defined__braid_App structure |
| :--- | :--- |
| $u$ | vector to be accessed |
| status | can be querried for info like the current XBraid Iteration |

14.3.2.3 braid_PtFcnBufAlloc typedef braid_Int (* braid_PtFcnBufAlloc) (braid_App app, void **buffer, braid_Int nbytes, braid_BufferStatus status)

This allows the user (not XBraid) to allocate the MPI buffer for a certain number of bytes. This routine is optional, but can be useful, if the MPI buffer needs to be allocated in a special way, e.g., on a device/accelerator

Parameters

| app | user-defined_braid_App structure |
| :--- | :--- |
| buffer | pointer to the void $*$ MPI Buffer |
| nbytes | number of bytes to allocate |
| status | can be querried for info on the current message type |

### 14.3.2.4 braid_PtFcnBufFree typedef braid_Int (* braid_PtFcnBufFree) (braid_App app, void **buffer)

This allows XBraid to free a user allocated MPI buffer

Parameters

| app | user-defined_braid_App structure |
| :--- | :--- |
| buffer | pointer to the void $*$ MPI Buffer |

14.3.2.5 braid_PtFcnBufPack typedef braid_Int(* braid_PtFcnBufPack) (braid_App app, braid_Vector u, void *buffer, braid_BufferStatus status)

This allows XBraid to send messages containing braid_Vectors. This routine packs a vector $u$ into a void $*$ bufferfor MPI. The status structure holds information regarding the message. This is accessed through the braid_BufferStatusGet**(..) routines. Optionally, the user can set the message size through the status structure.

## Parameters

| app | user-defined_braid_App structure |
| :--- | :--- |
| $u$ | vector to back into buffer |
| buffer | output, MPI buffer containing u |
| status | can be queeried for info on the message type required |

14.3.2.6 braid_PtFcnBufSize typedef braid_Int (* braid_PtFcnBufSize) (braid_App app, braid_Int
*size_ptr, braid_BufferStatus status)

This routine tells XBraid message sizes by computing an upper bound in bytes for an arbitrary braid_Vector. This size must be an upper bound for what BufPack and BufUnPack will assume.

## Parameters

| app | user-defined _braid_App structure |
| :--- | :--- |
| size_ptr | upper bound on vector size in bytes |
| status | can be querried for info on the message type |

14.3.2.7 braid_PtFcnBufUnpack typedef braid_Int (* braid_PtFcnBufUnpack) (braid_App app, void *buffer, braid_Vector *u_ptr, braid_BufferStatus status)

This allows XBraid to receive messages containing braid_Vectors. This routine unpacks a void $*$ buffer from MPI into a braid_Vector. The status structure, contains information conveying the type of message inside the buffer. This can be accessed through the braid_BufferStatusGet**(..) routines.

## Parameters

| app | user-defined _braid_App structure |
| :--- | :--- |
| buffer | MPI Buffer to unpack and place in u_ptr |
| u_ptr | output, braid_Vector containing buffer's data |
| status | can be querried for info on the current message type |

14.3.2.8 braid_PtFcnClone typedef braid_Int(* braid_PtFcnClone) (braid_App app, braid_Vector u, braid_Vector *v_ptr)

Clone $u$ into $v \_p t r$
Parameters

| $a p p$ | user-defined_braid_App structure |
| :--- | :--- |
| $u$ | vector to clone |
| $v \_p t r$ | output, newly allocated and cloned vector |

### 14.3.2.9 braid_PtFcnFree typedef braid_Int (* braid_PtFcnFree) (braid_App app, braid_Vector u)

Free and deallocate $u$
Parameters

| app | user-defined_braid_App structure |
| :--- | :--- |
| $u$ | vector to free |

14.3.2.10 braid_PtFcnInit typedef braid_Int (* braid_PtFcnInit) (braid_App app, braid_Real t, braid_Vector
*u_ptr)

Initializes a vector $u \_p$ tr at time $t$

Parameters

| app | user-defined_braid_App structure |
| :--- | :--- |
| $t$ | time value for $u \_p t r$ |
| $u \_p t r$ | output, newly allocated and initialized vector |

14.3.2.11 braid_PtFcnInitBasis typedef braid_Int (* braid_PtFcnInitBasis) (braid_App app, braid_Real t, braid_Int index, braid_Vector *u_ptr)
(optional) Initializes a Delta correction basis vector $u \_p t r$ at time $t$ and spatial index index. The spatial index is simply used to distinguish between the different basis vectors at a given time point.

## Parameters

| app | user-defined _braid_App structure |
| :--- | :--- |
| $t$ | time value for $u \_p t r$ |
| index | spatial index of basis vector |
| u_ptr | output, newly allocated and initialized vector |

14.3.2.12 braid_PtFcnInnerProd typedef braid_Int (* braid_PtFcnInnerProd) (braid_App app, braid_Vector
u, braid_Vector v, braid_Real *prod_ptr)
(optional) Compute an inner (scalar) product between two braid_Vectors prod_ptr $=\langle u, v\rangle$ Only needed when using Delta correction

The most common choice would be the standard dot product. Vectors are normalized under the norm induced by this inner product, not the function defined in SpatialNorm, which is only used for halting

Parameters

| app | user-defined_braid_App structure |
| :--- | :--- |
| $u$ | first vector |
| $v$ | second vector |
| prod_ptr | output, result of inner product |

14.3.2.13 braid_PtFcnResidual typedef braid_Int (* braid_PtFcnResidual) (braid_App app, braid_Vector ustop, braid_vector r, braid_StepStatus status)

This function (optional) computes the residual $r$ at time tstop. On input, $r$ holds the value of $u$ at tstart, and $u s t o p$ is the value of $u$ at tstop. If used, set with braid_SetResidual.

Query the status structure with braid_StepStatusGetTstart(status, \&tstart) and braid_StepStatusGetTstop(status, \&tstop) to get tstart and tstop.

Parameters

| app | user-defined_braid_App structure |
| :--- | :--- |
| ustop | input, u vector at tstop |
| $r$ | output, residual at tstop (at input, equals $u$ at $t$ start) |
| status | query this struct for info about u (e.g., tstart and tstop) |

14.3.2.14 braid_PtFcnSClone typedef braid_Int (* braid_PtFcnSClone) (braid_App app, braid_Vector
u, braid_Vector *v_ptr)
Shell clone (optional)
Parameters

| app | user-defined _braid_App structure |
| :--- | :--- |
| $u$ | vector to clone |
| $v \_p t r$ | output, newly allocated and cloned vector shell |

14.3.2.15 braid_PtFcnSCoarsen typedef braid_Int(* braid_PtFcnSCoarsen) (braid_App app, braid_Vector fu, braid_Vector *cu_ptr, braid_CoarsenRefStatus status)

Spatial coarsening (optional). Allows the user to coarsen when going from a fine time grid to a coarse time grid. This function is called on every vector at each level, thus you can coarsen the entire space time domain. The action of this function should match the braid_PtFcnSRefine function.

The user should query the status structure at run time with braid_CoarsenRefGet**() calls in order to determine how to coarsen.
For instance, status tells you what the current time value is, and what the time step sizes on the fine and coarse levels are.

Parameters

| app | user-defined_braid_App structure |
| :--- | :--- |
| $f u$ | braid_Vector to refine |
| cu_ptr | output, refined vector |
| status | query this struct for info about fu and cu (e.g., where in time fu and cu are) |

### 14.3.2.16 braid_PtFcnSFree typedef braid_Int (* braid_PtFcnSFree) (braid_App app, braid_Vector u)

Free the data of $u$, keep its shell (optional)
Parameters

| app | user-defined_braid_App structure |
| :--- | :--- |
| $u$ | vector to free (keeping the shell) |

14.3.2.17 braid_PtFcnSInit typedef braid_Int (* braid_PtFcnSInit) (braid_App app, braid_Real t,

```
braid_Vector *u_ptr)
```

Shell initialization (optional)
Parameters

| app | user-defined_braid_App structure |
| :--- | :--- |
| $t$ | time value for $u \_p t r$ |
| $u \_p t r$ | output, newly allocated and initialized vector shell |

14.3.2.18 braid_PtFenSpatialNorm typedef braid_Int (* braid_PtFcnSpatialNorm) (braid_App app, braid_Vector u, braid_Real *norm_ptr)

Carry out a spatial norm by taking the norm of a braid_Vector norm_ptr $=\|u\|$ A common choice is the standard Euclidean norm, but many other choices are possible, such as an L2-norm based on a finite element space. See braid_SetTemporalNorm for information on how the spatial norm is combined over time for a global space-time residual norm. This global norm then controls halting.

## Parameters

| app | user-defined _braid_App structure |
| :--- | :--- |
| $u$ | vector to norm |
| norm_ptr | output, norm of braid_Vector (this is a spatial norm) |

14.3.2.19 braid_PtFcnSRefine typedef braid_Int (* braid_PtFcnSRefine) (braid_App app, braid_Vector
cu, braid_Vector $*$ fu_ptr, braid_CoarsenRefStatus status)

Spatial refinement (optional). Allows the user to refine when going from a coarse time grid to a fine time grid. This function is called on every vector at each level, thus you can refine the entire space time domain. The action of this function should match the braid_PtFcnSCoarsen function.

The user should query the status structure at run time with braid_CoarsenRefGet**() calls in order to determine how to coarsen.
For instance, status tells you what the current time value is, and what the time step sizes on the fine and coarse levels are.

## Parameters

| app | user-defined_braid_App structure |
| :--- | :--- |
| $c u$ | braid_Vector to refine |
| $f u \_p t r$ | output, refined vector |
| status | query this struct for info about fu and cu (e.g., where in time fu and cu are) |

14.3.2.20 braid_PtFcnStep typedef braid_Int(* braid_PtFcnStep) (braid_App app, braid_Vector ustop, braid_Vector fstop, braid_Vector u, braid_StepStatus status)

Defines the central time stepping function that the user must write.

The user must advance the vector $u$ from time tstart to tstop. The time step is taken assuming the right-hand-side vector fstop at time tstop. The vector ustop may be the same vector as $u$ (in the case where not all unknowns are stored). The vector fstop is set to NULL to indicate a zero right-hand-side.

Query the status structure with braid_StepStatusGetTstart(status, \&tstart) and braid_StepStatusGetTstop(status, \&tstop) to get tstart and tstop. The status structure also allows for steering. For example, braid_StepStatusSet $\hookleftarrow$ RFactor(...) allows for setting a refinement factor, which tells XBraid to refine this time interval.

Parameters

| app | user-defined_braid_App structure |
| :--- | :--- |
| ustop | input, u vector at tstop |
| fstop | input, right-hand-side at tstop |
| $u$ | input/output, initially u vector at tstart, upon exit, u vector at tstop |
| status | query this struct for info about u (e.g., tstart and tstop), allows for steering (e.g., set rfactor) |

14.3.2.21 braid_PtFcnSum typedef braid_Int (* braid_PtFcnSum) (braid_App app, braid_Real alpha, braid_Vector x, braid_Real beta, braid_Vector y)

AXPY, alpha $x+$ beta $y-->y$

## Parameters

| app | user-defined_braid_App structure |
| :--- | :--- |
| alpha | scalar for AXPY |
| $x$ | vector for AXPY |
| beta | scalar for AXPY |
| $y$ | output and vector for AXPY |

14.3.2.22 braid_PtFcnSync typedef braid_Int(* braid_PtFcnSync) (braid_App app, braid_SyncStatus status)

Gives user access to XBraid and to the user's app at various points (primarily once per iteration inside FRefine and outside in the main cycle loop). This function is called once per-processor (not for every state vector stored on the processor, like access).

## Parameters

| app | user-defined_braid_App structure |
| :--- | :--- |
| status | can be querried for info like the current XBraid Iteration |

14.3.2.23 braid_PtFcnTimeGrid typedef braid_Int(* braid_PtFcnTimeGrid) (braid_App app, braid_Real *ta, braid_Int *ilower, braid_Int *iupper)

Set time values for temporal grid on level 0 (time slice per processor)
Parameters

| app | user-defined_braid_App structure |
| :--- | :--- |
| ta | temporal grid on level 0 (slice per processor) |
| ilower | lower time index value for this processor |
| iupper | upper time index value for this processor |

### 14.3.2.24 braid_Vector typedef struct _braid_Vector_struct* braid_Vector

This defines (roughly) a state vector at a certain time value.
It could also contain any other information related to this vector which is needed to evolve the vector to the next time value, like mesh information.

### 14.4 User-written routines for XBraid_Adjoint

## Typedefs

- typedef braid_Int(* braid_PtFcnObjectiveT) (braid_App app, braid_Vector u, braid_ObjectiveStatus ostatus, braid_Real *objectiveT_ptr)
- typedef braid_Int(* braid_PtFcnObjectiveTDiff) (braid_App app, braid_Vector u, braid_Vector u_bar, braid_Real F_bar, braid_ObjectiveStatus ostatus)
- typedef braid_Int(* braid_PtFcnPostprocessObjective) (braid_App app, braid_Real sum_obj, braid_Real *postprocess_ptr)
- typedef braid_Int(* braid_PtFcnPostprocessObjective_diff) (braid_App app, braid_Real sum_obj, braid_Real $* F_{\hookleftarrow}$ _bar_ptr)
- typedef braid_Int(* braid_PtFcnStepDiff) (braid_App app, braid_Vector ustop, braid_Vector u, braid_Vector ustop_bar, braid_Vector u_bar, braid_StepStatus status)
- typedef braid_Int(* braid_PtFcnResetGradient) (braid_App app)


### 14.4.1 Detailed Description

These are all the user-written routines needed to use XBraid_Adjoint. There are no new user-written data structures here. But, the braid_App structure will typically be used to store some things like optimization parameters and gradients.

### 14.4.2 Typedef Documentation

14.4.2.1 braid_PtFcnObjectiveT typedef braid_Int (* braid_PtFcnObjectiveT) (braid_App app, braid_Vector
u, braid_ObjectiveStatus ostatus, braid_Real *objectiveT_ptr)
This routine evaluates the time-dependent part of the objective function, at a current time $t$, i.e. the integrand. Query the braid_ObjectiveStatus structure for information about the current time and status of XBraid_Adjoint.

Parameters

| app | user-defined_braid_App structure |
| :--- | :--- |
| $u$ | input: state vector at current time |
| ostatus | status structure for querying time, index, etc. |
| objectiveT_ptr | output: objective function at current time |

14.4.2.2 braid_PtFcnObjectiveTDiff typedef braid_Int (* braid_PtFcnObjectiveTDiff) (braid_App app, braid_Vector u, braid_Vector u_bar, braid_Real F_bar, braid_ObjectiveStatus ostatus)

This is the differentiated version of the braid_PtFcnObjectiveT routine. It provides the derivatives of ObjectiveT() multiplied by the scalar input $F$ _bar.

First output: the derivative with respect to the state vector must be returned to XBraid_Adjoint in $u \_b a r$.

Second output: The derivative with respect to the design must update the gradient, which is stored in the braid_App.

## Parameters

| app | input / output: user-defined_braid_App structure, used to store gradient |
| :--- | :--- |
| $u$ | input: state vector at current time |
| u_bar | output: adjoint vector, holding the derivative wrt u |
| F_bar | scalar input, multiply the derivative with this |
| ostatus | query this for about $t$, tindex, etc |

14.4.2.3 braid_PtFcnPostprocessObjective typedef braid_Int(* braid_PtFcnPostprocessobjective)
(braid_App app, braid_Real sum_obj, braid_Real *postprocess_ptr)
(Optional) This function can be used to postprocess the time-integral objective function. For example, when inverse design problems are considered, you can use a tracking-type objective function by substracting a target value from postprocess_ptr, and squaring the result. Relaxation or penalty terms can also be added to postprocess_ptr. For a description of the postprocessing routine, see the Section Objective function evaluation .

Parameters

| app | user-defined_braid_App structure |
| :--- | :--- |
| sum_obj | input: sum over time of the local time-dependent ObjectiveT values |
| postprocess_ptr | output: Postprocessed objective, e.g. tracking type function |

14.4.2.4 braid_PtFcnPostprocessObjective_diff typedef braid_Int(* braid_PtFcnPostprocessobjective $\hookleftarrow$ _diff) (braid_App app, braid_Real sum_obj, braid_Real *F_bar_ptr)
(Optional) Differentiated version of the Postprocessing routine.

First output: Return the partial derivative of the braid_PtFcnPostprocessObjective routine with respect to the timeintegral objective function, and placing the result in the scalar value $F_{-}$bar_ptr

Second output: Update the gradient with the partial derivative with respect to the design. Gradients are usually stored in braid_App .

For a description of the postprocessing routine, see the Section Objective function evaluation.
Parameters

| app | user-defined _braid_App structure |
| :--- | :--- |
| sum_obj | input: sum over time of the local time-dependent ObjectiveT values |
| F_bar_ptr | output: partial derivative of the postprocessed objective with respect to sum_obj |

### 14.4.2.5 braid_PtFcnResetGradient typedef braid_Int (* braid_PtFcnResetGradient) (braid_App app)

Set the gradient to zero, which is usually stored in braid_App .
Parameters

```
app output: user-defined _braid_App structure, used to store gradient
```

14.4.2.6 braid_PtFcnStepDiff typedef braid_Int (* braid_PtFcnStepDiff) (braid_App app, braid_Vector ustop, braid_Vector u, braid_Vector ustop_bar, braid_Vector u_bar, braid_StepStatus status)

This is the differentiated version of the time-stepping routine. It provides the transposed derivatives of Step() multiplied by the adjoint input vector $u \_b a r$ (or ustop_bar).

First output: the derivative with respect to the state $u$ updates the adjoint vector $u \_$bar (or ustop_bar).
Second output: The derivative with respect to the design must update the gradient, which is stored in braid_App .

Parameters

| app | input / output: user-defined _braid_App structure, used to store gradient |
| :--- | :--- |
| ustop | input, u vector at tstop |
| u | input, u vector at tstart |
| ustop_bar | input / output, adjoint vector for ustop |
| u_bar | input / output, adjoint vector for u |
| status | query this struct for info about u (e.g., tstart and tstop) |

### 14.5 User interface routines

## Modules

- General Interface routines
- Interface routines for XBraid_Adjoint
- XBraid status structures
- XBraid status routines
- Inherited XBraid status routines
- XBraid status macros


### 14.5.1 Detailed Description

These are all the user interface routines.

### 14.6 General Interface routines

## Macros

- \#define braid_RAND_MAX 32768


## Typedefs

- typedef struct _braid_Core_struct * braid_Core


## Functions

- braid_Int braid_Init (MPI_Comm comm_world, MPI_Comm comm, braid_Real tstart, braid_Real tstop, braid_Int ntime, braid_App app, braid_PtFcnStep step, braid_PtFcnInit init, braid_PtFcnClone clone, braid_PtFcnFree free, braid_PtFcnSum sum, braid_PtFcnSpatialNorm spatialnorm, braid_PtFcnAccess access, braid_PtFcnBufSize bufsize, braid_PtFcnBufPack bufpack, braid_PtFcnBufUnpack bufunpack, braid_Core *core_ptr)
- braid_Int braid_Drive (braid_Core core)
- braid_Int braid_Destroy (braid_Core core)
- braid_Int braid_PrintStats (braid_Core core)
- braid_Int braid_SetTimerFile (braid_Core core, braid_Int length, const char *filestem)
- braid_Int braid_PrintTimers (braid_Core core)
- braid_Int braid_ResetTimer (braid_Core core)
- braid_Int braid_WriteConvHistory (braid_Core core, const char *filename)
- braid_Int braid_SetMaxLevels (braid_Core core, braid_Int max_levels)
- braid_Int braid_SetIncrMaxLevels (braid_Core core)
- braid_Int braid_SetSkip (braid_Core core, braid_Int skip)
- braid_Int braid_SetRefine (braid_Core core, braid_Int refine)
- braid_Int braid_SetMaxRefinements (braid_Core core, braid_Int max_refinements)
- braid_Int braid_SetTPointsCutoff (braid_Core core, braid_Int tpoints_cutoff)
- braid_Int braid_SetMinCoarse (braid_Core core, braid_Int min_coarse)
- braid_Int braid_SetRelaxOnlyCG (braid_Core core, braid_Int relax_only_cg)
- braid_Int braid_SetAbsTol (braid_Core core, braid_Real atol)
- braid_Int braid_SetRelTol (braid_Core core, braid_Real rtol)
- braid_Int braid_SetNRelax (braid_Core core, braid_Int level, braid_Int nrelax)
- braid_Int braid_SetCRelaxWt (braid_Core core, braid_Int level, braid_Real Cwt)
- braid_Int braid_SetCFactor (braid_Core core, braid_Int level, braid_Int cfactor)
- braid_Int braid_SetMaxIter (braid_Core core, braid_Int max_iter)
- braid_Int braid_SetFMG (braid_Core core)
- braid_Int braid_SetNFMG (braid_Core core, braid_Int k)
- braid_Int braid_SetNFMGVcyc (braid_Core core, braid_Int nfmg_Vcyc)
- braid_Int braid_SetStorage (braid_Core core, braid_Int storage)
- braid_Int braid_SetTemporalNorm (braid_Core core, braid_Int tnorm)
- braid_Int braid_SetResidual (braid_Core core, braid_PtFcnResidual residual)
- braid_Int braid_SetFullRNormRes (braid_Core core, braid_PtFcnResidual residual)
- braid_Int braid_SetTimeGrid (braid_Core core, braid_PtFcnTimeGrid tgrid)
- braid_Int braid_SetPeriodic (braid_Core core, braid_Int periodic)
- braid_Int braid_SetSpatialCoarsen (braid_Core core, braid_PtFcnSCoarsen scoarsen)
- braid_Int braid_SetSpatialRefine (braid_Core core, braid_PtFcnSRefine srefine)
- braid_Int braid_SetSync (braid_Core core, braid_PtFcnSync sync)
- braid_Int braid_SetInnerProd (braid_Core core, braid_PtFcnInnerProd inner_prod)
- braid_Int braid_SetPrintLevel (braid_Core core, braid_Int print_level)
- braid_Int braid_SetFileIOLevel (braid_Core core, braid_Int io_level)
- braid_Int braid_SetPrintFile (braid_Core core, const char *printfile_name)
- braid_Int braid_SetDefaultPrintFile (braid_Core core)
- braid_Int braid_SetAccessLevel (braid_Core core, braid_Int access_level)
- braid_Int braid_SetFinalFCRelax (braid_Core core)
- braid_Int braid_SetBufAllocFree (braid_Core core, braid_PtFcnBufAlloc bufalloc, braid_PtFcnBufFree buffree)
- braid_Int braid_SplitCommworld (const MPI_Comm *comm_world, braid_Int px, MPI_Comm $*$ comm_x, MPI_ $\hookleftarrow$ Comm *comm_t)
- braid_Int braid_SetShell (braid_Core core, braid_PtFcnSInit sinit, braid_PtFcnSClone sclone, braid_PtFcnSFree sfree)
- braid_Int braid_GetNumlter (braid_Core core, braid_Int *niter_ptr)
- braid_Int braid_GetRNorms (braid_Core core, braid_Int *nrequest_ptr, braid_Real *rnorms)
- braid_Int braid_GetNLevels (braid_Core core, braid_Int *nlevels_ptr)
- braid_Int braid_GetSpatialAccuracy (braid_StepStatus status, braid_Real loose_tol, braid_Real tight_tol, braid_Real *tol_ptr)
- braid_Int braid_SetSeqSoln (braid_Core core, braid_Int seq_soln)
- braid_Int braid_SetRichardsonEstimation (braid_Core core, braid_Int est_error, braid_Int richardson, braid_Int local_order)
- braid_Int braid_SetDeltaCorrection (braid_Core core, braid_Int rank, braid_PtFcnInitBasis basis_init, braid_PtFcnInnerProd inner_prod)
- braid_Int braid_SetDeferDelta (braid_Core core, braid_Int level, braid_Int iter)
- braid_Int braid_SetLyapunovEstimation (braid_Core core, braid_Int relax, braid_Int cglv, braid_Int exponents)
- braid_Int braid_SetTimings (braid_Core core, braid_Int timing_level)
- braid_Int braid_GetMyID (braid_Core core, braid_Int *myid_ptr)
- braid_Int braid_Rand (void)


### 14.6.1 Detailed Description

These are general interface routines, e.g., routines to initialize and run a XBraid solver, or to split a communicator into spatial and temporal components.

### 14.6.2 Macro Definition Documentation

### 14.6.2.1 braid_RAND_MAX \#define braid_RAND_MAX 32768

Machine independent pseudo-random number generator is defined in Braid.c

### 14.6.3 Typedef Documentation

14.6.3.1 braid_Core typedef struct _braid_Core_struct* braid_Core
points to the core structure defined in _braid.h

### 14.6.4 Function Documentation

14.6.4.1 braid_Destroy() braid_Int braid_Destroy (
braid_Core core )
Clean up and destroy core.

## Parameters

core braid_Core (_braid_Core) struct
14.6.4.2 braid_Drive() braid_Int braid_Drive (

```
    braid_Core core )
```

Carry out a simulation with XBraid. Integrate in time.

Parameters
core $\quad$ braid_Core (_braid_Core) struct
14.6.4.3 braid_GetMyID() braid_Int braid_GetMyID (
braid_core core,
braid_Int * myid_ptr )

Get the processor's rank.
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| myid_ptr | output: rank of the processor. |

14.6.4.4 braid_GetNLevels() braid_Int braid_GetNLevels (
braid_Core core,
braid_Int * nlevels_ptr )
After Drive() finishes, this returns the number of XBraid levels

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| nlevels_ptr | output, holds the number of XBraid levels |

14.6.4.5 braid_GetNumlter() braid_Int braid_GetNumIter (

```
braid_Core core,
braid_Int * niter_ptr )
```

After Drive() finishes, this returns the number of iterations taken.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| niter_ptr | output, holds number of iterations taken |

14.6.4.6 braid_GetRNorms() braid_Int braid_GetRNorms (

```
braid_Core core,
braid_Int * nrequest_ptr,
braid_Real * rnorms )
```

After Drive() finishes, this returns XBraid residual history. If nrequest_ptr is negative, return the last nrequest_ptr residual norms. If positive, return the first nrequest_ptr residual norms. Upon exit, nrequest_ptr holds the number of residuals actually returned.

Parameters

| Core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| nrequest_ptr | input/output, input: num requested resid norms, output: num actually returned |
| rnorms | output, holds residual norm history array |

### 14.6.4.7 braid_GetSpatialAccuracy() braid_Int braid_GetSpatialAccuracy (

```
braid_StepStatus status,
braid_Real loose_tol,
braid_Real tight_tol,
braid_Real * tol_ptr )
```

Example function to compute a tapered stopping tolerance for implicit time stepping routines, i.e., a tolerance tol_ptr for the spatial solves. This tapering only occurs on the fine grid.

This rule must be followed. The same tolerance must be returned over all processors, for a given XBraid and XBraid level. Different levels may have different tolerances and the same level may vary its tolerance from iteration to iteration, but for the same iteration and level, the tolerance must be constant.

This additional rule must be followed. The fine grid tolerance is never reduced (this is important for convergence)
On the fine level,the spatial stopping tolerance tol_ptr is interpolated from loose_tol to tight_tol based on the relationship between rnorm / rnorm0 and tol.
Remember when rnorm / rnorm0 < tol, XBraid halts. Thus, this function lets us have a loose stopping tolerance while the Braid residual is still relatively large, and then we transition to a tight stopping tolerance as the Braid residual is reduced.

If the user has not defined a residual function, tight_tol is always returned.
The loose_tol is always used on coarse grids, excepting the above mentioned residual computations.
This function will normally be called from the user's step routine.
This function is also meant as a guide for users to develop their own routine.
Parameters

| status | Current XBraid step status |
| :--- | :--- |
| loose_tol | Loosest allowed spatial solve stopping tol on fine grid |
| tight_tol | Tightest allowed spatial solve stopping tol on fine grid |
| tol_ptr | output, holds the computed spatial solve stopping tol |

14.6.4.8 braid_Init() braid_Int braid_Init (

```
MPI_Comm comm_world,
MPI_Comm comm,
braid_Real tstart,
braid_Real tstop,
braid_Int ntime,
braid_App app,
braid_PtFcnStep step,
braid_PtFcnInit init,
braid_PtFcnClone clone,
braid_PtFcnFree free,
braid_PtFcnSum sum,
braid_PtFcnSpatialNorm spatialnorm,
braid_PtFcnAccess access,
braid_PtFcnBufSize bufsize,
braid_PtFcnBufPack bufpack,
braid_PtFcnBufUnpack bufunpack,
    braid_Core * core_ptr )
```

Create a core object with the required initial data.
This core is used by XBraid for internal data structures. The output is core_ptr which points to the newly created braid_Core structure.

## Parameters

| comm_world | Global communicator for space and time |
| :--- | :--- |
| comm | Communicator for temporal dimension |
| tstart | start time |
| tstop | End time |
| ntime | Initial number of temporal grid values |
| app | User-defined_braid_App structure |
| step | User time stepping routine to advance a braid_Vector forward one step |

Parameters

| init | Initialize a braid_Vector on the finest temporal grid |
| :--- | :--- |
| clone | Clone a braid_Vector |
| free | Free a braid_Vector |
| sum | Compute vector sum of two braid_Vectors |
| spatialnorm | Compute norm of a braid_Vector, this is a norm only over space |
| access | Allows access to XBraid and current braid_Vector |
| bufsize | Computes size for MPI buffer for one braid_Vector |
| bufpack | Packs MPI buffer to contain one braid_Vector |
| bufunpack | Unpacks MPI buffer into a braid_Vector |
| core_ptr | Pointer to braid_Core (_braid_Core) struct |

14.6.4.9 braid_PrintStats() braid_Int braid_PrintStats (
braid_Core core )

Print statistics after a XBraid run.
Parameters

```
core braid_Core (_braid_Core) struct
```

14.6.4.10 braid_PrintTimers() braid_Int braid_PrintTimers (

```
braid_Core core )
```

Print timers after a XBraid run, note these timers do not include any adjoint routines or Richardson routines

Parameters
core $\quad$ braid_Core (_braid_Core) struct

### 14.6.4.11 braid_Rand() braid_Int braid_Rand ( <br> void )

Define a machine independent random number generator
14.6.4.12 braid_ResetTimer() braid_Int braid_ResetTimer (
braid_Core core )
Reset timers to 0

## Parameters

core braid_Core (_braid_Core) struct
14.6.4.13 braid_SetAbsTol() braid_Int braid_SetAbsTol (
braid_Core core,
braid_Real atol )

Set absolute stopping tolerance.
Recommended option over relative tolerance
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| atol | absolute stopping tolerance |

14.6.4.14 braid_SetAccessLevel() braid_Int braid_SetAccessLevel (
braid_Core core,
braid_Int access_level )

Set access level for XBraid. This controls how often the user's access routine is called.

- Level 0: Never call the user's access routine
- Level 1: Only call the user's access routine after XBraid is finished
- Level 2: Call the user's access routine every iteration and on every level. This is during _braid_FRestrict, during the down-cycle part of a XBraid iteration.

Default is level 1.
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| access_level | desired access_level |

14.6.4.15 braid_SetBufAllocFree() braid_Int braid_SetBufAllocFree (

```
        braid_Core core,
```

```
braid_PtFcnBufAlloc bufalloc,
braid_PtFcnBufFree buffree )
```

Set user-defined allocation and free routines for the MPI buffer. If these routines are not set, the default is to malloc and free with standard C.

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| bufalloc | (optional) user-allocate an MPI buffer for a certain number of bytes |
| buffree | (optional) free a user-allocated MPI buffer |

14.6.4.16 braid_SetCFactor() braid_Int braid_SetCFactor (

```
braid_Core core,
    braid_Int level,
    braid_Int cfactor )
```

Set the coarsening factor cfactor on grid level (level 0 is the finest grid). The default factor is 2 on all levels. To change the default factor, use level $=-1$.

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| level | level to set coarsening factor on |
| cfactor | desired coarsening factor |

14.6.4.17 braid_SetCRelaxWt() braid_Int braid_SetCRelaxWt (

```
braid_Core core,
    braid_Int level,
braid_Real Cwt )
```

Set the C-relaxation weight on grid level (level 0 is the finest grid). The default is 1.0 on all levels. To change the default factor,
use level $*=-1$.
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| level | level to set Cwt on |
| Cwt | C-relaxation weight to use on level |

### 14.6.4.18 braid_SetDefaultPrintFile() braid_Int braid_SetDefaultPrintFile (

```
braid_Core core )
```

Use default filename, braid_runtime.out for runtime print messages. This function is particularly useful for Fortran codes, where passing filename strings between C and Fortran is troublesome. Level of printing is controlled by braid_SetPrintLevel.

## Parameters

core $\quad$ braid_Core (_braid_Core) struct

### 14.6.4.19 braid_SetDeferDelta() braid_Int braid_SetDeferDelta (

```
braid_Core core,
braid_Int level,
braid_Int iter )
```

Defer the low-rank Delta correction to a coarse level or to a later iteration. To mitigate some of the cost of Delta correction, it may be turned off on the first few fine-grids, or turned off for the first few iterations.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| level | Integer, Delta correction will be deferred to this level (Default 0) |
| iter | Integer, Delta correction will be deferred until this iteration (Default 1) |

### 14.6.4.20 braid_SetDeltaCorrection() braid_Int braid_SetDeltaCorrection (

```
braid_Core core,
    braid_Int rank,
    braid_PtFcnInitBasis basis_init,
    braid_PtFcnInnerProd inner_prod )
```

Turn on low-rank Delta correction. This uses Jacobians of the fine-grid time-stepper as a linear correction to the coarse time-stepper. This can potentially greatly accelerate convergence for nonlinear systems.

The action of the Jacobian will be computed on a (low-rank) time-dependent basis initialized by the user.
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| rank | Integer, sets number of Lyapunov vectors to store |
| basis_init | Function pointer to routine for initializing basis vectors |
| inner_prod | Function pointer to routine for computing inner product between two vectors (needed for Gram-Schmidt <br> orthonormalization) |

14.6.4.21 braid_SetFileIOLevel() braid_Int braid_SetFileIOLevel (
braid_Core core,
braid_Int io_level )

Set output level for XBraid. This controls how much information is saved to files .

- Level 0: no output
- Level 1: save the cycle in braid.out.cycle

Default is level 1.
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| io_level | desired output-to-file level |

### 14.6.4.22 braid_SetFinalFCRelax() braid_Int braid_SetFinalFCRelax (

braid_Core core )

Perform a final FCRelax after XBraid finishes. This can be useful in order to

- Store the last time-point vector in 'ulast', which can then be retrieved by calling _braid_UGetLast()
- Gather gradient information when solving the adjoint equation with XBraid, so that you only need to gather/compute the gradient information once, after XBraid is finished. To do this, the users 'my_step' function for the adjoint time-stepper should compute gradients only if braid's 'done' flag is true
14.6.4.23 braid_SetFMG() braid_Int braid_SetFMG ( braid_Core core )

Once called, XBraid will use FMG (i.e., F-cycles.
Parameters
core $\quad$ braid_Core (_braid_Core) struct
14.6.4.24 braid_SetFullRNormRes() braid_Int braid_SetFullRNormRes (

```
braid_Core core,
braid_PtFcnResidual residual )
```

Set user-defined residual routine for computing full residual norm (all C/F points).

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| residual | function pointer to residual routine |

14.6.4.25 braid_SetIncrMaxLevels() braid_Int braid_SetIncrMaxLevels (
braid_Core core )
Increase the max number of multigrid levels after performing a refinement.

```
14.6.4.26 braid_SetInnerProd() braid_Int braid_SetInnerProd (
    braid_Core core,
    braid_PtFcnInnerProd inner_prod )
```

Set InnerProd routine with user-defined routine.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| inner_prod | function pointer to inner product routine |

### 14.6.4.27 braid_SetLyapunovEstimation() braid_Int braid_SetLyapunovEstimation (

```
braid_Core core,
    braid_Int relax,
    braid_Int cglv,
    braid_Int exponents )
```

Turn on Lyapunov vector estimation for Delta correction. The computed backward Lyapunov vectors will be used to update the time-dependent basis used by the low-rank Delta correction, and may be retrieved via the user's Access function. This can work particularly well for chaotic systems, where the Lyapunov vectors converge to a basis for the unstable manifold of the system, thus the Delta correction can target problematic unstable modes.
if Delta correction is not set, this will have no effect. if relax is set to 1 , the Lyapunov vectors will be propagated during FCRelax, potentially resolving them enough to be useful. if cglv is set to 1 , the Lyapunov vectors will be propagated during the sequential solve on the coarse grid, and they will be much better estimates. if both are set to 0 , no estimation of Lyapunov vectors will be computed, and the basis vectors will only be propagated during FRestrict.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| relax | Integer, if 1, turns on propagation of Lyapunov vectors during FCRelax (default 0) |
| cglv | Integer, if 1, turns on propagation of Lyapunov vectors during coarse-grid solve (default 1) |
| exponents | Integer, if 1, turns on estimation of Lyapunov exponents at C-points on the finest grid (default 0) |

14.6.4.28 braid_SetMaxIter() braid_Int braid_SetMaxIter (

```
braid_Core core,
braid_Int max_iter )
```

Set max number of multigrid iterations.

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| max_iter | maximum iterations to allow |

### 14.6.4.29 braid_SetMaxLevels() braid_Int braid_SetMaxLevels (

> braid_Core core, braid_Int max_levels )

Set max number of multigrid levels.

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| max_levels | maximum levels to allow |

14.6.4.30 braid_SetMaxRefinements() braid_Int braid_SetMaxRefinements (

```
braid_Core core,
braid_Int max_refinements )
```

Set the max number of time grid refinement levels allowed.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| max_refinements | maximum refinement levels allowed |

14.6.4.31 braid_SetMinCoarse() braid_Int braid_SetMinCoarse (
braid_core core, braid_Int min_coarse )

Set minimum allowed coarse grid size. XBraid stops coarsening whenever creating the next coarser grid will result in a grid smaller than min_coarse. The maximum possible coarse grid size will be min_coarse*coarsening_factor.

