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## Index

## 1 Abstract

This package implements an optimal-scaling multigrid solver for the (non)linear systems that arise from the discretization of problems with evolutionary behavior. Typically, solution algorithms for evolution equations are based on a timemarching approach, solving sequentially for one time step after the other. Parallelism in these traditional time-integration techniques is limited to spatial parallelism. However, current trends in computer architectures are leading towards systems with more, but not faster, processors, i.e., clock speeds are stagnate. Therefore, faster overall runtimes must come from greater parallelism. One approach to achieve parallelism in time is with multigrid, but extending classical multigrid methods for elliptic operators to this setting is a significant achievement. In this software, we implement a nonintrusive, optimal-scaling time-parallel method based on multigrid reduction techniques. The examples in the package demonstrate optimality of our multigrid-reduction-in-time algorithm (MGRIT) for solving a variety of equations in two and three spatial dimensions. These examples can also be used to show that MGRIT can achieve significant speedup in comparison to sequential time marching on modern architectures.

It is strongly recommended that you also read Parallel Time Integration with Multigrid after reading the Overview of the XBraid Algorithm. It is a more in depth discussion of the algorithm and associated experiments.

## 2 XBraid Quickstart, User Advice, and License

## What is XBraid?

XBraid is a parallel-in-time software package. It implements an optimal-scaling multigrid solver for the (non)linear systems that arise from the discretization of problems with evolutionary behavior.

This code and associated algorithms are developed at Lawrence Livermore National Laboratory, and at collaborating academic institutions.

For our publication list, please go here. There you will papers on XBraid and various application areas where XBraid has been applied, e.g., fluid dynamics, machine learning, parabolic equations, Burgers' equation, powergrid systems, etc.

## About XBraid

Typically, solution algorithms for evolution equations are based on a time-marching approach, solving sequentially for one time step after the other. Parallelism in these traditional time-integration techniques is limited to spatial parallelism. However, current trends in computer architectures are leading towards systems with more, but not faster, processors, i.e., clock speeds are stagnate. Therefore, faster overall runtimes must come from greater parallelism. Our approach to achieve such parallelism in time is with multigrid.

In this software, we implement a non-intrusive, optimal-scaling time-parallel method based on multigrid reduction techniques (multigrid-reduction-in-time or MGRIT). A few important points about XBraid are as follows.

- The algorithm enables a scalable parallel-in-time approach by applying multigrid to the time dimension.
- It is designed to be nonintrusive. That is, users apply their existing sequential time-stepping code according to our interface, and then XBraid does the rest. Users have spent years, sometimes decades, developing the right time-stepping scheme for their problem. XBraid allows users to keep their schemes, but enjoy parallelism in the time dimension.
- XBraid solves exactly the same problem that the existing sequential time-stepping scheme does.
- XBraid is flexible, allowing for a variety of time stepping, relaxation, and temporal and spatial coarsening options.
- The full approximation scheme multigrid approach is used to accommodate nonlinear problems.
- XBraid written in MPI/C with C++, Fortran 90, and Python interfaces.
- XBraid is released under LGPL 2.1.


## Documentation

- For examples of using XBraid, see the examples / and drivers/ directories, and in particular examples/ex-01-*
- See the release page for links to precompiled documentation PDFs that go through, step-by-step, how to use XBraid.
- For tutorials, see the bottom of our publications page.
- For citing XBraid, see here.


## Advice to Users

The field of parallel-in-time methods is in many ways under development, and success has been shown primarily for problems with some parabolic character. While there are ongoing projects (here and elsewhere) looking at varied applications such as hyperbolic problems, computational fluid dynamics, power grids, medical applications, and so on, expectations should take this fact into account. That being said, we strongly encourage new users to try our code for their application. Every new application has its own issues to address and this will help us to improve both the algorithm and the software. Please see our project publications website for our recent publications concerning some of these varied applications.

For bug reporting, please use the issue tracker here on Github. Please include as much relevant information as possible, including all the information in the "VERSION" file located in the bottom most XBraid directory. For compile and runtime problems, please also include the machine type, operating system, MPI implementation, compiler, and any error messages produced.

## Building XBraid

- To specify the compilers, flags and options for your machine, edit makefile.inc. For now, we keep it simple and avoid using configure or cmake.
- To make the library, libbraid.a,

```
$ make
```

- To make the examples
\$ make all
- The makefile lets you pass some parameters like debug with

```
$ make debug=yes
```

or
\$ make all debug=yes
It would also be easy to add additional parameters, e.g., to compile with insure.

- To set compilers and library locations, look in makefile.inc where you can set up an option for your machine to define simple stuff like

```
CC = mpicc
MPICC = mpicc
MPICXX = mpiCC
LFLAGS = -lm
```


## Meaning of the name

We chose the package name XBraid to stand for Time-Braid, where $X$ is the first letter in the Greek word for time, Chronos. The algorithm braids together time-grids of different granularity in order to create a multigrid method and achieve parallelism in the time dimension.

## License

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LLNL Release Number: LLNL-CODE-660355

## 3 Introduction

### 3.1 Overview of the XBraid Algorithm

The goal of XBraid is to solve a problem faster than a traditional time marching algorithm. Instead of sequential time marching, XBraid solves the problem iteratively by simultaneously updating a space-time solution guess over all time values. The initial solution guess can be anything, even a random function over space-time. The iterative updates to the solution guess are done by constructing a hierarchy of temporal grids, where the finest grid contains all of the time values for the simulation. Each subsequent grid is a coarser grid with fewer time values. The coarsest grid has a trivial number of time steps and can be quickly solved exactly. The effect is that solutions to the time marching problem on the coarser (i.e., cheaper) grids can be used to correct the original finest grid solution. Analogous to spatial multigrid, the coarse grid correction only corrects and accelerates convergence to the finest grid solution. The coarse grid does not need to represent an accurate time discretization in its own right. Thus, a problem with many time steps (thousands, tens of thousands or more) can be solved with 10 or 15 XBraid iterations, and the overall time to solution can be greatly sped up. However, this is achieved at the cost of more computational resources.

To understand how XBraid differs from traditional time marching, consider the simple linear advection equation, $u_{t}=$ $-c u_{x}$. The next figure depicts how one would typically evolve a solution here with sequential time stepping. The initial condition is a wave, and this wave propagates sequentially across space as time increases.

XBraid instead begins with a solution guess over all of space-time, which for demonstration, we let be random. An XBraid iteration does


Figure 1 Sequential time stepping.

1. Relaxation on the fine grid, i.e., the grid that contains all of the desired time values. Relaxation is just a local application of the time stepping scheme, e.g., backward Euler.
2. Restriction to the first coarse grid, i.e., interpolate the problem to a grid that contains fewer time values, say every second or every third time value.
3. Relaxation on the first coarse grid
4. Restriction to the second coarse grid and so on...
5. When a coarse grid of trivial size (say 2 time steps) is reached, it is solved exactly.
6. The solution is then interpolated from the coarsest grid to the finest grid

One XBraid iteration is called a cycle and these cycles continue until the solution is accurate enough. This is depicted in the next figure, where only a few iterations are required for this simple problem.


Figure 2 XBraid iterations.

There are a few important points to make.

- The coarse time grids allow for global propagation of information across space-time with only one XBraid iteration. This is visible in the above figure by observing how the solution is updated from iteration 0 to iteration 1.
- Using coarser (cheaper) grids to correct the fine grid is analogous to spatial multigrid.
- Only a few XBraid iterations are required to find the solution over 1024 time steps. Therefore if enough processors are available to parallelize XBraid, we can see a speedup over traditional time stepping (more on this later).
- This is a simple example, with evenly space time steps. XBraid is structured to handle variable time step sizes and adaptive time step sizes.

To firm up our understanding, let's do a little math. Assume that you have a general system of ordinary differential equations (ODEs),

$$
\boldsymbol{u}^{\prime}(t)=\boldsymbol{f}(t, \boldsymbol{u}(t)), \quad \boldsymbol{u}(0)=\boldsymbol{u}_{0}, \quad t \in[0, T]
$$

Next, let $t_{i}=i \delta t, i=0,1, \ldots, N$ be a temporal mesh with spacing $\delta t=T / N$, and $u_{i}$ be an approximation to $u\left(t_{i}\right)$. A general one-step time discretization is now given by

$$
\begin{aligned}
\boldsymbol{u}_{0} & =g_{0} \\
\boldsymbol{u}_{i} & =\Phi_{i}\left(\boldsymbol{u}_{i-1}\right)+\boldsymbol{g}_{i}, \quad i=1,2, \ldots, N
\end{aligned}
$$

Traditional time marching would first solve for $i=1$, then solve for $i=2$, and so on. For linear time propagators $\left\{\Phi_{i}\right\}$, this can also be expressed as applying a direct solver (a forward solve) to the following system:

$$
A \mathbf{u} \equiv\left(\begin{array}{cccc}
I & & & \\
-\Phi_{1} & I & & \\
& \ddots & \ddots & \\
& & -\Phi_{N} & I
\end{array}\right)\left(\begin{array}{c}
\boldsymbol{u}_{0} \\
\boldsymbol{u}_{1} \\
\vdots \\
\boldsymbol{u}_{N}
\end{array}\right)=\left(\begin{array}{c}
\boldsymbol{g}_{0} \\
\boldsymbol{g}_{1} \\
\vdots \\
\boldsymbol{g}_{N}
\end{array}\right) \equiv \mathbf{g}
$$

or

$$
A \mathbf{u}=\mathbf{g}
$$

This process is optimal and $\mathrm{O}(\mathrm{N})$, but it is sequential. XBraid achieves parallelism in time by replacing this sequential solve with an optimal multigrid reduction iterative method ${ }^{1}$ applied to only the time dimension. This approach is

- nonintrusive, in that it coarsens only in time and the user defines $\Phi$. Thus, users can continue using existing time stepping codes by wrapping them into our framework.
- optimal and $O(N)$, but $O(N)$ with a higher constant than time stepping. Thus with enough computational resources, XBraid will outperform sequential time stepping.
- highly parallel

We now describe the two-grid process in more detail, with the multilevel analogue being a recursive application of the process. We also assume that $\Phi$ is constant for notational simplicity. XBraid coarsens in the time dimension with factor $m>1$ to yield a coarse time grid with $N_{\Delta}=N / m$ points and time step $\Delta T=m \delta t$. The corresponding coarse grid problem,

$$
A_{\Delta}=\left(\begin{array}{cccc}
I & & & \\
-\Phi_{\Delta} & I & & \\
& \ddots & \ddots & \\
& & -\Phi_{\Delta} & I
\end{array}\right)
$$

is obtained by defining coarse grid propagators $\left\{\Phi_{\Delta}\right\}$ which are at least as cheap to apply as the fine scale propagators $\{\Phi\}$. The matrix $A_{\Delta}$ has fewer rows and columns than $A$, e.g., if we are coarsening in time by $2, A_{\Delta}$ has one half as many rows and columns.

This coarse time grid induces a partition of the fine grid into C-points (associated with coarse grid points) and F-points, as visualized next. C-points exist on both the fine and coarse time grid, but F-points exist only on the fine time scale.

[^0]

Every multigrid algorithm requires a relaxation method and an approach to transfer values between grids. Our relaxation scheme alternates between so-called F-relaxation and C-relaxation as illustrated next. F-relaxation updates the F-point values $\left\{u_{j}\right\}$ on interval $\left(T_{i}, T_{i+1}\right)$ by simply propagating the C-point value $u_{m i}$ across the interval using the time propagator $\{\Phi\}$. While this is a sequential process, each F-point interval update is independent from the others and can be computed in parallel. Similarly, C-relaxation updates the C-point value $u_{m i}$ based on the F-point value $u_{m i-1}$ and these updates can also be computed in parallel. This approach to relaxation can be thought of as line relaxation in space in that the residual is set to 0 for an entire time step.

The F updates are done simultaneously in parallel, as depicted next.


Figure 3 Update all F-point intervals in parallel, using the time propagator $\Phi$.

Following the F sweep, the C updates are also done simultaneously in parallel, as depicted next.


Figure 4 Update all C-points in parallel, using the time propagator $\Phi$.

In general, FCF- and F-relaxation will refer to the relaxation methods used in XBraid. We can say

- FCF- or F-relaxation is highly parallel.
- But, a sequential component exists equaling the number of F-points between two C-points.
- XBraid uses regular coarsening factors, i.e., the spacing of C-points happens every $m$ points.

After relaxation, comes forming the coarse grid error correction. To move quantities to the coarse grid, we use the restriction operator $R$ which simply injects values at C-points from the fine grid to the coarse grid,

$$
R=\left(\begin{array}{ccc}
I & & \\
0 & & \\
\vdots & & \\
0 & & \\
& I & \\
& 0 & \\
& \vdots & \\
& 0 & \\
& & \ddots
\end{array}\right) .
$$

The spacing between each $I$ is $m-1$ block rows. While injection is simple, XBraid always does an F-relaxation sweep before the application of $R$, which is equivalent to using the transpose of harmonic interpolation for restriction (see Parallel Time Integration with Multigrid). Another interpretation is that the F-relaxation compresses the residual into the C-points, i.e., the residual at all F-points after an F-relaxation is 0 . Thus, it makes sense for restriction to be injection.

To define the coarse grid equations, we apply the Full Approximation Scheme (FAS) method, which is a nonlinear version of multigrid. This is to accommodate the general case where $f$ is a nonlinear function. In FAS, the solution guess and residual (i.e., $\mathbf{u}, \mathbf{g}-A \mathbf{u}$ ) are restricted. This is in contrast to linear multigrid which typically restricts only the residual equation to the coarse grid. This algorithmic change allows for the solution of general nonlinear problems. For more details, see this PDF by Van Henson for a good introduction to FAS. However, FAS was originally invented by Achi Brandt.

A central question in applying FAS is how to form the coarse grid matrix $A_{\Delta}$, which in turn asks how to define the coarse grid time stepper $\Phi_{\Delta}$. One of the simplest choices (and one frequently used in practice) is to let $\Phi_{\Delta}$ simply be $\Phi$ but with the coarse time step size $\Delta T=m \delta t$. For example, if $\Phi=(I-\delta t A)^{-1}$ for some backward Euler scheme, then $\Phi_{\Delta}=(I-m \delta t A)^{-1}$ would be one choice.

With this $\Phi_{\Delta}$ and letting $\mathbf{u}_{\Delta}$ be the restricted fine grid solution and $\mathbf{r}_{\Delta}$ be the restricted fine grid residual, the coarse grid equation

$$
A_{\Delta}\left(\mathbf{v}_{\Delta}\right)=A_{\Delta}\left(\mathbf{u}_{\Delta}\right)+\mathbf{r}_{\Delta}
$$

is then solved. Finally, FAS defines a coarse grid error approximation $\mathbf{e}_{\Delta}=\mathbf{v}_{\Delta}-\mathbf{u}_{\Delta}$, which is interpolated with $P_{\Phi}$ back to the fine grid and added to the current solution guess. Interpolation is equivalent to injecting the coarse grid to the C-points on the fine grid, followed by an F-relaxation sweep (i.e., it is equivalent to harmonic interpolation, as mentioned
above about restriction). That is,

$$
P_{\Phi}=\left(\begin{array}{ccc}
I & & \\
\Phi & & \\
\Phi^{2} & & \\
\vdots & & \\
\Phi^{m-1} & & \\
& I & \\
& \Phi & \\
& \Phi^{2} & \\
& \vdots & \\
& \Phi^{m-1} & \\
& & \ddots
\end{array}\right)
$$

where $m$ is the coarsening factor. See Two-Grid Algorithm for a concise description of the FAS algorithm for MGRIT.

### 3.1.1 Two-Grid Algorithm

The two-grid FAS process is captured with this algorithm. Using a recursive coarse grid solve (i.e., step 3 becomes a recursive call) makes the process multilevel. Halting is done based on a residual tolerance. If the operator is linear, this FAS cycle is equivalent to standard linear multigrid. Note that we represent $A$ as a function below, whereas the above notation was simplified for the linear case.

1. Relax on $A(\mathbf{u})=\mathbf{g}$ using FCF-relaxation
2. Restrict the fine grid approximation and its residual:

$$
\mathbf{u}_{\Delta} \leftarrow R \mathbf{u}, \quad \mathbf{r}_{\Delta} \leftarrow R(\mathbf{g}-A(\mathbf{u})
$$

which is equivalent to updating each individual time step according to

$$
u_{\Delta, i} \leftarrow u_{m i}, \quad r_{\Delta, i} \leftarrow g_{m i}-A(\mathbf{u})_{m i} \quad \text { for } \quad i=0, \ldots, N_{\Delta}
$$

3. Solve $A_{\Delta}\left(\mathbf{v}_{\Delta}\right)=A_{\Delta}\left(\mathbf{u}_{\Delta}\right)+\mathbf{r}_{\Delta}$
4. Compute the coarse grid error approximation: $\mathbf{e}_{\Delta}=\mathbf{v}_{\Delta}-\mathbf{u}_{\Delta}$
5. Correct: $\mathbf{u} \leftarrow \mathbf{u}+P \mathbf{e}_{\Delta}$

This is equivalent to updating each individual time step by adding the error to the values of $\mathbf{u}$ at the C -points:

$$
u_{m i}=u_{m i}+e_{\Delta, i}
$$

followed by an F-relaxation sweep applied to $\mathbf{u}$.

### 3.1.2 Summary

In summary, a few points are

- XBraid is an iterative solver for the global space-time problem.
- The user defines the time stepping routine $\Phi$ and can wrap existing code to accomplish this.
- XBraid convergence will depend heavily on how well $\Phi_{\Delta}$ approximates $\Phi^{m}$, that is how well a time step size of $m \delta t=\Delta T$ will approximate $m$ applications of the same time integrator for a time step size of $\delta t$. This is a subject of research, but this approximation need not capture fine scale behavior, which is instead captured by relaxation on the fine grid.
- The coarsest grid is solved exactly, i.e., sequentially, which can be a bottleneck for two-level methods like Parareal,
${ }^{2}$ but not for a multilevel scheme like XBraid where the coarsest grid is of trivial size.
- By forming the coarse grid to have the same sparsity structure and time stepper as the fine grid, the algorithm can recur easily and efficiently.
- Interpolation is ideal or exact, in that an application of interpolation leaves a zero residual at all F-points.
- The process is applied recursively until a trivially sized temporal grid is reached, e.g., 2 or 3 time points. Thus, the coarsening rate $m$ determines how many levels there are in the hierarchy. For instance in this figure, a 3 level hierarchy is shown. Three levels are chosen because there are six time points, $m=2$ and $m^{2}<6 \leq m^{3}$. If the coarsening rate had been $m=4$ then there would only be two levels because there would be no more points to coarsen!

> - $F$-point (fine grid only)
> - $C$-point (coarse grid)


By default, XBraid will subdivide the time domain into evenly sized time steps. XBraid is structured to handle variable time step sizes and adaptive time step sizes.

### 3.2 Overview of the XBraid Code

XBraid is designed to run in conjunction with an existing application code that can be wrapped per our interface. This application code will implement some time marching simulation like fluid flow. Essentially, the user has to take their application code and extract a stand-alone time-stepping function $\Phi$ that can evolve a solution from one time value to another, regardless of time step size. After this is done, the XBraid code takes care of the parallelism in the time dimension.

XBraid

- is written in C and can easily interface with Fortran, C++, and Python
- uses MPI for parallelism
- self documents through comments in the source code and through $*$.md files
- functions and structures are prefixed by braid
- User routines are prefixed by braid_
- Developer routines are prefixed by _braid_

3.2.1 Parallel decomposition and memory
- XBraid decomposes the problem in parallel as depicted next. As you can see, traditional time stepping only stores one time step at a time, but only enjoys a spatial data decomposition and spatial parallelism. On the other hand, XBraid stores multiple time steps simultaneously and each processor holds a space-time chunk reflecting both the spatial and temporal parallelism.
- XBraid only handles temporal parallelism and is agnostic to the spatial decomposition. See braid_Split $\hookleftarrow$ Commworld. Each processor owns a certain number of CF intervals of points. In the following figure, processor 1 and processor 2 each own 2 CF intervals. XBraid distributes intervals evenly on the finest grid.

- XBraid increases the parallelism significantly, but now several time steps need to be stored, requiring more memory. XBraid employs two strategies to address the increased memory costs.
- First, one need not solve the whole problem at once. Storing only one space-time slab is advisable. That is, solve for as many time steps (say $k$ time steps) as you have available memory for. Then move on to the next $k$ time steps.
- Second, XBraid provides support for storing only C-points. Whenever an F-point is needed, it is generated by F-relaxation. More precisely, only the red C-point time values in the previous figure are stored. Coarsening is usually aggressive with $m=8,16,32, \ldots$, so the storage requirements of XBraid are significantly reduced when compared to storing all of the time values.

Overall, the memory multiplier per processor when using XBraid is $O(1)$ if space-time coarsening (see The Simplest Example) is used and $O\left(\log _{m} N\right)$ for time-only coarsening. The time-only coarsening option is the default and requires no user-written spatial interpolation/restriction routines (which is the case for space-time coasrening). We note that the base of the logarithm is $m$, which can be quite large.

[^1]
### 3.2.2 Cycling and relaxation strategies

There are two main cycling strategies available in XBraid, F-and V-cycles. These two cycles differ in how often and the order in which coarse levels are visited. A V-cycle is depicted next, and is a simple recursive application of the Two-Grid Algorithm.


V-cycle

An F-cycle visits coarse grids more frequently and in a different order. Essentially, an F-cycle uses a V-cycle as the post-smoother, which is an expensive choice for relaxation. But, this extra work gives you a closer approximation to a two-grid cycle, and a faster convergence rate at the extra expense of more work. The effectiveness of a V-cycle as a relaxation scheme can be seen in Figure 2, where one V-cycle globally propagates and smoothes the error. The cycling strategy of an F-cycle is depicted next.


Next, we make a few points about F-versus V-cycles.

- One V-cycle iteration is cheaper than one F-cycle iteration.
- But, F-cycles often converge more quickly. For some test cases, this difference can be quite large. The cycle choice for the best time to solution will be problem dependent. See Scaling Study with this Example for a case study of cycling strategies.
- For exceptionally strong F-cycles, the option braid_SetNFMGVcyc can be set to use multiple V-cycles as relaxation. This has proven useful for some problems with a strongly advective nature.

The number of FC relaxation sweeps is another important algorithmic setting. Note that at least one F-relaxation sweep is always done on a level. A few summary points about relaxation are as follows.

- Using FCF, FCFCF, or FCFCFCF relaxation corresponds to passing braid_SetNRelax a value of 1,2 or 3 respectively, and will result in an XBraid cycle that converges more quickly as the number of relaxations grows.
- But as the number of relaxations grows, each XBraid cycle becomes more expensive. The optimal relaxation strategy for the best time to solution will be problem dependent.
- However, a good first step is to try FCF on all levels (i.e., braid_SetNRelax(core, -1, 1) ).
- A common optimization is to first set FCF on all levels (i.e., braid_setnrelax(core, -1, 1) ), but then overwrite the FCF option on level 0 so that only F-relaxation is done on level 0, (i.e., braid_setnrelax(core, 0, 1) ). Another strategy is to use F-relaxation on all levels together with F-cycles.
- See Scaling Study with this Example for a case study of relaxation strategies.

There is also a weighted relaxation option, which applies weighted-Jacobi at the C-points during the C-relaxation. Experiments with the 1D heat equation and 1D advection showed iteration gains of $10-25 \%$ for V-cycles when the experimentally optimal weight was used.

- For the heat equation, a weight of around 1.3 was experimentally optimal
- For the advection equation, weights between 1.4 and 1.8 were experimentally optimal
- Set this option with braid_SetCRelaxWt, which allows you to set a global relaxation weight, or an individual weight for each level. In general, under-relaxation (weight $<1.0$ ) never improved performance, but over-relxation ( $1.0<$ weight $<2.0$ ) often offered some improvement.

Last, Parallel Time Integration with Multigrid has a more in depth case study of cycling and relaxation strategies

### 3.2.3 Overlapping communication and computation

XBraid effectively overlaps communication and computation. The main computational kernel of XBraid is one relaxation sweep touching all the CF intervals. At the start of a relaxation sweep, each process first posts a non-blocking receive at its left-most point. It then carries out F-relaxation in each interval, starting with the right-most interval to send the data to the neighboring process as soon as possible. If each process has multiple CF intervals at this XBraid level, the strategy allows for complete overlap.


### 3.2.4 Configuring the XBraid Hierarchy

Some of the more basic XBraid function calls allow you to control aspects discussed here.

- braid_SetFMG: switches between using F- and V-cycles.
- braid_SetMaxIter: sets the maximum number of XBraid iterations
- braid_SetCFactor: sets the coarsening factor for any (or all levels)
- braid_SetNRelax: sets the number of CF-relaxation sweeps for any (or all levels)
- braid_SetRelTol, braid_SetAbsTol: sets the stopping tolerance
- braid_SetMinCoarse: sets the minimum possible coarse grid size
- braid_SetMaxLevels: sets the maximum number of levels in the XBraid hierarchy

Another important configuration aspect regards setting a residual halting tolerance. Setting a tolerance involves these three XBraid options:

1. braid_PtFenSpatialNorm

This user-defined function carries out a spatial norm by taking the norm of a braid_Vector. A common choice is the standard Eucliden norm (2-norm), but many other choices are possible, such as an L2-norm based on a finite element space.
2. braid_SetTemporalNorm

This option determines how to obtain a global space-time residual norm. That is, this decides how to combine the spatial norms returned by braid_PtFcnSpatialNorm at each time step to obtain a global norm over space and time. It is this global norm that then controls halting.
There are three tnorm options supported by braid_SetTemporalNorm. We let the summation index $i$ be over all C-point values on the fine time grid, $k$ refer to the current XBraid iteration, $r$ be residual values, space_time norms be a norm over the entire space-time domain and spatial_norm be the user-defined spatial norm from braid_Pt $\hookleftarrow$ FcnSpatialNorm. Thus, $r_{i}$ is the residual at the ith C-point, and $r^{(k)}$ is the residual at the $k t h \mathrm{XBraid}$ iteration. The three options are then defined as,

- tnorm=1: One-norm summation of spatial norms

$$
\left\|r^{(k)}\right\|_{\text {space_time }}=\Sigma_{i}\left\|r_{i}^{(k)}\right\|_{\text {spatial_norm }}
$$

If braid_PtFcnSpatialNorm is the one-norm over space, then this is equivalent to the one-norm of the global space-time residual vector.

- tnorm=2: Two-norm summation of spatial norms

$$
\left\|r^{(k)}\right\|_{\text {space_time }}=\left(\Sigma_{i}\left\|r_{i}^{(k)}\right\|_{\text {spatial_norm }}^{2}\right)^{1 / 2}
$$

If braid_PtFcnSpatialNorm is the Euclidean norm (two-norm) over space, then this is equivalent to the Euclidean-norm of the global space-time residual vector.

- tnorm=3: Infinity-norm combination of spatial norms

$$
\left\|r^{(k)}\right\|_{\text {space_time }}=\max _{i}\left\|r_{i}^{(k)}\right\|_{\text {spatial_norm }}
$$

If braid_PtFcnSpatialNorm is the infinity-norm over space, then this is equivalent to the infinity-norm of the global space-time residual vector.

## The default choice is tnorm=2

3. braid_SetAbsTol, braid_SetRelTol

- If an absolute tolerance is used, then

$$
\left\|r^{(k)}\right\|_{\text {space_time }}<\text { tol }
$$

defines when to halt.

- If a relative tolerance is used, then

$$
\frac{\left\|r^{(k)}\right\|_{\text {space_time }}}{\left\|r^{(0)}\right\|_{\text {space_time }}}<\text { tol }
$$

defines when to halt. That is, the current $k$ th residual is scaled by the initial residual before comparison to the halting tolerance. This is similar to typical relative residual halting tolerances used in spatial multigrid, but can be a dangerous choice in this setting.

Care should be practiced when choosing a halting tolerance. For instance, if a relative tolerance is used, then issues can arise when the initial guess is zero for large numbers of time steps. Taking the case where the initial guess (defined by braid_PtFcnInit) is 0 for all time values $t>0$, the initial residual norm will essentially only be nonzero at the first time value,

$$
\left\|r^{(0)}\right\|_{\text {space_time }} \approx\left\|r_{1}^{(k)}\right\|_{\text {spatial_norm }}
$$

This will skew the relative halting tolerance, especially if the number of time steps increases, but the initial residual norm does not.

A better strategy is to choose an absolute tolerance that takes your space-time domain size into account, as in Section Scaling Study with this Example, or to use an infinity-norm temporal norm option.
3.2.6 Debugging XBraid

Wrapping and debugging a code with XBraid typically follows a few steps.

- Test your wrapped functions with XBraid test functions, e.g., braid_TestClone or braid_TestSum.
- Set max levels to 1 (braid_SetMaxLevels) and run an XBraid simulation. You should get the exact same answer as that achieved with sequential time stepping. If you make sure that the time-grids used by XBraid and by sequential time stepping are bit-wise the same (by using the user-defined time grid option braid_SetTimeGrid ), then the agreement of their solutions should be bit-wise the same.
- Continue with max levels equal to 1, but switch to two processors in time. Check that the answer again exactly matches sequential time stepping. This test checks that the information in braid_Vector is sufficient to correctly start the simulation on the second processor in time.
- Set max levels to 2, halting tolerance to 0.0 (braid_SetAbsTol), max iterations to 3 (braid_SetMaxIter) and turn on the option braid_SetSeqSoln. This will use the solution from sequential time-stepping as the initial guess for XBraid and then run 3 iterations. The residual should be exactly 0 each iteration, verifying the fixed-point nature of XBraid and a (hopefully!) correct implementation. The residual may be on the order of machine epsilon (or smaller). Repeat this test for multiple processors in time (and space if possible).
- A similar test turns on debug level printing by passing a print level of 3 to braid_SetPrintLevel. This will print out the residual norm at each C-point. XBraid with FCF-relaxation has the property that the exact solution is propagated forward two C-points each iteration. Thus, this should be reflected by numerically zero residual values for the first so many time points. Repeat this test for multiple processors in time (and space if possible).
- Finally, run some multilevel tests, making sure that the XBraid results are within the halting tolerance of the solutions generated by sequential time-stepping. Repeat this test for multiple processors in time (and space if possible).
- Congratulations! Your code is now verified.


### 3.3 Computing Derivatives with XBraid_Adjoint

XBraid_Adjoint has been developed in collaboration with the Scientific Computing group at TU Kaiserslautern, Germany, and in particular with Dr. Stefanie Guenther and Prof. Nicolas Gauger.

In many application scenarios, the ODE system is driven by some independent design parameters $\rho$. These can be any time-dependent or time-independent parameters that uniquely determine the solution of the ODE (e.g. a boundary condition, material coefficients, etc.). In a discretized ODE setting, the user's time-stepping routine might then be written as

$$
u_{i}=\Phi_{i}\left(u_{i-1}, \rho\right), \quad \forall i=1, \ldots N
$$

where the time-stepper $\Phi_{i}$, which propagates a state $u_{i-1}$ at a time $t_{i-1}$ to the next time step at $t_{i}$, now also depends on the design parameters $\rho$. In order to quantify the simulation output for the given design, a real-valued objective function can then be set up that measures the quality of the ODE solution:

$$
J(\mathbf{u}, \rho) \in \mathrm{R}
$$

Here, $\mathbf{u}=\left(u_{0}, \ldots, u_{N}\right)$ denotes the space-time state solution for a given design.
XBraid_Adjoint is a consistent discrete time-parallel adjoint solver for XBraid which provides sensitivity information of the output quantity $J$ with respect to the user-defined design parameters $\rho$. The ability to compute sensitivities can greatly improve and enhance the simulation tool, for example for solving

- Design optimization problems,
- Optimal control problems,
- Parameter estimation for validation and verification purposes,
- Error estimation,
- Uncertainty quantification techniques.

XBraid_Adjoint is non-intrusive with respect to the adjoint time-stepping scheme so that existing time-serial adjoint codes can be integrated easily though an extended user-interface.

### 3.3.1 Short Introduction to Adjoint-based Sensitivity Computation

Adjoint-based sensitivities compute the total derivative of $J$ with respect to changes in the design parameters $\rho$ by solving additional so-called adjoint equations. We will briefly introduce the idea in the following. You can skip this section, if you are familiar with adjoint sensitivity computation in general and move to Overview of the XBraid_Adjoint Algorithm immedately. Information on the adjoint method can be found in [Giles, Pierce, 2000] ${ }^{3}$ amongst many others.

Consider an augmented (so-called Lagrange) funtion

$$
L(\mathbf{u}, \rho)=J(\mathbf{u}, \rho)+\overline{\mathbf{u}}^{T} A(\mathbf{u}, \rho)
$$

where the discretized time-stepping ODE equations in

$$
A(\mathbf{u}, \rho):=\left(\begin{array}{c}
\Phi_{1}\left(u_{0}, \rho\right)-u_{1} \\
\vdots \\
\Phi_{N}\left(u_{N-1}, \rho\right)-u_{N}
\end{array}\right)
$$

[^2]have been added to the objective function, and multiplied with so-called adjoint variables $\overline{\mathbf{u}}=\left(\bar{u}_{1}, \ldots, \bar{u}_{N}\right)$. Since the added term is zero for all design and state variables that satisfy the discrete ODE equations, the total derivative of $J$ and $L$ with respect to the design match. Using the chain rule of differentiation, this derivative can be expressed as
$$
\frac{\mathrm{d} J}{\mathrm{~d} \rho}=\frac{\mathrm{d} L}{\mathrm{~d} \rho}=\frac{\partial J}{\partial \mathbf{u}} \frac{\mathrm{~d} \mathbf{u}}{\mathrm{~d} \rho}+\frac{\partial J}{\partial \rho}+\overline{\mathbf{u}}^{T}\left(\frac{\partial A}{\partial \mathbf{u}} \frac{\mathrm{~d} \mathbf{u}}{\mathrm{~d} \rho}+\frac{\partial A}{\partial \rho}\right)
$$
where $\partial$ denotes partial derivatives - in contrast to the total derivative (i.e. the sensitivity) denoted by d .

When computing this derivative, the terms in red are the ones that are computationally most expensive. In fact, the cost for computing these sensitivities scale linearly with the number of design parameters, i.e. the dimension of $\rho$. These costs can grow quickly. For example, consider a finite differencing setting, where a re-computation of the entire spacetime state would be necessary for each design variable, because a perturbation of the design must be computed in all the unit directions of the design space. In order to avoid these costs, the adjoint method aims to set the adjoint variable $\overline{\mathbf{u}}$ such that these red terms add up to zero in the above expression. Hence, if we solve first for

$$
\left(\frac{\partial J}{\partial \mathbf{u}}\right)^{T}+\left(\frac{\partial A}{\partial \mathbf{u}}\right)^{T} \overline{\mathbf{u}}=0
$$

for the adjoint variable $\overline{\mathbf{u}}$, then the so-called reduced gradient of $J$, which is the transpose of the total derivative of $J$ with respect to the design, is given by

$$
\left(\frac{\mathrm{d} J}{\mathrm{~d} \rho}\right)^{T}=\left(\frac{\partial J}{\partial \rho}\right)^{T}+\left(\frac{\partial A}{\partial \rho}\right)^{T} \overline{\mathbf{u}}
$$

The advantage of this strategy is, that in order to compute the sensitivity of $J$ with respect to $\rho$, only one additional space-time equation (adjoint) for $\overline{\mathbf{u}}$ has to be solved, in addition to evaluating the partial derivatives. The computational cost for computing $\mathrm{d} J / \mathrm{d} \rho$ therefore does not scale in this setting with the number of design parameters.

For the time-dependent discrete ODE problem, the adjoint equation from above reads

$$
\text { unsteady adjoint: } \quad \bar{u}_{i}=\partial_{u_{i}} J(\mathbf{u}, \rho)^{T}+\left(\partial_{u_{i}} \Phi_{i+1}\left(u_{i}, \rho\right)\right)^{T} \bar{u}_{i+1} \quad \forall i=N \ldots, 1
$$

using the terminal condition $u_{N+1}:=0$. The reduced gradient is given by
reduced gradient:

$$
\left(\frac{\partial J}{\partial \rho}\right)^{T}=\partial_{\rho} J(\mathbf{u}, \rho)^{T}+\sum_{i=1}^{N}\left(\partial_{\rho} \Phi_{i}\left(u_{i-1}, \rho\right)\right)^{T} \bar{u}_{i}
$$

### 3.3.2 Overview of the XBraid_Adjoint Algorithm

The unsteady adjoint equations can in principle be solved "backwards in time" in a time-serial manner, starting from the terminal condition $\bar{u}_{N+1}=0$. However, the parallel-in-time XBraid_Adjoint solver offers speedup by distributing the backwards-in-time phase onto multiple processors along the time domain. Its implementation is based on techniques of the reverse-mode of Automatic Differentiation applied to one primal XBraid iteration. To that end, each primal iteration is augmented by an objective function evaluation, followed by updates for the space-time adjoint variable $\overline{\mathbf{u}}$, as well as evaluation of the reduced gradient denoted by $\bar{\rho}$. In particular, the following so-called piggy-back iteration is performed:

1. XBraid: update the state and evaluate the objective function

$$
\mathbf{u}^{(k+1)} \leftarrow \operatorname{XBraid}\left(\mathbf{u}^{(k)}, \rho\right), \quad J \leftarrow J\left(\mathbf{u}^{(k)}, \rho\right)
$$

2. XBraid_Adjoint: update the adjoint and evaluate the reduced gradient

$$
\overline{\mathbf{u}}^{(k+1)} \leftarrow \text { XBraid_Adjoint }\left(\mathbf{u}^{(k)}, \overline{\mathbf{u}}^{(k)}, \rho\right), \quad \bar{\rho} \leftarrow\left(\frac{\mathrm{d} J\left(\mathbf{u}^{(k)}, \rho\right)}{\mathrm{d} \rho}\right)^{T}
$$

Each XBraid_Adjoint iteration moves backwards though the primal XBraid multigrid cycle. It collects local partial derivatives of the elemental XBraid operations in reverse order and concatenates them using the chain rule of differentiation. This is the basic idea of the reverse mode of Automatic Differentiation (AD). This yields a consistent discrete time-parallel adjoint solver that inherits the parallel scaling properties of the primal XBraid solver.

Further, XBraid_Adjoint is non-intrusive for existing adjoint methods based on sequential time marching schemes. It adds additional user-defined routines to the primal XBraid interface, in order to define the propagation of sensitivities of the forward time stepper backwards-in-time and the evaluation of partial derivatives of the local objective function at each time step. In cases where a time-serial unsteady adjoint solver is already available, this backwards time stepping capability can be easily wrapped according to the adjoint user interface with little extra coding.

The adjoint solve in the above piggy-back iteration converges at the same convergence rate as the primal state variables. However since the adjoint equations depend on the state solution, the adjoint convergence will slightly lag behind the convergence of the state. More information on convergence results and implementational details for XBraid_Adjoint can be found in [Gunther, Gauger, Schroder, 2017]. ${ }^{4}$

### 3.3.3 Overview of the XBraid_Adjoint Code

XBraid_Adjoint offers a non-intrusive approach for time-parallelization of existing time-serial adjoint codes. To that end, an extended user-interface allows the user to wrap their existing code for evaluating the objective function and performing a backwards-in-time adjoint step into routines according to the XBraid_Adjoint interface.

### 3.3.3.1 Objective function evaluation

The user-interface for XBraid_Adjoint allows for objective functions of the following type:

$$
J=F\left(\int_{t_{0}}^{t^{1}} f(u(t), \rho) \mathrm{d} t\right)
$$

This involves a time-integral part of some time-dependent quantity of interest $f$ as well as a postprocessing function $F$. The time-interval boundaries $t_{0}, t_{1}$ can be set using the options braid_SetTStartObjective and braid_SetTStopObjective, otherwise the entire time domain will be considered. Note that these options can be used for objective functions that are only evaluated at one specific time instance by setting $t_{0}=t_{1}$ (e.g. in cases where only the last time step is of interest). The postprocessing function $F$ offers the possibility to further modify the time-integral, e.g. for setting up a tracking-type objective function (substract a target value and square), or for adding relaxation or penalty terms. While defining $f$ is mandatory for XBraid_Adjoint, the postprocessing routine $F$ is optional and is passed to XBraid_Adjoint though the optional braid_SetPostprocessObjective and braid_SetPostprocessObjective_diff routines. XBraid_Adjoint will perform the time-integration by summing up the $f$ evaluations in the given time-domain

$$
I \leftarrow \sum_{i=i_{0}}^{i_{1}} f\left(u_{i}, \rho\right)
$$

followed by a call to the postprocessing function $F$, if set:

$$
J \leftarrow F(I, \rho)
$$

Note that any integration rule for computing $I$, e.g. for scaling contributions from $f()$, must be done by the user.

[^3]Partial derivatives of user-routines

The user needs to provide the derivatives of the time-stepper $\Phi$ and function evaluation $f$ (and potentially $F$ ) for $X_{\hookleftarrow}$ Braid_Adjoint. Those are provided in terms of transposed matrix-vector products in the following way:

## 1. Derivatives of the objective function $J$ :

- Time-dependent part $f$ : The user provides a routine that evaluates the following transposed partial derivatives of $f$ multiplied with the scalar input $\bar{F}$ :

$$
\begin{gathered}
\bar{u}_{i} \leftarrow\left(\frac{\partial f\left(u_{i}, \rho\right)}{\partial u_{i}}\right)^{T} \bar{F} \\
\bar{\rho} \leftarrow \bar{\rho}+\left(\frac{\partial f\left(u_{i}, \rho\right)}{\partial \rho}\right)^{T} \bar{F}
\end{gathered}
$$

The scalar input $\bar{F}$ equals 1.0, if no postpocessing function $F$ has been set.

