ECNU ICPC
Team Reference Document

F0RE1GNERS

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Contents

1 First Thing First

1.1 Header

```
#include <bits/stdc++.h>
using namespace std;
using LL = long long;
#define FOR(i, x, y) for (decay<decltype(y)>::type i = (x), _##i
        = (y); i < _##i; ++i)
#define FORD(i, x, y) for (decay<decltype(x)>::type i = (x), _##
        i = (y); i > _##i; --i)
#ifdef zerol
#define dbg(x...) do { cout << "\033[32;1m" << #x << " -> "; err
        (x); } while (0)
void err() { cout << "\033[39;0m" << endl; }
template<template<typename...> class T, typename t, typename...
        A>
void err(T<t> a, A... x) { for (auto v: a) cout << v << ' '; err
        (x...); }
template<typename T, typename... A>
void err(T a, A... x) { cout << a << ' '; err(x...); }
#else
#define dbg(...)
#endif

// for CLion
// set(CMAKE_CXX_FLAGS "${CMAKE_CXX_FLAGS} -O2 -Ddebug -Wall -
        DULTMASTER -DKBlack -Dzerol")

// for University of Porto
// setxkbmap us
```

1.2 55kai

```
inline char nc() {
    static char buf[100000], *p1 = buf, *p2 = buf;
    return p1 == p2 && (p2 = (p1 = buf) + fread(buf, 1, 100000,
           stdin), p1 == p2) ? EOF : *p1++;
}
template <typename T>
bool rn(T& v) {
    static char ch;
    while (ch != EOF && !isdigit(ch)) ch = nc();
    if (ch == EOF) return false;
    for (v = 0; isdigit(ch); ch = nc())
        v = v * 10 + ch - '0';
    return true;
}

template <typename T>
void o(T p) {
    static int stk[70], tp;
    if (p == 0) { putchar('0'); return; }
    if (p < 0) { p = -p; putchar('-'); }
    while (p) stk[++tp] = p % 10, p /= 10;
    while (tp) putchar(stk[tp--] + '0');
}
```

2 Data Structure

2.1 RMQ

```
int f[maxn][maxn][10][10];
inline int highbit(int x) { return 31 - __builtin_clz(x); }
inline int calc(int x, int y, int xx, int yy, int p, int q) {
    return max(
        max(f[x][y][p][q], f[xx - (1 << p) + 1][yy - (1 << q) +
              1][p][q]),
        max(f[xx - (1 << p) + 1][y][p][q], f[x][yy - (1 << q) +
              1][p][q])
    );
}
void init() {
    FOR (x, 0, highbit(n) + 1)
    FOR (y, 0, highbit(m) + 1)
        FOR (i, 0, n - (1 << x) + 1)
        FOR (j, 0, m - (1 << y) + 1) {
            if (!x && !y) { f[i][j][x][y] = a[i][j]; continue; }
            f[i][j][x][y] = calc(
                i, j,
                i + (1 << x) - 1, j + (1 << y) - 1,
                max(x - 1, 0), max(y - 1, 0)
            );
        }
}
inline int get_max(int x, int y, int xx, int yy) {
    return calc(x, y, xx, yy, highbit(xx - x + 1), highbit(yy -
           y + 1));
}

struct RMQ {
    int f[22][M];
    inline int highbit(int x) { return 31 - __builtin_clz(x); }
    void init(int* v, int n) {
        FOR (i, 0, n) f[0][i] = v[i];
        FOR (x, 1, highbit(n) + 1)
            FOR (i, 0, n - (1 << x) + 1)
                f[x][i] = min(f[x - 1][i], f[x - 1][i + (1 << (x
                      - 1))]);
    }
    int get_min(int l, int r) {
        assert(l <= r);
        int t = highbit(r - l + 1);
        return min(f[t][l], f[t][r - (1 << t) + 1]);
    }
} rmq;
```

2.2 Segment Tree Beats

```
namespace R {
#define lson o * 2, l, (l + r) / 2
#define rson o * 2 + 1, (l + r) / 2 + 1, r
    int m1[N], m2[N], cm1[N];
    LL sum[N];
    void up(int o) {
        int lc = o * 2, rc = lc + 1;
        m1[o] = max(m1[lc], m1[rc]);
        sum[o] = sum[lc] + sum[rc];
        if (m1[lc] == m1[rc]) {
            cm1[o] = cm1[lc] + cm1[rc];
            m2[o] = max(m2[lc], m2[rc]);
        } else {
            cm1[o] = m1[lc] > m1[rc] ? cm1[lc] : cm1[rc];
            m2[o] = max(min(m1[lc], m1[rc]), max(m2[lc], m2[rc])
                  );
        }
    }
    void mod(int o, int x) {
        if (x >= m1[o]) return;
        assert(x > m2[o]);
        sum[o] -= 1LL * (m1[o] - x) * cm1[o];
        m1[o] = x;
    }
    void down(int o) {
        int lc = o * 2, rc = lc + 1;
        mod(lc, m1[o]); mod(rc, m1[o]);
    }
    void build(int o, int l, int r) {
        if (l == r) { int t; read(t); sum[o] = m1[o] = t; m2[o]
              = -INF; cm1[o] = 1; }
        else { build(lson); build(rson); up(o); }
    }
    void update(int ql, int qr, int x, int o, int l, int r) {
        if (r < ql || qr < l || m1[o] <= x) return;
        if (ql <= l && r <= qr && m2[o] < x) { mod(o, x); return
              ; }
        down(o);
        update(ql, qr, x, lson); update(ql, qr, x, rson);
        up(o);
    }
    int qmax(int ql, int qr, int o, int l, int r) {
        if (r < ql || qr < l) return -INF;
        if (ql <= l && r <= qr) return m1[o];
        down(o);
        return max(qmax(ql, qr, lson), qmax(ql, qr, rson));
    }
    LL qsum(int ql, int qr, int o, int l, int r) {
        if (r < ql || qr < l) return 0;
        if (ql <= l && r <= qr) return sum[o];
        down(o);
        return qsum(ql, qr, lson) + qsum(ql, qr, rson);
    }
}
```

2.3 Segment Tree

```
// set + add

struct IntervalTree {
#define ls o * 2, l, m
#define rs o * 2 + 1, m + 1, r
    static const LL M = maxn * 4, RS = 1E18 - 1;
    LL addv[M], setv[M], minv[M], maxv[M], sumv[M];
    void init() {
        memset(addv, 0, sizeof addv);
        fill(setv, setv + M, RS);
        memset(minv, 0, sizeof minv);
        memset(maxv, 0, sizeof maxv);
        memset(sumv, 0, sizeof sumv);
    }
    void maintain(LL o, LL l, LL r) {
        if (l < r) {
            LL lc = o * 2, rc = o * 2 + 1;
            sumv[o] = sumv[lc] + sumv[rc];
            minv[o] = min(minv[lc], minv[rc]);
            maxv[o] = max(maxv[lc], maxv[rc]);
        } else sumv[o] = minv[o] = maxv[o] = 0;
        if (setv[o] != RS) { minv[o] = maxv[o] = setv[o]; sumv[o
              ] = setv[o] * (r - l + 1); }
        if (addv[o]) { minv[o] += addv[o]; maxv[o] += addv[o];
              sumv[o] += addv[o] * (r - l + 1); }
    }
    void build(LL o, LL l, LL r) {
        if (l == r) addv[o] = a[l];
        else {
            LL m = (l + r) / 2;
            build(ls); build(rs);
        }
        maintain(o, l, r);
    }
    void pushdown(LL o) {
        LL lc = o * 2, rc = o * 2 + 1;
        if (setv[o] != RS) {
            setv[lc] = setv[rc] = setv[o];
            addv[lc] = addv[rc] = 0;
            setv[o] = RS;
        }
        if (addv[o]) {
            addv[lc] += addv[o]; addv[rc] += addv[o];
            addv[o] = 0;
        }
    }
    void update(LL p, LL q, LL o, LL l, LL r, LL v, LL op) {
        if (p <= r && l <= q)
        if (p <= l && r <= q) {
            if (op == 2) { setv[o] = v; addv[o] = 0; }
            else addv[o] += v;
        } else {
            pushdown(o);
            LL m = (l + r) / 2;
            update(p, q, ls, v, op); update(p, q, rs, v, op);
        }
        maintain(o, l, r);
    }
    void query(LL p, LL q, LL o, LL l, LL r, LL add, LL& ssum,
          LL& smin, LL& smax) {
        if (p > r || l > q) return;
        if (setv[o] != RS) {
            LL v = setv[o] + add + addv[o];
            ssum += v * (min(r, q) - max(l, p) + 1);
            smin = min(smin, v);
            smax = max(smax, v);
        } else if (p <= l && r <= q) {
            ssum += sumv[o] + add * (r - l + 1);
            smin = min(smin, minv[o] + add);
            smax = max(smax, maxv[o] + add);
        } else {
            LL m = (l + r) / 2;
            query(p, q, ls, add + addv[o], ssum, smin, smax);
            query(p, q, rs, add + addv[o], ssum, smin, smax);
        }
    }
} IT;
```

```
// persistent

namespace tree {
#define mid ((l + r) >> 1)
#define lson ql, qr, l, mid
#define rson ql, qr, mid + 1, r
    struct P {
        LL add, sum;
        int ls, rs;
    } tr[maxn * 45 * 2];
    int sz = 1;
    int N(LL add, int l, int r, int ls, int rs) {
        tr[sz] = {add, tr[ls].sum + tr[rs].sum + add * (len[r] -
              len[l - 1]), ls, rs};
        return sz++;
    }
    int update(int o, int ql, int qr, int l, int r, LL add) {
        if (ql > r || l > qr) return o;
        const P& t = tr[o];
        if (ql <= l && r <= qr) return N(add + t.add, l, r, t.ls
              , t.rs);
        return N(t.add, l, r, update(t.ls, lson, add), update(t.
              rs, rson, add));
    }
    LL query(int o, int ql, int qr, int l, int r, LL add = 0) {
        if (ql > r || l > qr) return 0;
        const P& t = tr[o];
        if (ql <= l && r <= qr) return add * (len[r] - len[l -
              1]) + t.sum;
        return query(t.ls, lson, add + t.add) + query(t.rs, rson
              , add + t.add);
    }
}
```