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| min_coarse | minimum coarse grid size |

14.6.4.32 braid_SetNFMG() braid_Int braid_SetNFMG (

```
braid_Core core,
    braid_Int k )
```

Once called, XBraid will use FMG (i.e., F-cycles.
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| $k$ | number of initial F-cycles to do before switching to V-cycles |

14.6.4.33 braid_SetNFMGVcyc() braid_Int braid_SetNFMGVcyc (

> braid_Core core, braid_Int nfmg_Vcyc )

Set number of V-cycles to use at each FMG level (standard is 1 )
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| nfmg_Vcyc | number of V-cycles to do each FMG level |

14.6.4.34 braid_SetNRelax() braid_Int braid_SetNRelax (

[^4]```
braid_Int level,
braid_Int nrelax )
```

Set the number of relaxation sweeps nrelax on grid level (level 0 is the finest grid). The default is 1 on all levels. To change the default factor, use level $=-1$. One sweep is a CF relaxation sweep.

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| level | level to set nrelax on |
| nrelax | number of relaxations to do on level |

14.6.4.35 braid_SetPeriodic() braid_Int braid_SetPeriodic (

```
braid_Core core,
    braid_Int periodic )
```

Set periodic time grid. The periodicity on each grid level is given by the number of points on each level. Requirements: The number of points on the finest grid level must be evenly divisible by the product of the coarsening factors between each grid level. Currently, the coarsening factors must be the same on all grid levels. Also, braid_SetSeqSoln must not be used.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| periodic | boolean to specify if periodic |

14.6.4.36 braid_SetPrintFile() braid_Int braid_SetPrintFile (
braid_Core core, const char * printfile_name )

Set output file for runtime print messages. Level of printing is controlled by braid_SetPrintLevel. Default is stdout.
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| printfile_name | output file for XBraid runtime output |

14.6.4.37 braid_SetPrintLevel() braid_Int braid_SetPrintLevel (

```
braid_Core core,
braid_Int print_level )
```

Set print level for XBraid. This controls how much information is printed to the XBraid print file (braid_SetPrintFile).

- Level 0: no output
- Level 1: print runtime information like the residual history
- Level 2: level 1 output, plus post-Braid run statistics (default)
- Level 3: level 2 output, plus debug level output.

Default is level 1.
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| print_level | desired print level |

14.6.4.38 braid_SetRefine() braid_Int braid_SetRefine (
braid_Core core,
braid_Int refine )

Turn time refinement on (refine $=1$ ) or off (refine $=0$ ).
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| refine | boolean, refine in time or not |

14.6.4.39 braid_SetRelaxOnlyCG() braid_Int braid_SetRelaxOnlyCG (
braid_Core core,
braid_Int relax_only_cg )
Set whether the coarsest grid is solved only with relaxation. The default is to solve the coarsest grid with sequential time-stepping (relax_only_cg ==0). This default is generally recommended.

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| relax_only_cg | boolean for relaxation-only coarse-grid solve |

```
14.6.4.40 braid_SetRelTol() braid_Int braid_SetRelTol (
    braid_Core core,
    braid_Real rtol )
```

Set relative stopping tolerance, relative to the initial residual. Be careful. If your initial guess is all zero, then the initial residual may only be nonzero over one or two time values, and this will skew the relative tolerance. Absolute tolerances are recommended.

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| rtol | relative stopping tolerance |

14.6.4.41 braid_SetResidual() braid_Int braid_SetResidual (
braid_Core core,
braid_PtFcnResidual residual )
Set user-defined residual routine.
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| residual | function pointer to residual routine |

14.6.4.42 braid_SetRichardsonEstimation() braid_Int braid_SetRichardsonEstimation (

```
braid_Core core,
    braid_Int est_error,
    braid_Int richardson,
    braid_Int local_order )
```

Turn on built-in Richardson-based error estimation and/or extrapolation with XBraid. When enabled, the Richardson extrapolation (RE) option (richardson $==1$ ) is used to improve the accuracy of the solution at the C-points on the finest level. When the built-in error estimate option is turned on (est_error ==1), RE is used to estimate the local truncation error at each point. These estimates can be accessed through StepStatus and AccessStatus functions.

The last parameter is local_order, which represents the LOCAL order of the time integration scheme. e.g. local_order = 2 for Backward Euler.

Also, the Richardson error estimate is only available after roughly 1 Braid iteration. The estimate is given a dummy value of -1.0 , until an actual estimate is available. Thus after an adaptive refinement, and a new hierarchy is formed, another iteration must pass before the error estimates are available again.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| est_error | Boolean, if 1 compute Richardson-based error estimates, if 0, then do not |
| richardson | Boolean, if 1 carry out Richardson-based extrapolation to enhance accuracy on the fine-grid, if 0, <br> then do not |
| local_order | Local order of the time integration scheme, e.g., local _order=2 for backward Euler |

### 14.6.4.43 braid_SetSeqSoln() braid_Int braid_SetSeqSoln (

braid_Core core,
braid_Int seq_soln )
Set the initial guess to XBraid as the sequential time stepping solution. This is primarily for debugging. When used with storage $=-2$, the initial residual should evaluate to exactly 0 . The residual can also be 0 for other storage options if the time stepping is exact, e.g., the implicit solve in Step is done to full precision.

The value seq_soln is a Boolean

- 0 : The user's Init() function initializes the state vector (default)
- 1: Sequential time stepping, with the user's initial condition from Init $(\mathrm{t}=0)$ initializes the state vector

Default is 0 .
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| seq_soln | 1: Init with sequential time stepping soln, 0: Use user's Init() |

14.6.4.44 braid_SetShell() braid_Int braid_SetShell (

```
braid_Core core,
    braid_PtFcnSInit sinit,
    braid_PtFcnSClone sclone,
    braid_PtFcnSFree sfree )
```

Activate the shell vector feature, and set the various functions that are required :

- sinit : create a shell vector
- sclone : clone the shell of a vector
- sfree : free the data of a vector, keeping its shell This feature should be used with storage option $=-1$. It allows the used to keep metadata on all points (including F-points) without storing the all vector everywhere. With these options, the vectors are fully stored on C-points, but only the vector shell is kept on F-points.
14.6.4.45 braid_SetSkip() braid_Int braid_SetSkip (
braid_Core core, braid_Int skip )

Set whether to skip all work on the first down cycle (skip =1). On by default.

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| skip | boolean, whether to skip all work on first down-cycle |

14.6.4.46 braid_SetSpatialCoarsen() braid_Int braid_SetSpatialCoarsen (
braid_Core core,
braid_PtFcnSCoarsen scoarsen )

Set spatial coarsening routine with user-defined routine. Default is no spatial refinment or coarsening.
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| scoarsen | function pointer to spatial coarsening routine |

14.6.4.47 braid_SetSpatialRefine() braid_Int braid_SetSpatialRefine (
braid_Core core,
braid_PtFcnSRefine srefine )

Set spatial refinement routine with user-defined routine. Default is no spatial refinment or coarsening.
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| srefine | function pointer to spatial refinement routine |

14.6.4.48 braid_SetStorage() braid_Int braid_SetStorage (
braid_Core core,
braid_Int storage )

Sets the storage properties of the code. -1 : Default, store only C-points 0 : Full storage of $C$ - and F -Points on all levels $x>0$ : Full storage on all levels $>=x$

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| storage | storage property |

### 14.6.4.49 braid_SetSync() braid_Int braid_SetSync ( <br> braid_Core core, <br> braid_PtFcnSync sync )

Set sync routine with user-defined routine. Sync gives user access to XBraid and the user's app at various points (primarily once per iteration inside FRefine and outside in the main cycle loop). This function is called once perprocessor (instead of for every state vector on the processor, like access). The use case is to allow the user to update their app once-per iteration based on information from XBraid, for example to maintain the space-time grid when doing time-space adaptivity. Default is no sync routine.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| sync | function pointer to sync routine |

### 14.6.4.50 braid_SetTemporalNorm() braid_Int braid_SetTemporalNorm (

$$
\begin{aligned}
& \text { braid_Core core, } \\
& \text { braid_Int tnorm ) }
\end{aligned}
$$

Sets XBraid temporal norm.

This option determines how to obtain a global space-time residual norm. That is, this decides how to combine the spatial norms returned by braid_PtFcnSpatialNorm at each time step to obtain a global norm over space and time. It is this global norm that then controls halting.

There are three options for setting tnorm. See section Halting tolerance for a more detailed discussion (in Introduction.md).

- tnorm=1: One-norm summation of spatial norms
- tnorm=2: Two-norm summation of spatial norms
- tnorm=3: Infinity-norm combination of spatial norms


## The default choice is tnorm=2

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| tnorm | choice of temporal norm |

14.6.4.51 braid_SetTimeGrid() braid_Int braid_SetTimeGrid (
braid_Core core,
braid_PtFcnTimeGrid tgrid )

Set user-defined time points on finest grid
Parameters

| core | braid_Core (_braid_Core) struct |
| :---: | :--- |
| tgrid | function pointer to time grid routine |

14.6.4.52 braid_SetTimerFile() braid_Int braid_SetTimerFile (
braid_Core core,
braid_Int length,
const char * filestem )

Set file name stem for timing infomation output. Timings are output to timerfile_name_\#\#\#\#.txt, where \#\#\#\# is MPI rank. Default is braid_timings_\#\#\#\#.txt

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| length | length of file name string, not including null terminator |
| filestem | file name stem for timing output |

14.6.4.53 braid_SetTimings() braid_Int braid_SetTimings (

> braid_core core,
> braid_Int timing_level )

Control level of Braid internal timings. timing_level $==0$, no timings are taken anywhere in Braid timing_level $==1$, timings are taken only around Braid iterations timing_level $==2$, more intrusive timings are taken of individual user routines and printed to file

### 14.6.4.54 braid_SetTPointsCutoff() braid_Int braid_SetTPointsCutoff (

```
    braid_Core core,
    braid_Int tpoints_cutoff )
```

Set the number of time steps, beyond which refinements stop. If num(tpoints) > tpoints_cutoff, then stop doing refinements.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| tpoints_cutoff | cutoff for stopping refinements |

14.6.4.55 braid_SplitCommworld() braid_Int braid_SplitCommworld (

```
const MPI_Comm * comm_world,
braid_Int px,
MPI_Comm * comm_x,
MPI_Comm * comm_t )
```

Split MPI commworld into comm_x and comm_t, the spatial and temporal communicators. The total number of processors will equal $\mathrm{Px} * \mathrm{Pt}$, there Px is the number of procs in space, and Pt is the number of procs in time.

Parameters

| comm_world | Global communicator to split |
| :--- | :--- |
| $p x$ | Number of processors parallelizing space for a single time step |
| comm_x $x$ | Spatial communicator (written as output) |
| comm_$t$ | Temporal communicator (written as output) |

14.6.4.56 braid_WriteConvHistory() braid_Int braid_WriteConvHistory (
braid_Core core,
const char * filename )

After Drive() finishes, this function can be called to write out the convergence history (residuals for each iteration) to a file

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| filename | Output file name |

### 14.7 Interface routines for XBraid_Adjoint

## Functions

- braid_Int braid_InitAdjoint (braid_PtFcnObjectiveT objectiveT, braid_PtFcnObjectiveTDiff objectiveT_diff, braid_PtFcnStepDiff step_diff, braid_PtFcnResetGradient reset_gradient, braid_Core *core_ptr)
- braid_Int braid_SetTStartObjective (braid_Core core, braid_Real tstart_obj)
- braid_Int braid_SetTStopObjective (braid_Core core, braid_Real tstop_obj)
- braid_Int braid_SetPostprocessObjective (braid_Core core, braid_PtFcnPostprocessObjective post_fcn)
- braid_Int braid_SetPostprocessObjective_diff (braid_Core core, braid_PtFcnPostprocessObjective_diff post_ $\hookleftarrow$ fcn_diff)
- braid_Int braid_SetAbsToIAdjoint (braid_Core core, braid_Real tol_adj)
- braid_Int braid_SetRelTolAdjoint (braid_Core core, braid_Real rtol_adj)
- braid_Int braid_SetObjectiveOnly (braid_Core core, braid_Int boolean)
- braid_Int braid_SetRevertedRanks (braid_Core core, braid_Int boolean)
- braid_Int braid_GetObjective (braid_Core core, braid_Real *objective_ptr)
- braid_Int braid_GetRNormAdjoint (braid_Core core, braid_Real *rnorm_adj)


### 14.7.1 Detailed Description

These are interface routines for computing adjoint sensitivities, i.e., adjoint-based gradients. These routines initialize the XBraid_Adjoint solver, and allow the user to set XBraid_Adjoint solver parameters.

### 14.7.2 Function Documentation

14.7.2.1 braid_GetObjective() braid_Int braid_GetObjective (

```
braid_Core core,
braid_Real * objective_ptr )
```

After braid_Drive has finished, this returns the objective function value.

Parameters

| core | braid_Core struct |
| :--- | :--- |
| objective_ptr | output: value of the objective function |

### 14.7.2.2 braid_GetRNormAdjoint() braid_Int braid_GetRNormAdjoint (

```
braid_Core core,
braid_Real * rnorm_adj )
```

After braid_Drive has finished, this returns the residual norm after the last XBraid iteration.
Parameters

| core | braid_Core struct |
| :--- | :--- |
| rnorm_adj | output: adjoint residual norm of last iteration |

```
14.7.2.3 braid_InitAdjoint() braid_Int braid_InitAdjoint (
braid_PtFcnObjectiveT objectiveT,
    braid_PtFcnObjectiveTDiff objectiveT_diff,
    braid_PtFcnStepDiff step_diff,
    braid_PtFcnResetGradient reset_gradient,
    braid_Core * core_ptr )
```

Initialize the XBraid_Adjoint solver for computing adjoint sensitivities. Once this function is called, braid_Drive will then compute gradient information alongside the primal XBraid computations.

## Parameters

| objectiveT | user-routine: evaluates the time-dependent objective function value at time $t$ |
| :--- | :--- |
| objectiveT_diff | user-routine: differentiated version of the objectiveT function |
| step_diff | user-routine: differentiated version of the step function |
| reset_gradient | user-routine: set the gradient to zero (storage location of gradient up to user) |
| core_ptr | pointer to braid_Core (_braid_Core) struct |

14.7.2.4 braid_SetAbsTolAdjoint() braid_Int braid_SetAbsTolAdjoint (

```
braid_Core core,
braid_Real tol_adj )
```

Set an absolute halting tolerance for the adjoint residuals. XBraid_Adjoint stops iterating when the adjoint residual is below this value.

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| tol_adj | absolute stopping tolerance for adjoint solve |

14.7.2.5 braid_SetObjectiveOnly() braid_Int braid_SetobjectiveOnly (
braid_Core core,
braid_Int boolean )

Set this option with boolean = 1, and then braid_Drive(core) will skip the gradient computation and only compute the forward ODE solution and objective function value.
Reset this option with boolean $=0$ to turn the adjoint solve and gradient computations back on.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| boolean | set to '1' for computing objective function only, '0' for computing objective function AND gradients |

14.7.2.6 braid_SetPostprocessObjective() braid_Int braid_SetPostprocessobjective (
braid_Core core,
braid_PtFcnPostprocessObjective post_fcn )

Pass the postprocessing objective function $F$ to XBraid_Adjoint. For a description of $F$, see the Section Objective function evaluation.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| post_fcn | function pointer to postprocessing routine |

14.7.2.7 braid_SetPostprocessObjective_diff() braid_Int braid_SetPostprocessobjective_diff (
braid_Core core,
braid_PtFcnPostprocessObjective_diff post_fcn_diff )

Pass the differentiated version of the postprocessing objective function $F$ to XBraid_Adjoint. For a description of $F$, see the Section Objective function evaluation .

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| post_fcn_diff | function pointer to differentiated postprocessing routine |

14.7.2.8 braid_SetRelTolAdjoint() braid_Int braid_SetRelTolAdjoint (

```
braid_Core core,
braid_Real rtol_adj )
```

Set a relative stopping tolerance for adjoint residuals. XBraid_Adjoint will stop iterating when the relative residual drops below this value. Be careful when using a relative stopping criterion. The initial residual may already be close to zero, and this will skew the relative tolerance. Absolute tolerances are recommended.

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| rtol_adj | relative stopping tolerance for adjoint solve |

14.7.2.9 braid_SetRevertedRanks() braid_Int braid_SetRevertedRanks (

> braid_Core core, braid_Int boolean )

Set reverted ranks, so that Braid solves "backwards" in time, e.g., when solving and adjoint equation in time.
14.7.2.10 braid_SetTStartObjective() braid_Int braid_SetTStartObjective (

```
braid_Core core,
braid_Real tstart_obj )
```

Set a start time for integrating the objective function over time. Default is tstart of the primal XBraid run.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| tstart_obj | time value for starting the time-integration of the objective function |

14.7.2.11 braid_SetTStopObjective() braid_Int braid_SetTStopObjective (
braid_Core core,

```
    braid_Real tstop_obj )
```

Set the end-time for integrating the objective function over time.
Default is tstop of the primal XBraid run
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| tstop_obj | time value for stopping the time-integration of the objective function |

### 14.8 XBraid status structures

Define the different status types.

### 14.9 XBraid status routines

## Functions

- braid_Int braid_StatusGetT (braid_Status status, braid_Real *t_ptr)
- braid_Int braid_StatusGetTIndex (braid_Status status, braid_Int *idx_ptr)
- braid_Int braid_StatusGetlter (braid_Status status, braid_Int *iter_ptr)
- braid_Int braid_StatusGetLevel (braid_Status status, braid_Int *level_ptr)
- braid_Int braid_StatusGetNLevels (braid_Status status, braid_Int *nlevels_ptr)
- braid_Int braid_StatusGetNRefine (braid_Status status, braid_Int *nrefine_ptr)
- braid_Int braid_StatusGetNTPoints (braid_Status status, braid_Int *ntpoints_ptr)
- braid_Int braid_StatusGetResidual (braid_Status status, braid_Real *rnorm_ptr)
- braid_Int braid_StatusGetDone (braid_Status status, braid_Int *done_ptr)
- braid_Int braid_StatusGetTIUL (braid_Status status, braid_Int *iloc_upper, braid_Int *iloc_lower, braid_Int level)
- braid_Int braid_StatusGetTimeValues (braid_Status status, braid_Real **tvalues_ptr, braid_Int i_upper, braid_Int i_lower, braid_Int level)
- braid_Int braid_StatusGetTILD (braid_Status status, braid_Real *t_ptr, braid_Int *iter_ptr, braid_Int *level_ptr, braid_Int *done_ptr)
- braid_Int braid_StatusGetWrapperTest (braid_Status status, braid_Int *wtest_ptr)
- braid_Int braid_StatusGetCallingFunction (braid_Status status, braid_Int *cfunction_ptr)
- braid_Int braid_StatusGetDeltaRank (braid_Status status, braid_Int *rank_ptr)
- braid_Int braid_StatusGetBasisVec (braid_Status status, braid_Vector *v_ptr, braid_Int index)
- braid_Int braid_StatusGetLocalLyapExponents (braid_Status status, braid_Real *exp_ptr, braid_Int *num_↔ returned)
- braid_Int braid_StatusGetCTprior (braid_Status status, braid_Real *ctprior_ptr)
- braid_Int braid_StatusGetCTstop (braid_Status status, braid_Real *ctstop_ptr)
- braid_Int braid_StatusGetFTprior (braid_Status status, braid_Real *ftprior_ptr)
- braid_Int braid_StatusGetFTstop (braid_Status status, braid_Real *ftstop_ptr)
- braid_Int braid_StatusGetTpriorTstop (braid_Status status, braid_Real *t_ptr, braid_Real *ftprior_ptr, braid_Real *ftstop_ptr, braid_Real *ctprior_ptr, braid_Real *ctstop_ptr)
- braid_Int braid_StatusGetTstop (braid_Status status, braid_Real *tstop_ptr)
- braid_Int braid_StatusGetTstartTstop (braid_Status status, braid_Real *tstart_ptr, braid_Real *tstop_ptr)
- braid_Int braid_StatusGetTol (braid_Status status, braid_Real *tol_ptr)
- braid_Int braid_StatusGetRNorms (braid_Status status, braid_Int *nrequest_ptr, braid_Real *rnorms_ptr)
- braid_Int braid_StatusGetProc (braid_Status status, braid_Int *proc_ptr, braid_Int level, braid_Int index)
- braid_Int braid_StatusGetOldFineTolx (braid_Status status, braid_Real *old_fine_tolx_ptr)
- braid_Int braid_StatusSetOldFineTolx (braid_Status status, braid_Real old_fine_tolx)
- braid_Int braid_StatusSetTightFineTolx (braid_Status status, braid_Real tight_fine_tolx)
- braid_Int braid_StatusSetRFactor (braid_Status status, braid_Real rfactor)
- braid_Int braid_StatusSetRefinementDtValues (braid_Status status, braid_Real rfactor, braid_Real *dtarray)
- braid_Int braid_StatusSetRSpace (braid_Status status, braid_Real r_space)
- braid_Int braid_StatusGetMessageType (braid_Status status, braid_Int *messagetype_ptr)
- braid_Int braid_StatusSetSize (braid_Status status, braid_Real size)
- braid_Int braid_StatusSetBasisSize (braid_Status status, braid_Real size)
- braid_Int braid_StatusGetSingleErrorEstStep (braid_Status status, braid_Real *estimate)
- braid_Int braid_StatusGetSingleErrorEstAccess (braid_Status status, braid_Real *estimate)
- braid_Int braid_StatusGetNumErrorEst (braid_Status status, braid_Int *npoints)
- braid_Int braid_StatusGetAllErrorEst (braid_Status status, braid_Real *error_est)
- braid_Int braid_StatusGetTComm (braid_Status status, MPI_Comm *comm_ptr)


### 14.9.1 Detailed Description

XBraid status structures and associated Get/Set routines are what tell the user the status of the simulation when their routines (step, coarsen/refine, access) are called.

### 14.9.2 Function Documentation

### 14.9.2.1 braid_StatusGetAlIErrorEst() braid_Int braid_StatusGetAllerrorEst (

```
braid_Status status,
braid_Real * error_est )
```

Get All the Richardson based error estimates, e.g. from inside Sync. Use this function in conjuction with GetNumError $\hookleftarrow$ Est(). Workflow: use GetNumErrorEst() to get the size of the needed user-array that will hold the error estimates, then pre-allocate array, then call this function to write error estimates to the user-array, then post-process array in user-code. This post-processing will often occur in the Sync function. See examples/ex-06.c.

The error_est array must be user-allocated.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| error_est | output, user-allocated error estimate array, written by Braid, equals -1 if not available yet (e.g., before <br> iteration 1, or after refinement) |

14.9.2.2 braid_StatusGetBasisVec() braid_Int braid_StatusGetBasisVec (

```
braid_Status status,
    braid_Vector * v_ptr,
    braid_Int index )
```

Return a reference to the basis vector at the current time value and given spatial index
Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| $\vee \_p t r$ | output, reference to basis vector |
| index | input, spatial index (column) of desired basis vector |

14.9.2.3 braid_StatusGetCallingFunction() braid_Int braid_StatusGetCallingFunction (
braid_Status status,
braid_Int * cfunction_ptr )

Return flag indicating from which function the vector is accessed
Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| cfunction_ptr | output, function number ( $0=$ FInterp, $1=$ FRestrict, 2=FRefine, 3=FAccess, $4=$ FRefine after <br> refinement, $5=$ Drive Top of Cycle) |

14.9.2.4 braid_StatusGetCTprior() braid_Int braid_StatusGetCTprior (
braid_Status status,
braid_Real * ctprior_ptr )
Return the coarse grid time value to the left of the current time value from the Status structure.

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| ctprior_ptr | output, time value to the left of current time value on coarse grid |

14.9.2.5 braid_StatusGetCTstop() braid_Int braid_StatusGetCTstop (

```
braid_Status status,
    braid_Real * ctstop_ptr )
```

Return the coarse grid time value to the right of the current time value from the Status structure.
Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| ctstop_ptr | output, time value to the right of current time value on coarse grid |

14.9.2.6 braid_StatusGetDeltaRank() braid_Int braid_StatusGetDeltaRank (

```
braid_Status status,
    braid_Int * rank_ptr )
```

Return the current rank of Delta correction being used

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| rank_ptr | output, rank of Delta correction, number of tracked basis vectors |

14.9.2.7 braid_StatusGetDone() braid_Int braid_StatusGetDone (
braid_Status status,
braid_Int * done_ptr )
Return whether XBraid is done for the current simulation.
done_ptr = 1 indicates that XBraid has finished iterating, (either maxiter has been reached, or the tolerance has been met).

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| done_ptr | output, $=1$ if XBraid has finished, else $=0$ |

14.9.2.8 braid_StatusGetFTprior() braid_Int braid_StatusGetFTprior (
braid_Status status,
braid_Real * ftprior_ptr )
Return the fine grid time value to the left of the current time value from the Status structure.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| ftprior_ptr | output, time value to the left of current time value on fine grid |

14.9.2.9 braid_StatusGetFTstop() braid_Int braid_StatusGetFTstop (
braid_Status status,
braid_Real * ftstop_ptr)
Return the fine grid time value to the right of the current time value from the Status structure.
Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| ftstop_ptr | output, time value to the right of current time value on fine grid |

14.9.2.10 braid_StatusGetlter() braid_Int braid_StatusGetIter (

> braid_Status status,
braid_Int * iter_ptr )
Return the current iteration from the Status structure.
Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| iter_ptr | output, current XBraid iteration number |

14.9.2.11 braid_StatusGetLevel() braid_Int braid_StatusGetLevel (
braid_Status status,
braid_Int * level_ptr )

Return the current XBraid level from the Status structure.

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| level_ptr | output, current level in XBraid |

14.9.2.12 braid_StatusGetLocalLyapExponents() braid_Int braid_StatusGetLocalLyapExponents (
braid_Status status,
braid_Real * exp_ptr,
braid_Int * num_returned )

Return a reference to an array of local exponents, with each exponent $j$ corresponding to the total growth over the previous C-interval in the direction of the $* j *$ th Lyapunov exponent (These are only available after the final FCRelax)

Parameters

| status | structure containing the current simulation info |
| :--- | :--- |
| exp_ptr | output, reference to array containing (num_returned) exponents |
| num_returned | output, number of exponents contained in exp_ptr |

14.9.2.13 braid_StatusGetMessageType() braid_Int braid_StatusGetMessageType (

```
braid_Status status,
braid_Int * messagetype_ptr )
```

Return the current message type from the Status structure.

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| messagetype_ptr | output, type of message, 0: for Step(), 1: for load balancing |

14.9.2.14 braid_StatusGetNLevels() braid_Int braid_StatusGetNLevels (

```
braid_Status status,
braid_Int * nlevels_ptr )
```

Return the total number of XBraid levels from the Status structure.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| nlevels_ptr | output, number of levels in XBraid |

14.9.2.15 braid_StatusGetNRefine() braid_Int braid_StatusGetNRefine (

```
braid_Status status,
    braid_Int * nrefine_ptr )
```

Return the number of refinements done.
Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| nrefine_ptr | output, number of refinements done |

### 14.9.2.16 braid_StatusGetNTPoints() braid_Int braid_StatusGetNTPoints (

```
braid_Status status,
    braid_Int * ntpoints_ptr )
```

Return the global number of time points on the fine grid.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| ntpoints_ptr | output, number of time points on the fine grid |

14.9.2.17 braid_StatusGetNumErrorEst() braid_Int braid_StatusGetNumErrorEst (

```
braid_Status status,
    braid_Int * npoints )
```

Get the number of local Richardson-based error estimates stored on this processor. Use this function in conjuction with GetAllErrorEst(). Workflow: use this function to get the size of the needed user-array that will hold the error estimates, then pre-allocate array, then call GetAllErrorEst() to write error estimates to the user-array, then post-process array in user-code. This post-processing will often occur in the Sync function. See examples/ex-06.c.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| npoints | output, number of locally stored Richardson error estimates |

14.9.2.18 braid_StatusGetOldFineTolx() braid_Int braid_StatusGetOldFineTolx (

```
braid_Status status,
braid_Real * old_fine_tolx_ptr )
```

Return the previous old_fine_tolx set through braid_StatusSetOldFineTolx This is used especially by *braid_Get $\hookleftarrow$ SpatialAccuracy

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| old_fine_tolx_ptr | output, previous old_fine_tolx, set through braid_StepStatusSetOldFineTolx |

14.9.2.19 braid_StatusGetProc() braid_Int braid_StatusGetProc (

```
braid_Status status,
    braid_Int * proc_ptr,
    braid_Int level,
    braid_Int index )
```

Returns the processor number in proc_ptr on which the time step index lives for the given level. Returns -1 if index is out of range. This is used especially by the _braid_SyncStatus functionality

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| proc_ptr | output, the processor number corresponding to the level and time point index inputs |
| level | input, level for the desired processor |
| index | input, the global time point index for the desired processor |

14.9.2.20 braid_StatusGetResidual() braid_Int braid_StatusGetResidual (
braid_Status status,
braid_Real * rnorm_ptr )
Return the current residual norm from the Status structure.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| rnorm_ptr | output, current residual norm |

### 14.9.2.21 braid_StatusGetRNorms() braid_Int braid_StatusGetRNorms ( <br> ```braid_Status status, \\ braid_Int * nrequest_ptr, \\ braid_Real * rnorms_ptr )```

Return the current XBraid residual history. If nrequest_ptr is negative, return the last nrequest_ptr residual norms. If positive, return the first nrequest_ptr residual norms. Upon exit, nrequest_ptr holds the number of residuals actually returned.

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| nrequest_ptr | input/output, input: number of requested residual norms, output: number actually copied |
| rnorms_ptr | output, XBraid residual norm history, of length nrequest_ptr |

14.9.2.22 braid_StatusGetSingleErrorEstAccess() braid_Int braid_StatusGetSingleErrorEstAccess (

> braid_Status status,
> braid_Real * estimate )

Get the Richardson based error estimate at the single time point currently accessible from Access.

Note that Access needs specific logic distinct from Step, hence please use braid_StepStatusGetSingleErrorEstStep for the user Step() function.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| estimate | output, error estimate, equals -1 if not available yet (e.g., before iteration 1, or after refinement) |

### 14.9.2.23 braid_StatusGetSingleErrorEstStep() braid_Int braid_StatusGetSingleErrorEstStep ( <br> braid_Status status, <br> braid_Real * estimate )

Get the Richardson based error estimate at the single time point currently being "Stepped", i.e., return the current error estimate for the time point at "tstart".

Note that Step needs specific logic distinct from Access, hence please use braid_AccessStatusGetSingleErrorEstAccess for the user Access() function.

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| estimate | output, error estimate, equals -1 if not available yet (e.g., before iteration 1, or after refinement) |

14.9.2.24 braid_StatusGetT() braid_Int braid_StatusGet T (

> braid_Status status,
> braid_Real * t_ptr )

Return the current time from the Status structure.
Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| $t \_p t r$ | output, current time |

### 14.9.2.25 braid_StatusGetTComm() braid_Int braid_StatusGetTComm (

> braid_Status status,
> MPI_Comm * comm_ptr $)$

Gets accces to the temporal communicator. Allows this processor to access other temporal processors. This is used especially by Sync.

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| comm_ptr | output, temporal communicator |

14.9.2.26 braid_StatusGetTILD() braid_Int braid_StatusGet TILD (
braid_Status status,
braid_Real * t_ptr,
braid_Int * iter_ptr,
braid_Int * level_ptr,
braid_Int * done_ptr )
Return XBraid status for the current simulation. Four values are returned.

TILD : time, iteration, level, done
These values are also available through individual Get routines. These individual routines are the location of detailed documentation on each parameter, e.g., see braid_StatusGetDone for more information on the done value.

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| $t$ _ptr | output, current time |
| iter_ptr | output, current XBraid iteration number |
| level_ptr | output, current level in XBraid |
| done_ptr | output, $=1$ if XBraid has finished, else $=0$ |

14.9.2.27 braid_StatusGetTimeValues() braid_Int braid_StatusGetTimeValues (

```
braid_Status status,
    braid_Real ** tvalues_ptr,
    braid_Int i_upper,
    braid_Int i_lower,
    braid_Int level )
```

Returns an array of time values corresponding to the given inputs. The inputs are the level you want the time values from, the upper time point index you want the value of, and the lower time point index you want the time value of. The output is then filled with all time values from the upper index to the lower index, inclusive.

The caller is responsible for allocating and managing the memory for the array. Time values are filled in so that tvalues $\hookleftarrow$ _ptr[0] corresponds to the lower time index.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| tvalues_ptr | output, time point values for the requested range of indices |
| i_upper | input, upper index of the desired time value range (inclusive) |
| i_lower | input, lower index of the desired time value range (inclusive) |
| level | input, level for the desired time values |

14.9.2.28 braid_StatusGetTIndex() braid_Int braid_StatusGetTIndex (
braid_Status status,
braid_Int * idx_ptr )
Return the index value corresponding to the current time value from the Status structure.

For Step(), this corresponds to the time-index of "tstart", as this is the time-index of the input vector. That is, NOT the time-index of "tstop". For Access, this corresponds just simply to the time-index of the input vector.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| idx_ptr | output, global index value corresponding to current time value |

14.9.2.29 braid_StatusGetTIUL() braid_Int braid_StatusGettIUL (
braid_Status status,
braid_Int * iloc_upper,
braid_Int * iloc_lower, braid_Int level )

Returns upper and lower time point indices on this processor. Two values are returned. Requires the user to specify which level they want the time point indices from.

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| iloc_upper | output, the upper time point index on this processor |
| iloc_lower | output, the lower time point index on this processor |
| level | input, level for the desired indices |

### 14.9.2.30 braid_StatusGetTol() braid_Int braid_StatusGetTol (

braid_Status status,
braid_Real * tol_ptr )

Return the current XBraid stopping tolerance

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| tol_ptr | output, current XBraid stopping tolerance |

14.9.2.31 braid_StatusGetTpriorTstop() braid_Int braid_StatusGetTpriorTstop (

```
braid_Status status,
braid_Real * t_ptr,
braid_Real * ftprior_ptr,
braid_Real * ftstop_ptr,
braid_Real * ctprior_ptr,
braid_Real * ctstop_ptr
```

Return XBraid status for the current simulation. Five values are returned, tstart, f_tprior, f_tstop, c_tprior, c_tstop.
These values are also available through individual Get routines. These individual routines are the location of detailed documentation on each parameter, e.g., see braid_StatusGetCTprior for more information on the c_tprior value.

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| t_ptr | output, current time |
| ftprior_ptr | output, time value to the left of current time value on fine grid |
| ftstop_ptr | output, time value to the right of current time value on fine grid |
| ctprior_ptr | output, time value to the left of current time value on coarse grid |
| ctstop_ptr | output, time value to the right of current time value on coarse grid |

### 14.9.2.32 braid_StatusGetTstartTstop() braid_Int braid_StatusGetTstartTstop (

```
braid_Status status,
braid_Real * tstart_ptr,
braid_Real * tstop_ptr )
```

Return XBraid status for the current simulation. Two values are returned, tstart and tstop.
These values are also available through individual Get routines. These individual routines are the location of detailed documentation on each parameter, e.g., see braid_StatusGetTstart for more information on the tstart value.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| tstart_ptr | output, current time |
| tstop_ptr | output, next time value to evolve towards |

### 14.9.2.33 braid_StatusGetTstop() braid_Int braid_StatusGetTstop (

```
    braid_Status status,
    braid_Real * tstop_ptr )
```

Return the time value to the right of the current time value from the Status structure.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| tstop_ptr | output, next time value to evolve towards |

14.9.2.34 braid_StatusGetWrapperTest() braid_Int braid_StatusGetWrapperTest (

```
braid_Status status,
braid_Int * wtest_ptr )
```

Return whether this is a wrapper test or an XBraid run

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| wtest_ptr | output, =1 if this is a wrapper test, =0 if XBraid run |

14.9.2.35 braid_StatusSetBasisSize() braid_Int braid_StatusSetBasisSize (

```
braid_Status status,
braid_Real size )
```

Set the size of the buffer for basis vectors. If set by user, the send buffer will allocate "size" bytes of space for each basis vector. If not, BufSize is used for the size of each basis vector

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| size | input, size of the send buffer |

### 14.9.2.36 braid_StatusSetOldFineTolx() braid_Int braid_StatusSetoldFineTolx (

$$
\begin{aligned}
& \text { braid_Status status, } \\
& \text { braid_Real old_fine_tolx ) }
\end{aligned}
$$

Set old_fine_tolx, available for retrieval through braid_StatusGetOldFineTolx This is used especially by *braid_Get $\hookleftarrow$ SpatialAccuracy

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| old_fine_tolx | input, the last used fine_tolx |

14.9.2.37 braid_StatusSetRefinementDtValues() braid_Int braid_StatusSetRefinementDtValues (

```
braid_Status status,
braid_Real rfactor,
braid_Real * dtarray )
```

Set time step sizes for refining the time interval non-uniformly.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| rfactor | input, number of subintervals |
| dtarray | input, array of dt values for non-uniform refinement |

### 14.9.2.38 braid_StatusSetRFactor() braid_Int braid_StatusSetRFactor (

> braid_Status status,
> braid_Real rfactor )

Set the rfactor, a desired refinement factor for this interval. rfactor=1 indicates no refinement, otherwise, this inteval is subdivided rfactor times (uniform refinement).

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| rfactor | input, user-determined desired rfactor |

### 14.9.2.39 braid_StatusSetRSpace() braid_Int braid_StatusSetRSpace

```
braid_Status status,
```

braid_Real r_space )

Set the r_space flag. When set $=1$, spatial coarsening will be called, for all local time points, following the completion of the current iteration, provided rfactors are not set at any global time point. This allows for spatial refinment without temporal refinment

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| r_space | input, if 1, call spatial refinement on finest grid after this iter |

14.9.2.40 braid_StatusSetSize() braid_Int braid_StatusSetSize (