- Postprocessing $F$ : If the postprocessing routine has been set, the user needs to provide it's transposed partial derivatives in the following way:

$$
\begin{gathered}
\bar{F} \leftarrow \frac{\partial F(I, \rho)}{\partial I} \\
\bar{\rho} \leftarrow \rho+\frac{\partial F(I, \rho)}{\partial \rho}
\end{gathered}
$$

2. Derivatives of the time-stepper $\Phi_{i}$ : The user provides a routine that computes the following transposed partial derivatives of $\Phi_{i}$ multiplied with the adjoint input vector $\bar{u}_{i}$ :

$$
\begin{gathered}
\bar{u}_{i} \leftarrow\left(\frac{\partial \Phi\left(u_{i}, \rho\right)}{\partial u_{i}}\right)^{T} \bar{u}_{i} \\
\bar{\rho} \leftarrow \bar{\rho}+\left(\frac{\partial \Phi\left(u_{i}, \rho\right)}{\partial \rho}\right)^{T} \bar{u}_{i}
\end{gathered}
$$

Note that the partial derivatives with respect to $\rho$ always update the reduced gradient $\bar{\rho}$ instead of overwriting it (i.e. they are a plus-equal operation, $+=$ ). Therefore, the gradient needs to be reset to zero before each iteration of XBraid_ $\hookleftarrow$ Adjoint, which is taken care of by XBraid_Adjoint calling an additional user-defined routine braid_PtFcnResetGradient.

Depending on the nature of the design variables, it is neccessary to gather gradient information in $\bar{\rho}$ from all timeprocessors after XBraid_Adjoint has finished. It is the user's responsibility to do that, if needed, e.g. through a call to MPI_Allreduce.

## Halting tolerance

Similar to the primal XBraid algorithm, the user can choose a halting tolerance for XBraid_Adjoint which is based on the adjoint residual norm. An absolute tolerance (braid_SetAbsTolAdjoint)

$$
\left\|\overline{\mathbf{u}}^{(k)}-\overline{\mathbf{u}}^{(k-1)}\right\|_{\text {space_time }}<\text { tol_adjoint }
$$

or a relative tolerance (braid_SetRelTolAdjoint)

$$
\frac{\left\|\overline{\mathbf{u}}^{(k)}-\overline{\mathbf{u}}^{(k-1)}\right\|_{\text {space_time }}}{\left\|\overline{\mathbf{u}}^{(1)}-\overline{\mathbf{u}}^{(0)}\right\|_{\text {space_time }}}<\text { tol_adjoint }
$$

can be chosen.

## Finite Difference Testing

You can verify the gradient computed from XBraid_Adjoint using Finite Differences. Let $e_{i}$ denote the $i$-th unit vector in the design space, then the i-th entry of the gradient should match with

$$
i \text {-th Finite Difference: } \frac{J\left(\mathbf{u}_{\rho+h e_{i}}, \rho+h e_{i}\right)-J(\mathbf{u}, \rho)}{h}
$$

for a small perturbation $h>0$. Here, $\mathbf{u}_{\rho+h e_{i}}$ denotes the new state solution for the perturbed design variable. Keep in mind, that round-off errors have to be considered when computing the Finite Differences for very small perturbations $h \rightarrow 0$. Hence, you should vary the parameter to find the best fit.

In order to save some computational work while computing the perturbed objective function value, XBraid_Adjoint can run in ObjectiveOnly mode, see braid_SetObjectiveOnly. When in this mode, XBraid_Adjoint will only solve the ODE system and evaluate the objective function, without actually computing its derivative. This option might also be useful within an optimization framework e.g. for implementing a line-search procedure.

## Getting started

- Look at the simple example Simplest XBraid_Adjoint example in order to get started. This example is in examples/ex-01-adjoint.c, which implements XBraid_Adjoint sensitivity computation for a scalar ODE.


### 3.4 Citing XBraid

To cite XBraid, please state in your text the version number from the VERSION file, and please cite the project website in your bibliography as
[1] XBraid: Parallel multigrid in time. http://llnl.gov/casc/xbraid.
The corresponding BibTex entry is

```
@misc{xbraid-package,
    title = {{XB}raid: Parallel multigrid in time},
    howpublished = {\url{http://llnl.gov/casc/xbraid}}
    }
```


### 3.5 Summary

- XBraid applies multigrid to the time dimension.
- This exposes concurrency in the time dimension.
- The potential for speedup is large, 10x, 100x, ...
- This is a non-intrusive approach, with an unchanged time discretization defined by user.
- Parallel time integration is only useful beyond some scale. This is evidenced by the experimental results below. For smaller numbers of cores sequential time stepping is faster, but at larger core counts XBraid is much faster.
- The more time steps that you can parallelize over, the better your speedup will be.
- XBraid is optimal for a variety of parabolic problems (see the examples directory).
- XBraid_Adjoint provides time-parallel adjoint-based sensitivities of output quantities with respect to user-defined design variables
- It is non-intrusive with respect to existing adjoint time-marching schemes
- It inherits parallel scaling properties from XBraid


## 4 Examples

This section is the chief tutorial of XBraid, illustrating how to use it through a sequence of progressively more sophisticated examples.

### 4.1 The Simplest Example

User Defined Structures and Wrappers

The user must wrap their existing time stepping routine per the XBraid interface. To do this, the user must define two data structures and some wrapper routines. To make the idea more concrete, we now give these function definitions from examples/ex-01, which implements a scalar ODE,

$$
u_{t}=\lambda u
$$

The two data structures are:

1. App: This holds a wide variety of information and is global in that it is passed to every function. This structure holds everything that the user will need to carry out a simulation. Here for illustration, this is just an integer storing a processor's rank.
```
typedef struct _braid_App_struct
{
    int rank;
} my_App;
```

2. Vector: this defines (roughly) a state vector at a certain time value. It could also contain any other information related to this vector which is needed to evolve the vector to the next time value, like mesh information. Here, the vector is just a scalar double.
```
typedef struct _braid_Vector_struct
{
    double value;
} my_Vector;
```

The user must also define a few wrapper routines. Note, that the app structure is the first argument to every function.

1. Step: This function tells XBraid how to take a time step, and is the core user routine. The user must advance the vector $u$ from time tstart to time tstop. Note how the time values are given to the user through the status structure and associated Get routine. Important note: the $g_{i}$ function from Overview of the XBraid Algorithm must be incorporated into Step, so that the following equation is solved by default.

$$
\Phi\left(u_{i}\right)=0
$$

The ustop parameter serves as an approximation to the solution at time tstop and is not needed here. It can be useful for implicit schemes that require an initial guess for a linear or nonlinear solver. The use of fstop is an advanced parameter (not required) and forms the the right-hand side of the nonlinear problem on the given time grid. This value is only nonzero when providing a residual with braid_SetResidual. More information on how to use this optional feature is given below.

Here advancing the solution just involves the scalar $\lambda$.

```
int
my_Step(braid_App app,
            braid_Vector ustop,
            braid_Vector fstop,
            braid_Vector u,
            braid_StepStatus status)
{
    double tstart; /* current time */
    double tstop; /* evolve to this time*/
    braid_StepStatusGetTstartTstop(status, &tstart, &tstop);
    /* Use backward Euler to propagate solution */
    (u->value) = 1./(1. + tstop-tstart)*(u->value);
    return 0;
}
```

2. Init: This function tells XBraid how to initialize a vector at time $t$. Here that is just allocating and setting a scalar on the heap.
```
int
my_Init(braid_App app,
        double t,
        braid_Vector *u_ptr)
{
    my_Vector *u;
    u = (my_Vector *) malloc(sizeof(my_Vector));
    if (t == 0.0) /* Initial condition */
    {
        (u->value) = 1.0;
    }
    else /* All other time points set to arbitrary value */
    {
        (u->value) = 0.456;
    }
    *u_ptr = u;
    return 0;
}
```

3. Clone: This function tells XBraid how to clone a vector into a new vector.
```
int
my_Clone(braid_App app,
    braid_Vector u,
    braid_Vector *v_ptr)
{
    my_Vector *v;
    v = (my_Vector *) malloc(sizeof(my_Vector));
    (v->value) = (u->value);
    *v_ptr = v;
    return 0;
}
```

4. Free: This function tells XBraid how to free a vector.
```
int
my_Free(braid_App app,
    braid_Vector u)
{
    free(u);
    return 0;
}
```

5. Sum: This function tells XBraid how to sum two vectors (AXPY operation).
```
int
my_Sum(braid_App app,
        double alpha,
        braid_Vector x,
        double beta,
        braid_Vector y)
{
    (y->value) = alpha*(x->value) + beta*(y->value);
    return 0;
}
```

6. SpatialNorm: This function tells XBraid how to take the norm of a braid_Vector and is used for halting. This norm is only over space. A common norm choice is the standard Euclidean norm, but many other choices are possible, such as an L2-norm based on a finite element space. The norm choice should be based on what makes sense for your problem. How to accumulate spatial norm values to obtain a global space-time residual norm for halting decisions is controlled by braid_SetTemporalNorm.
```
int
my_SpatialNorm(braid_App app,
    braid_Vector u,
    double *norm_ptr)
{
    double dot;
    dot = (u->value)*(u->value);
    *norm_ptr = sqrt(dot);
    return 0;
}
```

7. Access: This function allows the user access to XBraid and the current solution vector at time $t$. This is most commonly used to print solution(s) to screen, file, etc... The user defines what is appropriate output. Notice how you are told the time value $t$ of the vector $u$ and even more information in astatus. This lets you tailor the output to only certain time values at certain XBraid iterations. Querying astatus for such information is done through braid_AccessStatusGet**(..) routines.

The frequency of the calls to access is controlled through braid_SetAccessLevel. For instance, if access_level is set to 2 , then access is called every XBraid iteration and on every XBraid level. In this case, querying astatus to determine the current XBraid level and iteration will be useful. This scenario allows for even more detailed tracking of the simulation. The default access_level is 1 and gives the user access only after the simulation ends and only on the finest time-grid.

Eventually, this routine will allow for broader access to XBraid and computational steering.

See examples/ex-03 and drivers/drive-diffusion for more advanced uses of the access function. In drive-diffusion, access is used to write solution vectors to a GLVIS visualization port, and ex-03 uses access to write to .vtu files.

```
int
my_Access(braid_App app,
    braid_Vector u,
    braid_AccessStatus astatus)
{
    int index;
    char filename[255];
    FILE *file;
    braid_AccessStatusGetTIndex(astatus, &index);
    sprintf(filename, "%s.%04d.%03d", "ex-01.out", index, app->rank);
```

```
    file = fopen(filename, "w");
    fprintf(file, "%.14e\n", (u->value));
    fflush(file);
    fclose(file);
    return 0;
}
```

8. BufSize, BufPack, BufUnpack: These three routines tell XBraid how to communicate vectors between processors. BufPack packs a vector into a void $*$ buffer for MPI and then BufUnPack unpacks the void $*$ buffer into a vector. Here doing that for a scalar is trivial. BufSize computes the upper bound for the size of an arbitrary vector.

Note how BufPack also sets the size in bstatus. This value is optional, but if set it should be the exact number of bytes packed, while BufSize should provide only an upper-bound on a possible buffer size. This flexibility allows for the buffer to be allocated the fewest possible times, but smaller messages to be sent when needed. For instance, this occurs when using variable spatial grid sizes. To avoid MPI issues, it is very important that BufSize be pessimistic, provide an upper bound, and return the same value across processors.

In general, the buffer should be self-contained. The receiving processor should be able to pull all necessary information from the buffer in order to properly interpret and unpack the buffer.

```
int
my_BufSize(braid_App app,
    int *size_ptr,
    braid_BufferStatus bstatus)
{
    *size_ptr = sizeof(double);
    return 0;
}
int
my_BufPack(braid_App app,
            braid_Vector u,
            void *buffer,
            braid_BufferStatus bstatus)
{
    double *dbuffer = buffer;
    dbuffer[0] = (u->value);
    braid_BufferStatusSetSize( bstatus, sizeof(double) );
    return 0;
}
int
my_BufUnpack(braid_App app,
                    void *buffer,
                    braid_Vector *u_ptr,
                    braid_BufferStatus bstatus)
{
    double *dbuffer = buffer;
    my_Vector *u;
    u = (my_Vector *) malloc(sizeof(my_Vector));
    (u->value) = dbuffer[0];
    *u_ptr = u;
    return 0;
}
```

4.1.1 Running XBraid for the Simplest Example

A typical flow of events in the main function is to first initialize the app structure.

```
/* set up app structure */
app = (my_App *) malloc(sizeof(my_App));
(app->rank) = rank;
```

Then, the data structure definitions and wrapper routines are passed to XBraid. The core structure is used by XBraid for internal data structures.

```
braid_Core core;
braid_Init(MPI_COMM_WORLD, comm, tstart, tstop, ntime, app,
    my_Step, my_Init, my_Clone, my_Free, my_Sum, my_SpatialNorm,
    my_Access, my_BufSize, my_BufPack, my_BufUnpack, &core);
```

Then, XBraid options are set.

```
braid_SetPrintLevel( core, 1);
braid_SetMaxLevels(core, max_levels);
braid_SetAbsTol(core, tol);
braid_SetCFactor(core, -1, cfactor);
```

Then, the simulation is run.

```
braid_Drive(core);
```

Then, we clean up.
braid_Destroy(core);

Finally, to run ex-01, type
ex-01

### 4.2 Some Advanced Features

We now give an overview of some optional advanced features that will be implemented in some of the following examples.

1. SCoarsen, SRestrict: These are advanced options that allow for coarsening in space while you coarsen in time. This is useful for maintaining stable explicit schemes on coarse time scales and is not needed here. See examples/ex-02 for a simple example of this feature, and then drivers/drive-diffusion and drivers/drive-diffusion-2D for more advanced examples of this feature.

These functions allow you to vary the spatial mesh size on XBraid levels as depicted here where the spatial and temporal grid sizes are halved every level.

2. Residual: A user-defined residual can be provided with the function braid_SetResidual and can result in substantial computational savings, as explained below. However to use this advanced feature, one must first understand how XBraid measures the residual. XBraid computes residuals of this equation,

$$
A_{i}\left(u_{i}, u_{i-1}\right)=f_{i}
$$

where $A_{i}($,$) evaluates one block-row of the the global space-time operator A$. The forcing $f_{i}$ is the XBraid forcing, which is the FAS right-hand-side term on coarse grids and 0 on the finest grid. The PDE forcing goes inside of $A_{i}$.
Since XBraid assumes one-step methods, $A_{i}()$ is defined to be

$$
A_{i}\left(u_{i}, u_{i-1}\right)=-\Phi\left(u_{i-1}\right)+\Psi\left(u_{i}\right)
$$

i.e., the subdiagonal and diagonal blocks of $A$.

Default setting: In the default XBraid setting (no residual option used), the user only implements Step() and Step() will simply apply $\Phi()$, because $\Psi()$ is assumed to be the identity. Thus, XBraid can compute the residual using only the user-defined Step() function by combining Step() with the Sum() function, i.e.

$$
r_{i}=f_{i}+\Phi\left(u_{i-1}\right)-u_{i}
$$

The fstop parameter in Step() corresponds to $f_{i}$, but is always passed in as NULL to the user in this setting and should be ignored. This is because XBraid can compute the contribution of $f_{i}$ to the residual on its own using the Sum() function.
An implication of this is that the evaluation of $\Phi()$ on the finest grid must be very accurate, or the residual will not be accurate. This leads to a nonintrusive, but expensive algorithm. The accuracy of $\Phi()$ can be relaxed on coarser grids to save computations.

Residual setting: The alternative to the above default least-intrusive strategy is to have the user define

$$
A_{i}\left(u_{i}, u_{i-1}\right)=-\Phi\left(u_{i-1}\right)+\Psi\left(u_{i}\right)
$$

directly, which is what the Residual function implements (set with braid_PtFcnResidual). In other words, the user now defines each block-row of the space-time operator, rather than only defining $\Phi()$. The user Residual() function computes $A_{i}\left(u_{i}, u_{i-1}\right)$ and XBraid then subtracts this from $f_{i}$ to compute $r_{i}$.
However, more care must now be taken when defining the Step() function. In particular, the fstop value (i.e., the $f_{i}$ value) must be taken into account. Essentially, the definition of Step() changes so that it no longer defines $\Phi()$, but instead defines a (possibly inexact) solve of the equation defined by

$$
A_{i}\left(u_{i}, u_{i-1}\right)=f_{i}
$$

Thus, Step() must be compatible with Residual(). Expanding the previous equation, we say that Step() must now compute

$$
u_{i}=\Psi^{-1}\left(f_{i}+\Phi\left(u_{i-1}\right)\right)
$$

It is clear that the fstop value (i.e., the $f_{i}$ value) must now be given to the $\operatorname{Step}($ ) function so that this equation can be solved by the user. In other words, fstop is now no longer NULL.
Essentially, one can think of Residual() as defining the equation, and Step() defining a preconditioner for that row of the equation, or an inexact solve for $u_{i}$.
As an example, let $\Psi=(I+\Delta t L)$, where $L$ is a Laplacian and $\Phi=I$. The application of the residual function will only be a sparse matrix-vector multiply, as opposed to the default case where an inversion is required for $\Phi=(I+\Delta t L)^{-1}$ and $\Psi=I$. This results in considerable computational savings. Moreover, the application of Step() now involves an inexact inversion of $\Psi$, e.g., by using just one spatial multigrid V-cycle. This again results in substantial computation savings when compared with the naive approach of a full matrix inversion.

Another way to think about the compatibility between $\Psi$ and $\Phi$ is that

$$
f_{i}-A_{i}\left(u_{i}, u_{i-1}\right)=0
$$

must hold exactly if $u_{i}$ is an exact propagation of $u_{i-1}$, that is,

$$
f_{i}-A_{i}\left(\operatorname{Step}\left(u_{i-1}, f_{i}\right), u_{i-1}\right)=0
$$

must hold. When the accuracy of the Step() function is reduced (as mentioned above), this exact equality with 0 is lost, but this should evaluate to something small. There is an XBraid test function braid_TestResidual that tests for this compatibility.

The residual feature is implemented in the examples examples/ex-01-expanded.c, examples/ex-02. $\hookleftarrow$ c, and examples/ex-03.c.
3. Adaptive and variable time stepping: This feature is available by first calling the function braid_SetRefine in the main driver and then using braid_StepStatusSetRFactor in the Step routine to set a refinement factor for interval [tstart, tstop]. In this way, user-defined criteria can subdivide intervals on the fly and adaptively refine in time. For instance, returning a refinement factor of 4 in Step will tell XBraid to subdivide that interval into 4 evenly spaced smaller intervals for the next iteration. Refinement can only be done on the finest XBraid level.

The final time grid is constructed adaptively in an FMG-like cycle by refining the initial grid according to the requested refinement factors. Refinement stops when the requested factors are all one or when various upper bounds are reached such as the max number of time points or max number of time grid refinement levels allowed. No restriction on the refinement factors is applied within XBraid, so the user may want to apply his own upper bound on the refinement factors to avoid over-refinement. See examples/ex-01-refinement.c and examples/ex-03.c for an implementation of this.
4. Richardson-based Error Estimation and Extrapolation: This feature allows the user to access built-in Richardson-based error estimates and accuracy improving extrapolation. The error estimates and/or extrapolation can be turned on by using braid_SetRichardsonEstimation. Moreover, this feature can be used in conjunction with the above discussed function, braid_StepStatusSetRFactor, to achieve easy-to-use adaptive refinement in time.
Essentially, Richardson extrapolation (RE) is used to improve the accuracy of the solution at the C-points on the finest level. When the built-in error estimate option is turned on, RE is used to estimate the local truncation error at each point. These estimates can be accessed through StepStatus and AccessStatus functions.
The Richardson-based error estimates and extrapolation are only available after the first Braid iteration, in that the coarse level solution must be available to compute the error estimate and/or extrapolation. Thus, after an adaptive refinement (and new hierarchy is constructed), another iteration is again required for the error estimates to be available. If the error estimate isn't available, Braid returns a value of -1 . See this example for more details

```
examples/ex-06.c
```

5. Shell-vector: This feature supports the use of multi-step methods. The strategy for BDF-K methods is to allow for the lumping of $k$ time points into a single XBraid vector. So, if the problem had 100 time points and the timestepper was BDF-2, then XBraid would only see 50 time points but each XBraid vector would contain two separate time points. By lumping 2 time points into one vector, the BDF-2 scheme remains one-step and compatible with XBraid.

However, the time-point spacing between the two points internal to the vector stays the same on all time grids, while the spacing between vectors grows on coarse time grids. This creates an irregular spacing which is problematic for BDF-k methods. Thus the shell-vector strategy lets meta-data be stored at all time points, even for F-points which are usually not stored, so that the irregular spacings can be tracked and accounted for with the BDF method. (Note, there are other possible uses for shell-vectors.)
There are many strategies for handling the coarse time-grids with BDF methods (dropping the BDF order, adjusting time-point spacings inside the lumped vectors, etc...). Prospective users are encouraged to contact the devlopers through the XBraid Github page and issue tracker. This area is active research.

See examples/ex-01-expanded-bdf2.c.
6. Storage: This option (see braid_SetStorage) allows the user to specify storage at all time points ( C and F ) or only at C-points. This extra storage is useful for implicit methods, where the solution value from the previous XBraid iteration for time step $i$ can be used as the initial guess when computing step $i$ with the implicit solver. This is often a better initial guess than using the solution value from the previous time step $i-1$. The default is to store only C-point values, thus the better initial guess is only available at C-points in the default setting. When storage is turned on at F-points, the better initial guess becomes available everywhere.

In general, the user should always use the ustop parameter in Step() as the initial guess for an implicit solve. If storage is turned on (i.e., set to 0 ), then this value will always be the improved initial guess for C - and F -points. If storage is not turned on, then this will be the improved guess only for C-points. For F-points, it will equal the solution from the previous time step.
See examples/ex-03 for an example which uses this feature.

### 4.3 Simplest example expanded

These examples build on The Simplest Example, but still solve the scalar ODE,

$$
u_{t}=\lambda u
$$

The goal here is to show more advanced features of XBraid.

- examples/ex-01-expanded.c: same as ex-01.c but adds more XBraid features such as the residual feature, the user defined initial time-grid and full multigrid cycling.
- examples/ex-01-expanded-bdf2.c: same as ex-01-expanded.c, but uses BDF2 instead of backward Euler. This example makes use of the advanced shell-vector feature in order to implement BDF2.
- examples/ex-01-expanded-f.f90: same as ex-01-expanded.c, but implemented in f90.
- examples/ex-01-refinement.c: same as ex-01.c, but adds the refinement feature of XBraid. The refinement can be arbitrary or based on error estimate.


### 4.4 One-Dimensional Heat Equation

In this example, we assume familiarity with The Simplest Example. This example is a time-only parallel example that implements the 1D heat equation,

$$
\delta / \delta_{t} u(x, t)=\Delta u(x, t)+g(x, t)
$$

as opposed to The Simplest Example, which implements only a scalar ODE for one degree-of-freedom in space. There is no spatial parallelism, as a serial cyclic reduction algorithm is used to invert the tri-diagonal spatial operators. The space-time discretization is the standard 3 -point finite difference stencil ( $[-1,2,-1]$ ), scaled by mesh widths. Backward Euler is used in time.

This example consists of three files and two executables.

- examples/ex-02-serial.c: This file compiles into its own executable ex-02-serial and represents a simple example user application that does sequential time-stepping. This file represents where a new XBraid user would start, in terms of converting a sequential time-stepping code to XBraid.
- examples/ex-02.c: This file compiles into its own executable ex-02 and represents a time-parallel XBraid wrapping of the user application ex-02-serial.
- ex-02-lib.c: This file contains shared functions used by the time-serial version and the time-parallel version. This file provides the basic functionality of this problem. For instance, take_step(u, tstart, tstop, ...) carries out a step, moving the vector $u$ from time tstart to time tstop.


### 4.5 Two-Dimensional Heat Equation

In this example, we assume familiarity with The Simplest Example and describe the major ways in which this example differs. This example is a full space-time parallel example, as opposed to The Simplest Example, which implements only a scalar ODE for one degree-of-freedom in space. We solve the heat equation in 2D,

$$
\delta / \delta_{t} u(x, y, t)=\Delta u(x, y, t)+g(x, y, t)
$$

For spatial parallelism, we rely on the hypre package where the SemiStruct interface is used to define our spatial discretization stencil and form our time stepping scheme, the backward Euler method. The spatial discretization is just the standard 5 -point finite difference stencil ( $[-1 ;-1,4,-1 ;-1]$ ), scaled by mesh widths, and the PFMG solver is used for the solves required by backward Euler. Please see the hypre manual and examples for more information on the SemiStruct interface and PFMG. Although, the hypre specific calls have mostly been abstracted away for this example, and so it is not necessary to be familiar with the SemiStruct interface for this example.

This example consists of three files and two executables.

- examples/ex-03-serial.c: This file compiles into its own executable ex-03-serial and represents a simple example user application. This file supports only parallelism in space and represents a basic approach to doing efficient sequential time stepping with the backward Euler scheme. Note that the hypre solver used (PFMG) to carry out the time stepping is highly efficient.
- examples/ex-03.c: This file compiles into its own executable ex-03 and represents a basic example of wrapping the user application ex-03-serial. We will go over the wrappers below.
- ex-03-lib.c: This file contains shared functions used by the time-serial version and the time-parallel version. This is where most of the hypre specific calls reside. This file provides the basic functionality of this problem. For instance, take_step( $u$, tstart, tstop, ...) carries out a step, moving the vector $u$ from time tstart to time tstop and setUpImplicitMatrix(...) constructs the matrix to be inverted by PFMG for the backward Euler method.


## User Defined Structures and Wrappers

We now discuss in more detail the important data structures and wrapper routines in examples/ex-03.c. The actual code for this example is quite simple and it is recommended to read through it after this overview.

The two data structures are:

1. App: This holds a wide variety of information and is global in that it is passed to every user function. This structure holds everything that the user will need to carry out a simulation. One important structure contained in the app is the simulation_manager. This is a structure native to the user code ex-03-lib.c. This structure conveniently holds the information needed by the user code to carry out a time step. For instance,
```
app->man->A
```

is the time stepping matrix,

```
app->man->solver
```

is the hypre PFMG solver object,

```
app->man->dt
```

is the current time step size. The app is defined as

```
typedef struct _braid_App_struct {
    MPI_Comm comm; /* global communicator */
    MPI_Comm comm_t; /* communicator for parallelizing in time */
    MPI_Comm comm_x; /* communicator for parallelizing in space */
    int pt; /* number of processors in time */
    simulation_manager *man; /* user's simulation manager structure */
    HYPRE_SStructVector e; /* temporary vector used for error computations */
    int nA; /* number of spatial matrices created */
    HYPRE_SStructMatrix *A; /* array of spatial matrices, size nA, one per level*/
    double *dt_A; /* array of time step sizes, size nA, one per level*/
    HYPRE_StructSolver *solver; /* array of PFMG solvers, size nA, one per level*/
    int use_rand; /* binary value, use random or zero initial guess */
    int *runtime_max_iter; /* runtime info for number of PFMG iterations*/
    int *max_iter_x; /* maximum iteration limits for PFMG */
} my_App;
```

The app contains all the information needed to take a time step with the user code for an arbitrary time step size. See the Step function below for more detail.

1. Vector: this defines a state vector at a certain time value. Here, the vector is a structure containing a native hypre data-type, the SStructVector, which describes a vector over the spatial grid. Note that my_Vector is used to define braid_Vector.
```
typedef struct _braid_Vector_struct {
    HYPRE_SStructVector x;
} my_Vector;
```

The user must also define a few wrapper routines. Note, that the app structure is the first argument to every function.

1. Step: This function tells XBraid how to take a time step, and is the core user routine. This function advances the vector $u$ from time tstart to time tstop. A few important things to note are as follows.

- The time values are given to the user through the status structure and associated Get routines.
- The basic strategy is to see if a matrix and solver already exist for this $d t$ value. If not, generate a new matrix and solver and store them in the app structure. If they do already exist, then re-use the data.
- To carry out a step, the user routines from ex-03-lib. c rely on a few crucial data members man->dt, man- $>A$ and man-solver. We overwrite these members with the correct information for the time step size in question. Then, we pass man and $u$ to the user function take_step(...) which evolves $u$.
- The forcing term $g_{i}$ is wrapped into the take_step(...) function. Thus, $\Phi\left(u_{i}\right) \rightarrow u_{i+1}$.

```
int my_Step(braid_App app,
                    braid_Vector u,
                    braid_StepStatus status)
{
    double tstart; /* current time */
    double tstop; /* evolve u to this time*/
    int i, A_idx;
    int iters_taken = -1;
    /* Grab status of current time step */
    braid_StepStatusGetTstartTstop(status, &tstart, &tstop);
    /* Check matrix lookup table to see if this matrix already exists*/
    A_idx = -1.0;
    for( i = 0; i < app->nA; i++ ){
        if( fabs( app->dt_A[i] - (tstop-tstart) )/(tstop-tstart) < 1e-10) {
            A_idx = i;
            break;
        }
    }
    /* We need to "trick" the user's manager with the new dt */
    app->man->dt = tstop - tstart;
    /* Set up a new matrix and solver and store in app */
    if( A_idx == -1.0 ) {
        A_idx = i;
        app->nA++;
        app->dt_A[A_idx] = tstop-tstart;
        setUpImplicitMatrix( app->man );
        app->A[A_idx] = app->man->A;
        setUpStructSolver( app->man, u->x, u->x );
        app->solver[A_idx] = app->man->solver;
    }
    /* Time integration to next time point: Solve the system Ax = b.
        * First, "trick" the user's manager with the right matrix and solver */
    app->man->A = app->A[A_idx];
    app->man->solver = app->solver[A_idx];
    ..
    /* Take step */
    take_step(app->man, u->x, tstart, tstop);
    return 0;
}
```

2. There are other functions, Init, Clone, Free, Sum, SpatialNorm, Access, BufSize, BufPack and BufUnpack, which also must be written. These functions are all simple for this example, as for the case of The Simplest Example. All we do here is standard operations on a spatial vector such as initialize, clone, take an inner-product, pack, etc... We refer the reader to ex-03. c.

## Running XBraid for this Example

To initialize and run XBraid, the procedure is similar to The Simplest Example. Only here, we have to both initialize the user code and XBraid. The code that is specific to the user's application comes directly from the existing serial
simulation code. If you compare ex-03-serial. c and ex-03.c, you will see that most of the code setting up the user's data structures and defining the wrapper functions are simply lifted from the serial simulation.

Taking excerpts from the function main() in ex-03.c, we first initialize the user's simulation manager with code like

```
app->man->px = 1; /* my processor number in the x-direction */
app->man->py = 1; /* my processor number in the y-direction */
    /* px*py=num procs in space */
app->man->nx = 17; /* number of points in the x-dim */
app->man->ny = 17; /* number of points in the y-dim */
app->man->nt = 32; /* number of time steps */
```

We also define default XBraid parameters with code like

```
max_levels = 15; /* Max levels for XBraid solver */
min_coarse = 3; /* Minimum possible coarse grid size */
nrelax = 1; /* Number of CF relaxation sweeps on all levels */
```

The XBraid app must also be initialized with code like

```
app->comm = comm;
app->tstart = tstart;
app->tstop = tstop;
app->ntime = ntime;
```

Then, the data structure definitions and wrapper routines are passed to XBraid.

```
braid_Core core;
braid_Init(MPI_COMM_WORLD, comm, tstart, tstop, ntime, app,
    my_Step, my_Init, my_Clone, my_Free, my_Sum, my_SpatialNorm,
    my_Access, my_BufSize, my_BufPack, my_BufUnpack, &core);
```

Then, XBraid options are set with calls like

```
braid_SetPrintLevel( core, 1);
braid_SetMaxLevels(core, max_levels);
braid_SetNRelax(core, -1, nrelax);
```

Then, the simulation is run.

```
braid_Drive(core);
```

Then, we clean up.
braid_Destroy(core);
Finally, to run ex-03, type
ex-03 -help
As a simple example, try the following.

```
mpirun -np 8 ex-03 -pgrid 2 2 2 -nt 256
```



### 4.5.1 Scaling Study with this Example

Here, we carry out a simple strong scaling study for this example. The "time stepping" data set represents sequential time stepping and was generated using examples/ex-03-serial. The time-parallel data set was generated using examples/ex-03. The problem setup is as follows.

- Backwards Euler is used as the time stepper. This is the only time stepper supported by ex-03.
- We used a Linux cluster with 4 cores per node, a Sandybridge Intel chipset, and a fast Infiniband interconnect.
- The space-time problem size was $129^{2} \times 16,192$ over the unit cube $[0,1] \times[0,1] \times[0,1]$.
- The coarsening factor was $m=16$ on the finest level and $m=2$ on coarser levels.
- Since 16 processors optimized the serial time stepping approach, 16 processors in space are also used for the XBraid experiments. So for instance 512 processrs in the plot corresponds to 16 processors in space and 32 processors in time, $16 * 32=512$. Thus, each processor owns a space-time hypercube of $\left(129^{2} / 16\right) \times$ $(16,192 / 32)$. See Parallel decomposition and memory for a depiction of how XBraid breaks the problem up.
- Various relaxation and V and F cycling strategies are experimented with.
- V-cycle, FCF denotes V-cycles and FCF-relaxation on each level.
- V-cycle, F-FCF denotes V-cycles and F-relaxation on the finest level and FCF-relaxation on all coarser levels.
- F-cycle, F denotes F-cycles and F-relaxation on each level.
- The initial guess at time values for $t>0$ is zero, which is typical.
- The halting tolerance corresponds to a discrete L2-norm and was

$$
\text { tol }=\frac{10^{-8}}{\sqrt{\left(h_{x}\right)^{2} h_{t}}}
$$

where $h_{x}$ and $h_{t}$ are the spatial and temporal grid spacings, respectively.
This corresponds to passing tol to braid_SetAbsTol, passing 2 to braid_SetTemporalNorm and defining braid $\hookleftarrow$ _PtFcnSpatialNorm to be the standard Euclidean 2-norm. All together, this appropriately scales the space-time residual in way that is relative to the number of space-time grid points (i.e., it approximates the L2-norm).

To re-run this scaling study, a sample run string for ex-03 is

```
mpirun -np 64 ex-03 -pgrid 4 4 4 -nx 129 129 -nt 16129 -cf0 16 -cf 2 -nu 1 -use_rand 0
```

To re-run the baseline sequential time stepper, ex-03-serial, try

```
mpirun -np 64 ex-03-serial -pgrid 8 8 -nx 129 129 -nt 16129
```

For explanations of the command line parameters, type

$$
\begin{aligned}
& \text { ex-03-serial -help } \\
& \text { ex-03 -help }
\end{aligned}
$$

Regarding the performance, we can say

- The best speedup is $10 x$ and this would grow if more processors were available.
- Although not shown, the iteration counts here are about 10-15 XBraid iterations. See Parallel Time Integration with Multigrid for the exact iteration counts.
- At smaller core counts, serial time stepping is faster. But at about 256 processors, there is a crossover and XBraid is faster.
- You can see the impact of the cycling and relaxation strategies discussed in Cycling and relaxation strategies. For instance, even though V-cycle, F-FCF is a weaker relaxation strategy than V-cycle, FCF (i.e., the XBraid convergence is slower), $V$-cycle, F-FCF has a faster time to solution than V-cycle, FCF because each cycle is cheaper.
- In general, one level of aggressive coarsening (here by a factor 16) followed by slower coarsening was found to be best on this machine.

Achieving the best speedup can require some tuning, and it is recommended to read Parallel Time Integration with Multigrid where this 2D heat equation example is explored in much more detail.

### 4.6 Simplest XBraid_Adjoint example

The file examples/ex-01-adjoint.c extends the simple scalar ODE example in ex-01.c for computing adjoint-based sensitivities. See The Simplest Example. The scalar ODE is

$$
u_{t}(t)=\lambda u(t) \quad \forall t \in(0, T)
$$

where $\lambda$ is considered the design variable. We consider an objective function of the form

$$
J(u, \lambda)=\int_{0}^{T} \frac{1}{T}\|u(t)\|^{2} d t
$$

## User Defined Structures and Wrappers

The two user-defined data structures are:

1. Vector: This structure is unchanged from The Simplest Example, and contains a single scalar representing the state at a given time.
```
typedef struct _braid_Vector_struct
{
    double value;
} my_Vector;
```

2. App: This structure holds two additional elements when compared to The Simplest Example : the design and the reduced gradient. This ensures that both are accessible in all user routines.
```
typedef struct __braid_App_struct
{
    int rank;
    double design;
    double gradient;
} my_App;
```

The user must also define a few additional wrapper routines. Note, that the app structure continues to be the first argument to every function.

1. All user-defined routines from examples/ex-01.c stay the same, except Step (), which must be changed to account for the new design parameter in app.
2. The user's Step routine queries the app to get the design and propagates the braid_Vector $u$ forward in time for one time step:
```
int
my_Step(braid_App app,
            braid_Vector ustop,
            braid_Vector fstop,
            braid_Vector u,
            braid_StepStatus status)
{
    double tstart; /* current time */
    double tstop; /* evolve to this time*/
    braid_StepStatusGetTstartTstop(status, &tstart, &tstop);
    /* Get the design variable from the app */
    double lambda = app->design;
    /* Use backward Euler to propagate the solution */
    (u->value) = 1./(1. - lambda * (tstop-tstart))*(u->value);
    return 0;
}
```

3. ObjectiveT: This new routine evaluates the time-dependent part of the objective function at a local time $t_{i}$, i.e. it returns the integrand $f\left(u_{i}, \lambda\right)=\frac{1}{T}\left\|u_{i}\right\|_{2}^{2}$.
```
int
my_ObjectiveT(braid_App app,
    braid_Vector u,
    braid_ObjectiveStatus ostatus,
    double *objectiveT_ptr)
{
    /* Get the total number of time steps */
    braid_ObjectiveStatusGetNTPoints(ostatus, &ntime);
    /* Evaluate the local objective: 1/N u(t)^2 */
    objT = 1. / ntime * (u->value) * (u->value);
    *objectiveT_ptr = objT;
    return 0;
}
```

The ObjectiveStatus can be queried for information about the current status of XBraid (e.g., what is the current time value, time-index, number of time steps, current iteration number, etc...).
XBraid_Adjoint calls the ObjectiveT function on the finest time-grid level during the down-cycle of the multigrid algorithm and adds the value to a global objective function value with a simple summation. Thus, any user-specific integration formula of the objective function must be here.
4. ObjectiveT_diff: This new routine updates the adjoint variable u_bar and the reduced gradient with the transposed partial derivatives of Ob jectiveT multiplied by the scalar input $\bar{F}$, i.e.,

$$
\bar{u}_{i}={\frac{\partial f\left(u_{i}, \lambda\right)^{T}}{\partial u_{i}}} \bar{F} \quad \text { and } \quad \bar{\rho}+=\frac{\partial f\left(u_{i}, \lambda\right)^{T}}{\partial \rho} \bar{F}
$$

Note that $\bar{u}_{i}$ gets overwritten (" ="), whereas $\rho$ is updated (" $+=$ ").

```
int
my_ObjectiveT_diff(braid_App app,
    braid_Vector u,
    braid_Vector u_bar,
    braid_Real F_bar,
    braid_ObjectiveStatus ostatus)
{
        int ntime;
        double ddu; /* Derivative wrt u */
    double ddesign; /* Derivative wrt design */
    /* Get the total number of time steps */
    braid_ObjectiveStatusGetNTPoints(ostatus, &ntime);
    /* Partial derivative with respect to u times F_bar */
    ddu = 2. / ntime * u->value * F_bar;
    /* Partial derivative with respect to design times F_bar*/
    ddesign = 0.0 * F_bar;
    /* Update u_bar and gradient */
    u_bar->value = ddu;
    app->gradient += ddesign;
    return 0;
}
```

5. Step_diff: This new routine computes transposed partial derivatives of the Step routine multiplied with the adjoint vector u_bar $\left(\bar{u}_{i}\right)$, i.e.,

$$
\bar{u}_{i}=\left(\frac{\partial \Phi_{i+1}\left(u_{i}, \rho\right)}{\partial u_{i}}\right)^{T} \bar{u}_{i} \quad \text { and } \quad \bar{\rho}+=\left(\frac{\partial \Phi_{i+1}\left(u_{i}, \rho\right)}{\partial \rho}\right)^{T} \bar{u}_{i} \text {. }
$$

```
int
```

my_Step_diff(braid_App app,
braid_Vector ustop,
braid_Vector u,
braid_Vector ustop_bar,
braid_Vector u_bar,
braid_StepStatus status)
\{

```
double ddu; /* Derivative wrt u */
double ddesign; /* Derivative wrt design */
/* Get the time step size */
double tstop, tstart, deltat;
braid_StepStatusGetTstartTstop(status, &tstart, &tstop);
deltat = tstop - tstart;
/* Get the design from the app */
double lambda = app->design;
/* Transposed derivative of step wrt u times u_bar */
ddu = 1./(1. - lambda * deltat) * (u_bar->value);
/* Transposed derivative of step wrt design times u_bar */
ddesign = (deltat * (u->value)) / pow(1. - deltat*lambda,2) * (u_bar->value);
```

```
    /* Update u_bar and gradient */
    u__bar->value = ddu;
    app->gradient += ddesign;
    return 0;
}
```

Important note on the usage of ustop: If the Step routine uses the input vector ustop instead of $u$ (typically for initializing a (non-)linear solve within $\Phi$ ), then Step_diff must update ustop_bar instead of u_bar and set u_bar to zero:

$$
\overline{u s t o p}+=\left(\frac{\partial \Phi_{i+1}(\text { ustop, } \rho)}{\partial \text { ustop }}\right)^{T} \bar{u}_{i} \quad \text { and } \quad \bar{u}_{i}=0.0 .
$$

6. ResetGradient: This new routine sets the gradient to zero.
```
int
my_ResetGradient(braid_App app)
{
    app->gradient = 0.0;
    return 0;
}
```

XBraid_Adjoint calls this routine before each iteration such that old gradient information is removed properly.