2.4 K-D Tree

```
// global variable pruning
// visit L/R with more potential
namespace kd {
    const int K = 2, inf = 1E9, M = N;
    const double lim = 0.7;
    struct P {
        int d[K], l[K], r[K], sz, val;
        LL sum;
        P *ls, *rs;
        P* up() {
            sz = ls->sz + rs->sz + 1;
            sum = ls->sum + rs->sum + val;
            FOR (i, 0, K) {
                l[i] = min(d[i], min(ls->l[i], rs->l[i]));
                r[i] = max(d[i], max(ls->r[i], rs->r[i]));
            }
            return this;
        }
    } pool[M], *null = new P, *pit = pool;
    static P *tmp[M], **pt;
    void init() {
        null->ls = null->rs = null;
        FOR (i, 0, K) null->l[i] = inf, null->r[i] = -inf;
        null->sum = null->val = 0;
        null->sz = 0;
    }

    P* build(P** l, P** r, int d = 0) { // [l, r)
        if (d == K) d = 0;
        if (l >= r) return null;
        P** m = l + (r - l) / 2; assert(l <= m && m < r);
        nth_element(l, m, r, [&](const P* a, const P* b){
            return a->d[d] < b->d[d];
        });
        P* o = *m;
        o->ls = build(l, m, d + 1); o->rs = build(m + 1, r, d +
              1);
        return o->up();
    }
    P* Build() {
        pt = tmp; FOR (it, pool, pit) *pt++ = it;
        return build(tmp, pt);
    }
    inline bool inside(int p[], int q[], int l[], int r[]) {
        FOR (i, 0, K) if (r[i] < q[i] || p[i] < l[i]) return
              false;
        return true;
    }
    LL query(P* o, int l[], int r[]) {
        if (o == null) return 0;
        FOR (i, 0, K) if (o->r[i] < l[i] || r[i] < o->l[i])
              return 0;
        if (inside(o->l, o->r, l, r)) return o->sum;
        return query(o->ls, l, r) + query(o->rs, l, r) +
               (inside(o->d, o->d, l, r) ? o->val : 0);
    }
    void dfs(P* o) {
        if (o == null) return;
        *pt++ = o; dfs(o->ls); dfs(o->rs);
    }
    P* ins(P* o, P* x, int d = 0) {
        if (d == K) d = 0;
        if (o == null) return x->up();
        P*& oo = x->d[d] <= o->d[d] ? o->ls : o->rs;
        if (oo->sz > o->sz * lim) {
            pt = tmp; dfs(o); *pt++ = x;
            return build(tmp, pt, d);
        }
        oo = ins(oo, x, d + 1);
        return o->up();
    }
}
```

2.5 STL+

```
// priority_queue

// binary_heap_tag
// pairing_heap_tag: support editing
// thin_heap_tag: fast when increasing, can't join
#include<ext/pb_ds/priority_queue.hpp>
using namespace __gnu_pbds;

typedef __gnu_pbds::priority_queue<LL, less<LL>,
      pairing_heap_tag> PQ;
__gnu_pbds::priority_queue<int, cmp, pairing_heap_tag>::
      point_iterator it;
PQ pq, pq2;

int main() {
    auto it = pq.push(2);
    pq.push(3);
    assert(pq.top() == 3);
    pq.modify(it, 4);
    assert(pq.top() == 4);
    pq2.push(5);
    pq.join(pq2);
    assert(pq.top() == 5);
}

// BBT

// ov_tree_tag
// rb_tree_tag
// splay_tree_tag

// mapped: null_typeor or null_mapped_type (old) is null
// Node_Update should be tree_order_statistics_node_update to
      use find_by_order & order_of_key
// find_by_order: find the element with order+1 (0-based)
// order_of_key: number of elements lt r_key
// support join & split

#include <ext/pb_ds/assoc_container.hpp>
using namespace __gnu_pbds;
using Tree = tree<int, null_type, less<int>, rb_tree_tag,
      tree_order_statistics_node_update>;
Tree t;

// Persistent BBT

#include <ext/rope>
using namespace __gnu_cxx;
rope<int> s;

int main() {
    FOR (i, 0, 5) s.push_back(i); // 0 1 2 3 4
    s.replace(1, 2, s); // 0 (0 1 2 3 4) 3 4
    auto ss = s.substr(2, 2); // 1 2
    s.erase(2, 2); // 0 1 4
    s.insert(2, s); // equal to s.replace(2, 0, s)
    assert(s[2] == s.at(2)); // 2
}

// Hash Table

#include<ext/pb_ds/assoc_container.hpp>
#include<ext/pb_ds/hash_policy.hpp>
using namespace __gnu_pbds;

gp_hash_table<int, int> mp;
cc_hash_table<int, int> mp;
```

2.6 BIT

```
namespace bit {
    LL c[M];
    inline int lowbit(int x) { return x & -x; }
    void add(int x, LL v) {
        for (; x < M; x += lowbit(x))
            c[x] += v;
    }
    LL sum(int x) {
        LL ret = 0;
        for (; x > 0; x -= lowbit(x))
            ret += c[x];
        return ret;
    }
    int kth(LL k) {
        int p = 0;
        for (int lim = 1 << 20; lim; lim /= 2)
            if (p + lim < M && c[p + lim] < k) {
                p += lim;
                k -= c[p];
            }
        return p + 1;
    }
}
namespace bit {
    int c[maxn], cc[maxn];
    inline int lowbit(int x) { return x & -x; }
    void add(int x, int v) {
        for (int i = x; i <= n; i += lowbit(i)) {
            c[i] += v; cc[i] += x * v;
        }
    }
    void add(int l, int r, int v) { add(l, v); add(r + 1, -v); }
    int sum(int x) {
        int ret = 0;
        for (int i = x; i > 0; i -= lowbit(i))
            ret += (x + 1) * c[i] - cc[i];
        return ret;
    }
    int sum(int l, int r) { return sum(r) - sum(l - 1); }
}
namespace bit {
    LL c[N], cc[N], ccc[N];
    inline LL lowbit(LL x) { return x & -x; }
    void add(LL x, LL v) {
        for (LL i = x; i < N; i += lowbit(i)) {
            c[i] = (c[i] + v) % MOD;
            cc[i] = (cc[i] + x * v) % MOD;
            ccc[i] = (ccc[i] + x * x % MOD * v) % MOD;
        }
    }
    void add(LL l, LL r, LL v) { add(l, v); add(r + 1, -v); }
    LL sum(LL x) {
        static LL INV2 = (MOD + 1) / 2;
        LL ret = 0;
        for (LL i = x; i > 0; i -= lowbit(i))
            ret += (x + 1) * (x + 2) % MOD * c[i] % MOD
                   - (2 * x + 3) * cc[i] % MOD
                   + ccc[i];
        return ret % MOD * INV2 % MOD;
    }
    LL sum(LL l, LL r) { return sum(r) - sum(l - 1); }
}
```

2.7 Trie

```
namespace trie {
    const int M = 31;
    int ch[N * M][2], sz;
    void init() { memset(ch, 0, sizeof ch); sz = 2; }
    void ins(LL x) {
        int u = 1;
        FORD (i, M, -1) {
            bool b = x & (1LL << i);
            if (!ch[u][b]) ch[u][b] = sz++;
            u = ch[u][b];
        }
    }
}

// persistent
// !!! sz = 1
struct P { int w, ls, rs; };
P tr[M] = {{0, 0, 0}};
int sz;

int _new(int w, int ls, int rs) { tr[sz] = {w, ls, rs}; return
     sz++; }
int ins(int oo, int v, int d = 30) {
    P& o = tr[oo];
    if (d == -1) return _new(o.w + 1, 0, 0);
    bool u = v & (1 << d);
    return _new(o.w + 1, u == 0 ? ins(o.ls, v, d - 1) : o.ls, u
          == 1 ? ins(o.rs, v, d - 1) : o.rs);
}
int query(int pp, int qq, int v, int d = 30) {
    if (d == -1) return 0;
    bool u = v & (1 << d);
    P &p = tr[pp], &q = tr[qq];
    int lw = tr[q.ls].w - tr[p.ls].w;
    int rw = tr[q.rs].w - tr[p.rs].w;

    int ret = 0;
    if (u == 0) {
        if (rw) { ret += 1 << d; ret += query(p.rs, q.rs, v, d -
             1); }
        else ret += query(p.ls, q.ls, v, d - 1);
    } else {
        if (lw) { ret += 1 << d; ret += query(p.ls, q.ls, v, d -
             1); }
        else ret += query(p.rs, q.rs, v, d - 1);
    }
    return ret;
}
```