```
braid_Status status,
braid_Real size )
```

Set the size of the buffer. If set by user, the send buffer will be "size" bytes in length. If not, BufSize is used.

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| size | input, size of the send buffer |

14.9.2.41 braid_StatusSetTightFineTolx() braid_Int braid_StatusSetTightFineTolx (
braid_Status status,
braid_Real tight_fine_tolx )

Set tight_fine_tolx, boolean variable indicating whether the tightest tolerance has been used for spatial solves (implicit schemes). This value must be 1 in order for XBraid to halt (unless maxiter is reached)

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| tight_fine_tolx | input, boolean indicating whether the tight tolx has been used |

### 14.10 Inherited XBraid status routines

## Functions

- braid_Int braid_AccessStatusGetT (braid_AccessStatus s, braid_Real $* \mathrm{~V} 1$ )
- braid_Int braid_AccessStatusGetTIndex (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetlter (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetLevel (braid_AccessStatus s, braid_Int *V1)
- braid_Int braid_AccessStatusGetNLevels (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetNRefine (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetNTPoints (braid_AccessStatus s, braid_Int *V1)
- braid_Int braid_AccessStatusGetResidual (braid_AccessStatus s, braid_Real *v1)
- braid_Int braid_AccessStatusGetDone (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetTILD (braid_AccessStatus s, braid_Real *v1, braid_Int *v2, braid_Int *v3, braid_Int *v4)
- braid_Int braid_AccessStatusGetWrapperTest (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetCallingFunction (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetSingleErrorEstAccess (braid_AccessStatus s, braid_Real *v1)
- braid_Int braid_AccessStatusGetDeltaRank (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetLocalLyapExponents (braid_AccessStatus s, braid_Real *V1, braid_Int *V2)
- braid_Int braid_AccessStatusGetBasisVec (braid_AccessStatus s, braid_Vector *v1, braid_Int v2)
- braid_Int braid_SyncStatusGetTIUL (braid_SyncStatus s, braid_Int $* \mathrm{~V} 1$, braid_Int $* \mathrm{~V} 2$, braid_Int v3)
- braid_Int braid_SyncStatusGetTimeValues (braid_SyncStatus s, braid_Real $* * v 1$, braid_Int v2, braid_Int v3, braid_Int v4)
- braid_Int braid_SyncStatusGetProc (braid_SyncStatus s, braid_Int *v1, braid_Int v2, braid_Int v3)
- braid_Int braid_SyncStatusGetlter (braid_SyncStatus s, braid_Int *V1)
- braid_Int braid_SyncStatusGetLevel (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetNLevels (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetNRefine (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetNTPoints (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetDone (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetCallingFunction (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetNumErrorEst (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetAllErrorEst (braid_SyncStatus s, braid_Real $*$ v1)
- braid_Int braid_SyncStatusGetTComm (braid_SyncStatus s, MPI_Comm *v1)
- braid_Int braid_CoarsenRefStatusGetT (braid_CoarsenRefStatus s, braid_Real $* \mathrm{v} 1$ )
- braid_Int braid_CoarsenRefStatusGetTIndex (braid_CoarsenRefStatus s, braid_Int *V1)
- braid_Int braid_CoarsenRefStatusGetlter (braid_CoarsenRefStatus s, braid_Int *v1)
- braid_Int braid_CoarsenRefStatusGetLevel (braid_CoarsenRefStatus s, braid_Int *v1)
- braid_Int braid_CoarsenRefStatusGetNLevels (braid_CoarsenRefStatus s, braid_Int *v1)
- braid_Int braid_CoarsenRefStatusGetNRefine (braid_CoarsenRefStatus s, braid_Int *v1)
- braid_Int braid_CoarsenRefStatusGetNTPoints (braid_CoarsenRefStatus s, braid_Int *V1)
- braid_Int braid_CoarsenRefStatusGetCTprior (braid_CoarsenRefStatus s, braid_Real *v1)
- braid_Int braid_CoarsenRefStatusGetCTstop (braid_CoarsenRefStatus s, braid_Real *V1)
- braid_Int braid_CoarsenRefStatusGetFTprior (braid_CoarsenRefStatus s, braid_Real $* \mathrm{~V} 1$ )
- braid_Int braid_CoarsenRefStatusGetFTstop (braid_CoarsenRefStatus s, braid_Real *v1)
- braid_Int braid_CoarsenRefStatusGetTpriorTstop (braid_CoarsenRefStatus s, braid_Real *v1, braid_Real *v2, braid_Real $* \mathrm{v} 3$, braid_Real $* \mathrm{v} 4$, braid_Real $* \mathrm{v} 5$ )
- braid_Int braid_StepStatusGetTIUL (braid_StepStatus s, braid_Int *v1, braid_Int $*$ v2, braid_Int v3)
- braid_Int braid_StepStatusGetT (braid_StepStatus s, braid_Real *v1)
- braid_Int braid_StepStatusGetTIndex (braid_StepStatus s, braid_Int *V1)
- braid_Int braid_StepStatusGetlter (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetLevel (braid_StepStatus s, braid_Int *V1)
- braid_Int braid_StepStatusGetNLevels (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetNRefine (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetNTPoints (braid_StepStatus s, braid_Int $* \mathrm{~V} 1$ )
- braid_Int braid_StepStatusGetTstop (braid_StepStatus s, braid_Real *v1)
- braid_Int braid_StepStatusGetTstartTstop (braid_StepStatus s, braid_Real *v1, braid_Real *v2)
- braid_Int braid_StepStatusGetTol (braid_StepStatus s, braid_Real *v1)
- braid_Int braid_StepStatusGetRNorms (braid_StepStatus s, braid_Int *v1, braid_Real *v2)
- braid_Int braid_StepStatusGetOldFineTolx (braid_StepStatus s, braid_Real $*$ v1)
- braid_Int braid_StepStatusSetOldFineTolx (braid_StepStatus s, braid_Real v1)
- braid_Int braid_StepStatusSetTightFineTolx (braid_StepStatus s, braid_Real v1)
- braid_Int braid_StepStatusSetRFactor (braid_StepStatus s, braid_Real v1)
- braid_Int braid_StepStatusSetRSpace (braid_StepStatus s, braid_Real v1)
- braid_Int braid_StepStatusGetDone (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetSingleErrorEstStep (braid_StepStatus s, braid_Real *v1)
- braid_Int braid_StepStatusGetCallingFunction (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetDeltaRank (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetBasisVec (braid_StepStatus s, braid_Vector *v1, braid_Int v2)
- braid_Int braid_BufferStatusGetMessageType (braid_BufferStatus s, braid_Int *v1)
- braid_Int braid_BufferStatusGetTIndex (braid_BufferStatus s, braid_Int *v1)
- braid_Int braid_BufferStatusGetLevel (braid_BufferStatus s, braid_Int *v1)
- braid_Int braid_BufferStatusSetSize (braid_BufferStatus s, braid_Real v1)
- braid_Int braid_BufferStatusSetBasisSize (braid_BufferStatus s, braid_Real v1)
- braid_Int braid_ObjectiveStatusGetT (braid_ObjectiveStatus s, braid_Real *v1)
- braid_Int braid_ObjectiveStatusGetTIndex (braid_ObjectiveStatus s, braid_Int *v1)
- braid_Int braid_ObjectiveStatusGetlter (braid_ObjectiveStatus s, braid_Int *v1)
- braid_Int braid_ObjectiveStatusGetLevel (braid_ObjectiveStatus s, braid_Int *v1)
- braid_Int braid_ObjectiveStatusGetNLevels (braid_ObjectiveStatus s, braid_Int *v1)
- braid_Int braid_ObjectiveStatusGetNRefine (braid_ObjectiveStatus s, braid_Int *v1)
- braid_Int braid_ObjectiveStatusGetNTPoints (braid_ObjectiveStatus s, braid_Int *v1)
- braid_Int braid_ObjectiveStatusGetTol (braid_ObjectiveStatus s, braid_Real *v1)


### 14.10.1 Detailed Description

These are the 'inherited' Status Get/Set functions. See the XBraid status routines section for the description of each function. For example, for braid_StepStatusGetT(...), you would look up braid_StatusGetT(...)

### 14.10.2 Function Documentation

14.10.2.1 braid_AccessStatusGetBasisVec() braid_Int braid_AccessStatusGetBasisVec (

```
braid_AccessStatus s,
braid_Vector * v1,
braid_Int v2 )
```

14.10.2.2 braid_AccessStatusGetCallingFunction() braid_Int braid_AccessStatusGetCallingFunction (
braid_AccessStatus s, braid_Int * v1 )
14.10.2.3 braid_AccessStatusGetDeltaRank() braid_Int braid_AccessStatusGetDeltaRank (
braid_AccessStatus s, braid_Int * v1 )
14.10.2.4 braid_AccessStatusGetDone() braid_Int braid_AccessStatusGetDone ( braid_AccessStatus s, braid_Int * v1 )
14.10.2.5 braid_AccessStatusGetler() braid_Int braid_AccessStatusGetIter ( braid_AccessStatus s, braid_Int * v1 )
14.10.2.6 braid_AccessStatusGetLevel() braid_Int braid_AccessStatusGetLevel (

```
    braid_AccessStatus s,
    braid_Int * v1 )
```

14.10.2.7 braid_AccessStatusGetLocalLyapExponents() braid_Int braid_AccessStatusGetLocalLyap $\hookleftarrow$ Exponents (

```
braid_AccessStatus s,
braid_Real * v1,
braid_Int * v2 )
```

14.10.2.8 braid_AccessStatusGetNLevels() braid_Int braid_AccessStatusGetNLevels ( braid_AccessStatus s, braid_Int * vI )
14.10.2.9 braid_AccessStatusGetNRefine() braid_Int braid_AccessStatusGetNRefine ( braid_AccessStatus s, braid_Int * v1 )
14.10.2.10 braid_AccessStatusGetNTPoints() braid_Int braid_AccessStatusGetNTPoints ( braid_AccessStatus s, braid_Int * v1 )
14.10.2.11 braid_AccessStatusGetResidual() braid_Int braid_AccessStatusGetResidual (
braid_AccessStatus s, braid_Real * v1 )
14.10.2.12 braid_AccessStatusGetSingleErrorEstAccess() braid_Int braid_AccessStatusGetSingleError $\hookleftarrow$ EstAccess (

```
    braid_AccessStatus s,
```

    braid_Real * v1 )
    14.10.2.13 braid_AccessStatusGetT() braid_Int braid_AccessStatusGetT (

```
    braid_AccessStatus s,
    braid_Real * v1 )
```

14.10.2.14 braid_AccessStatusGetTILD() braid_Int braid_AccessStatusGetTILD (
braid_AccessStatus s,
braid_Real * v1,
braid_Int * v2,
braid_Int * v3,
braid_Int * v4 )
14.10.2.15 braid_AccessStatusGetTIndex() braid_Int braid_AccessStatusGetTIndex ( braid_AccessStatus s, braid_Int * v1 )
14.10.2.16 braid_AccessStatusGetWrapperTest() braid_Int braid_AccessStatusGetWrapperTest ( braid_AccessStatus s, braid_Int * v1 )
14.10.2.17 braid_BufferStatusGetLevel() braid_Int braid_BufferStatusGetLevel ( braid_BufferStatus s, braid_Int * v1 )
14.10.2.18 braid_BufferStatusGetMessageType() braid_Int braid_BufferStatusGetMessageType ( braid_BufferStatus s, braid_Int * v1 )
14.10.2.19 braid_BufferStatusGetTIndex() braid_Int braid_BufferStatusGetTIndex ( braid_BufferStatus s, braid_Int * v1 )
14.10.2.20 braid_BufferStatusSetBasisSize() braid_Int braid_BufferStatusSetBasisSize ( braid_BufferStatus s, braid_Real v1 )

```
14.10.2.21 braid_BufferStatusSetSize() braid_Int braid_BufferStatusSetSize (
    braid_BufferStatus s,
    braid_Real v1 )
14.10.2.22 braid_CoarsenRefStatusGetCTprior() braid_Int braid_CoarsenRefStatusGetCTprior (
        braid_CoarsenRefStatus s,
    braid_Real * v1 )
14.10.2.23 braid_CoarsenRefStatusGetCTstop() braid_Int braid_CoarsenRefStatusGetCTstop ( braid_CoarsenRefStatus s, braid_Real * v1 )
```

14.10.2.24 braid_CoarsenRefStatusGetFTprior() braid_Int braid_CoarsenRefStatusGetFTprior ( braid_CoarsenRefStatus $s$, braid_Real * v1 )
14.10.2.25 braid_CoarsenRefStatusGetFTstop() braid_Int braid_CoarsenRefStatusGetFTstop ( braid_CoarsenRefStatus $s$, braid_Real * v1 )
14.10.2.26 braid_CoarsenRefStatusGetlter() braid_Int braid_CoarsenRefStatusGetIter ( braid_CoarsenRefStatus s, braid_Int * v1 )
14.10.2.27 braid_CoarsenRefStatusGetLevel() braid_Int braid_CoarsenRefStatusGetLevel ( braid_CoarsenRefStatus $s$, braid_Int * v1 )
14.10.2.28 braid_CoarsenRefStatusGetNLevels() braid_Int braid_CoarsenRefStatusGetNLevels ( braid_CoarsenRefStatus s, braid_Int * v1 )
14.10.2.29 braid_CoarsenRefStatusGetNRefine() braid_Int braid_CoarsenRefStatusGetNRefine ( braid_CoarsenRefStatus s, braid_Int * v1 )
14.10.2.30 braid_CoarsenRefStatusGetNTPoints() braid_Int braid_CoarsenRefStatusGetNTPoints ( braid_CoarsenRefStatus $s$, braid_Int * v1 )
14.10.2.31 braid_CoarsenRefStatusGetT() braid_Int braid_CoarsenRefStatusGet $T$ (
braid_CoarsenRefStatus s, braid_Real * v1 )
14.10.2.32 braid_CoarsenRefStatusGetTIndex() braid_Int braid_CoarsenRefStatusGetTIndex ( braid_CoarsenRefStatus s, braid_Int * v1 )
14.10.2.33 braid_CoarsenRefStatusGetTpriorTstop() braid_Int braid_CoarsenRefStatusGetTpriorTstop (
braid_CoarsenRefStatus s,
braid_Real * v1,
braid_Real * v2,
braid_Real * v3,
braid_Real * $V 4$,
braid_Real * v5 )
14.10.2.34 braid_ObjectiveStatusGetlter() braid_Int braid_ObjectiveStatusGetIter ( braid_ObjectiveStatus $s$, braid_Int * v1 )
14.10.2.35 braid_ObjectiveStatusGetLevel() braid_Int braid_ObjectiveStatusGetLevel (

```
    braid_ObjectiveStatus s,
    braid_Int * v1 )
```

```
14.10.2.36 braid_ObjectiveStatusGetNLevels() braid_Int braid_ObjectiveStatusGetNLevels ( braid_ObjectiveStatus s, braid_Int * v1 )
```

14.10.2.37 braid_ObjectiveStatusGetNRefine() braid_Int braid_ObjectiveStatusGetNRefine ( braid_ObjectiveStatus $s$, braid_Int * v1 )
14.10.2.38 braid_ObjectiveStatusGetNTPoints() braid_Int braid_ObjectiveStatusGetNTPoints ( braid_ObjectiveStatus $s$, braid_Int * v1 )
14.10.2.39 braid_ObjectiveStatusGetT() braid_Int braid_ObjectiveStatusGetT (
braid_ObjectiveStatus s, braid_Real * v1 )
14.10.2.40 braid_ObjectiveStatusGetTIndex() braid_Int braid_ObjectiveStatusGetTIndex ( braid_ObjectiveStatus $s$, braid_Int * v1 )
14.10.2.41 braid_ObjectiveStatusGetTol() braid_Int braid_ObjectiveStatusGetTol ( braid_ObjectiveStatus $s$, braid_Real * v1 )
14.10.2.42 braid_StepStatusGetBasisVec() braid_Int braid_StepStatusGetBasisVec (

```
    braid_StepStatus s,
    braid_Vector * v1,
    braid_Int v2 )
```

14.10.2.43 braid_StepStatusGetCallingFunction() braid_Int braid_StepStatusGetCallingFunction (
braid_StepStatus s,
braid_Int * v1 )

```
14.10.2.44 braid_StepStatusGetDeltaRank() braid_Int braid_StepStatusGetDeltaRank (
    braid_StepStatus s,
```

    braid_Int * v1 )
    14.10.2.45 braid_StepStatusGetDone() braid_Int braid_StepStatusGetDone (
braid_StepStatus s,
braid_Int * v1 )
14.10.2.46 braid_StepStatusGetler() braid_Int braid_StepStatusGetIter ( braid_StepStatus s, braid_Int * v1 )
14.10.2.47 braid_StepStatusGetLevel() braid_Int braid_StepStatusGetLevel ( braid_StepStatus s, braid_Int * v1 )
14.10.2.48 braid_StepStatusGetNLevels() braid_Int braid_StepStatusGetNLevels ( braid_StepStatus s, braid_Int * v1 )
14.10.2.49 braid_StepStatusGetNRefine() braid_Int braid_StepStatusGetNRefine ( braid_StepStatus s, braid_Int * v1 )
14.10.2.50 braid_StepStatusGetNTPoints() braid_Int braid_StepStatusGetNTPoints ( braid_StepStatus s, braid_Int * v1 )
14.10.2.51 braid_StepStatusGetOldFineTolx() braid_Int braid_StepStatusGetoldFineTolx (
braid_StepStatus s, braid_Real * v1 )

```
14.10.2.52 braid_StepStatusGetRNorms() braid_Int braid_StepStatusGetRNorms (
braid_StepStatus s,
braid_Int * v1,
braid_Real * v2 )
```

14.10.2.53 braid_StepStatusGetSingleErrorEstStep() braid_Int braid_StepStatusGetSingleErrorEstStep (

```
braid_StepStatus s,
braid_Real * v1 )
```

14.10.2.54 braid_StepStatusGetT() braid_Int braid_StepStatusGetT (
braid_StepStatus s, braid_Real * v1 )
14.10.2.55 braid_StepStatusGetTIndex() braid_Int braid_StepStatusGetTIndex (
braid_StepStatus s, braid_Int * v1 )
14.10.2.56 braid_StepStatusGetTIUL() braid_Int braid_StepStatusGetTIUL (

```
    braid_StepStatus s,
    braid_Int * v1,
    braid_Int * v2,
    braid_Int v3 )
```

14.10.2.57 braid_StepStatusGetTol() braid_Int braid_StepStatusGetTol (

```
    braid_StepStatus s,
```

    braid_Real * v1 )
    14.10.2.58 braid_StepStatusGetTstartTstop() braid_Int braid_StepStatusGetTstartTstop ( braid_StepStatus s, braid_Real * v1, braid_Real * v2 )

```
14.10.2.59 braid_StepStatusGetTstop() braid_Int braid_StepStatusGetTstop (
    braid_StepStatus s,
```

    braid_Real * v1 )
    14.10.2.60 braid_StepStatusSetOldFineTolx() braid_Int braid_StepStatusSetOldFineTolx (
braid_StepStatus s,
braid_Real v1 )
14.10.2.61 braid_StepStatusSetRFactor() braid_Int braid_StepStatusSetRFactor ( braid_StepStatus s, braid_Real v1 )
14.10.2.62 braid_StepStatusSetRSpace() braid_Int braid_StepStatusSetRSpace ( braid_StepStatus s, braid_Real v1 )
14.10.2.63 braid_StepStatusSetTightFineTolx() braid_Int braid_StepStatusSetTightFineTolx ( braid_StepStatus s, braid_Real v1 )
14.10.2.64 braid_SyncStatusGetAllErrorEst() braid_Int braid_SyncStatusGetAllErrorEst ( braid_SyncStatus s, braid_Real * v1 )
14.10.2.65 braid_SyncStatusGetCallingFunction() braid_Int braid_SyncStatusGetCallingFunction ( braid_SyncStatus s, braid_Int * v1 )
14.10.2.66 braid_SyncStatusGetDone() braid_Int braid_SyncStatusGetDone ( braid_SyncStatus s, braid_Int * v1 )
14.10.2.67 braid_SyncStatusGetlter() braid_Int braid_SyncStatusGetIter (
braid_SyncStatus s, braid_Int * v1 )
14.10.2.68 braid_SyncStatusGetLevel() braid_Int braid_SyncStatusGetLevel (
braid_SyncStatus s, braid Int * v1 )
14.10.2.69 braid_SyncStatusGetNLevels() braid_Int braid_SyncStatusGetNLevels ( braid_SyncStatus s, braid_Int * v1 )
14.10.2.70 braid_SyncStatusGetNRefine() braid_Int braid_SyncStatusGetNRefine ( braid_SyncStatus s, braid_Int * v1
14.10.2.71 braid_SyncStatusGetNTPoints() braid_Int braid_SyncStatusGetNTPoints ( braid_SyncStatus s, braid_Int * v1 )
14.10.2.72 braid_SyncStatusGetNumErrorEst() braid_Int braid_SyncStatusGetNumErrorEst ( braid_SyncStatus s, braid_Int * v1 )
14.10.2.73 braid_SyncStatusGetProc() braid_Int braid_SyncStatusGetProc (
braid_SyncStatus s, braid_Int * v1, braid_Int v2, braid_Int v3 )
14.10.2.74 braid_SyncStatusGetTComm() braid_Int braid_SyncStatusGetTComm (

```
braid_SyncStatus s,
    MPI_Comm * v1 )
```

14.10.2.75 braid_SyncStatusGetTimeValues() braid_Int braid_SyncStatusGetTimeValues (
braid_SyncStatus s,
braid_Real ** v1,
braid_Int v2,
braid_Int v3,
braid_Int v4 )
14.10.2.76 braid_SyncStatusGetTIUL() braid_Int braid_SyncStatusGetTIUL (

```
    braid_SyncStatus s,
    braid_Int * v1,
    braid_Int * v2,
    braid_Int v3 )
```


### 14.11 XBraid status macros

## Macros

- \#define braid_ASCaller_FInterp 0
- \#define braid_ASCaller_FRestrict 1
- \#define braid_ASCaller_FRefine 2
- \#define braid_ASCaller_FAccess 3
- \#define braid_ASCaller_FRefine_AfterInitHier 4
- \#define braid_ASCaller_Drive_TopCycle 5
- \#define braid_ASCaller_FCRelax 6
- \#define braid_ASCaller_Drive_AfterInit 7
- \#define braid_ASCaller_BaseStep_diff 8
- \#define braid_ASCaller_ComputeFullRNorm 9
- \#define braid_ASCaller_FASResidual 10
- \#define braid_ASCaller_Residual 11
- \#define braid_ASCaller_InitGuess 12


### 14.11.1 Detailed Description

Macros defining Status values that the user can obtain during runtime, which will tell the user where in Braid the current cycle is, e.g. in the FInterp function.

### 14.11.2 Macro Definition Documentation

```
14.11.2.1 braid_ASCaller_BaseStep_diff #define braid_ASCaller_BaseStep_diff 8
When CallingFunction equals 8, Braid is in BaseStep_diff
14.11.2.2 braid_ASCaller_ComputeFulIRNorm #define braid_ASCaller_ComputeFullRNorm 9
When CallingFunction equals 9, Braid is in ComputeFullRNorm
```

14.11.2.3 braid_ASCaller_Drive_AfterInit \#define braid_ASCaller_Drive_AfterInit 7
When CallingFunction equals 7, Braid just finished initialization
14.11.2.4 braid_ASCaller_Drive_TopCycle \#define braid_ASCaller_Drive_TopCycle 5
When CallingFunction equals 5, Braid is at the top of the cycle
14.11.2.5 braid_ASCaller_FAccess \#define braid_ASCaller_FAccess 3
When CallingFunction equals 3, Braid is in FAccess
14.11.2.6 braid_ASCaller_FASResidual \#define braid_ASCaller_FASResidual 10
When CallingFunction equals 10, Braid is in FASResidual
14.11.2.7 braid_ASCaller_FCRelax \#define braid_ASCaller_FCRelax 6
When CallingFunction equals 6, Braid is in FCrelax
14.11.2.8 braid_ASCaller_FInterp \#define braid_ASCaller_FInterp 0
When CallingFunction equals 0 , Braid is in FInterp
14.11.2.9 braid_ASCaller_FRefine \#define braid_ASCaller_FRefine 2

When CallingFunction equals 2, Braid is in FRefine
14.11.2.10 braid_ASCaller_FRefine_AfterInitHier \#define braid_ASCaller_FRefine_AfterInitHier 4

When CallingFunction equals 4, Braid is inside FRefine after the new finest level has been initialized

### 14.11.2.11 braid_ASCaller_FRestrict \#define braid_ASCaller_FRestrict 1

When CallingFunction equals 1, Braid is in FRestrict
14.11.2.12 braid_ASCaller_InitGuess \#define braid_ASCaller_InitGuess 12

When CallingFunction equals 12, Braid is in InitGuess
14.11.2.13 braid_ASCaller_Residual \#define braid_ASCaller_Residual 11

When CallingFunction equals 11, Braid is in Residual, immediately after restriction

### 14.12 XBraid test routines

## Functions

- braid_Int braid_TestlnitAccess (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_PtFcnInit init, braid_PtFcnAccess access, braid_PtFcnFree free)
- braid_Int braid_TestClone (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_PtFcnlnit init, braid_PtFcnAccess access, braid_PtFcnFree free, braid_PtFcnClone clone)
- braid_Int braid_TestSum (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_PtFcnInit init, braid_PtFcnAccess access, braid_PtFcnFree free, braid_PtFcnClone clone, braid_PtFcnSum sum)
- braid_Int braid_TestSpatialNorm (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_PtFcnInit init, braid_PtFcnFree free, braid_PtFcnClone clone, braid_PtFcnSum sum, braid_PtFcnSpatialNorm spatialnorm)
- braid_Int braid_TestInnerProd (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t1, braid_Real t2, braid_PtFcnInit init, braid_PtFcnFree free, braid_PtFcnSum sum, braid_PtFcnInnerProd inner_prod)
- braid_Int braid_TestBuf (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_PtFcnInit init, braid_PtFcnFree free, braid_PtFcnSum sum, braid_PtFcnSpatialNorm spatialnorm, braid_PtFcnBufSize bufsize, braid_PtFcnBufPack bufpack, braid_PtFcnBufUnpack bufunpack)
- braid_Int braid_TestCoarsenRefine (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_Real fdt, braid_Real cdt, braid_PtFcnInit init, braid_PtFcnAccess access, braid_PtFcnFree free, braid_PtFcnClone clone, braid_PtFcnSum sum, braid_PtFcnSpatialNorm spatialnorm, braid_PtFcnSCoarsen coarsen, braid_PtFcnSRefine refine)
- braid_Int braid_TestResidual (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_Real dt, braid_PtFcnInit myinit, braid_PtFcnAccess myaccess, braid_PtFcnFree myfree, braid_PtFcnClone clone, braid_PtFcnSum sum, braid_PtFcnSpatialNorm spatialnorm, braid_PtFcnResidual residual, braid_PtFcnStep step)
- braid_Int braid_TestAll (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_Real fdt, braid_Real cdt, braid_PtFcnInit init, braid_PtFcnFree free, braid_PtFcnClone clone, braid_PtFcnSum sum, braid_PtFcnSpatialNorm spatialnorm, braid_PtFcnBufSize bufsize, braid_PtFcnBufPack bufpack, braid_PtFcnBufUnpack bufunpack, braid_PtFcnSCoarsen coarsen, braid_PtFcnSRefine refine, braid_PtFcnResidual residual, braid_PtFcnStep step)
- braid_Int braid_TestDelta (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_Real dt, braid_Int rank, braid_PtFcnInit myinit, braid_PtFcnInitBasis myinit_basis, braid_PtFcnAccess myaccess, braid_PtFcnFree myfree, braid_PtFcnClone myclone, braid_PtFcnSum mysum, braid_PtFcnBufSize bufsize, braid_PtFcnBufPack bufpack, braid_PtFcnBufUnpack bufunpack, braid_PtFcnInnerProd myinner_prod, braid_PtFcnStep mystep)


### 14.12.1 Detailed Description

These are sanity check routines to help a user test their XBraid code.

### 14.12.2 Function Documentation

14.12.2.1 braid_TestAll() braid_Int braid_TestAll (

```
braid_App app,
MPI_Comm comm_x,
FILE * fp,
braid_Real t,
braid_Real fdt,
braid_Real cdt,
braid_PtFcnInit init,
braid_PtFcnFree free,
braid_PtFcnClone clone,
braid_PtFcnSum sum,
braid_PtFcnSpatialNorm spatialnorm,
braid_PtFcnBufSize bufsize,
braid_PtFcnBufPack bufpack,
braid_PtFcnBufUnpack bufunpack,
braid_PtFcnSCoarsen coarsen,
braid_PtFcnSRefine refine,
braid_PtFcnResidual residual,
braid_PtFcnStep step )
```

Runs all of the individual braid_Test* routines

- Returns 0 if the tests fail
- Returns 1 if the tests pass
- Check the log messages to see details of which tests failed.


## Parameters

| app | User defined App structure |
| :--- | :--- |
| comm_ $x$ | Spatial communicator |
| $f p$ | File pointer (could be stdout or stderr) for log messages |
| $t$ | Time value to initialize test vectors with |
| $f d t$ | Fine time step value that you spatially coarsen from |
| $c d t$ | Coarse time step value that you coarsen to |
| init | Initialize a braid_Vector on finest temporal grid |
| free | Free a braid_Vector |
| clone | Clone a braid_Vector |
| sum | Compute vector sum of two braid_Vectors |

## Parameters

| spatialnorm | Compute norm of a braid_Vector, this is a norm only over space |
| :--- | :--- |
| bufsize | Computes size in bytes for one braid_Vector MPI buffer |
| bufpack | Packs MPI buffer to contain one braid_Vector |
| bufunpack | Unpacks MPI buffer into a braid_Vector |
| coarsen | Spatially coarsen a vector. If NULL, test is skipped. |
| refine | Spatially refine a vector. If NULL, test is skipped. |
| residual | Compute a residual given two consectuive braid_Vectors |
| step | Compute a time step with a braid_Vector |

### 14.12.2.2 braid_TestBuf() braid_Int braid_TestBuf (

```
braid_App app,
MPI_Comm comm_x,
FILE * fp,
braid_Real t,
braid_PtFcnInit init,
braid_PtFcnFree free,
braid_PtFcnSum sum,
braid_PtFcnSpatialNorm spatialnorm,
braid_PtFcnBufSize bufsize,
braid_PtFcnBufPack bufpack,
braid_PtFcnBufUnpack bufunpack )
```

Test the BufPack, BufUnpack and BufSize functions.
A vector is initialized at time $t$, packed into a buffer, then unpacked from a buffer. The unpacked result must equal the original vector.

- Returns 0 if the tests fail
- Returns 1 if the tests pass
- Check the log messages to see details of which tests failed.

Parameters

| app | User defined App structure |
| :--- | :--- |
| comm_ $x$ | Spatial communicator |
| $f p$ | File pointer (could be stdout or stderr) for log messages |
| $t$ | Time value to test Buffer routines (used to initialize the vectors) |
| init | Initialize a braid_Vector on finest temporal grid |
| free | Free a braid_Vector |
| sum | Compute vector sum of two braid_Vectors |
| spatialnorm | Compute norm of a braid_Vector, this is a norm only over space |
| bufsize | Computes size in bytes for one braid_Vector MPI buffer |
| bufpack | Packs MPI buffer to contain one braid_Vector |
| GdaldataleqayatoxygeriUnpacks MPI buffer containing one braid_Vector |  |

### 14.12.2.3 braid_TestClone() braid_Int braid_TestClone (

```
braid_App app,
MPI_Comm comm_x,
FILE * fp,
braid_Real t,
braid_PtFcnInit init,
braid_PtFcnAccess access,
braid_PtFcnFree free,
braid_PtFcnClone clone )
```

Test the clone function.
A vector is initialized at time $t$, cloned, and both vectors are written. Then both vectors are free-d. The user is to check (via the access function) to see if it is identical.

## Parameters

| app | User defined App structure |
| :--- | :--- |
| comm <br> $\_$ | Spatial communicator |
| $f p$ | File pointer (could be stdout or stderr) for log messages |
| $t$ | Time value to test clone with (used to initialize the vectors) |
| init | Initialize a braid_Vector on finest temporal grid |
| access | Allows access to XBraid and current braid_Vector (can be NULL for no writing) |
| free | Free a braid_Vector |
| clone | Clone a braid_Vector |

14.12.2.4 braid_TestCoarsenRefine() braid_Int braid_TestCoarsenRefine (

```
braid_App app,
MPI_Comm comm_x,
FILE * fp,
braid_Real t,
braid_Real fdt,
braid_Real cdt,
braid_PtFcnInit init,
braid_PtFcnAccess access,
braid_PtFcnFree free,
braid_PtFcnClone clone,
braid_PtFcnSum sum,
braid_PtFcnSpatialNorm spatialnorm,
braid_PtFcnSCoarsen coarsen,
braid_PtFcnSRefine refine )
```

Test the Coarsen and Refine functions.
A vector is initialized at time $t$, and various sanity checks on the spatial coarsening and refinement routines are run.

- Returns 0 if the tests fail
- Returns 1 if the tests pass
- Check the log messages to see details of which tests failed.


## Parameters

| app | User defined App structure |
| :--- | :--- |
| comm_ $x$ | Spatial communicator |
| $f p$ | File pointer (could be stdout or stderr) for log messages |
| $t$ | Time value to initialize test vectors |
| fdt | Fine time step value that you spatially coarsen from |
| $c d t$ | Coarse time step value that you coarsen to |
| init | Initialize a braid_Vector on finest temporal grid |
| access | Allows access to XBraid and current braid_Vector (can be NULL for no writing) |
| free | Free a braid_Vector |
| clone | Clone a braid_Vector |
| sum | Compute vector sum of two braid_Vectors |
| spatialnorm | Compute norm of a braid_Vector, this is a norm only over space |
| coarsen | Spatially coarsen a vector |
| refine | Spatially refine a vector |

14.12.2.5 braid_TestDelta() braid_Int braid_TestDelta (

```
braid_App app,
MPI_Comm comm_x,
FILE * Ep,
braid_Real t,
braid_Real dt,
braid_Int rank,
braid_PtFcnInit myinit,
braid_PtFcnInitBasis myinit_basis,
braid_PtFcnAccess myaccess,
braid_PtFcnFree myfree,
braid_PtFcnClone myclone,
braid_PtFcnSum mysum,
braid_PtFcnBufSize bufsize,
braid_PtFcnBufPack bufpack,
braid_PtFcnBufUnpack bufunpack,
braid_PtFcnInnerProd myinner_prod,
braid_PtFcnstep mystep )
```

Test functions required for Delta correction. Initializes a braid_Vector and braid_Basis at time 0 , then tests the inner product function with braid_TestInnerProd, then checks that the basis vectors are not linearly dependent with the Gram Schmidt process. Finally, compares the user propagation of tangent vectors against a finite difference approximation: [step_du(u)] Psi_i - (step(u + eps Psi_i) - step(u)/eps ~= 0

Parameters

| app | User defined App structure |
| :--- | :--- |
| comm_x | Spatial communicator |
| $f p$ | File pointer (could be stdout or stderr) for log messages |
| $t$ | Time value to initialize test vectors with |
| $d t$ | time step size |
| rank | rank (number of columns) of basis |
| myinit | Initialize a braid_Vector |
| myinit_basis | Initialize the ith column of a basis set of braid_Vectors |
| myaccess | Allows access to XBraid and current braid_Vector and braid_Basis |
| myfree | Free a braid_Vector |
| myclone | Clone a braid_Vector |
| mysum | Compute vector sum of two braid_Vectors |
| bufsize | Computes size in bytes for one braid_Vector MPI buffer |
| bufpack | Packs MPI buffer to contain one braid_Vector |
| bufunpack | Unpacks MPI buffer containing one braid_Vector |
| myinner_prod | Compute inner product of two braid_Vectors |
| mystep | Compute a time step with a braid_Vector |

14.12.2.6 braid_TestInitAccess() braid_Int braid_TestInitAccess (

```
braid_App app,
MPI_Comm comm_x,
FILE * fp,
braid_Real t,
braid_PtFcnInit init,
braid_PtFcnAccess access,
braid_PtFcnFree free )
```

Test the init, access and free functions.
A vector is initialized at time $t$, written, and then free-d
Parameters

| app | User defined App structure |
| :--- | :--- |
| comm <br> $\_$ <br> $\hookleftarrow$ | Spatial communicator |
| $f p$ | File pointer (could be stdout or stderr) for log messages |
| $t$ | Time value to test init with (used to initialize the vectors) |
| init | Initialize a braid_Vector on finest temporal grid |
| access | Allows access to XBraid and current braid_Vector (can be NULL for no writing) |
| free | Free a braid_Vector |

### 14.12.2.7 braid_TestInnerProd() braid_Int braid_TestInnerProd (

```
braid_App app,
MPI_Comm comm_x,
FILE * fp,
braid_Real t1,
braid_Real t2,
braid_PtFcnInit init,
braid_PtFcnFree free,
braid_PtFcnSum sum,
braid_PtFcnInnerProd inner_prod )
```

Test the inner_prod function.
A vector is initialized at time $t 1$, then the vector is normalized under the norm induced by inner_prod. A second vector is initialized at time t2, and the Gram Schmidt process removes the component of the second vector along the direction of the first. The test is inconclusive unless both vectors are nonzero and not orthogonal.

- Returns 0 if the tests fail
- Returns 1 if the tests pass
- Check the log messages to see details of which tests failed.


## Parameters

| app | User defined App structure |
| :--- | :--- |
| comm_x | Spatial communicator |
| $f p$ | File pointer (could be stdout or stderr) for log messages |
| $t 1$ | Time value used to initialize the 1st vector |
| $t 2$ | Time value used to initialize the 2nd vector (t1 ! = t2) |
| init | Initialize a braid_Vector on finest temporal grid |
| free | Free a braid_Vector |
| sum | Compute vector sum of two braid_Vectors |
| inner_prod | Compute inner product of two braid_Vectors |

14.12.2.8 braid_TestResidual() braid_Int braid_TestResidual (

```
braid_App app,
MPI_Comm comm_x,
FILE * fp,
braid_Real t,
braid_Real dt,
braid_PtFcnInit myinit,
    braid_PtFcnAccess myaccess,
    braid_PtFcnFree myfree,
    braid_PtFcnClone clone,
    braid_PtFcnSum sum,
    braid_PtFcnSpatialNorm spatialnorm,
```

```
braid_PtFcnResidual residual,
braid_PtFcnStep step )
```

Test compatibility of the Step and Residual functions.
A vector is initialized at time $t$, step is called with $d t$, followed by an evaluation of residual, to test the condition fstop residual( step(u, fstop), u) approx. 0

- Check the log messages to determine if test passed. The result should approximately be zero. The more accurate the solution for $u$ is computed in step, the closer the result will be to 0 .
- The residual is also written to file

Parameters

| app | User defined App structure |
| :--- | :--- |
| comm_x | Spatial communicator |
| $f p$ | File pointer (could be stdout or stderr) for log messages |
| $t$ | Time value to initialize test vectors |
| $d t$ | Time step value to use in step |
| myinit | Initialize a braid_Vector on finest temporal grid |
| myaccess | Allows access to XBraid and current braid_Vector (can be NULL for no writing) |
| myfree | Free a braid_Vector |
| clone | Clone a braid_Vector |
| sum | Compute vector sum of two braid_Vectors |
| spatialnorm | Compute norm of a braid_Vector, this is a norm only over space |
| residual | Compute a residual given two consectuive braid_Vectors |
| step | Compute a time step with a braid_Vector |

14.12.2.9 braid_TestSpatialNorm() braid_Int braid_TestSpatialNorm (

```
braid_App app,
MPI_Comm comm_x,
FILE * fp,
braid_Real t,
braid_PtFcnInit init,
braid_PtFcnFree free,
braid_PtFcnClone clone,
braid_PtFcnSum sum,
braid_PtFcnSpatialNorm spatialnorm )
```

Test the spatialnorm function.
A vector is initialized at time $t$ and then cloned. Various norm evaluations like $\|3 \mathrm{v}\| /\|\mathrm{v}\|$ with known output are then done.

- Returns 0 if the tests fail
- Returns 1 if the tests pass
- Check the log messages to see details of which tests failed.

Parameters

| app | User defined App structure |
| :--- | :--- |
| comm_x | Spatial communicator |
| $f p$ | File pointer (could be stdout or stderr) for log messages |
| $t$ | Time value to test SpatialNorm with (used to initialize the vectors) |
| init | Initialize a braid_Vector on finest temporal grid |
| free | Free a braid_Vector |
| clone | Clone a braid_Vector |
| sum | Compute vector sum of two braid_Vectors |
| spatialnorm | Compute norm of a braid_Vector, this is a norm only over space |

```
14.12.2.10 braid_TestSum() braid_Int braid_TestSum (
    braid_App app,
    MPI_Comm comm_x,
    FILE * fp,
    braid_Real t,
    braid_PtFcnInit init,
    braid_PtFcnAccess access,
    braid_PtFcnFree free,
    braid_PtFcnClone clone,
    braid_PtFcnSum sum )
```

Test the sum function.
A vector is initialized at time $t$, cloned, and then these two vectors are summed a few times, with the results written. The vectors are then free-d. The user is to check (via the access function) that the output matches the sum of the two original vectors.

Parameters

| app | User defined App structure |
| :--- | :--- |
| $c^{\prime}$ <br> - <br> $x$ | Spatial communicator |
| $f p$ | File pointer (could be stdout or stderr) for log messages |
| $t$ | Time value to test Sum with (used to initialize the vectors) |
| init | Initialize a braid_Vector on finest temporal grid |
| access | Allows access to XBraid and current braid_Vector (can be NULL for no writing) |
| free | Free a braid_Vector |
| clone | Clone a braid_Vector |
| sum | Compute vector sum of two braid_Vectors |

## 15 Data Structure Documentation

## 15.1 _braid_Action Struct Reference