## Running XBraid_Adjoint for this example

The workflow for computing adjoint sensitivities with XBraid_Adjoint alongside the primal state computation closely follows XBraid's workflow. The user's main file will first set up the app structure, holding the additional information on an initial design and zero gradient. Then, all the setup calls done in Running XBraid for the Simplest Example will also be done.

The XBraid_Adjoint specific calls are as follows. After braid_Init ( . . . ) is called, the user initializes XBraid_Adjoint by calling

```
/* Initialize XBraid_Adjoint */
braid_InitAdjoint( my_ObjectiveT, my_ObjectiveT_diff, my_Step_diff, my_ResetGradient, &core);
```

Next, in addition to the usual XBraid options for controlling the multigrid iterations, the adjoint solver's accuracy is set by calling

```
braid_SetAbsTolAdjoint(core, 1e-6);
```

After that, one call to

```
/* Run simulation and adjoint-based gradient computation */
braid_Drive(core);
```

runs the multigrid iterations with additional adjoint sensitivity computations (i.e. the piggy-back iterations). After it finishes, the objective function value can be accessed by calling

```
/* Get the objective function value from XBraid */
braid_GetObjective(core, &objective);
```

Further, the reduced gradient, which is stored in the user's App structure, holds the sensitivity information $d J / d \rho$. As this information is local to all the time-processors, the user is responsible for summing up the gradients from all time-processors, if necessary. This usually involves an MP I_Allreduce call as in

```
/* Collect sensitivities from all processors */
double mygradient = app->gradient;
MPI_Allreduce(&mygradient, &(app->gradient), 1, MPI_DOUBLE, MPI_SUM, MPI_COMM_WORLD);
```

Lastly, the gradient computed with XBraid_Adjoint is verified using Finite Differences. See the source code examples/ex-01-adjoint.c for details.

### 4.7 Optimization with the Simplest Example

The file examples/ex-01-optimization.c implements a simple optimization iteration by extending examples/ex-01-adjoint.c, described in Simplest XBraid_Adjoint example. This example solves an inverse design problem for the simple scalar ODE example:

$$
\begin{gathered}
\min \quad \frac{1}{2}\left(\int_{0}^{T} \frac{1}{T}\|u(t)\|^{2} d t-J_{\text {Target }}\right)^{2}+\frac{\gamma}{2}\|\lambda\|^{2} \\
\text { s.t. } \quad \frac{\partial}{\partial t} u(t)=\lambda u(t) \quad \forall t \in(0, T)
\end{gathered}
$$

where $J_{\text {Target }}$ is a fixed and precomputed target value and $\gamma>0$ is a fixed relaxation parameter. Those fixed values are stored within the App.

## User Defined Structures and Wrappers

In order to evaluate the time-independent part of the objective function (e.g. the postprocessing function $F$ ) and its derivative, two additional user routines are necessary. There are no new user-defined data structures.

1. PostprocessObjective: This function evaluates the tracking-type objective function and the regularization term. The input variable integral contains the integral-part of the objective and returns the objective that is to be minimized $F(I)$ :
```
/* Evaluate the time-independent part of the objective function */
int
my_PostprocessObjective(braid_App app,
    double integral,
    double *postprocess
    )
{
    double F;
    /* Tracking-type functional */
    F = 1./2. * pow(integral - app->target,2);
    /* Regularization term */
    F += (app->gamma) / 2. * pow(app->design,2);
    *postprocess = F;
        return 0;
}
```

1. PostprocessObjective_diff: This provides XBraid_Adjoint with the partial derivatives of the Postprocess $\leftarrow$ Objective routine, i.e.

$$
\bar{F}=\frac{\partial F(I, \lambda)}{\partial I} \quad \text { and } \quad \bar{\rho}+=\frac{\partial F(I, \lambda)}{\partial \lambda}
$$

int
my_PostprocessObjective_diff(braid_App app,
double integral,
double *F_bar
)
\{
/* Derivative of tracking type function */
*F_bar $=$ integral - app->target;
/* Derivative of regularization term */
app->gradient += (app->gamma) * (app->design);
return 0;
\}

These routines are optional for XBraid_Adjoint. Therefore, they need to be passed to XBraid_Adjoint after the initialization with braid_Init (. . .) and braid_InitAdjoint (. . .) in the user's main file:

```
/* Optional: Set the tracking type objective function and derivative */
braid_SetPostprocessObjective(core, my_PostprocessObjective);
braid_SetPostprocessObjective_diff(core, my_PostprocessObjective_diff);
```


## Running an Optimization Cycle with XBraid_Adjoint

XBraid_Adjoint does not natively implement any optimization algorithms. Instead, we provide examples showing how one can easily use XBraid_Adjoint inside an optimization cycle. Here, one iteration of the optimization cycle consists of the following steps:

1. First, we run XBraid_Adjoint to solve the primal and adjoint dynamics:
```
braid_Drive(core);
```

2. Get the value of the objective function with
```
braid_GetObjective(core, &objective);
```

3. Gradient information is stored in the app structure. Since it is local to all temporal processors, we need to invoke an MP I_Allreduce call which sums up the local sensitivities:
```
mygradient = app->gradient;
MPI_Allreduce(&mygradient, &app->gradient, 1, MPI_DOUBLE, MPI_SUM, MPI_COMM_WORLD);
```

Note: For time-dependent design variables, summing over all processors might not be necessary, since information is needed only locally in time. See examples/ex-04.c for a time-dependent design example.
4. Update the design variable using the gradient information. Here, we implement a simple steepest descent update into the direction of the negative gradient:

```
app->design -= stepsize * app->gradient;
```

Here, a fixed step size is used to update the design variable. Usually, a line-search procedure should be implemented in order to find a suitable step length that minimizes the objective function along the update direction. However to carry out a line search, we must re-evaluate the objective function for different design value(s). Thus, the option braid_SetObjectiveOnly(core, 1) can be used. After this option has been set, any further call to braid_Drive (core) will then only run a primal XBraid simulation and carry out an objective function evaluation. No gradients will be computed, which saves computational time. After the line search, make sure to reset XBraid_Adjoint for gradient computation with braid_SetobjectiveOnly (core, 0).
5. The optimization iterations are stopped when the norm of the gradient is below a prescribed tolerance.

### 4.8 A Simple Optimal Control Problem

This example demonstrates the use of XBraid_Adjoint for solving an optimal control problem with time-dependent design variables:

$$
\begin{array}{rlr}
\min \int_{0}^{1} u_{1}(t)^{2}+u_{2}(t)^{2}+\gamma c(t)^{2} d t & \\
& & \\
\text { s.t. } & \frac{\partial}{\partial t} u_{1}(t)=u_{2}(t) & \forall t \in(0,1) \\
\frac{\partial}{\partial t} u_{2}(t)=-u_{2}(t)+c(t) & \forall t \in(0,1)
\end{array}
$$

with initial condition $u_{1}(0)=0, u_{2}(0)=-1$ and piecewise constant control (design) variable $c(t)$.
The example consists of three files, meant to indicate how one can take a time-serial implementation for an optimal control problem and create a corresponding XBraid_Adjoint implementation.

- examples/ex-04-serial.c: Compiles into its own executable examples/ex-04-serial, which solves the optimal control problem using time-serial forward-propagation of state variables and time-serial backward-propagation of the adjoint variables in each iteration of an outer optimization cycle.
- examples/ex-04.c: Compiles into ex-04. This solves the same optimization problem in time-parallel by replacing the forward- and backward-propagation of state and adjoint by the time-parallel XBraid and XBraid_ Adjoint solvers.
- examples/ex-04-lib.c: Contains the routines that are shared by both the serial and the time-parallel implementation. Study this file, and discover that most of the important code setting up the user-defined data structures and wrapper routines are simply lifted from the serial simulation.


### 4.9 Running and Testing XBraid

The best overall test for XBraid, is to set the maximum number of levels to 1 (see braid_SetMaxLevels ) which will carry out a sequential time stepping test. Take the output given to you by your Access function and compare it to output from a non-XBraid run. Is everything OK? Once this is complete, repeat for multilevel XBraid, and check that the solution is correct (that is, it matches a serial run to within tolerance).

At a lower level, to do sanity checks of your data structures and wrapper routines, there are also XBraid test functions, which can be easily run. The test routines also take as arguments the app structure, spatial communicator comm_ $x$, a stream like stdout for test output and a time step size $d t$ to test. After these arguments, function pointers to wrapper routines are the rest of the arguments. Some of the tests can return a boolean variable to indicate correctness.

```
/* Test init(), access(), free() */
braid_TestInitAccess( app, comm_x, stdout, dt, my_Init, my_Access, my_Free);
/* Test clone() */
braid_TestClone( app, comm_x, stdout, dt, my_Init, my_Access, my_Free, my_Clone);
/* Test sum() */
braid_TestSum( app, comm_x, stdout, dt, my_Init, my_Access, my_Free, my_Clone, my_Sum);
/* Test spatialnorm() */
correct = braid_TestSpatialNorm( app, comm_x, stdout, dt, my_Init, my_Free, my_Clone,
                                    my_Sum, my_SpatialNorm);
/* Test bufsize(), bufpack(), bufunpack() */
correct = braid_TestBuf( app, comm_x, stdout, dt, my_Init, my_Free, my_Sum, my_SpatialNorm,
    my_BufSize, my_BufPack, my_BufUnpack);
/* Test coarsen and refine */
correct = braid_TestCoarsenRefine(app, comm_x, stdout, 0.0, dt, 2*dt, my_Init,
    my_Access, my_Free, my_Clone, my_Sum, my_SpatialNorm,
    my_CoarsenInjection, my_Refine);
correct = braid_TestCoarsenRefine(app, comm_x, stdout, 0.0, dt, 2*dt, my_Init,
    my_Access, my_Free, my_Clone, my_Sum, my_SpatialNorm,
    my_CoarsenBilinear, my_Refine);
```


### 4.10 Fortan90 Interface, C++ Interface, Python Interface, and More Complicated Examples

We have Fortran90, C++, and Python interfaces. For Fortran 90, see examples/ex-01f.f90. For C++ see braid.hpp and examples/ex-01-pp.cpp For more complicated C++ examples, see the various C++ examples in drivers/drive-**.cpp. For Python, see the directories examples/ex-01-cython and examples/ex-01-cython-alt.

For a discussion of more complex problems please see our project publications website for our recent publications concerning some of these varied applications.

## 5 Examples: compiling and running

For C/C++/Fortran examples, type

```
ex-* -help
```

for instructions on how to run. To run the C/C++/Fortran examples, type

```
mpirun -np 4 ex-* [args]
```

For the Cython examples, see the corresponding $*$.pyx file.

1. ex-01 is the simplest example. It implements a scalar ODE and can be compiled and run with no outside dependencies. See Section (The Simplest Example) for more discussion of this example. There are seven versions of this example,

- ex-01.c: simplest possible implementation, start reading this example first
- ex-01-expanded.c: same as ex-01.c but adds more XBraid features
- ex-01-expanded-bdf2.c: same as ex-01-expanded.c, but uses BDF2 instead of backward Euler
- ex-01-expanded-f.f90: same as ex-01-expanded.c, but implemented in f90
- ex-01-refinement.c: same as ex-01.c, but adds the refinement feature
- ex-01-adjoint.c: adds adjoint-based gradient computation to ex-01.c
- ex-01-optimization.c: gradient-based optimization cycle for ex-01-c
- ex-01-cython/: is a directory containing an example using the Braid-Cython interface defined in braid. $\hookleftarrow$ pyx ( braid/braid.pyx ). It solves the same scalar ODE equation as the ex-01 series described above. This example uses a Python-like syntax, in contrast to the ex-01-cython-alt example, which uses a C-style syntax. For instructions on running and compiling, see
examples/ex-01-cython/ex_01.pyx
and
examples/ex-01-cython/ex_01-setup.py
- ex-01-cython-alt/: is a directory containing another example using the Braid-Cython interface defined in braid.pyx ( braid/braid.pyx ). It solves the same scalar ODE equation as the ex-01 series described above. This example uses a lower-level C-like syntax for most of it's code, in contrast to the ex-01-cython example, which uses a Python-style syntax. For instructions on running and compiling, see
examples/ex-01-cython-alt/ex_01_alt.pyx
and
examples/ex-01-cython-alt/ex_01_alt-setup.py

2. ex-02 implements the 1D heat equation on a regular grid, using a very simple implementation. This is the next example to read after the various ex-01 cases.
3. ex-03 implements the 2D heat equation on a regular grid. You must have hypre installed and these variables in examples/Makefile set correctly
```
HYPRE_DIR = ../../linear_solvers/hypre
HYPRE_FLAGS = -I\$(HYPRE_DIR)/include
HYPRE_LIB \(=-L \$\left(H Y P R E \_D I R\right) / l i b-1 H Y P R E\)
```

Only implicit time stepping (backward Euler) is supported. See Section (Two-Dimensional Heat Equation) for more discussion of this example. The driver
drivers/drive-diffusion
is a more sophisticated version of this simple example that supports explicit time stepping and spatial coarsening.
4. ex-04 solves a simple optimal control problem with time-dependent design variable using a simple steepestdescent optimization iteration.
5. Directory ex-05-cython/ solves a simple 1D heat equation using the Cython interface
examples/ex-05-cython/ex_05.pyx
and
examples/ex-05-cython/ex_05-setup.py
6. ex-06 solve a simple scalar ODE, but allows for use of the built-in Richardson-based error estimator and accuracy improving extrapolation. With the "-refinet" option, the error estimator allows for adaptive refinement in time, and with the "-richardson" option, Richardson extrapolation is used improve the solution at fine-level C-points.
The viz script,
examples/viz-ex-06.py
allows you to visualize the solution, error, and error estimate. The use of "-richardson" notably improves the accuracy of the solution.

The Richardson-based error estimates and/or extrapolation are only available after the first Braid iteration, in that the coarse level solution must be available to compute the error estimate and extrapolation. Thus, after an adaptive refinement (and new hierarchy is constructed), another iteration is again required for the error estimate to be available. If the error estimate isn't available, Braid returns a value of -1 . See this example and the comments therein for more details.

## 6 Drivers: compiling and running

Type
drive-* -help
for instructions on how to run any driver.
To run the examples, type

```
mpirun -np 4 drive-* [args]
```

1. drive-diffusion-2D implements the 2D heat equation on a regular grid. You must have hypre installed and these variables in examples/Makefile set correctly
```
HYPRE_DIR = ../../linear_solvers/hypre
HYPRE_FLAGS = -I$(HYPRE_DIR)/include
HYPRE_LIB = -L$(HYPRE_DIR)/lib -lHYPRE
```

This driver also support spatial coarsening and explicit time stepping. This allows you to use explicit time stepping on each Braid level, regardless of time step size.
2. drive-burgers-1D implements Burger's equation (and also linear advection) in 1D using forward or backward Euler in time and Lax-Friedrichs in space. Spatial coarsening is supported, allowing for stable time stepping on coarse time-grids.
See also viz-burgers.py for visualizing the output.
3. drive-Iorenz implements the Lorenz equation, with it's trademark attractors. This problem has not been researched very extensively, and XBraid's behavior is not yet well understood. Convergence stagnates, but is the solution "good enough" from a statistical point-of-view?
See also viz-lorenz.py for visualizing the output.
4. drive-diffusion is a sophisticated test bed for finite element discretizations of the heat equation. It relies on the mfem package to create general finite element discretizations for the spatial problem. Other packages must be installed in this order.

- Unpack and install Met is
- Unpack and install hypre
- Unpack mfem. Then make sure to set these variables correctly in the mfem Makefile:

```
USE_METIS_5 = YES
HYPRE_DIR = where_ever_linear_solvers_is/hypre
```

- Make the parallel version of mfem first by typing

```
make parallel
```

- Make GLVIS. Set these variables in the glvis makefile

```
MFEM_DIR = mfem_location
MFEM_LIB = -L$(MFEM_DIR) -lmfem
```

- Go to braid/examples and set these Makefile variables,

```
METIS_DIR = ../../metis-5.1.0/lib
MFEM_DIR = ../../mfem
MFEM_FLAGS = -I$(MFEM_DIR)
MFEM_LIB = -L$ (MFEM_DIR) -lmfem -L$(METIS_DIR) -lmetis
then type
make drive-diffusion
```

- To run drive-diffusion and glvis, open two windows. In one, start a glvis session
./glvis

Then, in the other window, run drive-diffusion

```
mpirun -np ... drive-diffusion [args]
```

Glvis will listen on a port to which drive-diffusion will dump visualization information.
5. The other drive-.cpp files use MFEM to implement other PDEs

- drive-adv-diff-DG: implements advection(-diffusion) with a discontinuous Galerkin discretization. This driver is under developement.
- drive-diffusion-1D-moving-mesh: implements the 1D heat equation, but with a moving mesh that adapts to the forcing function so that the mesh equidistributes the arc-length of the solution.
- drive-diffusion-1D-moving-mesh-serial: implements a serial time-stepping version of the above problem.
- drive-pLaplacian: implements the 2D the $p$-Laplacian (nonlinear diffusion).
- drive-diffusion-ben: implements the 2D/3D diffusion equation with time-dependent coefficients. This is essentially equivalent to drive-diffusion, and could be removed, but we're keeping it around because it implements linear diffusion in the same way that the p-Laplacian driver implemented nonlinear diffusion. This makes it suitable for head-to-head timings.
- drive-lin-elasticity: implements time-dependent linearized elasticity and is under development.
- drive-nonlin-elasticity: implements time-dependent nonlinear elasticity and is under development.

6. Directory drive-adv-diff-1D-Cython/ solves a simple 1D advection-diffussion equation using the Cython interface and numerous spatial and temporal discretizations
drivers/drive-adv-diff-1D-Cython/drive_adv_diff_1D.pyx
and
drivers/drive-adv-diff-1D-Cython/drive_adv_diff_1D-setup.py

## 7 Coding Style

Code should follow the ellemtel style. See braid/misc/sample_c_code.c, and for emacs and vim style files, see braid $/ \mathrm{misc} /$ sample.vimrc, and braid $/ \mathrm{misc} /$ sample.emacs.

## 8 File naming conventions

These are the general filenaming conventions for Braid

User interface routines in braid begin with braid_ and all other internal non-user routines begin with _braid_. This helps to prevent name clashes when working with other libraries and helps to clearly distinguish user routines that are supported and maintained.

To keep things somewhat organized, all user header files and implementation files should have names that begin with braid, for example, braid.h, braid.c, braid_status.c, ... There should be no user interface prototypes or implementations that appear elsewhere.

Note that it is okay to include internal prototypes and implementations in these user interface files when it makes sense (say, as supporting routines), but this should generally be avoided.

An attempt has been made to simplify header file usage as much as possible by requiring only one header file for users, braid.h, and one header file for developers, _braid.h.

## 9 Using Doxygen

To build the documentation, doxygen must be version 1.8 or greater. XBraid documentation uses a markdown syntax both in source file comments and in $*$.md files.

To make the documentation,

```
$ make user_manual
$ acroread user_manual.pdf
```

or to make a more extensive reference manual for developers,

```
$ make developer_manual
$ acroread developer_manual.pdf
```

Developers can run doxygen from a precompiled binary, which may or may not work for your machine,

```
/usr/casc/hypre/braid/share/doxygen/bin/doxygen
```

or build doxygen from
/usr/casc/hypre/braid/share/doxygen.tgz

- Compiling doxygen requires a number of dependencies like Bison, GraphViz and Flex. Configure will tell you what you're missing
- Unpack doxygen.tgz, then from the doxygen directory

```
./configure --prefix some_dir_in_your_path
make
make install
```


## Documentation Strategy

- The doxygen comments are to be placed in the header files.
- A sample function declaration using the documenation approach using markdown (including typesetting equations) is in braid.h for the function braid_Init()
- A sample structure documentation is in _braid.h for _braid_Core_struct
- Descriptors for files can also be added, as at the top of braid.h
- The Doxygen manual is at http://www.stack.nl/~dimitri/doxygen/manual/index.html


## XBraid Doxygen details

The user and developer manuals are ultimately produced by Latex. The formatting of the manuals is configured according to the following.

- docs/local_doxygen.sty
- Latex style file used
- docs/user_manual_header.tex
- User manual title page and header info
- docs/developer_manual_header.tex
- Developer manual title page and header info
- *.md
- Any file ending in .md is extra documentation in markdown format, like Introduction.md or the various Readme.md files in each directory. This material can be read in plain-text or when it's compiled by Doxygen and Latex.
- docs/user_manual.conf
- Doxygen configure file for the user manual
- The FILE_NAMES tag is a filter to only include the user interface routines in braid.h
- The INPUT tag orders the processing of the files and hence the section ordering
- docs/reference_manual.conf
- Same as user_manual.conf, but the FILE_NAMES tag does not exclude any file from processing.
- docs/img
- Contains the images
- To regenerate generic doxygen latex files, type
\$ doxygen -w latex header.tex footer.tex doxygen. sty doxy.conf
If this is done, then the .conf file must be changed to use the new header file and to copy the local_doxygen.sty file to the latex directory.


## 10 Regression Testing

## Overview

- There are three levels in the testing framework. At each level, the fine-grain output from a testscript. sh is dumped into a directory testscript. dir, with the standard out and error stored in testscript. out and testscript.err. The test testscript.sh passes if testscript.err is empty (nothing is written to standard error).
- Basic instructions: run a test with a command like

```
$ ./test.sh diffusion2D.sh
```

Then, see if diffusion2D. err is of size 0 . If it is not, look at it's contents to see which test failed.

- To add a new regression test, create a new lowest level script like diffusion2D.sh and then call it from a machine script at level 2.
- Regression tests should be run before pushing code. It is recommended to run the basic (lowest level) tests like diffusion 2 d. sh or machine test like machine-tux.sh


## Lowest Level Test Scripts

As an example, here we look at one of the lowest level tests, the diffusion2d test.
Files used:

- test.sh
- diffusion2D.sh
- diffusion2D.saved


## Output:

- diffusion2D.dir
- diffusion2D.err
- diffusion2D.out

At this level, we execute
\$ ./test.sh diffusion2D.sh
or just
\$ ./diffusion2D.sh

The script diffusion2D.sh must create diffusion2D.dir and place all fine-grain test output in this directory. test.sh captures the standard out and error in diffusion2D.out and diffusion2D.err. The test diffusion2D.sh passes if diffusion2D.err is empty (nothing is written to standard error).

The strategy for low level scripts like diffusion2D.sh is to run a sequence of tests such as

```
$ mpirun -np 1 ../examples/ex-02 -pgrid 1 1 1 -nt 256
$ mpirun -np 4 ../examples/ex-02 -pgrid 1 1 4 -nt 256
```

The output from the first mpirun test must then be written to files named

```
diffusion2D.dir/unfiltered.std.out.0
diffusion2D.dir/std.out.0
diffusion2D.dir/std.err.0
```

and the second mpirun test similarly writes the files

```
diffusion2D.dir/unfiltered.std.out.1
diffusion2D.dir/std.out.1
diffusion2D.dir/std.err.1
```

Subsequent tests are written to higher numbered files. The unfiltered.std.out. num file contains all of the standard out for the test, while std. out . num contains filtered output (usually from a grep command) and could contain the output lines such as iteration numbers and number of levels. The file std.err. num contains the standard error output.

To see if a test ran correctly, std. out. num is compared to saved output in diffusion2D.saved. The file diffusion2D.saved contains the concatenated output from all the tests that diffusion2D.sh will run. For the above example, this file could look like

```
# Begin Test 1
number of levels = 6
iterations = 16
# Begin Test 2
number of levels = 4
iterations = 8
```

This saved output is split into an individual file for each test (using \# Begin Test as a delimiter) and these new files are placed in diffusion2D.dir. So, after running these two regression tests, diffusion2D. dir will contain

```
diffusion2D.saved.0
diffusion2D.saved.1
unfiltered.std.out.0
std.out.0
std.err.0
unfiltered.std.out.1
std.out.1
std.err.1
```

An individual test has passed if std.err.num is empty. The file std.err.num contains a diff between diffusion2D.save. num and std. out. num (the diff ignores whitespace and the delimiter \# Begin Test).

Last in the directy where you ran ./test.sh diffusion2d.sh, the files

```
diffusion2D.err
diffusion2D.out
```

will be created. If all the tests passed then diffusion2D. err will be empty. Otherwise, it will contain the filenames of the std.err. num files that are non-empty, representing failed tests.

## Level 2 Scripts

As an example, here we look at one of the Level 2 tests, the machine-tux test that Jacob runs.
Files used:

- machine-tux.sh

Output:

- machine-tux.dir
- machine-tux.err (only generated if autotest. sh is used to run machine-tux.sh)
- machine-tux. out (only generated if autotest.sh is used to run machine-tux.sh)

At this level, we execute

```
./machine-tux.sh
```

The autotest framework (autotest.sh) calls machine scripts in this way. Each machine script should be short and call lower-level scripts like diffusion2D.sh. The output from lower-level scripts must be moved to machine-tux.dir like this:

```
$ ./test.sh diffusion2D.sh
$ mv -f diffusion2D.dir machine-tux.dir
mv -f diffusion2D.out machine-tux.dir
mv -f diffusion2D.err machine-tux.dir
```

All error files from diffusion2D. sh will be placed in machine-tux. dir, so if machine-tux. dir has all zero *. err files, then the machine-tux test has passed.

To begin testing on a new machine, like vulcan, add a new machine script similar to machine-tux. sh and change autotest. sh to recognize and run the new machine test. To then use autotest. sh with the machine script, you'll have to set up a passwordless connection from the new machine to

## Level 3 Script

Here we look at the highest level, where autotest. sh runs all of the level 2 machine tests and emails out the results.

Files used:

- autotest.sh

Output:

- test/autotest_finished
- /usr/casc/hypre/braid/testing/AUTOTEST-20**.**.**-Day
- Email to recipients listed in autotest.sh

At the highest level sits autotest. sh and is called automatically as a cronjob. If you just want to check to see if you've broken anything with a commit, just use lower level scripts.

There are four steps to running autotest.

- Step 1
\$ ./autotesh.sh -init
will do a pull from master for the current working repository and recompile Braid. The autotest output files (autotest.err and autotest.out) and the output directory (autotest_finished) are initialized.
- Step 2

```
$ ./autotest.sh -tux343
```

will run the autotests on tux343. This command will look for a machine-tux.sh, and execute it, moving the resulting

```
machine-tux.dir
machine-tux.err
machine-tux.out
```

into test/autotest_finished.

- Step 3

```
$ ./autotest.sh -remote-copy
```

will copy /test/autotest_finished/* to a time-stamped directory such as
/usr/casc/hypre/braid/testing/AUTOTEST-2013.11.18-Mon
Alternatively,
\$ ./autotesh.sh -remote-copy tux343
will ssh through tux343 to copy to /usr/casc. Multiple machines may independently be running regression tests and then copy to AUTOTEST-2013.11.18-Mon.

- Step 4

```
$ ./autotest.sh -summary-email
```

will email everyone listed in the \$email_list (an autotest. sh variable)

## Cronfile

To add entries to your crontab, First, put your new cronjob lines into cronfile. Then see what you already have in your crontab file with

```
$ crontab -1
```

Next, append to cronfile whatever you already have

```
$ crontab -l >> cronfile
```

Finally, tell crontab to use your cronfile

```
$ crontab cronfile
```

Then make sure it took affect with

```
$ crontab -1
```

Crontab entry format uses ' $*$ ' to mean "every" and ' $* / m$ ' to mean "every m-th". The first five entries on each line correspond respectively to:

- minute (0-56)
- hour (0-23)
- day of month (1-31)
- month (1-12)
- day of week (0-6)(0=Sunday)

Jacob's crontab (on tux343):

```
00 01 * * * source /etc/profile; source $HOME/.bashrc; cd $HOME/joint_repos/braid/test; ./autotest.sh -init
10 01 * * * source /etc/profile; source $HOME/.bashrc; cd $HOME/joint_repos/braid/test; ./autotest.sh -tux343
40 01 * * * source /etc/profile; source $HOME/.bashrc; cd $HOME/joint_repos/braid/test; ./autotest.sh -remote-c
50 01 * * * source /etc/profile; source $HOME/.bashrc; cd $HOME/joint_repos/braid/test; ./autotest.sh -summary-
O0 02 * * * source /etc/profile; source $HOME/.bashrc; cd $HOME/joint_repos/braid/test; ./autotest.sh -create-t
```


## 11 Module Index

### 11.1 Modules

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## 13 File Index

### 13.1 File List

Here is a list of all files with brief descriptions:

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Definitions of braid types, error flags, etc.. 188
braid_status.h
Define headers for the user-interface with the XBraid status structures, allowing the user to get/set status structure values
braid_test.h
Define headers for XBraid user-test routines
mpistubs.h
XBraid internal headers to define fake MPI stubs. This ultimately allows the user to generate purely serial codes without MPI

## status.h

Define the XBraid internal headers for the XBraid status structure routines, and define the status structures themselves
tape.h
Define the XBraid internal headers for the action-tape routines (linked list for AD)
util.h
Define XBraid internal headers for utility routines

## 14 Module Documentation

### 14.1 Fortran 90 interface options

## Macros

- \#define braid_FMANGLE 1
- \#define braid_Fortran_SpatialCoarsen 0
- \#define braid_Fortran_Residual 1
- \#define braid_Fortran_TimeGrid 1
- \#define braid_Fortran_Sync 1
14.1.1 Detailed Description

Allows user to manually, at compile-time, turn on Fortran 90 interface options
14.1.2 Macro Definition Documentation
14.1.2.1 braid_FMANGLE
\#define braid_FMANGLE 1

Define Fortran name-mangling schema, there are four supported options, see braid_F90_iface.c
14.1.2.2 braid_Fortran_Residual
\#define braid_Fortran_Residual 1

Turn on the optional user-defined residual function
14.1.2.3 braid_Fortran_SpatialCoarsen
\#define braid_Fortran_SpatialCoarsen 0

Turn on the optional user-defined spatial coarsening and refinement functions
14.1.2.4 braid_Fortran_Sync
\#define braid_Fortran_Sync 1

Turn on the optional user-defined sync function
14.1.2.5 braid_Fortran_TimeGrid
\#define braid_Fortran_TimeGrid 1

Turn on the optional user-defined time-grid function

### 14.2 Error Codes

## Macros

- \#define braid_INVALID_RNORM -1
- \#define braid_ERROR_GENERIC $1 / *$ generic error $* /$
- \#define braid_ERROR_MEMORY 2 /* unable to allocate memory */
- \#define braid_ERROR_ARG 4 /* argument error */


### 14.2.1 Detailed Description

14.2.2 Macro Definition Documentation
14.2.2.1 braid_ERROR_ARG
\#define braid_ERROR_ARG 4 /* argument error */
14.2.2.2 braid_ERROR_GENERIC
\#define braid_ERROR_GENERIC 1 /* generic error */
14.2.2.3 braid_ERROR_MEMORY
\#define braid_ERROR_MEMORY 2 /* unable to allocate memory */
14.2.2.4 braid_INVALID_RNORM
\#define braid_INVALID_RNORM -1

Value used to represent an invalid residual norm

### 14.3 User-written routines

Modules

- User-written routines for XBraid_Adjoint


## Typedefs

- typedef struct _braid_App_struct $*$ braid_App
- typedef struct _braid_Vector_struct * braid_Vector
- typedef braid_Int(* braid_PtFcnStep) (braid_App app, braid_Vector ustop, braid_Vector fstop, braid_Vector u, braid_StepStatus status)
- typedef braid_Int(* braid_PtFcnInit) (braid_App app, braid_Real t, braid_Vector *u_ptr)
- typedef braid_Int(* braid_PtFcnClone) (braid_App app, braid_Vector u, braid_Vector *V_ptr)
- typedef braid_Int(* braid_PtFcnFree) (braid_App app, braid_Vector u)
- typedef braid_Int(* braid_PtFcnSum) (braid_App app, braid_Real alpha, braid_Vector x, braid_Real beta, braid $\hookleftarrow$ _Vector y)
- typedef braid_Int(* braid_PtFcnSpatialNorm) (braid_App app, braid_Vector u, braid_Real *norm_ptr)
- typedef braid_Int(* braid_PtFcnAccess) (braid_App app, braid_Vector u, braid_AccessStatus status)
- typedef braid_Int(* braid_PtFcnSync) (braid_App app, braid_SyncStatus status)
- typedef braid_Int(* braid_PtFcnBufSize) (braid_App app, braid_Int *size_ptr, braid_BufferStatus status)
- typedef braid_Int(* braid_PtFcnBufPack) (braid_App app, braid_Vector u, void *buffer, braid_BufferStatus status)
- typedef braid_Int(* braid_PtFcnBufUnpack) (braid_App app, void *buffer, braid_Vector *u_ptr, braid_BufferStatus status)
- typedef braid_Int(* braid_PtFcnResidual) (braid_App app, braid_Vector ustop, braid_Vector r, braid_StepStatus status)
- typedef braid_Int(* braid_PtFcnSCoarsen) (braid_App app, braid_Vector fu, braid_Vector $*$ cu_ptr, braid_ $\hookleftarrow$ CoarsenRefStatus status)
- typedef braid_Int(* braid_PtFcnSRefine) (braid_App app, braid_Vector cu, braid_Vector $*$ fu_ptr, braid_Coarsen $\hookleftarrow$ RefStatus status)
- typedef braid_Int(* braid_PtFcnSInit) (braid_App app, braid_Real t, braid_Vector *u_ptr)
- typedef braid_Int(* braid_PtFcnSClone) (braid_App app, braid_Vector u, braid_Vector *v_ptr)
- typedef braid_Int(* braid_PtFcnSFree) (braid_App app, braid_Vector u)
- typedef braid_Int(* braid_PtFcnTimeGrid) (braid_App app, braid_Real *ta, braid_Int *ilower, braid_Int *iupper)


### 14.3.1 Detailed Description

These are all the user-written data structures and routines. There are two data structures (braid_App and braid_Vector) for the user to define. And, there are a variety of function interfaces (defined through function pointer declarations) that the user must implement.
14.3.2 Typedef Documentation

### 14.3.2.1 braid_App

typedef struct _braid_App_struct* braid_App

This holds a wide variety of information and is global in that it is passed to every function. This structure holds everything that the user will need to carry out a simulation. For a simple example, this could just hold the global MPI communicator and a few values describing the temporal domain.

### 14.3.2.2 braid_PtFcnAccess

typedef braid_Int (* braid_PtFcnAccess) (braid_App app, braid_Vector u, braid_AccessStatus status)

Gives user access to XBraid and to the current vector $u$ at time $t$. Most commonly, this lets the user write the vector to screen, file, etc... The user decides what is appropriate. Note how you are told the time value $t$ of the vector $u$ and other information in status. This lets you tailor the output, e.g., for only certain time values at certain XBraid iterations. Querrying status for such information is done through braid_AccessStatusGet**(..) routines.

The frequency of XBraid's calls to access is controlled through braid_SetAccessLevel. For instance, if access_level is set to 3 , then access is called every XBraid iteration and on every XBraid level. In this case, querrying status to determine the current XBraid level and iteration will be useful. This scenario allows for even more detailed tracking of the simulation.

Eventually, access will be broadened to allow the user to steer XBraid.
14.3.2.3 braid_PtFcnBufPack

```
typedef braid_Int(* braid_PtFcnBufPack) (braid_App app, braid_Vector u, void *buffer, braid_\hookleftarrow
```

BufferStatus status)

This allows XBraid to send messages containing braid_Vectors. This routine packs a vector $u$ into a void $*$ bufferfor MPI. The status structure holds information regarding the message. This is accessed through the braid_BufferStatusGet**(..) routines. Optionally, the user can set the message size through the status structure.

### 14.3.2.4 braid_PtFcnBufSize

typedef braid_Int(* braid_PtFcnBufSize) (braid_App app, braid_Int *size_ptr, braid_BufferStatus status)

This routine tells XBraid message sizes by computing an upper bound in bytes for an arbitrary braid_Vector. This size must be an upper bound for what BufPack and BufUnPack will assume.
14.3.2.5 braid_PtFcnBufUnpack

```
typedef braid_Int(* braid_PtFcnBufUnpack) (braid_App app, void *buffer, braid_Vector *u_ptr, braid\hookleftarrow
_BufferStatus status)
```

This allows XBraid to receive messages containing braid_Vectors. This routine unpacks a void $*$ buffer from MPI into a braid_Vector. The status structure, contains information conveying the type of message inside the buffer. This can be accessed through the braid_BufferStatusGet**(..) routines.
14.3.2.6 braid_PtFcnClone
typedef braid_Int (* braid_PtFcnClone) (braid_App app, braid_Vector u, braid_Vector *V_ptr)

Clone $u$ into $v \_p t r$
14.3.2.7 braid_PtFcnFree
typedef braid_Int(* braid_PtFcnFree) (braid_App app, braid_Vector u)

Free and deallocate $u$
14.3.2.8 braid_PtFcnlnit
typedef braid_Int(* braid_PtFcnInit) (braid_App app, braid_Real t, braid_Vector *u_ptr)

Initializes a vector $u \_p t r$ at time $t$
14.3.2.9 braid_PtFcnResidual
typedef braid_Int (* braid_PtFcnResidual) (braid_App app, braid_Vector ustop, braid_Vector r, braid↔ _StepStatus status)

This function (optional) computes the residual $r$ at time tstop. On input, $r$ holds the value of $u$ at $t s t a r t$, and $u s t o p$ is the value of $u$ at tstop. If used, set with braid_SetResidual.

Query the status structure with braid_StepStatusGetTstart(status, \&tstart) and braid_StepStatusGetTstop(status, \&tstop) to get tstart and tstop.
14.3.2.10 braid_PtFcnSClone
typedef braid_Int (* braid_PtFcnSClone) (braid_App app, braid_Vector u, braid_Vector *V_ptr)

Shell clone (optional)
14.3.2.11 braid_PtFcnSCoarsen
typedef braid_Int (* braid_PtFcnSCoarsen) (braid_App app, braid_Vector fu, braid_Vector *cu_ptr,
braid_CoarsenRefStatus status)

Spatial coarsening (optional). Allows the user to coarsen when going from a fine time grid to a coarse time grid. This function is called on every vector at each level, thus you can coarsen the entire space time domain. The action of this function should match the braid_PtFcnSRefine function.

The user should query the status structure at run time with braid_CoarsenRefGet**() calls in order to determine how to coarsen. For instance, status tells you what the current time value is, and what the time step sizes on the fine and coarse levels are.
14.3.2.12 braid_PtFcnSFree
typedef braid_Int (* braid_PtFcnSFree) (braid_App app, braid_Vector u)

Free the data of $u$, keep its shell (optional)
14.3.2.13 braid_PtFenSInit
typedef braid_Int (* braid_PtFcnSInit) (braid_App app, braid_Real t, braid_Vector *u_ptr)

Shell initialization (optional)
14.3.2.14 braid_PtFcnSpatialNorm
typedef braid_Int (* braid_PtFcnSpatialNorm) (braid_App app, braid_Vector u, braid_Real *norm_ptr)

Carry out a spatial norm by taking the norm of a braid_Vector norm_ptr $=\|u\|$ A common choice is the standard Eucliden norm, but many other choices are possible, such as an L2-norm based on a finite element space. See braid $\hookleftarrow$ SetTemporalNorm for information on how the spatial norm is combined over time for a global space-time residual norm. This global norm then controls halting.
14.3.2.15 braid_PtFcnSRefine

```
typedef braid_Int(* braid_PtFcnSRefine) (braid_App app, braid_Vector cu, braid_Vector *fu_ptr,
```

braid_CoarsenRefStatus status)

Spatial refinement (optional). Allows the user to refine when going from a coarse time grid to a fine time grid. This function is called on every vector at each level, thus you can refine the entire space time domain. The action of this function should match the braid_PtFenSCoarsen function.

The user should query the status structure at run time with braid_CoarsenRefGet**() calls in order to determine how to coarsen. For instance, status tells you what the current time value is, and what the time step sizes on the fine and coarse levels are.

### 14.3.2.16 braid_PtFcnStep

typedef braid_Int (* braid_PtFcnStep) (braid_App app, braid_Vector ustop, braid_Vector fstop, braid↔
_Vector u, braid_StepStatus status)

Defines the central time stepping function that the user must write.

The user must advance the vector $u$ from time tstart to tstop. The time step is taken assuming the right-hand-side vector fstop at time tstop. The vector ustop may be the same vector as $u$ (in the case where not all unknowns are stored). The vector fstop is set to NULL to indicate a zero right-hand-side.