2.8 Treap

```
// set
namespace treap {
    const int M = maxn * 17;
    extern struct P* const null;
    struct P {
        P *ls, *rs;
        int v, sz;
        unsigned rd;
        P(int v): ls(null), rs(null), v(v), sz(1), rd(rnd()) {}
        P(): sz(0) {}

        P* up() { sz = ls->sz + rs->sz + 1; return this; }
        int lower(int v) {
            if (this == null) return 0;
            return this->v >= v ? ls->lower(v) : rs->lower(v) +
                 ls->sz + 1;
        }
        int upper(int v) {
            if (this == null) return 0;
            return this->v > v ? ls->upper(v) : rs->upper(v) +
                 ls->sz + 1;
        }
    } *const null = new P, pool[M], *pit = pool;

    P* merge(P* l, P* r) {
        if (l == null) return r; if (r == null) return l;
        if (l->rd < r->rd) { l->rs = merge(l->rs, r); return l->
             up(); }
        else { r->ls = merge(l, r->ls); return r->up(); }
    }

    void split(P* o, int rk, P*& l, P*& r) {
        if (o == null) { l = r = null; return; }
        if (o->ls->sz >= rk) { split(o->ls, rk, l, o->ls); r = o
             ->up(); }
        else { split(o->rs, rk - o->ls->sz - 1, o->rs, r); l = o
             ->up(); }
    }
}
// persistent set
namespace treap {
    const int M = maxn * 17 * 12;
    extern struct P* const null, *pit;
    struct P {
        P *ls, *rs;
        int v, sz;
        LL sum;
        P(P* ls, P* rs, int v): ls(ls), rs(rs), v(v), sz(ls->sz
              + rs->sz + 1),
                                                      sum(ls->sum
                                                             + rs
                                                            ->sum
                                                             + v)
                                                             {}
        P() {}

        void* operator new(size_t _) { return pit++; }
        template<typename T>
        int rk(int v, T&& cmp) {
            if (this == null) return 0;
            return cmp(this->v, v) ? ls->rk(v, cmp) : rs->rk(v,
                  cmp) + ls->sz + 1;
        }
        int lower(int v) { return rk(v, greater_equal<int>()); }
        int upper(int v) { return rk(v, greater<int>()); }
    } pool[M], *pit = pool, *const null = new P;
    P* merge(P* l, P* r) {
        if (l == null) return r; if (r == null) return l;
        if (rnd() % (l->sz + r->sz) < l->sz) return new P{l->ls,
               merge(l->rs, r), l->v};
        else return new P{merge(l, r->ls), r->rs, r->v};
    }
    void split(P* o, int rk, P*& l, P*& r) {
        if (o == null) { l = r = null; return; }
        if (o->ls->sz >= rk) { split(o->ls, rk, l, r); r = new P
              {r, o->rs, o->v}; }
        else { split(o->rs, rk - o->ls->sz - 1, l, r); l = new P
              {o->ls, l, o->v}; }
    }
}
// persistent set with pushdown
int now;
namespace Treap {
    const int M = 10000000;
    extern struct P* const null, *pit;
    struct P {
        P *ls, *rs;
        int sz, time;
        LL cnt, sc, pos, add;
        bool rev;

        P* up() { sz = ls->sz + rs->sz + 1; sc = ls->sc + rs->sc
               + cnt; return this; } // MOD
        P* check() {
            if (time == now) return this;
            P* t = new(pit++) P; *t = *this; t->time = now;
                  return t;
        };
        P* _do_rev() { rev ^= 1; add *= -1; pos *= -1; swap(ls,
              rs); return this; } // MOD
        P* _do_add(LL v) { add += v; pos += v; return this; } //
               MOD
        P* do_rev() { if (this == null) return this; return
              check()->_do_rev(); } // FIX & MOD
        P* do_add(LL v) { if (this == null) return this; return
              check()->_do_add(v); } // FIX & MOD
        P* _down() { // MOD
            if (rev) { ls = ls->do_rev(); rs = rs->do_rev(); rev
                   = 0; }
            if (add) { ls = ls->do_add(add); rs = rs->do_add(add
                  ); add = 0; }
            return this;
        }
        P* down() { return check()->_down(); } // FIX & MOD
        void _split(LL p, P*& l, P*& r) { // MOD
            if (pos >= p) { ls->split(p, l, r); ls = r; r = up()
                  ; }
            else          { rs->split(p, l, r); rs = l; l = up()
                  ; }
        }
        void split(LL p, P*& l, P*& r) { // FIX & MOD
            if (this == null) l = r = null;
            else down()->_split(p, l, r);
        }
    } pool[M], *pit = pool, *const null = new P;
    P* merge(P* a, P* b) {
        if (a == null) return b; if (b == null) return a;
        if (rand() % (a->sz + b->sz) < a->sz) { a = a->down(); a
              ->rs = merge(a->rs, b); return a->up(); }
        else                                  { b = b->down(); b
              ->ls = merge(a, b->ls); return b->up(); }
    }
}
// sequence with add, sum
namespace treap {
    const int M = 8E5 + 100;
    extern struct P*const null;
    struct P {
        P *ls, *rs;
        int sz, val, add, sum;
        P(int v, P* ls = null, P* rs = null): ls(ls), rs(rs), sz
              (1), val(v), add(0), sum(v) {}
        P(): sz(0), val(0), add(0), sum(0) {}

        P* up() {
            assert(this != null);
            sz = ls->sz + rs->sz + 1;
            sum = ls->sum + rs->sum + val + add * sz;
            return this;
        }
        void upd(int v) {
            if (this == null) return;
            add += v;
            sum += sz * v;
        }
        P* down() {
            if (add) {
                ls->upd(add); rs->upd(add);
                val += add;
                add = 0;
            }
            return this;
        }

        P* select(int rk) {
            if (rk == ls->sz + 1) return this;
            return ls->sz >= rk ? ls->select(rk) : rs->select(rk
                   - ls->sz - 1);
        }
    } pool[M], *pit = pool, *const null = new P, *rt = null;

    P* merge(P* a, P* b) {
        if (a == null) return b->up();
        if (b == null) return a->up();
        if (rand() % (a->sz + b->sz) < a->sz) {
            a->down()->rs = merge(a->rs, b);
            return a->up();
        } else {
            b->down()->ls = merge(a, b->ls);
            return b->up();
        }
    }

    void split(P* o, int rk, P*& l, P*& r) {
        if (o == null) { l = r = null; return; }
        o->down();
        if (o->ls->sz >= rk) {
            split(o->ls, rk, l, o->ls);
            r = o->up();
        } else {
            split(o->rs, rk - o->ls->sz - 1, o->rs, r);
            l = o->up();
        }
    }

    inline void insert(int k, int v) {
        P *l, *r;
        split(rt, k - 1, l, r);
        rt = merge(merge(l, new (pit++) P(v)), r);
    }

    inline void erase(int k) {
        P *l, *r, *_, *t;
        split(rt, k - 1, l, t);
        split(t, 1, _, r);
        rt = merge(l, r);
    }

    P* build(int l, int r, int* a) {
        if (l > r) return null;
        if (l == r) return new(pit++) P(a[l]);
        int m = (l + r) / 2;
        return (new(pit++) P(a[m], build(l, m - 1, a), build(m +
               1, r, a)))->up();
    }
};
// persistent sequence
namespace treap {
    struct P;
    extern P*const null;
    P* N(P* ls, P* rs, LL v, bool fill);
    struct P {
        P *const ls, *const rs;
        const int sz, v;
        const LL sum;
        bool fill;
        int cnt;

        void split(int k, P*& l, P*& r) {
            if (this == null) { l = r = null; return; }
            if (ls->sz >= k) {
                ls->split(k, l, r);
                r = N(r, rs, v, fill);
            } else {
                rs->split(k - ls->sz - fill, l, r);
                l = N(ls, l, v, fill);
            }
        }


    } *const null = new P{0, 0, 0, 0, 0, 0, 1};

    P* N(P* ls, P* rs, LL v, bool fill) {
        ls->cnt++; rs->cnt++;
        return new P{ls, rs, ls->sz + rs->sz + fill, v, ls->sum
              + rs->sum + v, fill, 1};
    }

    P* merge(P* a, P* b) {
        if (a == null) return b;
        if (b == null) return a;
        if (rand() % (a->sz + b->sz) < a->sz)
            return N(a->ls, merge(a->rs, b), a->v, a->fill);
        else
            return N(merge(a, b->ls), b->rs, b->v, b->fill);
    }

    void go(P* o, int x, int y, P*& l, P*& m, P*& r) {
        o->split(y, l, r);
        l->split(x - 1, l, m);
    }
}
```

2.9 Cartesian Tree

```
void build() {
    static int s[N], last;
    int p = 0;
    FOR (x, 1, n + 1) {
        last = 0;
        while (p && val[s[p - 1]] > val[x]) last = s[--p];
        if (p) G[s[p - 1]][1] = x;
        if (last) G[x][0] = last;
        s[p++] = x;
    }
    rt = s[0];
}
```