```
#include <tape.h>
```


## Data Fields

- _braid_Call braidCall
- braid_Core core
- braid_Real inTime
- braid_Real outTime
- braid_Int inTimeldx
- braid_Real sum_alpha
- braid_Real sum_beta
- braid_Int send_recv_rank
- braid_Int braid_iter
- braid_Int myid
- braid_Int level
- braid_Int nrefine
- braid_Int gupper
- braid_Real tol
- braid_Int messagetype
- braid_Int size_buffer


### 15.1.1 Detailed Description

XBraid Action structure

Holds information for the called user routines

### 15.1.2 Field Documentation

15.1.2.1 braid_iter braid_Int braid_iter iteration number of xBraid
15.1.2.2 braidCall
_braid_Call braidCall
type of the user routine
15.1.2.3 core braid_Core core
pointer to braid's core structure
15.1.2.4 gupper braid_Int gupper
global size of the fine grid
15.1.2.5 inTime braid_Real inTime
time of the input vector
15.1.2.6 inTimeldx braid_Int inTimeIdx
index of time of input vector
15.1.2.7 level braid_Int level
current level in Braid
15.1.2.8 messagetype braid_Int messagetype
message type, 0: for Step(), 1: for load balancing
15.1.2.9 myid braid_Int myid
processors id
15.1.2.10 nrefine braid_Int nrefine
number of refinements done
15.1.2.11 outTime braid_Real outTime
time of the output vector
15.1.2.12 send_recv_rank braid_Int send_recv_rank
processor rank of sender / receiver in my_bufpack / my_bufunpack
15.1.2.13 size_buffer braid_Int size_buffer
if set by user, size of send buffer is "size" bytes
15.1.2.14 sum_alpha braid_Real sum_alpha
first coefficient of my_sum
15.1.2.15 sum_beta braid_Real sum_beta
second coefficient of my_sum
15.1.2.16 tol braid_Real tol
primal stopping tolerance

The documentation for this struct was generated from the following file:

- tape.h


## 15.2 _braid_BaseVector Struct Reference

\#include <_braid.h>

## Data Fields

- braid_Vector userVector
- braid_VectorBar bar
- braid_Basis basis


### 15.2.1 Detailed Description

Braid vector used for storage of all state and (if needed) adjoint information. Stores the user's primal vector (braid_ $\leftarrow$ Vector type) and the associated bar vector (braid_VectorBar type) if the adjoint functionality is being used, as well as basis vectors used by Delta correction. If adjoint is not being used, bar==NULL, and if Delta correction is not used, basis==NULL.

For Delta correction, basis stores the Lyapunov vectors and low-rank Delta corrections.

### 15.2.2 Field Documentation

### 15.2.2.1 bar braid_VectorBar bar

holds the bar vector (shared pointer implementation)
15.2.2.2 basis braid_Basis basis
local basis of variable rank, stored as the user's vector type
15.2.2.3 userVector braid_Vector userVector
holds the users primal vector

The documentation for this struct was generated from the following file:

- _braid.h


## 15.3 _braid_Basis Struct Reference

```
#include <_braid.h>
```


## Data Fields

- braid_Vector * userVecs
- braid_Int rank


### 15.3.1 Detailed Description

This contains an array of braid_Vector objects which should be thought of as a basis for the state space, along with the number of vectors stored in each (rank). Only initialized when using Delta correction. The vectors should be initialized using the user's InitBasis function.

### 15.3.2 Field Documentation

15.3.2.1 rank braid_Int rank
15.3.2.2 userVecs braid_Vector* userVecs

The documentation for this struct was generated from the following file:

- _braid.h


## 15.4 _braid_CommHandle Struct Reference

```
#include <_braid.h>
```


## Data Fields

- braid_Int request_type
- braid_Int num_requests
- braid_Int index
- braid_Int level
- MPI_Request $*$ requests
- MPI_Status * status
- void $*$ buffer
- braid_BaseVector * vector_ptr


### 15.4.1 Detailed Description

XBraid comm handle structure

Used for initiating and completing nonblocking communication to pass braid_BaseVectors between processors.

### 15.4.2 Field Documentation

### 15.4.2.1 buffer void* buffer

Buffer for message
15.4.2.2 index braid_Int index
time index of the time point corresponding to this handle
15.4.2.3 level braid_Int level
level of the time point corresponding to this handle

### 15.4.2.4 num_requests braid_Int num_requests

number of active requests for this handle, usually 1
15.4.2.5 request_type braid_Int request_type
two values: recv type $=1$, and send type $=0$
15.4.2.6 requests MPI_Request* requests

MPI request structure
15.4.2.7 status MPI_Status* status

MPI status
15.4.2.8 vector_ptr braid_BaseVector* vector_ptr
braid_vector being sent/received

The documentation for this struct was generated from the following file:

- _braid.h


## 15.5 _braid_Core Struct Reference

\#include <_braid.h>

## Data Fields

- MPI_Comm comm_world
- MPI_Comm comm
- braid_Int myid_world
- braid_Int myid
- braid_Real tstart
- braid_Real tstop
- braid_Int ntime
- braid_App app
- braid_PtFcnStep step
- braid_PtFcnInit init
- braid_PtFcnInitBasis init_basis
- braid_PtFcnSInit sinit
- braid_PtFcnClone clone
- braid_PtFcnSClone sclone
- braid_PtFcnFree free
- braid_PtFcnSFree sfree
- braid_PtFcnSum sum
- braid_PtFcnSpatialNorm spatialnorm
- braid_PtFcnInnerProd inner_prod
- braid_PtFcnAccess access
- braid_PtFcnBufSize bufsize
- braid_PtFcnBufPack bufpack
- braid_PtFcnBufUnpack bufunpack
- braid_PtFcnResidual residual
- braid_PtFcnSCoarsen scoarsen
- braid_PtFcnSRefine srefine
- braid_PtFcnSync sync
- braid_PtFcnTimeGrid tgrid
- braid_PtFcnBufAlloc bufalloc
- braid_PtFcnBufFree buffree
- braid_Int periodic
- braid_Int initiali
- braid_Int access_level
- braid_Int finalFCrelax
- braid_Int print_level
- braid_Int io_level
- braid_Int seq_soln
- braid_Int max_levels
- braid_Int incr_max_levels
- braid_Int min_coarse
- braid_Int relax_only_cg
- braid_Real tol
- braid_Int rtol
- braid_Int * nrels
- braid_Int nrdefault
- braid_Real * CWts
- braid_Real CWt_default
- braid_Int $*$ cfactors
- braid_Int cfdefault
- braid_Int max_iter
- braid_Int niter
- braid_Int fmg
- braid_Int nfmg
- braid_Int nfmg_Vcyc
- braid_Int warm_restart
- braid_Int tnorm
- braid_Real * tnorm_a
- braid_Real rnorm0
- braid_Real * rnorms
- braid_PtFcnResidual full_rnorm_res
- braid_Real full_rnorm0
- braid_Real * full_rnorms
- braid_Int storage
- braid_Int useshell
- braid_Int gupper
- braid_Int refine
- braid Int * rfactors
- braid_Real ** rdtvalues
- braid_Int r_space
- braid_Int rstopped
- braid_Int nrefine
- braid_Int max_refinements
- braid_Int tpoints_cutoff
- braid_Int skip
- braid_Int nlevels
- _braid_Grid ** grids
- braid_Real localtime
- braid_Real globaltime
- braid_Int delta_correct
- braid_Int delta_rank
- braid_Int delta_defer_|v|
- braid_Int delta_defer_iter
- braid_Int estimate_lyap
- braid_Int relax_lyap
- braid_Int lyap_exp
- braid_Real ** local_exponents
- braid_Int richardson
- braid_Int est_error
- braid_Int order
- braid_Real * dtk
- braid_Real * estimate
- braid_Optim optim
- braid_Int adjoint
- braid_Int record
- braid_Int obj_only
- braid_Int reverted_ranks
- braid_Int verbose_adj
- _braid_Tape $*$ actionTape
- _braid_Tape $*$ userVectorTape
- _braid_Tape $*$ barTape
- braid_PtFcnObjectiveT objectiveT
- braid_PtFcnStepDiff step_diff
- braid_PtFcnObjectiveTDiff objT_diff
- braid_PtFcnResetGradient reset_gradient
- braid_PtFcnPostprocessObjective postprocess_obj
- braid_PtFcnPostprocessObjective_diff postprocess_obj_diff
- braid_Real t
- braid_Int idx
- braid_Int level
- braid_Real rnorm
- braid_Int done
- braid_Int wrapper_test
- braid_Int calling_function
- braid_Real f_tprior
- braid_Real f_tstop
- braid_Real c_tprior
- braid_Real c_tstop
- braid_Real tnext
- braid_Real old_fine_tolx
- braid_Int tight_fine_tolx
- braid_Int rfactor
- braid_Basis Ivectors
- braid_Int messagetype
- braid_Int size_buffer
- braid_Int size_basis
- braid_Int send_recv_rank
- braid_Int timings
- braid_Real timer_MPI_wait
- braid_Real timer_MPI_wait_coarse
- braid_Real timer_MPI_send
- braid_Real timer_MPI_recv
- braid_Real timer_coarse_solve
- braid_Real timer_drive_init
- braid_Real timer_user_step
- braid_Real timer_user_init
- braid_Real timer_user_clone
- braid_Real timer_user_free
- braid_Real timer_user_sum
- braid_Real timer_user_spatialnorm
- braid_Real timer_user_access
- braid_Real timer_user_bufsize
- braid_Real timer_user_bufpack
- braid_Real timer_user_bufunpack
- braid_Real timer_user_residual
- braid_Real timer_user_scoarsen
- braid_Real timer_user_srefine
- braid_Real timer_user_sync
- braid_Real timer_user_innerprod
- char $*$ timer_file_stem
- braid_Int timer_file_stem_len


### 15.5.1 Detailed Description

The typedef _braid_Core struct is a critical part of XBraid and is passed to each routine in XBraid. It thus allows each routine access to XBraid attributes.

### 15.5.2 Field Documentation

15.5.2.1 access braid_PtFcnAccess access
user access function to XBraid and current vector
15.5.2.2 access_level braid_Int access_level
determines how often to call the user's access routine
15.5.2.3 actionTape _braid_Tape* actionTape
tape storing the actions while recording
15.5.2.4 adjoint braid_Int adjoint
determines if adjoint run is performed (1) or not (0)
15.5.2.5 app braid_App app
application data for the user
15.5.2.6 barTape _braid_Tape* barTape
tape storing intermediate AD-bar variables while recording
15.5.2.7 bufalloc braid_PtFcnBufAlloc bufalloc
(optional) user-allocated MPI buffer for a certain number of bytes
15.5.2.8 buffree braid_PtFcnBufFree buffree (optional) free a user-allocated MPI buffer
15.5.2.9 bufpack braid_PtFcnBufPack bufpack pack a buffer

### 15.5.2.10 bufsize braid_PtFcnBufSize bufsize

return buffer size
15.5.2.11 bufunpack braid_PtFcnBufUnpack bufunpack
unpack a buffer
15.5.2.12 c_tprior braid_Real c_tprior
time value to the left of tstart on coarse grid
15.5.2.13 c_tstop braid_Real c_tstop
time value to the right of tstart on coarse grid
15.5.2.14 calling_function braid_Int calling_function
from which function are we accessing the vector
15.5.2.15 cfactors braid_Int* cfactors coarsening factors
15.5.2.16 cfdefault braid_Int cfdefault
default coarsening factor
15.5.2.17 clone braid_PtFcnClone clone clone a vector
15.5.2.18 comm MPI_Comm comm
communicator for the time dimension
15.5.2.19 comm_world MPI_Comm comm_world
15.5.2.20 CWt_default braid_Real CWt_default
default C-relaxtion weight

### 15.5.2.21 CWts braid_Real* CWts

C-relaxation weight for each level
15.5.2.22 delta_correct braid_Int delta_correct

Delta correction and Lyapunov vector estimation turns on Delta correction to potentially accelerate convergence
15.5.2.23 delta_defer_iter braid_Int delta_defer_iter

Delta correction will be turned off until this iteration
15.5.2.24 delta_defer_IvI braid_Int delta_defer_lvl

Delta correction will be turned off until this coarse level
15.5.2.25 delta_rank braid_Int delta_rank
for low rank Delta correction
15.5.2.26 done braid_Int done
boolean describing whether XBraid has finished
15.5.2.27 dtk braid_Real* dtk
holds value of sum_\{i\} dt_i^k for each C-interval
15.5.2.28 est_error braid_Int est_error
turns on embedded error estimation, e.g., for refinement

### 15.5.2.29 estimate braid_Real* estimate

holds value of the error estimate at each fine grid point
15.5.2.30 estimate_lyap braid_Int estimate_lyap
turns on estimation of Lyapunov vectors, via coarse grid solve, otherwise the basis at each C-point remains fixed

### 15.5.2.31 f_tprior braid_Real f_tprior

CoarsenRefStatus properties time value to the left of tstart on fine grid
15.5.2.32 f_tstop braid_Real f_tstop
time value to the right of tstart on fine grid
15.5.2.33 finalFCrelax braid_Int finalFCrelax
determines if a final FCrelax is performed (default $0=$ false)
15.5.2.34 fmg braid_Int fmg
use FMG cycle
15.5.2.35 free braid_PtFcnFree free free up a vector
15.5.2.36 full_rnorm0 braid_Real full_rnorm0
(optional) initial full residual norm
15.5.2.37 full_rnorm_res braid_PtFcnResidual full_rnorm_res
(optional) used to compute full residual norm
15.5.2.38 full_rnorms braid_Real* full_rnorms
(optional) full residual norm history
15.5.2.39 globaltime braid_Real globaltime
global wall time for braid_Drive()

### 15.5.2.40 grids _braid_Grid** grids

pointer to temporal grid structures for each level

### 15.5.2.41 gupper braid_Int gupper

global size of the fine grid
15.5.2.42 idx braid_Int idx
time point index value corresponding to $t$ on the global time grid
15.5.2.43 incr_max_levels braid_Int incr_max_levels

After doing refinement, increase the max number of levels by 1 ( $0=$ false, $1=$ true )
15.5.2.44 init braid_PtFcnInit init
return an initialized braid_BaseVector
15.5.2.45 init_basis braid_PtFcnInitBasis init_basis
(optional) return an initialized braid_Vector for initializing braid_Basis
15.5.2.46 initiali braid_Int initiali
initial condition grid index ( 0 : default; -1 : periodic )
15.5.2.47 inner_prod braid_PtFcnInnerProd inner_prod
(optional) compute a spatial inner product between two vectors
15.5.2.48 io_level braid_Int io_level
determines amount of output printed to files $(0,1)$
15.5.2.49 level braid_Int level
current level in XBraid
15.5.2.50 local_exponents braid_Real** local_exponents
holds local Lyapunov exponents at each C-point
15.5.2.51 localtime braid_Real localtime
local wall time for braid_Drive()
15.5.2.52 Ivectors braid_Basis lvectors
if Delta correction is set, contains reference to a braid_Basis object for giving user access to lyapunov vectors
15.5.2.53 lyap_exp braid_Int lyap_exp
turns on estimation of Lyapunov exponents
15.5.2.54 max_iter braid_Int max_iter
maximum number of multigrid in time iterations
15.5.2.55 max_levels braid_Int max_levels
maximum number of temporal grid levels
15.5.2.56 max_refinements braid_Int max_refinements
maximum number of refinements

### 15.5.2.57 messagetype braid_Int messagetype

BufferStatus properties message type, 0: for Step(), 1: for load balancing

### 15.5.2.58 min_coarse braid_Int min_coarse

minimum possible coarse grid size

### 15.5.2.59 myid braid_Int myid

my rank in the time communicator
15.5.2.60 myid_world braid_Int myid_world
my rank in the world communicator
15.5.2.61 nfmg braid_Int nfmg
number of fmg cycles to do initially before switching to V-cycles
15.5.2.62 nfmg_Vcyc braid_Int nfmg_Vcyc
number of V -cycle calls at each level in FMG
15.5.2.63 niter braid_Int niter
number of iterations
15.5.2.64 nlevels braid_Int nlevels
number of temporal grid levels
15.5.2.65 nrdefault braid_Int nrdefault
default number of pre-relaxations
15.5.2.66 nrefine braid_Int nrefine
number of refinements done
15.5.2.67 nrels braid_Int* nrels
number of pre-relaxations on each level

### 15.5.2.68 ntime braid_Int ntime

initial number of time intervals
15.5.2.69 obj_only braid_Int obj_only
determines if adjoint code computes ONLY objective, no gradients.
15.5.2.70 objectiveT braid_PtFcnObjectiveT objectiveT

User function: evaluate objective function at time $t$

### 15.5.2.71 objT_diff braid_PtFcnObjectiveTDiff objT_diff

User function: apply differentiated objective function
15.5.2.72 old_fine_tolx braid_Real old_fine_tolx

Allows for storing the previously used fine tolerance from GetSpatialAccuracy
15.5.2.73 optim braid_Optim optim
structure that stores optimization variables (objective function, etc.)
15.5.2.74 order braid_Int order
local order of time integration scheme
15.5.2.75 periodic braid_Int periodic
determines if periodic
15.5.2.76 postprocess_obj braid_PtFcnPostprocessObjective postprocess_obj

Optional user function: Modify the time-averaged objective function, e.g. for inverse design problems, adding relaxation term etc.

### 15.5.2.77 postprocess_obj_diff braid_PtFcnPostprocessObjective_diff postprocess_obj_diff

Optional user function: Derivative of postprocessing function

### 15.5.2.78 print_level braid_Int print_level

determines amount of output printed to screen ( $0,1,2,3$ )
15.5.2.79 r_space braid_Int r_space
spatial refinement flag
15.5.2.80 rdtvalues braid_Real** rdtvalues

Array of pointers to arrays of dt values for non-uniform refinement

### 15.5.2.81 record braid_Int record

determines if actions are recorded to the tape or not. This separate flag from adjoint is needed, because the final FAccess call should not be recorded unless nlevels==1, but the adjoint flag must be true even if nlevels==1.
15.5.2.82 refine braid_Int refine
refine in time $($ refine $=1)$
15.5.2.83 relax_lyap braid_Int relax_lyap
turns on propagation of Lyapunov vectors during FCRelax (to hopefully better resolve Lyapunov vectors)
15.5.2.84 relax_only_cg braid_Int relax_only_cg

Use relaxation only on coarsest grid (alternative to serial solve)
15.5.2.85 reset_gradient braid_PtFcnResetGradient reset_gradient

User function: Set the gradient to zero. Is called before each iteration
15.5.2.86 residual braid_PtFcnResidual residual
(optional) compute residual
15.5.2.87 reverted_ranks braid_Int reverted_ranks
15.5.2.88 rfactor braid_Int rfactor
if set by user, allows for subdivision of this interval for better time accuracy
15.5.2.89 rfactors braid_Int* rfactors
refinement factors for finest grid (if any)

### 15.5.2.90 richardson braid_Int richardson

Richardson-based error estimation and refinement turns on Richardson extrapolation for accuracy
15.5.2.91 rnorm braid_Real rnorm

AccessStatus properties residual norm
15.5.2.92 rnorm0 braid_Real rnorm0
initial residual norm
15.5.2.93 rnorms braid_Real* rnorms
residual norm history
15.5.2.94 rstopped braid_Int rstopped
refinement stopped at iteration rstopped
15.5.2.95 rtol braid_Int rtol
use relative tolerance
15.5.2.96 sclone braid_PtFcnSClone sclone
(optional) clone the shell of a vector
15.5.2.97 scoarsen braid_PtFcnSCoarsen scoarsen
(optional) return a spatially coarsened vector
15.5.2.98 send_recv_rank braid_Int send_recv_rank
holds the rank of the source / receiver from MPI_Send / MPI_Recv calls.
15.5.2.99 seq_soln braid_Int seq_soln
boolean, controls if the initial guess is from sequential time stepping
15.5.2.100 sfree braid_PtFcnSFree sfree
(optional) free up the data of a vector, keep the shell
15.5.2.101 sinit braid_PtFcnSInit sinit
(optional) return an initialized shell of braid_BaseVector
15.5.2.102 size_basis braid_Int size_basis
if Delta correction, send buffer will be of length (size_buffer + rank*size_basis)
15.5.2.103 size_buffer braid_Int size_buffer size of buffer, in bytes
15.5.2.104 skip braid_Int skip boolean, controls skipping any work on first down-cycle
15.5.2.105 spatialnorm braid_PtFenSpatialNorm spatialnorm

Compute norm of a braid_BaseVector, this is a norm only over space
15.5.2.106 srefine braid_PtFcnSRefine srefine
(optional) return a spatially refined vector
15.5.2.107 step braid_PtFcnStep step
apply step function
15.5.2.108 step_diff braid_PtFenStepDiff step_diff

User function: apply differentiated step function

### 15.5.2.109 storage braid_Int storage

storage $=0$ (C-points),$=1$ (all)
15.5.2.110 sum braid_PtFcnSum sum
vector sum

### 15.5.2.111 Sync braid_PtFcnSync sync

(optional) user access to app once-per-processor
15.5.2.112 t braid_Real t

Data elements required for the Status structures Common Status properties current time
15.5.2.113 tgrid braid_PtFcnTimeGrid tgrid
(optional) return time point values on level 0
15.5.2.114 tight_fine_tolx braid_Int tight_fine_tolx

Boolean, indicating whether the tightest fine tolx has been used, condition for halting
15.5.2.115 timer_coarse_solve braid_Real timer_coarse_solve
15.5.2.116 timer_drive_init braid_Real timer_drive_init
15.5.2.117 timer_file_stem char* timer_file_stem
15.5.2.118 timer_file_stem_len braid_Int timer_file_stem_len
15.5.2.119 timer_MPI_recv braid_Real timer_MPI_recv
15.5.2.120 timer_MPI_send braid_Real timer_MPI_send
15.5.2.121 timer_MPI_wait braid_Real timer_MPI_wait
15.5.2.122 timer_MPI_wait_coarse braid_Real timer_MPI_wait_coarse
15.5.2.123 timer_user_access braid_Real timer_user_access
15.5.2.124 timer_user_bufpack braid_Real timer_user_bufpack
15.5.2.125 timer_user_bufsize braid_Real timer_user_bufsize
15.5.2.126 timer_user_bufunpack braid_Real timer_user_bufunpack
15.5.2.127 timer_user_clone braid_Real timer_user_clone
15.5.2.128 timer_user_free braid_Real timer_user_free
15.5.2.129 timer_user_init braid_Real timer_user_init
15.5.2.130 timer_user_innerprod braid_Real timer_user_innerprod
15.5.2.131 timer_user_residual braid_Real timer_user_residual
15.5.2.132 timer_user_scoarsen braid_Real timer_user_scoarsen
15.5.2.133 timer_user_spatialnorm braid_Real timer_user_spatialnorm
15.5.2.134 timer_user_srefine braid_Real timer_user_srefine
15.5.2.135 timer_user_step braid_Real timer_user_step
15.5.2.136 timer_user_sum braid_Real timer_user_sum
15.5.2.137 timer_user_sync braid_Real timer_user_sync
15.5.2.138 timings braid_Int timings

Timers for various key parts of the code

### 15.5.2.139 tnext braid_Real tnext

StepStatus properties time value to evolve towards, time value to the right of tstart
15.5.2.140 tnorm braid_Int tnorm
choice of temporal norm
15.5.2.141 tnorm_a braid_Real* tnorm_a
local array of residual norms on a proc's interval, used for inf-norm
15.5.2.142 tol braid_Real tol
stopping tolerance
15.5.2.143 tpoints_cutoff braid_Int tpoints_cutoff
refinements halt after the number of time steps exceed this value
15.5.2.144 tstart braid_Real tstart
start time
15.5.2.145 tstop braid_Real tstop
stop time
15.5.2.146 userVectorTape _braid_Tape* userVectorTape
tape storing primal braid_vectors while recording
15.5.2.147 useshell braid_Int useshell
activate the shell structure of vectors
15.5.2.148 verbose_adj braid_Int verbose_adj
verbosity of the adjoint tape, displays the actions that are pushed / popped to the tape
15.5.2.149 warm_restart braid_Int warm_restart
boolean, indicates whether this is a warm restart of an existing braid_Core
15.5.2.150 wrapper_test braid_Int wrapper_test
boolean describing whether this call is only a wrapper test
The documentation for this struct was generated from the following file:

- _braid.h


## 15.6 _braid_Grid Struct Reference

```
#include <_braid.h>
```


## Data Fields

- braid_Int level
- braid_Int ilower
- braid_Int iupper
- braid_Int clower
- braid_Int cupper
- braid_Int gupper
- braid_Int cfactor
- braid_Int ncpoints
- braid_Int nupoints
- braid_BaseVector * ua
- braid_Real * ta
- braid_BaseVector * va
- braid_BaseVector $*$ fa
- braid_Int recv_index
- braid_Int send_index
- _braid_CommHandle $*$ recv_handle
- _braid_CommHandle $*$ send_handle
- braid_BaseVector * ua_alloc
- braid_Real * ta_alloc
- braid_BaseVector * va_alloc
- braid_BaseVector $*$ fa_alloc
- braid_BaseVector ulast


### 15.6.1 Detailed Description

XBraid Grid structure for a certain time level

Holds all the information for a processor related to the temporal grid at this level.

### 15.6.2 Field Documentation

15.6.2.1 cfactor braid_Int cfactor
coarsening factor
15.6.2.2 clower braid_Int clower
smallest C point index
15.6.2.3 cupper braid_Int cupper
largest $C$ point index
15.6.2.4 fa braid_BaseVector* fa rhs vectors f (all points, NULL on level 0 )
15.6.2.5 fa_alloc braid_BaseVector* fa_alloc
original memory allocation for fa
15.6.2.6 gupper braid_Int gupper
global size of the grid
15.6.2.7 ilower braid_Int ilower
smallest time index at this level
15.6.2.8 iupper braid_Int iupper
largest time index at this level
15.6.2.9 level braid_Int level

Level that grid is on
15.6.2.10 ncpoints braid_Int ncpoints
number of $C$ points

### 15.6.2.11 nupoints braid_Int nupoints

number of unknown vector points

### 15.6.2.12 recv_handle _braid_CommHandle* recv_handle

Handle for nonblocking receives of braid_BaseVectors

### 15.6.2.13 recv_index braid_Int recv_index

-1 means no receive

### 15.6.2.14 send_handle _braid_CommHandle* send_handle

Handle for nonblocking sends of braid_BaseVectors
15.6.2.15 send_index braid_Int send_index
-1 means no send
15.6.2.16 ta braid_Real* ta
time values (all points)
15.6.2.17 ta_alloc braid_Real* ta_alloc original memory allocation for ta
15.6.2.18 ua braid_BaseVector* ua
unknown vectors (C-points at least)
15.6.2.19 ua_alloc braid_BaseVector* ua_alloc
original memory allocation for ua
15.6.2.20 ulast braid_BaseVector ulast
stores vector at last time step, only set in FAccess and FCRelax if done is True
15.6.2.21 va braid_BaseVector* va
restricted unknown vectors (all points, NULL on level 0)
15.6.2.22 va_alloc braid_BaseVector* va_alloc
original memory allocation for va
The documentation for this struct was generated from the following file:

- _braid.h


## 15.7 _braid_Status Struct Reference

\#include <status.h>

## Data Fields

- _braid_Core core


### 15.7.1 Detailed Description

This is the main Status structure, that contains the properties of all the status. The user does not have access to this structure, but only to the derived Status structures. This class is accessed only inside XBraid code.

### 15.7.2 Field Documentation

### 15.7.2.1 core _braid_Core core

The documentation for this struct was generated from the following file:

- status.h


## 15.8 _braid_Tape Struct Reference

```
#include <tape.h>
```


## Data Fields

- int size
- void $*$ data_ptr
- struct _braid_tape_struct * next


### 15.8.1 Detailed Description

C-Implementation of a linked list storing pointers to generic data This structure represents one tape element, holding a pointer to data and a pointer to the next element int size holds the number of all elements in the tape

### 15.8.2 Field Documentation

15.8.2.1 data_ptr void* data_ptr
15.8.2.2 next struct _braid_tape_struct* next
15.8.2.3 size int size

The documentation for this struct was generated from the following file:

- tape.h


## 15.9 _braid_VectorBar Struct Reference

\#include <_braid.h>

## Data Fields

- braid_Vector userVector
- braid_Int useCount


### 15.9.1 Detailed Description

Braid Vector Structures:

There are four vector structures braid_BaseVector Defined below braid_Vector Defined in braid.h _braid_VectorBar Defined below braid_Basis Defined below

The braid_BaseVector is the main internal Vector class, which is stored at each time point. It wraps the Vector, braid $\hookleftarrow$ _VectorBar, and braid_Basis objects. The braid_VectorBar is only used if the adjoint capability is used, when it stores adjoint variables. It's basically a smart pointer wrapper around a braid_Vector. Note that it is always the braid_Vector that's passed to user-routines. The braid_Basis wraps (braid_Vector*) and can be considered a two-dimensional array. It is only used for Delta correction and when estimating the Lyapunov vectors. Shared pointer implementation for storing the intermediate AD-bar variables while taping. This is essentially the same as a userVector, except we need shared pointer capabilities to know when to delete.

### 15.9.2 Field Documentation

15.9.2.1 useCount braid_Int useCount
counts the number of pointers to this struct
15.9.2.2 userVector braid_Vector userVector
holds the u_bar data

The documentation for this struct was generated from the following file:

- _braid.h


### 15.10 braid AccessStatus Struct Reference

\#include <status.h>

## Data Fields

- _braid_Status status


### 15.10.1 Detailed Description

AccessStatus structure which defines the status of XBraid at a given instant on some level during a run. The user accesses it through braid_AccessStatusGet**() functions. This is just a pointer to the braid_Status.

### 15.10.2 Field Documentation

### 15.10.2.1 status _braid_Status status

The documentation for this struct was generated from the following file:

- status.h


### 15.11 braid_BufferStatus Struct Reference

\#include <status.h>

Data Fields

- _braid_Status status


### 15.11.1 Detailed Description

The user's bufpack, bufunpack and bufsize routines will receive a BufferStatus structure, which defines the status of XBraid at a given buff (un)pack instance. The user accesses it through braid_BufferStatusGet**() functions. This is just a pointer to the braid_Status.

### 15.11.2 Field Documentation

### 15.11.2.1 status _braid_Status status

The documentation for this struct was generated from the following file:

- status.h


### 15.12 braid_CoarsenRefStatus Struct Reference

\#include <status.h>

## Data Fields

- _braid_Status status


### 15.12.1 Detailed Description

The user coarsen and refine routines will receive a CoarsenRefStatus structure, which defines the status of XBraid at a given instant of coarsening or refinement on some level during a run. The user accesses it through braid_Coarsen $\hookleftarrow$ RefStatusGet**() functions. This is just a pointer to the braid_Status.

### 15.12.2 Field Documentation

15.12.2.1 status

The documentation for this struct was generated from the following file:

- status.h


### 15.13 braid_ObjectiveStatus Struct Reference