Query the status structure with braid_StepStatusGetTstart(status, \&tstart) and braid_StepStatusGetTstop(status, \&tstop) to get tstart and tstop. The status structure also allows for steering. For example, braid_StepStatusSetR $\hookleftarrow$ Factor(...) allows for setting a refinement factor, which tells XBraid to refine this time interval.
14.3.2.17 braid_PtFcnSum
typedef braid_Int (* braid_PtFcnSum) (braid_App app, braid_Real alpha, braid_Vector x, braid_Real
beta, braid_Vector y)

AXPY, alpha $x+$ beta $y->y$

### 14.3.2.18 braid_PtFcnSync

typedef braid_Int (* braid_PtFcnSync) (braid_App app, braid_SyncStatus status)
Gives user access to XBraid and to the user's app at various points (primarily once per iteration inside FRefine and outside in the main cycle loop). This function is called once per-processor (not for every state vector stored on the processor, like access).
14.3.2.19 braid_PtFcnTimeGrid
typedef braid_Int (* braid_PtFcnTimeGrid) (braid_App app, braid_Real *ta, braid_Int *ilower, braid $\hookleftarrow$ _Int *iupper)

Set time values for temporal grid on level 0 (time slice per processor)
14.3.2.20 braid_Vector
typedef struct _braid_Vector_struct* braid_Vector

This defines (roughly) a state vector at a certain time value. It could also contain any other information related to this vector which is needed to evolve the vector to the next time value, like mesh information.

### 14.4 User-written routines for XBraid_Adjoint

## Typedefs

- typedef braid_Int(* braid_PtFcnObjectiveT) (braid_App app, braid_Vector u, braid_ObjectiveStatus ostatus, braid_Real *objectiveT_ptr)
- typedef braid_Int(* braid_PtFcnObjectiveTDiff) (braid_App app, braid_Vector u, braid_Vector u_bar, braid_Real F_bar, braid_ObjectiveStatus ostatus)
- typedef braid_Int(* braid_PtFcnPostprocessObjective) (braid_App app, braid_Real sum_obj, braid_Real *postprocess_ptr)
- typedef braid_Int(* braid_PtFcnPostprocessObjective_diff) (braid_App app, braid_Real sum_obj, braid_Real $*$ F $\hookleftarrow$ _bar_ptr)
- typedef braid_Int(* braid_PtFcnStepDiff) (braid_App app, braid_Vector ustop, braid_Vector u, braid_Vector ustop_bar, braid_Vector u_bar, braid_StepStatus status)
- typedef braid_Int(* braid_PtFcnResetGradient) (braid_App app)


### 14.4.1 Detailed Description

These are all the user-written routines needed to use XBraid_Adjoint. There are no new user-written data structures here. But, the braid_App structure will typically be used to store some things like optimization parameters and gradients.

### 14.4.2 Typedef Documentation

### 14.4.2.1 braid_PtFcnObjectiveT

typedef braid_Int (* braid_PtFcnObjectiveT) (braid_App app, braid_Vector u, braid_ObjectiveStatus
ostatus, braid_Real *objectiveT_ptr)

This routine evaluates the time-dependent part of the objective function, at a current time $t$, i.e. the integrand. Query the braid_ObjectiveStatus structure for information about the current time and status of XBraid_Adjoint.
14.4.2.2 braid_PtFcnObjectiveTDiff

```
typedef braid_Int(* braid_PtFcnObjectiveTDiff) (braid_App app, braid_Vector u, braid_Vector u_bar,
```

braid_Real F_bar, braid_objectiveStatus ostatus)

This is the differentiated version of the braid_PtFcnObjectiveT routine. It provides the derivatives of ObjectiveT() multiplied by the scalar input $F_{-}$bar.

First output: the derivative with respect to the state vector must be returned to XBraid_Adjoint in $u$ _bar.

Second output: The derivative with respect to the design must update the gradient, which is stored in the braid_App.
14.4.2.3 braid_PtFcnPostprocessObjective
typedef braid_Int (* braid_PtFcnPostprocessObjective) (braid_App app, braid_Real sum_obj, braid_↔
Real *postprocess_ptr)
(Optional) This function can be used to postprocess the time-integral objective function. For example, when inverse design problems are considered, you can use a tracking-type objective function by substracting a target value from postprocess_ptr, and squaring the result. Relaxation or penalty terms can also be added to postprocess_ptr. For a description of the postprocessing routine, see the Section Objective function evaluation.
14.4.2.4 braid_PtFcnPostprocessObjective_diff
typedef braid_Int(* braid_PtFcnPostprocessObjective_diff) (braid_App app, braid_Real sum_obj, braid_Real *F_bar_ptr)
(Optional) Differentiated version of the Postprocessing routine.

First output: Return the partial derivative of the braid_PtFcnPostprocessObjective routine with respect to the timeintegral objective function, and placing the result in the scalar value $F_{-} b a r \_p t r$

Second output: Update the gradient with the partial derivative with respect to the design. Gradients are usually stored in braid_App .

For a description of the postprocessing routine, see the Section Objective function evaluation.
14.4.2.5 braid_PtFcnResetGradient
typedef braid_Int(* braid_PtFcnResetGradient) (braid_App app)

Set the gradient to zero, which is usually stored in braid_App .

### 14.4.2.6 braid_PtFcnStepDiff

typedef braid_Int (* braid_PtFcnStepDiff) (braid_App app, braid_Vector ustop, braid_Vector u, braid↔ _Vector ustop_bar, braid_Vector u_bar, braid_StepStatus status)

This is the differentiated version of the time-stepping routine. It provides the transposed derivatives of Step() multiplied by the adjoint input vector u_bar (or ustop_bar).

First output: the derivative with respect to the state $u$ updates the adjoint vector $u \_b a r$ (or ustop_bar).

Second output: The derivative with respect to the design must update the gradient, which is stored in braid_App .

### 14.5 User interface routines

Modules

- General Interface routines
- Interface routines for XBraid_Adjoint
- XBraid status structures
- XBraid status routines
- Inherited XBraid status routines
- XBraid status macros
14.5.1 Detailed Description

These are all the user interface routines.

### 14.6 General Interface routines

## Macros

- \#define braid_RAND_MAX 32768


## Typedefs

- typedef struct _braid_Core_struct * braid_Core


## Functions

- braid_Int braid_Init (MPI_Comm comm_world, MPI_Comm comm, braid_Real tstart, braid_Real tstop, braid_↔ Int ntime, braid_App app, braid_PtFcnStep step, braid_PtFcnInit init, braid_PtFcnClone clone, braid_PtFcnFree free, braid_PtFcnSum sum, braid_PtFcnSpatialNorm spatialnorm, braid_PtFcnAccess access, braid_PtFcnBuf $\hookleftarrow$ Size bufsize, braid_PtFcnBufPack bufpack, braid_PtFcnBufUnpack bufunpack, braid_Core $*$ core_ptr)
- braid_Int braid_Drive (braid_Core core)
- braid_Int braid_Destroy (braid_Core core)
- braid_Int braid_PrintStats (braid_Core core)
- braid_Int braid_WriteConvHistory (braid_Core core, const char *filename)
- braid_Int braid_SetMaxLevels (braid_Core core, braid_Int max_levels)
- braid_Int braid_SetIncrMaxLevels (braid_Core core)
- braid_Int braid_SetSkip (braid_Core core, braid_Int skip)
- braid_Int braid_SetRefine (braid_Core core, braid_Int refine)
- braid_Int braid_SetMaxRefinements (braid_Core core, braid_Int max_refinements)
- braid_Int braid_SetTPointsCutoff (braid_Core core, braid_Int tpoints_cutoff)
- braid_Int braid_SetMinCoarse (braid_Core core, braid_Int min_coarse)
- braid_Int braid_SetAbsTol (braid_Core core, braid_Real atol)
- braid_Int braid_SetRelTol (braid_Core core, braid_Real rtol)
- braid_Int braid_SetNRelax (braid_Core core, braid_Int level, braid_Int nrelax)
- braid_Int braid_SetCRelaxWt (braid_Core core, braid_Int level, braid_Real Cwt)
- braid_Int braid_SetCFactor (braid_Core core, braid_Int level, braid_Int cfactor)
- braid_Int braid_SetMaxIter (braid_Core core, braid_Int max_iter)
- braid_Int braid_SetFMG (braid_Core core)
- braid_Int braid_SetNFMG (braid_Core core, braid_Int k)
- braid_Int braid_SetNFMGVcyc (braid_Core core, braid_Int nfmg_Vcyc)
- braid_Int braid_SetStorage (braid_Core core, braid_Int storage)
- braid_Int braid_SetTemporalNorm (braid_Core core, braid_Int tnorm)
- braid_Int braid_SetResidual (braid_Core core, braid_PtFcnResidual residual)
- braid_Int braid_SetFullRNormRes (braid_Core core, braid_PtFcnResidual residual)
- braid_Int braid_SetTimeGrid (braid_Core core, braid_PtFcnTimeGrid tgrid)
- braid_Int braid_SetPeriodic (braid_Core core, braid_Int periodic)
- braid_Int braid_SetSpatialCoarsen (braid_Core core, braid_PtFcnSCoarsen scoarsen)
- braid_Int braid_SetSpatialRefine (braid_Core core, braid_PtFcnSRefine srefine)
- braid_Int braid_SetSync (braid_Core core, braid_PtFcnSync sync)
- braid_Int braid_SetPrintLevel (braid_Core core, braid_Int print_level)
- braid_Int braid_SetFileIOLevel (braid_Core core, braid_Int io_level)
- braid_Int braid_SetPrintFile (braid_Core core, const char *printfile_name)
- braid_Int braid_SetDefaultPrintFile (braid_Core core)
- braid_Int braid_SetAccessLevel (braid_Core core, braid_Int access_level)
- braid_Int braid_SplitCommworld (const MPI_Comm *comm_world, braid_Int px, MPI_Comm *comm_x, MPI_↔ Comm * comm_t)
- braid_Int braid_SetShell (braid_Core core, braid_PtFcnSInit sinit, braid_PtFcnSClone sclone, braid_PtFcnSFree sfree)
- braid_Int braid_GetNumlter (braid_Core core, braid_Int *niter_ptr)
- braid_Int braid_GetRNorms (braid_Core core, braid_Int *nrequest_ptr, braid_Real *rnorms)
- braid_Int braid_GetNLevels (braid_Core core, braid_Int *nlevels_ptr)
- braid_Int braid_GetSpatialAccuracy (braid_StepStatus status, braid_Real loose_tol, braid_Real tight_tol, braid $\hookleftarrow$ Real *tol_ptr)
- braid_Int braid_SetSeqSoln (braid_Core core, braid_Int seq_soln)
- braid_Int braid_GetMyID (braid_Core core, braid_Int *myid_ptr)
- braid_Int braid_Rand (void)


### 14.6.1 Detailed Description

These are general interface routines, e.g., routines to initialize and run a XBraid solver, or to split a communicator into spatial and temporal components.
14.6.2 Macro Definition Documentation

### 14.6.2.1 braid_RAND_MAX

\#define braid_RAND_MAX 32768
Machine independent pseudo-random number generator is defined in Braid.c
14.6.3 Typedef Documentation
14.6.3.1 braid_Core
typedef struct _braid_Core_struct* braid_Core
points to the core structure defined in _braid.h
14.6.4 Function Documentation
14.6.4.1 braid_Destroy()
braid_Int braid_Destroy (
braid_Core core )
Clean up and destroy core.

## Parameters

core $\quad$ braid_Core (_braid_Core) struct
14.6.4.2 braid_Drive()
braid_Int braid_Drive (

```
    braid_Core core )
```

Carry out a simulation with XBraid. Integrate in time.
Parameters
core $\quad$ braid_Core (_braid_Core) struct
14.6.4.3 braid_GetMyID()
braid_Int braid_GetMyID (
braid_Core core,
braid_Int * myid_ptr )

Get the processor's rank.
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| myid_ptr | output: rank of the processor. |

14.6.4.4 braid_GetNLevels()
braid_Int braid_GetNLevels (
braid_Core core,
braid_Int * nlevels_ptr )

After Drive() finishes, this returns the number of XBraid levels
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| nlevels_ptr | output, holds the number of XBraid levels |

14.6.4.5 braid_GetNumlter()
braid_Int braid_GetNumIter (
braid_Core core,
braid_Int * niter_ptr )
After Drive() finishes, this returns the number of iterations taken.
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| niter_ptr | output, holds number of iterations taken |

14.6.4.6 braid_GetRNorms()
braid_Int braid_GetRNorms (
braid_Core core,
braid_Int * nrequest_ptr,
braid_Real * rnorms )

After Drive() finishes, this returns XBraid residual history. If nrequest_ptr is negative, return the last nrequest_ptr residual norms. If positive, return the first nrequest_ptr residual norms. Upon exit, nrequest_ptr holds the number of residuals actually returned.

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| nrequest_ptr | input/output, input: num requested resid norms, output: num actually returned |
| rnorms | output, holds residual norm history array |

### 14.6.4.7 braid_GetSpatialAccuracy()

```
braid_Int braid_GetSpatialAccuracy (
    braid_StepStatus status,
    braid_Real loose_tol,
    braid_Real tight_tol,
    braid_Real * tol_ptr )
```

Example function to compute a tapered stopping tolerance for implicit time stepping routines, i.e., a tolerance tol_ptr for the spatial solves. This tapering only occurs on the fine grid.

This rule must be followed. The same tolerance must be returned over all processors, for a given XBraid and XBraid level. Different levels may have different tolerances and the same level may vary its tolerance from iteration to iteration, but for the same iteration and level, the tolerance must be constant.

This additional rule must be followed. The fine grid tolerance is never reduced (this is important for convergence)
On the fine level,the spatial stopping tolerance tol_ptr is interpolated from loose_tol to tight_tol based on the relationship between rnorm / rnorm0 and tol. Remember when rnorm / rnorm0 $<t o l$, XBraid halts. Thus, this function lets us have a loose stopping tolerance while the Braid residual is still relatively large, and then we transition to a tight stopping tolerance as the Braid residual is reduced.

If the user has not defined a residual function, tight_tol is always returned.

The loose_tol is always used on coarse grids, excepting the above mentioned residual computations.

This function will normally be called from the user's step routine.
This function is also meant as a guide for users to develop their own routine.
Parameters

| status | Current XBraid step status |
| :--- | :--- |
| loose_tol | Loosest allowed spatial solve stopping tol on fine grid |
| tight_tol | Tightest allowed spatial solve stopping tol on fine grid |
| tol_ptr | output, holds the computed spatial solve stopping tol |

### 14.6.4.8 braid_Init()

```
braid_Int braid_Init (
    MPI_Comm comm_world,
    MPI_Comm comm,
    braid_Real tstart,
    braid_Real tstop,
    braid_Int ntime,
    braid_App app,
    braid_PtFcnStep step,
    braid_PtFcnInit init,
    braid_PtFcnClone clone,
    braid_PtFcnFree free,
    braid_PtFcnSum sum,
    braid_PtFcnSpatialNorm spatialnorm,
    braid_PtFcnAccess access,
    braid_PtFcnBufSize bufsize,
    braid_PtFcnBufPack bufpack,
    braid_PtFcnBufUnpack bufunpack,
    braid_Core * core_ptr )
```

Create a core object with the required initial data.
This core is used by XBraid for internal data structures. The output is core_ptr which points to the newly created braid_Core structure.

## Parameters

| comm_world | Global communicator for space and time |
| :--- | :--- |
| comm | Communicator for temporal dimension |
| tstart | start time |
| tstop | End time |
| ntime | Initial number of temporal grid values |
| app | User-defined_braid_App structure |
| step | User time stepping routine to advance a braid_Vector forward one step |
| init | Initialize a braid_Vector on the finest temporal grid |
| clone | Clone a braid_Vector |
| free | Free a braid_Vector |
| sum | Compute vector sum of two braid_Vectors |
| spatialnorm | Compute norm of a braid_Vector, this is a norm only over space |
| access | Allows access to XBraid and current braid_Vector |
| bufsize | Computes size for MPI buffer for one braid_Vector |
| bufpack | Packs MPI buffer to contain one braid_Vector |
| bufunpack | Unpacks MPI buffer into a braid_Vector |
| core_ptr | Pointer to braid_Core __braid_Core) struct |

14.6.4.9 braid_PrintStats()
braid_Int braid_PrintStats (

> braid_Core core )

Print statistics after a XBraid run.
Parameters
core braid_Core (_braid_Core) struct

### 14.6.4.10 braid_Rand()

braid_Int braid_Rand (
void )
Define a machine independent random number generator
14.6.4.11 braid_SetAbsTol()
braid_Int braid_SetAbsTol (
braid_Core core,
braid_Real atol )

Set absolute stopping tolerance.
Recommended option over relative tolerance

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| atol | absolute stopping tolerance |

### 14.6.4.12 braid_SetAccessLevel()

```
braid_Int braid_SetAccessLevel (
    braid_Core core,
    braid_Int access_level )
```

Set access level for XBraid. This controls how often the user's access routine is called.

- Level 0: Never call the user's access routine
- Level 1: Only call the user's access routine after XBraid is finished
- Level 2: Call the user's access routine every iteration and on every level. This is during _braid_FRestrict, during the down-cycle part of a XBraid iteration.

Default is level 1.
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| access_level | desired access_level |

### 14.6.4.13 braid_SetCFactor()

```
braid_Int braid_SetCFactor (
    braid_Core core,
    braid_Int level,
    braid_Int cfactor )
```

Set the coarsening factor cfactor on grid level (level 0 is the finest grid). The default factor is 2 on all levels. To change the default factor, use level $=-1$.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| level | level to set coarsening factor on |
| cfactor | desired coarsening factor |

14.6.4.14 braid_SetCRelaxWt()
braid_Int braid_SetCRelaxWt (
braid_Core core,
braid_Int level,
braid_Real Cwt )

Set the C-relaxation weight on grid level (level 0 is the finest grid). The default is 1.0 on all levels. To change the default factor, use level $*=-1$.

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| level | level to set Cwt on |
| Cwt | C-relaxation weight to use on level |

### 14.6.4.15 braid_SetDefaultPrintFile()

```
braid_Int braid_SetDefaultPrintFile (
    braid_Core core )
```

Use default filename, braid_runtime.out for runtime print messages. This function is particularly useful for Fortran codes, where passing filename strings between C and Fortran is troublesome. Level of printing is controlled by braid_Set $\hookleftarrow$ PrintLevel.

## Parameters

core braid_Core (_braid_Core) struct

```
14.6.4.16 braid_SetFileIOLevel()
braid_Int braid_SetFileIOLevel (
    braid_Core core,
    braid_Int io_level )
```

Set output level for XBraid. This controls how much information is saved to files .

- Level 0: no output
- Level 1: save the cycle in braid.out.cycle

Default is level 1.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| io_level | desired output-to-file level |

14.6.4.17 braid_SetFMG()
braid_Int braid_SetFMG (
braid_Core core )

Once called, XBraid will use FMG (i.e., F-cycles.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :---: | :--- |

14.6.4.18 braid_SetFullRNormRes()
braid_Int braid_SetFullRNormRes (
braid_Core core,
braid_PtFcnResidual residual )

Set user-defined residual routine for computing full residual norm (all C/F points).
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| residual | function pointer to residual routine |

### 14.6.4.19 braid_SetIncrMaxLevels()

braid_Int braid_SetIncrMaxLevels (
braid_Core core )
Increase the max number of multigrid levels after performing a refinement.
14.6.4.20 braid_SetMaxlter()

```
braid_Int braid_SetMaxIter (
    braid_Core core,
    braid_Int max_iter )
```

Set max number of multigrid iterations.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| max_iter | maximum iterations to allow |

### 14.6.4.21 braid_SetMaxLevels()

```
braid_Int braid_SetMaxLevels (
    braid_Core core,
    braid_Int max_levels )
```

Set max number of multigrid levels.
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| max_levels | maximum levels to allow |

14.6.4.22 braid_SetMaxRefinements()
braid_Int braid_SetMaxRefinements (
braid_Core core,
braid_Int max_refinements )

Set the max number of time grid refinement levels allowed.

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| max_refinements | maximum refinement levels allowed |

14.6.4.23 braid_SetMinCoarse()
braid_Int braid_SetMinCoarse (
braid_Core core,
braid_Int min_coarse )

Set minimum allowed coarse grid size. XBraid stops coarsening whenever creating the next coarser grid will result in a grid smaller than min_coarse. The maximum possible coarse grid size will be min_coarse*coarsening_factor.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| min_coarse | minimum coarse grid size |

### 14.6.4.24 braid_SetNFMG()

```
braid_Int braid_SetNFMG (
    braid_Core core,
    braid_Int k )
```

Once called, XBraid will use FMG (i.e., F-cycles.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| $k$ | number of initial F-cycles to do before switching to V-cycles |

### 14.6.4.25 braid_SetNFMGVcyc()

```
braid_Int braid_SetNFMGVcyc (
    braid_Core core,
    braid_Int nfmg_Vcyc )
```

Set number of V-cycles to use at each FMG level (standard is 1 )
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| nfmg_Vcyc | number of V-cycles to do each FMG level |

### 14.6.4.26 braid_SetNRelax()

```
braid_Int braid_SetNRelax (
    braid_Core core,
    braid_Int level,
    braid_Int nrelax )
```

Set the number of relaxation sweeps nrelax on grid level (level 0 is the finest grid). The default is 1 on all levels. To change the default factor, use level $=-1$. One sweep is a CF relaxation sweep.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| level | level to set nrelax on |
| nrelax | number of relaxations to do on level |

```
14.6.4.27 braid_SetPeriodic()
braid_Int braid_SetPeriodic (
    braid_Core core,
    braid_Int periodic )
```

Set periodic time grid. The periodicity on each grid level is given by the number of points on each level. Requirements: The number of points on the finest grid level must be evenly divisible by the product of the coarsening factors between each grid level. Currently, the coarsening factors must be the same on all grid levels. Also, braid_SetSeqSoln must not be used.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| periodic | boolean to specify if periodic |

14.6.4.28 braid_SetPrintFile()
braid_Int braid_SetPrintFile (
braid_Core core,
const char * printfile_name )
Set output file for runtime print messages. Level of printing is controlled by braid_SetPrintLevel. Default is stdout.
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| printfile_name | output file for XBraid runtime output |

### 14.6.4.29 braid_SetPrintLevel()

```
braid_Int braid_SetPrintLevel (
    braid_Core core,
    braid_Int print_level )
```

Set print level for XBraid. This controls how much information is printed to the XBraid print file (braid_SetPrintFile).

- Level 0: no output
- Level 1: print runtime information like the residual history
- Level 2: level 1 output, plus post-Braid run statistics (default)
- Level 3: level 2 output, plus debug level output.

Default is level 1.
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| print_level | desired print level |

### 14.6.4.30 braid_SetRefine()

braid_Int braid_SetRefine (
braid_Core core,
braid_Int refine )
Turn time refinement on (refine $=1$ ) or off (refine $=0$ ).
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| refine | boolean, refine in time or not |

14.6.4.31 braid_SetRelTol()

```
braid_Int braid_SetRelTol (
    braid_Core core,
    braid_Real rtol )
```

Set relative stopping tolerance, relative to the initial residual. Be careful. If your initial guess is all zero, then the initial residual may only be nonzero over one or two time values, and this will skew the relative tolerance. Absolute tolerances are recommended.

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| rtol | relative stopping tolerance |

14.6.4.32 braid_SetResidual()
braid_Int braid_SetResidual (
braid_Core core,
braid_PtFcnResidual residual )

Set user-defined residual routine.
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| residual | function pointer to residual routine |

### 14.6.4.33 braid_SetSeqSoln()

```
braid_Int braid_SetSeqSoln (
    braid_Core core,
    braid_Int seq_soln )
```

Set the initial guess to XBraid as the sequential time stepping solution. This is primarily for debugging. When used with storage $=-2$, the initial residual should evaluate to exactly 0 . The residual can also be 0 for other storage options if the time stepping is exact, e.g., the implicit solve in Step is done to full precision.

The value seq_soln is a Boolean

- 0 : The user's Init() function initializes the state vector (default)
- 1: Sequential time stepping, with the user's initial condition from Init(t=0) initializes the state vector

Default is 0 .
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| seq_soln | 1: Init with sequential time stepping soln, 0: Use user's Init() |

```
14.6.4.34 braid_SetShell()
braid_Int braid_SetShell (
    braid_Core core,
    braid_PtFcnSInit sinit,
    braid_PtFcnSClone sclone,
    braid_PtFcnSFree sfree )
```

Activate the shell vector feature, and set the various functions that are required :

- sinit : create a shell vector
- sclone : clone the shell of a vector
- sfree : free the data of a vector, keeping its shell This feature should be used with storage option $=-1$. It allows the used to keep metadata on all points (including F-points) without storing the all vector everywhere. With these options, the vectors are fully stored on C-points, but only the vector shell is kept on F-points.

```
14.6.4.35 braid_SetSkip()
```

braid_Int braid_SetSkip (
braid_Core core,
braid_Int skip )

Set whether to skip all work on the first down cycle (skip =1). On by default.
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| skip | boolean, whether to skip all work on first down-cycle |

14.6.4.36 braid_SetSpatialCoarsen()

```
braid_Int braid_SetSpatialCoarsen (
    braid_Core core,
    braid_PtFcnSCoarsen scoarsen )
```

Set spatial coarsening routine with user-defined routine. Default is no spatial refinment or coarsening.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| scoarsen | function pointer to spatial coarsening routine |

### 14.6.4.37 braid_SetSpatialRefine()

```
braid_Int braid_SetSpatialRefine (
    braid_Core core,
    braid_PtFcnSRefine srefine )
```

Set spatial refinement routine with user-defined routine. Default is no spatial refinment or coarsening.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| srefine | function pointer to spatial refinement routine |

### 14.6.4.38 braid_SetStorage()

```
braid_Int braid_SetStorage (
    braid_Core core,
    braid_Int storage )
```

Sets the storage properties of the code. -1 : Default, store only C-points 0 : Full storage of C - and F -Points on all levels $x>0$ : Full storage on all levels $>=x$

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| storage | storage property |

### 14.6.4.39 braid_SetSync()

```
braid_Int braid_SetSync (
    braid_Core core,
    braid_PtFcnSync sync )
```

Set sync routine with user-defined routine. Sync gives user access to XBraid and the user's app at various points (primarily once per iteration inside FRefine and outside in the main cycle loop). This function is called once perprocessor (instead of for every state vector on the processor, like access). The use case is to allow the user to update their app once-per iteration based on information from XBraid, for example to maintain the space-time grid when doing time-space adaptivity. Default is no sync routine.

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| sync | function pointer to sync routine |

```
14.6.4.40 braid_SetTemporalNorm()
braid_Int braid_SetTemporalNorm (
    braid_Core core,
    braid_Int tnorm )
```

Sets XBraid temporal norm.
This option determines how to obtain a global space-time residual norm. That is, this decides how to combine the spatial norms returned by braid_PtFcnSpatialNorm at each time step to obtain a global norm over space and time. It is this global norm that then controls halting.

There are three options for setting tnorm. See section Halting tolerance for a more detailed discussion (in Introduction. $\hookleftarrow$ md ).

- tnorm=1: One-norm summation of spatial norms
- tnorm=2: Two-norm summation of spatial norms
- tnorm=3: Infinity-norm combination of spatial norms


## The default choice is tnorm=2

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| tnorm | choice of temporal norm |

### 14.6.4.41 braid_SetTimeGrid()

```
braid_Int braid_SetTimeGrid (
    braid_Core core,
    braid_PtFcnTimeGrid tgrid )
```

Set user-defined time points on finest grid
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| tgrid | function pointer to time grid routine |

### 14.6.4.42 braid_SetTPointsCutoff()

braid_Int braid_SetTPointsCutoff (
braid_Core core,
braid_Int tpoints_cutoff )

Set the number of time steps, beyond which refinements stop. If num(tpoints) $>$ tpoints_cutoff, then stop doing refinements.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| tpoints_cutoff | cutoff for stopping refinements |

### 14.6.4.43 braid_SplitCommworld()

```
braid_Int braid_SplitCommworld (
    const MPI_Comm * comm_world,
    braid_Int px,
    MPI_Comm * comm_x,
    MPI_Comm * comm_t )
```

Split MPI commworld into comm_x and comm_t, the spatial and temporal communicators. The total number of processors will equal $\mathrm{Px} * \mathrm{Pt}$, there Px is the number of procs in space, and Pt is the number of procs in time.

Parameters

| comm_world | Global communicator to split |
| :--- | :--- |
| $p x$ | Number of processors parallelizing space for a single time step |
| comm_ $x$ | Spatial communicator (written as output) |
| comm_ $t$ | Temporal communicator (written as output) |

14.6.4.44 braid_WriteConvHistory()

```
braid_Int braid_WriteConvHistory (
    braid_Core core,
    const char * filename )
```

After Drive() finishes, this function can be called to write out the convergence history (residuals for each iteration) to a file

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| filename | Output file name |

### 14.7 Interface routines for XBraid_Adjoint

## Functions

- braid_Int braid_InitAdjoint (braid_PtFcnObjectiveT objectiveT, braid_PtFcnObjectiveTDiff objectiveT_diff, braid $\hookleftarrow$ _PtFcnStepDiff step_diff, braid_PtFcnResetGradient reset_gradient, braid_Core $*$ core_ptr)
- braid_Int braid_SetTStartObjective (braid_Core core, braid_Real tstart_obj)
- braid_Int braid_SetTStopObjective (braid_Core core, braid_Real tstop_obj)
- braid_Int braid_SetPostprocessObjective (braid_Core core, braid_PtFcnPostprocessObjective post_fcn)
- braid_Int braid_SetPostprocessObjective_diff (braid_Core core, braid_PtFcnPostprocessObjective_diff post_↔ fcn_diff)
- braid_Int braid_SetAbsToIAdjoint (braid_Core core, braid_Real tol_adj)
- braid_Int braid_SetRelTolAdjoint (braid_Core core, braid_Real rtol_adj)
- braid_Int braid_SetObjectiveOnly (braid_Core core, braid_Int boolean)
- braid_Int braid_GetObjective (braid_Core core, braid_Real *objective_ptr)
- braid_Int braid_GetRNormAdjoint (braid_Core core, braid_Real *rnorm_adj)
- braid_Int braid_SetRichardsonEstimation (braid_Core core, braid_Int est_error, braid_Int richardson, braid_Int local_order)


### 14.7.1 Detailed Description

These are interface routines for computing adjoint sensitivities, i.e., adjoint-based gradients. These routines initialize the XBraid_Adjoint solver, and allow the user to set XBraid_Adjoint solver parameters.

### 14.7.2 Function Documentation

### 14.7.2.1 braid_GetObjective()

braid_Int braid_GetObjective (
braid_Core core,
braid_Real * objective_ptr )

After braid_Drive has finished, this returns the objective function value.

Parameters

| core | braid_Core struct |
| :--- | :--- |
| objective_ptr | output: value of the objective function |

### 14.7.2.2 braid_GetRNormAdjoint()

braid_Int braid_GetRNormAdjoint (

```
braid_Core core,
braid_Real * rnorm_adj )
```

After braid_Drive has finished, this returns the residual norm after the last XBraid iteration.

Parameters

| core | braid_Core struct |
| :--- | :--- |
| rnorm_adj | output: adjoint residual norm of last iteration |

### 14.7.2.3 braid_InitAdjoint()

```
braid_Int braid_InitAdjoint (
    braid_PtFcnObjectiveT objectiveT,
    braid_PtFcnObjectiveTDiff objectiveT_diff,
    braid_PtFcnStepDiff step_diff,
    braid_PtFcnResetGradient reset_gradient,
    braid_Core * core_ptr )
```

Initialize the XBraid_Adjoint solver for computing adjoint sensitivities. Once this function is called, braid_Drive will then compute gradient information alongside the primal XBraid computations.

Parameters

| objectiveT | user-routine: evaluates the time-dependent objective function value at time $t$ |
| :--- | :--- |
| objectiveT_diff | user-routine: differentiated version of the objectiveT function |
| step_diff | user-routine: differentiated version of the step function |
| reset_gradient | user-routine: set the gradient to zero (storage location of gradient up to user) |
| core_ptr | pointer to braid_Core (_braid_Core) struct |

```
14.7.2.4 braid_SetAbsToIAdjoint()
braid_Int braid_SetAbsTolAdjoint (
    braid_Core core,
    braid_Real tol_adj )
```

Set an absolute halting tolerance for the adjoint residuals. XBraid_Adjoint stops iterating when the adjoint residual is below this value.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| tol_adj | absolute stopping tolerance for adjoint solve |

14.7.2.5 braid_SetObjectiveOnly()

```
braid_Int braid_SetObjectiveOnly (
    braid_Core core,
    braid_Int boolean )
```

Set this option with boolean = 1, and then braid_Drive(core) will skip the gradient computation and only compute the forward ODE solution and objective function value. Reset this option with boolean $=0$ to turn the adjoint solve and gradient computations back on.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| boolean | set to '1' for computing objective function only, '0' for computing objective function AND gradients |

```
14.7.2.6 braid_SetPostprocessObjective()
braid_Int braid_SetPostprocessObjective (
    braid_Core core,
    braid_PtFcnPostprocessObjective post_fcn )
```

Pass the postprocessing objective function Fto XBraid_Adjoint. For a description of $F$, see the Section Objective function evaluation.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| post_fcn | function pointer to postprocessing routine |

14.7.2.7 braid_SetPostprocessObjective_diff()

```
braid_Int braid_SetPostprocessObjective_diff (
    braid_Core core,
    braid_PtFcnPostprocessObjective_diff post_fcn_diff )
```

Pass the differentiated version of the postprocessing objective function $F$ to XBraid_Adjoint. For a description of $F$, see the Section Objective function evaluation .

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| post_fcn_diff | function pointer to differentiated postprocessing routine |

14.7.2.8 braid_SetRelToIAdjoint()

```
braid_Int braid_SetRelTolAdjoint (
    braid_Core core,
    braid_Real rtol_adj )
```

Set a relative stopping tolerance for adjoint residuals. XBraid_Adjoint will stop iterating when the relative residual drops below this value. Be careful when using a relative stopping criterion. The initial residual may already be close to zero, and this will skew the relative tolerance. Absolute tolerances are recommended.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| rtol_adj | relative stopping tolerance for adjoint solve |

### 14.7.2.9 braid_SetRichardsonEstimation()

braid_Int braid_SetRichardsonEstimation (
braid_Core core,
braid_Int est_error,
braid_Int richardson,
braid_Int local_order )
Turn on built-in Richardson-based error estimation and/or extrapolation with XBraid. When enabled, the Richardson extrapolation (RE) option (richardson $==1$ ) is used to improve the accuracy of the solution at the C-points on the finest level. When the built-in error estimate option is turned on (est_error == 1), RE is used to estimate the local truncation error at each point. These estimates can be accessed through StepStatus and AccessStatus functions.

The last parameter is local_order, which represents the LOCAL order of the time integration scheme. e.g. local_order = 2 for Backward Euler.

Also, the Richardson error estimate is only available after roughly 1 Braid iteration. The estimate is given a dummy value of -1.0 , until an actual estimate is available. Thus after an adaptive refinement, and a new hierarchy is formed, another iteration must pass before the error estimates are available again.

Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| est_error | Boolean, if 1 compute Richardson-based error estimates, if 0, then do not |
| richardson | Boolean, if 1 carry out Richardson-based extrapolation to enhance accuracy on the fine-grid, if 0, <br> then do not |
| local_order | Local order of the time integration scheme, e.g., local_order=2 for backward Euler |

14.7.2.10 braid_SetTStartObjective()
braid_Int braid_SetTStartObjective (
braid_Core core,
braid_Real tstart_obj )
Set a start time for integrating the objective function over time. Default is tstart of the primal XBraid run.
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| tstart_obj | time value for starting the time-integration of the objective function |

14.7.2.11 braid_SetTStopObjective()

```
braid_Int braid_SetTStopObjective (
    braid_Core core,
    braid_Real tstop_obj )
```

Set the end-time for integrating the objective function over time. Default is tstop of the primal XBraid run
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| tstop_obj | time value for stopping the time-integration of the objective function |

### 14.8 XBraid status structures

Define the different status types.

### 14.9 XBraid status routines

## Functions

- braid_Int braid_StatusGetT (braid_Status status, braid_Real *t_ptr)
- braid_Int braid_StatusGetTIndex (braid_Status status, braid_Int *idx_ptr)
- braid_Int braid_StatusGetlter (braid_Status status, braid_Int *iter_ptr)
- braid_Int braid_StatusGetLevel (braid_Status status, braid_Int *level_ptr)
- braid_Int braid_StatusGetNLevels (braid_Status status, braid_Int *nlevels_ptr)
- braid_Int braid_StatusGetNRefine (braid_Status status, braid_Int *nrefine_ptr)
- braid_Int braid_StatusGetNTPoints (braid_Status status, braid_Int *ntpoints_ptr)
- braid_Int braid_StatusGetResidual (braid_Status status, braid_Real *rnorm_ptr)
- braid_Int braid_StatusGetDone (braid_Status status, braid_Int *done_ptr)
- braid_Int braid_StatusGetTIUL (braid_Status status, braid_Int *iloc_upper, braid_Int *iloc_lower, braid_Int level)
- braid_Int braid_StatusGetTimeValues (braid_Status status, braid_Real **tvalues_ptr, braid_Int i_upper, braid_Int i_lower, braid_Int level)
- braid_Int braid_StatusGetTILD (braid_Status status, braid_Real *t_ptr, braid_Int *iter_ptr, braid_Int *level_ptr, braid_Int *done_ptr)
- braid_Int braid_StatusGetWrapperTest (braid_Status status, braid_Int *wtest_ptr)
- braid_Int braid_StatusGetCallingFunction (braid_Status status, braid_Int *cfunction_ptr)
- braid_Int braid_StatusGetCTprior (braid_Status status, braid_Real *ctprior_ptr)
- braid_Int braid_StatusGetCTstop (braid_Status status, braid_Real *ctstop_ptr)
- braid_Int braid_StatusGetFTprior (braid_Status status, braid_Real *ftprior_ptr)
- braid_Int braid_StatusGetFTstop (braid_Status status, braid_Real *ftstop_ptr)
- braid_Int braid_StatusGetTpriorTstop (braid_Status status, braid_Real *t_ptr, braid_Real *ftprior_ptr, braid_Real *ftstop_ptr, braid_Real *ctprior_ptr, braid_Real *ctstop_ptr)
- braid_Int braid_StatusGetTstop (braid_Status status, braid_Real *tstop_ptr)
- braid_Int braid_StatusGetTstartTstop (braid_Status status, braid_Real *tstart_ptr, braid_Real *tstop_ptr)
- braid_Int braid_StatusGetTol (braid_Status status, braid_Real *tol_ptr)
- braid_Int braid_StatusGetRNorms (braid_Status status, braid_Int *nrequest_ptr, braid_Real *rnorms_ptr)
- braid_Int braid_StatusGetOldFineTolx (braid_Status status, braid_Real *old_fine_tolx_ptr)
- braid_Int braid_StatusSetOldFineTolx (braid_Status status, braid_Real old_fine_tolx)
- braid_Int braid_StatusSetTightFineTolx (braid_Status status, braid_Real tight_fine_tolx)
- braid_Int braid_StatusSetRFactor (braid_Status status, braid_Real rfactor)
- braid_Int braid_StatusSetRefinementDtValues (braid_Status status, braid_Real rfactor, braid_Real *dtarray)
- braid_Int braid_StatusSetRSpace (braid_Status status, braid_Real r_space)
- braid_Int braid_StatusGetMessageType (braid_Status status, braid_Int *messagetype_ptr)
- braid_Int braid_StatusSetSize (braid_Status status, braid_Real size)
- braid_Int braid_StatusGetSingleErrorEstStep (braid_Status status, braid_Real *estimate)
- braid_Int braid_StatusGetSingleErrorEstAccess (braid_Status status, braid_Real *estimate)
- braid_Int braid_StatusGetNumErrorEst (braid_Status status, braid_Int *npoints)
- braid_Int braid_StatusGetAllErrorEst (braid_Status status, braid_Real *error_est)


### 14.9.1 Detailed Description

XBraid status structures and associated Get/Set routines are what tell the user the status of the simulation when their routines (step, coarsen/refine, access) are called.
14.9.2 Function Documentation
14.9.2.1 braid_StatusGetAllErrorEst()

```
braid_Int braid_StatusGetAllErrorEst (
    braid_Status status,
    braid_Real * error_est )
```

Get All the Richardson based error estimates, e.g. from inside Sync. Use this function in conjuction with GetNumError $\hookleftarrow$ Est(). Workflow: use GetNumErrorEst() to get the size of the needed user-array that will hold the error estimates, then pre-allocate array, then call this function to write error estimates to the user-array, then post-process array in user-code. This post-processing will often occur in the Sync function. See examples/ex-06.c.

The error_est array must be user-allocated.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| error_est | output, user-allocated error estimate array, written by Braid, equals -1 if not available yet (e.g., before <br> iteration 1, or after refinement) |

14.9.2.2 braid_StatusGetCallingFunction()
braid_Int braid_StatusGetCallingFunction (
braid_Status status,
braid_Int * cfunction_ptr )

Return flag indicating from which function the vector is accessed

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| cfunction_ptr | output, function number (0=FInterp, 1=FRestrict, 2=FRefine, 3=FAccess, 4=FRefine after <br> refinement, 5=Drive Top of Cycle) |

### 14.9.2.3 braid_StatusGetCTprior()

```
braid_Int braid_StatusGetCTprior (
    braid_Status status,
    braid_Real * ctprior_ptr )
```

Return the coarse grid time value to the left of the current time value from the Status structure.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| ctprior_ptr | output, time value to the left of current time value on coarse grid |

14.9.2.4 braid_StatusGetCTstop()
braid_Int braid_StatusGetCTstop (
braid_Status status,
braid_Real * ctstop_ptr )
Return the coarse grid time value to the right of the current time value from the Status structure.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| ctstop_ptr | output, time value to the right of current time value on coarse grid |

```
14.9.2.5 braid_StatusGetDone()
braid_Int braid_StatusGetDone (
    braid_Status status,
    braid_Int * done_ptr )
```

Return whether XBraid is done for the current simulation.
done_ptr = 1 indicates that XBraid has finished iterating, (either maxiter has been reached, or the tolerance has been met).