2.10 LCT

```
// do not forget down when findint L/R most son
// make_root if not sure

namespace lct {
    extern struct P *const null;
    const int M = N;
    struct P {
        P *fa, *ls, *rs;
        int v, maxv;
        bool rev;

        bool has_fa() { return fa->ls == this || fa->rs == this;
             }
        bool d() { return fa->ls == this; }
        P*& c(bool x) { return x ? ls : rs; }
        void do_rev() {
            if (this == null) return;
            rev ^= 1;
            swap(ls, rs);
        }
        P* up() {
            maxv = max(v, max(ls->maxv, rs->maxv));
            return this;
        }
        void down() {
            if (rev) {
                rev = 0;
                ls->do_rev(); rs->do_rev();
            }
        }
        void all_down() { if (has_fa()) fa->all_down(); down();
             }
    } *const null = new P{0, 0, 0, 0, 0, 0}, pool[M], *pit =
         pool;

    void rot(P* o) {
        bool dd = o->d();
        P *f = o->fa, *t = o->c(!dd);
        if (f->has_fa()) f->fa->c(f->d()) = o; o->fa = f->fa;
        if (t != null) t->fa = f; f->c(dd) = t;
        o->c(!dd) = f->up(); f->fa = o;
    }
    void splay(P* o) {
        o->all_down();
        while (o->has_fa()) {
            if (o->fa->has_fa())
                rot(o->d() ^ o->fa->d() ? o : o->fa);
            rot(o);
        }
        o->up();
    }
    void access(P* u, P* v = null) {
        if (u == null) return;
        splay(u); u->rs = v;
        access(u->up()->fa, u);
    }
    void make_root(P* o) {
        access(o); splay(o); o->do_rev();
    }
    void split(P* o, P* u) {
        make_root(o); access(u); splay(u);
    }
    void link(P* u, P* v) {
        make_root(u); u->fa = v;
    }
    void cut(P* u, P* v) {
        split(u, v);
        u->fa = v->ls = null; v->up();
    }
    bool adj(P* u, P* v) {
        split(u, v);
        return v->ls == u && u->ls == null && u->rs == null;
    }
    bool linked(P* u, P* v) {
        split(u, v);
        return u == v || u->fa != null;
    }
    P* findrt(P* o) {
        access(o); splay(o);
        while (o->ls != null) o = o->ls;
        return o;
    }
    P* findfa(P* rt, P* u) {
        split(rt, u);
        u = u->ls;
        while (u->rs != null) {
            u = u->rs;
            u->down();
        }
        return u;
    }
}
// maintain subtree size
P* up() {
    sz = ls->sz + rs->sz + _sz + 1;
    return this;
}
void access(P* u, P* v = null) {
    if (u == null) return;
    splay(u);
    u->_sz += u->rs->sz - v->sz;
    u->rs = v;
    access(u->up()->fa, u);
}
void link(P* u, P* v) {
    split(u, v);
    u->fa = v; v->_sz += u->sz;
    v->up();
}

//////////////////////
// latest spanning tree
//////////////////////
namespace lct {
    extern struct P* null;
    struct P {
        P *fa, *ls, *rs;
        int v;
        P *minp;
        bool rev;

        bool has_fa() { return fa->ls == this || fa->rs == this;
             }
        bool d() { return fa->ls == this; }
        P*& c(bool x) { return x ? ls : rs; }
        void do_rev() { if (this == null) return; rev ^= 1; swap
             (ls, rs); }

        P* up() {
            minp = this;
            if (minp->v > ls->minp->v) minp = ls->minp;
            if (minp->v > rs->minp->v) minp = rs->minp;
            return this;
        }
        void down() { if (rev) { rev = 0; ls->do_rev(); rs->
             do_rev(); }}
        void all_down() { if (has_fa()) fa->all_down(); down();
             }
    } *null = new P{0, 0, 0, INF, 0, 0}, pool[maxm], *pit = pool
         ;
    void rot(P* o) {
        bool dd = o->d();
        P *f = o->fa, *t = o->c(!dd);
        if (f->has_fa()) f->fa->c(f->d()) = o; o->fa = f->fa;
        if (t != null) t->fa = f; f->c(dd) = t;
        o->c(!dd) = f->up(); f->fa = o;
    }
    void splay(P* o) {
        o->all_down();
        while (o->has_fa()) {
            if (o->fa->has_fa()) rot(o->d() ^ o->fa->d() ? o : o
                 ->fa);
            rot(o);
        }
        o->up();
    }
    void access(P* u, P* v = null) {
        if (u == null) return;
        splay(u); u->rs = v;
        access(u->up()->fa, u);
    }
    void make_root(P* o) { access(o); splay(o); o->do_rev(); }
    void split(P* u, P* v) { make_root(u); access(v); splay(v);
         }
    bool linked(P* u, P* v) { split(u, v); return u == v || u->
         fa != null; }
    void link(P* u, P* v) { make_root(u); u->fa = v; }
    void cut(P* u, P* v) { split(u, v); u->fa = v->ls = null; v
         ->up(); }
}

using namespace lct;
int n, m;
P *p[maxn];
struct Q {
    int tp, u, v, l, r;
};
vector<Q> q;

int main() {
    null->minp = null;
    cin >> n >> m;
    FOR (i, 1, n + 1) p[i] = new (pit++) P{null, null, null, INF
         , p[i], 0};
    int clk = 0;
    map<pair<int, int>, int> mp;
    FOR (_, 0, m) {
        int tp, u, v; scanf("%d%d%d", &tp, &u, &v);
        if (u > v) swap(u, v);
        if (tp == 0) mp.insert({{u, v}, clk});
        else if (tp == 1) {
            auto it = mp.find({u, v}); assert(it != mp.end());
            q.push_back({1, u, v, it->second, clk});
            mp.erase(it);
        } else q.push_back({0, u, v, clk, clk});
        ++clk;
    }
    for (auto& x: mp) q.push_back({1, x.first.first, x.first.
         second, x.second, clk});
    sort(q.begin(), q.end(), [](const Q& a, const Q& b)->bool {
         return a.l < b.l; });
    map<P*, int> mp2;
    FOR (i, 0, q.size()) {
        Q& cur = q[i];
        int u = cur.u, v = cur.v;
        if (cur.tp == 0) {
            if (!linked(p[u], p[v])) puts("N");
            else puts(p[v]->minp->v >= cur.r ? "Y" : "N");
            continue;
        }
        if (linked(p[u], p[v])) {
            P* t = p[v]->minp;
            if (t->v > cur.r) continue;
            Q& old = q[mp2[t]];
            cut(p[old.u], t); cut(p[old.v], t);
        }
        P* t = new (pit++) P {null, null, null, cur.r, t, 0};
        mp2[t] = i;
        link(t, p[u]); link(t, p[v]);
    }
}
```

2.11 Mo’s Algorithm On Tree

```
struct Q {
    int u, v, idx;
    bool operator < (const Q& b) const {
        const Q& a = *this;
        return blk[a.u] < blk[b.u] || (blk[a.u] == blk[b.u] &&
             in[a.v] < in[b.v]);
    }
};

void dfs(int u = 1, int d = 0) {
    static int S[maxn], sz = 0, blk_cnt = 0, clk = 0;
    in[u] = clk++;
    dep[u] = d;
    int btm = sz;
    for (int v: G[u]) {
        if (v == fa[u]) continue;
        fa[v] = u;
        dfs(v, d + 1);
        if (sz - btm >= B) {
            while (sz > btm) blk[S[--sz]] = blk_cnt;
            ++blk_cnt;
        }
    }
    S[sz++] = u;
    if (u == 1) while (sz) blk[S[--sz]] = blk_cnt - 1;
}

void flip(int k) {
    dbg(k);
    if (vis[k]) {
        // ...
    } else {
        // ...
    }
    vis[k] ^= 1;
}

void go(int& k) {
    if (bug == -1) {
        if (vis[k] && !vis[fa[k]]) bug = k;
        if (!vis[k] && vis[fa[k]]) bug = fa[k];
    }
    flip(k);
    k = fa[k];
}

void mv(int a, int b) {
    bug = -1;
    if (vis[b]) bug = b;
    if (dep[a] < dep[b]) swap(a, b);
    while (dep[a] > dep[b]) go(a);
    while (a != b) {
        go(a); go(b);
    }
    go(a); go(bug);
}

for (Q& q: query) {
    mv(u, q.u); u = q.u;
    mv(v, q.v); v = q.v;
    ans[q.idx] = Ans;
}
```

2.12 CDQ's Divide and Conquer

```
const int maxn = 2E5 + 100;
struct P {
    int x, y;
    int* f;
    bool d1, d2;
} a[maxn], b[maxn], c[maxn];
int f[maxn];

void go2(int l, int r) {
    if (l + 1 == r) return;
    int m = (l + r) >> 1;
    go2(l, m); go2(m, r);
    FOR (i, l, m) b[i].d2 = 0;
    FOR (i, m, r) b[i].d2 = 1;
    merge(b + l, b + m, b + m, b + r, c + l, [](const P& a,
          const P& b)->bool {
            if (a.y != b.y) return a.y < b.y;
            return a.d2 > b.d2;
        });
    int mx = -1;
    FOR (i, l, r) {
        if (c[i].d1 && c[i].d2) *c[i].f = max(*c[i].f, mx + 1);
        if (!c[i].d1 && !c[i].d2) mx = max(mx, *c[i].f);
    }
    FOR (i, l, r) b[i] = c[i];
}

void go1(int l, int r) { // [l, r)
    if (l + 1 == r) return;
    int m = (l + r) >> 1;
    go1(l, m);
    FOR (i, l, m) a[i].d1 = 0;
    FOR (i, m, r) a[i].d1 = 1;
    copy(a + l, a + r, b + l);
    sort(b + l, b + r, [](const P& a, const P& b)->bool {
            if (a.x != b.x) return a.x < b.x;
            return a.d1 > b.d1;
        });
    go2(l, r);
    go1(m, r);
}
```