```
#include <status.h>
```


## Data Fields

- _braid_Status status


### 15.13.1 Detailed Description

The user's objectiveT and PostprocessObjective will receive an ObjectiveStatus structure, which defines the status of XBraid at a given instance of evaluating the objective function. The user accesses it through braid_ObjectiveStatus $\hookleftarrow$ Get** () functions. This is just a pointer to the braid_Status.

### 15.13.2 Field Documentation

### 15.13.2.1 status _braid_Status status

The documentation for this struct was generated from the following file:

- status.h


### 15.14 braid_Optim Struct Reference

\#include <_braid.h>

## Data Fields

- braid_Real sum_user_obj
- braid_Real objective
- braid_Real tstart_obj
- braid_Real tstop_obj
- braid_Real f_bar
- braid_Real rnorm_adj
- braid_Real rnorm0_adj
- braid_Real rnorm
- braid_Real rnorm0
- braid_Real tol_adj
- braid_Int rtol_adj
- braid_Vector * adjoints
- braid_VectorBar * tapeinput
- void $*$ sendbuffer
- MPI_Request * request


### 15.14.1 Detailed Description

Data structure for storing the optimization variables

### 15.14.2 Field Documentation

15.14.2.1 adjoints braid_Vector* adjoints vector for the adjoint variables

### 15.14.2.2 f_bar braid_Real f_bar

contains the seed for tape evaluation
15.14.2.3 objective braid_Real objective
global objective function value
15.14.2.4 request MPI_Request* request
helper: Storing the MPI request of BufUnPackDiff
15.14.2.5 rnorm braid_Real rnorm
norm of the state residual
15.14.2.6 rnorm0 braid_Real rnorm0
initial norm of the state residual
15.14.2.7 rnorm0_adj braid_Real rnorm0_adj
initial norm of the adjoint residual
15.14.2.8 rnorm_adj braid_Real rnorm_adj
norm of the adjoint residual
15.14.2.9 rtol_adj braid_Int rtol_adj
flag: use relative tolerance for adjoint
15.14.2.10 sendbuffer void* sendbuffer
helper: Memory for BufUnPackDiff communication
15.14.2.11 sum_user_obj braid_Real sum_user_obj sum of user's objective function values over time

### 15.14.2.12 tapeinput braid_VectorBar* tapeinput

helper: store pointer to input of one braid iteration
15.14.2.13 tol_adj braid_Real tol_adj
tolerance of adjoint residual
15.14.2.14 tstart_obj braid_Real tstart_obj
starting time for evaluating the user's local objective
15.14.2.15 tstop_obj braid_Real tstop_obj
stopping time for evaluating the user's local objective

The documentation for this struct was generated from the following file:

- _braid.h


### 15.15 braid_StepStatus Struct Reference

\#include <status.h>

Data Fields

- _braid_Status status


### 15.15.1 Detailed Description

The user's step routine routine will receive a StepStatus structure, which defines the status of XBraid at the given instant for step evaluation on some level during a run. The user accesses it through braid_StepStatusGet**() functions. This is just a pointer to the braid_Status.

### 15.15.2 Field Documentation

### 15.15.2.1 status _braid_Status status

The documentation for this struct was generated from the following file:

- status.h


### 15.16 braid_SyncStatus Struct Reference

\#include <status.h>

## Data Fields

- _braid_Status status


### 15.16.1 Detailed Description

SyncStatus structure which provides the status of XBraid at a given instant on some level during a run. This is vector independent and called once per processor. The user accesses it through braid_SyncStatusGet**() functions. This is just a pointer to the braid_Status.

### 15.16.2 Field Documentation

### 15.16.2.1 status

The documentation for this struct was generated from the following file:

- status.h


## 16 File Documentation

## 16.1 _braid.h File Reference

## Data Structures

- struct _braid_VectorBar
- struct _braid_Basis
- struct _braid_BaseVector
- struct braid_Optim
- struct _braid_CommHandle
- struct _braid_Grid
- struct _braid_Core


## Macros

- \#define _braid_Error(IERR, msg) _braid_ErrorHandler(__FILE__, _LINE__, IERR, msg)
- \#define _braid_ErrorlnArg(IARG, msg) _braid_Error(HYPRE_ERROR_ARG|IARG<<3, msg)
- \#define _braid_TAlloc(type, count) ( (type *)malloc((size_t)(sizeof(type) * (count))) )
- \#define _braid_CTAlloc(type, count) ( (type *)calloc((size_t)(count), (size_t)sizeof(type)) )
- \#define _braid_TReAlloc(ptr, type, count) ( (type *)realloc((braid_Byte *)ptr, (size_t)(sizeof(type) * (count))) )
- \#define _braid_TFree(ptr) ( free((braid_Byte *)ptr), ptr = NULL )
- \#define _braid_max(a, b) $(((\mathrm{a})<(\mathrm{b}))$ ? (b) : (a))
- \#define _braid_min(a, b) (((a)<(b)) ? (a) : (b))
- \#define _braid_isnan(a) (a != a)
- \#define _braid_SendlndexNull -2
- \#define _braid_RecvIndexNull -2
- \#define _braid_MapPeriodic(index, npoints) (index = ((index)+(npoints)) \% (npoints) ) /* this also handles negative indexes $* /$
- \#define _braid_CommHandleElt(handle, elt) ((handle) -> elt)
- \#define _braid_GridEIt(grid, elt) ((grid) -> elt)
- \#define _braid_CoreElt(core, elt) ( (core) -> elt )
- \#define _braid_CoreFcn(core, fcn) ( $*((\operatorname{core})->\mathrm{fcn}))$
- \#define _braid_MapFineToCoarse(findex, cfactor, cindex) ( cindex = (findex)/(cfactor) )
- \#define _braid_MapCoarseToFine(cindex, cfactor, findex) ( findex = (cindex)*(cfactor) )
- \#define _braid_IsFPoint(index, cfactor) ( (index)\%(cfactor) )
- \#define _braid_IsCPoint(index, cfactor) (!_braid_IsFPoint(index, cfactor) )
- \#define _braid_NextCPoint(index, cfactor) ( ((braid_Int)((index)+(cfactor)-1)/(cfactor))*(cfactor) )
- \#define _braid_PriorCPoint(index, cfactor) ( ((braid_Int)(index)/(cfactor))*(cfactor) )


## Typedefs

- typedef _braid_VectorBar * braid_VectorBar
- typedef _braid_Basis * braid_Basis
- typedef _braid_BaseVector * braid_BaseVector


## Functions

- void _braid_ErrorHandler (const char *filename, braid_Int line, braid_Int ierr, const char *msg)
- braid_Int _braid_GetBlockDistInterval (braid_Int npoints, braid_Int nprocs, braid_Int proc, braid_Int *ilower_ptr, braid_Int *iupper_ptr)
- braid_Int _braid_GetBlockDistProc (braid_Int npoints, braid_Int nprocs, braid_Int index, braid_Int periodic, braid_Int *proc_ptr)
- braid_Int _braid_GetDistribution (braid_Core core, braid_Int *ilower_ptr, braid_Int *iupper_ptr)
- braid_Int _braid_GetProc (braid_Core core, braid_Int level, braid_Int index, braid_Int *proc_ptr)
- braid_Int _braid_CommRecvInit (braid_Core core, braid_Int level, braid_Int index, braid_BaseVector *vector_ptr, _braid_CommHandle $* *$ handle_ptr)
- braid_Int _braid_CommSendlnit (braid_Core core, braid_Int level, braid_Int index, braid_BaseVector vector, _braid_CommHandle $* *$ handle_ptr)
- braid_Int _braid_CommWait (braid_Core core, _braid_CommHandle $* *$ handle_ptr)
- braid_Int _braid_UGetIndex (braid_Core core, braid_Int level, braid_Int index, braid_Int *uindex_ptr, braid_Int *store_flag_ptr)
- braid_Int _braid_UGetVectorRef (braid_Core core, braid_Int level, braid_Int index, braid_BaseVector *u_ptr)
- braid_Int _braid_USetVectorRef (braid_Core core, braid_Int level, braid_Int index, braid_BaseVector u)
- braid_Int _braid_UGetVector (braid_Core core, braid_Int level, braid_Int index, braid_BaseVector *u_ptr)
- braid_Int _braid_USetVector (braid_Core core, braid_Int level, braid_Int index, braid_BaseVector u, braid_Int move)
- braid_Int _braid_UCommInitBasic (braid_Core core, braid_Int level, braid_Int recv_msg, braid_Int send_msg, braid_Int send_now)
- braid_Int _braid_UCommInit (braid_Core core, braid_Int level)
- braid_Int _braid_UCommInitF (braid_Core core, braid_Int level)
- braid_Int _braid_UCommWait (braid_Core core, braid_Int level)
- braid_Int _braid_UGetLast (braid_Core core, braid_BaseVector *u_ptr)
- braid_Int_braid_Step (braid_Core core, braid_Int level, braid_Int index, braid_Int calling_function, braid_BaseVector ustop, braid_BaseVector u)
- braid_Int _braid_GetUInit (braid_Core core, braid_Int level, braid_Int index, braid_BaseVector u, braid_BaseVector *ustop_ptr)
- braid_Int _braid_Residual (braid_Core core, braid_Int level, braid_Int index, braid_Int calling_function, braid_BaseVector ustop, braid_BaseVector r)
- braid_Int _braid_FASResidual (braid_Core core, braid_Int level, braid_Int index, braid_BaseVector ustop, braid_BaseVector r)
- braid_Int_braid_Coarsen (braid_Core core, braid_Int level, braid_Int f_index, braid_Int c_index, braid_BaseVector fvector, braid_BaseVector *cvector)
- braid_Int _braid_RefineBasic (braid_Core core, braid_Int level, braid_Int c_index, braid_Real *f_ta, braid_Real *c_ta, braid_BaseVector cvector, braid_BaseVector *fvector)
- braid_Int _braid_Refine (braid_Core core, braid_Int level, braid_Int f_index, braid_Int c_index, braid_BaseVector cvector, braid_BaseVector *fvector)
- braid_Int _braid_FRefineSpace (braid_Core core, braid_Int *refined_ptr)
- braid_Int _braid_GridInit (braid_Core core, braid_Int level, braid_Int ilower, braid_Int iupper, _braid_Grid $* *$ grid $\hookleftarrow$ _ptr)
- braid_Int _braid_GridClean (braid_Core core, _braid_Grid *grid)
- braid_Int _braid_GridDestroy (braid_Core core, _braid_Grid *grid)
- braid_Int _braid_SetRNorm (braid_Core core, braid_Int iter, braid_Real rnorm)
- braid_Int _braid_GetRNorm (braid_Core core, braid_Int iter, braid_Real *rnorm_ptr)
- braid_Int _braid_SetFullRNorm (braid_Core core, braid_Int iter, braid_Real rnorm)
- braid_Int _braid_GetFullRNorm (braid_Core core, braid_Int iter, braid_Real *rnorm_ptr)
- braid_Int _braid_ComputeFullRNorm (braid_Core core, braid_Int level, braid_Real *return_rnorm)
- braid_Int _braid_PrintSpatialNorms (braid_Core core, braid_Real *rnorms, braid_Int n)
- braid_Int _braid_FCRelax (braid_Core core, braid_Int level)
- braid_Int _braid_FRestrict (braid_Core core, braid_Int level)
- braid_Int _braid_FInterp (braid_Core core, braid_Int level)
- braid_Int _braid_FRefine (braid_Core core, braid_Int *refined_ptr)
- braid_Int _braid_FAccess (braid_Core core, braid_Int level, braid_Int done)
- braid_Int _braid_AccessVector (braid_Core core, braid_AccessStatus status, braid_BaseVector u)
- braid_Int _braid_Sync (braid_Core core, braid_SyncStatus status)
- braid_Int _braid_InitHierarchy (braid_Core core, _braid_Grid *fine_grid, braid_Int refined)
- braid_Int _braid_FinalizeErrorEstimates (braid_Core core, braid_Real *estimate, braid_Int length)
- braid_Int _braid_GetDtk (braid_Core core)
- braid_Int _braid_GetCFactor (braid_Core core, braid_Int level, braid_Int *cfactor_ptr)
- braid_Int _braid_InitGuess (braid_Core core, braid_Int level)
- braid_Int _braid_CopyFineToCoarse (braid_Core core)
- braid_Int _braid_Drive (braid_Core core, braid_Real localtime)


## Variables

- braid_Int _braid_error_flag
- FILE * _braid_printfile


### 16.1.1 Detailed Description

Define headers for XBraid internal (developer) routines and XBraid internal structure declarations.

This file contains the headers for XBraid internal (developer) routines and structure declarations.

### 16.1.2 Macro Definition Documentation

16.1.2.1 _braid_CommHandleElt \#define _braid_CommHandleElt(
handle,
elt ) ((handle) -> elt)
Accessor for _braid_CommHandle attributes
16.1.2.2 _braid_CoreElt \#define _braid_CoreElt (
core,
elt ) ( (core) -> elt )
Accessor for _braid_Core attributes
16.1.2.3 _braid_CoreFcn \#define _braid_CoreFcn(
core,
fcn ) (* ((core) -> fon))
Accessor for _braid_Core functions
16.1.2.4 _braid_CTAlloc \#define _braid_CTAlloc(
type, count ) ( (type *) calloc((size_t) (count), (size_t)sizeof(type)) )

Allocation macro
16.1.2.5 _braid_Error \#define _braid_Error(

IERR,
msg ) _braid_ErrorHandler(__FILE__, __LINE__, IERR, msg)
16.1.2.6 _braid_ErrorInArg \#define _braid_ErrorInArg (

IARG,
msg ) _braid_Error (HYPRE_ERROR_ARG | IARG<<3, msg)
16.1.2.7 _braid_GridElt \#define _braid_GridElt(
grid, elt ) ((grid) -> elt)

Accessor for _braid_Grid attributes
16.1.2.8 _braid_IsCPoint \#define _braid_IsCPoint (
index,
cfactor ) ( !_braid_IsFPoint(index, cfactor) )
Boolean, returns whether a time index is an C-point
16.1.2.9 _braid_IsFPoint \#define _braid_IsFPoint (
index,
cfactor ) ( (index) \%(cfactor) )
Boolean, returns whether a time index is an F-point
16.1.2.10 _braid_isnan \#define _braid_isnan(
a ) (a ! = a)
16.1.2.11 _braid_MapCoarseToFine \#define _braid_MapCoarseToFine(
cindex,
cfactor,
findex ) ( findex $=($ (cindex) $*(c f a c t o r)$ )
Map a coarse time index to a fine time index, assumes a uniform coarsening factor.
16.1.2.12 _braid_MapFineToCoarse \#define _braid_MapFineToCoarse(
findex,
cfactor,
cindex ) ( cindex = (findex)/(cfactor) )
Map a fine time index to a coarse time index, assumes a uniform coarsening factor.
16.1.2.13 _braid_MapPeriodic \#define _braid_MapPeriodic(
index,
npoints ) ( index $=((i n d e x)+(n p o i n t s))$ \% (npoints) ) /* this also handles negative
indexes */
16.1.2.14 _braid_max \#define _braid_max (
a,
$b) \quad(((a)<(b))$ ?
(b) :
(a))
16.1.2.15 _braid_min \#define _braid_min(
a,
$b$ ) $(((a)<(b))$ ?
(a) :
(b) )
16.1.2.16 _braid_NextCPoint \#define _braid_NextCPoint(
index,
cfactor ) ( ((braid_Int) ((index) +(cfactor)-1)/(cfactor))*(cfactor) )
Returns the index for the next C-point to the right of index (inclusive)
16.1.2.17 _braid_PriorCPoint \#define _braid_PriorCPoint(
index,
cfactor ) ( ((braid_Int)(index)/(cfactor))*(cfactor) )
Returns the index for the previous C-point to the left of index (inclusive)
16.1.2.18 _braid_RecvIndexNull \#define _braid_RecvIndexNull -2
16.1.2.19 _braid_SendIndexNull \#define _braid_SendIndexNull -2
16.1.2.20 _braid_TAlloc \#define _braid_TAlloc (
type,
count ) ( (type *) malloc((size_t) (sizeof(type) * (count))) )
Allocation macro
16.1.2.21 _braid_TFree \#define _braid_TFree(
ptr ) ( free((braid_Byte *)ptr), ptr = NULL )

Free memory macro
16.1.2.22 _braid_TReAlloc \#define _braid_TReAlloc(
ptr,
type,
count ) ( (type *) realloc ((braid_Byte *)ptr, (size_t) (sizeof(type) * (count))) )
Re-allocation macro

### 16.1.3 Typedef Documentation

16.1.3.1 braid_BaseVector typedef _braid_BaseVector* braid_BaseVector
16.1.3.2 braid_Basis typedef _braid_Basis* braid_Basis
16.1.3.3 braid_VectorBar typedef _braid_VectorBar* braid_VectorBar

### 16.1.4 Function Documentation

16.1.4.1 _braid_AccessVector() braid_Int _braid_AccessVector (
braid_Core core,
braid_AccessStatus status,
braid_BaseVector u )
Call user's access function in order to give access to XBraid and the current vector. Most commonly, this lets the user write $u$ to screen, disk, etc... The vector $u$ corresponds to time step index on level. status holds state information about the current XBraid iteration, time value, etc...
16.1.4.2 _braid_Coarsen() braid_Int _braid_Coarsen (
braid_Core core,
braid_Int level,
braid_Int f_index,
braid_Int c_index,
braid_BaseVector fvector,
braid_BaseVector * cvector )

Coarsen in space on level by calling the user's coarsen function. The vector corresponding to the time step index f_index on the fine grid is coarsened to the time step index c_index on the coarse grid. The output goes in cvector and the input vector is fvector.

### 16.1.4.3 _braid_CommRecvInit()

braid_Int _braid_CommRecvInit (
braid_Core core,
braid_Int level,
braid_Int index,
braid_BaseVector * vector_ptr,
_braid_CommHandle ** handle_ptr )
Initialize a receive to go into vector_ptr for the given time index on level. Also return a comm handle handle_ptr for querying later, to see if the receive has occurred.

### 16.1.4.4 _braid_CommSendInit() braid_Int _braid_CommSendInit (

braid_Core core,
braid_Int level,
braid_Int index,
braid_BaseVector vector,
_braid_CommHandle ** handle_ptr )
Initialize a send of vector for the given time index on level.
Also return a comm handle handle_ptr for querying later, to see if the send has occurred.
16.1.4.5 _braid_CommWait() braid_Int _braid_CommWait (
braid_Core core,
_braid_CommHandle ** handle_ptr )
Block on the comm handle handle_ptr until the MPI operation (send or recv) has completed

### 16.1.4.6 _braid_ComputeFullRNorm() braid_Int _braid_ComputeFullRNorm (

braid_Core core,
braid_Int level,
braid_Real * return_rnorm )
Compute full temporal residual norm with user-provided residual routine. Output goes in *return_rnorm.

```
16.1.4.7 _braid_CopyFineToCoarse() braid_Int _braid_CopyFineToCoarse (
braid_Core core )
```

Copy the initialized C-points on the fine grid, to all coarse levels. For instance, if a point $k$ on level $m$ corresponds to point $p$ on level 0 , then they are equivalent after this function. The only exception is any spatial coarsening the user decides to do. This function allows XBraid to skip all work on the first down cycle and start in FMG style on the coarsest level. Assumes level 0 C-points are initialized.
16.1.4.8 _braid_Drive() braid_Int _braid_Drive (
braid_Core core, braid_Real localtime )

Main loop for MGRIT
16.1.4.9 _braid_ErrorHandler() void _braid_ErrorHandler (

```
const char * filename,
```

    braid_Int line,
    braid_Int ierr,
    const char * msg )
    16.1.4.10 _braid_FAccess() braid_Int _braid_FAccess (

$$
\begin{aligned}
& \text { braid_Core core, } \\
& \text { braid_Int level, } \\
& \text { braid_Int done ) }
\end{aligned}
$$

Call the user's access function in order to give access to XBraid and the current vector at grid level and iteration *iter. Most commonly, this lets the user write solutions to screen, disk, etc... The quantity rnorm denotes the last computed residual norm, and done is a boolean indicating whether XBraid has finished iterating and this is the last Access call.

### 16.1.4.11 _braid_FASResidual() braid_Int _braid_FASResidual (

braid_Core core,
braid_Int level,
braid_Int index,
braid_BaseVector ustop,
braid_BaseVector r )
Compute FAS residual $=f-$ residual
16.1.4.12 _braid_FCRelax() braid_Int _braid_FCRelax (
braid_Core core,
braid_Int level )
Do nu sweeps of F-then-C relaxation on level
16.1.4.13 _braid_FinalizeErrorEstimates() braid_Int _braid_FinalizeErrorEstimates (
braid_Core core,
braid_Real * estimate,
braid_Int length )
Finalize Richardson error estimates
16.1.4.14 _braid_FInterp() braid_Int _braid_FInterp (
braid_Core core,
braid_Int level )
F-Relax on level and interpolate to level-1
The output is set in the braid_Grid in core, so that the vector $u$ on level is created by interpolating from level+1.
If the user has set spatial refinement, then this user-defined routine is also called.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| level | interp from level to level +1 |

### 16.1.4.15 _braid_FRefine() braid_Int _braid_FRefine (

braid_Core core,
braid_Int * refined_ptr )
Create a new fine grid (level 0) and corresponding grid hierarchy by refining the current fine grid based on user-provided refinement factors. Return the boolean refined_ptr to indicate whether grid refinement was actually done. To simplify the algorithm, refinement factors are automatically capped to be no greater than the coarsening factor (for level 0 ). The grid data is also redistributed to achieve good load balance in the temporal dimension. If the refinement factor is 1 in each time interval, no refinement is done.
16.1.4.16 _braid_FRefineSpace() braid_Int _braid_FRefineSpace (
braid_core core,
braid_Int * refined_ptr )

Call spatial refinement on all local time steps if $r$ _space has been set on the local processor. Returns refined_ptr $==2$ if refinment was completed at any point globally, otherwise returns 0 . This is a helper function for _braid_FRefine().
16.1.4.17 _braid_FRestrict() braid_Int _braid_FRestrict (
braid_Core core,
braid_Int level )
F-Relax on level and then restrict to level+1
The output is set in the braid_Grid in core, so that the restricted vectors va and fa will be created, representing level+1 versions of the unknown and rhs vectors.

If the user has set spatial coarsening, then this user-defined routine is also called.
If level==0, then rnorm_ptr will contain the residual norm.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| level | restrict from level to level+1 |

16.1.4.18 _braid_GetBlockDistInterval() braid_Int _braid_GetBlockDistInterval (

```
braid_Int npoints,
```

```
braid_Int nprocs,
braid_Int proc,
braid_Int * ilower_ptr,
braid_Int * iupper_ptr )
```

Returns the index interval for proc in a blocked data distribution.
16.1.4.19 _braid_GetBlockDistProc() braid_Int _braid_GetBlockDistProc (

```
braid_Int npoints,
    braid_Int nprocs,
    braid_Int index,
    braid_Int periodic,
    braid_Int * proc_ptr )
```

Returns the processor that owns index in a blocked data distribution (returns -1 if index is out of range).

### 16.1.4.20 _braid_GetCFactor() braid_Int _braid_GetCFactor (

```
braid_Core core,
braid_Int level,
braid_Int * cfactor_ptr )
```

Returns the coarsening factor to use on grid level.
16.1.4.21 _braid_GetDistribution() braid_Int _braid_GetDistribution (

```
    braid_Core core,
    braid_Int * ilower_ptr,
    braid_Int * iupper_ptr )
```

Returns the index interval for my processor on the finest grid level. For the processor rank calling this function, it returns the smallest and largest time indices (ilower_ptr and iupper_ptr) that belong to that processor (the indices may be F or C points).
16.1.4.22 _braid_GetDtk() braid_Int _braid_GetDtk (
braid_Core core )

Propagate time step information required to compute the Richardson error estimate at each C-point. This can be done at any time, but does require some communication. This fills in error_factors at the C points.
16.1.4.23 _braid_GetFullRNorm() braid_Int _braid_GetFullRNorm (

```
    braid_Core core,
    braid_Int iter,
    braid_Real * rnorm_ptr )
```

Same as GetRNorm, but gets full residual norm.
16.1.4.24 _braid_GetProc() braid_Int _braid_GetProc (
braid_Core core,
braid_Int level,
braid_Int index,
braid_Int * proc_ptr )

Returns the processor number in proc_ptr on which the time step index lives for the given level. Returns - 1 if index is out of range.
16.1.4.25 _braid_GetRNorm() braid_Int _braid_GetRNorm (
braid_Core core,
braid_Int iter, braid_Real * rnorm_ptr )

Get the residual norm for iteration iter. If iter $<0$, get the rnorm for the last iteration minus $\mid$ iter $\mid-1$.
16.1.4.26 _braid_GetUInit() braid_Int _braid_GetUInit (
braid_Core core,
braid_Int level,
braid_Int index,
braid_BaseVector u,
braid_BaseVector * ustop_ptr )
Return an initial guess in ustop_ptr to use in the step routine for implicit schemes. The value returned depends on the storage options used. If the return value is NULL, no initial guess is available.
16.1.4.27 _braid_GridClean() braid_Int _braid_GridClean (
braid_Core core,
_braid_Grid * grid)

Destroy the vectors on grid
16.1.4.28 _braid_GridDestroy() braid_Int _braid_GridDestroy (

$$
\begin{aligned}
& \text { braid_Core core, } \\
& \text { _braid_Grid } * \text { grid) }
\end{aligned}
$$

Destroy grid
16.1.4.29 _braid_GridInit() braid_Int _braid_GridInit (
braid_Core core,
braid_Int level,
braid_Int ilower,
braid_Int iupper,
_braid_Grid ** grid_ptr )
Create a new grid object grid_ptr with level indicator level. Arguments ilower and iupper correspond to the lower and upper time index values for this processor on this grid.

```
16.1.4.30 _braid_InitGuess() braid_Int _braid_InitGuess (
    braid_Core core,
    braid_Int level )
```

Set initial guess on level. Only C-pts are initialized on level 0 , otherwise stored values are initialized based on restricted fine-grid values.
16.1.4.31 _braid_InitHierarchy() braid_Int _braid_InitHierarchy (

```
braid_Core core,
_braid_Grid * fine_grid,
braid_Int refined )
```

Initialize grid hierarchy with fine_grid serving as the finest grid. Boolean refined indicates whether fine_grid was created by refining a coarser grid (in the FRefine() routine), which has implications on how to define CF-intervals.

### 16.1.4.32 _braid_PrintSpatialNorms() braid_Int _braid_PrintSpatialNorms (

```
braid_Core core,
    braid_Real * rnorms,
    braid_Int n )
```

Print out the residual norm for every C-point. Processor 0 gathers all the rnorms and prints them in order through a gatherv operator

```
16.1.4.33 _braid_Refine() braid_Int _braid_Refine (
    braid_Core core,
    braid_Int level,
    braid_Int f_index,
    braid_Int c_index,
    braid_BaseVector cvector,
    braid_BaseVector * fvector )
```

Refine in space on level by calling the user's refine function. The vector corresponding to the time step index c_index on the coarse grid is refined to the time step index $f_{-}$index on the fine grid. The output goes in fvector and the input vector is cuector.
16.1.4.34 _braid_RefineBasic() braid_Int _braid_RefineBasic (

```
braid_Core core,
    braid_Int level,
    braid_Int c_index,
    braid_Real * f_ta,
    braid_Real * c_ta,
    braid_BaseVector cvector,
    braid_BaseVector * fvector )
```

Refine in space (basic routine)
16.1.4.35 _braid_Residual() braid_Int _braid_Residual ( braid_Core core, braid_Int level, braid_Int index, braid_Int calling_function, braid_BaseVector ustop, braid_BaseVector r )

Compute residual $r$
16.1.4.36 _braid_SetFullRNorm() braid_Int _braid_SetFullRNorm (
braid_Core core,
braid_Int iter,
braid_Real rnorm )
Same as SetRNorm, but sets full residual norm.
16.1.4.37 _braid_SetRNorm() braid_Int _braid_SetRNorm (
braid_Core core,
braid_Int iter,
braid_Real rnorm )
Set the residual norm for iteration iter. If iter $<0$, set the rnorm for the last iteration minus $\mid$ iter $\mid-1$. Also set the initial residual norm.

```
16.1.4.38 _braid_Step() braid_Int _braid_Step (
    braid_Core core,
    braid_Int level,
    braid_Int index,
    braid_Int calling_function,
    braid_BaseVector ustop,
    braid_BaseVector u )
```

Integrate one time step at time step index to time step index+1.
16.1.4.39 _braid_Sync() braid_Int _braid_Sync (
braid_Core core,
braid_SyncStatus status )
Call user's sync function in order to give access to XBraid and the user's app. This is called once-per-processor at various points in XBraid in order to allow the user to perform any book-keeping operations. status provides state information about the current XBraid status and processor.
16.1.4.40 _braid_UCommInit() braid_Int _braid_UCommInit (
braid_Core core,
braid_Int level )
This routine initiates communication under the assumption that work will be done on all intervals ( F or C ) on level. It posts a receive for the point to the left of ilower (regardless whether ilower is F or C ), and it posts a send of iupper if iupper is a C point.
16.1.4.41 _braid_UCommInitBasic() braid_Int _braid_UCommInitBasic (

```
braid_Core core,
braid_Int level,
braid_Int recv_msg,
braid_Int send_msg,
braid_Int send_now )
```

Basic communication (from the left, to the right). Arguments recv_msg and send_msg are booleans that indicate whether or not to initiate a receive from the left and a send to the right respectively. Argument send_now indicates that the send should be initiated immediately.

### 16.1.4.42 _braid_UCommInitF() braid_Int _braid_UCommInitF (

```
    braid_Core core,
```

    braid_Int level )
    This routine initiates communication under the assumption that work will be done on only F-pt intervals on level. It only posts a receive for the point to the left of ilower if ilower is an F point, and it posts a send of iupper if iupper is a C point.
16.1.4.43 _braid_UCommWait() braid_Int _braid_UCommWait (

$$
\begin{aligned}
& \text { braid_Core core, } \\
& \text { braid_Int level ) }
\end{aligned}
$$

Finish up communication. On level, wait on both the recv and send handles at this level.

```
16.1.4.44 _braid_UGetIndex() braid_Int _braid_UGetIndex (
    braid_Core core,
    braid_Int level,
    braid_Int index,
    braid_Int * uindex_ptr,
    braid_Int * store_flag_ptr )
```

Returns an index into the local u-vector for grid level at point index, and information on the storage status of the point. If nothing is stored at that point, uindex $=-1$ and store_flag $=-2$. If only the shell is stored store_flag $=-1$, and if the whole $u$-vector is stored, store_flag $=0$.
16.1.4.45 _braid_UGetLast() braid_Int _braid_UGetLast (

```
    braid_Core core,
    braid_BaseVector * u_ptr )
```

Retrieve uvector at last time-step

```
16.1.4.46 _braid_UGetVector() braid_Int _braid_UGetVector (
    braid_Core core,
    braid_Int level,
    braid_Int index,
    braid_BaseVector * u_ptr )
```

Returns a copy of the u-vector on grid level at point index. If index is my "receive index" (as set by UCommlnit(), for example), the $u$-vector will be received from a neighbor processor. If the $u$-vector is not stored, NULL is returned.
16.1.4.47 _braid_UGetVectorRef() braid_Int _braid_UGetVectorRef (

```
braid_Core core,
braid_Int level,
braid_Int index,
braid_BaseVector * u_ptr )
```

Returns a reference to the local $u$-vector on grid level at point index. If the $u$-vector is not stored, returns NULL.
16.1.4.48 _braid_USetVector() braid_Int _braid_USetVector (
braid_Core core,
braid_Int level,
braid_Int index,
braid_BaseVector $u$,
braid_Int move )

Stores the $u$-vector on grid level at point index. If index is my "send index", a send is initiated to a neighbor processor. If move is true, the u-vector is moved into core storage instead of copied. If the u-vector is not stored, nothing is done.
16.1.4.49 _braid_USetVectorRef() braid_Int _braid_USetVectorRef (

```
braid_Core core,
    braid_Int level,
    braid_Int index,
    braid_BaseVector u )
```

Stores a reference to the local u-vector on grid level at point index. If the u-vector is not stored, nothing is done.

### 16.1.5 Variable Documentation

### 16.1.5.1 _braid_error_flag braid_Int _braid_error_flag [extern]

This is the global XBraid error flag. If it is ever nonzero, an error has occurred.
16.1.5.2 _braid_printfile FILE* _braid_printfile [extern]

This is the print file for redirecting stdout for all XBraid screen output

## 16.2 adjoint.h File Reference

## Functions

- braid_Int _braid_VectorBarCopy (braid_VectorBar bar, braid_VectorBar *bar_ptr)
- braid_Int _braid_VectorBarDelete (braid_Core core, braid_VectorBar bar)
- braid_Int _braid_OptimDestroy (braid_Core core)
- braid_Int _braid_UpdateAdjoint (braid_Core core, braid_Real *rnorm_adj_ptr)
- braid_Int _braid_SetRNormAdjoint (braid_Core core, braid_Int iter, braid_Real rnorm_adj)
- braid_Int _braid_AddToObjective (braid_Core core, braid_BaseVector u, braid_ObjectiveStatus ostatus)
- braid_Int _braid_EvalObjective (braid_Core core)
- braid_Int _braid_EvalObjective_diff (braid_Core core)
- braid_Int _braid_InitAdjointVars (braid_Core core, _braid_Grid *fine_grid)
- braid_Int _braid_AdjointFeatureCheck (braid_Core core)


### 16.2.1 Detailed Description

Define internal XBraid headers for the adjoint feature.
This file contains the internal XBriad headers for the adjoint feature, e.g., the functions to wrap and call the users objective function, and allocate adjoint (bar) variables.

### 16.2.2 Function Documentation

16.2.2.1 _braid_AddToObjective() braid_Int _braid_AddToobjective (
braid_Core core,
braid_BaseVector u,
braid_ObjectiveStatus ostatus )

Evaluate the user's local objective function at time $t$ and add it to the time-averaged objective function
16.2.2.2 _braid_AdjointFeatureCheck() braid_Int _braid_AdjointFeatureCheck (
braid_Core core )

Sanity check for non-supported adjoint features
16.2.2.3 _braid_EvalObjective() braid_Int _braid_EvalObjective (
braid_Core core )

Evaluate the objective function: MPI_Allreduce the time average and postprocess the objective
16.2.2.4 _braid_EvalObjective_diff() braid_Int _braid_EvalObjective_diff (
braid_Core core )

Differentiated objective function
16.2.2.5 _braid_InitAdjointVars() braid_Int _braid_InitAdjointVars (
braid_Core core,
_braid_Grid * fine_grid )

Allocate and initialize the adjoint variables
16.2.2.6 _braid_OptimDestroy() braid_Int _braid_OptimDestroy (
braid_Core core )

Free memory of the optimization structure
16.2.2.7 _braid_SetRNormAdjoint() braid_Int _braid_SetRNormAdjoint (

```
braid_Core core,
braid_Int iter,
braid_Real rnorm_adj )
```

Set adjoint residual norm
16.2.2.8 _braid_UpdateAdjoint() braid_Int _braid_UpdateAdjoint (

```
braid_Core core,
    braid_Real * rnorm_adj_ptr )
```

Update the adjoint variables and compute adjoint residual norm Returns the tnorm of adjoint residual
16.2.2.9 _braid_VectorBarCopy() braid_Int _braid_VectorBarCopy (
braid_VectorBar bar,
braid_VectorBar * bar_ptr )
Shallow copy a braid_VectorBar shared pointer, bar_ptr is set to bar and the useCount is incremented by one.
16.2.2.10 _braid_VectorBarDelete() braid_Int _braid_VectorBarDelete (
braid_Core core,
braid_VectorBar bar )
Reduce the useCount of a braid_VectorBar shared pointer Free the pointer memory if useCount is zero.

## 16.3 base.h File Reference

## Functions

- braid_Int _braid_BaseStep (braid_Core core, braid_App app, braid_BaseVector ustop, braid_BaseVector fstop, braid_BaseVector u, braid_Int level, braid_StepStatus status)
- braid_Int _braid_Baselnit (braid_Core core, braid_App app, braid_Real t, braid_BaseVector *u_ptr)
- braid_Int _braid_BaseInitBasis (braid_Core core, braid_App app, braid_Real t, braid_Basis *psi_ptr)
- braid_Int _braid_BaseClone (braid_Core core, braid_App app, braid_BaseVector u, braid_BaseVector *v_ptr)
- braid_Int _braid_BaseCloneBasis (braid_Core core, braid_App app, braid_Basis A, braid_Basis *B_ptr)
- braid_Int _braid_BaseFree (braid_Core core, braid_App app, braid_BaseVector u)
- braid_Int _braid_BaseFreeBasis (braid_Core core, braid_App app, braid_Basis b)
- braid_Int _braid_BaseSum (braid_Core core, braid_App app, braid_Real alpha, braid_BaseVector x, braid_Real beta, braid_BaseVector y)
- braid_Int _braid_BaseSumBasis (braid_Core core, braid_App app, braid_Real alpha, braid_Basis A, braid_Real beta, braid_Basis B)
- braid_Int _braid_BaseSpatialNorm (braid_Core core, braid_App app, braid_BaseVector u, braid_Real *norm_ptr)
- braid_Int _braid_BaselnnerProd (braid_Core core, braid_App app, braid_Vector u, braid_Vector v, braid_Real *prod_ptr)
- braid_Int _braid_BaseAccess (braid_Core core, braid_App app, braid_BaseVector u, braid_AccessStatus status)
- braid_Int _braid_BaseSync (braid_Core core, braid_App app, braid_SyncStatus status)
- braid_Int _braid_BaseBufSize (braid_Core core, braid_App app, braid_Int *size_ptr, braid_BufferStatus status)
- braid_Int _braid_BaseBufPack (braid_Core core, braid_App app, braid_BaseVector u, void *buffer, braid_Buffer $\hookleftarrow$ Status status)
- braid_Int _braid_BaseBufUnpack (braid_Core core, braid_App app, void *buffer, braid_BaseVector *u_ptr, braid_BufferStatus status)
- braid_Int _braid_BaseBufAlloc (braid_Core core, braid_App app, void $* *$ buffer, braid_Int nbytes, braid_Buffer $\hookleftarrow$ Status status)
- braid_Int _braid_BaseBufFree (braid_Core core, braid_App app, void **buffer)
- braid_Int _braid_BaseObjectiveT (braid_Core core, braid_App app, braid_BaseVector u, braid_ObjectiveStatus ostatus, braid_Real *objT_ptr)
- braid_Int _braid_BaseResidual (braid_Core core, braid_App app, braid_BaseVector ustop, braid_BaseVector r, braid_StepStatus status)
- braid_Int _braid_BaseFullResidual (braid_Core core, braid_App app, braid_BaseVector r, braid_BaseVector u, braid_StepStatus status)
- braid_Int _braid_BaseSCoarsen (braid_Core core, braid_App app, braid_BaseVector fu, braid_BaseVector *cu $\hookleftarrow$ _ptr, braid_CoarsenRefStatus status)
- braid_Int_braid_BaseSRefine (braid_Core core, braid_App app, braid_BaseVector cu, braid_BaseVector *fu_ptr, braid_CoarsenRefStatus status)
- braid_Int _braid_BaseSInit (braid_Core core, braid_App app, braid_Real t, braid_BaseVector *u_ptr)
- braid_Int _braid_BaseSClone (braid_Core core, braid_App app, braid_BaseVector u, braid_BaseVector *v_ptr)
- braid_Int _braid_BaseSFree (braid_Core core, braid_App app, braid_BaseVector u)
- braid_Int _braid_BaseTimeGrid (braid_Core core, braid_App app, braid_Real $*$ ta, braid_Int *ilower, braid_Int *iupper)
- braid_Int _braid_BaseStep_diff (_braid_Action *action)
- braid_Int _braid_BaseClone_diff (_braid_Action *action)
- braid_Int_braid_BaseSum_diff (_braid_Action *action)
- braid_Int _braid_BaseObjectiveT_diff (_braid_Action *action)
- braid_Int _braid_BaseBufPack_diff (_braid_Action *action)
- braid_Int _braid_BaseBufUnpack_diff (_braid_Action *action)
- braid_Int _braid_Baselnit_diff (_braid_Action *action)


### 16.3.1 Detailed Description

Define XBraid internal headers for wrapper routines of user-defined functions.
The XBraid internal headers defined here wrap the user-defined routines. If this is a normal XBraid run (i.e., no adjoint), then the wrappers serve no function, and just call the user's routines. If this is an XBraid_Adjoint run, then these routines record themselves to the action tape and push state and bar vectors to the primal and the bar tape, respectively. These vectors are then later popped from the tape and passed to the user diff routines in order to compute the differentiated actions. This is a form of automatic differentiation to compute the adjoint cycle.

### 16.3.2 Function Documentation

16.3.2.1 _braid_BaseAccess() braid_Int _braid_BaseAccess (
braid_Core core, braid_App app, braid_BaseVector $u$, braid_AccessStatus status )

This calls the user's Access routine. If (adjoint): also record the action

## Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| $u$ | vector to be accessed |
| status | can be querried for info like the current XBraid Iteration |

16.3.2.2 _braid_BaseBufAlloc() braid_Int _braid_BaseBufAlloc (

```
braid_Core core,
braid_App app,
void ** buffer,
braid_Int nbytes,
braid_BufferStatus status )
```

This calls the user's BufAlloc routine for MPI buffer allocation
Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| buffer | MPI buffer for user to allocate |
| nbytes | number of bytes to allocate |
| status | can be querried for info about the message type |

16.3.2.3 _braid_BaseBufFree() braid_Int _braid_BaseBufFree (

> braid_Core core,
> braid_App app,
> void ** buffer )

This calls the user's BufFree routine for MPI buffer de-allocation
Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| buffer | user-allocated MPI buffer to free |

16.3.2.4 _braid_BaseBufPack() braid_Int _braid_BaseBufPack (

```
braid_App app,
braid_BaseVector u,
void * buffer,
braid_BufferStatus status )
```

This calls the user's BufPack routine. If (adjoint): also record the action, and push to the bar tape.

Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined _braid_App structure |
| $u$ | vector to pack into buffer |
| buffer | output, MPI buffer containing u |
| status | can be querried for info about the message |

```
16.3.2.5 _braid_BaseBufPack_diff() braid_Int _braid_BaseBufPack_diff (
    _braid_Action * action )
```

This pops the bar vector from the tape, and then performs the differentiated BufPack action using that vector as input: MPI_Recv(utmp) ubar += utmp

## Parameters

action $\quad$ _braid_Action structure, holds information about the primal XBraid action

```
16.3.2.6 _braid_BaseBufSize() braid_Int _braid_BaseBufSize (
    braid_Core core,
    braid_App app,
    braid_Int * size_ptr,
    braid_BufferStatus status )
```

This calls the user's BufSize routine. If (adjoint): nothing If (Delta correction): compute extra space needed for basis vectors

Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| size_ptr | upper bound on vector size in bytes |
| status | can be querried for info about the message type |

16.3.2.7 _braid_BaseBufUnpack() braid_Int _braid_BaseBufUnpack (

```
braid_Core core,
braid_App app,
void * buffer,
braid_BaseVector * u_ptr,
braid_BufferStatus status )
```

This calls the user's BufUnPack routine. If (adjoint): also record the action, initialize the bar vector with zero and push it to the bar tape.

Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| buffer | MPI Buffer to unpack and place in u_ptr |
| u_ptr | output, braid_Vector containing buffer's data |
| status | can be querried for info about the message type |

16.3.2.8 _braid_BaseBufUnpack_diff() braid_Int _braid_BaseBufUnpack_diff (
_braid_Action * action )

This pops the bar vector from the tape, and then performs the differentiated BufUnPack action using that vector as input: MPI_Send(ubar); ubar = 0.0

Parameters
action $\quad$ _braid_Action structure, holds information about the primal XBraid action
16.3.2.9 _braid_BaseClone() braid_Int _braid_BaseClone (
braid_Core core,
braid_App app,
braid_BaseVector $u$,
braid_BaseVector * v_ptr )
This initializes a braid_BaseVector and calls the user's clone routine. If (adjoint): also record the action, initialize a barVector with zero and push to the bar tape

## Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| $u$ | vector to clone |
| v_ptr | output, newly allocated and cloned vector |

16.3.2.10 _braid_BaseClone_diff() braid_Int _braid_BaseClone_diff (
_braid_Action * action )
This pops bar vectors from the tape, and then performs the differentiated clone action using those vectors as input: ubar += vbar vbar $=0.0$

Parameters
action __braid_Action structure, holds information about the primal XBraid action
16.3.2.11 _braid_BaseCloneBasis() braid_Int _braid_BaseCloneBasis (

```
braid_Core core,
braid_App app,
braid_Basis A,
braid_Basis * B_ptr )
```

This initializes a braid_Basis and calls the user's clone routine to initialize each column vector, cloning $A$ into $B$.
Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined _braid_App structure |
| A | basis to clone |
| B_ptr | output, newly allocated and cloned basis |

16.3.2.12 _braid_BaseFree() braid_Int _braid_BaseFree (

```
braid_Core core,
    braid_App app,
    braid_BaseVector u )
```

This calls the user's free routine. If (adjoint): also record the action, and free the bar vector.
Parameters

| core | braid_Core structure |
| :--- | :--- |
| $a p p$ | user-defined_braid_App structure |
| $u$ | vector to free |

16.3.2.13 _braid_BaseFreeBasis() braid_Int _braid_BaseFreeBasis (

> braid_Core core,
> braid_App app,
> braid_Basis b )

This calls the user's free routine on each vector in the braid_Basis.

## Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| $b$ | basis to free |

### 16.3.2.14 _braid_BaseFullResidual() braid_Int _braid_BaseFullResidual (

```
braid_Core core,
braid_App app,
braid_BaseVector r,
braid_BaseVector u,
braid_StepStatus status )
```

This calls the user's FullResidual routine (full_rnorm_res). If (adjoint): nothing
Parameters

| core | braid_Core structure |
| :--- | :--- |
| $a p p$ | user-defined_braid_App structure |
| $r$ | output, residual at tstop |
| $u$ | input, $u$ vector at tstop |
| status | braid_Status structure (pointer to the core) |

16.3.2.15 _braid_Baselnit() braid_Int _braid_BaseInit (

```
braid_Core core,
    braid_App app,
    braid_Real t,
    braid_BaseVector * u_ptr )
```

This initializes a braid_BaseVector and calls the user's init routine. If (adjoint): also record the action, initialize barVector with zero and push to the bar tape.

Parameters

| core | braid_Core structure |
| :--- | :--- |
| $a p p$ | user-defined_braid_App structure |
| $t$ | current time value for $u \_p t r$ |
| $u \_p t r$ | output, newly allocated and initialized vector |

16.3.2.16 _braid_Baselnit_diff() braid_Int _braid_BaseInit_diff (
_braid_Action * action )

This pops the bar vector from the tape, and then call's the user's differentiated init routine (init_diff) using that vector as input. Note: init_diff is optional for the user.

Parameters
action braid_Action structure, holds information about the primal XBraid action
16.3.2.17 _braid_BaseInitBasis() braid_Int _braid_BaseInitBasis (
braid_Core core,
braid_App app,
braid_Real t,
braid_Basis * psi_ptr )
This initializes a braid_Basis and calls the user's InitBasis routine.
Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| $t$ | current time value for $u \_p t r$ |
| psi_ptr | output, newly allocated and initialized basis |

16.3.2.18 _braid_BaselnnerProd() braid_Int _braid_BaseInnerProd (
braid_Core core,
braid_App app,
braid_Vector $u$,
braid_Vector v,
braid_Real * prod_ptr )
This calls the user's InnerProd routine
Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| $u$ | first vector for inner product |
| $v$ | second vector for inner product |
| prod_ptr | output, result of inner product |

16.3.2.19 _braid_BaseObjectiveT() braid_Int _braid_BaseObjectiveT (

```
braid_Core core,
    braid_App app,
    braid_BaseVector u,
    braid_ObjectiveStatus ostatus,
    braid_Real * objT_ptr )
```

If (adjoint): This calls the user's ObjectiveT routine, records the action, and pushes to the state and bar tapes.
Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| $u$ | input, state vector at current time |
| ostatus | status structure for querying time, index, etc. |
| objT_ptr | output, objective function value at current time |

16.3.2.20 _braid_BaseObjectiveT_diff() braid_Int _braid_BaseObjectiveT_diff (