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| done_ptr | output, $=1$ if XBraid has finished, else $=0$ |

### 14.9.2.6 braid_StatusGetFTprior()

```
braid_Int braid_StatusGetFTprior (
    braid_Status status,
    braid_Real * ftprior_ptr )
```

Return the fine grid time value to the left of the current time value from the Status structure.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| ftprior_ptr | output, time value to the left of current time value on fine grid |

14.9.2.7 braid_StatusGetFTstop()
braid_Int braid_StatusGetFTstop (
braid_Status status,
braid_Real * ftstop_ptr )

Return the fine grid time value to the right of the current time value from the Status structure.
Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| ftstop_ptr | output, time value to the right of current time value on fine grid |

```
14.9.2.8 braid_StatusGetlter()
braid_Int braid_StatusGetIter (
    braid_Status status,
    braid_Int * iter_ptr )
```

Return the current iteration from the Status structure.
Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| iter_ptr | output, current XBraid iteration number |

14.9.2.9 braid_StatusGetLevel()
braid_Int braid_StatusGetLevel (
braid_Status status,
braid_Int * level_ptr )
Return the current XBraid level from the Status structure.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| level_ptr | output, current level in XBraid |

### 14.9.2.10 braid_StatusGetMessageType()

braid_Int braid_StatusGetMessageType (
braid_Status status,
braid_Int * messagetype_ptr )

Return the current message type from the Status structure.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| messagetype_ptr | output, type of message, 0: for Step(), 1: for load balancing |

14.9.2.11 braid_StatusGetNLevels()

```
braid_Int braid_StatusGetNLevels (
    braid_Status status,
    braid_Int * nlevels_ptr )
```

Return the total number of XBraid levels from the Status structure.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| nlevels_ptr | output, number of levels in XBraid |

14.9.2.12 braid_StatusGetNRefine()
braid_Int braid_StatusGetNRefine (
braid_Status status,
braid_Int * nrefine_ptr )

Return the number of refinements done.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| nrefine_ptr | output, number of refinements done |

### 14.9.2.13 braid_StatusGetNTPoints()

```
braid_Int braid_StatusGetNTPoints (
    braid_Status status,
    braid_Int * ntpoints_ptr )
```

Return the global number of time points on the fine grid.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| ntpoints_ptr | output, number of time points on the fine grid |

### 14.9.2.14 braid_StatusGetNumErrorEst()

```
braid_Int braid_StatusGetNumErrorEst (
    braid_Status status,
    braid_Int * npoints )
```

Get the number of local Richardson-based error estimates stored on this processor. Use this function in conjuction with GetAllErrorEst(). Workflow: use this function to get the size of the needed user-array that will hold the error estimates, then pre-allocate array, then call GetAllErrorEst() to write error estimates to the user-array, then post-process array in user-code. This post-processing will often occur in the Sync function. See examples/ex-06.c.

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| npoints | output, number of locally stored Richardson error estimates |

### 14.9.2.15 braid_StatusGetOldFineTolx()

```
braid_Int braid_StatusGetOldFineTolx (
    braid_Status status,
    braid_Real * old_fine_tolx_ptr )
```

Return the previous old_fine_tolx set through braid_StatusSetOldFineTolx This is used especially by *braid_Get $\hookleftarrow$ SpatialAccuracy

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| old_fine_tolx_ptr | output, previous old_fine_tolx, set through braid_StepStatusSetOldFineTolx |

14.9.2.16 braid_StatusGetResidual()
braid_Int braid_StatusGetResidual (
braid_Status status,
braid_Real * rnorm_ptr )
Return the current residual norm from the Status structure.
Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| rnorm_ptr | output, current residual norm |

14.9.2.17 braid_StatusGetRNorms()

```
braid_Int braid_StatusGetRNorms (
    braid_Status status,
    braid_Int * nrequest_ptr,
    braid_Real * rnorms_ptr )
```

Return the current XBraid residual history. If nrequest_ptr is negative, return the last nrequest_ptr residual norms. If positive, return the first nrequest_ptr residual norms. Upon exit, nrequest_ptr holds the number of residuals actually returned.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| nrequest_ptr | input/output, input: number of requested residual norms, output: number actually copied |
| rnorms_ptr | output, XBraid residual norm history, of length nrequest_ptr |

```
14.9.2.18 braid_StatusGetSingleErrorEstAccess()
braid_Int braid_StatusGetSingleErrorEstAccess (
    braid_Status status,
    braid_Real * estimate )
```

Get the Richardson based error estimate at the single time point currently accessible from Access.
Note that Access needs specific logic distinct from Step, hence please use braid_StepStatusGetSingleErrorEstStep for the user Step() function.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| estimate | output, error estimate, equals -1 if not available yet (e.g., before iteration 1, or after refinement) |

### 14.9.2.19 braid_StatusGetSingleErrorEstStep()

```
braid_Int braid_StatusGetSingleErrorEstStep (
    braid_Status status,
    braid_Real * estimate )
```

Get the Richardson based error estimate at the single time point currently being "Stepped", i.e., return the current error estimate for the time point at "tstart".

Note that Step needs specific logic distinct from Access, hence please use braid_AccessStatusGetSingleErrorEst $\hookleftarrow$ Access for the user Access() function.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| estimate | output, error estimate, equals -1 if not available yet (e.g., before iteration 1, or after refinement) |

14.9.2.20 braid_StatusGetT()

```
braid_Int braid_StatusGetT (
    braid_Status status,
    braid_Real * t_ptr )
```

Return the current time from the Status structure.
Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| $t \_p t r$ | output, current time |

### 14.9.2.21 braid_StatusGetTILD()

braid_Int braid_StatusGetTILD (

```
braid_Status status,
braid_Real * t_ptr,
braid_Int * iter_ptr,
braid_Int * level_ptr,
braid_Int * done_ptr )
```

Return XBraid status for the current simulation. Four values are returned.

TILD : time, iteration, level, done

These values are also available through individual Get routines. These individual routines are the location of detailed documentation on each parameter, e.g., see braid_StatusGetDone for more information on the done value.

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| $t \_p t r$ | output, current time |
| iter_ptr | output, current XBraid iteration number |
| level_ptr | output, current level in XBraid |
| done_ptr | output, $=1$ if XBraid has finished, else $=0$ |

14.9.2.22 braid_StatusGetTimeValues()

```
braid_Int braid_StatusGetTimeValues (
    braid_Status status,
    braid_Real ** tvalues_ptr,
    braid_Int i_upper,
    braid_Int i_lower,
    braid_Int level )
```

Returns an array of time values corresponding to the given inputs. The inputs are the level you want the time values from, the upper time point index you want the value of, and the lower time point index you want the time value of. The output is then filled with all time values from the upper index to the lower index, inclusive.

The caller is responsible for allocating and managing the memory for the array. Time values are filled in so that tvalues $\hookleftarrow$ _ptr[0] corresponds to the lower time index.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| tvalues_ptr | output, time point values for the requested range of indices |
| i_upper | input, upper index of the desired time value range (inclusive) |
| i_lower | input, lower index of the desired time value range (inclusive) |
| level | input, level for the desired time values |

14.9.2.23 braid_StatusGetTIndex()

```
braid_Int braid_StatusGetTIndex (
    braid_Status status,
    braid_Int * idx_ptr )
```

Return the index value corresponding to the current time value from the Status structure.

For Step(), this corresponds to the time-index of "tstart", as this is the time-index of the input vector. That is, NOT the time-index of "tstop". For Access, this corresponds just simply to the time-index of the input vector.

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| idx_ptr | output, global index value corresponding to current time value |

### 14.9.2.24 braid_StatusGetTIUL()

```
braid_Int braid_StatusGetTIUL (
    braid_Status status,
    braid_Int * iloc_upper,
    braid_Int * iloc_lower,
    braid_Int level )
```

Returns upper and lower time point indices on this processor. Two values are returned. Requires the user to specify which level they want the time point indices from.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| iloc_upper | output, the upper time point index on this processor |
| iloc_lower | output, the lower time point index on this processor |
| level | input, level for the desired indices |

14.9.2.25 braid_StatusGetTol()

```
braid_Int braid_StatusGetTol (
    braid_Status status,
    braid_Real * tol_ptr )
```

Return the current XBraid stopping tolerance

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| tol_ptr | output, current XBraid stopping tolerance |

14.9.2.26 braid_StatusGetTpriorTstop()

```
braid_Int braid_StatusGetTpriorTstop (
    braid_Status status,
    braid_Real * t_ptr,
    braid_Real * ftprior_ptr,
    braid_Real * ftstop_ptr,
    braid_Real * ctprior_ptr,
    braid_Real * ctstop_ptr )
```

Return XBraid status for the current simulation. Five values are returned, tstart, f_tprior, f_tstop, c_tprior, c_tstop.
These values are also available through individual Get routines. These individual routines are the location of detailed documentation on each parameter, e.g., see braid_StatusGetCTprior for more information on the c_tprior value.

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| t_ptr | output, current time |
| ftprior_ptr | output, time value to the left of current time value on fine grid |
| ftstop_ptr | output, time value to the right of current time value on fine grid |
| ctprior_ptr | output, time value to the left of current time value on coarse grid |
| ctstop_ptr | output, time value to the right of current time value on coarse grid |

### 14.9.2.27 braid_StatusGetTstartTstop()

```
braid_Int braid_StatusGetTstartTstop (
    braid_Status status,
    braid_Real * tstart_ptr,
    braid_Real * tstop_ptr )
```

Return XBraid status for the current simulation. Two values are returned, tstart and tstop.
These values are also available through individual Get routines. These individual routines are the location of detailed documentation on each parameter, e.g., see braid_StatusGetTstart for more information on the tstart value.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| tstart_ptr | output, current time |
| tstop_ptr | output, next time value to evolve towards |

14.9.2.28 braid_StatusGetTstop()

```
braid_Int braid_StatusGetTstop (
    braid_Status status,
    braid_Real * tstop_ptr )
```

Return the time value to the right of the current time value from the Status structure.

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| tstop_ptr | output, next time value to evolve towards |

### 14.9.2.29 braid_StatusGetWrapperTest()

```
braid_Int braid_StatusGetWrapperTest (
    braid_Status status,
    braid_Int * wtest_ptr )
```

Return whether this is a wrapper test or an XBraid run

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| wtest_ptr | output, $=1$ if this is a wrapper test, $=0$ if XBraid run |

### 14.9.2.30 braid_StatusSetOldFineTolx()

braid_Int braid_StatusSetOldFineTolx (
braid_Status status,
braid_Real old_fine_tolx )

Set old_fine_tolx, available for retrieval through braid_StatusGetOldFineTolx This is used especially by *braid_Get $\hookleftarrow$ SpatialAccuracy

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| old_fine_tolx | input, the last used fine_tolx |

14.9.2.31 braid_StatusSetRefinementDtValues()

```
braid_Int braid_StatusSetRefinementDtValues (
    braid_Status status,
    braid_Real rfactor,
    braid_Real * dtarray )
```

Set time step sizes for refining the time interval non-uniformly.
Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| rfactor | input, number of subintervals |
| dtarray | input, array of dt values for non-uniform refinement |

14.9.2.32 braid_StatusSetRFactor()

```
braid_Int braid_StatusSetRFactor (
    braid_Status status,
    braid_Real rfactor )
```

Set the rfactor, a desired refinement factor for this interval. rfactor=1 indicates no refinement, otherwise, this inteval is subdivided rfactor times (uniform refinement).

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| rfactor | input, user-determined desired rfactor |

14.9.2.33 braid_StatusSetRSpace()

```
braid_Int braid_StatusSetRSpace (
    braid_Status status,
    braid_Real r_space )
```

Set the r_space flag. When set $=1$, spatial coarsening will be called, for all local time points, following the completion of the current iteration, provided rfactors are not set at any global time point. This allows for spatial refinment without temporal refinment

Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| $r_{-}$space | input, if 1, call spatial refinement on finest grid after this iter |

14.9.2.34 braid_StatusSetSize()
braid_Int braid_StatusSetSize (
braid_Status status,
braid_Real size )

Set the size of the buffer. If set by user, the send buffer will be "size" bytes in length. If not, BufSize is used.
Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| size | input, size of the send buffer |

14.9.2.35 braid_StatusSetTightFineTolx()
braid_Int braid_StatusSetTightFineTolx (
braid_Status status,
braid_Real tight_fine_tolx )

Set tight_fine_tolx, boolean variable indicating whether the tightest tolerance has been used for spatial solves (implicit schemes). This value must be 1 in order for XBraid to halt (unless maxiter is reached)

## Parameters

| status | structure containing current simulation info |
| :--- | :--- |
| tight_fine_tolx | input, boolean indicating whether the tight tolx has been used |

### 14.10 Inherited XBraid status routines

## Functions

- braid_Int braid_AccessStatusGetT (braid_AccessStatus s, braid_Real $* \mathrm{~V} 1$ )
- braid_Int braid_AccessStatusGetTIndex (braid_AccessStatus s, braid_Int *V1)
- braid_Int braid_AccessStatusGetlter (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetLevel (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetNLevels (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetNRefine (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetNTPoints (braid_AccessStatus s, braid_Int *V1)
- braid_Int braid_AccessStatusGetResidual (braid_AccessStatus s, braid_Real *v1)
- braid_Int braid_AccessStatusGetDone (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetTILD (braid_AccessStatus s, braid_Real $* \mathrm{v} 1$, braid_Int $* \mathrm{v} 2$, braid_Int $* \mathrm{v} 3$, braid_Int *V4)
- braid_Int braid_AccessStatusGetWrapperTest (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetCallingFunction (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetSingleErrorEstAccess (braid_AccessStatus s, braid_Real $*$ v1)
- braid_Int braid_SyncStatusGetTIUL (braid_SyncStatus s, braid_Int $* \mathrm{v} 1$, braid_Int $* \mathrm{v} 2$, braid_Int v3)
- braid_Int braid_SyncStatusGetTimeValues (braid_SyncStatus s, braid_Real **v1, braid_Int v2, braid_Int v3, braid_Int v4)
- braid_Int braid_SyncStatusGetlter (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetLevel (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetNLevels (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetNRefine (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetNTPoints (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetDone (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetCallingFunction (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetNumErrorEst (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetAllErrorEst (braid_SyncStatus s, braid_Real *v1)
- braid_Int braid_CoarsenRefStatusGetT (braid_CoarsenRefStatus s, braid_Real *v1)
- braid_Int braid_CoarsenRefStatusGetTIndex (braid_CoarsenRefStatus s, braid_Int *V1)
- braid_Int braid_CoarsenRefStatusGetlter (braid_CoarsenRefStatus s, braid_Int *v1)
- braid_Int braid_CoarsenRefStatusGetLevel (braid_CoarsenRefStatus s, braid_Int *v1)
- braid_Int braid_CoarsenRefStatusGetNLevels (braid_CoarsenRefStatus s, braid_Int *V1)
- braid_Int braid_CoarsenRefStatusGetNRefine (braid_CoarsenRefStatus s, braid_Int *v1)
- braid_Int braid_CoarsenRefStatusGetNTPoints (braid_CoarsenRefStatus s, braid_Int *v1)
- braid_Int braid_CoarsenRefStatusGetCTprior (braid_CoarsenRefStatus s, braid_Real *v1)
- braid_Int braid_CoarsenRefStatusGetCTstop (braid_CoarsenRefStatus s, braid_Real *v1)
- braid_Int braid_CoarsenRefStatusGetFTprior (braid_CoarsenRefStatus s, braid_Real *v1)
- braid_Int braid_CoarsenRefStatusGetFTstop (braid_CoarsenRefStatus s, braid_Real *v1)
- braid_Int braid_CoarsenRefStatusGetTpriorTstop (braid_CoarsenRefStatus s, braid_Real *v1, braid_Real $* \mathrm{v} 2$, braid_Real $* \mathrm{v} 3$, braid_Real $* \mathrm{v} 4$, braid_Real $* \mathrm{v} 5$ )
- braid_Int braid_StepStatusGetT (braid_StepStatus s, braid_Real *v1)
- braid_Int braid_StepStatusGetTIndex (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetlter (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetLevel (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetNLevels (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetNRefine (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetNTPoints (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetTstop (braid_StepStatus s, braid_Real *v1)
- braid_Int braid_StepStatusGetTstartTstop (braid_StepStatus s, braid_Real *v1, braid_Real *v2)
- braid_Int braid_StepStatusGetTol (braid_StepStatus s, braid_Real *v1)
- braid_Int braid_StepStatusGetRNorms (braid_StepStatus s, braid_Int *v1, braid_Real *v2)
- braid_Int braid_StepStatusGetOIdFineTolx (braid_StepStatus s, braid_Real *v1)
- braid_Int braid_StepStatusSetOldFineTolx (braid_StepStatus s, braid_Real v1)
- braid_Int braid_StepStatusSetTightFineTolx (braid_StepStatus s, braid_Real v1)
- braid_Int braid_StepStatusSetRFactor (braid_StepStatus s, braid_Real v1)
- braid_Int braid_StepStatusSetRSpace (braid_StepStatus s, braid_Real v1)
- braid_Int braid_StepStatusGetSingleErrorEstStep (braid_StepStatus s, braid_Real *v1)
- braid_Int braid_BufferStatusGetMessageType (braid_BufferStatus s, braid_Int $*$ V1)
- braid_Int braid_BufferStatusSetSize (braid_BufferStatus s, braid_Real v1)
- braid_Int braid_ObjectiveStatusGetT (braid_ObjectiveStatus s, braid_Real *v1)
- braid_Int braid_ObjectiveStatusGetTIndex (braid_ObjectiveStatus s, braid_Int *v1)
- braid_Int braid_ObjectiveStatusGetlter (braid_ObjectiveStatus s, braid_Int *v1)
- braid_Int braid_ObjectiveStatusGetLevel (braid_ObjectiveStatus s, braid_Int *V1)
- braid_Int braid_ObjectiveStatusGetNLevels (braid_ObjectiveStatus s, braid_Int *v1)
- braid_Int braid_ObjectiveStatusGetNRefine (braid_ObjectiveStatus s, braid_Int *v1)
- braid_Int braid_ObjectiveStatusGetNTPoints (braid_ObjectiveStatus s, braid_Int *v1)
- braid_Int braid_ObjectiveStatusGetTol (braid_ObjectiveStatus s, braid_Real *v1)
14.10.1 Detailed Description

These are the 'inherited' Status Get/Set functions. See the XBraid status routines section for the description of each function. For example, for braid_StepStatusGetT(...), you would look up braid_StatusGetT(...)
14.10.2 Function Documentation
14.10.2.1 braid_AccessStatusGetCallingFunction()
braid_Int braid_AccessStatusGetCallingFunction (
braid_AccessStatus s,
braid_Int * v1 )
14.10.2.2 braid_AccessStatusGetDone()
braid_Int braid_AccessStatusGetDone (
braid_AccessStatus $s$,
braid_Int * v1 )
14.10.2.3 braid_AccessStatusGetlter()

```
braid_Int braid_AccessStatusGetIter (
    braid_AccessStatus s,
    braid_Int * v1 )
```

14.10.2.4 braid_AccessStatusGetLevel()
braid_Int braid_AccessStatusGetLevel (
braid_AccessStatus s, braid_Int * v1 )
14.10.2.5 braid_AccessStatusGetNLevels()

```
braid_Int braid_AccessStatusGetNLevels (
    braid_AccessStatus s,
    braid_Int * vI )
```

14.10.2.6 braid_AccessStatusGetNRefine()
braid_Int braid_AccessStatusGetNRefine (
braid_AccessStatus s,
braid_Int * v1 )
14.10.2.7 braid_AccessStatusGetNTPoints()
braid_Int braid_AccessStatusGetNTPoints (
braid_AccessStatus s,
braid_Int * v1 )
14.10.2.8 braid_AccessStatusGetResidual()
braid_Int braid_AccessStatusGetResidual (
braid_Accessstatus s,
braid_Real * v1 )
14.10.2.9 braid_AccessStatusGetSingleErrorEstAccess()

```
braid_Int braid_AccessStatusGetSingleErrorEstAccess (
    braid_AccessStatus s,
    braid_Real * v1 )
```

14.10.2.10 braid_AccessStatusGetT()

```
braid_Int braid_AccessStatusGetT (
```

    braid_AccessStatus s,
    braid_Real * v1 )
    14.10.2.11 braid_AccessStatusGetTILD()

```
braid_Int braid_AccessStatusGetTILD (
    braid_AccessStatus s,
    braid_Real * v1,
    braid_Int * v2,
    braid_Int * v3,
    braid_Int * v4 )
```

14.10.2.12 braid_AccessStatusGetTIndex()
braid_Int braid_AccessStatusGetTIndex (
braid_AccessStatus s, braid_Int * v1 )
14.10.2.13 braid_AccessStatusGetWrapperTest()

```
braid_Int braid_AccessStatusGetWrapperTest (
    braid_AccessStatus s,
    braid_Int * v1 )
```

14.10.2.14 braid_BufferStatusGetMessageType()
braid_Int braid_BufferStatusGetMessageType (
braid_BufferStatus s, braid_Int * v1 )
14.10.2.15 braid_BufferStatusSetSize()

```
braid_Int braid_BufferStatusSetSize (
    braid_BufferStatus s,
    braid_Real v1 )
```

14.10.2.16 braid_CoarsenRefStatusGetCTprior()
braid_Int braid_CoarsenRefStatusGetCTprior (
braid_CoarsenRefStatus s,
braid_Real * v1 )
14.10.2.17 braid_CoarsenRefStatusGetCTstop()

```
braid_Int braid_CoarsenRefStatusGetCTstop
    braid_CoarsenRefStatus s,
    braid_Real * v1 )
```

14.10.2.18 braid_CoarsenRefStatusGetFTprior()
braid_Int braid_CoarsenRefStatusGetFTprior
braid_CoarsenRefStatus s, braid_Real * v1 )
14.10.2.19 braid_CoarsenRefStatusGetFTstop()
braid_Int braid_CoarsenRefStatusGetFTstop (
braid_CoarsenRefStatus s, braid_Real * v1 )
14.10.2.20 braid_CoarsenRefStatusGetlter()
braid_Int braid_CoarsenRefStatusGetIter (
braid_CoarsenRefStatus s,
braid_Int * vI )
14.10.2.21 braid_CoarsenRefStatusGetLevel()

```
braid Int braid CoarsenRefStatusGetLevel (
    braid_CoarsenRefStatus s,
    braid_Int * v1 )
```

14.10.2.22 braid_CoarsenRefStatusGetNLevels()
braid_Int braid_CoarsenRefStatusGetNLevels (
braid_CoarsenRefStatus s, braid_Int * v1 )
14.10.2.23 braid_CoarsenRefStatusGetNRefine()
braid_Int braid_CoarsenRefStatusGetNRefine (
braid_CoarsenRefStatus s, braid_Int * v1 )
14.10.2.24 braid_CoarsenRefStatusGetNTPoints()
braid_Int braid_CoarsenRefStatusGetNTPoints (
braid_CoarsenRefStatus s,
braid_Int * v1 )
14.10.2.25 braid_CoarsenRefStatusGetT()
braid_Int braid_CoarsenRefStatusGetT (
braid_CoarsenRefStatus s, braid_Real * v1 )
14.10.2.26 braid_CoarsenRefStatusGetTIndex()
braid_Int braid_CoarsenRefStatusGetTIndex (

$$
\text { braid_CoarsenRefStatus } s \text {, }
$$

braid_Int * v1 )
14.10.2.27 braid_CoarsenRefStatusGetTpriorTstop()
braid_Int braid_CoarsenRefStatusGetTpriorTstop ( braid_CoarsenRefStatus $s$, braid_Real * v1, braid_Real * v2, braid_Real * v3, braid_Real * v4, braid_Real * v5 )
14.10.2.28 braid_ObjectiveStatusGetlter()
braid_Int braid_ObjectiveStatusGetIter ( braid_ObjectiveStatus $s$, braid_Int * v1 )
14.10.2.29 braid_ObjectiveStatusGetLevel()
braid_Int braid_ObjectiveStatusGetLevel (
braid_ObjectiveStatus s, braid_Int * v1 )
14.10.2.30 braid_ObjectiveStatusGetNLevels()
braid_Int braid_ObjectiveStatusGetNLevels (
braid_ObjectiveStatus $s$, braid_Int * v1 )
14.10.2.31 braid_ObjectiveStatusGetNRefine()
braid_Int braid_ObjectiveStatusGetNRefine (
braid_ObjectiveStatus $s$,
braid_Int * v1 )
14.10.2.32 braid_ObjectiveStatusGetNTPoints()
braid_Int braid_ObjectiveStatusGetNTPoints (
braid_ObjectiveStatus $s$,
braid_Int * v1 )
14.10.2.33 braid_ObjectiveStatusGetT()

```
braid_Int braid_ObjectiveStatusGetT (
    braid_ObjectiveStatus s,
    braid_Real * v1 )
```

14.10.2.34 braid_ObjectiveStatusGetTIndex()
braid_Int braid_ObjectiveStatusGetTIndex (
braid_ObjectiveStatus s,
braid_Int * v1 )
14.10.2.35 braid_ObjectiveStatusGetTol()
braid_Int braid_ObjectiveStatusGetTol ( braid_ObjectiveStatus s, braid_Real * v1 )
14.10.2.36 braid_StepStatusGetlter()
braid_Int braid_StepStatusGetIter (
braid_StepStatus s, braid_Int * v1 )
14.10.2.37 braid_StepStatusGetLevel()
braid_Int braid_StepStatusGetLevel (

```
    braid_StepStatus s,
```

    braid_Int * v1 )
    14.10.2.38 braid_StepStatusGetNLevels()
braid_Int braid_StepStatusGetNLevels (

> braid_StepStatus s, braid_Int * v1 )
14.10.2.39 braid_StepStatusGetNRefine()

```
braid Int braid StepStatusGetNRefine (
    braid_StepStatus s,
    braid_Int * v1 )
```

14.10.2.40 braid_StepStatusGetNTPoints()

```
braid_Int braid_StepStatusGetNTPoints (
```

    braid_StepStatus s,
    braid_Int * v1 )
    14.10.2.41 braid_StepStatusGetOldFineTolx()

```
braid_Int braid_StepStatusGetOldFineTolx (
    braid_StepStatus s,
    braid_Real * v1 )
```

14.10.2.42 braid_StepStatusGetRNorms()
braid_Int braid_StepStatusGetRNorms (

```
    braid_StepStatus s,
    braid_Int * v1,
    braid_Real * v2 )
```

14.10.2.43 braid_StepStatusGetSingleErrorEstStep()

```
braid_Int braid_StepStatusGetSingleErrorEstStep (
    braid_StepStatus s,
    braid_Real * v1 )
```

14.10.2.44 braid_StepStatusGetT()

```
braid_Int braid_StepStatusGetT (
    braid_StepStatus s,
    braid_Real * v1 )
```

14.10.2.45 braid_StepStatusGetTIndex()

```
braid_Int braid_StepStatusGetTIndex (
    braid_StepStatus s,
    braid_Int * v1 )
```

14.10.2.46 braid_StepStatusGetTol()

```
braid_Int braid_StepStatusGetTol (
```

    braid_StepStatus s,
    braid_Real * v1 )
    14.10.2.47 braid_StepStatusGetTstartTstop()

```
braid_Int braid_StepStatusGetTstartTstop (
    braid_StepStatus s,
    braid_Real * vI,
    braid_Real * v2 )
```

14.10.2.48 braid_StepStatusGetTstop()
braid_Int braid_StepStatusGetTstop (
braid_Stepstatus s,
braid_Real * v1 )
14.10.2.49 braid_StepStatusSetOIdFineTolx()
braid_Int braid_StepStatusSetOldFineTolx (
braid_Stepstatus $s$, braid_Real v1 )
14.10.2.50 braid_StepStatusSetRFactor()
braid_Int braid_StepStatusSetRFactor (
braid_Stepstatus s,
braid_Real v1 )
14.10.2.51 braid_StepStatusSetRSpace()

```
braid_Int braid_StepStatusSetRSpace (
    braid_StepStatus s,
    braid_Real v1 )
```

14.10.2.52 braid_StepStatusSetTightFineTolx()

```
braid_Int braid_StepStatusSetTightFineTolx
    braid_StepStatus s,
    braid_Real v1 )
```

14.10.2.53 braid_SyncStatusGetAIIErrorEst()
braid_Int braid_SyncStatusGetAllErrorEst (
braid_SyncStatus s, braid_Real * v1
14.10.2.54 braid_SyncStatusGetCallingFunction()
braid_Int braid_SyncStatusGetCallingFunction (
braid_SyncStatus s, braid_Int * v1 )
14.10.2.55 braid_SyncStatusGetDone()
braid_Int braid_SyncStatusGetDone

> braid_SyncStatus s,
braid_Int * v1
14.10.2.56 braid_SyncStatusGetlter()
braid_Int braid_SyncStatusGetIter (

> braid_SyncStatus s, braid_Int * v1 )

```
14.10.2.57 braid_SyncStatusGetLevel()
```

```
braid_Int braid_SyncStatusGetLevel (
```

braid_Int braid_SyncStatusGetLevel (
braid_SyncStatus s,
braid_SyncStatus s,
braid_Int * v1 )

```
    braid_Int * v1 )
```

14.10.2.58 braid_SyncStatusGetNLevels()
braid_Int braid_SyncStatusGetNLevels (
braid_SyncStatus s,
braid_Int * v1 )
14.10.2.59 braid_SyncStatusGetNRefine()
braid_Int braid_SyncStatusGetNRefine (
braid_SyncStatus s,
braid_Int * v1 )
14.10.2.60 braid_SyncStatusGetNTPoints()
braid_Int braid_SyncStatusGetNTPoints (
braid_SyncStatus s, braid_Int * v1 )
14.10.2.61 braid_SyncStatusGetNumErrorEst()
braid_Int braid_SyncStatusGetNumErrorEst (
braid_SyncStatus s,
braid_Int * v1 )
14.10.2.62 braid_SyncStatusGetTimeValues()
braid_Int braid_SyncStatusGetTimeValues (
braid_SyncStatus s,
braid_Real ** v1,
braid_Int v2,
braid_Int v3,
braid_Int v4 )
14.10.2.63 braid_SyncStatusGetTIUL()
braid_Int braid_SyncStatusGetTIUL (
braid_SyncStatus s,
braid_Int * v1,
braid_Int * v2,
braid_Int v3 )

### 14.11 XBraid status macros

## Macros

- \#define braid_ASCaller_FInterp 0
- \#define braid_ASCaller_FRestrict 1
- \#define braid_ASCaller_FRefine 2
- \#define braid_ASCaller_FAccess 3
- \#define braid_ASCaller_FRefine_AfterlnitHier 4
- \#define braid_ASCaller_Drive_TopCycle 5


### 14.11.1 Detailed Description

Macros defining Status values that the user can obtain during runtime, which will tell the user where in Braid the current cycle is, e.g. in the FInterp function.
14.11.2 Macro Definition Documentation
14.11.2.1 braid_ASCaller_Drive_TopCycle
\#define braid_ASCaller_Drive_TopCycle 5
When CallingFunction equals 5, Braid is at the top of the cycle
14.11.2.2 braid_ASCaller_FAccess
\#define braid_ASCaller_FAccess 3
When CallingFunction equals 0 , Braid is in FAccess
14.11.2.3 braid_ASCaller_FInterp
\#define braid_ASCaller_FInterp 0
When CallingFunction equals 0, Braid is in FInterp
14.11.2.4 braid_ASCaller_FRefine
\#define braid_ASCaller_FRefine 2
When CallingFunction equals 0, Braid is in FRefine
14.11.2.5 braid_ASCaller_FRefine_AfterInitHier
\#define braid_ASCaller_FRefine_AfterInitHier 4
When CallingFunction equals 4, Braid is inside FRefine after the new finest level has been initialized
14.11.2.6 braid_ASCaller_FRestrict
\#define braid_ASCaller_FRestrict 1
When CallingFunction equals 0 , Braid is in FRestrict

### 14.12 XBraid test routines

## Functions

- braid_Int braid_TestlnitAccess (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_PtFcnInit init, braid_PtFcnAccess access, braid_PtFcnFree free)
- braid_Int braid_TestClone (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_PtFcnInit init, braid_PtFcnAccess access, braid_PtFcnFree free, braid_PtFcnClone clone)
- braid_Int braid_TestSum (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_PtFcnInit init, braid_PtFcnAccess access, braid_PtFcnFree free, braid_PtFcnClone clone, braid_PtFcnSum sum)
- braid_Int braid_TestSpatialNorm (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_PtFcnlnit init, braid_PtFcnFree free, braid_PtFcnClone clone, braid_PtFcnSum sum, braid_PtFcnSpatialNorm spatialnorm)
- braid_Int braid_TestBuf (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_PtFcnInit init, braid_PtFcnFree free, braid_PtFcnSum sum, braid_PtFcnSpatialNorm spatialnorm, braid_PtFcnBufSize bufsize, braid_PtFcnBufPack bufpack, braid_PtFcnBufUnpack bufunpack)
- braid_Int braid_TestCoarsenRefine (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_Real fdt, braid_Real cdt, braid_PtFcnInit init, braid_PtFcnAccess access, braid_PtFcnFree free, braid_PtFcnClone clone, braid_PtFcnSum sum, braid_PtFcnSpatialNorm spatialnorm, braid_PtFcnSCoarsen coarsen, braid_Pt $\hookleftarrow$ FcnSRefine refine)
- braid_Int braid_TestResidual (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_Real dt, braid $\hookleftarrow$ $\_$PtFcnInit myinit, braid_PtFcnAccess myaccess, braid_PtFcnFree myfree, braid_PtFcnClone clone, braid_Pt $\hookleftarrow$ FcnSum sum, braid_PtFcnSpatialNorm spatialnorm, braid_PtFcnResidual residual, braid_PtFcnStep step)
- braid_Int braid_TestAll (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_Real fdt, braid_ $\hookleftarrow$ Real cdt, braid_PtFcnInit init, braid_PtFcnFree free, braid_PtFcnClone clone, braid_PtFcnSum sum, braid_Pt $\hookleftarrow$ FcnSpatialNorm spatialnorm, braid_PtFcnBufSize bufsize, braid_PtFcnBufPack bufpack, braid_PtFcnBufUnpack bufunpack, braid_PtFcnSCoarsen coarsen, braid_PtFcnSRefine refine, braid_PtFcnResidual residual, braid_ $\leftarrow$ PtFcnStep step)


### 14.12.1 Detailed Description

These are sanity check routines to help a user test their XBraid code.
14.12.2 Function Documentation
14.12.2.1 braid_TestAll()

```
braid_Int braid_TestAll (
    braid_App app,
    MPI_Comm comm_x,
    FILE * fp,
    braid_Real t,
    braid_Real fdt,
    braid_Real cdt,
    braid_PtFcnInit init,
    braid_PtFcnFree free,
    braid_PtFcnClone clone,
    braid_PtFcnSum sum,
```

```
braid_PtFcnSpatialNorm spatialnorm,
braid_PtFcnBufSize bufsize,
braid_PtFcnBufPack bufpack,
braid_PtFcnBufUnpack bufunpack,
braid_PtFcnSCoarsen coarsen,
braid_PtFcnSRefine refine,
braid_PtFcnResidual residual,
braid_PtFcnStep step )
```

Runs all of the individual braid_Test* routines

- Returns 0 if the tests fail
- Returns 1 if the tests pass
- Check the log messages to see details of which tests failed.

Parameters

| app | User defined App structure |
| :--- | :--- |
| comm_x | Spatial communicator |
| $f p$ | File pointer (could be stdout or stderr) for log messages |
| $t$ | Time value to initialize test vectors with |
| fdt | Fine time step value that you spatially coarsen from |
| cdt | Coarse time step value that you coarsen to |
| init | Initialize a braid_Vector on finest temporal grid |
| free | Free a braid_Vector |
| clone | Clone a braid_Vector |
| sum | Compute vector sum of two braid_Vectors |
| spatialnorm | Compute norm of a braid_Vector, this is a norm only over space |
| bufsize | Computes size in bytes for one braid_Vector MPI buffer |
| bufpack | Packs MPI buffer to contain one braid_Vector |
| bufunpack | Unpacks MPI buffer into a braid_Vector |
| coarsen | Spatially coarsen a vector. If NULL, test is skipped. |
| refine | Spatially refine a vector. If NULL, test is skipped. |
| residual | Compute a residual given two consectuive braid_Vectors |
| step | Compute a time step with a braid_Vector |

14.12.2.2 braid_TestBuf()

```
braid_Int braid_TestBuf (
    braid_App app,
    MPI_Comm comm_x,
    FILE * fp,
    braid_Real t,
    braid_PtFcnInit init,
```

```
braid_PtFcnFree free,
braid_PtFcnSum sum,
braid_PtFcnSpatialNorm spatialnorm,
braid_PtFcnBufSize bufsize,
braid_PtFcnBufPack bufpack,
braid_PtFcnBufUnpack bufunpack )
```

Test the BufPack, BufUnpack and BufSize functions.
A vector is initialized at time $t$, packed into a buffer, then unpacked from a buffer. The unpacked result must equal the original vector.

- Returns 0 if the tests fail
- Returns 1 if the tests pass
- Check the log messages to see details of which tests failed.

Parameters

| app | User defined App structure |
| :--- | :--- |
| comm_x | Spatial communicator |
| $f p$ | File pointer (could be stdout or stderr) for log messages |
| $t$ | Time value to test Buffer routines (used to initialize the vectors) |
| init | Initialize a braid_Vector on finest temporal grid |
| free | Free a braid_Vector |
| sum | Compute vector sum of two braid_Vectors |
| spatialnorm | Compute norm of a braid_Vector, this is a norm only over space |
| bufsize | Computes size in bytes for one braid_Vector MPI buffer |
| bufpack | Packs MPI buffer to contain one braid_Vector |
| bufunpack | Unpacks MPI buffer containing one braid_Vector |

14.12.2.3 braid_TestClone()

```
braid_Int braid_TestClone (
    braid_App app,
    MPI_Comm comm_x,
    FILE * fp,
    braid_Real t,
    braid_PtFcnInit init,
    braid_PtFcnAccess access,
    braid_PtFcnFree free,
    braid_PtFcnClone clone )
```

Test the clone function.
A vector is initialized at time $t$, cloned, and both vectors are written. Then both vectors are free-d. The user is to check (via the access function) to see if it is identical.

## Parameters

| app | User defined App structure |
| :--- | :--- |
| comm <br> $\_$ <br> $x$ | Spatial communicator |
| $f p$ | File pointer (could be stdout or stderr) for log messages |
| $t$ | Time value to test clone with (used to initialize the vectors) |
| init | Initialize a braid_Vector on finest temporal grid |
| access | Allows access to XBraid and current braid_Vector (can be NULL for no writing) |
| free | Free a braid_Vector |
| clone | Clone a braid_Vector |

### 14.12.2.4 braid_TestCoarsenRefine()

```
braid_Int braid_TestCoarsenRefine (
    braid_App app,
    MPI_Comm comm_x,
    FILE * fp,
    braid_Real t,
    braid_Real fdt,
    braid_Real cdt,
    braid_PtFcnInit init,
    braid_PtFcnAccess access,
    braid PtFcnFree free,
    braid PtFcnClone clone,
    braid PtFcnSum sum,
    braid_PtFcnSpatialNorm spatialnorm,
    braid_PtFcnSCoarsen coarsen,
    braid_PtFcnSRefine refine )
```

Test the Coarsen and Refine functions.
A vector is initialized at time $t$, and various sanity checks on the spatial coarsening and refinement routines are run.

- Returns 0 if the tests fail
- Returns 1 if the tests pass
- Check the log messages to see details of which tests failed.