2.13 Persistent Segment Tree

```
namespace tree {
#define mid ((l + r) >> 1)
#define lson l, mid
#define rson mid + 1, r
    const int MAGIC = M * 30;
    struct P {
        int sum, ls, rs;
    } tr[MAGIC] = {{0, 0, 0}};
    int sz = 1;
    int N(int sum, int ls, int rs) {
        if (sz == MAGIC) assert(0);
        tr[sz] = {sum, ls, rs};
        return sz++;
    }
    int ins(int o, int x, int v, int l = 1, int r = ls) {
        if (x < l || x > r) return o;
        const P& t = tr[o];
        if (l == r) return N(t.sum + v, 0, 0);
        return N(t.sum + v, ins(t.ls, x, v, lson), ins(t.rs, x,
             v, rson));
    }
    int query(int o, int ql, int qr, int l = 1, int r = ls) {
        if (ql > r || l > qr) return 0;
        const P& t = tr[o];
        if (ql <= l && r <= qr) return t.sum;
        return query(t.ls, ql, qr, lson) + query(t.rs, ql, qr,
             rson);
    }
}
// kth
int query(int pp, int qq, int l, int r, int k) { // (pp, qq]
    if (l == r) return l;
    const P &p = tr[pp], &q = tr[qq];
    int w = tr[q.ls].w - tr[p.ls].w;
    if (k <= w) return query(p.ls, q.ls, lson, k);
    else return query(p.rs, q.rs, rson, k - w);
}

////////////////////////
// with bit
////////////////////////

typedef vector<int> VI;
struct TREE {
#define mid ((l + r) >> 1)
#define lson l, mid
#define rson mid + 1, r
    struct P {
        int w, ls, rs;
    } tr[maxn * 20 * 20];
    int sz = 1;
    TREE() { tr[0] = {0, 0, 0}; }
    int N(int w, int ls, int rs) {
        tr[sz] = {w, ls, rs};
        return sz++;
    }
    int add(int tt, int l, int r, int x, int d) {
        if (x < l || r < x) return tt;
        const P& t = tr[tt];
        if (l == r) return N(t.w + d, 0, 0);
        return N(t.w + d, add(t.ls, lson, x, d), add(t.rs, rson,
             x, d));
    }
    int ls_sum(const VI& rt) {
        int ret = 0;
        FOR (i, 0, rt.size())
            ret += tr[tr[rt[i]].ls].w;
        return ret;
    }
    inline void ls(VI& rt) { transform(rt.begin(), rt.end(), rt.
         begin(), [&](int x)->int{ return tr[x].ls; }); }
    inline void rs(VI& rt) { transform(rt.begin(), rt.end(), rt.
         begin(), [&](int x)->int{ return tr[x].rs; }); }
    int query(VI& p, VI& q, int l, int r, int k) {
        if (l == r) return l;
        int w = ls_sum(q) - ls_sum(p);
        if (k <= w) {
            ls(p); ls(q);
            return query(p, q, lson, k);
        }
        else {
            rs(p); rs(q);
            return query(p, q, rson, k - w);
        }
    }
} tree;
struct BIT {
    int root[maxn];
    void init() { memset(root, 0, sizeof root); }
    inline int lowbit(int x) { return x & -x; }
    void update(int p, int x, int d) {
        for (int i = p; i <= m; i += lowbit(i))
            root[i] = tree.add(root[i], 1, m, x, d);
    }
    int query(int l, int r, int k) {
        VI p, q;
        for (int i = l - 1; i > 0; i -= lowbit(i)) p.push_back(
             root[i]);
        for (int i = r; i > 0; i -= lowbit(i)) q.push_back(root[
             i]);
        return tree.query(p, q, 1, m, k);
    }
} bit;

void init() {
    m = 10000;
    tree.sz = 1;
    bit.init();
    FOR (i, 1, m + 1)
        bit.update(i, a[i], 1);
}
```

2.14 Persistent Union Find

```
namespace uf {
    int fa[maxn], sz[maxn];
    int undo[maxn], top;
    void init() { memset(fa, -1, sizeof fa); memset(sz, 0,
         sizeof sz); top = 0; }
    int findset(int x) { while (fa[x] != -1) x = fa[x]; return x
          ; }
    bool join(int x, int y) {
        x = findset(x); y = findset(y);
        if (x == y) return false;
        if (sz[x] > sz[y]) swap(x, y);
        undo[top++] = x;
        fa[x] = y;
        sz[y] += sz[x] + 1;
        return true;
    }
    inline int checkpoint() { return top; }
    void rewind(int t) {
        while (top > t) {
            int x = undo[--top];
            sz[fa[x]] -= sz[x] + 1;
            fa[x] = -1;
        }
    }
}
```

3 Math

3.1 Multiplication, Powers

```
LL mul(LL u, LL v, LL p) {
    return (u * v - LL((long double) u * v / p) * p + p) % p;
}
LL mul(LL u, LL v, LL p) { // better constant
    LL t = u * v - LL((long double) u * v / p) * p;
    return t < 0 ? t + p : t;
}
LL bin(LL x, LL n, LL MOD) {
    n %= (MOD - 1); // if MOD is prime
    LL ret = MOD != 1;
    for (x %= MOD; n; n >>= 1, x = mul(x, x, MOD))
        if (n & 1) ret = mul(ret, x, MOD);
    return ret;
}
```

3.2 Matrix Power

```
struct Mat {
    static const LL M = 2;
    LL v[M][M];
    Mat() { memset(v, 0, sizeof v); }
    void eye() { FOR (i, 0, M) v[i][i] = 1; }
    LL* operator [] (LL x) { return v[x]; }
    const LL* operator [] (LL x) const { return v[x]; }
    Mat operator * (const Mat& B) {
        const Mat& A = *this;
        Mat ret;
        FOR (k, 0, M)
            FOR (i, 0, M) if (A[i][k])
                FOR (j, 0, M)
                    ret[i][j] = (ret[i][j] + A[i][k] * B[k][j])
                         % MOD;
        return ret;
    }
    Mat pow(LL n) const {
        Mat A = *this, ret; ret.eye();
        for (; n; n >>= 1, A = A * A)
            if (n & 1) ret = ret * A;
        return ret;
    }
    Mat operator + (const Mat& B) {
        const Mat& A = *this;
        Mat ret;
        FOR (i, 0, M)
            FOR (j, 0, M)
                 ret[i][j] = (A[i][j] + B[i][j]) % MOD;
        return ret;
    }
    void prt() const {
        FOR (i, 0, M)
            FOR (j, 0, M)
                printf("%lld%c", (*this)[i][j], j == M - 1 ? '\
                      n' : ' ');
    }
};
```

3.3 Sieve

```
const LL p_max = 1E5 + 100;
LL phi[p_max];
void get_phi() {
    phi[1] = 1;
    static bool vis[p_max];
    static LL prime[p_max], p_sz, d;
    FOR (i, 2, p_max) {
        if (!vis[i]) {
            prime[p_sz++] = i;
            phi[i] = i - 1;
        }
        for (LL j = 0; j < p_sz && (d = i * prime[j]) < p_max;
             ++j) {
            vis[d] = 1;
            if (i % prime[j] == 0) {
                phi[d] = phi[i] * prime[j];
                break;
            }
            else phi[d] = phi[i] * (prime[j] - 1);
        }
    }
}
// mobius
const LL p_max = 1E5 + 100;
LL mu[p_max];
void get_mu() {
    mu[1] = 1;
    static bool vis[p_max];
    static LL prime[p_max], p_sz, d;
    mu[1] = 1;
    FOR (i, 2, p_max) {
        if (!vis[i]) {
            prime[p_sz++] = i;
            mu[i] = -1;
        }
        for (LL j = 0; j < p_sz && (d = i * prime[j]) < p_max;
             ++j) {
            vis[d] = 1;
            if (i % prime[j] == 0) {
                mu[d] = 0;
                break;
            }
            else mu[d] = -mu[i];
        }
    }
}
// min_25
namespace min25 {
    const int M = 1E6 + 100;
    LL B, N;
    // g(x)
    inline LL pg(LL x) { return 1; }
    inline LL ph(LL x) { return x % MOD; }
    // Sum[g(i),{x,2,x}]
    inline LL psg(LL x) { return x % MOD - 1; }
    inline LL psh(LL x) {
        static LL inv2 = (MOD + 1) / 2;
        x = x % MOD;
        return x * (x + 1) % MOD * inv2 % MOD - 1;
    }
    // f(pp=p^k)
    inline LL fpk(LL p, LL e, LL pp) { return (pp - pp / p) %
         MOD; }
    // f(p) = fgh(g(p), h(p))
    inline LL fgh(LL g, LL h) { return h - g; }

    LL pr[M], pc, sg[M], sh[M];
    void get_prime(LL n) {
        static bool vis[M]; pc = 0;
        FOR (i, 2, n + 1) {
            if (!vis[i]) {
                pr[pc++] = i;
                sg[pc] = (sg[pc - 1] + pg(i)) % MOD;
                sh[pc] = (sh[pc - 1] + ph(i)) % MOD;
            }
            FOR (j, 0, pc) {
                if (pr[j] * i > n) break;
                vis[pr[j] * i] = 1;
                if (i % pr[j] == 0) break;
            }
        }
    }
    LL w[M];
    LL id1[M], id2[M], h[M], g[M];
    inline LL id(LL x) { return x <= B ? id1[x] : id2[N / x]; }
    LL go(LL x, LL k) {
        if (x <= 1 || (k >= 0 && pr[k] > x)) return 0;
        LL t = id(x);
        LL ans = fgh((g[t] - sg[k + 1]), (h[t] - sh[k + 1]));
        FOR (i, k + 1, pc) {
            LL p = pr[i];
            if (p * p > x) break;
            ans -= fgh(pg(p), ph(p));
            for (LL pp = p, e = 1; pp <= x; ++e, pp = pp * p)
                ans += fpk(p, e, pp) * (1 + go(x / pp, i)) % MOD
                      ;
        }
        return ans % MOD;
    }
    LL solve(LL _N) {
        N = _N;
        B = sqrt(N + 0.5);
        get_prime(B);
        int sz = 0;
        for (LL l = 1, v, r; l <= N; l = r + 1) {
            v = N / l; r = N / v;
            w[sz] = v; g[sz] = psg(v); h[sz] = psh(v);
            if (v <= B) id1[v] = sz; else id2[r] = sz;
            sz++;
        }
        FOR (k, 0, pc) {
            LL p = pr[k];
            FOR (i, 0, sz) {
                LL v = w[i]; if (p * p > v) break;
                LL t = id(v / p);
                g[i] = (g[i] - (g[t] - sg[k]) * pg(p)) % MOD;
                h[i] = (h[i] - (h[t] - sh[k]) * ph(p)) % MOD;
            }
        }
        return (go(N, -1) % MOD + MOD + 1) % MOD;
    }
}

// see cheatsheet for instructions
namespace dujiao {
    const int M = 5E6;
    LL f[M] = {0, 1};
    void init() {
        static bool vis[M];
        static LL pr[M], p_sz, d;
        FOR (i, 2, M) {
            if (!vis[i]) { pr[p_sz++] = i; f[i] = -1; }
            FOR (j, 0, p_sz) {
                if ((d = pr[j] * i) >= M) break;
                vis[d] = 1;
                if (i % pr[j] == 0) {
                    f[d] = 0;
                    break;
                } else f[d] = -f[i];
            }
        }
        FOR (i, 2, M) f[i] += f[i - 1];
    }
    inline LL s_fg(LL n) { return 1; }
    inline LL s_g(LL n) { return n; }
    LL N, rd[M];
    bool vis[M];
    LL go(LL n) {
        if (n < M) return f[n];
        LL id = N / n;
        if (vis[id]) return rd[id];
        vis[id] = true;
        LL& ret = rd[id] = s_fg(n);
        for (LL l = 2, v, r; l <= n; l = r + 1) {
            v = n / l; r = n / v;
            ret -= (s_g(r) - s_g(l - 1)) * go(v);
        }
        return ret;
    }
    LL solve(LL n) {
        N = n;
        memset(vis, 0, sizeof vis);
        return go(n);
    }
}
```