```
    _braid_Action * action )
```

This pops state and bar vectors from the tape, and then calls the user's differentiated ObjectiveT routine (objT_diff) using those vectors as input: ubar $=(\mathrm{d}(\text { objective } \mathrm{T}) / \mathrm{d}(\mathrm{u}))^{\wedge} \mathrm{T} * \mathrm{f}$ bar

Parameters
action __braid_Action structure, holds information about the primal XBraid action
16.3.2.21 _braid_BaseResidual() braid_Int _braid_BaseResidual (
braid_Core core,
braid_App app,
braid_BaseVector ustop,
braid_BaseVector r,
braid_Stepstatus status )

This calls the user's Residual routine. If (adjoint): nothing
Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| ustop | input, $u$ vector at tstop |

## Parameters

| $r$ | output, residual at tstop (at input, equals $u$ at tstart) |
| :--- | :--- |
| status | braid_Status structure (pointer to the core) |

16.3.2.22 _braid_BaseSClone() braid_Int _braid_BaseSClone (
braid_Core core,
braid_App app,
braid_BaseVector u,
braid_BaseVector * v_ptr )

This clones a shell baseVector and call's the user's SClone routine. If (adjoint): nothing

Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined _braid_App structure |
| $u$ | vector to clone |
| v_ptr | output, newly allocated and cloned vector shell |

16.3.2.23 _braid_BaseSCoarsen() braid_Int _braid_BaseSCoarsen (
braid_Core core,
braid_App app,
braid_BaseVector fu,
braid_BaseVector * cu_ptr,
braid_CoarsenRefStatus status )
This initializes a baseVector and calls the user's SCoarsen routine. If (adjoint): nothing
Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| $f u$ | braid_BaseVector to refine |
| cu_ptr | output, refined vector |
| status | braid_Status structure (pointer to the core) |

16.3.2.24 _braid_BaseSFree() braid_Int _braid_BaseSFree (

```
braid_Core core,
```

```
braid_App app,
braid_BaseVector u )
```

Call the user's shell free (SFree) routine. If (adjoint): nothing
Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| $u$ | vector to free (keeping the shell) |

16.3.2.25 _braid_BaseSInit() braid_Int _braid_BaseSInit (
braid_Core core
braid_App app,
braid_Real t,
braid_BaseVector * u_ptr )

This initializes a shell baseVector and call's the user's SInit routine. If (adjoint): nothing
Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| $t$ | time value for $u \_p t r$ |
| $u \_p t r$ | output, newly allocated and initialized vector shell |

16.3.2.26 _braid_BaseSpatialNorm() braid_Int _braid_BaseSpatialNorm (
braid_Core core,
braid_App app,
braid_BaseVector $u$,
braid_Real * norm_ptr )

This calls the user's SpatialNorm routine. If (adjoint): nothing
Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined _braid_App structure |
| $u$ | vector to norm |
| norm_ptr | output, norm of braid_Vector (this is a spatial norm) |

16.3.2.27 _braid_BaseSRefine() braid_Int _braid_BaseSRefine (

```
braid_Core core,
```

braid_App app,
braid_BaseVector cu,
braid_BaseVector * fu_ptr,
braid_CoarsenRefStatus status )

This initializes a baseVector and calls the user's SRefine routine. If (adjoint): nothing

## Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| cu | braid_BaseVector to refine |
| fu_ptr | output, refined vector |
| status | braid_Status structure (pointer to the core) |

16.3.2.28 _braid_BaseStep() braid_Int _braid_BaseStep (
braid_Core core,
braid_App app,
braid_BaseVector ustop,
braid_BaseVector fstop,
braid_BaseVector u,
braid_Int level,
braid_StepStatus status )
This calls the user's step routine. If (adjoint): also record the action, and push state and bar vector to primal and bar tapes.

Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| ustop | input, $u$ vector at tstop |
| fstop | input, right-hand-side at tstop |
| $u$ | input/output, initially $u$ vector at tstart, upon exit, $u$ vector at tstop |
| level | current time grid level |
| status | braid_Status structure (pointer to the core) |

16.3.2.29 _braid_BaseStep_diff() braid_Int _braid_BaseStep_diff (
_braid_Action * action )

This pops state and bar vectors from the tape, and then calls the user's differentiated step routine (step_diff) using those vectors as input. ubar $=(\mathrm{d}(\mathrm{step}) / \mathrm{d}(\mathrm{u}))^{\wedge} \mathrm{T} *$ ubar

## Parameters

action __braid_Action structure, holds information about the primal XBraid action
16.3.2.30 _braid_BaseSum() braid_Int _braid_BaseSum (

```
braid_Core core,
braid_App app,
braid_Real alpha,
braid_BaseVector x,
braid_Real beta,
braid_BaseVector y )
```

This calls the user's sum routine. If (adjoint): also record the action, and push to the bar tape.
Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| alpha | scalar for AXPY |
| $x$ | vector for AXPY |
| beta | scalar for AXPY |
| $y$ | output and vector for AXPY |

16.3.2.31 _braid_BaseSum_diff() braid_Int _braid_BaseSum_diff (
_braid_Action * action )
This pops bar vectors from the tape, and then performs the differentiated sum action using those vectors as input: xbar += alpha $*$ ybar ybar $=$ beta $*$ ybar

Parameters
action _braid_Action structure, holds information about the primal XBraid action
16.3.2.32 _braid_BaseSumBasis() braid_Int _braid_BaseSumBasis (
braid_Core core,
braid_App app,
braid_Real alpha,
braid_Basis A,
braid_Real beta,
braid_Basis B )

This calls the user's sum routine on the columns of the bases $A, B$.
Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| alpha | scalar for AXPY |
| A | basis for AXPY |
| beta | scalar for AXPY |
| B | output and basis for AXPY |

16.3.2.33 _braid_BaseSync() braid_Int _braid_BaseSync (

> braid_Core core,
braid_App app,
braid_SyncStatus status )

This calls the user's Sync routine. If (adjoint): nothing
Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined _braid_App structure |
| status | can be querried for info like the current XBraid Iteration |

16.3.2.34 _braid_BaseTimeGrid() braid_Int _braid_BaseTimeGrid (

```
braid_Core core,
braid_App app,
braid_Real * ta,
braid_Int * ilower,
braid_Int * iupper )
```

This calls the user's TimeGrid routine, which allows the user to explicitly define the initial time grid. If (adjoint): nothing
Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| ta | temporal grid on level 0 (slice per processor) |
| ilower | lower time index value for this processor |
| iupper | upper time index value for this processor |

## 16.4 braid.h File Reference

## Macros

- \#define braid_FMANGLE 1
- \#define braid_Fortran_SpatialCoarsen 0
- \#define braid_Fortran_Residual 1
- \#define braid_Fortran_TimeGrid 1
- \#define braid_Fortran_Sync 1
- \#define braid_INVALID_RNORM -1
- \#define braid_ERROR_GENERIC $1 / *$ generic error $* /$
- \#define braid_ERROR_MEMORY $2 / *$ unable to allocate memory $* /$
- \#define braid_ERROR_ARG $4 / *$ argument error $* /$
- \#define braid_RAND_MAX 32768


## Typedefs

- typedef struct _braid_App_struct * braid_App
- typedef struct _braid_Vector_struct * braid_Vector
- typedef braid_Int(* braid_PtFcnStep) (braid_App app, braid_Vector ustop, braid_Vector fstop, braid_Vector u, braid_StepStatus status)
- typedef braid_Int(* braid_PtFcnInit) (braid_App app, braid_Real t, braid_Vector *u_ptr)
- typedef braid_Int(* braid_PtFcnInitBasis) (braid_App app, braid_Real t, braid_Int index, braid_Vector *u_ptr)
- typedef braid_Int(* braid_PtFcnClone) (braid_App app, braid_Vector u, braid_Vector *v_ptr)
- typedef braid_Int(* braid_PtFcnFree) (braid_App app, braid_Vector u)
- typedef braid_Int(* braid_PtFcnSum) (braid_App app, braid_Real alpha, braid_Vector x, braid_Real beta, braid_Vector y)
- typedef braid_Int(* braid_PtFcnSpatialNorm) (braid_App app, braid_Vector u, braid_Real *norm_ptr)
- typedef braid_Int(* braid_PtFcnInnerProd) (braid_App app, braid_Vector u, braid_Vector v, braid_Real *prod_ptr)
- typedef braid_Int(* braid_PtFcnAccess) (braid_App app, braid_Vector u, braid_AccessStatus status)
- typedef braid_Int(* braid_PtFcnSync) (braid_App app, braid_SyncStatus status)
- typedef braid_Int(* braid_PtFcnBufSize) (braid_App app, braid_Int *size_ptr, braid_BufferStatus status)
- typedef braid_Int(* braid_PtFcnBufPack) (braid_App app, braid_Vector u, void *buffer, braid_BufferStatus status)
- typedef braid_Int(* braid_PtFcnBufUnpack) (braid_App app, void *buffer, braid_Vector *u_ptr, braid_BufferStatus status)
- typedef braid_Int(* braid_PtFcnBufAlloc) (braid_App app, void **buffer, braid_Int nbytes, braid_BufferStatus status)
- typedef braid_Int(* braid_PtFcnBufFree) (braid_App app, void $* *$ buffer)
- typedef braid_Int(* braid_PtFcnResidual) (braid_App app, braid_Vector ustop, braid_Vector r, braid_StepStatus status)
- typedef braid_Int(* braid_PtFcnSCoarsen) (braid_App app, braid_Vector fu, braid_Vector $*$ cu_ptr, braid_ $\hookleftarrow$ CoarsenRefStatus status)
- typedef braid_Int(* braid_PtFcnSRefine) (braid_App app, braid_Vector cu, braid_Vector $* f u \_p t r$, braid_Coarsen $\hookleftarrow$ RefStatus status)
- typedef braid_Int(* braid_PtFcnSInit) (braid_App app, braid_Real t, braid_Vector *u_ptr)
- typedef braid_Int(* braid_PtFcnSClone) (braid_App app, braid_Vector u, braid_Vector $*$ v_ptr)
- typedef braid_Int(* braid_PtFcnSFree) (braid_App app, braid_Vector u)
- typedef braid_Int(* braid_PtFcnTimeGrid) (braid_App app, braid_Real *ta, braid_Int *ilower, braid_Int *iupper)
- typedef braid_Int(* braid_PtFcnObjectiveT) (braid_App app, braid_Vector u, braid_ObjectiveStatus ostatus, braid_Real *objectiveT_ptr)
- typedef braid_Int(* braid_PtFcnObjectiveTDiff) (braid_App app, braid_Vector u, braid_Vector u_bar, braid_Real F_bar, braid_ObjectiveStatus ostatus)
- typedef braid_Int(* braid_PtFcnPostprocessObjective) (braid_App app, braid_Real sum_obj, braid_Real *postprocess_ptr)
- typedef braid_Int(* braid_PtFcnPostprocessObjective_diff) (braid_App app, braid_Real sum_obj, braid_Real $*$ F $\hookleftarrow$ _bar_ptr)
- typedef braid_Int(* braid_PtFcnStepDiff) (braid_App app, braid_Vector ustop, braid_Vector u, braid_Vector ustop_bar, braid_Vector u_bar, braid_StepStatus status)
- typedef braid_Int(* braid_PtFcnResetGradient) (braid_App app)
- typedef struct _braid_Core_struct * braid_Core


## Functions

- braid_Int braid_Init (MPI_Comm comm_world, MPI_Comm comm, braid_Real tstart, braid_Real tstop, braid_Int ntime, braid_App app, braid_PtFcnStep step, braid_PtFcnInit init, braid_PtFcnClone clone, braid_PtFcnFree free, braid_PtFcnSum sum, braid_PtFcnSpatialNorm spatialnorm, braid_PtFcnAccess access, braid_PtFcnBufSize bufsize, braid_PtFcnBufPack bufpack, braid_PtFcnBufUnpack bufunpack, braid_Core *core_ptr)
- braid_Int braid_Drive (braid_Core core)
- braid_Int braid_Destroy (braid_Core core)
- braid_Int braid_PrintStats (braid_Core core)
- braid_Int braid_SetTimerFile (braid_Core core, braid_Int length, const char *filestem)
- braid_Int braid_PrintTimers (braid_Core core)
- braid_Int braid_ResetTimer (braid_Core core)
- braid_Int braid_WriteConvHistory (braid_Core core, const char *filename)
- braid_Int braid_SetMaxLevels (braid_Core core, braid_Int max_levels)
- braid_Int braid_SetIncrMaxLevels (braid_Core core)
- braid_Int braid_SetSkip (braid_Core core, braid_Int skip)
- braid_Int braid_SetRefine (braid_Core core, braid_Int refine)
- braid_Int braid_SetMaxRefinements (braid_Core core, braid_Int max_refinements)
- braid_Int braid_SetTPointsCutoff (braid_Core core, braid_Int tpoints_cutoff)
- braid_Int braid_SetMinCoarse (braid_Core core, braid_Int min_coarse)
- braid_Int braid_SetRelaxOnlyCG (braid_Core core, braid_Int relax_only_cg)
- braid_Int braid_SetAbsTol (braid_Core core, braid_Real atol)
- braid_Int braid_SetRelTol (braid_Core core, braid_Real rtol)
- braid_Int braid_SetNRelax (braid_Core core, braid_Int level, braid_Int nrelax)
- braid_Int braid_SetCRelaxWt (braid_Core core, braid_Int level, braid_Real Cwt)
- braid_Int braid_SetCFactor (braid_Core core, braid_Int level, braid_Int cfactor)
- braid_Int braid_SetMaxIter (braid_Core core, braid_Int max_iter)
- braid_Int braid_SetFMG (braid_Core core)
- braid_Int braid_SetNFMG (braid_Core core, braid_Int k)
- braid_Int braid_SetNFMGVcyc (braid_Core core, braid_Int nfmg_Vcyc)
- braid_Int braid_SetStorage (braid_Core core, braid_Int storage)
- braid_Int braid_SetTemporalNorm (braid_Core core, braid_Int tnorm)
- braid_Int braid_SetResidual (braid_Core core, braid_PtFcnResidual residual)
- braid_Int braid_SetFullRNormRes (braid_Core core, braid_PtFcnResidual residual)
- braid_Int braid_SetTimeGrid (braid_Core core, braid_PtFcnTimeGrid tgrid)
- braid_Int braid_SetPeriodic (braid_Core core, braid_Int periodic)
- braid_Int braid_SetSpatialCoarsen (braid_Core core, braid_PtFcnSCoarsen scoarsen)
- braid_Int braid_SetSpatialRefine (braid_Core core, braid_PtFcnSRefine srefine)
- braid_Int braid_SetSync (braid_Core core, braid_PtFcnSync sync)
- braid_Int braid_SetInnerProd (braid_Core core, braid_PtFcnInnerProd inner_prod)
- braid_Int braid_SetPrintLevel (braid_Core core, braid_Int print_level)
- braid_Int braid_SetFilelOLevel (braid_Core core, braid_Int io_level)
- braid_Int braid_SetPrintFile (braid_Core core, const char *printfile_name)
- braid_Int braid_SetDefaultPrintFile (braid_Core core)
- braid_Int braid_SetAccessLevel (braid_Core core, braid_Int access_level)
- braid_Int braid_SetFinalFCRelax (braid_Core core)
- braid_Int braid_SetBufAllocFree (braid_Core core, braid_PtFcnBufAlloc bufalloc, braid_PtFcnBufFree buffree)
- braid_Int braid_SplitCommworld (const MPI_Comm $*$ comm_world, braid_Int px, MPI_Comm $*$ comm_x, MPI_ $\hookleftarrow$ Comm *comm_t)
- braid_Int braid_SetShell (braid_Core core, braid_PtFcnSInit sinit, braid_PtFcnSClone sclone, braid_PtFcnSFree sfree)
- braid_Int braid_GetNumlter (braid_Core core, braid_Int *niter_ptr)
- braid_Int braid_GetRNorms (braid_Core core, braid_Int *nrequest_ptr, braid_Real *rnorms)
- braid_Int braid_GetNLevels (braid_Core core, braid_Int *nlevels_ptr)
- braid_Int braid_GetSpatialAccuracy (braid_StepStatus status, braid_Real loose_tol, braid_Real tight_tol, braid_Real *tol_ptr)
- braid_Int braid_SetSeqSoln (braid_Core core, braid_Int seq_soln)
- braid_Int braid_SetRichardsonEstimation (braid_Core core, braid_Int est_error, braid_Int richardson, braid_Int local_order)
- braid_Int braid_SetDeltaCorrection (braid_Core core, braid_Int rank, braid_PtFcnInitBasis basis_init, braid_PtFcnInnerProd inner_prod)
- braid_Int braid_SetDeferDelta (braid_Core core, braid_Int level, braid_Int iter)
- braid_Int braid_SetLyapunovEstimation (braid_Core core, braid_Int relax, braid_Int cglv, braid_Int exponents)
- braid_Int braid_SetTimings (braid_Core core, braid_Int timing_level)
- braid_Int braid_GetMyID (braid_Core core, braid_Int *myid_ptr)
- braid_Int braid_Rand (void)
- braid_Int braid_InitAdjoint (braid_PtFcnObjectiveT objectiveT, braid_PtFcnObjectiveTDiff objectiveT_diff, braid_PtFcnStepDiff step_diff, braid_PtFcnResetGradient reset_gradient, braid_Core $*$ core_ptr)
- braid_Int braid_SetTStartObjective (braid_Core core, braid_Real tstart_obj)
- braid_Int braid_SetTStopObjective (braid_Core core, braid_Real tstop_obj)
- braid_Int braid_SetPostprocessObjective (braid_Core core, braid_PtFcnPostprocessObjective post_fcn)
- braid_Int braid_SetPostprocessObjective_diff (braid_Core core, braid_PtFcnPostprocessObjective_diff post_↔ fcn_diff)
- braid_Int braid_SetAbsTolAdjoint (braid_Core core, braid_Real tol_adj)
- braid_Int braid_SetReIToIAdjoint (braid_Core core, braid_Real rtol_adj)
- braid_Int braid_SetObjectiveOnly (braid_Core core, braid_Int boolean)
- braid_Int braid_SetRevertedRanks (braid_Core core, braid_Int boolean)
- braid_Int braid_GetObjective (braid_Core core, braid_Real *objective_ptr)
- braid_Int braid_GetRNormAdjoint (braid_Core core, braid_Real *rnorm_adj)


### 16.4.1 Detailed Description

Define headers for user-interface routines.

This file contains user-routines used to allow the user to initialize, run and get and set options for a XBraid solver.

## 16.5 braid_defs.h File Reference

## Macros

- \#define braid_Int_Max INT_MAX;
- \#define braid_Int_Min INT_MIN;
- \#define braid_MPI_REAL MPI_DOUBLE
- \#define braid_MPI_INT MPI_INT
- \#define braid_MPI_Comm MPI_Comm


## Typedefs

- typedef int braid_Int
- typedef char braid_Byte
- typedef double braid_Real
- typedef struct _braid_Vector_struct _braid_Vector
- typedef _braid_Vector * braid_Vector


### 16.5.1 Detailed Description

Definitions of braid types, error flags, etc...
16.5.2 Macro Definition Documentation
16.5.2.1 braid_Int_Max \#define braid_Int_Max INT_MAX;
16.5.2.2 braid_Int_Min \#define braid_Int_Min INT_MIN;
16.5.2.3 braid_MPI_Comm \#define braid_MPI_Comm MPI_Comm
16.5.2.4 braid_MPI_INT \#define braid_MPI_INT MPI_INT
16.5.2.5 braid_MPI_REAL \#define braid_MPI_REAL MPI_DOUBLE

### 16.5.3 Typedef Documentation

16.5.3.1 _braid_Vector typedef struct _braid_Vector_struct _braid_Vector
16.5.3.2 braid_Byte typedef char braid_Byte

Defines byte type (can be any type, but sizeof(braid_Byte) MUST be 1)
16.5.3.3 braid_Int typedef int braid_Int

Defines integer type

### 16.5.3.4 braid_Real typedef double braid_Real

Defines floating point type Switch beween single and double precision by un-/commenting lines.
16.5.3.5 braid_Vector typedef _braid_Vector* braid_Vector

This defines (roughly) a state vector at a certain time value.
It could also contain any other information related to this vector which is needed to evolve the vector to the next time value, like mesh information. reproduced here from braid.h to give braid_status access to the braid_Vector typedef

## 16.6 braid_status.h File Reference

## Macros

- \#define ACCESSOR_HEADER_GET1(stype, param, vtype1) braid_Int braid_\#\#stype\#\#StatusGet\#\#param(braid $\hookleftarrow$ _\#\#stype\#\#Status s, braid_\#\#vtype1 *v1);
- \#define ACCESSOR_HEADER_GET1_IN1(stype, param, vtype1, vtype2) braid_Int braid_\#\#stype\#\#Status $\hookleftarrow$ Get\#\#param(braid_\#\#stype\#\#Status s, braid_\#\#vtype1 *v1, braid_\#\#vtype2 v2);
- \#define ACCESSOR_HEADER_GET1_IN2(stype, param, vtype1, vtype2, vtype3) braid_Int braid_↔ \#\#stype\#\#StatusGet\#\#param(braid_\#\#stype\#\#Status s, braid_\#\#vtype1 *v1, braid_\#\#vtype2 v2, braid_\#\#vtype3 v3);
- \#define ACCESSOR_HEADER_GET1_IN3(stype, param, vtype1, vtype2, vtype3, vtype4) braid_Int braid_↔ \#\#stype\#\#StatusGet\#\#param(braid_\#\#stype\#\#Status s, braid_\#\#vtype1 *v1, braid_\#\#vtype2 v2, braid_\#\#vtype3 v3, braid_\#\#vtype4 v4);
- \#define ACCESSOR_HEADER_GET2(stype, param, vtype1, vtype2) braid_Int braid_\#\#stype\#\#Status $\hookleftarrow$ Get\#\#param(braid_\#\#stype\#\#Status s, braid_\#\#vtype1 *v1, braid_\#\#vtype2 *v2);
- \#define ACCESSOR_HEADER_GET2_IN1(stype, param, vtype1, vtype2, vtype3) braid_Int braid_↔ \#\#stype\#\#StatusGet\#\#param(braid_\#\#stype\#\#Status s, braid_\#\#vtype1 *v1, braid_\#\#vtype2 *v2, braid_↔ \#\#vtype3 v3);
- \#define ACCESSOR_HEADER_GET3(stype, param, vtype1, vtype2, vtype3) braid_Int braid_\#\#stype\#\#Status $\hookleftarrow$ Get\#\#param(braid_\#\#stype\#\#Status s, braid_\#\#vtype1 *v1, braid_\#\#vtype2 *v2, braid_\#\#vtype3 *v3);
- \#define ACCESSOR_HEADER_GET4(stype, param, vtype1, vtype2, vtype3, vtype4) braid_Int braid_↔ \#\#stype\#\#StatusGet\#\#param(braid_\#\#stype\#\#Status s, braid_\#\#vtype1 *v1, braid_\#\#vtype2 *v2, braid_↔ \#\#vtype3 *v3, braid_\#\#vtype4 *v4);
- \#define ACCESSOR_HEADER_GET5(stype, param, vtype1, vtype2, vtype3, vtype4, vtype5) braid_Int braid $\hookleftarrow$ _\#\#stype\#\#StatusGet\#\#param(braid_\#\#stype\#\#Status s, braid_\#\#vtype1 *v1, braid_\#\#vtype2 *v2, braid_↔ \#\#vtype3 $*$ v3, braid_\#\#vtype4 $*$ v4, braid_\#\#vtype5 $* \mathrm{v} 5$ );
- \#define ACCESSOR_HEADER_SET1(stype, param, vtype1) braid_Int braid_\#\#stype\#\#StatusSet\#\#param(braid $\hookleftarrow$ _\#\#stype\#\#Status s, braid_\#\#vtype1 v1);
- \#define braid_ASCaller_FInterp 0
- \#define braid_ASCaller_FRestrict 1
- \#define braid_ASCaller_FRefine 2
- \#define braid_ASCaller_FAccess 3
- \#define braid_ASCaller_FRefine_AfterInitHier 4
- \#define braid_ASCaller_Drive_TopCycle 5
- \#define braid_ASCaller_FCRelax 6
- \#define braid_ASCaller_Drive_Afterlnit 7
- \#define braid_ASCaller_BaseStep_diff 8
- \#define braid_ASCaller_ComputeFullRNorm 9
- \#define braid_ASCaller_FASResidual 10
- \#define braid_ASCaller_Residual 11
- \#define braid_ASCaller_InitGuess 12


## Functions

- braid_Int braid_StatusGetT (braid_Status status, braid_Real *t_ptr)
- braid_Int braid_StatusGetTIndex (braid_Status status, braid_Int *idx_ptr)
- braid_Int braid_StatusGetlter (braid_Status status, braid_Int *iter_ptr)
- braid_Int braid_StatusGetLevel (braid_Status status, braid_Int *level_ptr)
- braid_Int braid_StatusGetNLevels (braid_Status status, braid_Int *nlevels_ptr)
- braid_Int braid_StatusGetNRefine (braid_Status status, braid_Int *nrefine_ptr)
- braid_Int braid_StatusGetNTPoints (braid_Status status, braid_Int *ntpoints_ptr)
- braid_Int braid_StatusGetResidual (braid_Status status, braid_Real $*$ rnorm_ptr)
- braid_Int braid_StatusGetDone (braid_Status status, braid_Int *done_ptr)
- braid_Int braid_StatusGetTIUL (braid_Status status, braid_Int *iloc_upper, braid_Int *iloc_lower, braid_Int level)
- braid_Int braid_StatusGetTimeValues (braid_Status status, braid_Real **tvalues_ptr, braid_Int i_upper, braid_Int i_lower, braid_Int level)
- braid_Int braid_StatusGetTILD (braid_Status status, braid_Real *t_ptr, braid_Int *iter_ptr, braid_Int *level_ptr, braid_Int *done_ptr)
- braid_Int braid_StatusGetWrapperTest (braid_Status status, braid_Int *wtest_ptr)
- braid_Int braid_StatusGetCallingFunction (braid_Status status, braid_Int *cfunction_ptr)
- braid_Int braid_StatusGetDeltaRank (braid_Status status, braid_Int *rank_ptr)
- braid_Int braid_StatusGetBasisVec (braid_Status status, braid_Vector *v_ptr, braid_Int index)
- braid_Int braid_StatusGetLocalLyapExponents (braid_Status status, braid_Real *exp_ptr, braid_Int *num_↔ returned)
- braid_Int braid_StatusGetCTprior (braid_Status status, braid_Real *ctprior_ptr)
- braid_Int braid_StatusGetCTstop (braid_Status status, braid_Real *ctstop_ptr)
- braid_Int braid_StatusGetFTprior (braid_Status status, braid_Real *ftprior_ptr)
- braid_Int braid_StatusGetFTstop (braid_Status status, braid_Real *ftstop_ptr)
- braid_Int braid_StatusGetTpriorTstop (braid_Status status, braid_Real *t_ptr, braid_Real *ftprior_ptr, braid_Real *ftstop_ptr, braid_Real *ctprior_ptr, braid_Real *ctstop_ptr)
- braid_Int braid_StatusGetTstop (braid_Status status, braid_Real *tstop_ptr)
- braid_Int braid_StatusGetTstartTstop (braid_Status status, braid_Real *tstart_ptr, braid_Real *tstop_ptr)
- braid_Int braid_StatusGetTol (braid_Status status, braid_Real *tol_ptr)
- braid_Int braid_StatusGetRNorms (braid_Status status, braid_Int *nrequest_ptr, braid_Real *rnorms_ptr)
- braid_Int braid_StatusGetProc (braid_Status status, braid_Int *proc_ptr, braid_Int level, braid_Int index)
- braid_Int braid_StatusGetOldFineTolx (braid_Status status, braid_Real *old_fine_tolx_ptr)
- braid_Int braid_StatusSetOldFineTolx (braid_Status status, braid_Real old_fine_tolx)
- braid_Int braid_StatusSetTightFineTolx (braid_Status status, braid_Real tight_fine_tolx)
- braid_Int braid_StatusSetRFactor (braid_Status status, braid_Real rfactor)
- braid_Int braid_StatusSetRefinementDtValues (braid_Status status, braid_Real rfactor, braid_Real *dtarray)
- braid_Int braid_StatusSetRSpace (braid_Status status, braid_Real r_space)
- braid_Int braid_StatusGetMessageType (braid_Status status, braid_Int *messagetype_ptr)
- braid_Int braid_StatusSetSize (braid_Status status, braid_Real size)
- braid_Int braid_StatusSetBasisSize (braid_Status status, braid_Real size)
- braid_Int braid_StatusGetSingleErrorEstStep (braid_Status status, braid_Real *estimate)
- braid_Int braid_StatusGetSingleErrorEstAccess (braid_Status status, braid_Real *estimate)
- braid_Int braid_StatusGetNumErrorEst (braid_Status status, braid_Int *npoints)
- braid_Int braid_StatusGetAllErrorEst (braid_Status status, braid_Real *error_est)
- braid_Int braid_StatusGetTComm (braid_Status status, MPI_Comm *comm_ptr)
- braid_Int braid_AccessStatusGetT (braid_AccessStatus s, braid_Real *v1)
- braid_Int braid_AccessStatusGetTIndex (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetlter (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetLevel (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetNLevels (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetNRefine (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetNTPoints (braid_AccessStatus s, braid_Int *V1)
- braid_Int braid_AccessStatusGetResidual (braid_AccessStatus s, braid_Real *v1)
- braid_Int braid_AccessStatusGetDone (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetTILD (braid_AccessStatus s, braid_Real $* \mathrm{v} 1$, braid_Int $* \mathrm{v} 2$, braid_Int $* \mathrm{v} 3$, braid_Int *V4)
- braid_Int braid_AccessStatusGetWrapperTest (braid_AccessStatus s, braid_Int $* \mathrm{v} 1$ )
- braid_Int braid_AccessStatusGetCallingFunction (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetSingleErrorEstAccess (braid_AccessStatus s, braid_Real *v1)
- braid_Int braid_AccessStatusGetDeltaRank (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetLocalLyapExponents (braid_AccessStatus s, braid_Real $* \mathrm{~V} 1$, braid_Int $* \mathrm{v} 2$ )
- braid_Int braid_AccessStatusGetBasisVec (braid_AccessStatus s, braid_Vector *v1, braid_Int v2)
- braid_Int braid_SyncStatusGetTIUL (braid_SyncStatus s, braid_Int $*$ v1, braid_Int $*$ v2, braid_Int v3)
- braid_Int braid_SyncStatusGetTimeValues (braid_SyncStatus s, braid_Real **v1, braid_Int v2, braid_Int v3, braid_Int v4)
- braid_Int braid_SyncStatusGetProc (braid_SyncStatus s, braid_Int *v1, braid_Int v2, braid_Int v3)
- braid_Int braid_SyncStatusGetlter (braid_SyncStatus s, braid_Int *V1)
- braid_Int braid_SyncStatusGetLevel (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetNLevels (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetNRefine (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetNTPoints (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetDone (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetCallingFunction (braid_SyncStatus s, braid_Int *V1)
- braid_Int braid_SyncStatusGetNumErrorEst (braid_SyncStatus s, braid_Int *V1)
- braid_Int braid_SyncStatusGetAllErrorEst (braid_SyncStatus s, braid_Real $*$ V1)
- braid_Int braid_SyncStatusGetTComm (braid_SyncStatus s, MPI_Comm *v1)
- braid_Int braid_CoarsenRefStatusGetT (braid_CoarsenRefStatus s, braid_Real *v1)
- braid_Int braid_CoarsenRefStatusGetTIndex (braid_CoarsenRefStatus s, braid_Int *v1)
- braid_Int braid_CoarsenRefStatusGetlter (braid_CoarsenRefStatus s, braid_Int *v1)
- braid_Int braid_CoarsenRefStatusGetLevel (braid_CoarsenRefStatus s, braid_Int *v1)
- braid_Int braid_CoarsenRefStatusGetNLevels (braid_CoarsenRefStatus s, braid_Int *v1)
- braid_Int braid_CoarsenRefStatusGetNRefine (braid_CoarsenRefStatus s, braid_Int *v1)
- braid_Int braid_CoarsenRefStatusGetNTPoints (braid_CoarsenRefStatus s, braid_Int *v1)
- braid_Int braid_CoarsenRefStatusGetCTprior (braid_CoarsenRefStatus s, braid_Real *v1)
- braid_Int braid_CoarsenRefStatusGetCTstop (braid_CoarsenRefStatus s, braid_Real *v1)
- braid_Int braid_CoarsenRefStatusGetFTprior (braid_CoarsenRefStatus s, braid_Real $* \mathrm{~V} 1$ )
- braid_Int braid_CoarsenRefStatusGetFTstop (braid_CoarsenRefStatus s, braid_Real *v1)
- braid_Int braid_CoarsenRefStatusGetTpriorTstop (braid_CoarsenRefStatus s, braid_Real *v1, braid_Real *v2, braid_Real $* \mathrm{v} 3$, braid_Real $* \mathrm{v} 4$, braid_Real $* \mathrm{v} 5$ )
- braid_Int braid_StepStatusGetTIUL (braid_StepStatus s, braid_Int *v1, braid_Int $*$ v2, braid_Int v3)
- braid_Int braid_StepStatusGetT (braid_StepStatus s, braid_Real *v1)
- braid_Int braid_StepStatusGetTIndex (braid_StepStatus s, braid_Int *V1)
- braid_Int braid_StepStatusGetlter (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetLevel (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetNLevels (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetNRefine (braid_StepStatus s, braid_Int *V1)
- braid_Int braid_StepStatusGetNTPoints (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetTstop (braid_StepStatus s, braid_Real *v1)
- braid_Int braid_StepStatusGetTstartTstop (braid_StepStatus s, braid_Real *v1, braid_Real *v2)
- braid_Int braid_StepStatusGetTol (braid_StepStatus s, braid_Real *v1)
- braid_Int braid_StepStatusGetRNorms (braid_StepStatus s, braid_Int *v1, braid_Real *v2)
- braid_Int braid_StepStatusGetOIdFineTolx (braid_StepStatus s, braid_Real $*$ v1)
- braid_Int braid_StepStatusSetOldFineTolx (braid_StepStatus s, braid_Real v1)
- braid_Int braid_StepStatusSetTightFineTolx (braid_StepStatus s, braid_Real v1)
- braid_Int braid_StepStatusSetRFactor (braid_StepStatus s, braid_Real v1)
- braid_Int braid_StepStatusSetRSpace (braid_StepStatus s, braid_Real v1)
- braid_Int braid_StepStatusGetDone (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetSingleErrorEstStep (braid_StepStatus s, braid_Real *v1)
- braid_Int braid_StepStatusGetCallingFunction (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetDeltaRank (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetBasisVec (braid_StepStatus s, braid_Vector *v1, braid_Int v2)
- braid_Int braid_BufferStatusGetMessageType (braid_BufferStatus s, braid_Int *v1)
- braid_Int braid_BufferStatusGetTIndex (braid_BufferStatus s, braid_Int $* \mathrm{~V} 1$ )
- braid_Int braid_BufferStatusGetLevel (braid_BufferStatus s, braid_Int *v1)
- braid_Int braid_BufferStatusSetSize (braid_BufferStatus s, braid_Real v1)
- braid_Int braid_BufferStatusSetBasisSize (braid_BufferStatus s, braid_Real v1)
- braid_Int braid_ObjectiveStatusGetT (braid_ObjectiveStatus s, braid_Real *v1)
- braid_Int braid_ObjectiveStatusGetTIndex (braid_ObjectiveStatus s, braid_Int *v1)
- braid_Int braid_ObjectiveStatusGetlter (braid_ObjectiveStatus s, braid_Int *v1)
- braid_Int braid_ObjectiveStatusGetLevel (braid_ObjectiveStatus s, braid_Int *V1)
- braid_Int braid_ObjectiveStatusGetNLevels (braid_ObjectiveStatus s, braid_Int *v1)
- braid_Int braid_ObjectiveStatusGetNRefine (braid_ObjectiveStatus s, braid_Int *v1)
- braid_Int braid_ObjectiveStatusGetNTPoints (braid_ObjectiveStatus s, braid_Int *v1)
- braid_Int braid_ObjectiveStatusGetTol (braid_ObjectiveStatus s, braid_Real *v1)


### 16.6.1 Detailed Description

Define headers for the user-interface with the XBraid status structures, allowing the user to get/set status structure values.

### 16.6.2 Macro Definition Documentation

16.6.2.1 ACCESSOR_HEADER_GET1
\#define ACCESSOR_HEADER_GET1(
stype,
param,
vtype1 ) braid_Int braid_\#\#stype\#\#StatusGet\#\#param(braid_\#\#stype\#\#Status s, braid↔
_\#\#vtype1 *v1);

Macros allowing for auto-generation of 'inherited' StatusGet functions