## Parameters

| app | User defined App structure |
| :--- | :--- |
| comm_ $x$ | Spatial communicator |
| $f p$ | File pointer (could be stdout or stderr) for log messages |
| $t$ | Time value to initialize test vectors |
| $f d t$ | Fine time step value that you spatially coarsen from |
| $c d t$ | Coarse time step value that you coarsen to |

## Parameters

| init | Initialize a braid_Vector on finest temporal grid |
| :--- | :--- |
| access | Allows access to XBraid and current braid_Vector (can be NULL for no writing) |
| free | Free a braid_Vector |
| clone | Clone a braid_Vector |
| sum | Compute vector sum of two braid_Vectors |
| spatialnorm | Compute norm of a braid_Vector, this is a norm only over space |
| coarsen | Spatially coarsen a vector |
| refine | Spatially refine a vector |

### 14.12.2.5 braid_TestlnitAccess()

```
braid_Int braid_TestInitAccess (
    braid_App app,
    MPI_Comm comm_x,
    FILE * fp,
    braid_Real t,
    braid_PtFcnInit init,
    braid_PtFcnAccess access,
    braid_PtFcnFree free )
```

Test the init, access and free functions.
A vector is initialized at time $t$, written, and then free-d
Parameters

| app | User defined App structure |
| :--- | :--- |
| comm <br> $\_$ | Spatial communicator |
| $f p$ | File pointer (could be stdout or stderr) for log messages |
| $t$ | Time value to test init with (used to initialize the vectors) |
| init | Initialize a braid_Vector on finest temporal grid |
| access | Allows access to XBraid and current braid_Vector (can be NULL for no writing) |
| free | Free a braid_Vector |

14.12.2.6 braid_TestResidual()

```
braid_Int braid_TestResidual (
    braid_App app,
    MPI_Comm comm_x,
    FILE * fp,
    braid_Real t,
```

```
braid_Real dt,
braid_PtFcnInit myinit,
braid_PtFcnAccess myaccess,
braid_PtFcnFree myfree,
braid_PtFcnClone clone,
braid_PtFcnSum sum,
braid_PtFcnSpatialNorm spatialnorm,
braid_PtFcnResidual residual,
braid_PtFcnStep step )
```

Test compatibility of the Step and Residual functions.
A vector is initialized at time $t$, step is called with $d t$, followed by an evaluation of residual, to test the condition fstop residual( step(u, fstop), u) approx. 0

- Check the log messages to determine if test passed. The result should approximately be zero. The more accurate the solution for $u$ is computed in step, the closer the result will be to 0 .
- The residual is also written to file


## Parameters

| app | User defined App structure |
| :--- | :--- |
| comm_x | Spatial communicator |
| $f p$ | File pointer (could be stdout or stderr) for log messages |
| $t$ | Time value to initialize test vectors |
| $d t$ | Time step value to use in step |
| $m y i n i t$ | Initialize a braid_Vector on finest temporal grid |
| myaccess | Allows access to XBraid and current braid_Vector (can be NULL for no writing) |
| myfree | Free a braid_Vector |
| clone | Clone a braid_Vector |
| sum | Compute vector sum of two braid_Vectors |
| spatialnorm | Compute norm of a braid_Vector, this is a norm only over space |
| residual | Compute a residual given two consectuive braid_Vectors |
| step | Compute a time step with a braid_Vector |

14.12.2.7 braid_TestSpatialNorm()

```
braid_Int braid_TestSpatialNorm (
    braid_App app,
    MPI_Comm comm_x,
    FILE * fp,
    braid_Real t,
    braid_PtFcnInit init,
    braid_PtFcnFree free,
    braid_PtFcnClone clone,
    braid_PtFcnSum sum,
    braid_PtFcnSpatialNorm spatialnorm )
```

Test the spatialnorm function.
A vector is initialized at time $t$ and then cloned. Various norm evaluations like $\|3 \mathrm{v}\| /\|\mathrm{v}\|$ with known output are then done.

- Returns 0 if the tests fail
- Returns 1 if the tests pass
- Check the log messages to see details of which tests failed.


## Parameters

| app | User defined App structure |
| :--- | :--- |
| comm_x | Spatial communicator |
| $f p$ | File pointer (could be stdout or stderr) for log messages |
| $t$ | Time value to test SpatialNorm with (used to initialize the vectors) |
| init | Initialize a braid_Vector on finest temporal grid |
| free | Free a braid_Vector |
| clone | Clone a braid_Vector |
| sum | Compute vector sum of two braid_Vectors |
| spatialnorm | Compute norm of a braid_Vector, this is a norm only over space |

14.12.2.8 braid_TestSum()

```
braid_Int braid_TestSum (
    braid_App app,
    MPI_Comm comm_x,
    FILE * fp,
    braid_Real t,
    braid_PtFcnInit init,
    braid_PtFcnAccess access,
    braid_PtFcnFree free,
    braid_PtFcnClone clone,
    braid_PtFcnSum sum )
```

Test the sum function.
A vector is initialized at time $t$, cloned, and then these two vectors are summed a few times, with the results written. The vectors are then free-d. The user is to check (via the access function) that the output matches the sum of the two original vectors.

Parameters

| app | User defined App structure |
| :--- | :--- |
| comm <br> $\_$ | Spatial communicator |
| $f p$ | File pointer (could be stdout or stderr) for log messages |
| $t$ | Time value to test Sum with (used to initialize the vectors) |

## Parameters

| init | Initialize a braid_Vector on finest temporal grid |
| :--- | :--- |
| access | Allows access to XBraid and current braid_Vector (can be NULL for no writing) |
| free | Free a braid_Vector |
| clone | Clone a braid_Vector |
| sum | Compute vector sum of two braid_Vectors |

## 15 Data Structure Documentation

## 15.1 _braid_Action Struct Reference

Data Fields

- _braid_Call braidCall
- braid_Core core
- braid_Real inTime
- braid_Real outTime
- braid_Int inTimeldx
- braid_Real sum_alpha
- braid_Real sum_beta
- braid_Int send_recv_rank
- braid_Int braid_iter
- braid_Int myid
- braid_Int level
- braid_Int nrefine
- braid_Int gupper
- braid_Real tol
- braid_Int messagetype
- braid_Int size_buffer
15.1.1 Detailed Description

XBraid Action structure

Holds information for the called user routines

### 15.1.2 Field Documentation

15.1.2.1 braid_iter

```
braid_Int braid_iter
```

iteration number of xBraid
15.1.2.2 braidCall
_braid_Call braidCall
type of the user routine
15.1.2.3 core
braid_Core core
pointer to braid's core structure
15.1.2.4 gupper
braid_Int gupper
global size of the fine grid
15.1.2.5 inTime
braid_Real inTime
time of the input vector
15.1.2.6 inTimeldx
braid_Int inTimeIdx
index of time of input vector
15.1.2.7 level
braid_Int level
current level in Braid
15.1.2.8 messagetype
braid_Int messagetype
message type, 0: for Step(), 1: for load balancing
15.1.2.9 myid
braid_Int myid
processors id
15.1.2.10 nrefine
braid_Int nrefine
number of refinements done
15.1.2.11 outTime
braid_Real outTime
time of the output vector
15.1.2.12 send_recv_rank
braid_Int send_recv_rank
processor rank of sender / receiver in my_bufpack / my_bufunpack
15.1.2.13 size_buffer
braid_Int size_buffer
if set by user, size of send buffer is "size" bytes
15.1.2.14 sum_alpha
braid_Real sum_alpha
first coefficient of my_sum
15.1.2.15 sum_beta
braid_Real sum_beta
second coefficient of my_sum
15.1.2.16 tol
braid_Real tol
primal stopping tolerance
The documentation for this struct was generated from the following file:

- tape.h


## 15.2 _braid_CommHandle Struct Reference

Data Fields

- braid_Int request_type
- braid_Int num_requests
- MPI_Request $*$ requests
- MPI_Status * status
- void $*$ buffer
- braid_BaseVector $*$ vector_ptr


### 15.2.1 Detailed Description

XBraid comm handle structure

Used for initiating and completing nonblocking communication to pass braid_BaseVectors between processors.
15.2.2 Field Documentation
15.2.2.1 buffer
void* buffer

Buffer for message
15.2.2.2 num_requests
braid_Int num_requests
number of active requests for this handle, usually 1
15.2.2.3 request_type
braid_Int request_type
two values: recv type $=1$, and send type $=0$
15.2.2.4 requests

MP I_Request* requests

MPI request structure
15.2.2.5 status

MPI_Status* status

MPI status
15.2.2.6 vector_ptr
braid_BaseVector* vector_ptr
braid_vector being sent/received
The documentation for this struct was generated from the following file:

- _braid.h


## 15.3 _braid_Core Struct Reference

Data Fields

- MPI_Comm comm_world
- MPI_Comm comm
- braid_Int myid_world
- braid_Int myid
- braid_Real tstart
- braid_Real tstop
- braid_Int ntime
- braid_App app
- braid_PtFcnStep step
- braid_PtFcnlnit init
- braid_PtFcnSInit sinit
- braid_PtFcnClone clone
- braid_PtFcnSClone sclone
- braid_PtFcnFree free
- braid_PtFcnSFree sfree
- braid_PtFcnSum sum
- braid_PtFenSpatialNorm spatialnorm
- braid_PtFcnAccess access
- braid_PtFcnBufSize bufsize
- braid_PtFcnBufPack bufpack
- braid_PtFcnBufUnpack bufunpack
- braid_PtFcnResidual residual
- braid_PtFcnSCoarsen scoarsen
- braid_PtFcnSRefine srefine
- braid_PtFcnSync sync
- braid_PtFcnTimeGrid tgrid
- braid_Int periodic
- braid_Int initiali
- braid_Int access_level
- braid_Int print_level
- braid_Int io_level
- braid_Int seq_soln
- braid_Int max_levels
- braid_Int incr_max_levels
- braid_Int min_coarse
- braid_Real tol
- braid_Int rtol
- braid_Int * nrels
- braid_Int nrdefault
- braid_Real * CWts
- braid_Real CWt_default
- braid_Int * cfactors
- braid_Int cfdefault
- braid_Int max_iter
- braid_Int niter
- braid_Int fmg
- braid_Int nfmg
- braid_Int nfmg_Vcyc
- braid_Int warm_restart
- braid_Int tnorm
- braid_Real * tnorm_a
- braid_Real rnorm0
- braid_Real * rnorms
- braid_PtFcnResidual full_rnorm_res
- braid_Real full_rnorm0
- braid_Real * full_rnorms
- braid_Int storage
- braid_Int useshell
- braid_Int gupper
- braid_Int refine
- braid_Int $*$ rfactors
- braid_Real ** rdtvalues
- braid_Int r_space
- braid_Int rstopped
- braid_Int nrefine
- braid_Int max_refinements
- braid_Int tpoints_cutoff
- braid_Int skip
- braid_Int nlevels
- _braid_Grid ** grids
- braid_Real localtime
- braid_Real globaltime
- braid_Int richardson
- braid_Int est_error
- braid_Int order
- braid_Real * dtk
- braid_Real * estimate
- braid_Optim optim
- braid_Int adjoint
- braid_Int record
- braid_Int obj_only
- braid_Int verbose_adj
- _braid_Tape $*$ actionTape
- _braid_Tape $*$ userVectorTape
- _braid_Tape $*$ barTape
- braid_PtFcnObjectiveT objectiveT
- braid_PtFcnStepDiff step_diff
- braid_PtFcnObjectiveTDiff objT_diff
- braid_PtFcnResetGradient reset_gradient
- braid_PtFcnPostprocessObjective postprocess_obj
- braid_PtFcnPostprocessObjective_diff postprocess_obj_diff
- braid_Real t
- braid_Int idx
- braid_Int level
- braid_Real rnorm
- braid_Int done
- braid_Int wrapper_test
- braid_Int calling_function
- braid_Real f_tprior
- braid_Real f_tstop
- braid_Real c_tprior
- braid_Real c_tstop
- braid_Real tnext
- braid_Real old_fine_tolx
- braid_Int tight_fine_tolx
- braid_Int rfactor
- braid_Int messagetype
- braid Int size buffer
- braid_Int send_recv_rank
15.3.1 Detailed Description

The typedef _braid_Core struct is a critical part of XBraid and is passed to each routine in XBraid. It thus allows each routine access to XBraid attributes.
15.3.2 Field Documentation
15.3.2. access
braid_PtFcnAccess access
user access function to XBraid and current vector
15.3.2.2 access_level
braid_Int access_level
determines how often to call the user's access routine
15.3.2.3 actionTape
_braid_Tape* actionTape
tape storing the actions while recording
15.3.2.4 adjoint
braid_Int adjoint
determines if adjoint run is performed (1) or not (0)
15.3.2.5 app
braid_App app
application data for the user
15.3.2.6 barTape
_braid_Tape* barTape
tape storing intermediate AD-bar variables while recording
15.3.2.7 bufpack
braid_PtFcnBufPack bufpack
pack a buffer
15.3.2.8 bufsize
braid_PtFcnBufSize bufsize
return buffer size
15.3.2.9 bufunpack
braid_PtFcnBufUnpack bufunpack
unpack a buffer
15.3.2.10 c_tprior
braid_Real c_tprior
time value to the left of tstart on coarse grid
15.3.2.11 c_tstop
braid_Real c_tstop
time value to the right of tstart on coarse grid
15.3.2.12 calling_function
braid_Int calling_function
from which function are we accessing the vector
15.3.2.13 cfactors
braid_Int* cfactors
coarsening factors
15.3.2.14 cfdefault
braid_Int cfdefault
default coarsening factor
15.3.2.15 clone
braid_PtFcnClone clone
clone a vector
15.3.2.16 comm

MP I_Comm comm
communicator for the time dimension
15.3.2.17 comm_world

MPI_Comm comm_world
15.3.2.18 CWt default
braid_Real CWt_default
default C-relaxtion weight
15.3.2.19 CWts
braid_Real* CWts

C-relaxation weight for each level
15.3.2.20 done
braid_Int done
boolean describing whether XBraid has finished
15.3.2.21 dtk
braid_Real* dtk
holds value of sum_\{i\} dt_i^k for each C-interval
15.3.2.22 est_error
braid_Int est_error
turns on embedded error estimation, e.g., for refinement
15.3.2.23 estimate
braid_Real* estimate
holds value of the error estimate at each fine grid point
15.3.2.24 f_tprior
braid_Real f_tprior

CoarsenRefStatus properties time value to the left of tstart on fine grid
15.3.2.25 f_tstop
braid_Real f_tstop
time value to the right of tstart on fine grid
15.3.2.26 fmg
braid_Int fmg
use FMG cycle
15.3.2.27 free
braid_PtFcnFree free
free up a vector
15.3.2.28 full_rnorm0
braid_Real full_rnorm0
(optional) initial full residual norm
15.3.2.29 full rnorm res
braid_PtFcnResidual full_rnorm_res
(optional) used to compute full residual norm
15.3.2.30 full_rnorms
braid_Real* full_rnorms
(optional) full residual norm history
15.3.2.31 globaltime
braid_Real globaltime
global wall time for braid_Drive()
15.3.2.32 grids
_braid_Grid** grids
pointer to temporal grid structures for each level
15.3.2.33 gupper
braid_Int gupper
global size of the fine grid
15.3.2.34 idx
braid_Int idx
time point index value corresponding to $t$ on the global time grid
15.3.2.35 incr_max_levels
braid_Int incr_max_levels

After doing refinement, increase the max number of levels by 1 ( $0=$ false, $1=$ true )
15.3.2.36 init
braid_PtFcnInit init
return an initialized braid_BaseVector
15.3.2.37 initiali
braid_Int initiali
initial condition grid index ( 0 : default; -1 : periodic )
15.3.2.38 io_level
braid_Int io_level
determines amount of output printed to files $(0,1)$
15.3.2.39 level
braid_Int level
current level in XBraid
15.3.2.40 localtime
braid_Real localtime
local wall time for braid_Drive()
15.3.2.41 max_iter
braid_Int max_iter
maximum number of multigrid in time iterations
15.3.2.42 max_levels
braid_Int max_levels
maximum number of temporal grid levels
15.3.2.43 max_refinements
braid_Int max_refinements
maximum number of refinements
15.3.2.44 messagetype
braid_Int messagetype

BufferStatus properties message type, 0: for Step(), 1: for load balancing
15.3.2.45 min_coarse
braid_Int min_coarse
minimum possible coarse grid size
15.3.2.46 myid
braid_Int myid
my rank in the time communicator
15.3.2.47 myid_world
braid_Int myid_world
my rank in the world communicator
15.3.2.48 nfmg
braid_Int nfmg
number of fmg cycles to do initially before switching to V-cycles
15.3.2.49 nfmg_Vcyc
braid_Int nfmg_Vcyc
number of V -cycle calls at each level in FMG
15.3.2.50 niter
braid_Int niter
number of iterations
15.3.2.51 nlevels
braid_Int nlevels
number of temporal grid levels
15.3.2.52 nrdefault
braid_Int nrdefault
default number of pre-relaxations
15.3.2.53 nrefine
braid_Int nrefine
number of refinements done
15.3.2.54 nrels
braid_Int* nrels
number of pre-relaxations on each level
15.3.2.55 ntime
braid_Int ntime
initial number of time intervals
15.3.2.56 obj_only
braid_Int obj_only
determines if adjoint code computes ONLY objective, no gradients.
15.3.2.57 objectiveT
braid_PtFcnObjectiveT objectiveT

User function: evaluate objective function at time $t$
15.3.2.58 objT_diff
braid_PtFcnObjectiveTDiff objT_diff

User function: apply differentiated objective function
15.3.2.59 old_fine_tolx
braid_Real old_fine_tolx

Allows for storing the previously used fine tolerance from GetSpatialAccuracy
15.3.2.60 optim
braid_Optim optim
structure that stores optimization variables (objective function, etc.)
15.3.2.61 order
braid_Int order
local order of time integration scheme
15.3.2.62 periodic
braid_Int periodic
determines if periodic
15.3.2.63 postprocess_obj
braid_PtFcnPostprocessobjective postprocess_obj
Optional user function: Modify the time-averaged objective function, e.g. for inverse design problems, adding relaxation term etc.
15.3.2.64 postprocess_obj_diff
braid_PtFcnPostprocessobjective_diff postprocess_obj_diff
Optional user function: Derivative of postprocessing function
15.3.2.65 print_level
braid_Int print_level
determines amount of output printed to screen ( $0,1,2,3$ )
15.3.2.66 r_space
braid_Int r_space
spatial refinement flag
15.3.2.67 rdtvalues
braid_Real** rdtvalues
Array of pointers to arrays of dt values for non-uniform refinement
15.3.2.68 record
braid_Int record
determines if actions are recorded to the tape or not. This separate flag from adjoint is needed, because the final FAccess call should not be recorded unless nlevels $==1$, but the adjoint flag must be true even if nlevels==1.
15.3.2.69 refine
braid_Int refine
refine in time $($ refine $=1$ )
15.3.2.70 reset_gradient
braid_PtFcnResetGradient reset_gradient

User function: Set the gradient to zero. Is called before each iteration
15.3.2.71 residual
braid_PtFcnResidual residual
(optional) compute residual
15.3.2.72 rfactor
braid_Int rfactor
if set by user, allows for subdivision of this interval for better time accuracy
15.3.2.73 rfactors
braid_Int* rfactors
refinement factors for finest grid (if any)
15.3.2.74 richardson
braid_Int richardson

Richardson-based error estimation and refinement turns on Richardson extrapolation for accuracy
15.3.2.75 rnorm
braid_Real rnorm

AccessStatus properties residual norm
15.3.2.76 rnorm0
braid_Real rnorm0
initial residual norm
15.3.2.77 rnorms
braid_Real* rnorms
residual norm history
15.3.2.78 rstopped
braid_Int rstopped
refinement stopped at iteration rstopped
15.3.2.79 rtol
braid_Int rtol
use relative tolerance
15.3.2.80 sclone
braid_PtFcnSClone sclone
(optional) clone the shell of a vector
15.3.2.81 scoarsen
braid_PtFcnSCoarsen scoarsen
(optional) return a spatially coarsened vector
15.3.2.82 send_recv_rank
braid_Int send_recv_rank
15.3.2.83 seq_soln
braid_Int seq_soln
boolean, controls if the initial guess is from sequential time stepping
15.3.2.84 sfree
braid_PtFcnSFree sfree
(optional) free up the data of a vector, keep the shell
15.3.2.85 sinit
braid_PtFcnSInit sinit
(optional) return an initialized shell of braid_BaseVector
15.3.2.86 size_buffer
braid_Int size_buffer
if set by user, send buffer will be "size" bytes in length
15.3.2.87 skip
braid_Int skip
boolean, controls skipping any work on first down-cycle
15.3.2.88 spatialnorm
braid_PtFcnSpatialNorm spatialnorm

Compute norm of a braid_BaseVector, this is a norm only over space
15.3.2.89 srefine
braid_PtFcnSRefine srefine
(optional) return a spatially refined vector
15.3.2.90 step
braid_PtFcnStep step
apply step function
15.3.2.91 step_diff
braid_PtFcnStepDiff step_diff

User function: apply differentiated step function
15.3.2.92 storage
braid_Int storage
storage $=0$ (C-points), $=1$ (all)
15.3.2.93 sum
braid_PtFcnSum sum
vector sum
15.3.2.94 sync
braid_PtFenSync sync
(optional) user access to app once-per-processor
15.3.2.95 t
braid_Real t

Data elements required for the Status structures Common Status properties current time
15.3.2.96 tgrid
braid_PtFcnTimeGrid tgrid
(optional) return time point values on level 0
15.3.2.97 tight_fine_tolx
braid_Int tight_fine_tolx

Boolean, indicating whether the tightest fine tolx has been used, condition for halting
15.3.2.98 tnext
braid_Real tnext
StepStatus properties time value to evolve towards, time value to the right of tstart
15.3.2.99 tnorm
braid_Int tnorm
choice of temporal norm
15.3.2.100 tnorm_a
braid_Real* tnorm_a
local array of residual norms on a proc's interval, used for inf-norm
15.3.2.101 tol
braid_Real tol
stopping tolerance
15.3.2.102 tpoints_cutoff
braid_Int tpoints_cutoff
refinements halt after the number of time steps exceed this value
15.3.2.103 tstart
braid_Real tstart
start time
15.3.2.104 tstop
braid_Real tstop
stop time
15.3.2.105 userVectorTape
_braid_Tape* userVectorTape
tape storing primal braid_vectors while recording
15.3.2.106 useshell
braid_Int useshell
activate the shell structure of vectors
15.3.2.107 verbose_adj
braid_Int verbose_adj
verbosity of the adjoint tape, displays the actions that are pushed / popped to the tape
15.3.2.108 warm_restart
braid_Int warm_restart
boolean, indicates whether this is a warm restart of an existing braid_Core
15.3.2.109 wrapper_test
braid_Int wrapper_test
boolean describing whether this call is only a wrapper test
The documentation for this struct was generated from the following file:

- _braid.h


## 15.4 _braid_Grid Struct Reference

## Data Fields

- braid_Int level
- braid_Int ilower
- braid_Int iupper
- braid_Int clower
- braid_Int cupper
- braid_Int gupper
- braid_Int cfactor
- braid_Int ncpoints
- braid_Int nupoints
- braid_BaseVector * ua
- braid_Real $*$ ta
- braid_BaseVector * va
- braid_BaseVector $*$ fa
- braid_Int recv_index
- braid_Int send_index
- _braid_CommHandle $*$ recv_handle
- _braid_CommHandle $*$ send_handle
- braid_BaseVector $*$ ua_alloc
- braid_Real * ta_alloc
- braid_BaseVector * va_alloc
- braid_BaseVector $*$ fa_alloc
15.4.1 Detailed Description

XBraid Grid structure for a certain time level

Holds all the information for a processor related to the temporal grid at this level.
15.4.2 Field Documentation
15.4.2. cfactor
braid_Int cfactor
coarsening factor
15.4.2.2 clower
braid_Int clower
smallest C point index
15.4.2.3 cupper
braid_Int cupper
largest C point index
15.4.2.4 fa
braid_BaseVector* fa
rhs vectors f (all points, NULL on level 0)
15.4.2.5 fa_alloc
braid_BaseVector* fa_alloc
original memory allocation for fa
15.4.2.6 gupper
braid_Int gupper
global size of the grid
15.4.2.7 ilower
braid_Int ilower
smallest time index at this level
15.4.2.8 iupper
braid_Int iupper
largest time index at this level
15.4.2.9 level
braid_Int level

Level that grid is on
15.4.2.10 nepoints
braid_Int ncpoints
number of $C$ points
15.4.2.11 nupoints
braid_Int nupoints
number of unknown vector points
15.4.2.12 recv_handle
__braid_CommHandle* recv_handle

Handle for nonblocking receives of braid_BaseVectors
15.4.2.13 recv_index
braid_Int recv_index
-1 means no receive
15.4.2.14 send_handle
_braid_CommHandle* send_handle
Handle for nonblocking sends of braid_BaseVectors
15.4.2.15 send_index
braid_Int send_index
-1 means no send
15.4.2.16 ta
braid_Real* ta
time values (all points)

```
15.4.2.17 ta alloc
braid_Real* ta_alloc
original memory allocation for ta
15.4.2.18 ua
braid_BaseVector* ua
unknown vectors (C-points at least)
```


### 15.4.2.19 ua_alloc

```
braid_BaseVector* ua_alloc
original memory allocation for ua
15.4.2.20 va
braid_BaseVector* va
restricted unknown vectors (all points, NULL on level 0)
15.4.2.21 va_alloc
braid_BaseVector* va_alloc
```

original memory allocation for va

The documentation for this struct was generated from the following file:

- braid.h


## 15.5 _braid_Status Struct Reference

## Data Fields

- _braid_Core core


### 15.5.1 Detailed Description

This is the main Status structure, that contains the properties of all the status. The user does not have access to this structure, but only to the derived Status structures. This class is accessed only inside XBraid code.
15.5.2 Field Documentation
15.5.2.1 core
_braid_Core core

The documentation for this struct was generated from the following file:

- status.h


## 15.6 _braid_Tape Struct Reference

## Data Fields

- int size
- void $*$ data_ptr
- struct _braid_tape_struct * next
15.6.1 Detailed Description

C-Implementation of a linked list storing pointers to generic data This structure represents one tape element, holding a pointer to data and a pointer to the next element int size holds the number of all elements in the tape
15.6.2 Field Documentation
15.6.2.1 data_ptr
void* data_ptr
15.6.2.2 next
struct _braid_tape_struct* next
15.6.2.3 size
int size

The documentation for this struct was generated from the following file:

- tape.h


## 15.7 braid_AccessStatus Struct Reference

Data Fields

- _braid_Status status


### 15.7.1 Detailed Description

AccessStatus structure which defines the status of XBraid at a given instant on some level during a run. The user accesses it through braid_AccessStatusGet $* *$ () functions. This is just a pointer to the braid_Status.

### 15.7.2 Field Documentation

15.7.2.1 status
_braid_Status status
The documentation for this struct was generated from the following file:

- status.h


## 15.8 braid BaseVector Struct Reference

Data Fields

- braid_Vector userVector
- braid_VectorBar bar


### 15.8.1 Detailed Description

Braid vector used for storage of all state and (if needed) adjoint information. Stores both the user's primal vector (braid $\hookleftarrow$ _Vector type) and the associated bar vector (braid_VectorBar type) if the adjoint functionality is being used. If adjoint is not being used, bar==NULL.
15.8.2 Field Documentation
15.8.2.1 bar
braid_VectorBar bar
holds the bar vector (shared pointer implementation)
15.8.2.2 userVector
braid_Vector userVector
holds the users primal vector

The documentation for this struct was generated from the following file:

- _braid.h


## 15.9 braid BufferStatus Struct Reference

Data Fields

- _braid_Status status


### 15.9.1 Detailed Description

The user's bufpack, bufunpack and bufsize routines will receive a BufferStatus structure, which defines the status of XBraid at a given buff (un)pack instance. The user accesses it through braid_BufferStatusGet**() functions. This is just a pointer to the braid_Status.
15.9.2 Field Documentation
15.9.2.1 status
_braid_Status status

The documentation for this struct was generated from the following file:

- status.h


### 15.10 braid_CoarsenRefStatus Struct Reference

Data Fields

- _braid_Status status


### 15.10.1 Detailed Description

The user coarsen and refine routines will receive a CoarsenRefStatus structure, which defines the status of XBraid at a given instant of coarsening or refinement on some level during a run. The user accesses it through braid_Coarsen $\hookleftarrow$ RefStatusGet**() functions. This is just a pointer to the braid_Status.

### 15.10.2 Field Documentation

15.10.2. status
_braid_Status status

The documentation for this struct was generated from the following file:

- status.h


### 15.11 braid_ObjectiveStatus Struct Reference

Data Fields

- _braid_Status status
15.11.1 Detailed Description

The user's objectiveT and PostprocessObjective will receive an ObjectiveStatus structure, which defines the status of XBraid at a given instance of evaluating the objective function. The user accesses it through braid_ObjectiveStatus $\hookleftarrow$ Get**() functions. This is just a pointer to the braid_Status.
15.11.2 Field Documentation

### 15.11.2.1 status

_braid_Status status
The documentation for this struct was generated from the following file:

- status.h


### 15.12 braid_Optim Struct Reference

## Data Fields

- braid_Real sum_user_obj
- braid_Real objective
- braid_Real tstart_obj
- braid_Real tstop_obj
- braid_Real f_bar
- braid_Real rnorm_adj
- braid_Real rnorm0_adj
- braid_Real rnorm
- braid_Real rnorm0
- braid_Real tol_adj
- braid_Int rtol_adj
- braid_Vector $*$ adjoints
- braid_VectorBar * tapeinput
- void $*$ sendbuffer
- MPI_Request * request
15.12.1 Detailed Description

Data structure for storing the optimization variables
15.12.2 Field Documentation
15.12.2.1 adjoints
braid_Vector* adjoints
vector for the adjoint variables
15.12.2.2 f_bar
braid_Real f_bar
contains the seed for tape evaluation
15.12.2.3 objective
braid_Real objective
global objective function value
15.12.2.4 request

MPI_Request* request
helper: Storing the MPI request of BufUnPackDiff
15.12.2.5 rnorm
braid_Real rnorm
norm of the state residual
15.12.2.6 rnorm0
braid_Real rnorm0
initial norm of the state residual
15.12.2.7 rnorm0_adj
braid_Real rnorm0_adj
initial norm of the adjoint residual
15.12.2.8 rnorm_adj
braid_Real rnorm_adj
norm of the adjoint residual
15.12.2.9 rtol_adj
braid_Int rtol_adj
flag: use relative tolerance for adjoint
15.12.2.10 sendbuffer
void* sendbuffer
helper: Memory for BufUnPackDiff communication
15.12.2.11 sum_user_obj
braid_Real sum_user_obj
sum of user's objective function values over time
15.12.2.12 tapeinput
braid_VectorBar* tapeinput
helper: store pointer to input of one braid iteration
15.12.2.13 tol_adj
braid_Real tol_adj
tolerance of adjoint residual
15.12.2.14 tstart_obj
braid_Real tstart_obj
starting time for evaluating the user's local objective
15.12.2.15 tstop_obj
braid_Real tstop_obj
stopping time for evaluating the user's local objective

The documentation for this struct was generated from the following file:

- _braid.h


### 15.13 braid_StepStatus Struct Reference

Data Fields

- _braid_Status status


### 15.13.1 Detailed Description

The user's step routine routine will receive a StepStatus structure, which defines the status of XBraid at the given instant for step evaluation on some level during a run. The user accesses it through braid_StepStatusGet**() functions. This is just a pointer to the braid_Status.

### 15.13.2 Field Documentation

### 15.13.2.1 status

```
_braid_Status status
```

The documentation for this struct was generated from the following file:

- status.h


### 15.14 braid_SyncStatus Struct Reference

## Data Fields

- _braid_Status status


### 15.14.1 Detailed Description

SyncStatus structure which provides the status of XBraid at a given instant on some level during a run. This is vector independent and called once per processor. The user accesses it through braid_SyncStatusGet** () functions. This is just a pointer to the braid_Status.
15.14.2 Field Documentation
15.14.2.1 status
_braid_Status status

The documentation for this struct was generated from the following file:

- status.h


### 15.15 braid_VectorBar Struct Reference

Data Fields

- braid_Vector userVector
- braid_Int useCount
15.15.1 Detailed Description

Braid Vector Structures:

There are three vector structures _braid_VectorBar Defined below braid_Vector Defined in braid.h braid_BaseVector Defined below

The braid_BaseVector is the main internal Vector class, which is stored at each time point. It basically wraps the Vector and braid_VectorBar (see below). The braid_VectorBar is only used if the adjoint capability is used, when it stores adjoint variables. It's basically a smart pointer wrapper around a braid_Vector. Note that it is always the braid_Vector that's passed to user-routines. Shared pointer implementation for storing the intermediat AD-bar variables while taping. This is essentially the same as a userVector, except we need shared pointer capabilities to know when to delete.

### 15.15.2 Field Documentation

### 15.15.2.1 useCount

braid_Int useCount
counts the number of pointers to this struct
15.15.2.2 userVector
braid_Vector userVector
holds the u_bar data

The documentation for this struct was generated from the following file:

- _braid.h


## 16 File Documentation

## 16.1 _braid.h File Reference

Data Structures

- struct braid_VectorBar
- struct braid_BaseVector
- struct braid_Optim
- struct _braid_CommHandle
- struct _braid_Grid
- struct _braid_Core


## Macros

- \#define _braid_Error(IERR, msg) _braid_ErrorHandler(__FILE__, _LINE__, IERR, msg)
- \#define _braid_ErrorlnArg(IARG, msg) _braid_Error(HYPRE_ERROR_ARG|IARG<<3, msg)
- \#define _braid_TAlloc(type, count) ( (type *)malloc((size_t)(sizeof(type) * (count))) )
- \#define _braid_CTAlloc(type, count) ( (type *)calloc((size_t)(count), (size_t)sizeof(type)) )
- \#define _braid_TReAlloc(ptr, type, count) ( (type *)realloc((char *)ptr, (size_t)(sizeof(type) * (count))) )
- \#define _braid_TFree(ptr) ( free((char *)ptr), ptr = NULL )
- \#define _braid_max(a, b) (((a)<(b)) ? (b) : (a))
- \#define _braid_min(a, b) (((a)<(b)) ? (a) : (b))
- \#define _braid_isnan(a) (a != a)
- \#define _braid_SendIndexNull -2
- \#define _braid_RecvIndexNull -2
- \#define _braid_MapPeriodic(index, npoints) (index = ((index)+(npoints)) \% (npoints) ) /* this also handles negative indexes $*$ /
- \#define _braid_CommHandleElt(handle, elt) ((handle) -> elt)
- \#define _braid_GridEIt(grid, elt) ((grid) -> elt)
- \#define _braid_CoreElt(core, elt) ( (core) -> elt )
- \#define _braid_CoreFcn(core, fcn) (*((core) -> fcn))
- \#define _braid_MapFineToCoarse(findex, cfactor, cindex) ( cindex = (findex)/(cfactor) )
- \#define _braid_MapCoarseToFine(cindex, cfactor, findex) ( findex = (cindex)*(cfactor) )
- \#define _braid_IsFPoint(index, cfactor) ( (index)\%(cfactor) )
- \#define _braid_IsCPoint(index, cfactor) (!_braid_IsFPoint(index, cfactor) )
- \#define _braid_NextCPoint(index, cfactor) ( ((braid_Int)((index)+(cfactor)-1)/(cfactor))*(cfactor) )
- \#define _braid_PriorCPoint(index, cfactor) ( ((braid_Int)(index)/(cfactor))*(cfactor) )


## Functions

- void _braid_ErrorHandler (const char *filename, braid_Int line, braid_Int ierr, const char *msg)
- braid_Int _braid_GetBlockDistInterval (braid_Int npoints, braid_Int nprocs, braid_Int proc, braid_Int *ilower_ptr, braid_Int *iupper_ptr)
- braid_Int _braid_GetBlockDistProc (braid_Int npoints, braid_Int nprocs, braid_Int index, braid_Int periodic, braid $\hookleftarrow$ _Int *proc_ptr)
- braid_Int _braid_GetDistribution (braid_Core core, braid_Int *ilower_ptr, braid_Int *iupper_ptr)
- braid_Int _braid_GetProc (braid_Core core, braid_Int level, braid_Int index, braid_Int *proc_ptr)
- braid_Int _braid_CommRecvInit (braid_Core core, braid_Int level, braid_Int index, braid_BaseVector *vector_ptr, _braid_CommHandle $* *$ handle_ptr)
- braid_Int _braid_CommSendlnit (braid_Core core, braid_Int level, braid_Int index, braid_BaseVector vector, _ $\hookleftarrow$ braid_CommHandle $* *$ handle_ptr)
- braid_Int _braid_CommWait (braid_Core core, _braid_CommHandle **handle_ptr)
- braid_Int _braid_UGetIndex (braid_Core core, braid_Int level, braid_Int index, braid_Int *uindex_ptr, braid_Int *store_flag_ptr)
- braid_Int _braid_UGetVectorRef (braid_Core core, braid_Int level, braid_Int index, braid_BaseVector *u_ptr)
- braid_Int _braid_USetVectorRef (braid_Core core, braid_Int level, braid_Int index, braid_BaseVector u)
- braid_Int _braid_UGetVector (braid_Core core, braid_Int level, braid_Int index, braid_BaseVector *u_ptr)
- braid_Int _braid_USetVector (braid_Core core, braid_Int level, braid_Int index, braid_BaseVector u, braid_Int move)
- braid_Int _braid_UCommInitBasic (braid_Core core, braid_Int level, braid_Int recv_msg, braid_Int send_msg, braid_Int send_now)
- braid_Int _braid_UCommInit (braid_Core core, braid_Int level)
- braid_Int _braid_UCommInitF (braid_Core core, braid_Int level)
- braid_Int_braid_UCommWait (braid_Core core, braid_Int level)
- braid_Int _braid_Step (braid_Core core, braid_Int level, braid_Int index, braid_BaseVector ustop, braid_Base $\hookleftarrow$ Vector u)
- braid_Int _braid_GetUlnit (braid_Core core, braid_Int level, braid_Int index, braid_BaseVector u, braid_Base $\hookleftarrow$ Vector *ustop_ptr)
- braid_Int _braid_Residual (braid_Core core, braid_Int level, braid_Int index, braid_BaseVector ustop, braid_↔ BaseVector r)
- braid_Int _braid_FASResidual (braid_Core core, braid_Int level, braid_Int index, braid_BaseVector ustop, braid $\hookleftarrow$ _BaseVector r)
- braid_Int _braid_Coarsen (braid_Core core, braid_Int level, braid_Int f_index, braid_Int c_index, braid_BaseVector fvector, braid_BaseVector *cvector)
- braid_Int _braid_RefineBasic (braid_Core core, braid_Int level, braid_Int c_index, braid_Real *f_ta, braid_Real *c_ta, braid_BaseVector cvector, braid_BaseVector *fvector)
- braid_Int _braid_Refine (braid_Core core, braid_Int level, braid_Int f_index, braid_Int c_index, braid_BaseVector cvector, braid_BaseVector *fvector)
- braid_Int _braid_FRefineSpace (braid_Core core, braid_Int *refined_ptr)
- braid_Int _braid_GridInit (braid_Core core, braid_Int level, braid_Int ilower, braid_Int iupper, _braid_Grid **grid $\hookleftarrow$ _ptr)
- braid_Int _braid_GridClean (braid_Core core, _braid_Grid *grid)
- braid_Int _braid_GridDestroy (braid_Core core, _braid_Grid *grid)
- braid_Int _braid_SetRNorm (braid_Core core, braid_Int iter, braid_Real rnorm)
- braid_Int _braid_GetRNorm (braid_Core core, braid_Int iter, braid_Real *rnorm_ptr)
- braid_Int _braid_SetFullRNorm (braid_Core core, braid_Int iter, braid_Real rnorm)
- braid_Int _braid_GetFullRNorm (braid_Core core, braid_Int iter, braid_Real $*$ rnorm_ptr)
- braid_Int _braid_ComputeFullRNorm (braid_Core core, braid_Int level, braid_Real *return_rnorm)
- braid_Int _braid_PrintSpatialNorms (braid_Core core, braid_Real *rnorms, braid_Int n)
- braid_Int _braid_FCRelax (braid_Core core, braid_Int level)
- braid_Int _braid_FRestrict (braid_Core core, braid_Int level)
- braid_Int _braid_FInterp (braid_Core core, braid_Int level)
- braid_Int _braid_FRefine (braid_Core core, braid_Int *refined_ptr)
- braid_Int _braid_FAccess (braid_Core core, braid_Int level, braid_Int done)
- braid_Int _braid_AccessVector (braid_Core core, braid_AccessStatus status, braid_BaseVector u)
- braid_Int _braid_Sync (braid_Core core, braid_SyncStatus status)
- braid_Int _braid_InitHierarchy (braid_Core core, _braid_Grid *fine_grid, braid_Int refined)
- braid_Int _braid_FinalizeErrorEstimates (braid_Core core, braid_Real *estimate, braid_Int length)
- braid_Int _braid_GetDtk (braid_Core core)
- braid_Int _braid_GetCFactor (braid_Core core, braid_Int level, braid_Int *cfactor_ptr)
- braid_Int _braid_InitGuess (braid_Core core, braid_Int level)
- braid_Int _braid_CopyFineToCoarse (braid_Core core)
- braid_Int _braid_Drive (braid_Core core, braid_Real localtime)


## Variables

- braid_Int _braid_error_flag
- FILE * _braid_printfile


### 16.1.1 Detailed Description

Define headers for XBraid internal (developer) routines and XBraid internal structure declarations.
This file contains the headers for XBraid internal (developer) routines and structure declarations.
16.1.2 Macro Definition Documentation
16.1.2.1 _braid_CommHandleElt

```
#define _braid_CommHandleElt(
    handle,
    elt ) ((handle) -> elt)
```

Accessor for _braid_CommHandle attributes
16.1.2.2 _braid_CoreElt

```
#define _braid_CoreElt(
    core,
    elt ) ( (core) -> elt )
```

Accessor for _braid_Core attributes
16.1.2.3 braid_CoreFcn

```
#define _braid_CoreFcn(
    core,
    fcn ) (*((core) -> fcn))
```

Accessor for _braid_Core functions
16.1.2.4 _braid_CTAlloc

```
#define _braid_CTAlloc(
    type,
    count ) ( (type *)calloc((size_t)(count), (size_t)sizeof(type)) )
```

Allocation macro
16.1.2.5 _braid_Error

```
#define _braid_Error(
    IERR,
    msg ) _braid_ErrorHandler(__FILE__, __LINE__, IERR, msg)
```


### 16.1.2.6 _braid_ErrorlnArg

```
#define _braid_ErrorInArg(
    IARG,
    msg ) _braid_Error(HYPRE_ERROR_ARG | IARG<<3, msg)
```

16.1.2.7 _braid_GridElt

```
#define _braid_GridElt(
    grid,
    elt ) ((grid) -> elt)
```

Accessor for _braid_Grid attributes
16.1.2.8 _braid_IsCPoint
\#define _braid_IsCPoint(
index,
cfactor ) ( !_braid_IsFPoint(index, cfactor) )

Boolean, returns whether a time index is an C-point
16.1.2.9 _braid_IsFPoint
\#define _braid_IsFPoint(
index,
cfactor ) ( (index) \%(cfactor) )

Boolean, returns whether a time index is an F-point
16.1.2.10 _braid_isnan
\#define _braid_isnan(
a) $(\mathrm{a}!=\mathrm{a})$
16.1.2.11 _braid_MapCoarseToFine
\#define _braid_MapCoarseToFine(
cindex,
cfactor,
findex ) ( findex $=($ cindex)*(cfactor) )

Map a coarse time index to a fine time index, assumes a uniform coarsening factor.
16.1.2.12 _braid_MapFineToCoarse
\#define _braid_MapFineToCoarse(
findex,
cfactor,
cindex ) ( cindex = (findex)/(cfactor) )

Map a fine time index to a coarse time index, assumes a uniform coarsening factor.
16.1.2.13 _braid_MapPeriodic

```
#define _braid_MapPeriodic(
    index,
    npoints ) ( index = ((index)+(npoints)) % (npoints) ) /* this also handles negative
indexes */
```

16.1.2.14 braid max
\#define _braid_max(
$a$,
$b) \quad(((a)<(b))$ ?
(b) :
(a) )
16.1.2.15 _braid_min
\#define _braid_min(
$a$,
$b) \quad(((a)<(b))$ ?
(a) :
(b) )
16.1.2.16 _braid_NextCPoint

```
#define _braid_NextCPoint(
    index,
    cfactor ) ( ((braid_Int)((index) +(cfactor)-1)/(cfactor))*(cfactor) )
```

Returns the index for the next C-point to the right of index (inclusive)
16.1.2.17 braid PriorCPoint

```
#define _braid_PriorCPoint(
    index,
    cfactor ) ( ((braid_Int) (index)/(cfactor))*(cfactor)
```

Returns the index for the previous C-point to the left of index (inclusive)
16.1.2.18 braid RecvIndexNull
\#define _braid_RecvIndexNull -2
16.1.2.19 braid SendIndexNull

```
#define _braid_SendIndexNull -2
```

16.1.2.20 braid_TAlloc

```
#define _braid_TAlloc(
    type,
    count ) ( (type *)malloc((size_t)(sizeof(type) * (count))) )
```

Allocation macro
16.1.2.21 _braid_TFree
\#define _braid_TFree(
ptr ) ( free((char *)ptr), ptr = NULL )

Free memory macro
16.1.2.22 _braid_TReAlloc

```
#define _braid_TReAlloc(
```

    ptr,
    type,
    count ) ( (type *)realloc ((char *)ptr, (size_t) (sizeof(type) * (count))) )
    Re-allocation macro
16.1.3 Function Documentation
16.1.3.1 _braid_AccessVector()

```
braid_Int _braid_AccessVector (
    braid_Core core,
    braid_AccessStatus status,
    braid_BaseVector u )
```

Call user's access function in order to give access to XBraid and the current vector. Most commonly, this lets the user write $u$ to screen, disk, etc... The vector $u$ corresponds to time step index on level. status holds state information about the current XBraid iteration, time value, etc...
16.1.3.2 _braid_Coarsen()

```
braid_Int _braid_Coarsen (
    braid_Core core,
    braid_Int level,
    braid_Int f_index,
    braid_Int c_index,
    braid_BaseVector fvector,
    braid_BaseVector * cvector )
```

Coarsen in space on level by calling the user's coarsen function. The vector corresponding to the time step index f_index on the fine grid is coarsened to the time step index c_index on the coarse grid. The output goes in cvector and the input vector is fvector.