3.4 Prime Test

```
bool checkQ(LL a, LL n) {
    if (n == 2 || a >= n) return 1;
    if (n == 1 || !(n & 1)) return 0;
    LL d = n - 1;
    while (!(d & 1)) d >>= 1;
    LL t = bin(a, d, n);  // usually needs mul-on-LL
    while (d != n - 1 && t != 1 && t != n - 1) {
        t = mul(t, t, n);
        d <<= 1;
    }
    return t == n - 1 || d & 1;
}

bool primeQ(LL n) {
    static vector<LL> t = {2, 325, 9375, 28178, 450775, 9780504,
          1795265022};
    if (n <= 1) return false;
    for (LL k: t) if (!checkQ(k, n)) return false;
    return true;
}
```

3.5 Pollard-Rho

```
mt19937 mt(time(0));
LL pollard_rho(LL n, LL c) {
    LL x = uniform_int_distribution<LL>(1, n - 1)(mt), y = x;
    auto f = [&](LL v) { LL t = mul(v, v, n) + c; return t < n ?
          t : t - n; };
    while (1) {
        x = f(x); y = f(f(y));
        if (x == y) return n;
        LL d = gcd(abs(x - y), n);
        if (d != 1) return d;
    }
}

LL fac[100], fcnt;
void get_fac(LL n, LL cc = 19260817) {
    if (n == 4) { fac[fcnt++] = 2; fac[fcnt++] = 2; return; }
    if (primeQ(n)) { fac[fcnt++] = n; return; }
    LL p = n;
    while (p == n) p = pollard_rho(n, --cc);
    get_fac(p); get_fac(n / p);
}
```

3.6 Berlekamp-Massey

```
namespace BerlekampMassey {
    inline void up(LL& a, LL b) { (a += b) %= MOD; }
    V mul(const V&a, const V& b, const V& m, int k) {
        V r; r.resize(2 * k - 1);
        FOR (i, 0, k) FOR (j, 0, k) up(r[i + j], a[i] * b[j]);
        FORD (i, k - 2, -1) {
            FOR (j, 0, k) up(r[i + j], r[i + k] * m[j]);
            r.pop_back();
        }
        return r;
    }
    V pow(LL n, const V& m) {
        int k = (int) m.size() - 1; assert (m[k] == -1 || m[k]
              == MOD - 1);
        V r(k), x(k); r[0] = x[1] = 1;
        for (; n; n >>= 1, x = mul(x, x, m, k))
            if (n & 1) r = mul(x, r, m, k);
        return r;
    }
    LL go(const V& a, const V& x, LL n) {
        // a: (-1, a1, a2, ..., ak).reverse
        // x: x1, x2, ..., xk
        // x[n] = sum[a[i]*x[n-i],{i,1,k}]
        int k = (int) a.size() - 1;
        if (n <= k) return x[n - 1];
        if (a.size() == 2) return x[0] * bin(a[0], n - 1, MOD) %
              MOD;
        V r = pow(n - 1, a);
        LL ans = 0;
        FOR (i, 0, k) up(ans, r[i] * x[i]);
        return (ans + MOD) % MOD;
    }
    V BM(const V& x) {
        V a = {-1}, b = {233}, t;
        FOR (i, 1, x.size()) {
            b.push_back(0);
            LL d = 0, la = a.size(), lb = b.size();
            FOR (j, 0, la) up(d, a[j] * x[i - la + 1 + j]);
            if (d == 0) continue;
            t.clear(); for (auto& v: b) t.push_back(d * v % MOD)
                  ;
            FOR (_, 0, la - lb) t.push_back(0);
            lb = max(la, lb);
            FOR (j, 0, la) up(t[lb - 1 - j], a[la - 1 - j]);
            if (lb > la) {
                b.swap(a);
                LL inv = -get_inv(d, MOD);
                for (auto& v: b) v = v * inv % MOD;
            }
            a.swap(t);
        }
        for (auto& v: a) up(v, MOD);
        return a;
    }
}
```

3.7 Extended Euclidean

```
LL ex_gcd(LL a, LL b, LL &x, LL &y) {
    if (b == 0) { x = 1; y = 0; return a; }
    LL ret = ex_gcd(b, a % b, y, x);
    y -= a / b * x;
    return ret;
}
////////////////////////////////
inline int ctz(LL x) { return __builtin_ctzll(x); }
LL gcd(LL a, LL b) {
    if (!a) return b; if (!b) return a;
    int t = ctz(a | b);
    a >>= ctz(a);
    do {
        b >>= ctz(b);
        if (a > b) swap(a, b);
        b -= a;
    } while (b);
    return a << t;
}
```

3.8 Inverse

```
// if p is prime
inline LL get_inv(LL x, LL p) { return bin(x, p - 2, p); }
// if p is not prime
LL get_inv(LL a, LL M) {
    static LL x, y;
    assert(exgcd(a, M, x, y) == 1);
    return (x % M + M) % M;
}
/////////////////////////
LL inv[N];
void inv_init(LL n, LL p) {
    inv[1] = 1;
    FOR (i, 2, n)
        inv[i] = (p - p / i) * inv[p % i] % p;
}
/////////////////////////
LL invf[M], fac[M] = {1};
void fac_inv_init(LL n, LL p) {
    FOR (i, 1, n)
        fac[i] = i * fac[i - 1] % p;
    invf[n - 1] = bin(fac[n - 1], p - 2, p);
    FORD (i, n - 2, -1)
        invf[i] = invf[i + 1] * (i + 1) % p;
}
```

3.9 Binomial Numbers

```
inline LL C(LL n, LL m) { // n >= m >= 0
    return n < m || m < 0 ? 0 : fac[n] * invf[m] % MOD * invf[n
          - m] % MOD;
}
// The following code reverses n and m
LL C(LL n, LL m) { // m >= n >= 0
    if (m - n < n) n = m - n;
    if (n < 0) return 0;
    LL ret = 1;
    FOR (i, 1, n + 1)
        ret = ret * (m - n + i) % MOD * bin(i, MOD - 2, MOD) %
              MOD;
    return ret;
}
LL Lucas(LL n, LL m) { // m >= n >= 0
    return m ? C(n % MOD, m % MOD) * Lucas(n / MOD, m / MOD) %
          MOD : 1;
}
// precalculations
LL C[M][M];
void init_C(int n) {
    FOR (i, 0, n) {
        C[i][0] = C[i][i] = 1;
        FOR (j, 1, i)
            C[i][j] = (C[i - 1][j] + C[i - 1][j - 1]) % MOD;
    }
}
```

3.10 NTT, FFT, FWT

```
// NTT
LL wn[N << 2], rev[N << 2];
int NTT_init(int n_) {
    int step = 0; int n = 1;
    for ( ; n < n_; n <<= 1) ++step;
    FOR (i, 1, n)
        rev[i] = (rev[i >> 1] >> 1) | ((i & 1) << (step - 1));
    int g = bin(G, (MOD - 1) / n, MOD);
    wn[0] = 1;
    for (int i = 1; i <= n; ++i)
        wn[i] = wn[i - 1] * g % MOD;
    return n;
}
void NTT(LL a[], int n, int f) {
    FOR (i, 0, n) if (i < rev[i])
        std::swap(a[i], a[rev[i]]);
    for (int k = 1; k < n; k <<= 1) {
        for (int i = 0; i < n; i += (k << 1)) {
            int t = n / (k << 1);
            FOR (j, 0, k) {
                LL w = f == 1 ? wn[t * j] : wn[n - t * j];
                LL x = a[i + j];
                LL y = a[i + j + k] * w % MOD;
                a[i + j] = (x + y) % MOD;
                a[i + j + k] = (x - y + MOD) % MOD;
            }
        }
    }
    if (f == -1) {
        LL ninv = get_inv(n, MOD);
        FOR (i, 0, n)
            a[i] = a[i] * ninv % MOD;
    }
}

// FFT
// n needs to be power of 2
typedef double LD;
const LD PI = acos(-1);
struct C {
    LD r, i;
    C(LD r = 0, LD i = 0): r(r), i(i) {}
};
C operator + (const C& a, const C& b) {
    return C(a.r + b.r, a.i + b.i);
}
C operator - (const C& a, const C& b) {
    return C(a.r - b.r, a.i - b.i);
}
C operator * (const C& a, const C& b) {
    return C(a.r * b.r - a.i * b.i, a.r * b.i + a.i * b.r);
}
void FFT(C x[], int n, int p) {
    for (int i = 0, t = 0; i < n; ++i) {
        if (i > t) swap(x[i], x[t]);
        for (int j = n >> 1; (t ^= j) < j; j >>= 1);
    }
    for (int h = 2; h <= n; h <<= 1) {
        C wn(cos(p * 2 * PI / h), sin(p * 2 * PI / h));
        for (int i = 0; i < n; i += h) {
            C w(1, 0), u;
            for (int j = i, k = h >> 1; j < i + k; ++j) {
                u = x[j + k] * w;
                x[j + k] = x[j] - u;
                x[j] = x[j] + u;
                w = w * wn;
            }
        }
    }
    if (p == -1)
        FOR (i, 0, n)
            x[i].r /= n;
}
void conv(C a[], C b[], int n) {
    FFT(a, n, 1);
    FFT(b, n, 1);
    FOR (i, 0, n)
        a[i] = a[i] * b[i];
    FFT(a, n, -1);
}

// FWT
// C_k = \sum_{i \oplus j=k} A_i B_j
template<typename T>
void fwt(LL a[], int n, T f) {
    for (int d = 1; d < n; d *= 2)
        for (int i = 0, t = d * 2; i < n; i += t)
            FOR (j, 0, d)
                f(a[i + j], a[i + j + d]);
}

void AND(LL& a, LL& b) { a += b; }
void OR(LL& a, LL& b) { b += a; }
void XOR (LL& a, LL& b) {
    LL x = a, y = b;
    a = (x + y) % MOD;
    b = (x - y + MOD) % MOD;
}
void rAND(LL& a, LL& b) { a -= b; }
void rOR(LL& a, LL& b) { b -= a; }
void rXOR(LL& a, LL& b) {
    static LL INV2 = (MOD + 1) / 2;
    LL x = a, y = b;
    a = (x + y) * INV2 % MOD;
    b = (x - y + MOD) * INV2 % MOD;
}

/*
FWT subset convolution
a[popcount(x)][x] = A[x]
b[popcount(x)][x] = B[x]
fwt(a[i]) fwt(b[i])
c[i + j][x] += a[i][x] * b[j][x]
rfwt(c[i])
ans[x] = c[popcount(x)][x]
*/
```