```
16.6.2.2 ACCESSOR_HEADER_GET1_IN1 #define ACCESSOR_HEADER_GET1_IN1(
    stype,
    param,
    vtype1,
    vtype2 ) braid_Int braid_##stype##StatusGet##param(braid_##stype##Status s, braid\hookleftarrow
_##vtype1 *v1, braid_##vtype2 v2);
```

16.6.2.3 ACCESSOR_HEADER_GET1_IN2 \#define ACCESSOR_HEADER_GET1_IN2 (
stype,
param,
vtype1,
vtype2,
vtype3 ) braid_Int braid_\#\#stype\#\#StatusGet\#\#param(braid_\#\#stype\#\#Status s, braid↔ _\#\#vtype1 *v1, braid_\#\#vtype2 v2, braid_\#\#vtype3 v3);
16.6.2.4 ACCESSOR_HEADER_GET1_IN3 \#define ACCESSOR_HEADER_GET1_IN3(
stype,
param,
vtype1,
vtype2,
vtype3,
vtype4 ) braid_Int braid_\#\#stype\#\#StatusGet\#\#param(braid_\#\#stype\#\#Status s, braid↔ _\#\#vtype1 *v1, braid_\#\#vtype2 v2, braid_\#\#vtype3 v3, braid_\#\#vtype4 v4);
16.6.2.5 ACCESSOR_HEADER_GET2 \#define ACCESSOR_HEADER_GET2 (
stype,
param,
vtype1,
vtype2 ) braid_Int braid_\#\#stype\#\#StatusGet\#\#param(braid_\#\#stype\#\#Status s, braid $\hookleftarrow$ _\#\#vtype1 *v1, braid_\#\#vtype2 *v2);
16.6.2.6 ACCESSOR_HEADER_GET2_IN1 \#define ACCESSOR_HEADER_GET2_IN1 (
stype,
param,
vtype1,
vtype2,
vtype3 ) braid_Int braid_\#\#stype\#\#StatusGet\#\#param(braid_\#\#stype\#\#Status s, braid↔ _\#\#vtype1 *v1, braid_\#\#vtype2 *v2, braid_\#\#vtype3 v3);
16.6.2.7 ACCESSOR_HEADER_GET3 \#define ACCESSOR_HEADER_GET3(
stype,
param,
vtype1,
vtype2,
vtype3 ) braid_Int braid_\#\#stype\#\#StatusGet\#\#param(braid_\#\#stype\#\#Status s, braid↔
_\#\#vtype1 *v1, braid_\#\#vtype2 *v2, braid_\#\#vtype3 *v3);
16.6.2.8 ACCESSOR HEADER GET4 \#define ACCESSOR_HEADER_GET4 (
stype,
param,
vtype1,
vtype2,
vtype3,
vtype4 ) braid_Int braid_\#\#stype\#\#StatusGet\#\#param(braid_\#\#stype\#\#Status s, braid↔
_\#\#vtype1 *v1, braid_\#\#vtype2 *v2, braid_\#\#vtype3 *v3, braid_\#\#vtype4 *v4);
16.6.2.9 ACCESSOR_HEADER_GET5 \#define ACCESSOR_HEADER_GET5(
stype,
param,
vtype1,
vtype2,
vtype3,
vtype4,
vtype5 ) braid_Int braid_\#\#stype\#\#StatusGet\#\#param(braid_\#\#stype\#\#Status s, braid $\hookleftarrow$
_\#\#vtype1 *v1, braid_\#\#vtype2 *v2, braid_\#\#vtype3 *v3, braid_\#\#vtype4 *v4, braid_\#\#vtype5 *v5);
16.6.2.10 ACCESSOR_HEADER_SET1 \#define ACCESSOR_HEADER_SET1 (
stype,
param,
vtype1 ) braid_Int braid_\#\#stype\#\#StatusSet\#\#param(braid_\#\#stype\#\#Status s, braid↔
_\#\#vtype1 v1);

## 16.7 braid_test.h File Reference

## Functions

- braid_Int braid_TestInitAccess (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_PtFcnInit init, braid_PtFcnAccess access, braid_PtFcnFree free)
- braid_Int braid_TestClone (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_PtFcnlnit init, braid_PtFcnAccess access, braid_PtFcnFree free, braid_PtFcnClone clone)
- braid_Int braid_TestSum (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_PtFcnInit init, braid_PtFcnAccess access, braid_PtFcnFree free, braid_PtFcnClone clone, braid_PtFcnSum sum)
- braid_Int braid_TestSpatialNorm (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_PtFcnInit init, braid_PtFcnFree free, braid_PtFcnClone clone, braid_PtFcnSum sum, braid_PtFcnSpatialNorm spatialnorm)
- braid_Int braid_TestInnerProd (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t1, braid_Real t2, braid_PtFcnInit init, braid_PtFcnFree free, braid_PtFcnSum sum, braid_PtFcnInnerProd inner_prod)
- braid_Int braid_TestBuf (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_PtFcnInit init, braid_PtFcnFree free, braid_PtFcnSum sum, braid_PtFcnSpatialNorm spatialnorm, braid_PtFcnBufSize bufsize, braid_PtFcnBufPack bufpack, braid_PtFenBufUnpack bufunpack)
- braid_Int braid_TestCoarsenRefine (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_Real fdt, braid_Real cdt, braid_PtFcnInit init, braid_PtFcnAccess access, braid_PtFcnFree free, braid_PtFcnClone clone, braid_PtFcnSum sum, braid_PtFcnSpatialNorm spatialnorm, braid_PtFcnSCoarsen coarsen, braid_PtFcnSRefine refine)
- braid_Int braid_TestResidual (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_Real dt, braid_PtFcnInit myinit, braid_PtFcnAccess myaccess, braid_PtFcnFree myfree, braid_PtFcnClone clone, braid_PtFcnSum sum, braid_PtFcnSpatialNorm spatialnorm, braid_PtFcnResidual residual, braid_PtFcnStep step)
- braid_Int braid_TestAll (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_Real fdt, braid_Real cdt, braid_PtFcnlnit init, braid_PtFcnFree free, braid_PtFcnClone clone, braid_PtFcnSum sum, braid_PtFcnSpatialNorm spatialnorm, braid_PtFcnBufSize bufsize, braid_PtFcnBufPack bufpack, braid_PtFcnBufUnpack bufunpack, braid_PtFenSCoarsen coarsen, braid_PtFcnSRefine refine, braid_PtFcnResidual residual, braid_PtFcnStep step)
- braid_Int braid_TestDelta (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_Real dt, braid_Int rank, braid_PtFcnInit myinit, braid_PtFcnInitBasis myinit_basis, braid_PtFcnAccess myaccess, braid_PtFcnFree myfree, braid_PtFcnClone myclone, braid_PtFcnSum mysum, braid_PtFcnBufSize bufsize, braid_PtFcnBufPack bufpack, braid_PtFcnBufUnpack bufunpack, braid_PtFcnInnerProd myinner_prod, braid_PtFcnStep mystep)


### 16.7.1 Detailed Description

Define headers for XBraid user-test routines.

This file contains headers for the user to test their XBraid wrapper routines one-by-one.

## 16.8 delta.h File Reference

## Macros

- \#define _braid_DoDeltaCorrect(core, level, niter) ( _braid_CoreElt(core, delta_correct) \&\& niter >= _braid_CoreElt(core, delta_defer_iter) \&\& level >= _braid_CoreElt(core, delta_defer_IvI) )
- \#define _braid_UseDeltaCorrect(core, level, niter) ( _braid_CoreElt(core, delta_correct) \&\& niter >= _braid_CoreElt(core, delta_defer_iter) \&\& level > _braid_CoreElt(core, delta_defer_Ivl) )


## Functions

- braid_Int _braid_LRDeltaDot (braid_Core core, braid_App app, braid_Vector u, braid_Basis delta, braid_Basis basis)
- braid_Int _braid_LRDeltaDotMat (braid_Core core, braid_App app, braid_Basis psi, braid_Basis delta, braid_Basis basis)
- braid_Int _braid_GramSchmidt (braid_Core core, braid_App app, braid_Basis basis, braid_Real *exps)
- braid_Int _braid_DeltaFeatureCheck (braid_Core core)


### 16.8.1 Detailed Description

Define internal XBraid headers for Delta correction.

This file contains the internal XBraid headers for Delta correction,

### 16.8.2 Macro Definition Documentation

```
16.8.2.1 _braid_DoDeltaCorrect #define _braid_DoDeltaCorrect(
    core,
    level,
    niter ) ( _braid_CoreElt(core, delta_correct) && niter >= __braid_CoreElt(core,
delta_defer_iter) && level >= _braid_CoreElt(core, delta_defer_lvl) )
```

macro for determining when to compute Delta correction
16.8.2.2 _braid_UseDeltaCorrect \#define _braid_UseDeltaCorrect (
core,
level,
niter ) ( _braid_CoreElt(core, delta_correct) \&\& niter >= _braid_CoreElt(core, delta_defer_iter) \&\& level > _braid_CoreElt(core, delta_defer_lvl) )
macro for determining when we can use computed Delta correction

### 16.8.3 Function Documentation

16.8.3.1 _braid_DeltaFeatureCheck() braid_Int _braid_DeltaFeatureCheck ( braid_Core core )

Sanity check for non-supported Delta correction features
16.8.3.2 _braid_GramSchmidt() braid_Int _braid_GramSchmidt (
braid_Core core,
braid_App app,
braid_Basis basis,
braid_Real * exps )

Perform modified Gram-Schmidt orthonormalization on a basis, while also computing local Lyapunov exponents
16.8.3.3 _braid_LRDeltaDot() braid_Int _braid_LRDeltaDot (
braid_Core core,
braid_App app,
braid_Vector u,
braid_Basis delta,
braid_Basis basis )

Compute the action of the low-rank approximation to Delta on a vector
16.8.3.4 _braid_LRDeltaDotMat() braid_Int _braid_LRDeltaDotMat (
braid_Core core,
braid_App app,
braid_Basis psi,
braid_Basis delta,
braid_Basis basis )

Compute the action of the low-rank approximation to Delta on a basis

## 16.9 mpistubs.h File Reference

### 16.9.1 Detailed Description

XBraid internal headers to define fake MPI stubs. This ultimately allows the user to generate purely serial codes without MPI.

### 16.10 status.h File Reference

## Data Structures

- struct _braid_Status
- struct braid_AccessStatus
- struct braid_SyncStatus
- struct braid_StepStatus
- struct braid_CoarsenRefStatus
- struct braid_BufferStatus
- struct braid_ObjectiveStatus


## Macros

- \#define _braid_StatusElt(status, elt) ( ((braid_Core)status) -> elt )


## Functions

- braid_Int _braid_StatusDestroy (braid_Status status)
- braid_Int _braid_AccessStatusInit (braid_Real t, braid_Int idx, braid_Real rnorm, braid_Int iter, braid_Int level, braid_Int nrefine, braid_Int gupper, braid_Int done, braid_Int wrapper_test, braid_Int calling_function, braid_Basis basis, braid_AccessStatus status)
- braid_Int _braid_SyncStatusInit (braid_Int iter, braid_Int level, braid_Int nrefine, braid_Int gupper, braid_Int done, braid_Int calling_function, braid_SyncStatus status)
- braid_Int_braid_CoarsenRefStatusInit (braid_Real tstart, braid_Real f_tprior, braid_Real f_tstop, braid_Real c_↔ tprior, braid_Real c_tstop, braid_Int level, braid_Int nrefine, braid_Int gupper, braid_Int c_index, braid_Coarsen $\hookleftarrow$ RefStatus status)
- braid_Int _braid_StepStatusInit (braid_Real tstart, braid_Real tstop, braid_Int idx, braid_Real tol, braid_Int iter, braid_Int level, braid_Int nrefine, braid_Int gupper, braid_Int calling_function, braid_Basis basis, braid_StepStatus status)
- braid_Int _braid_BufferStatusInit (braid_Int messagetype, braid_Int idx, braid_Int level, braid_Int size, braid_↔ BufferStatus status)
- braid_Int _braid_ObjectiveStatusInit (braid_Real tstart, braid_Int idx, braid_Int iter, braid_Int level, braid_Int nrefine, braid_Int gupper, braid_ObjectiveStatus status)


### 16.10.1 Detailed Description

Define the XBraid internal headers for the XBraid status structure routines, and define the status structures themselves.

### 16.10.2 Macro Definition Documentation

16.10.2.1 _braid_StatusElt \#define _braid_StatusElt (
status,
elt ) ( ((braid_Core)status) $->$ elt )

### 16.10.3 Function Documentation

16.10.3.1 _braid_AccessStatusInit() braid_Int _braid_AccessStatusInit (
braid_Real t,
braid_Int idx,
braid_Real rnorm,
braid_Int iter,
braid_Int level,
braid_Int nrefine,
braid_Int gupper,
braid_Int done,
braid_Int wrapper_test,
braid_Int calling_function,
braid_Basis basis,
braid_AccessStatus status )

Initialize a braid_AccessStatus structure
Parameters

| $t$ | current time |
| :--- | :--- |
| $i d x$ | time point index value corresponding to t on the global time grid |
| rnorm | current residual norm in XBraid |
| iter | current iteration in XBraid |
| level | current level in XBraid |
| nrefine | number of refinements done |
| gupper | global size of the fine grid |
| done | boolean describing whether XBraid has finished |
| wrapper_test | boolean describing whether this call is only a wrapper test |
| calling_function | from which function are we accessing the vector |
| basis | if Delta correction is set, basis vectors at this point |
| status | structure to initialize |

16.10.3.2 _braid_BufferStatusInit() braid_Int _braid_BufferStatusInit (
braid_Int messagetype,
braid_Int idx,
braid_Int level,
braid_Int size,
braid_BufferStatus status )

Initialize a braid_BufferStatus structure

## Parameters

| messagetype | message type, 0: for Step(), 1: for load balancing |
| :--- | :--- |
| idx | time point index value corresponding to this buffer |
| level | current level in XBraid |
| size | if set by user, size of send buffer is "size" bytes |
| status | structure to initialize |

16.10.3.3 _braid_CoarsenRefStatusInit() braid_Int _braid_CoarsenRefStatusInit (

```
braid_Real tstart,
braid_Real f_tprior,
braid_Real f_tstop,
braid_Real c_tprior,
braid_Real c_tstop,
braid_Int level,
braid_Int nrefine,
braid_Int gupper,
braid_Int c_index,
braid_CoarsenRefStatus status )
```

Initialize a braid_CoarsenRefStatus structure

Parameters

| tstart | time value for current vector |
| :--- | :--- |
| f_tprior | time value to the left of tstart on fine grid |
| f_tstop | time value to the right of tstart on fine grid |
| c_tprior | time value to the left of tstart on coarse grid |
| c_tstop | time value to the right of tstart on coarse grid |
| level | current fine level in XBraid |
| nrefine | number of refinements done |
| gupper | global size of the fine grid |
| c_index | coarse time index refining from |
| status | structure to initialize |

### 16.10.3.4 _braid_ObjectiveStatusInit() braid_Int _braid_ObjectiveStatusInit (

```
braid_Real tstart,
braid_Int idx,
braid_Int iter,
braid_Int level,
braid_Int nrefine,
braid_Int gupper,
braid_ObjectiveStatus status )
```

Initialize a braid_ObjectiveStatus structure
16.10.3.5 _braid_StatusDestroy() braid_Int _braid_StatusDestroy ( braid_Status status )
16.10.3.6 _braid_StepStatusInit() braid_Int _braid_StepStatusInit (
braid_Real tstart,
braid_Real tstop,
braid_Int idx,
braid_Real tol,
braid_Int iter,
braid_Int level,
braid_Int nrefine,
braid_Int gupper,
braid_Int calling_function,
braid_Basis basis,
braid_Stepstatus status )

Initialize a braid_StepStatus structure

Parameters

| tstart | current time value |
| :--- | :--- |
| tstop | time value to evolve towards, time value to the right of tstart |
| idx | time point index value corresponding to tstart on the global time grid |
| tol | Current XBraid stopping tolerance |
| iter | Current XBraid iteration (also equal to length of rnorms) |
| level | current level in XBraid |
| nrefine | number of refinements done |
| gupper | global size of the fine grid |
| calling_function | from which function are we accessing braid |
| basis | if Delta correction is set, tangent vectors to propagate across the interval |
| status | structure to initialize |

16.10.3.7 _braid_SyncStatusInit() braid_Int _braid_SyncStatusInit (

```
braid_Int iter,
braid_Int level,
braid_Int nrefine,
braid_Int gupper,
braid_Int done,
braid_Int calling_function,
braid_SyncStatus status )
```

Initialize a braid_SyncStatus structure

Parameters

| iter | current iteration in XBraid |
| :--- | :--- |
| level | current level in XBraid |
| nrefine | number of refinements done |
| gupper | global size of the fine grid |
| done | boolean describing whether XBraid has finished |
| calling_function | from which function are we accessing braid |
| status | structure to initialize |

### 16.11 tape.h File Reference

## Data Structures

- struct _braid_Tape
- struct _braid_Action


## Enumerations

- enum _braid_Call \{

STEP $=1$, INIT $=2$, CLONE $=3$, FREE $=4$,
SUM $=5$, BUFPACK $=6$, BUFUNPACK $=7$, ACCESS $=8$, OBJECTIVET = 9 \}

## Functions

- braid_Int _braid_TapeInit (_braid_Tape *head)
- _braid_Tape * _braid_TapePush (_braid_Tape $*$ head, void $*$ ptr)
- _braid_Tape * _braid_TapePop (_braid_Tape *head)
- braid_Int _braid_TapelsEmpty (_braid_Tape *head)
- braid_Int _braid_TapeGetSize (_braid_Tape *head)
- braid_Int _braid_TapeDisplayBackwards (braid_Core core, _braid_Tape *head, void(*fctptr)(braid_Core core, void *data_ptr))
- braid_Int _braid_TapeEvaluate (braid_Core core)
- braid_Int _braid_DiffCall (_braid_Action *action)
- braid_Int _braid_TapeSetSeed (braid_Core core)
- braid_Int _braid_TapeResetInput (braid_Core core)
- const char * _braid_CallGetName (_braid_Call call)


### 16.11.1 Detailed Description

Define the XBraid internal headers for the action-tape routines (linked list for AD)

### 16.11.2 Enumeration Type Documentation

16.11.2.1 _braid_Call enum _braid_Call

Enumerator for identifying performed action

Enumerator

| STEP |  |
| ---: | :--- |
| INIT |  |
| CLONE |  |
| FREE |  |
| SUM |  |
| BUFPACK |  |
| BUFUNPACK |  |
| ACCESS |  |
| OBJECTIVET |  |

### 16.11.3 Function Documentation

16.11.3.1 _braid_CallGetName() const char * _braid_CallGetName ( _braid_Call call )

Return the name of a _braid_Call (action name)
16.11.3.2 _braid_DiffCall() braid_Int _braid_DiffCall (
_braid_Action * action )
Call differentiated action
16.11.3.3 _braid_TapeDisplayBackwards() braid_Int _braid_TapeDisplayBackwards (
braid_Core core,
_braid_Tape * head,
void(*) (braid_Core core, void *data_ptr) fctptr )
Display the tape in reverse order, calls the display function at each element Input: - pointer to the braid core

- pointer to the display function


### 16.11.3.4 _braid_TapeEvaluate() braid_Int _braid_TapeEvaluate (

braid_Core core )
Evaluate the action tape in reverse order. This will clear the action tape! Input: - pointer to the braid core

- pointer to the head of the action tape
16.11.3.5 _braid_TapeGetSize() braid_Int _braid_TapeGetSize (
_braid_Tape * head )
Returns the number of elements in the tape
16.11.3.6 _braid_Tapelnit() braid_Int _braid_TapeInit (
_braid_Tape * head )
Initialize the tape Set head to NULL
16.11.3.7 _braid_TapelsEmpty() braid_Int _braid_TapeIsEmpty ( _braid_Tape * head )

Test if tape is empty return 1 if tape is empty, otherwise returns 0
16.11.3.8 _braid_TapePop() _braid_Tape * _braid_TapePop (
_braid_Tape * head)
Pop an element from the tape Return pointer to head
16.11.3.9 _braid_TapePush() _braid_Tape * _braid_TapePush ( _braid_Tape * head, void * ptr )

Push data on the tape Return pointer to head
16.11.3.10 _braid_TapeResetInput() braid_Int _braid_TapeResetInput (
braid_Core core )
Set the pointers in tapeinput to the input of an xbraid iteration (ua).
16.11.3.11 _braid_TapeSetSeed() braid_Int _braid_TapeSetSeed (
braid_Core core )
Set the adjoint seed for tape evaluation, i.e., set $u->$ bar at stored points on level 0 to the values contained in core->optim->adjoints

### 16.12 util.h File Reference

## Functions

- braid_Int _braid_ProjectInterval (braid_Int ilower, braid_Int iupper, braid_Int index, braid_Int stride, braid_Int *pilower, braid_Int *piupper)
- braid_Int_braid_GetInterval (braid_Core core, braid_Int level, braid_Int interval_index, braid_Int *flo_ptr, braid_Int *fhi_ptr, braid_Int *ci_ptr)
- braid_Int _braid_SetVerbosity (braid_Core core, braid_Int verbose_adj)
- braid_Int _braid_printf (const char *format,...)
- braid_Int _braid_ParFprintfFlush (FILE *file, braid_Int myid, const char *message,...)
- braid_Int _braid_Max (braid_Real *array, braid_Int size, braid_Real *max_val)
- braid_Int _braid_GetNEntries (braid_Real *_array, braid_Int array_len, braid_Int *k_ptr, braid_Real *array)
- braid_Real _braid_MPI_Wtime (braid_Core core, braid_Int timing_level)


### 16.12.1 Detailed Description

Define XBraid internal headers for utility routines.
This file contains the headers for utility routines. Essentially, if a routine does not take braid_Core (or other XBraid specific structs) as an argument, then it's a utility routine.

### 16.12.2 Function Documentation

### 16.12.2.1 _braid_GetInterval() braid_Int _braid_GetInterval (

```
braid_Core core,
    braid_Int level,
    braid_Int interval_index,
    braid_Int * flo_ptr,
    braid_Int * fhi_ptr,
    braid_Int * ci_ptr )
```

Retrieve the time step indices at this level corresponding to a local FC interval given by interval_index. Argument ci_ptr is the time step index for the C-pt and flo_ptr and fhi_ptr are the smallest and largest F-pt indices in this interval. The C-pt is always to the right of the F-interval, but neither a C-pt or an F-interval are guaranteed. If the ci_ptr returns a-1, there is no C-pt. If the flo_ptr is greater than the fhi_ptr, there is no F-interval.

### 16.12.2.2 _braid_GetNEntries() braid_Int _braid_GetNEntries (

```
    braid_Real * _array,
    braid_Int array_len,
    braid_Int * k_ptr,
    braid_Real * array )
```

Copy $k$ entries from $*$ _array* into array. If $k$ is negative, return the last $k$ entries. If positive, return the first $k$ entries. Upon exit, $k$ holds the number of residuals actually returned (in the case that $|\mathrm{k}|>$ array_len.)

If no entries are copied, $k=0, \operatorname{array}[0]=-1.0$
16.12.2.3 _braid_Max() braid_Int _braid_Max (
braid_Real * array,
braid_Int size,
braid_Real * max_val )
This function finds the maximum value in a braid_Real array

### 16.12.2.4 _braid_MPI_Wtime() braid_Real _braid_MPI_Wtime ( <br> braid_Core core, <br> braid_Int timing_level )

Wrap MPI_Wtime. If core->timings >=timing_level, then return MPI_Wtime(), otherwise, return -1. This allows us to time Braid more intrusively, as timing_level increases.

```
16.12.2.5 _braid_ParFprintfFlush() braid_Int _braid_ParFprintfFlush (
    FILE * file,
    braid_Int myid,
    const char * message,
    ... )
```

This is a function that allows for "sane" printing of information in parallel. Currently, only myid $=0$ prints, but this can be updated as needs change.

The string message is printed and can be multiple parameters in the standard $*$ C-format, like message $=$ '\%1.2e is a format string', 1.24
16.12.2.6 _braid_printf() braid_Int _braid_printf (

> const char * format,
... )

If set, print to _braid_printfile and then flush.
Else print to standard out.
The string format can be multiple parameters in the standard * C-format, like
format $=$ '\%1.2e is a format string', 1.24
16.12.2.7 _braid_ProjectInterval() braid_Int _braid_ProjectInterval (

```
braid_Int ilower,
braid_Int iupper,
braid_Int index,
braid_Int stride,
braid_Int * pilower,
braid_Int * piupper )
```

Project an interval onto a strided index space that contains the index 'index' and has stride 'stride'. An empty projection is represented by ilower > iupper.
16.12.2.8 _braid_SetVerbosity() braid_Int _braid_SetVerbosity (

```
braid_Core core,
    braid_Int verbose_adj )
```

Switch for displaying the XBraid actions. Used for debugging only.

## Index

```
_braid.h, }17
    _braid_AccessVector, 180
    _braid_CTAlloc, 177
    _braid_Coarsen,180
    _braid_CommHandleElt, 177
    _braid_CommRecvInit, 181
    _braid_CommSendlnit, 181
    _braid_CommWait, 181
    _braid_ComputeFullRNorm, 181
    _braid_CopyFineToCoarse, 181
    _braid_CoreElt,177
    _braid_CoreFcn, 177
    _braid_Drive, 181
    _braid_Error, 178
    _braid_ErrorHandler, 182
    _braid_ErrorInArg, 178
    _braid_FASResidual, 182
    _braid_FAccess, 182
    _braid_FCRelax, 182
    _braid_FInterp, 182
    _braid_FRefine,183
    _braid_FRefineSpace,183
    _braid_FRestrict, 183
    _braid_FinalizeErrorEstimates,182
    _braid_GetBlockDistInterval, 183
    _braid_GetBlockDistProc,184
    _braid_GetCFactor, 184
    _braid_GetDistribution,184
    _braid_GetDtk, 184
    _braid_GetFullRNorm, 184
    _braid_GetProc, 184
    _braid_GetRNorm, 185
    _braid_GetUInit,185
    _braid_GridClean, 185
    _braid_GridDestroy, 185
    _braid_GridElt, 178
    _braid_Gridlnit,185
    _braid_InitGuess,185
    _braid_InitHierarchy,186
    _braid_IsCPoint,178
    _braid_IsFPoint, 178
    _braid_MapCoarseToFine,178
    _braid_MapFineToCoarse, 178
    _braid_MapPeriodic,179
    _braid_NextCPoint, 179
    _braid_PrintSpatialNorms,186
    _braid_PriorCPoint, 179
    _braid_RecvIndexNull, 179
    _braid_Refine, 186
    _braid_RefineBasic, 186
    _braid_Residual, 186
    _braid_SendlndexNull, 179
    _braid_SetFullRNorm, 187
    _braid_SetRNorm, 187
    _braid_Step,187
    _braid_Sync,187
    _braid_TAlloc, 179
    _braid_TFree, 180
    _braid_TReAlloc,180
    _braid_UCommlnit,187
    _braid_UCommInitBasic,187
    _braid_UCommInitF,188
    _braid_UCommWait, 188
    _braid_UGetIndex, 188
    _braid_UGetLast,188
    _braid_UGetVector,}18
    _braid_UGetVectorRef, 188
    _braid_USetVector,189
    _braid_USetVectorRef,189
    _braid_error_flag, 189
    _braid_isnan, 178
    _braid_max, 179
    _braid_min, 179
    _braid_printfile, 189
    braid_BaseVector, 180
    braid_Basis,180
    braid_VectorBar,180
_braid_AccessStatusInit
    status.h, }21
_braid_AccessVector
    _braid.h, 180
_braid_Action,140
    braid_iter, }14
    braidCall, 140
    core, }14
    gupper, }14
    inTime, 141
    inTimeldx, }14
    level, 141
    messagetype, }14
    myid, 141
    nrefine, }14
    outTime, 141
    send_recv_rank, 141
    size_buffer, 141
    sum_alpha, 142
    sum_beta, 142
    tol,}14
_braid_AddToObjective
    adjoint.h, }19
_braid_AdjointFeatureCheck
    adjoint.h, 190
```

_braid_BaseAccess base.h, 192
_braid_BaseBufAlloc base.h, 193
_braid_BaseBufFree base.h, 193
_braid_BaseBufPack base.h, 193
_braid_BaseBufPack_diff base.h, 194
_braid_BaseBufSize base.h, 194
_braid_BaseBufUnpack base.h, 194
_braid_BaseBufUnpack_diff base.h, 195
_braid_BaseClone base.h, 195
_braid_BaseCloneBasis base.h, 196
_braid_BaseClone_diff base.h, 196
_braid_BaseFree
base.h, 196
_braid_BaseFreeBasis base.h, 196
_braid_BaseFullResidual base.h, 197
_braid_Baselnit base.h, 197
_braid_BaselnitBasis base.h, 198
_braid_Baselnit_diff base.h, 198
_braid_BaselnnerProd base.h, 198
_braid_BaseObjectiveT base.h, 199
_braid_BaseObjectiveT_diff base.h, 199
_braid_BaseResidual base.h, 199
_braid_BaseSClone base.h, 200
_braid_BaseSCoarsen base.h, 200
_braid_BaseSFree base.h, 200
_braid_BaseSInit base.h, 201
_braid_BaseSRefine base.h, 201
_braid_BaseSpatialNorm base.h, 201
_braid_BaseStep
base.h, 202
_braid_BaseStep_diff base.h, 202
_braid_BaseSum base.h, 203
_braid_BaseSumBasis base.h, 203
_braid_BaseSum_diff base.h, 203
_braid_BaseSync base.h, 204
_braid_BaseTimeGrid base.h, 204
_braid_BaseVector, 142
bar, 142
basis, 143
userVector, 143
_braid_Basis, 143
rank, 143
userVecs, 143
_braid_BufferStatusInit status.h, 219
_braid_CTAlloc _braid.h, 177
_braid_Call
tape.h, 222
_braid_CallGetName tape.h, 223
_braid_Coarsen _braid.h, 180
_braid_CoarsenRefStatusInit status.h, 220
_braid_CommHandle, 144
buffer, 144
index, 144
level, 144
num_requests, 144
request_type, 145
requests, 145
status, 145
vector_ptr, 145
_braid_CommHandleElt
_braid.h, 177
_braid_CommRecvInit
_braid.h, 181
_braid_CommSendInit
_braid.h, 181
_braid_CommWait
_braid.h, 181
_braid_ComputeFullRNorm
_braid.h, 181
_braid_CopyFineToCoarse
_braid.h, 181