### 16.1.3.3 _braid_CommRecvinit()

```
braid_Int _braid_CommRecvInit (
    braid_Core core,
    braid_Int level,
    braid_Int index,
    braid_BaseVector * vector_ptr,
    _braid_CommHandle ** handle_ptr )
```

Initialize a receive to go into vector_ptr for the given time index on level. Also return a comm handle handle_ptr for querying later, to see if the receive has occurred.

### 16.1.3.4 _braid_CommSendlnit()

```
braid_Int __braid_CommSendInit (
    braid_Core core,
    braid_Int level,
    braid_Int index,
    braid_BaseVector vector,
    _braid_CommHandle ** handle_ptr )
```

Initialize a send of vector for the given time index on level. Also return a comm handle handle_ptr for querying later, to see if the send has occurred.
16.1.3.5 _braid_CommWait()

```
braid_Int _braid_CommWait (
    braid_Core core,
    _braid_CommHandle ** handle_ptr )
```

Block on the comm handle handle_ptr until the MPI operation (send or recv) has completed
16.1.3.6 _braid_ComputeFullRNorm()

```
braid_Int _braid_ComputeFullRNorm (
    braid_Core core,
    braid_Int level,
    braid_Real * return_rnorm )
```

Compute full temporal residual norm with user-provided residual routine. Output goes in *return_rnorm.

### 16.1.3.7 _braid_CopyFineToCoarse()

braid_Int _braid_CopyFineToCoarse (

```
    braid_Core core )
```

Copy the initialized C-points on the fine grid, to all coarse levels. For instance, if a point $k$ on level $m$ corresponds to point $p$ on level 0 , then they are equivalent after this function. The only exception is any spatial coarsening the user decides to do. This function allows XBraid to skip all work on the first down cycle and start in FMG style on the coarsest level. Assumes level 0 C-points are initialized.
16.1.3.8 _braid_Drive()
braid_Int _braid_Drive (
braid_Core core,
braid_Real localtime )

Main loop for MGRIT
16.1.3.9 _braid_ErrorHandler()

```
void _braid_ErrorHandler (
    const char * filename,
    braid_Int line,
    braid_Int ierr,
    const char * msg )
```

16.1.3.10 _braid_FAccess()
braid_Int _braid_FAccess (
braid_Core core,
braid_Int level,
braid_Int done )

Call the user's access function in order to give access to XBraid and the current vector at grid level and iteration *iter. Most commonly, this lets the user write solutions to screen, disk, etc... The quantity rnorm denotes the last computed residual norm, and done is a boolean indicating whether XBraid has finished iterating and this is the last Access call.
16.1.3.11 _braid_FASResidual()

```
braid_Int _braid_FASResidual (
    braid_Core core,
    braid_Int level,
    braid_Int index,
    braid_BaseVector ustop,
    braid_BaseVector r )
```

Compute FAS residual $=\mathrm{f}$ - residual
16.1.3.12 _braid_FCRelax()

```
braid_Int _braid_FCRelax (
    braid_Core core,
    braid_Int level )
```

Do nu sweeps of F-then-C relaxation on level
16.1.3.13 _braid_FinalizeErrorEstimates()
braid_Int _braid_FinalizeErrorEstimates (
braid_Core core,
braid_Real * estimate,
braid_Int length )

Finalize Richardson error estimates
16.1.3.14 _braid_FInterp()

```
braid_Int _braid_FInterp (
    braid_Core core,
    braid_Int level )
```

F-Relax on level and interpolate to level-1

The output is set in the braid_Grid in core, so that the vector $u$ on level is created by interpolating from level+1.
If the user has set spatial refinement, then this user-defined routine is also called.

## Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| level | interp from level to level +1 |

16.1.3.15 _braid_FRefine()

```
braid_Int _braid_FRefine (
    braid_Core core,
    braid_Int * refined_ptr )
```

Create a new fine grid (level 0) and corresponding grid hierarchy by refining the current fine grid based on user-provided refinement factors. Return the boolean refined_ptr to indicate whether grid refinement was actually done. To simplify the algorithm, refinement factors are automatically capped to be no greater than the coarsening factor (for level 0 ). The grid data is also redistributed to achieve good load balance in the temporal dimension. If the refinement factor is 1 in each time interval, no refinement is done.
16.1.3.16 _braid_FRefineSpace()

```
braid_Int _braid_FRefineSpace (
    braid_Core core,
    braid_Int * refined_ptr )
```

Call spatial refinement on all local time steps if $r$ _space has been set on the local processor. Returns refined_ptr $==2$ if refinment was completed at any point globally, otherwise returns 0 . This is a helper function for _braid_FRefine().
16.1.3.17 _braid_FRestrict()

```
braid_Int _braid_FRestrict (
    braid_Core core,
    braid_Int level )
```

F-Relax on level and then restrict to level+1

The output is set in the braid_Grid in core, so that the restricted vectors va and fa will be created, representing level+1 versions of the unknown and rhs vectors.

If the user has set spatial coarsening, then this user-defined routine is also called.

If level==0, then rnorm_ptr will contain the residual norm.
Parameters

| core | braid_Core (_braid_Core) struct |
| :--- | :--- |
| level | restrict from level to level +1 |

16.1.3.18 _braid_GetBlockDistInterval()

```
braid_Int _braid_GetBlockDistInterval (
    braid_Int npoints,
    braid_Int nprocs,
    braid_Int proc,
    braid_Int * ilower_ptr,
    braid_Int * iupper_ptr )
```

Returns the index interval for proc in a blocked data distribution.
16.1.3.19 _braid_GetBlockDistProc()

```
braid_Int _braid_GetBlockDistProc (
    braid_Int npoints,
    braid_Int nprocs,
    braid_Int index,
    braid_Int periodic,
    braid_Int * proc_ptr )
```

Returns the processor that owns index in a blocked data distribution (returns -1 if index is out of range).
16.1.3.20 _braid_GetCFactor()

```
braid_Int _braid_GetCFactor (
    braid_Core core,
    braid_Int level,
    braid_Int * cfactor_ptr )
```

Returns the coarsening factor to use on grid level.
16.1.3.21 _braid_GetDistribution()
braid_Int __braid_GetDistribution (
braid_Core core,
braid_Int * ilower_ptr,
braid_Int * iupper_ptr )

Returns the index interval for my processor on the finest grid level. For the processor rank calling this function, it returns the smallest and largest time indices (ilower_ptr and iupper_ptr) that belong to that processor (the indices may be F or C points).
16.1.3.22 _braid_GetDtk()
braid_Int _braid_GetDtk (

```
    braid_Core core )
```

Propagate time step information required to compute the Richardson error estimate at each C-point. This can be done at any time, but does require some communication. This fills in error_factors at the $C$ points.
16.1.3.23 _braid_GetFullRNorm()

```
braid_Int _braid_GetFullRNorm (
    braid_Core core,
    braid_Int iter,
    braid_Real * rnorm_ptr )
```

Same as GetRNorm, but gets full residual norm.
16.1.3.24 _braid_GetProc()

```
braid_Int _braid_GetProc (
    braid_Core core,
    braid_Int level,
    braid_Int index,
    braid_Int * proc_ptr )
```

Returns the processor number in proc_ptr on which the time step index lives for the given level. Returns -1 if index is out of range.
16.1.3.25 _braid_GetRNorm()

```
braid_Int _braid_GetRNorm (
    braid_Core core,
    braid_Int iter,
    braid_Real * rnorm_ptr )
```

Get the residual norm for iteration iter. If iter $<0$, get the rnorm for the last iteration minus $\mid$ iter $\mid-1$.
16.1.3.26 _braid_GetUlnit()

```
braid_Int _braid_GetUInit (
    braid_Core core,
    braid_Int level,
    braid_Int index,
    braid_BaseVector u,
    braid_BaseVector * ustop_ptr )
```

Return an initial guess in ustop_ptr to use in the step routine for implicit schemes. The value returned depends on the storage options used. If the return value is NULL, no initial guess is available.

```
16.1.3.27 _braid_GridClean()
```

```
braid_Int _braid_GridClean (
    braid_Core core,
    _braid_Grid * grid )
```

Destroy the vectors on grid
16.1.3.2 _braid_GridDestroy()

```
braid_Int _braid_GridDestroy (
    braid_Core core,
    _braid_Grid * grid
```

Destroy grid
16.1.3.29 _braid_GridInit()

```
braid_Int _braid_GridInit (
    braid_Core core,
    braid_Int level,
    braid_Int ilower,
    braid_Int iupper,
    _braid_Grid ** grid_ptr )
```

Create a new grid object grid_ptr with level indicator level. Arguments ilower and iupper correspond to the lower and upper time index values for this processor on this grid.
16.1.3.30 _braid_InitGuess()

```
braid_Int _braid_InitGuess (
    braid_Core core,
    braid_Int level )
```

Set initial guess on level. Only C-pts are initialized on level 0, otherwise stored values are initialized based on restricted fine-grid values.
16.1.3.31 _braid_InitHierarchy()

```
braid_Int _braid_InitHierarchy (
    braid_Core core,
    _braid_Grid * fine_grid,
    braid_Int refined )
```

Initialize grid hierarchy with fine_grid serving as the finest grid. Boolean refined indicates whether fine_grid was created by refining a coarser grid (in the FRefine() routine), which has implications on how to define CF-intervals.
16.1.3.32 _braid_PrintSpatialNorms()

```
braid_Int _braid_PrintSpatialNorms (
    braid_Core core,
    braid_Real * rnorms,
    braid_Int n )
```

Print out the residual norm for every C-point. Processor 0 gathers all the rnorms and prints them in order through a gatherv operator
16.1.3.33 _braid_Refine()

```
braid_Int _braid_Refine (
    braid_Core core,
    braid_Int level,
    braid_Int f_index,
    braid_Int c_index,
    braid_BaseVector cvector,
    braid_BaseVector * fvector )
```

Refine in space on level by calling the user's refine function. The vector corresponding to the time step index c_index on the coarse grid is refined to the time step index $f_{-}$index on the fine grid. The output goes in fvector and the input vector is cuector.
16.1.3.34 _braid_RefineBasic()

```
braid_Int _braid_RefineBasic (
    braid_Core core,
    braid_Int level,
    braid_Int c_index,
    braid_Real * f_ta,
    braid_Real * c_ta,
    braid_BaseVector cvector,
    braid_BaseVector * fvector )
```

Refine in space (basic routine)
16.1.3.35 _braid_Residual()

```
braid_Int _braid_Residual (
    braid_Core core,
    braid_Int level,
    braid_Int index,
    braid_BaseVector ustop,
    braid_BaseVector r )
```

Compute residual $r$
16.1.3.36 _braid_SetFullRNorm()

```
braid_Int _braid_SetFullRNorm (
    braid_Core core,
    braid_Int iter,
    braid_Real rnorm )
```

Same as SetRNorm, but sets full residual norm.
16.1.3.37 _braid_SetRNorm()

```
braid_Int _braid_SetRNorm (
    braid_Core core,
    braid_Int iter,
    braid_Real rnorm )
```

Set the residual norm for iteration iter. If iter $<0$, set the rnorm for the last iteration minus |iter|-1. Also set the initial residual norm.

### 16.1.3.38 _braid_Step()

```
braid_Int _braid_Step (
    braid_Core core,
    braid_Int level,
    braid_Int index,
    braid_BaseVector ustop,
    braid_BaseVector u )
```

Integrate one time step at time step index to time step index+1.
16.1.3.39 _braid_Sync()

```
braid_Int _braid_Sync (
    braid_Core core,
    braid_SyncStatus status )
```

Call user's sync function in order to give access to XBraid and the user's app. This is called once-per-processor at various points in XBraid in order to allow the user to perform any book-keeping operations. status provides state information about the current XBraid status and processor.
16.1.3.40 _braid_UCommInit()

```
braid_Int _braid_UCommInit (
    braid_Core core,
    braid_Int level )
```

This routine initiates communication under the assumption that work will be done on all intervals ( F or C ) on level. It posts a receive for the point to the left of ilower (regardless whether ilower is F or C ), and it posts a send of iupper if iupper is a C point.

```
16.1.3.41 _braid_UCommInitBasic()
braid_Int _braid_UCommInitBasic (
    braid_Core core,
    braid_Int level,
    braid_Int recv_msg,
    braid_Int send_msg,
    braid_Int send_now )
```

Basic communication (from the left, to the right). Arguments recv_msg and send_msg are booleans that indicate whether or not to initiate a receive from the left and a send to the right respectively. Argument send_now indicates that the send should be initiated immediately.

### 16.1.3.42 _braid_UCommInitF()

```
braid_Int __braid_UCommInitF (
    braid_Core core,
    braid_Int level )
```

This routine initiates communication under the assumption that work will be done on only F-pt intervals on level. It only posts a receive for the point to the left of ilower if ilower is an $F$ point, and it posts a send of iupper if iupper is a $C$ point.
16.1.3.43 _braid_UCommWait()

```
braid_Int _braid_UCommWait (
    braid_Core core,
    braid_Int level )
```

Finish up communication. On level, wait on both the recv and send handles at this level.
16.1.3.44 _braid_UGetIndex()

```
braid_Int _braid_UGetIndex (
    braid_Core core,
    braid_Int level,
    braid_Int index,
    braid_Int * uindex_ptr,
    braid_Int * store_flag_ptr )
```

Returns an index into the local u-vector for grid level at point index, and information on the storage status of the point. If nothing is stored at that point, uindex $=-1$ and store_flag $=-2$. If only the shell is stored store_flag $=-1$, and if the whole u -vector is stored, store_flag $=0$.
16.1.3.45 _braid_UGetVector()

```
braid_Int __braid_UGetVector (
    braid_Core core,
    braid_Int level,
    braid_Int index,
    braid_BaseVector * u_ptr )
```

Returns a copy of the u-vector on grid level at point index. If index is my "receive index" (as set by UCommlnit(), for example), the $u$-vector will be received from a neighbor processor. If the $u$-vector is not stored, NULL is returned.
16.1.3.46 _braid_UGetVectorRef()
braid_Int _braid_UGetVectorRef (
braid_Core core,
braid_Int level,
braid_Int index,
braid_BaseVector * u_ptr )

Returns a reference to the local u-vector on grid level at point index. If the u-vector is not stored, returns NULL.
16.1.3.47 _braid_USetVector()

```
braid_Int __braid_USetVector (
    braid_Core core,
    braid_Int level,
    braid_Int index,
    braid_BaseVector u,
    braid_Int move )
```

Stores the u-vector on grid level at point index. If index is my "send index", a send is initiated to a neighbor processor. If move is true, the u-vector is moved into core storage instead of copied. If the u-vector is not stored, nothing is done.
16.1.3.48 _braid_USetVectorRef()
braid_Int _braid_USetVectorRef (
braid_Core core,
braid_Int level,
braid_Int index,
braid_BaseVector u )

Stores a reference to the local $u$-vector on grid level at point index. If the $u$-vector is not stored, nothing is done.
16.1.4 Variable Documentation
16.1.4.1 _braid_error_flag
braid_Int _braid_error_flag

This is the global XBraid error flag. If it is ever nonzero, an error has occurred.
16.1.4.2 _braid_printfile

FILE* _braid_printfile
This is the print file for redirecting stdout for all XBraid screen output

## 16.2 adjoint.h File Reference

Functions

- braid_Int _braid_VectorBarCopy (braid_VectorBar bar, braid_VectorBar *bar_ptr)
- braid_Int _braid_VectorBarDelete (braid_Core core, braid_VectorBar bar)
- braid_Int _braid_OptimDestroy (braid_Core core)
- braid_Int _braid_UpdateAdjoint (braid_Core core, braid_Real *rnorm_adj_ptr)
- braid_Int _braid_SetRNormAdjoint (braid_Core core, braid_Int iter, braid_Real rnorm_adj)
- braid_Int _braid_AddToObjective (braid_Core core, braid_BaseVector u, braid_ObjectiveStatus ostatus)
- braid_Int _braid_EvalObjective (braid_Core core)
- braid_Int _braid_EvalObjective_diff (braid_Core core)
- braid_Int _braid_InitAdjointVars (braid_Core core, _braid_Grid *fine_grid)
- braid_Int _braid_AdjointFeatureCheck (braid_Core core)


### 16.2.1 Detailed Description

Define internal XBraid headers for the adjoint feature.
This file contains the internal XBriad headers for the adjoint feature, e.g., the functions to wrap and call the users objective function, and allocate adjoint (bar) variables.
16.2.2 Function Documentation
16.2.2.1 _braid_AddToObjective()
braid_Int _braid_AddToObjective (
braid_Core core,
braid_BaseVector $u$,
braid_ObjectiveStatus ostatus )
Evaluate the user's local objective function at time $t$ and add it to the time-averaged objective function
16.2.2.2 _braid_AdjointFeatureCheck()
braid_Int _braid_AdjointFeatureCheck (
braid_Core core )
Sanity check for non-supported adjoint features
16.2.2.3 _braid_EvalObjective()
braid_Int _braid_EvalObjective (
braid_Core core )
Evaluate the objective function: MPI_Allreduce the time average and postprocess the objective
16.2.2.4 _braid_EvalObjective_diff()
braid_Int _braid_EvalObjective_diff (
braid_Core core )
Differentiated objective function
16.2.2.5 _braid_InitAdjointVars()
braid_Int _braid_InitAdjointVars (
braid_Core core,
_braid_Grid * fine_grid )
Allocate and initialize the adjoint variables
16.2.2.6 _braid_OptimDestroy()
braid_Int _braid_OptimDestroy (
braid_Core core )

Free memory of the optimization structure
16.2.2.7 _braid_SetRNormAdjoint()

```
braid_Int _braid_SetRNormAdjoint (
    braid_Core core,
    braid_Int iter,
    braid_Real rnorm_adj )
```

Set adjoint residual norm
16.2.2.8 _braid_UpdateAdjoint()

```
braid_Int _braid_UpdateAdjoint (
    braid_Core core,
    braid_Real * rnorm_adj_ptr )
```

Update the adjoint variables and compute adjoint residual norm Returns the tnorm of adjoint residual
16.2.2.9 _braid_VectorBarCopy()

```
braid_Int _braid_VectorBarCopy (
    braid_VectorBar bar,
    braid_VectorBar * bar_ptr )
```

Shallow copy a braid_VectorBar shared pointer, bar_ptr is set to bar and the useCount is incremented by one.
16.2.2.10 _braid_VectorBarDelete()

```
braid_Int _braid_VectorBarDelete (
    braid_Core core,
    braid_VectorBar bar )
```

Reduce the useCount of a braid_VectorBar shared pointer Free the pointer memory if useCount is zero.

## 16.3 base.h File Reference

## Functions

- braid_Int _braid_BaseStep (braid_Core core, braid_App app, braid_BaseVector ustop, braid_BaseVector fstop, braid_BaseVector u, braid_Int level, braid_StepStatus status)
- braid_Int _braid_Baselnit (braid_Core core, braid_App app, braid_Real t, braid_BaseVector *u_ptr)
- braid_Int _braid_BaseClone (braid_Core core, braid_App app, braid_BaseVector u, braid_BaseVector *v_ptr)
- braid_Int _braid_BaseFree (braid_Core core, braid_App app, braid_BaseVector u)
- braid_Int _braid_BaseSum (braid_Core core, braid_App app, braid_Real alpha, braid_BaseVector x, braid_Real beta, braid_BaseVector y)
- braid_Int _braid_BaseSpatialNorm (braid_Core core, braid_App app, braid_BaseVector u, braid_Real *norm_ptr)
- braid_Int _braid_BaseAccess (braid_Core core, braid_App app, braid_BaseVector u, braid_AccessStatus status)
- braid_Int _braid_BaseSync (braid_Core core, braid_App app, braid_SyncStatus status)
- braid_Int _braid_BaseBufSize (braid_Core core, braid_App app, braid_Int *size_ptr, braid_BufferStatus status)
- braid_Int_braid_BaseBufPack (braid_Core core, braid_App app, braid_BaseVector u, void *buffer, braid_Buffer $\hookleftarrow$ Status status)
- braid_Int _braid_BaseBufUnpack (braid_Core core, braid_App app, void *buffer, braid_BaseVector *u_ptr, braid_BufferStatus status)
- braid_Int _braid_BaseObjectiveT (braid_Core core, braid_App app, braid_BaseVector u, braid_ObjectiveStatus ostatus, braid_Real *objT_ptr)
- braid_Int _braid_BaseResidual (braid_Core core, braid_App app, braid_BaseVector ustop, braid_BaseVector r, braid_StepStatus status)
- braid_Int _braid_BaseFullResidual (braid_Core core, braid_App app, braid_BaseVector r, braid_BaseVector u, braid_StepStatus status)
- braid_Int _braid_BaseSCoarsen (braid_Core core, braid_App app, braid_BaseVector fu, braid_BaseVector $*$ cu $\hookleftarrow$ _ptr, braid_CoarsenRefStatus status)
- braid_Int _braid_BaseSRefine (braid_Core core, braid_App app, braid_BaseVector cu, braid_BaseVector *fu_ptr, braid_CoarsenRefStatus status)
- braid_Int _braid_BaseSInit (braid_Core core, braid_App app, braid_Real t, braid_BaseVector $* u$ _ptr)
- braid_Int _braid_BaseSClone (braid_Core core, braid_App app, braid_BaseVector u, braid_BaseVector *v_ptr)
- braid_Int _braid_BaseSFree (braid_Core core, braid_App app, braid_BaseVector u)
- braid_Int _braid_BaseTimeGrid (braid_Core core, braid_App app, braid_Real *ta, braid_Int *ilower, braid_Int *iupper)
- braid_Int _braid_BaseStep_diff (_braid_Action *action)
- braid_Int _braid_BaseClone_diff (_braid_Action *action)
- braid_Int _braid_BaseSum_diff (_braid_Action *action)
- braid_Int _braid_BaseObjectiveT_diff (_braid_Action *action)
- braid_Int _braid_BaseBufPack_diff (_braid_Action *action)
- braid_Int _braid_BaseBufUnpack_diff (_braid_Action *action)
- braid_Int _braid_Baselnit_diff (_braid_Action *action)


### 16.3.1 Detailed Description

Define XBraid internal headers for wrapper routines of user-defined functions.

The XBraid internal headers defined here wrap the user-defined routines. If this is a normal XBraid run (i.e., no adjoint), then the wrappers serve no function, and just call the user's routines. If this is an XBraid_Adjoint run, then these routines record themselves to the action tape and push state and bar vectors to the primal and the bar tape, respectively. These vectors are then later popped from the tape and passed to the user diff routines in order to compute the differentiated actions. This is a form of automatic differentiation to compute the adjoint cycle.
16.3.2 Function Documentation
16.3.2.1 _braid_BaseAccess()

```
braid_Int _braid_BaseAccess (
    braid_Core core,
    braid_App app,
    braid_BaseVector u,
    braid_AccessStatus status )
```

This calls the user's Access routine. If (adjoint): also record the action
Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| $u$ | vector to be accessed |
| status | can be querried for info like the current XBraid Iteration |

16.3.2.2 _braid_BaseBufPack()

```
braid_Int _braid_BaseBufPack (
    braid_Core core,
    braid_App app,
```

```
braid_BaseVector u,
void * buffer,
braid_BufferStatus status )
```

This calls the user's BufPack routine. If (adjoint): also record the action, and push to the bar tape.
Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| $u$ | vector to pack into buffer |
| buffer | output, MPI buffer containing u |
| status | can be querried for info about the message |

16.3.2.3 _braid_BaseBufPack_diff()
braid_Int _braid_BaseBufPack_diff (
_braid_Action * action )

This pops the bar vector from the tape, and then performs the differentiated BufPack action using that vector as input: MPI_Recv(utmp) ubar += utmp

Parameters
action __braid_Action structure, holds information about the primal XBraid action
16.3.2.4 _braid_BaseBufSize()

```
braid_Int _braid_BaseBufSize (
    braid_Core core,
    braid_App app,
    braid_Int * size_ptr,
    braid_BufferStatus status )
```

This calls the user's BufSize routine. If (adjoint): nothing
Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| size_ptr | upper bound on vector size in bytes |
| status | can be querried for info about the message type |

16.3.2.5 _braid_BaseBufUnpack()

```
braid_Int _braid_BaseBufUnpack (
    braid_Core core,
    braid_App app,
    void * buffer,
    braid_BaseVector * u_ptr,
    braid_BufferStatus status )
```

This calls the user's BufUnPack routine. If (adjoint): also record the action, initialize the bar vector with zero and push it to the bar tape.

Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| buffer | MPI Buffer to unpack and place in u_ptr |
| u_ptr | output, braid_Vector containing buffer's data |
| status | can be querried for info about the message type |

16.3.2.6 _braid_BaseBufUnpack_diff()
braid_Int _braid_BaseBufUnpack_diff (
_braid_Action * action )

This pops the bar vector from the tape, and then performs the differentiated BufUnPack action using that vector as input: MPI_Send(ubar); ubar $=0.0$

## Parameters

```
action _braid_Action structure, holds information about the primal XBraid action
```


### 16.3.2.7 _braid_BaseClone()

```
braid_Int _braid_BaseClone (
    braid_Core core,
    braid_App app,
    braid_BaseVector u,
    braid_BaseVector * v_ptr )
```

This initializes a braid_BaseVector and calls the user's clone routine. If (adjoint): also record the action, initialize a barVector with zero and push to the bar tape

## Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| $u$ | vector to clone |
| v_ptr | output, newly allocated and cloned vector |

16.3.2.8 _braid_BaseClone_diff()
braid_Int _braid_BaseClone_diff (

```
                        _braid_Action * action )
```

This pops bar vectors from the tape, and then performs the differentiated clone action using those vectors as input: ubar += vbar vbar $=0.0$

## Parameters

action $\quad$ _braid_Action structure, holds information about the primal XBraid action
16.3.2.9 _braid_BaseFree()

```
braid_Int _braid_BaseFree (
    braid_Core core,
    braid_App app,
    braid_BaseVector u )
```

This calls the user's free routine. If (adjoint): also record the action, and free the bar vector.

## Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| $u$ | vector to free |

16.3.2.10 _braid_BaseFullResidual()

```
braid_Int _braid_BaseFullResidual (
    braid_Core core,
    braid_App app,
    braid_BaseVector r,
```

```
braid_BaseVector u,
braid_StepStatus status )
```

This calls the user's FullResidual routine (full_rnorm_res). If (adjoint): nothing
Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| $r$ | output, residual at tstop |
| $u$ | input, $u$ vector at tstop |
| status | braid_Status structure (pointer to the core) |

16.3.2.11 _braid_Baselnit()

```
braid_Int _braid_BaseInit (
    braid_Core core,
    braid_App app,
    braid_Real t,
    braid_BaseVector * u_ptr )
```

This initializes a braid_BaseVector and calls the user's init routine. If (adjoint): also record the action, initialize barVector with zero and push to the bar tape.

Parameters

| core | braid_Core structure |
| :--- | :--- |
| $a p p$ | user-defined_braid_App structure |
| $t$ | current time value for $u \_p t r$ |
| $u \_p t r$ | output, newly allocated and initialized vector |

16.3.2.12 _braid_Baselnit_diff()
braid_Int _braid_BaseInit_diff (
_braid_Action * action )

This pops the bar vector from the tape, and then call's the user's differentiated init routine (init_diff) using that vector as input. Note: init_diff is optional for the user.

Parameters

```
action _braid_Action structure, holds information about the primal XBraid action
```

16.3.2.13 _braid_BaseObjectiveT()

```
braid_Int _braid_BaseObjectiveT (
    braid_Core core,
    braid_App app,
    braid_BaseVector u,
    braid_ObjectiveStatus ostatus,
    braid_Real * objT_ptr )
```

If (adjoint): This calls the user's ObjectiveT routine, records the action, and pushes to the state and bar tapes.

## Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| $u$ | input, state vector at current time |
| ostatus | status structure for querying time, index, etc. |
| objT_ptr | output, objective function value at current time |

16.3.2.14 _braid_BaseObjectiveT_diff()
braid_Int _braid_BaseObjectiveT_diff (
_braid_Action * action )
This pops state and bar vectors from the tape, and then calls the user's differentiated ObjectiveT routine (objT_diff) using those vectors as input: ubar $=(\mathrm{d}(\text { objective } \mathrm{T}) / \mathrm{d}(\mathrm{u}))^{\wedge} \mathrm{T} * \mathrm{f}$ bar

## Parameters

```
action _braid_Action structure, holds information about the primal XBraid action
```


### 16.3.2.15 _braid_BaseResidual()

```
braid_Int __braid_BaseResidual (
    braid_Core core,
    braid_App app,
    braid_BaseVector ustop,
    braid_BaseVector r,
    braid_StepStatus status )
```

This calls the user's Residual routine. If (adjoint): nothing

## Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| ustop | input, $u$ vector at tstop |
| $r$ | output, residual at tstop (at input, equals $u$ at tstart) |
| status | braid_Status structure (pointer to the core) |

### 16.3.2.16 _braid_BaseSClone()

```
braid_Int _braid_BaseSClone (
    braid_Core core,
    braid_App app,
    braid_BaseVector u,
    braid_BaseVector * v_ptr )
```

This clones a shell baseVector and call's the user's SClone routine. If (adjoint): nothing

Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined _braid_App structure |
| $u$ | vector to clone |
| $v \_p t r$ | output, newly allocated and cloned vector shell |

### 16.3.2.17 _braid_BaseSCoarsen()

```
braid_Int _braid_BaseSCoarsen (
    braid_Core core,
    braid_App app,
    braid_BaseVector fu,
    braid_BaseVector * cu_ptr,
    braid_CoarsenRefStatus status )
```

This initializes a baseVector and calls the user's SCoarsen routine. If (adjoint): nothing

## Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| $f u$ | braid_BaseVector to refine |
| cu_ptr | output, refined vector |
| status | braid_Status structure (pointer to the core) |

16.3.2.18 _braid_BaseSFree()
braid_Int _braid_BaseSFree (
braid_Core core,
braid_App app,
braid_BaseVector $u$ )

Call the user's shell free (SFree) routine. If (adjoint): nothing
Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| $u$ | vector to free (keeping the shell) |

16.3.2.19 _braid_BaseSInit()
braid_Int _braid_BaseSInit (
braid_Core core,
braid_App app,
braid_Real t,
braid_BaseVector * u_ptr )
This initializes a shell baseVector and call's the user's SInit routine. If (adjoint): nothing

## Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| $t$ | time value for $u \_p t r$ |
| $u \_p t r$ | output, newly allocated and initialized vector shell |

16.3.2.20 _braid_BaseSpatialNorm()
braid_Int _braid_BaseSpatialNorm (
braid_Core core,
braid_App app,
braid_BaseVector u,
braid_Real * norm_ptr )

This calls the user's SpatialNorm routine. If (adjoint): nothing

## Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| $u$ | vector to norm |
| norm_ptr | output, norm of braid_Vector (this is a spatial norm) |

16.3.2.21 _braid_BaseSRefine()

```
braid_Int _braid_BaseSRefine (
    braid_Core core,
    braid_App app,
    braid_BaseVector cu,
    braid_BaseVector * fu_ptr,
    braid_CoarsenRefStatus status )
```

This initializes a baseVector and calls the user's SRefine routine. If (adjoint): nothing

## Parameters

| core | braid_Core structure |
| :--- | :--- |
| $a p p$ | user-defined _braid_App structure |
| cu | braid_BaseVector to refine |
| fu_ptr | output, refined vector |
| status | braid_Status structure (pointer to the core) |

16.3.2.22 _braid_BaseStep()

```
braid_Int _braid_BaseStep (
    braid_Core core,
    braid_App app,
    braid_BaseVector ustop,
    braid_BaseVector fstop,
    braid_BaseVector u,
    braid_Int level,
    braid_StepStatus status )
```

This calls the user's step routine. If (adjoint): also record the action, and push state and bar vector to primal and bar tapes. braid_Status structure (pointer to the core)

## Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined _braid_App structure |

## Parameters

| ustop | input, $u$ vector at tstop |
| :--- | :--- |
| fstop | input, right-hand-side at tstop |
| $u$ | input/output, initially $u$ vector at tstart, upon exit, $u$ vector at tstop |
| level | current time grid level |

16.3.2.23 _braid_BaseStep_diff()
braid_Int _braid_BaseStep_diff (
_braid_Action * action )

This pops state and bar vectors from the tape, and then calls the user's differentiated step routine (step_diff) using those vectors as input. ubar $=(\mathrm{d}(\mathrm{step}) / \mathrm{d}(\mathrm{u}))^{\wedge} \mathrm{T} *$ ubar

## Parameters

```
action _braid_Action structure, holds information about the primal XBraid action
```

16.3.2.24 _braid_BaseSum()

```
braid_Int _braid_BaseSum (
    braid_Core core,
    braid_App app,
    braid_Real alpha,
    braid_BaseVector x,
    braid_Real beta,
    braid_BaseVector y )
```

This calls the user's sum routine. If (adjoint): also record the action, and push to the bar tape.

## Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| alpha | scalar for AXPY |
| $x$ | vector for AXPY |
| beta | scalar for AXPY |
| $y$ | output and vector for AXPY |

16.3.2.25 _braid_BaseSum_diff()
braid_Int _braid_BaseSum_diff (
_braid_Action * action )

This pops bar vectors from the tape, and then performs the differentiated sum action using those vectors as input: xbar $+=$ alpha $*$ ybar ybar $=$ beta $*$ ybar

## Parameters

| action | _braid_Action structure, holds information about the primal XBraid action |
| :---: | :---: | :---: |

```
16.3.2.26 _braid_BaseSync()
braid_Int _braid_BaseSync (
    braid_Core core,
    braid_App app,
    braid_SyncStatus status )
```

This calls the user's Sync routine. If (adjoint): nothing

## Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| status | can be querried for info like the current XBraid Iteration |

```
16.3.2.27 _braid_BaseTimeGrid()
braid_Int __braid_BaseTimeGrid (
    braid_Core core,
    braid_App app,
    braid_Real * ta,
    braid_Int * ilower,
    braid_Int * iupper )
```

This calls the user's TimeGrid routine, which allows the user to explicitly define the initial time grid. If (adjoint): nothing

Parameters

| core | braid_Core structure |
| :--- | :--- |
| app | user-defined_braid_App structure |
| ta | temporal grid on level 0 (slice per processor) |
| ilower | lower time index value for this processor |
| iupper | upper time index value for this processor |

## 16.4 braid.h File Reference

## Macros

- \#define braid_FMANGLE 1
- \#define braid_Fortran_SpatialCoarsen 0
- \#define braid_Fortran_Residual 1
- \#define braid_Fortran_TimeGrid 1
- \#define braid_Fortran_Sync 1
- \#define braid_INVALID_RNORM -1
- \#define braid_ERROR_GENERIC $1 / *$ generic error $* /$
- \#define braid_ERROR_MEMORY $2 / *$ unable to allocate memory $* /$
- \#define braid_ERROR_ARG $4 / *$ argument error $* /$
- \#define braid_RAND_MAX 32768


## Typedefs

- typedef struct _braid_App_struct * braid_App
- typedef struct _braid_Vector_struct * braid_Vector
- typedef braid_Int(* braid_PtFcnStep) (braid_App app, braid_Vector ustop, braid_Vector fstop, braid_Vector u, braid_StepStatus status)
- typedef braid_Int(* braid_PtFcnlnit) (braid_App app, braid_Real t, braid_Vector $* \mathrm{u} \_$ptr)
- typedef braid_Int(* braid_PtFcnClone) (braid_App app, braid_Vector u, braid_Vector *v_ptr)
- typedef braid_Int(* braid_PtFcnFree) (braid_App app, braid_Vector u)
- typedef braid_Int(* braid_PtFcnSum) (braid_App app, braid_Real alpha, braid_Vector x, braid_Real beta, braid $\hookleftarrow$ _Vector y)
-     - typedef braid_Int(* braid_PtFcnSpatialNorm) (braid_App app, braid_Vector u, braid_Real *norm_ptr)
- typedef braid_Int(* braid_PtFcnAccess) (braid_App app, braid_Vector u, braid_AccessStatus status)
- typedef braid_Int(* braid_PtFcnSync) (braid_App app, braid_SyncStatus status)
- typedef braid_Int(* braid_PtFcnBufSize) (braid_App app, braid_Int *size_ptr, braid_BufferStatus status)
- typedef braid_Int $* *$ braid_PtFcnBufPack) (braid_App app, braid_Vector u, void *buffer, braid_BufferStatus status)
- typedef braid_Int(* braid_PtFcnBufUnpack) (braid_App app, void *buffer, braid_Vector *u_ptr, braid_BufferStatus status)
- typedef braid_Int(* braid_PtFcnResidual) (braid_App app, braid_Vector ustop, braid_Vector r, braid_StepStatus status)
- typedef braid_Int(* braid_PtFcnSCoarsen) (braid_App app, braid_Vector fu, braid_Vector $*$ cu_ptr, braid_ $\hookleftarrow$ CoarsenRefStatus status)
- typedef braid_Int(* braid_PtFcnSRefine) (braid_App app, braid_Vector cu, braid_Vector *fu_ptr, braid_Coarsen $\hookleftarrow$ RefStatus status)
- typedef braid_Int(* braid_PtFcnSInit) (braid_App app, braid_Real t, braid_Vector $*$ u_ptr)
- typedef braid_Int(* braid_PtFcnSClone) (braid_App app, braid_Vector u, braid_Vector *v_ptr)
- typedef braid_Int(* braid_PtFcnSFree) (braid_App app, braid_Vector u)
- typedef braid_Int(* braid_PtFcnTimeGrid) (braid_App app, braid_Real *ta, braid_Int *ilower, braid_Int *iupper)
- typedef braid_Int(* braid_PtFcnObjectiveT) (braid_App app, braid_Vector u, braid_ObjectiveStatus ostatus, braid_Real *objectiveT_ptr)
- typedef braid_Int(* braid_PtFcnObjectiveTDiff) (braid_App app, braid_Vector u, braid_Vector u_bar, braid_Real F_bar, braid_ObjectiveStatus ostatus)
- typedef braid_Int(* braid_PtFcnPostprocessObjective) (braid_App app, braid_Real sum_obj, braid_Real *postprocess_ptr)
- typedef braid_Int(* braid_PtFcnPostprocessObjective_diff) (braid_App app, braid_Real sum_obj, braid_Real $*$ F $\hookleftarrow$ _bar_ptr)
- typedef braid_Int(* braid_PtFcnStepDiff) (braid_App app, braid_Vector ustop, braid_Vector u, braid_Vector ustop_bar, braid_Vector u_bar, braid_StepStatus status)
- typedef braid_Int(* braid_PtFcnResetGradient) (braid_App app)
- typedef struct _braid_Core_struct * braid_Core


## Functions

- braid_Int braid_Init (MPI_Comm comm_world, MPI_Comm comm, braid_Real tstart, braid_Real tstop, braid_↔ Int ntime, braid_App app, braid_PtFcnStep step, braid_PtFcnInit init, braid_PtFcnClone clone, braid_PtFcnFree free, braid_PtFcnSum sum, braid_PtFcnSpatialNorm spatialnorm, braid_PtFcnAccess access, braid_PtFcnBuf $\hookleftarrow$ Size bufsize, braid_PtFcnBufPack bufpack, braid_PtFcnBufUnpack bufunpack, braid_Core *core_ptr)
- braid_Int braid_Drive (braid_Core core)
- braid_Int braid_Destroy (braid_Core core)
- braid_Int braid_PrintStats (braid_Core core)
- braid_Int braid_WriteConvHistory (braid_Core core, const char *filename)
- braid_Int braid_SetMaxLevels (braid_Core core, braid_Int max_levels)
- braid_Int braid_SetIncrMaxLevels (braid_Core core)
- braid_Int braid_SetSkip (braid_Core core, braid_Int skip)
- braid_Int braid_SetRefine (braid_Core core, braid_Int refine)
- braid_Int braid_SetMaxRefinements (braid_Core core, braid_Int max_refinements)
- braid_Int braid_SetTPointsCutoff (braid_Core core, braid_Int tpoints_cutoff)
- braid_Int braid_SetMinCoarse (braid_Core core, braid_Int min_coarse)
- braid_Int braid_SetAbsTol (braid_Core core, braid_Real atol)
- braid_Int braid_SetRelTol (braid_Core core, braid_Real rtol)
- braid_Int braid_SetNRelax (braid_Core core, braid_Int level, braid_Int nrelax)
- braid_Int braid_SetCRelaxWt (braid_Core core, braid_Int level, braid_Real Cwt)
- braid_Int braid_SetCFactor (braid_Core core, braid_Int level, braid_Int cfactor)
- braid_Int braid_SetMaxIter (braid_Core core, braid_Int max_iter)
- braid_Int braid_SetFMG (braid_Core core)
- braid_Int braid_SetNFMG (braid_Core core, braid_Int k)
- braid_Int braid_SetNFMGVcyc (braid_Core core, braid_Int nfmg_Vcyc)
- braid_Int braid_SetStorage (braid_Core core, braid_Int storage)
- braid_Int braid_SetTemporalNorm (braid_Core core, braid_Int tnorm)
- braid_Int braid_SetResidual (braid_Core core, braid_PtFcnResidual residual)
- braid_Int braid_SetFullRNormRes (braid_Core core, braid_PtFcnResidual residual)
- braid_Int braid_SetTimeGrid (braid_Core core, braid_PtFcnTimeGrid tgrid)
- braid_Int braid_SetPeriodic (braid_Core core, braid_Int periodic)
- braid_Int braid_SetSpatialCoarsen (braid_Core core, braid_PtFcnSCoarsen scoarsen)
- braid_Int braid_SetSpatialRefine (braid_Core core, braid_PtFcnSRefine srefine)
- braid_Int braid_SetSync (braid_Core core, braid_PtFcnSync sync)
- braid_Int braid_SetPrintLevel (braid_Core core, braid_Int print_level)
- braid_Int braid_SetFileIOLevel (braid_Core core, braid_Int io_level)
- braid_Int braid_SetPrintFile (braid_Core core, const char *printfile_name)
- braid_Int braid_SetDefaultPrintFile (braid_Core core)
- braid_Int braid_SetAccessLevel (braid_Core core, braid_Int access_level)
- braid_Int braid_SplitCommworld (const MPI_Comm *comm_world, braid_Int px, MPI_Comm *comm_x, MPI_↔ Comm *comm_t)
- braid_Int braid_SetShell (braid_Core core, braid_PtFcnSInit sinit, braid_PtFcnSClone sclone, braid_PtFcnSFree sfree)
- braid_Int braid_GetNumlter (braid_Core core, braid_Int *niter_ptr)
- braid_Int braid_GetRNorms (braid_Core core, braid_Int *nrequest_ptr, braid_Real *rnorms)
- braid_Int braid_GetNLevels (braid_Core core, braid_Int *nlevels_ptr)
- braid_Int braid_GetSpatialAccuracy (braid_StepStatus status, braid_Real loose_tol, braid_Real tight_tol, braid $\hookleftarrow$ _Real *tol_ptr)
- braid_Int braid_SetSeqSoln (braid_Core core, braid_Int seq_soln)
- braid_Int braid_GetMyID (braid_Core core, braid_Int *myid_ptr)
- braid_Int braid_Rand (void)
- braid_Int braid_InitAdjoint (braid_PtFcnObjectiveT objectiveT, braid_PtFcnObjectiveTDiff objectiveT_diff, braid $\hookleftarrow$ _PtFcnStepDiff step_diff, braid_PtFcnResetGradient reset_gradient, braid_Core $*$ core_ptr)
- braid_Int braid_SetTStartObjective (braid_Core core, braid_Real tstart_obj)
- braid_Int braid_SetTStopObjective (braid_Core core, braid_Real tstop_obj)
- braid_Int braid_SetPostprocessObjective (braid_Core core, braid_PtFcnPostprocessObjective post_fcn)
- braid_Int braid_SetPostprocessObjective_diff (braid_Core core, braid_PtFcnPostprocessObjective_diff post_ $\hookleftarrow$ fcn_diff)
- braid_Int braid_SetAbsToIAdjoint (braid_Core core, braid_Real tol_adj)
- braid_Int braid_SetRelTolAdjoint (braid_Core core, braid_Real rtol_adj)
- braid_Int braid_SetObjectiveOnly (braid_Core core, braid_Int boolean)
- braid_Int braid_GetObjective (braid_Core core, braid_Real *objective_ptr)
- braid_Int braid_GetRNormAdjoint (braid_Core core, braid_Real *rnorm_adj)
- braid_Int braid_SetRichardsonEstimation (braid_Core core, braid_Int est_error, braid_Int richardson, braid_Int local_order)


### 16.4.1 Detailed Description

Define headers for user-interface routines.

This file contains user-routines used to allow the user to initialize, run and get and set options for a XBraid solver.

## 16.5 braid_defs.h File Reference

## Macros

- \#define braid_Int_Max INT_MAX;
- \#define braid_Int_Min INT_MIN;
- \#define braid_MPI_REAL MPI_DOUBLE
- \#define braid_MPI_INT MPI_INT


## Typedefs

- typedef int braid_Int
- typedef double braid_Real
16.5.1 Detailed Description

Definitions of braid types, error flags, etc...
16.5.2 Macro Definition Documentation

### 16.5.2.1 braid_Int_Max

\#define braid_Int_Max INT_MAX;
16.5.2.2 braid_Int_Min
\#define braid_Int_Min INT_MIN;
16.5.2.3 braid_MPI_INT
\#define braid_MPI_INT MPI_INT
16.5.2.4 braid_MPI_REAL
\#define braid_MPI_REAL MPI_DOUBLE
16.5.3 Typedef Documentation
16.5.3.1 braid_Int
typedef int braid_Int

Defines integer type
16.5.3.2 braid_Real
typedef double braid_Real

Defines floating point type

## 16.6 braid_status.h File Reference

## Macros

- \#define ACCESSOR_HEADER_GET1(stype, param, vtype1) braid_Int braid_\#\#stype\#\#StatusGet\#\#param(braid $\hookleftarrow$ _\#\#stype\#\#Status s, braid_\#\#vtype1 *v1);
- \#define ACCESSOR_HEADER_GET1_IN3(stype, param, vtype1, vtype2, vtype3, vtype4) braid_Int braid_↔ \#\#stype\#\#StatusGet\#\#param(braid_\#\#stype\#\#Status s, braid_\#\#vtype1 *v1, braid_\#\#vtype2 v2, braid_\#\#vtype3 v3, braid_\#\#vtype4 v4);
- \#define ACCESSOR_HEADER_GET2(stype, param, vtype1, vtype2) braid_Int braid_\#\#stype\#\#Status $\hookleftarrow$ Get\#\#param(braid_\#\#stype\#\#Status s, braid_\#\#vtype1 *v1, braid_\#\#vtype2 *v2);
- \#define ACCESSOR_HEADER_GET2_IN1(stype, param, vtype1, vtype2, vtype3) braid_Int braid_↔ \#\#stype\#\#StatusGet\#\#param(braid_\#\#stype\#\#Status s, braid_\#\#vtype1 *v1, braid_\#\#vtype2 *v2, braid_↔ \#\#vtype3 v3);
- \#define ACCESSOR_HEADER_GET3(stype, param, vtype1, vtype2, vtype3) braid_Int braid_\#\#stype\#\#Status $\leftarrow$ Get\#\#param(braid_\#\#stype\#\#Status s, braid_\#\#vtype1 *v1, braid_\#\#vtype2 *v2, braid_\#\#vtype3 *v3);
- \#define ACCESSOR_HEADER_GET4(stype, param, vtype1, vtype2, vtype3, vtype4) braid_Int braid_↔ \#\#stype\#\#StatusGet\#\#param(braid_\#\#stype\#\#Status s, braid_\#\#vtype1 *v1, braid_\#\#vtype2 *v2, braid_ $\hookleftarrow$ \#\#vtype3 *v3, braid_\#\#vtype4 *v4);
- \#define ACCESSOR_HEADER_GET5(stype, param, vtype1, vtype2, vtype3, vtype4, vtype5) braid_Int braid $\hookleftarrow$ _\#\#stype\#\#StatusGet\#\#param(braid_\#\#stype\#\#Status s, braid_\#\#vtype1 *v1, braid_\#\#vtype2 *v2, braid_↔ \#\#vtype3 $*$ v3, braid_\#\#vtype4 $*$ v4, braid_\#\#vtype5 $* \mathrm{v} 5$ );
- \#define ACCESSOR_HEADER_SET1(stype, param, vtype1) braid_Int braid_\#\#stype\#\#StatusSet\#\#param(braid $\hookleftarrow$ _\#\#stype\#\#Status s, braid_\#\#vtype1 v1);
- \#define braid_ASCaller_FInterp 0
- \#define braid_ASCaller_FRestrict 1
- \#define braid_ASCaller_FRefine 2
- \#define braid_ASCaller_FAccess 3
- \#define braid_ASCaller_FRefine_AfterInitHier 4
- \#define braid_ASCaller_Drive_TopCycle 5


## Functions

- braid_Int braid_StatusGetT (braid_Status status, braid_Real *t_ptr)
- braid_Int braid_StatusGetTIndex (braid_Status status, braid_Int *idx_ptr)
- braid_Int braid_StatusGetlter (braid_Status status, braid_Int *iter_ptr)
- braid_Int braid_StatusGetLevel (braid_Status status, braid_Int *level_ptr)
- braid_Int braid_StatusGetNLevels (braid_Status status, braid_Int *nlevels_ptr)
- braid_Int braid_StatusGetNRefine (braid_Status status, braid_Int *nrefine_ptr)
- braid_Int braid_StatusGetNTPoints (braid_Status status, braid_Int *ntpoints_ptr)
- braid_Int braid_StatusGetResidual (braid_Status status, braid_Real *rnorm_ptr)
- braid_Int braid_StatusGetDone (braid_Status status, braid_Int *done_ptr)
- braid_Int braid_StatusGetTIUL (braid_Status status, braid_Int *iloc_upper, braid_Int *iloc_lower, braid_Int level)
- braid_Int braid_StatusGetTimeValues (braid_Status status, braid_Real **tvalues_ptr, braid_Int i_upper, braid_Int i_lower, braid_Int level)
- braid_Int braid_StatusGetTILD (braid_Status status, braid_Real *t_ptr, braid_Int *iter_ptr, braid_Int *level_ptr, braid_Int *done_ptr)
- braid_Int braid_StatusGetWrapperTest (braid_Status status, braid_Int *wtest_ptr)
- braid_Int braid_StatusGetCallingFunction (braid_Status status, braid_Int *cfunction_ptr)
- braid_Int braid_StatusGetCTprior (braid_Status status, braid_Real *ctprior_ptr)
- braid_Int braid_StatusGetCTstop (braid_Status status, braid_Real *ctstop_ptr)
- braid_Int braid_StatusGetFTprior (braid_Status status, braid_Real *ftprior_ptr)
- braid_Int braid_StatusGetFTstop (braid_Status status, braid_Real *ftstop_ptr)
- braid_Int braid_StatusGetTpriorTstop (braid_Status status, braid_Real *t_ptr, braid_Real *ftprior_ptr, braid_Real *ftstop_ptr, braid_Real *ctprior_ptr, braid_Real *ctstop_ptr)
- braid_Int braid_StatusGetTstop (braid_Status status, braid_Real *tstop_ptr)
- braid_Int braid_StatusGetTstartTstop (braid_Status status, braid_Real *tstart_ptr, braid_Real *tstop_ptr)
- braid_Int braid_StatusGetTol (braid_Status status, braid_Real *tol_ptr)
- braid_Int braid_StatusGetRNorms (braid_Status status, braid_Int *nrequest_ptr, braid_Real *rnorms_ptr)
- braid_Int braid_StatusGetOldFineTolx (braid_Status status, braid_Real *old_fine_tolx_ptr)
- braid_Int braid_StatusSetOldFineTolx (braid_Status status, braid_Real old_fine_tolx)
- braid_Int braid_StatusSetTightFineTolx (braid_Status status, braid_Real tight_fine_tolx)
- braid_Int braid_StatusSetRFactor (braid_Status status, braid_Real rfactor)
- braid_Int braid_StatusSetRefinementDtValues (braid_Status status, braid_Real rfactor, braid_Real *dtarray)
- braid_Int braid_StatusSetRSpace (braid_Status status, braid_Real r_space)
- braid_Int braid_StatusGetMessageType (braid_Status status, braid_Int *messagetype_ptr)
- braid_Int braid_StatusSetSize (braid_Status status, braid_Real size)
- braid_Int braid_StatusGetSingleErrorEstStep (braid_Status status, braid_Real *estimate)
- braid_Int braid_StatusGetSingleErrorEstAccess (braid_Status status, braid_Real *estimate)
- braid_Int braid_StatusGetNumErrorEst (braid_Status status, braid_Int *npoints)
- braid_Int braid_StatusGetAllErrorEst (braid_Status status, braid_Real *error_est)
- braid_Int braid_AccessStatusGetT (braid_AccessStatus s, braid_Real $* \mathrm{v} 1$ )
- braid_Int braid_AccessStatusGetTIndex (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetlter (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetLevel (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetNLevels (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetNRefine (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetNTPoints (braid_AccessStatus s, braid_Int *V1)
- braid_Int braid_AccessStatusGetResidual (braid_AccessStatus s, braid_Real *v1)
- braid_Int braid_AccessStatusGetDone (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetTILD (braid_AccessStatus s, braid_Real $* \mathrm{v} 1$, braid_Int $* \mathrm{v} 2$, braid_Int $* \mathrm{v} 3$, braid_Int *v4)
- braid_Int braid_AccessStatusGetWrapperTest (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetCallingFunction (braid_AccessStatus s, braid_Int *v1)
- braid_Int braid_AccessStatusGetSingleErrorEstAccess (braid_AccessStatus s, braid_Real $* \mathrm{~V} 1$ )
- braid_Int braid_SyncStatusGetTIUL (braid_SyncStatus s, braid_Int $* \mathrm{~V} 1$, braid_Int $* \mathrm{v} 2$, braid_Int v3)
- braid_Int braid_SyncStatusGetTimeValues (braid_SyncStatus s, braid_Real $* * \mathrm{~V} 1$, braid_Int v2, braid_Int v3, braid_Int v4)
- braid_Int braid_SyncStatusGetlter (braid_SyncStatus s, braid_Int *V1)
- braid_Int braid_SyncStatusGetLevel (braid_SyncStatus s, braid_Int *V1)
- braid_Int braid_SyncStatusGetNLevels (braid_SyncStatus s, braid_Int *V1)
- braid_Int braid_SyncStatusGetNRefine (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetNTPoints (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetDone (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetCallingFunction (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetNumErrorEst (braid_SyncStatus s, braid_Int *v1)
- braid_Int braid_SyncStatusGetAllErrorEst (braid_SyncStatus s, braid_Real *v1)
- braid_Int braid_CoarsenRefStatusGetT (braid_CoarsenRefStatus s, braid_Real $* \mathrm{v} 1$ )
- braid_Int braid_CoarsenRefStatusGetTIndex (braid_CoarsenRefStatus s, braid_Int *v1)
- braid_Int braid_CoarsenRefStatusGetlter (braid_CoarsenRefStatus s, braid_Int *v1)
- braid_Int braid_CoarsenRefStatusGetLevel (braid_CoarsenRefStatus s, braid_Int *v1)
- braid_Int braid_CoarsenRefStatusGetNLevels (braid_CoarsenRefStatus s, braid_Int *v1)
- braid_Int braid_CoarsenRefStatusGetNRefine (braid_CoarsenRefStatus s, braid_Int *v1)
- braid_Int braid_CoarsenRefStatusGetNTPoints (braid_CoarsenRefStatus s, braid_Int *V1)
- braid_Int braid_CoarsenRefStatusGetCTprior (braid_CoarsenRefStatus s, braid_Real *v1)
- braid_Int braid_CoarsenRefStatusGetCTstop (braid_CoarsenRefStatus s, braid_Real *V1)
- braid_Int braid_CoarsenRefStatusGetFTprior (braid_CoarsenRefStatus s, braid_Real *V1)
- braid_Int braid_CoarsenRefStatusGetFTstop (braid_CoarsenRefStatus s, braid_Real *v1)
- braid_Int braid_CoarsenRefStatusGetTpriorTstop (braid_CoarsenRefStatus s, braid_Real *v1, braid_Real $* \mathrm{v} 2$, braid_Real $* \mathrm{v} 3$, braid_Real $* \mathrm{v} 4$, braid_Real $* \mathrm{v} 5$ )
- braid_Int braid_StepStatusGetT (braid_StepStatus s, braid_Real *v1)
- braid_Int braid_StepStatusGetTIndex (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetlter (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetLevel (braid_StepStatus s, braid_Int *V1)
- braid_Int braid_StepStatusGetNLevels (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetNRefine (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetNTPoints (braid_StepStatus s, braid_Int *v1)
- braid_Int braid_StepStatusGetTstop (braid_StepStatus s, braid_Real *v1)
- braid_Int braid_StepStatusGetTstartTstop (braid_StepStatus s, braid_Real *v1, braid_Real *v2)
- braid_Int braid_StepStatusGetTol (braid_StepStatus s, braid_Real *v1)
- braid_Int braid_StepStatusGetRNorms (braid_StepStatus s, braid_Int *v1, braid_Real *v2)
- braid_Int braid_StepStatusGetOIdFineTolx (braid_StepStatus s, braid_Real $*$ v1)
- braid_Int braid_StepStatusSetOldFineTolx (braid_StepStatus s, braid_Real v1)
- braid_Int braid_StepStatusSetTightFineTolx (braid_StepStatus s, braid_Real v1)
- braid_Int braid_StepStatusSetRFactor (braid_StepStatus s, braid_Real v1)
- braid_Int braid_StepStatusSetRSpace (braid_StepStatus s, braid_Real v1)
- braid_Int braid_StepStatusGetSingleErrorEstStep (braid_StepStatus s, braid_Real *v1)
- braid_Int braid_BufferStatusGetMessageType (braid_BufferStatus s, braid_Int $*$ v1)
- braid_Int braid_BufferStatusSetSize (braid_BufferStatus s, braid_Real v1)
- braid_Int braid_ObjectiveStatusGetT (braid_ObjectiveStatus s, braid_Real *v1)
- braid_Int braid_ObjectiveStatusGetTIndex (braid_ObjectiveStatus s, braid_Int *v1)
- braid_Int braid_ObjectiveStatusGetlter (braid_ObjectiveStatus s, braid_Int *v1)
- braid_Int braid_ObjectiveStatusGetLevel (braid_ObjectiveStatus s, braid_Int *v1)
- braid_Int braid_ObjectiveStatusGetNLevels (braid_ObjectiveStatus s, braid_Int *v1)
- braid_Int braid_ObjectiveStatusGetNRefine (braid_ObjectiveStatus s, braid_Int *v1)
- braid_Int braid_ObjectiveStatusGetNTPoints (braid_ObjectiveStatus s, braid_Int *v1)
- braid_Int braid_ObjectiveStatusGetTol (braid_ObjectiveStatus s, braid_Real *V1)


### 16.6.1 Detailed Description

Define headers for the user-interface with the XBraid status structures, allowing the user to get/set status structure values.
16.6.2 Macro Definition Documentation

### 16.6.2.1 ACCESSOR_HEADER_GET1

```
#define ACCESSOR_HEADER_GET1(
    stype,
    param,
    vtype1 ) braid_Int braid_##stype##StatusGet##param(braid_##stype##Status s, braid_\hookleftarrow
##vtype1 *v1);
```

Macros allowing for auto-generation of 'inherited' StatusGet functions
16.6.2.2 ACCESSOR_HEADER_GET1_IN3
\#define ACCESSOR_HEADER_GET1_IN3(
stype,
param,
vtype1,
vtype2,
vtype3,
vtype4 ) braid_Int braid_\#\#stype\#\#StatusGet\#\#param(braid_\#\#stype\#\#Status s, braid_↔
\#\#vtype1 *v1, braid_\#\#vtype2 v2, braid_\#\#vtype3 v3, braid_\#\#vtype4 v4);
16.6.2.3 ACCESSOR_HEADER_GET2
\#define ACCESSOR_HEADER_GET2(
stype,
param,
vtype1,
vtype2 ) braid_Int braid_\#\#stype\#\#StatusGet\#\#param(braid_\#\#stype\#\#Status s, braid_↔
\#\#vtype1 *v1, braid_\#\#vtype2 *v2);
16.6.2.4 ACCESSOR_HEADER_GET2_IN1
\#define ACCESSOR_HEADER_GET2_IN1(
stype,
param,
vtype1,
vtype2,
vtype3 ) braid_Int braid_\#\#stype\#\#StatusGet\#\#param(braid_\#\#stype\#\#Status s, braid_↔
\#\#vtype1 *v1, braid_\#\#vtype2 *v2, braid_\#\#vtype3 v3);
16.6.2.5 ACCESSOR_HEADER_GET3
\#define ACCESSOR_HEADER_GET3(
stype,
param,
vtype1,
vtype2,
vtype3 ) braid_Int braid_\#\#stype\#\#StatusGet\#\#param(braid_\#\#stype\#\#Status s, braid_↔
\#\#vtype1 *v1, braid_\#\#vtype2 *v2, braid_\#\#vtype3 *v3);
16.6.2.6 ACCESSOR_HEADER_GET4

```
#define ACCESSOR_HEADER_GET4(
    stype,
    param,
    vtype1,
    vtype2,
    vtype3,
    vtype4 ) braid_Int braid_##stype##StatusGet##param(braid_##stype##Status s, braid_\hookleftarrow
##vtype1 *v1, braid_##vtype2 *v2, braid_##vtype3 *v3, braid_##vtype4 *v4);
```

16.6.2.7 ACCESSOR_HEADER_GET5

```
#define ACCESSOR_HEADER_GET5(
    stype,
    param,
    vtype1,
    vtype2,
    vtype3,
    vtype4,
    vtype5 ) braid_Int braid_##stype##StatusGet##param(braid_##stype##Status s, braid_\hookleftarrow
##vtype1 *v1, braid_##vtype2 *v2, braid_##vtype3 *v3, braid_##vtype4 *v4, braid_##vtype5 *v5);
```

16.6.2.8 ACCESSOR_HEADER_SET1
\#define ACCESSOR_HEADER_SET1(

```
    stype,
    param,
    vtype1 ) braid_Int braid_##stype##StatusSet##param(braid_##stype##Status s, braid_\hookleftarrow
```

\#\#vtype1 v1);

## 16.7 braid_test.h File Reference

## Functions

- braid_Int braid_TestlnitAccess (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_PtFcnInit init, braid_PtFcnAccess access, braid_PtFcnFree free)
- braid_Int braid_TestClone (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_PtFcnlnit init, braid_PtFcnAccess access, braid_PtFcnFree free, braid_PtFcnClone clone)
- braid_Int braid_TestSum (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_PtFcnlnit init, braid_PtFcnAccess access, braid_PtFcnFree free, braid_PtFcnClone clone, braid_PtFcnSum sum)
- braid_Int braid_TestSpatialNorm (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_PtFcnInit init, braid_PtFcnFree free, braid_PtFcnClone clone, braid_PtFcnSum sum, braid_PtFcnSpatialNorm spatialnorm)
- braid_Int braid_TestBuf (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_PtFcnInit init, braid_PtFcnFree free, braid_PtFcnSum sum, braid_PtFcnSpatialNorm spatialnorm, braid_PtFcnBufSize bufsize, braid_PtFcnBufPack bufpack, braid_PtFcnBufUnpack bufunpack)
- braid_Int braid_TestCoarsenRefine (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_Real fdt, braid_Real cdt, braid_PtFcnlnit init, braid_PtFcnAccess access, braid_PtFcnFree free, braid_PtFcnClone clone, braid_PtFcnSum sum, braid_PtFcnSpatialNorm spatialnorm, braid_PtFcnSCoarsen coarsen, braid_Pt $\hookleftarrow$ FcnSRefine refine)
- braid_Int braid_TestResidual (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_Real dt, braid $\hookleftarrow$ _PtFcnInit myinit, braid_PtFcnAccess myaccess, braid_PtFcnFree myfree, braid_PtFcnClone clone, braid_Pt $\leftarrow$ FcnSum sum, braid_PtFcnSpatialNorm spatialnorm, braid_PtFcnResidual residual, braid_PtFcnStep step)
- braid_Int braid_TestAll (braid_App app, MPI_Comm comm_x, FILE *fp, braid_Real t, braid_Real fdt, braid_↔ Real cdt, braid_PtFcnInit init, braid_PtFcnFree free, braid_PtFcnClone clone, braid_PtFcnSum sum, braid_Pt FcnSpatialNorm spatialnorm, braid_PtFcnBufSize bufsize, braid_PtFcnBufPack bufpack, braid_PtFcnBufUnpack bufunpack, braid_PtFcnSCoarsen coarsen, braid_PtFcnSRefine refine, braid_PtFcnResidual residual, braid_↔ PtFcnStep step)


### 16.7.1 Detailed Description

Define headers for XBraid user-test routines.

This file contains headers for the user to test their XBraid wrapper routines one-by-one.

## 16.8 mpistubs.h File Reference

16.8.1 Detailed Description

XBraid internal headers to define fake MPI stubs. This ultimately allows the user to generate purely serial codes without MPI.

## 16.9 status.h File Reference

Data Structures

- struct _braid_Status
- struct braid_AccessStatus
- struct braid_SyncStatus
- struct braid_StepStatus
- struct braid_CoarsenRefStatus
- struct braid_BufferStatus
- struct braid_ObjectiveStatus


## Macros

- \#define _braid_StatusElt(status, elt) ( ((braid_Core)status) -> elt )


## Functions

- braid_Int _braid_StatusDestroy (braid_Status status)
- braid_Int _braid_AccessStatusInit (braid_Real t, braid_Int idx, braid_Real rnorm, braid_Int iter, braid_Int level, braid_Int nrefine, braid_Int gupper, braid_Int done, braid_Int wrapper_test, braid_Int calling_function, braid_ $\leftarrow$ AccessStatus status)
- braid_Int _braid_SyncStatusInit (braid_Int iter, braid_Int level, braid_Int nrefine, braid_Int gupper, braid_Int done, braid_Int calling_function, braid_SyncStatus status)
- braid_Int_braid_CoarsenRefStatusInit (braid_Real tstart, braid_Real f_tprior, braid_Real f_tstop, braid_Real c_↔ tprior, braid_Real c_tstop, braid_Int level, braid_Int nrefine, braid_Int gupper, braid_Int c_index, braid_Coarsen $\hookleftarrow$ RefStatus status)
- braid_Int _braid_StepStatusInit (braid_Real tstart, braid_Real tstop, braid_Int idx, braid_Real tol, braid_Int iter, braid_Int level, braid_Int nrefine, braid_Int gupper, braid_StepStatus status)
- braid_Int _braid_BufferStatusInit (braid_Int messagetype, braid_Int size, braid_BufferStatus status)
- braid_Int _braid_ObjectiveStatusInit (braid_Real tstart, braid_Int idx, braid_Int iter, braid_Int level, braid_Int nrefine, braid_Int gupper, braid_ObjectiveStatus status)


### 16.9.1 Detailed Description

Define the XBraid internal headers for the XBraid status structure routines, and define the status structures themselves.
16.9.2 Macro Definition Documentation
16.9.2.1 _braid_StatusElt
\#define _braid_StatusElt(

> status,
elt ) ( ((braid_Core)status) -> elt )
16.9.3 Function Documentation
16.9.3.1 _braid_AccessStatusInit()

```
braid_Int _braid_AccessStatusInit (
    braid_Real t,
    braid_Int idx,
    braid_Real rnorm,
    braid_Int iter,
    braid_Int level,
    braid_Int nrefine,
    braid_Int gupper,
    braid_Int done,
    braid_Int wrapper_test,
    braid_Int calling_function,
    braid_AccessStatus status )
```

Initialize a braid_AccessStatus structure

## Parameters

| $t$ | current time |
| :--- | :--- |
| idx | time point index value corresponding to t on the global time grid |
| rnorm | current residual norm in XBraid |
| iter | current iteration in XBraid |
| level | current level in XBraid |
| nrefine | number of refinements done |
| gupper | global size of the fine grid |
| done | boolean describing whether XBraid has finished |
| wrapper_test | boolean describing whether this call is only a wrapper test |
| calling_function | from which function are we accessing the vector |
| status | structure to initialize |

### 16.9.3.2 _braid_BufferStatusInit()

```
braid_Int _braid_BufferStatusInit (
    braid_Int messagetype,
    braid_Int size,
    braid_BufferStatus status )
```

Initialize a braid_BufferStatus structure

Parameters

| messagetype | message type, 0 : for Step(), 1 : for load balancing |
| :--- | :--- |
| size | if set by user, size of send buffer is "size" bytes |
| status | structure to initialize |

### 16.9.3.3 _braid_CoarsenRefStatusInit()

```
braid_Int _braid_CoarsenRefStatusInit (
    braid_Real tstart,
    braid_Real f_tprior,
    braid_Real f_tstop,
    braid_Real c_tprior,
    braid_Real c_tstop,
    braid_Int level,
    braid_Int nrefine,
    braid_Int gupper,
    braid_Int c_index,
    braid_CoarsenRefStatus status )
```

Initialize a braid_CoarsenRefStatus structure

## Parameters

| tstart | time value for current vector |
| :--- | :--- |
| f_tprior | time value to the left of tstart on fine grid |
| f_tstop | time value to the right of tstart on fine grid |
| c_tprior | time value to the left of tstart on coarse grid |
| c_tstop | time value to the right of tstart on coarse grid |
| level | current fine level in XBraid |
| nrefine | number of refinements done |
| gupper | global size of the fine grid |
| c_index | coarse time index refining from |
| status | structure to initialize |

### 16.9.3.4 _braid_ObjectiveStatusInit()

```
braid_Int _braid_ObjectiveStatusInit (
    braid_Real tstart,
    braid_Int idx,
    braid_Int iter,
    braid_Int level,
    braid_Int nrefine,
    braid_Int gupper,
    braid_ObjectiveStatus status )
```

Initialize a braid_ObjectiveStatus structure
16.9.3.5 _braid_StatusDestroy()
braid_Int _braid_StatusDestroy (
braid_Status status )
16.9.3.6 _braid_StepStatusInit()

```
braid_Int _braid_StepStatusInit (
    braid_Real tstart,
    braid_Real tstop,
    braid_Int idx,
    braid_Real tol,
    braid_Int iter,
    braid_Int level,
    braid_Int nrefine,
    braid_Int gupper,
    braid_StepStatus status )
```

Initialize a braid_StepStatus structure

## Parameters

| tstart | current time value |
| :--- | :--- |
| tstop | time value to evolve towards, time value to the right of tstart |
| idx | time point index value corresponding to tstart on the global time grid |
| tol | Current XBraid stopping tolerance |
| iter | Current XBraid iteration (also equal to length of rnorms) |
| level | current level in XBraid |
| nrefine | number of refinements done |
| gupper | global size of the fine grid |
| status | structure to initialize |

### 16.9.3.7 _braid_SyncStatusInit()

```
braid_Int _braid_SyncStatusInit (
    braid_Int iter,
    braid_Int level,
    braid_Int nrefine,
    braid_Int gupper,
    braid_Int done,
    braid_Int calling_function,
    braid_SyncStatus status )
```

Initialize a braid_SyncStatus structure
Parameters

| iter | current iteration in XBraid |
| :--- | :--- |
| level | current level in XBraid |
| nrefine | number of refinements done |
| gupper | global size of the fine grid |
| done | boolean describing whether XBraid has finished |
| calling_function | from which function are we accessing braid |
| status | structure to initialize |

### 16.10 tape.h File Reference

Data Structures

- struct _braid_Tape
- struct _braid_Action


## Enumerations

- enum _braid_Call \{

STEP = 1, INIT = 2, CLONE = 3, $\mathrm{FREE}=4$,
SUM $=5$, BUFPACK $=6$, BUFUNPACK $=7$, ACCESS $=8$,
OBJECTIVET = 9$\}$

Functions

- braid_Int _braid_TapeInit (_braid_Tape *head)
- _braid_Tape * _braid_TapePush (_braid_Tape $*$ head, void $*$ ptr)
- _braid_Tape * _braid_TapePop (_braid_Tape *head)
- braid_Int _braid_TapelsEmpty (_braid_Tape *head)
- braid_Int _braid_TapeGetSize (_braid_Tape *head)
- braid_Int _braid_TapeDisplayBackwards (braid_Core core, _braid_Tape *head, void(*fctptr)(braid_Core core, void *data_ptr))
- braid_Int _braid_TapeEvaluate (braid_Core core)
- braid_Int _braid_DiffCall (_braid_Action *action)
- braid_Int _braid_TapeSetSeed (braid_Core core)
- braid_Int _braid_TapeResetInput (braid_Core core)
- const char * _braid_CallGetName (_braid_Call call)


### 16.10.1 Detailed Description

Define the XBraid internal headers for the action-tape routines (linked list for AD)

### 16.10.2 Enumeration Type Documentation

16.10.2.1 _braid_Call
enum _braid_Call

Enumerator for identifying performed action
Enumerator

| STEP |  |
| ---: | :--- |
| INIT |  |
| CLONE |  |
| FREE |  |
| SUM |  |
| BUFPACK |  |
| BUFUNPACK |  |
| ACCESS |  |
| OBJECTIVET |  |

### 16.10.3 Function Documentation

16.10.3.1 _braid_CallGetName()
const char* _braid_CallGetName (
$\ldots \quad$ braid_Call call )

Return the name of a _braid_Call (action name)
16.10.3.2 _braid_DiffCall()
braid_Int _braid_DiffCall (
_braid_Action * action )
Call differentiated action
16.10.3.3 _braid_TapeDisplayBackwards()
braid_Int _braid_TapeDisplayBackwards (
braid_Core core,
__braid_Tape * head,
void(*) (braid_Core core, void *data_ptr) fctptr )

Display the tape in reverse order, calls the display function at each element Input: - pointer to the braid core

- pointer to the display function
16.10.3.4 _braid_TapeEvaluate()
braid_Int _braid_TapeEvaluate (
braid_Core core )

Evaluate the action tape in reverse order. This will clear the action tape! Input: - pointer to the braid core

- pointer to the head of the action tape
16.10.3.5 _braid_TapeGetSize()
braid_Int _braid_TapeGetSize (
_braid_Tape * head )

Returns the number of elements in the tape
16.10.3.6 _braid_TapeInit()
braid_Int _braid_TapeInit (
_braid_Tape * head)

Initialize the tape Set head to NULL
16.10.3.7 _braid_TapelsEmpty()
braid_Int _braid_TapeIsEmpty (
_braid_Tape * head )

Test if tape is empty return 1 if tape is empty, otherwise returns 0
16.10.3.8 _braid_TapePop()

```
_braid_Tape* _braid_TapePop (
    _braid_Tape * head )
```

Pop an element from the tape Return pointer to head
16.10.3.9 _braid_TapePush()

```
_braid_Tape* __braid_TapePush (
    _braid_Tape * head,
    void * ptr )
```

Push data on the tape Return pointer to head
16.10.3.10 _braid_TapeResetInput()
braid_Int _braid_TapeResetInput (
braid_Core core )

Set the pointers in tapeinput to the input of an xbraid iteration (ua).
16.10.3.11 _braid_TapeSetSeed()
braid_Int _braid_TapeSetSeed (
braid_Core core )

Set the adjoint seed for tape evaluation, i.e., set $u->$ bar at stored points on level 0 to the values contained in core->optim->adjoints

### 16.11 util.h File Reference

## Functions

- braid_Int _braid_ProjectInterval (braid_Int ilower, braid_Int iupper, braid_Int index, braid_Int stride, braid_Int *pilower, braid_Int *piupper)
- braid_Int_braid_GetInterval (braid_Core core, braid_Int level, braid_Int interval_index, braid_Int *flo_ptr, braid_Int *fhi_ptr, braid_Int *ci_ptr)
- braid_Int _braid_SetVerbosity (braid_Core core, braid_Int verbose_adj)
- braid_Int _braid_printf (const char *format,...)
- braid_Int _braid_ParFprintfFlush (FILE *file, braid_Int myid, const char *message,...)
- braid_Int _braid_Max (braid_Real *array, braid_Int size, braid_Real *max_val)
- braid_Int _braid_GetNEntries (braid_Real *_array, braid_Int array_len, braid_Int *k_ptr, braid_Real *array)


### 16.11.1 Detailed Description

Define XBraid internal headers for utility routines.
This file contains the headers for utility routines. Essentially, if a routine does not take braid_Core (or other XBraid specific structs) as an argument, then it's a utility routine.
16.11.2 Function Documentation
16.11.2.1 _braid_GetInterval()

```
braid_Int _braid_GetInterval (
    braid_Core core,
    braid_Int level,
    braid_Int interval_index,
    braid_Int * flo_ptr,
    braid_Int * fhi_ptr,
    braid_Int * ci_ptr )
```

Retrieve the time step indices at this level corresponding to a local FC interval given by interval_index. Argument ci_ptr is the time step index for the C-pt and flo_ptr and fhi_ptr are the smallest and largest F-pt indices in this interval. The C-pt is always to the right of the F-interval, but neither a C-pt or an F-interval are guaranteed. If the ci_ptr returns a -1 , there is no C-pt. If the flo_ptr is greater than the fhi_ptr, there is no F-interval.
16.11.2.2 _braid_GetNEntries()

```
braid_Int _braid_GetNEntries (
    braid_Real * __array,
    braid_Int array_len,
    braid_Int * k_ptr,
    braid_Real * array )
```

Copy $k$ entries from $*$ _array $*$ into array. If $k$ is negative, return the last $k$ entries. If positive, return the first $k$ entries. Upon exit, $k$ holds the number of residuals actually returned (in the case that $|\mathrm{k}|>$ array_len.)

If no entries are copied, $k=0$, array[ 0$]=-1.0$

### 16.11.2.3 _braid_Max()

```
braid_Int _braid_Max (
    braid_Real * array,
    braid_Int size,
    braid_Real * max_val )
```

This function finds the maximum value in a braid_Real array

### 16.11.2.4 _braid_ParFprintfFlush()

```
braid_Int _braid_ParFprintfFlush (
    FILE * file,
    braid_Int myid,
    const char * message,
    ... )
```

This is a function that allows for "sane" printing of information in parallel. Currently, only myid $=0$ prints, but this can be updated as needs change.

The string message is printed and can be multiple parameters in the standard $*$ C-format, like

```
message = '%1.2e is a format string', 1.24
```

16.11.2.5 _braid_printf()

```
braid_Int _braid_printf (
    const char * format,
    ... )
```

If set, print to _braid_printfile and then flush. Else print to standard out.
The string format can be multiple parameters in the standard $* \mathrm{C}$-format, like format $=1 \% 1.2 \mathrm{e}$ is a format string', 1.24
16.11.2.6 _braid_ProjectInterval()

```
braid_Int _braid_ProjectInterval (
    braid_Int ilower,
    braid_Int iupper,
    braid_Int index,
    braid_Int stride,
    braid_Int * pilower,
    braid_Int * piupper )
```

Project an interval onto a strided index space that contains the index 'index' and has stride 'stride'. An empty projection is represented by ilower > iupper.
16.11.2.7 _braid_SetVerbosity()

```
braid_Int __braid_SetVerbosity (
    braid_Core core,
    braid_Int verbose_adj )
```

Switch for displaying the XBraid actions. Used for debugging only.

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