3.11 Simpson's Numerical Integration

```
LD simpson(LD l, LD r) {
    LD c = (l + r) / 2;
    return (f(l) + 4 * f(c) + f(r)) * (r - l) / 6;
}

LD asr(LD l, LD r, LD eps, LD S) {
    LD m = (l + r) / 2;
    LD L = simpson(l, m), R = simpson(m, r);
    if (fabs(L + R - S) < 15 * eps) return L + R + (L + R - S) /
      15;
    return asr(l, m, eps / 2, L) + asr(m, r, eps / 2, R);
}

LD asr(LD l, LD r, LD eps) { return asr(l, r, eps, simpson(l, r)
    ); }
```

3.12 Gauss Elimination

```
// n equations, m variables
// a is an n x (m + 1) augmented matrix
// free is an indicator of free variable
// return the number of free variables, -1 for "404"
int n, m;
LD a[maxn][maxn], x[maxn];
bool free_x[maxn];
inline int sgn(LD x) { return (x > eps) - (x < -eps); }
int gauss(LD a[maxn][maxn], int n, int m) {
  memset(free_x, 1, sizeof free_x); memset(x, 0, sizeof x);
  int r = 0, c = 0;
  while (r < n && c < m) {
    int m_r = r;
    FOR (i, r + 1, n)
      if (fabs(a[i][c]) > fabs(a[m_r][c])) m_r = i;
    if (m_r != r)
      FOR (j, c, m + 1)
         swap(a[r][j], a[m_r][j]);
    if (!sgn(a[r][c])) {
      a[r][c] = 0; ++c;
      continue;
    }
    FOR (i, r + 1, n)
      if (a[i][c]) {
        LD t = a[i][c] / a[r][c];
        FOR (j, c, m + 1) a[i][j] -= a[r][j] * t;
      }
    ++r; ++c;
  }
  FOR (i, r, n)
    if (sgn(a[i][m])) return -1;
  if (r < m) {
    FORD (i, r - 1, -1) {
      int f_cnt = 0, k = -1;
      FOR (j, 0, m)
        if (sgn(a[i][j]) && free_x[j]) {
          ++f_cnt; k = j;
        }
      if(f_cnt > 0) continue;
      LD s = a[i][m];
      FOR (j, 0, m)
        if (j != k) s -= a[i][j] * x[j];
      x[k] = s / a[i][k];
      free_x[k] = 0;
    }
    return m - r;
  }
  FORD (i, m - 1, -1) {
    LD s = a[i][m];
    FOR (j, i + 1, m)
      s -= a[i][j] * x[j];
    x[i] = s / a[i][i];
  }
  return 0;
}
```

3.13 Factor Decomposition

```
LL factor[30], f_sz, factor_exp[30];
void get_factor(LL x) {
    f_sz = 0;
    LL t = sqrt(x + 0.5);
    for (LL i = 0; pr[i] <= t; ++i)
        if (x % pr[i] == 0) {
            factor_exp[f_sz] = 0;
            while (x % pr[i] == 0) {
                x /= pr[i];
                ++factor_exp[f_sz];
            }
            factor[f_sz++] = pr[i];
        }
    if (x > 1) {
        factor_exp[f_sz] = 1;
        factor[f_sz++] = x;
    }
}
```

3.14 Primitive Root

```
LL find_smallest_primitive_root(LL p) {
    // p should be a prime
    get_factor(p - 1);
    FOR (i, 2, p) {
        bool flag = true;
        FOR (j, 0, f_sz)
            if (bin(i, (p - 1) / factor[j], p) == 1) {
                flag = false;
                break;
            }
        if (flag) return i;
    }
    assert(0); return -1;
}
```

3.15 Quadratic Residue

```
LL q1, q2, w;
struct P { // x + y * sqrt(w)
    LL x, y;
};
P pmul(const P& a, const P& b, LL p) {
    P res;
    res.x = (a.x * b.x + a.y * b.y % p * w) % p;
    res.y = (a.x * b.y + a.y * b.x) % p;
    return res;
}
P bin(P x, LL n, LL MOD) {
    P ret = {1, 0};
    for (; n; n >>= 1, x = pmul(x, x, MOD))
```

```
        if (n & 1) ret = pmul(ret, x, MOD);
    return ret;
}
LL Legendre(LL a, LL p) { return bin(a, (p - 1) >> 1, p); }
LL equation_solve(LL b, LL p) {
    if (p == 2) return 1;
    if ((Legendre(b, p) + 1) % p == 0)
        return -1;
    LL a;
    while (true) {
        a = rand() % p;
        w = ((a * a - b) % p + p) % p;
        if ((Legendre(w, p) + 1) % p == 0)
            break;
    }
    return bin({a, 1}, (p + 1) >> 1, p).x;
}
// Given a and prime p, find x such that x*x=a(mod p)
int main() {
    LL a, p; cin >> a >> p;
    a = a % p;
    LL x = equation_solve(a, p);
    if (x == -1) {
        puts("No root");
    } else {
        LL y = p - x;
        if (x == y) cout << x << endl;
        else cout << min(x, y) << " " << max(x, y) << endl;
    }
}
```

3.16 Chinese Remainder Theorem

```
LL CRT(LL *m, LL *r, LL n) {
    if (!n) return 0;
    LL M = m[0], R = r[0], x, y, d;
    FOR (i, 1, n) {
        d = ex_gcd(M, m[i], x, y);
        if ((r[i] - R) % d) return -1;
        x = (r[i] - R) / d * x % (m[i] / d);
        R += x * M;
        M = M / d * m[i];
        R %= M;
    }
    return R >= 0 ? R : R + M;
}
```

3.17 Bernoulli Numbers

```
namespace Bernoulli {
    LL inv[M] = {-1, 1};
    LL C[M][M];
    void init();
    LL B[M] = {1};
    void init() {
        inv_init(M, MOD);
        init_C(M);
        FOR (i, 1, M - 1) {
            LL& s = B[i] = 0;
            FOR (j, 0, i)
                s += C[i + 1][j] * B[j] % MOD;
            s = (s % MOD * -inv[i + 1] % MOD + MOD) % MOD;
        }
    }
    LL p[M] = {1};
    LL go(LL n, LL k) {
        n %= MOD;
        if (k == 0) return n;
        FOR (i, 1, k + 2)
            p[i] = p[i - 1] * (n + 1) % MOD;
        LL ret = 0;
        FOR (i, 1, k + 2)
            ret += C[k + 1][i] * B[k + 1 - i] % MOD * p[i] % MOD
                ;
        ret = ret % MOD * inv[k + 1] % MOD;
        return ret;
    }
}
```

3.18 Simplex Method

```
// x = 0 should satisfy the constraints
// initialize v to be 0
// n is dimension of vector, m is number of constraints
// min{ b x } / max { c x }
// A x >= c   / A x <= b
// x >= 0
namespace lp {
    int n, m;
    double a[M][N], b[M], c[N], v;

    void pivot(int l, int e) {
        b[l] /= a[l][e];
        FOR (j, 0, n) if (j != e) a[l][j] /= a[l][e];
        a[l][e] = 1 / a[l][e];

        FOR (i, 0, m)
            if (i != l && fabs(a[i][e]) > 0) {
                b[i] -= a[i][e] * b[l];
                FOR (j, 0, n)
                    if (j != e) a[i][j] -= a[i][e] * a[l][j];
                a[i][e] = -a[i][e] * a[l][e];
            }
        v += c[e] * b[l];
        FOR (j, 0, n) if (j != e) c[j] -= c[e] * a[l][j];
        c[e] = -c[e] * a[l][e];
    }
    double simplex() {
        while (1) {
            v = 0;
            int e = -1, l = -1;
            FOR (i, 0, n) if (c[i] > eps) { e = i; break; }
            if (e == -1) return v;
            double t = INF;
            FOR (i, 0, m)
                if (a[i][e] > eps && t > b[i] / a[i][e]) {
                    t = b[i] / a[i][e];
                    l = i;
                }
            if (l == -1) return INF;
            pivot(l, e);
        }
    }
}
```