```
_braid_Core, 145
    access, }14
    access_level, 148
    actionTape, }14
    adjoint,149
    app,}14
    barTape, 149
    bufalloc, 149
    buffree, }14
    bufpack, }14
    bufsize, 149
    bufunpack, }14
    c_tprior, 149
    c_tstop, 150
    calling_function, 150
    cfactors, 150
    cfdefault, 150
    clone, 150
    comm, }15
    comm_world, 150
    CWt_default, 150
    CWts, }15
    delta_correct, 150
    delta_defer_iter, 151
    delta_defer_Ivl, 151
    delta_rank, 151
    done, 151
    dtk, 151
    est_error, 151
    estimate, 151
    estimate_lyap, 151
    f_tprior, 151
    f_tstop, 151
    finalFCrelax, 152
    fmg, }15
    free, }15
    full_rnorm0, 152
    full_rnorm_res, 152
    full_rnorms, 152
    globaltime, 152
    grids, }15
    gupper, 152
    idx, 152
    incr_max_levels, 153
    init, 153
    init_basis, 153
    initiali, 153
    inner_prod, 153
    io_level, 153
    level, 153
    local_exponents, 153
    localtime, 153
    Ivectors, 153
    lyap_exp,154
```

max_iter, 154
max_levels, 154
max_refinements, 154
messagetype, 154
min_coarse, 154
myid, 154
myid_world, 154
nfmg, 154
nfmg_Vcyc, 154
niter, 155
nlevels, 155
nrdefault, 155
nrefine, 155
nrels, 155
ntime, 155
obj_only, 155
objectiveT, 155
objT_diff, 155
old_fine_tolx, 155
optim, 156
order, 156
periodic, 156
postprocess_obj, 156
postprocess_obj_diff, 156
print_level, 156
r_space, 156
rdtvalues, 156
record, 156
refine, 156
relax_lyap, 157
relax_only_cg, 157
reset_gradient, 157
residual, 157
reverted_ranks, 157
rfactor, 157
rfactors, 157
richardson, 157
rnorm, 157
rnorm0, 157
rnorms, 158
rstopped, 158
rtol, 158
sclone, 158
scoarsen, 158
send_recv_rank, 158
seq_soln, 158
sfree, 158
sinit, 158
size_basis, 158
size_buffer, 159
skip, 159
spatialnorm, 159
srefine, 159
step, 159
step_diff, 159
storage, 159
sum, 159
sync, 159
t, 159
tgrid, 160
tight_fine_tolx, 160
timer_coarse_solve, 160
timer_drive_init, 160
timer_file_stem, 160
timer_file_stem_len, 160
timer_MPI_recv, 160
timer_MPI_send, 160
timer_MPI_wait, 160
timer_MPI_wait_coarse, 160
timer_user_access, 161
timer_user_bufpack, 161
timer_user_bufsize, 161
timer_user_bufunpack, 161
timer_user_clone, 161
timer_user_free, 161
timer_user_init, 161
timer_user_innerprod, 161
timer_user_residual, 161
timer_user_scoarsen, 161
timer_user_spatialnorm, 162
timer_user_srefine, 162
timer_user_step, 162
timer_user_sum, 162
timer_user_sync, 162
timings, 162
tnext, 162
tnorm, 162
tnorm_a, 162
tol, 162
tpoints_cutoff, 163
tstart, 163
tstop, 163
userVectorTape, 163
useshell, 163
verbose_adj, 163
warm_restart, 163
wrapper_test, 163
_braid_CoreElt
_braid.h, 177
_braid_CoreFcn
_braid.h, 177
_braid_DeltaFeatureCheck
delta.h, 217
_braid_DiffCall
tape.h, 223
_braid_DoDeltaCorrect
delta.h, 216
_braid_Drive
_braid.h, 181
_braid_Error
_braid.h, 178
_braid_ErrorHandler
_braid.h, 182
_braid_ErrorlnArg
_braid.h, 178
_braid_EvalObjective
adjoint.h, 190
_braid_EvalObjective_diff adjoint.h, 190
_braid_FASResidual
_braid.h, 182
_braid_FAccess
_braid.h, 182
_braid_FCRelax
_braid.h, 182
_braid_FInterp
_braid.h, 182
_braid_FRefine
_braid.h, 183
_braid_FRefineSpace
_braid.h, 183
_braid_FRestrict
_braid.h, 183
_braid_FinalizeErrorEstimates
_braid.h, 182
_braid_GetBlockDistInterval
_braid.h, 183
_braid_GetBlockDistProc
_braid.h, 184
_braid_GetCFactor
_braid.h, 184
_braid_GetDistribution
_braid.h, 184
_braid_GetDtk
_braid.h, 184
_braid_GetFullRNorm
_braid.h, 184
_braid_GetInterval util.h, 225
_braid_GetNEntries util.h, 225
_braid_GetProc
_braid.h, 184
_braid_GetRNorm
_braid.h, 185
_braid_GetUInit
_braid.h, 185
_braid_GramSchmidt delta.h, 217
_braid_Grid, 163
cfactor, 164
clower, 164
cupper, 164
fa, 164
fa_alloc, 165
gupper, 165
ilower, 165
iupper, 165
level, 165
ncpoints, 165
nupoints, 165
recv_handle, 165
recv_index, 165
send_handle, 165
send_index, 166
ta, 166
ta_alloc, 166
ua, 166
ua_alloc, 166
ulast, 166
va, 166
va_alloc, 166
_braid_GridClean
_braid.h, 185
_braid_GridDestroy
_braid.h, 185
_braid_GridElt
_braid.h, 178
_braid_Gridlnit
_braid.h, 185
_braid_InitAdjointVars
adjoint.h, 190
_braid_InitGuess
_braid.h, 185
_braid_InitHierarchy
_braid.h, 186
_braid_IsCPoint
_braid.h, 178
_braid_IsFPoint
_braid.h, 178
_braid_LRDeltaDot
delta.h, 217
_braid_LRDeltaDotMat
delta.h, 217
_braid_MPI_Wtime
util.h, 225
_braid_MapCoarseToFine _braid.h, 178
_braid_MapFineToCoarse
_braid.h, 178
_braid_MapPeriodic
_braid.h, 179
_braid_Max
util.h, 225
_braid_NextCPoint
_braid.h, 179
_braid_ObjectiveStatusInit status.h, 220
_braid_OptimDestroy adjoint.h, 190
_braid_ParFprintfFlush util.h, 225
_braid_PrintSpatialNorms _braid.h, 186
_braid_PriorCPoint _braid.h, 179
_braid_ProjectInterval util.h, 226
_braid_RecvIndexNull _braid.h, 179
_braid_Refine _braid.h, 186
_braid_RefineBasic
_braid.h, 186
_braid_Residual _braid.h, 186
_braid_SendIndexNull _braid.h, 179
_braid_SetFullRNorm _braid.h, 187
_braid_SetRNorm _braid.h, 187
_braid_SetRNormAdjoint adjoint.h, 190
_braid_SetVerbosity util.h, 226
_braid_Status, 166 core, 167
_braid_StatusDestroy status.h, 220
_braid_StatusElt status.h, 218
_braid_Step _braid.h, 187
_braid_StepStatusInit status.h, 221
_braid_Sync _braid.h, 187
_braid_SyncStatusInit status.h, 221
_braid_TAlloc _braid.h, 179
_braid_TFree _braid.h, 180
_braid_TReAlloc _braid.h, 180
_braid_Tape, 167
data_ptr, 167
next, 167
size, 168
_braid_TapeDisplayBackwards tape.h, 223
_braid_TapeEvaluate tape.h, 223
_braid_TapeGetSize tape.h, 223
_braid_Tapelnit tape.h, 224
_braid_TapelsEmpty tape.h, 224
_braid_TapePop tape.h, 224
_braid_TapePush tape.h, 224
_braid_TapeResetInput tape.h, 224
_braid_TapeSetSeed tape.h, 224
_braid_UCommInit _braid.h, 187
_braid_UCommInitBasic _braid.h, 187
_braid_UCommInitF _braid.h, 188
_braid_UCommWait _braid.h, 188
_braid_UGetIndex _braid.h, 188
_braid_UGetLast _braid.h, 188
_braid_UGetVector _braid.h, 188
_braid_UGetVectorRef _braid.h, 188
_braid_USetVector _braid.h, 189
_braid_USetVectorRef _braid.h, 189
_braid_UpdateAdjoint adjoint.h, 191
_braid_UseDeltaCorrect delta.h, 216
_braid_Vector braid_defs.h, 209
_braid_VectorBar, 168 useCount, 168 userVector, 168
_braid_VectorBarCopy adjoint.h, 191
_braid_VectorBarDelete adjoint.h, 191
_braid_error_flag _braid.h, 189
_braid_isnan
_braid.h, 178
_braid_max
_braid.h, 179
_braid_min
_braid.h, 179
_braid_printf
util.h, 226
_braid_printfile _braid.h, 189

## ACCESS

tape.h, 223
access
_braid_Core, 148
access_level
_braid_Core, 148
ACCESSOR_HEADER_GET1
braid_status.h, 213
ACCESSOR_HEADER_GET1_IN1
braid_status.h, 213
ACCESSOR_HEADER_GET1_IN2
braid_status.h, 213
ACCESSOR_HEADER_GET1_IN3
braid_status.h, 213
ACCESSOR_HEADER_GET2
braid_status.h, 213
ACCESSOR_HEADER_GET2_IN1
braid_status.h, 214
ACCESSOR_HEADER_GET3
braid_status.h, 214
ACCESSOR_HEADER_GET4
braid_status.h, 214
ACCESSOR_HEADER_GET5
braid_status.h, 214
ACCESSOR_HEADER_SET1
braid_status.h, 214
actionTape
_braid_Core, 149
adjoint
_braid_Core, 149
adjoint.h, 189
_braid_AddToObjective, 190
_braid_AdjointFeatureCheck, 190
_braid_EvalObjective, 190
_braid_EvalObjective_diff, 190
_braid_InitAdjointVars, 190
_braid_OptimDestroy, 190
_braid_SetRNormAdjoint, 190
_braid_UpdateAdjoint, 191
_braid_VectorBarCopy, 191
_braid_VectorBarDelete, 191
adjoints
braid_Optim, 172
app
_braid_Core, 149
bar
_braid_BaseVector, 142
barTape
_braid_Core, 149
base.h, 191
_braid_BaseAccess, 192
_braid_BaseBufAlloc, 193
_braid_BaseBufFree, 193
_braid_BaseBufPack, 193
_braid_BaseBufPack_diff, 194
_braid_BaseBufSize, 194
_braid_BaseBufUnpack, 194
_braid_BaseBufUnpack_diff, 195
_braid_BaseClone, 195
_braid_BaseCloneBasis, 196
_braid_BaseClone_diff, 196
_braid_BaseFree, 196
_braid_BaseFreeBasis, 196
_braid_BaseFullResidual, 197
_braid_Baselnit, 197
_braid_BaselnitBasis, 198
_braid_Baselnit_diff, 198
_braid_BaseInnerProd, 198
_braid_BaseObjectiveT, 199
_braid_BaseObjectiveT_diff, 199
_braid_BaseResidual, 199
_braid_BaseSClone, 200
_braid_BaseSCoarsen, 200
_braid_BaseSFree, 200
_braid_BaseSInit, 201
_braid_BaseSRefine, 201
_braid_BaseSpatialNorm, 201
_braid_BaseStep, 202
_braid_BaseStep_diff, 202
_braid_BaseSum, 203
_braid_BaseSumBasis, 203
_braid_BaseSum_diff, 203
_braid_BaseSync, 204
_braid_BaseTimeGrid, 204
basis
_braid_BaseVector, 143
braid.h, 205
braid_AccessStatus, 169
status, 169
braid_AccessStatusGetBasisVec
Inherited XBraid status routines, 119
braid_AccessStatusGetCallingFunction
Inherited XBraid status routines, 119
braid_AccessStatusGetDeltaRank
Inherited XBraid status routines, 119
braid_AccessStatusGetDone
Inherited XBraid status routines, 119
braid_AccessStatusGetlter Inherited XBraid status routines, 119
braid_AccessStatusGetLevel Inherited XBraid status routines, 119
braid_AccessStatusGetLocalLyapExponents Inherited XBraid status routines, 119
braid_AccessStatusGetNLevels Inherited XBraid status routines, 120
braid_AccessStatusGetNRefine Inherited XBraid status routines, 120
braid_AccessStatusGetNTPoints Inherited XBraid status routines, 120
braid_AccessStatusGetResidual Inherited XBraid status routines, 120
braid_AccessStatusGetSingleErrorEstAccess Inherited XBraid status routines, 120
braid_AccessStatusGetT Inherited XBraid status routines, 120
braid_AccessStatusGetTILD Inherited XBraid status routines, 120
braid_AccessStatusGetTIndex Inherited XBraid status routines, 121
braid_AccessStatusGetWrapperTest Inherited XBraid status routines, 121
braid_App
User-written routines, 65
braid_ASCaller_BaseStep_diff XBraid status macros, 129
braid_ASCaller_ComputeFullRNorm XBraid status macros, 130
braid_ASCaller_Drive_AfterInit XBraid status macros, 130
braid_ASCaller_Drive_TopCycle XBraid status macros, 130
braid_ASCaller_FAccess XBraid status macros, 130
braid_ASCaller_FASResidual XBraid status macros, 130
braid_ASCaller_FCRelax XBraid status macros, 130
braid_ASCaller_FInterp XBraid status macros, 130
braid_ASCaller_FRefine XBraid status macros, 130
braid_ASCaller_FRefine_AfterInitHier XBraid status macros, 130
braid_ASCaller_FRestrict XBraid status macros, 130
braid_ASCaller_InitGuess XBraid status macros, 131
braid_ASCaller_Residual XBraid status macros, 131
braid_BaseVector _braid.h, 180
braid_Basis
_braid.h, 180
braid_BufferStatus, 169
status, 170
braid_BufferStatusGetLevel
Inherited XBraid status routines, 121
braid_BufferStatusGetMessageType
Inherited XBraid status routines, 121
braid_BufferStatusGetTIndex
Inherited XBraid status routines, 121
braid_BufferStatusSetBasisSize
Inherited XBraid status routines, 121
braid_BufferStatusSetSize
Inherited XBraid status routines, 121
braid_Byte
braid_defs.h, 209
braid_CoarsenRefStatus, 170
status, 170
braid_CoarsenRefStatusGetCTprior
Inherited XBraid status routines, 122
braid_CoarsenRefStatusGetCTstop
Inherited XBraid status routines, 122
braid_CoarsenRefStatusGetFTprior
Inherited XBraid status routines, 122
braid_CoarsenRefStatusGetFTstop Inherited XBraid status routines, 122
braid_CoarsenRefStatusGetlter
Inherited XBraid status routines, 122
braid_CoarsenRefStatusGetLevel Inherited XBraid status routines, 122
braid_CoarsenRefStatusGetNLevels Inherited XBraid status routines, 122
braid_CoarsenRefStatusGetNRefine Inherited XBraid status routines, 122
braid_CoarsenRefStatusGetNTPoints
Inherited XBraid status routines, 123
braid_CoarsenRefStatusGetT
Inherited XBraid status routines, 123
braid_CoarsenRefStatusGetTIndex
Inherited XBraid status routines, 123
braid_CoarsenRefStatusGetTpriorTstop
Inherited XBraid status routines, 123
braid_Core
General Interface routines, 78
braid_defs.h, 208
_braid_Vector, 209
braid_Byte, 209
braid_Int, 209
braid_Int_Max, 208
braid_Int_Min, 208
braid_MPI_Comm, 208
braid_MPI_INT, 208
braid_MPI_REAL, 208
braid_Real, 209
braid_Vector, 209
braid_Destroy
General Interface routines, 78
braid_Drive
General Interface routines, 79
braid_ERROR_ARG
Error Codes, 64
braid_ERROR_GENERIC
Error Codes, 64
braid_ERROR_MEMORY
Error Codes, 64
braid_FMANGLE
Fortran 90 interface options, 63
braid_Fortran_Residual
Fortran 90 interface options, 63
braid_Fortran_SpatialCoarsen
Fortran 90 interface options, 63
braid_Fortran_Sync
Fortran 90 interface options, 63
braid_Fortran_TimeGrid
Fortran 90 interface options, 63
braid_GetMyID
General Interface routines, 79
braid_GetNLevels
General Interface routines, 79
braid_GetNumlter
General Interface routines, 79
braid_GetObjective
Interface routines for XBraid_Adjoint, 98
braid_GetRNormAdjoint
Interface routines for XBraid_Adjoint, 98
braid_GetRNorms
General Interface routines, 80
braid_GetSpatialAccuracy
General Interface routines, 80
braid_Init
General Interface routines, 81
braid_InitAdjoint
Interface routines for XBraid_Adjoint, 99
braid_Int
braid_defs.h, 209
braid_Int_Max
braid_defs.h, 208
braid_Int_Min
braid_defs.h, 208
braid_INVALID_RNORM
Error Codes, 64
braid_iter
_braid_Action, 140
braid_MPI_Comm
braid_defs.h, 208
braid_MPI_INT
braid_defs.h, 208
braid_MPI_REAL
braid_defs.h, 208
braid_ObjectiveStatus, 171
status, 171
braid_ObjectiveStatusGetlter
Inherited XBraid status routines, 123
braid_ObjectiveStatusGetLevel
Inherited XBraid status routines, 123
braid_ObjectiveStatusGetNLevels
Inherited XBraid status routines, 123
braid_ObjectiveStatusGetNRefine
Inherited XBraid status routines, 124
braid_ObjectiveStatusGetNTPoints
Inherited XBraid status routines, 124
braid_ObjectiveStatusGetT
Inherited XBraid status routines, 124
braid_ObjectiveStatusGetTIndex
Inherited XBraid status routines, 124
braid_ObjectiveStatusGetTol
Inherited XBraid status routines, 124
braid_Optim, 171
adjoints, 172
f_bar, 172
objective, 172
request, 172
rnorm, 172
rnorm0, 172
rnorm0_adj, 172
rnorm_adj, 172
rtol_adj, 172
sendbuffer, 173
sum_user_obj, 173
tapeinput, 173
tol_adj, 173
tstart_obj, 173
tstop_obj, 173
braid_PrintStats
General Interface routines, 82
braid_PrintTimers
General Interface routines, 82
braid_PtFcnAccess
User-written routines, 65
braid_PtFcnBufAlloc
User-written routines, 66
braid_PtFcnBufFree
User-written routines, 66
braid_PtFcnBufPack
User-written routines, 67
braid_PtFcnBufSize
User-written routines, 67
braid_PtFcnBufUnpack
User-written routines, 67
braid_PtFcnClone
User-written routines, 68
braid_PtFcnFree

User-written routines, 68
braid_PtFcnInit
User-written routines, 68
braid_PtFcnInitBasis
User-written routines, 68
braid_PtFcnInnerProd
User-written routines, 69
braid_PtFcnObjectiveT
User-written routines for XBraid_Adjoint, 74
braid_PtFcnObjectiveTDiff
User-written routines for XBraid_Adjoint, 74
braid_PtFcnPostprocessObjective
User-written routines for XBraid_Adjoint, 74
braid_PtFcnPostprocessObjective_diff
User-written routines for XBraid_Adjoint, 75
braid_PtFcnResetGradient
User-written routines for XBraid_Adjoint, 75
braid_PtFcnResidual
User-written routines, 69
braid_PtFcnSClone
User-written routines, 69
braid_PtFcnSCoarsen
User-written routines, 70
braid_PtFcnSFree
User-written routines, 70
braid_PtFcnSInit
User-written routines, 70
braid_PtFcnSpatialNorm
User-written routines, 71
braid_PtFcnSRefine
User-written routines, 71
braid_PtFcnStep
User-written routines, 72
braid_PtFcnStepDiff
User-written routines for XBraid_Adjoint, 75
braid_PtFcnSum
User-written routines, 72
braid_PtFcnSync
User-written routines, 72
braid_PtFcnTimeGrid
User-written routines, 73
braid_Rand
General Interface routines, 82
braid_RAND_MAX
General Interface routines, 78
braid_Real
braid_defs.h, 209
braid_ResetTimer
General Interface routines, 82
braid_SetAbsTol
General Interface routines, 83
braid_SetAbsToIAdjoint
Interface routines for XBraid_Adjoint, 99
braid_SetAccessLevel

General Interface routines, 83
braid_SetBufAllocFree
General Interface routines, 83
braid_SetCFactor
General Interface routines, 84
braid_SetCRelaxWt
General Interface routines, 84
braid_SetDefaultPrintFile
General Interface routines, 84
braid_SetDeferDelta
General Interface routines, 85
braid_SetDeltaCorrection
General Interface routines, 85
braid_SetFileIOLevel
General Interface routines, 86
braid_SetFinalFCRelax
General Interface routines, 86
braid_SetFMG
General Interface routines, 86
braid_SetFullRNormRes
General Interface routines, 86
braid_SetIncrMaxLevels
General Interface routines, 87
braid_SetInnerProd
General Interface routines, 87
braid_SetLyapunovEstimation
General Interface routines, 87
braid_SetMaxIter
General Interface routines, 88
braid_SetMaxLevels
General Interface routines, 88
braid_SetMaxRefinements
General Interface routines, 88
braid_SetMinCoarse
General Interface routines, 89
braid_SetNFMG
General Interface routines, 89
braid_SetNFMGVcyc
General Interface routines, 89
braid_SetNRelax
General Interface routines, 89
braid_SetObjectiveOnly
Interface routines for XBraid_Adjoint, 99
braid_SetPeriodic
General Interface routines, 90
braid_SetPostprocessObjective
Interface routines for XBraid_Adjoint, 100
braid_SetPostprocessObjective_diff
Interface routines for XBraid_Adjoint, 100
braid_SetPrintFile
General Interface routines, 90
braid_SetPrintLevel
General Interface routines, 90
braid_SetRefine

General Interface routines, 91
braid_SetRelaxOnlyCG
General Interface routines, 91
braid_SetRelTol
General Interface routines, 91
braid_SetRelTolAdjoint
Interface routines for XBraid_Adjoint, 100
braid_SetResidual
General Interface routines, 92
braid_SetRevertedRanks
Interface routines for XBraid_Adjoint, 100
braid_SetRichardsonEstimation
General Interface routines, 92
braid_SetSeqSoln
General Interface routines, 93
braid_SetShell
General Interface routines, 93
braid_SetSkip
General Interface routines, 93
braid_SetSpatialCoarsen
General Interface routines, 94
braid_SetSpatialRefine
General Interface routines, 94
braid_SetStorage
General Interface routines, 94
braid_SetSync
General Interface routines, 95
braid_SetTemporalNorm
General Interface routines, 95
braid_SetTimeGrid
General Interface routines, 96
braid_SetTimerFile
General Interface routines, 96
braid_SetTimings
General Interface routines, 96
braid_SetTPointsCutoff
General Interface routines, 96
braid_SetTStartObjective
Interface routines for XBraid_Adjoint, 101
braid_SetTStopObjective
Interface routines for XBraid_Adjoint, 101
braid_SplitCommworld
General Interface routines, 97
braid_status.h, 209
ACCESSOR_HEADER_GET1, 213
ACCESSOR_HEADER_GET1_IN1, 213
ACCESSOR_HEADER_GET1_IN2, 213
ACCESSOR_HEADER_GET1_IN3, 213
ACCESSOR_HEADER_GET2, 213
ACCESSOR_HEADER_GET2_IN1, 214
ACCESSOR_HEADER_GET3, 214
ACCESSOR_HEADER_GET4, 214
ACCESSOR_HEADER_GET5, 214
ACCESSOR_HEADER_SET1, 214
braid_StatusGetAllErrorEst
XBraid status routines, 102
braid_StatusGetBasisVec
XBraid status routines, 103
braid_StatusGetCallingFunction
XBraid status routines, 103
braid_StatusGetCTprior
XBraid status routines, 103
braid_StatusGetCTstop
XBraid status routines, 104
braid_StatusGetDeltaRank
XBraid status routines, 104
braid_StatusGetDone
XBraid status routines, 104
braid_StatusGetFTprior
XBraid status routines, 105
braid_StatusGetFTstop
XBraid status routines, 105
braid_StatusGetlter
XBraid status routines, 105
braid_StatusGetLevel
XBraid status routines, 106
braid_StatusGetLocalLyapExponents
XBraid status routines, 106
braid_StatusGetMessageType
XBraid status routines, 106
braid_StatusGetNLevels
XBraid status routines, 106
braid_StatusGetNRefine
XBraid status routines, 107
braid_StatusGetNTPoints
XBraid status routines, 107
braid_StatusGetNumErrorEst
XBraid status routines, 107
braid_StatusGetOldFineTolx
XBraid status routines, 108
braid_StatusGetProc
XBraid status routines, 108
braid_StatusGetResidual
XBraid status routines, 108
braid_StatusGetRNorms
XBraid status routines, 109
braid_StatusGetSingleErrorEstAccess
XBraid status routines, 109
braid_StatusGetSingleErrorEstStep
XBraid status routines, 109
braid_StatusGetT
XBraid status routines, 110
braid_StatusGetTComm
XBraid status routines, 110
braid_StatusGetTILD
XBraid status routines, 110
braid_StatusGetTimeValues
XBraid status routines, 111
braid StatusGetTIndex
XBraid status routines, 111
braid_StatusGetTIUL XBraid status routines, 112
braid_StatusGetTol
XBraid status routines, 112
braid_StatusGetTpriorTstop
XBraid status routines, 112
braid_StatusGetTstartTstop
XBraid status routines, 113
braid_StatusGetTstop
XBraid status routines, 113
braid_StatusGetWrapperTest
XBraid status routines, 113
braid_StatusSetBasisSize
XBraid status routines, 114
braid_StatusSetOldFineTolx
XBraid status routines, 114
braid_StatusSetRefinementDtValues
XBraid status routines, 114
braid_StatusSetRFactor
XBraid status routines, 116
braid_StatusSetRSpace
XBraid status routines, 116
braid_StatusSetSize
XBraid status routines, 116
braid_StatusSetTightFineTolx
XBraid status routines, 117
braid_StepStatus, 173
status, 174
braid_StepStatusGetBasisVec
Inherited XBraid status routines, 124
braid_StepStatusGetCallingFunction
Inherited XBraid status routines, 124
braid_StepStatusGetDeltaRank
Inherited XBraid status routines, 124
braid_StepStatusGetDone
Inherited XBraid status routines, 125
braid_StepStatusGetIter
Inherited XBraid status routines, 125
braid_StepStatusGetLevel
Inherited XBraid status routines, 125
braid_StepStatusGetNLevels
Inherited XBraid status routines, 125
braid_StepStatusGetNRefine
Inherited XBraid status routines, 125
braid_StepStatusGetNTPoints
Inherited XBraid status routines, 125
braid_StepStatusGetOldFineTolx
Inherited XBraid status routines, 125
braid_StepStatusGetRNorms
Inherited XBraid status routines, 125
braid_StepStatusGetSingleErrorEstStep
Inherited XBraid status routines, 126
braid_StepStatusGetT
Inherited XBraid status routines, 126
braid_StepStatusGetTIndex
Inherited XBraid status routines, 126
braid_StepStatusGetTIUL
Inherited XBraid status routines, 126
braid_StepStatusGetTol
Inherited XBraid status routines, 126
braid_StepStatusGetTstartTstop
Inherited XBraid status routines, 126
braid_StepStatusGetTstop
Inherited XBraid status routines, 126
braid_StepStatusSetOldFineTolx
Inherited XBraid status routines, 127
braid_StepStatusSetRFactor
Inherited XBraid status routines, 127
braid_StepStatusSetRSpace
Inherited XBraid status routines, 127
braid_StepStatusSetTightFineTolx
Inherited XBraid status routines, 127
braid_SyncStatus, 174
status, 174
braid_SyncStatusGetAllErrorEst
Inherited XBraid status routines, 127
braid_SyncStatusGetCallingFunction
Inherited XBraid status routines, 127
braid_SyncStatusGetDone
Inherited XBraid status routines, 127
braid_SyncStatusGetlter
Inherited XBraid status routines, 127
braid_SyncStatusGetLevel
Inherited XBraid status routines, 128
braid_SyncStatusGetNLevels
Inherited XBraid status routines, 128
braid_SyncStatusGetNRefine
Inherited XBraid status routines, 128
braid_SyncStatusGetNTPoints
Inherited XBraid status routines, 128
braid_SyncStatusGetNumErrorEst
Inherited XBraid status routines, 128
braid_SyncStatusGetProc
Inherited XBraid status routines, 128
braid_SyncStatusGetTComm
Inherited XBraid status routines, 128
braid_SyncStatusGetTimeValues
Inherited XBraid status routines, 129
braid_SyncStatusGetTIUL
Inherited XBraid status routines, 129
braid_test.h, 215
braid_TestAll
XBraid test routines, 132
braid_TestBuf
XBraid test routines, 133
braid_TestClone

XBraid test routines, 134
braid_TestCoarsenRefine
XBraid test routines, 134
braid_TestDelta
XBraid test routines, 135
braid_TestInitAccess
XBraid test routines, 136
braid_TestInnerProd
XBraid test routines, 136
braid_TestResidual
XBraid test routines, 137
braid_TestSpatialNorm
XBraid test routines, 138
braid_TestSum
XBraid test routines, 139
braid_Vector
braid_defs.h, 209
User-written routines, 73
braid_VectorBar
_braid.h, 180
braid_WriteConvHistory
General Interface routines, 97
braidCall
_braid_Action, 140
bufalloc
_braid_Core, 149
buffer
_braid_CommHandle, 144
buffree
_braid_Core, 149
BUFPACK
tape.h, 223
bufpack
_braid_Core, 149
bufsize
_braid_Core, 149
BUFUNPACK
tape.h, 223
bufunpack
_braid_Core, 149
c_tprior
_braid_Core, 149
c_tstop
_braid_Core, 150
calling_function
_braid_Core, 150
cfactor
_braid_Grid, 164
cfactors
_braid_Core, 150
cfdefault
_braid_Core, 150
CLONE
tape.h, 223
clone
_braid_Core, 150
clower
_braid_Grid, 164
comm
_braid_Core, 150
comm_world
_braid_Core, 150
core
_braid_Action, 140
_braid_Status, 167
cupper
_braid_Grid, 164
CWt_default
_braid_Core, 150
CWts
_braid_Core, 150
data_ptr
_braid_Tape, 167
delta.h, 216
_braid_DeltaFeatureCheck, 217
_braid_DoDeltaCorrect, 216
_braid_GramSchmidt, 217
_braid_LRDeltaDot, 217
_braid_LRDeltaDotMat, 217
_braid_UseDeltaCorrect, 216
delta_correct
_braid_Core, 150
delta_defer_iter
_braid_Core, 151
delta_defer_Ivl
_braid_Core, 151
delta_rank
_braid_Core, 151
done
_braid_Core, 151
dtk
_braid_Core, 151
Error Codes, 64
braid_ERROR_ARG, 64
braid_ERROR_GENERIC, 64
braid_ERROR_MEMORY, 64
braid_INVALID_RNORM, 64
est_error
_braid_Core, 151
estimate
_braid_Core, 151
estimate_lyap
_braid_Core, 151
f_bar
braid_Optim, 172
f_tprior
_braid_Core, 151
f_tstop
_braid_Core, 151
fa
_braid_Grid, 164
fa_alloc
_braid_Grid, 165
finalFCrelax
_braid_Core, 152
fmg
_braid_Core, 152
Fortran 90 interface options, 63
braid_FMANGLE, 63
braid_Fortran_Residual, 63
braid_Fortran_SpatialCoarsen, 63
braid_Fortran_Sync, 63
braid_Fortran_TimeGrid, 63
FREE
tape.h, 223
free
_braid_Core, 152
full_rnorm0
_braid_Core, 152
full_rnorm_res
_braid_Core, 152
full_rnorms
_braid_Core, 152

General Interface routines, 76
braid_Core, 78
braid_Destroy, 78
braid_Drive, 79
braid_GetMyID, 79
braid_GetNLevels, 79
braid_GetNumlter, 79
braid_GetRNorms, 80
braid_GetSpatialAccuracy, 80
braid_Init, 81
braid_PrintStats, 82
braid_PrintTimers, 82
braid_Rand, 82
braid_RAND_MAX, 78
braid_ResetTimer, 82
braid_SetAbsTol, 83
braid_SetAccessLevel, 83
braid_SetBufAllocFree, 83
braid_SetCFactor, 84
braid_SetCRelaxWt, 84
braid_SetDefaultPrintFile, 84
braid_SetDeferDelta, 85
braid_SetDeltaCorrection, 85
braid_SetFileIOLevel, 86
braid_SetFinalFCRelax, 86
braid_SetFMG, 86
braid_SetFullRNormRes, 86
braid_SetIncrMaxLevels, 87
braid_SetInnerProd, 87
braid_SetLyapunovEstimation, 87
braid_SetMaxIter, 88
braid_SetMaxLevels, 88
braid_SetMaxRefinements, 88
braid_SetMinCoarse, 89
braid_SetNFMG, 89
braid_SetNFMGVcyc, 89
braid_SetNRelax, 89
braid_SetPeriodic, 90
braid_SetPrintFile, 90
braid_SetPrintLevel, 90
braid_SetRefine, 91
braid_SetRelaxOnlyCG, 91
braid_SetRelTol, 91
braid_SetResidual, 92
braid_SetRichardsonEstimation, 92
braid_SetSeqSoln, 93
braid_SetShell, 93
braid_SetSkip, 93
braid_SetSpatialCoarsen, 94
braid_SetSpatialRefine, 94
braid_SetStorage, 94
braid_SetSync, 95
braid_SetTemporalNorm, 95
braid_SetTimeGrid, 96
braid_SetTimerFile, 96
braid_SetTimings, 96
braid_SetTPointsCutoff, 96
braid_SplitCommworld, 97
braid_WriteConvHistory, 97
globaltime
_braid_Core, 152
grids
_braid_Core, 152
gupper
_braid_Action, 141
_braid_Core, 152
_braid_Grid, 165
idx
_braid_Core, 152
ilower
_braid_Grid, 165
incr_max_levels
_braid_Core, 153
index
_braid_CommHandle, 144
Inherited XBraid status routines, 117
braid_AccessStatusGetBasisVec, 119
braid_AccessStatusGetCallingFunction, 119
braid_AccessStatusGetDeltaRank, 119
braid_AccessStatusGetDone, 119
braid_AccessStatusGetlter, 119
braid_AccessStatusGetLevel, 119
braid_AccessStatusGetLocalLyapExponents, 119
braid_AccessStatusGetNLevels, 120
braid_AccessStatusGetNRefine, 120
braid_AccessStatusGetNTPoints, 120
braid_AccessStatusGetResidual, 120
braid_AccessStatusGetSingleErrorEstAccess, 120
braid_AccessStatusGetT, 120
braid_AccessStatusGetTILD, 120
braid_AccessStatusGetTIndex, 121
braid_AccessStatusGetWrapperTest, 121
braid_BufferStatusGetLevel, 121
braid_BufferStatusGetMessageType, 121
braid_BufferStatusGetTIndex, 121
braid_BufferStatusSetBasisSize, 121
braid_BufferStatusSetSize, 121
braid_CoarsenRefStatusGetCTprior, 122
braid_CoarsenRefStatusGetCTstop, 122
braid_CoarsenRefStatusGetFTprior, 122
braid_CoarsenRefStatusGetFTstop, 122
braid_CoarsenRefStatusGetlter, 122
braid_CoarsenRefStatusGetLevel, 122
braid_CoarsenRefStatusGetNLevels, 122
braid_CoarsenRefStatusGetNRefine, 122
braid_CoarsenRefStatusGetNTPoints, 123
braid_CoarsenRefStatusGetT, 123
braid_CoarsenRefStatusGetTIndex, 123
braid_CoarsenRefStatusGetTpriorTstop, 123
braid_ObjectiveStatusGetIter, 123
braid_ObjectiveStatusGetLevel, 123
braid_ObjectiveStatusGetNLevels, 123
braid_ObjectiveStatusGetNRefine, 124
braid_ObjectiveStatusGetNTPoints, 124
braid_ObjectiveStatusGetT, 124
braid_ObjectiveStatusGetTIndex, 124
braid_ObjectiveStatusGetTol, 124
braid_StepStatusGetBasisVec, 124
braid_StepStatusGetCallingFunction, 124
braid_StepStatusGetDeltaRank, 124
braid_StepStatusGetDone, 125
braid_StepStatusGetIter, 125
braid_StepStatusGetLevel, 125
braid_StepStatusGetNLevels, 125
braid_StepStatusGetNRefine, 125
braid_StepStatusGetNTPoints, 125
braid_StepStatusGetOldFineTolx, 125
braid_StepStatusGetRNorms, 125
braid_StepStatusGetSingleErrorEstStep, 126
braid_StepStatusGetT, 126
braid_StepStatusGetTIndex, 126
braid_StepStatusGetTIUL, 126
braid_StepStatusGetTol, 126
braid_StepStatusGetTstartTstop, 126
braid_StepStatusGetTstop, 126
braid_StepStatusSetOldFineTolx, 127
braid_StepStatusSetRFactor, 127
braid_StepStatusSetRSpace, 127
braid_StepStatusSetTightFineTolx, 127
braid_SyncStatusGetAIIErrorEst, 127
braid_SyncStatusGetCallingFunction, 127
braid_SyncStatusGetDone, 127
braid_SyncStatusGetIter, 127
braid_SyncStatusGetLevel, 128
braid_SyncStatusGetNLevels, 128
braid_SyncStatusGetNRefine, 128
braid_SyncStatusGetNTPoints, 128
braid_SyncStatusGetNumErrorEst, 128
braid_SyncStatusGetProc, 128
braid_SyncStatusGetTComm, 128
braid_SyncStatusGetTimeValues, 129
braid_SyncStatusGetTIUL, 129
INIT
tape.h, 223
init
_braid_Core, 153
init_basis
_braid_Core, 153
initiali
_braid_Core, 153
inner_prod
_braid_Core, 153
Interface routines for XBraid_Adjoint, 97
braid_GetObjective, 98
braid_GetRNormAdjoint, 98
braid_InitAdjoint, 99
braid_SetAbsTolAdjoint, 99
braid_SetObjectiveOnly, 99
braid_SetPostprocessObjective, 100
braid_SetPostprocessObjective_diff, 100
braid_SetRelTolAdjoint, 100
braid_SetRevertedRanks, 100
braid_SetTStartObjective, 101
braid_SetTStopObjective, 101
inTime
_braid_Action, 141
inTimeldx
_braid_Action, 141
io_level
_braid_Core, 153
iupper
_braid_Grid, 165
level
_braid_Action, 141
_braid_CommHandle, 144
_braid_Core, 153
_braid_Grid, 165
local_exponents
_braid_Core, 153
localtime
_braid_Core, 153
Ivectors
_braid_Core, 153
lyap_exp
_braid_Core, 154
max_iter
_braid_Core, 154
max_levels
_braid_Core, 154
max_refinements
_braid_Core, 154
messagetype
_braid_Action, 141
_braid_Core, 154
min_coarse
_braid_Core, 154
mpistubs.h, 217
myid
_braid_Action, 141
_braid_Core, 154
myid_world
_braid_Core, 154
ncpoints
_braid_Grid, 165
next
_braid_Tape, 167
nfmg
_braid_Core, 154
nfmg_Vcyc
_braid_Core, 154
niter
_braid_Core, 155
nlevels
_braid_Core, 155
nrdefault
_braid_Core, 155
nrefine
_braid_Action, 141
_braid_Core, 155
nrels
_braid_Core, 155
ntime
_braid_Core, 155
num_requests
_braid_CommHandle, 144
nupoints
_braid_Grid, 165
obj_only
_braid_Core, 155
objective
braid_Optim, 172
OBJECTIVET
tape.h, 223
objectiveT
_braid_Core, 155
objT_diff
_braid_Core, 155
old_fine_tolx
_braid_Core, 155
optim
_braid_Core, 156
order
_braid_Core, 156
outTime
_braid_Action, 141
periodic
_braid_Core, 156
postprocess_obj
_braid_Core, 156
postprocess_obj_diff
_braid_Core, 156
print_level
_braid_Core, 156
r_space
_braid_Core, 156
rank
_braid_Basis, 143
rdtvalues
_braid_Core, 156
record
_braid_Core, 156
recv_handle
_braid_Grid, 165
recv_index
_braid_Grid, 165
refine
_braid_Core, 156
relax_lyap
_braid_Core, 157
relax_only_cg
_braid_Core, 157
request
braid_Optim, 172
request_type
_braid_CommHandle, 145
requests
_braid_CommHandle, 145
reset_gradient
_braid_Core, 157
residual
braid Core, 157
reverted_ranks
_braid_Core, 157
rfactor
_braid_Core, 157
rfactors
_braid_Core, 157
richardson
_braid_Core, 157
rnorm
_braid_Core, 157
braid_Optim, 172
rnorm0
_braid_Core, 157
braid_Optim, 172
rnorm0_adj
braid_Optim, 172
rnorm_adj
braid_Optim, 172
rnorms
_braid_Core, 158
rstopped
_braid_Core, 158
rtol
_braid_Core, 158
rtol_adj
braid_Optim, 172
sclone
_braid_Core, 158
scoarsen
_braid_Core, 158
send_handle
_braid_Grid, 165
send_index
_braid_Grid, 166
send_recv_rank
_braid_Action, 141
_braid_Core, 158
sendbuffer
braid_Optim, 173
seq_soln
_braid_Core, 158
sfree
_braid_Core, 158
sinit
_braid_Core, 158
size
_braid_Tape, 168
size_basis
_braid_Core, 158
size_buffer
_braid_Action, 141
_braid_Core, 159
skip
_braid_Core, 159
spatialnorm
_braid_Core, 159
srefine
_braid_Core, 159
status
_braid_CommHandle, 145
braid_AccessStatus, 169
braid_BufferStatus, 170
braid_CoarsenRefStatus, 170
braid_ObjectiveStatus, 171
braid_StepStatus, 174
braid_SyncStatus, 174
status.h, 218
_braid_AccessStatusInit, 219
_braid_BufferStatusInit, 219
_braid_CoarsenRefStatusInit, 220
_braid_ObjectiveStatusInit, 220
_braid_StatusDestroy, 220
_braid_StatusElt, 218
_braid_StepStatusInit, 221
_braid_SyncStatusInit, 221
STEP
tape.h, 223
step
_braid_Core, 159
step_diff
_braid_Core, 159
storage
_braid_Core, 159
SUM
tape.h, 223
sum
_braid_Core, 159
sum_alpha
_braid_Action, 142
sum_beta
_braid_Action, 142
sum_user_obj
braid_Optim, 173
sync
_braid_Core, 159
t
_braid_Core, 159
ta
_braid_Grid, 166
ta_alloc
_braid_Grid, 166
tape.h, 222
_braid_Call, 222
_braid_CallGetName, 223
_braid_DiffCall, 223
_braid_TapeDisplayBackwards, 223
_braid_TapeEvaluate, 223
_braid_TapeGetSize, 223
_braid_Tapelnit, 224
_braid_TapelsEmpty, 224
_braid_TapePop, 224
_braid_TapePush, 224
_braid_TapeResetInput, 224
_braid_TapeSetSeed, 224
ACCESS, 223
BUFPACK, 223
BUFUNPACK, 223
CLONE, 223
FREE, 223
INIT, 223
OBJECTIVET, 223
STEP, 223
SUM, 223
tapeinput
braid_Optim, 173
tgrid
_braid_Core, 160
tight_fine_tolx
_braid_Core, 160
timer_coarse_solve
_braid_Core, 160
timer_drive_init
_braid_Core, 160
timer_file_stem
_braid_Core, 160
timer_file_stem_len
_braid_Core, 160
timer_MPI_recv
_braid_Core, 160
timer_MPI_send
_braid_Core, 160
timer_MPI_wait _braid_Core, 160
timer_MPI_wait_coarse
_braid_Core, 160
timer_user_access
_braid_Core, 161
timer_user_bufpack
_braid_Core, 161
timer_user_bufsize
_braid_Core, 161
timer_user_bufunpack
_braid_Core, 161
timer_user_clone
_braid_Core, 161
timer_user_free
_braid_Core, 161
timer_user_init
_braid_Core, 161
timer_user_innerprod
_braid_Core, 161
timer_user_residual
_braid_Core, 161
timer_user_scoarsen
_braid_Core, 161
timer_user_spatialnorm
_braid_Core, 162
timer_user_srefine
_braid_Core, 162
timer_user_step
_braid_Core, 162
timer_user_sum
_braid_Core, 162
timer_user_sync
_braid_Core, 162
timings
_braid_Core, 162
tnext
_braid_Core, 162
tnorm
_braid_Core, 162
tnorm_a
_braid_Core, 162
tol
_braid_Action, 142
_braid_Core, 162
tol_adj
braid_Optim, 173
tpoints_cutoff
_braid_Core, 163
tstart
_braid_Core, 163
tstart_obj
braid_Optim, 173
tstop
_braid_Core, 163
tstop_obj
braid_Optim, 173
ua
_braid_Grid, 166
ua_alloc
_braid_Grid, 166
ulast
_braid_Grid, 166
useCount
_braid_VectorBar, 168
User interface routines, 76
User-written routines, 64
braid_App, 65
braid_PtFcnAccess, 65
braid_PtFcnBufAlloc, 66
braid_PtFcnBufFree, 66
braid_PtFcnBufPack, 67
braid_PtFcnBufSize, 67
braid_PtFcnBufUnpack, 67
braid_PtFcnClone, 68
braid_PtFcnFree, 68
braid_PtFcnInit, 68
braid_PtFcnInitBasis, 68
braid_PtFcnInnerProd, 69
braid_PtFcnResidual, 69
braid_PtFcnSClone, 69
braid_PtFcnSCoarsen, 70
braid_PtFcnSFree, 70
braid_PtFcnSInit, 70
braid_PtFcnSpatialNorm, 71
braid_PtFcnSRefine, 71
braid_PtFcnStep, 72
braid_PtFcnSum, 72
braid_PtFcnSync, 72
braid_PtFcnTimeGrid, 73
braid_Vector, 73
User-written routines for XBraid_Adjoint, 73
braid_PtFcnObjectiveT, 74
braid_PtFcnObjectiveTDiff, 74
braid_PtFcnPostprocessObjective, 74
braid_PtFcnPostprocessObjective_diff, 75
braid_PtFcnResetGradient, 75
braid_PtFcnStepDiff, 75
userVecs
_braid_Basis, 143
userVector
_braid_BaseVector, 143
_braid_VectorBar, 168
userVectorTape
_braid_Core, 163
useshell
_braid_Core, 163
util.h, 224
_braid_GetInterval, 225
_braid_GetNEntries, 225
_braid_MPI_Wtime, 225
_braid_Max, 225
_braid_ParFprintfFlush, 225
_braid_ProjectInterval, 226
_braid_SetVerbosity, 226
_braid_printf, 226
va
_braid_Grid, 166
va_alloc
_braid_Grid, 166
vector_ptr
_braid_CommHandle, 145
verbose_adj
_braid_Core, 163
warm_restart
_braid_Core, 163
wrapper_test
_braid_Core, 163

XBraid status macros, 129
braid_ASCaller_BaseStep_diff, 129
braid_ASCaller_ComputeFullRNorm, 130
braid_ASCaller_Drive_AfterInit, 130
braid_ASCaller_Drive_TopCycle, 130
braid_ASCaller_FAccess, 130
braid_ASCaller_FASResidual, 130
braid_ASCaller_FCRelax, 130
braid_ASCaller_FInterp, 130
braid_ASCaller_FRefine, 130
braid_ASCaller_FRefine_AfterInitHier, 130
braid_ASCaller_FRestrict, 130
braid_ASCaller_InitGuess, 131
braid_ASCaller_Residual, 131
XBraid status routines, 101
braid_StatusGetAllErrorEst, 102
braid_StatusGetBasisVec, 103
braid_StatusGetCallingFunction, 103
braid_StatusGetCTprior, 103
braid_StatusGetCTstop, 104
braid_StatusGetDeltaRank, 104
braid_StatusGetDone, 104
braid_StatusGetFTprior, 105
braid_StatusGetFTstop, 105
braid_StatusGetlter, 105
braid_StatusGetLevel, 106
braid_StatusGetLocalLyapExponents, 106
braid_StatusGetMessageType, 106
braid_StatusGetNLevels, 106
braid_StatusGetNRefine, 107
braid_StatusGetNTPoints, 107
braid_StatusGetNumErrorEst, 107
braid_StatusGetOldFineTolx, 108
braid_StatusGetProc, 108
braid_StatusGetResidual, 108
braid_StatusGetRNorms, 109
braid_StatusGetSingleErrorEstAccess, 109
braid_StatusGetSingleErrorEstStep, 109
braid_StatusGetT, 110
braid_StatusGetTComm, 110
braid_StatusGetTILD, 110
braid_StatusGetTimeValues, 111
braid_StatusGetTIndex, 111
braid_StatusGetTIUL, 112
braid_StatusGetTol, 112
braid_StatusGetTpriorTstop, 112
braid_StatusGetTstartTstop, 113
braid_StatusGetTstop, 113
braid_StatusGetWrapperTest, 113
braid_StatusSetBasisSize, 114
braid_StatusSetOldFineTolx, 114
braid_StatusSetRefinementDtValues, 114
braid_StatusSetRFactor, 116
braid_StatusSetRSpace, 116
braid_StatusSetSize, 116
braid_StatusSetTightFineTolx, 117
XBraid status structures, 101
XBraid test routines, 131
braid_TestAll, 132
braid_TestBuf, 133
braid_TestClone, 134
braid_TestCoarsenRefine, 134
braid_TestDelta, 135
braid_TestInitAccess, 136
braid_TestInnerProd, 136
braid_TestResidual, 137
braid_TestSpatialNorm, 138
braid_TestSum, 139


[^0]:    ${ }^{1}$ Ries, Manfred, Ulrich Trottenberg, and Gerd Winter. "A note on MGR methods." Linear Algebra and its Applications 49 (1983): 1-26.

[^1]:    ${ }^{2}$ Lions, J., Yvon Maday, and Gabriel Turinici. "A"parareal"in time discretization of PDE's." Comptes Rendus de l'Academie des Sciences Series I Mathematics 332.7 (2001): 661-668.

[^2]:    ${ }^{3}$ Giles, M.B., Pierce, N.A.: "An introduction to the adjoint approach to design." Flow, Turbulence and Combustion 65(3), 393-415 (2000)

[^3]:    ${ }^{4}$ Günther, S., Gauger, N.R. and Schroder, J.B. "A Non-Intrusive Parallel-in-Time Adjoint Solver with the XBraid Library." Computing and Visualization in Science, Springer, (accepted), (2017)

[^4]:    braid_Core core,