3.19 BSGS

```
// p is a prime
LL BSGS(LL a, LL b, LL p) { // a^x = b (mod p)
    a %= p;
    if (!a && !b) return 1;
    if (!a) return -1;
    static map<LL, LL> mp; mp.clear();
    LL m = sqrt(p + 1.5);
    LL v = 1;
    FOR (i, 1, m + 1) {
        v = v * a % p;
        mp[v * b % p] = i;
    }
    LL vv = v;
    FOR (i, 1, m + 1) {
        auto it = mp.find(vv);
        if (it != mp.end()) return i * m - it->second;
        vv = vv * v % p;
    }
    return -1;
}
// p can be not a prime
LL exBSGS(LL a, LL b, LL p) { // a^x = b (mod p)
    a %= p; b %= p;
    if (a == 0) return b > 1 ? -1 : b == 0 && p != 1;
    LL c = 0, q = 1;
    while (1) {
        LL g = __gcd(a, p);
        if (g == 1) break;
        if (b == 1) return c;
        if (b % g) return -1;
        ++c; b /= g; p /= g; q = a / g * q % p;
    }
    static map<LL, LL> mp; mp.clear();
    LL m = sqrt(p + 1.5);
    LL v = 1;
    FOR (i, 1, m + 1) {
        v = v * a % p;
        mp[v * b % p] = i;
    }
    FOR (i, 1, m + 1) {
        q = q * v % p;
        auto it = mp.find(q);
        if (it != mp.end()) return i * m - it->second + c;
    }
    return -1;
}
```

4 Graph Theory

4.1 LCA

```
void dfs(int u, int fa) {
    pa[u][0] = fa; dep[u] = dep[fa] + 1;
    FOR (i, 1, SP) pa[u][i] = pa[pa[u][i - 1]][i - 1];
    for (int& v: G[u]) {
        if (v == fa) continue;
        dfs(v, u);
    }
}
int lca(int u, int v) {
    if (dep[u] < dep[v]) swap(u, v);
    int t = dep[u] - dep[v];
    FOR (i, 0, SP) if (t & (1 << i)) u = pa[u][i];
    FORD (i, SP - 1, -1) {
        int uu = pa[u][i], vv = pa[v][i];
        if (uu != vv) { u = uu; v = vv; }
    }
    return u == v ? u : pa[u][0];
}
```

4.2 Maximum Flow

```
struct E {
    int to, cp;
    E(int to, int cp): to(to), cp(cp) {}
};

struct Dinic {
    static const int M = 1E5 * 5;
    int m, s, t;
    vector<E> edges;
    vector<int> G[M];
    int d[M];
    int cur[M];
    void init(int n, int s, int t) {
        this->s = s; this->t = t;
        for (int i = 0; i <= n; i++) G[i].clear();
        edges.clear(); m = 0;
    }
    void addedge(int u, int v, int cap) {
        edges.emplace_back(v, cap);
        edges.emplace_back(u, 0);
        G[u].push_back(m++);
        G[v].push_back(m++);
    }
    bool BFS() {
        memset(d, 0, sizeof d);
        queue<int> Q;
        Q.push(s); d[s] = 1;
        while (!Q.empty()) {
            int x = Q.front(); Q.pop();
            for (int& i: G[x]) {
                E &e = edges[i];
                if (!d[e.to] && e.cp > 0) {
                    d[e.to] = d[x] + 1;
                    Q.push(e.to);
                }
            }
        }
        return d[t];
    }
    int DFS(int u, int cp) {
        if (u == t || !cp) return cp;
        int tmp = cp, f;
        for (int& i = cur[u]; i < G[u].size(); i++) {
            E& e = edges[G[u][i]];
            if (d[u] + 1 == d[e.to]) {
                f = DFS(e.to, min(cp, e.cp));
                e.cp -= f;
                edges[G[u][i] ^ 1].cp += f;
                cp -= f;
                if (!cp) break;
            }
        }
        return tmp - cp;
    }
    int go() {
        int flow = 0;
        while (BFS()) {
            memset(cur, 0, sizeof cur);
            flow += DFS(s, INF);
        }
        return flow;
    }
} DC;
```

4.3 Minimum Cost Maximum Flow

```
struct E {
    int from, to, cp, v;
    E() {}
    E(int f, int t, int cp, int v) : from(f), to(t), cp(cp), v(v
        ) {}
};
struct MCMF {
    int n, m, s, t;
    vector<E> edges;
    vector<int> G[maxn];
    bool inq[maxn];
    int d[maxn]; // shortest path
    int p[maxn]; // the last edge id of the path from s to i
    int a[maxn]; // least remaining capacity from s to i
    void init(int _n, int _s, int _t) {}
    void addedge(int from, int to, int cap, int cost) {
        edges.emplace_back(from, to, cap, cost);
        edges.emplace_back(to, from, 0, -cost);
        G[from].push_back(m++);
        G[to].push_back(m++);
    }
    bool BellmanFord(int &flow, int &cost) {
        FOR (i, 0, n + 1) d[i] = INF;
        memset(inq, 0, sizeof inq);
        d[s] = 0, a[s] = INF, inq[s] = true;
        queue<int> Q; Q.push(s);
        while (!Q.empty()) {
            int u = Q.front(); Q.pop();
            inq[u] = false;
            for (int& idx: G[u]) {
                E &e = edges[idx];
                if (e.cp && d[e.to] > d[u] + e.v) {
                    d[e.to] = d[u] + e.v;
                    p[e.to] = idx;
                    a[e.to] = min(a[u], e.cp);
                    if (!inq[e.to]) {
                        Q.push(e.to);
                        inq[e.to] = true;
                    }
                }
            }
        }
        if (d[t] == INF) return false;
        flow += a[t];
        cost += a[t] * d[t];
        int u = t;
        while (u != s) {
            edges[p[u]].cp -= a[t];
            edges[p[u] ^ 1].cp += a[t];
            u = edges[p[u]].from;
        }
        return true;
    }

    int go() {
        int flow = 0, cost = 0;
        while (BellmanFord(flow, cost));
        return cost;
    }
} MM;
```

4.4 Path Intersection on Trees

```
int intersection(int x, int y, int xx, int yy) {
    int t[4] = {lca(x, xx), lca(x, yy), lca(y, xx), lca(y, yy)};
    sort(t, t + 4);
    int r = lca(x, y), rr = lca(xx, yy);
    if (dep[t[0]] < min(dep[r], dep[rr]) || dep[t[2]] < max(dep[
        r], dep[rr]))
        return 0;
    int tt = lca(t[2], t[3]);
    int ret = 1 + dep[t[2]] + dep[t[3]] - dep[tt] * 2;
    return ret;
}
```

4.5 Centroid Decomposition (Divide-Conquer)

```
int get_rt(int u) {
    static int q[N], fa[N], sz[N], mx[N];
    int p = 0, cur = -1;
    q[p++] = u; fa[u] = -1;
    while (++cur < p) {
        u = q[cur]; mx[u] = 0; sz[u] = 1;
        for (int& v: G[u])
            if (!vis[v] && v != fa[u]) fa[q[p++] = v] = u;
    }
    FORD (i, p - 1, -1) {
        u = q[i];
        mx[u] = max(mx[u], p - sz[u]);
        if (mx[u] * 2 <= p) return u;
        sz[fa[u]] += sz[u];
        mx[fa[u]] = max(mx[fa[u]], sz[u]);
    }
    assert(0);
}

void dfs(int u) {
    u = get_rt(u);
    vis[u] = true;
    get_dep(u, -1, 0);
    // ...
    for (E& e: G[u]) {
        int v = e.to;
        if (vis[v]) continue;
        // ...
        dfs(v);
    }
}

///////////////////////////////
// dynamic divide and conquer
///////////////////////////////
const int maxn = 15E4 + 100, INF = 1E9;
struct E {
    int to, d;
};
vector<E> G[maxn];
int n, Q, w[maxn];
LL A, ans;

bool vis[maxn];
int sz[maxn];

int get_rt(int u) {
    static int q[N], fa[N], sz[N], mx[N];
    int p = 0, cur = -1;
    q[p++] = u; fa[u] = -1;
    while (++cur < p) {
        u = q[cur]; mx[u] = 0; sz[u] = 1;
        for (int& v: G[u])
            if (!vis[v] && v != fa[u]) fa[q[p++] = v] = u;
    }
    FORD (i, p - 1, -1) {
        u = q[i];
        mx[u] = max(mx[u], p - sz[u]);
        if (mx[u] * 2 <= p) return u;
        sz[fa[u]] += sz[u];
        mx[fa[u]] = max(mx[fa[u]], sz[u]);
    }
    assert(0);
}

int dep[maxn], md[maxn];
void get_dep(int u, int fa, int d) {
    dep[u] = d; md[u] = 0;
    for (E& e: G[u]) {
        int v = e.to;
        if (vis[v] || v == fa) continue;
        get_dep(v, u, d + e.d);
        md[u] = max(md[u], md[v] + 1);
    }
}

struct P {
    int w;
    LL s;
};
using VP = vector<P>;
struct R {
    VP *rt, *rt2;
    int dep;
};
VP pool[maxn << 1], *pit = pool;
vector<R> tr[maxn];

void go(int u, int fa, VP* rt, VP* rt2) {
    tr[u].push_back({rt, rt2, dep[u]});
    for (E& e: G[u]) {
        int v = e.to;
        if (v == fa || vis[v]) continue;
        go(v, u, rt, rt2);
    }
}

void dfs(int u) {
    u = get_rt(u);
    vis[u] = true;
    get_dep(u, -1, 0);
    VP* rt = pit++; tr[u].push_back({rt, nullptr, 0});
    for (E& e: G[u]) {
        int v = e.to;
        if (vis[v]) continue;
        go(v, u, rt, pit++);
        dfs(v);
    }
}

bool cmp(const P& a, const P& b) { return a.w < b.w; }

LL query(VP& p, int d, int l, int r) {
    l = lower_bound(p.begin(), p.end(), P{l, -1}, cmp) - p.begin
        ();
    r = upper_bound(p.begin(), p.end(), P{r, -1}, cmp) - p.begin
        () - 1;
    return p[r].s - p[l - 1].s + 1LL * (r - l + 1) * d;
}

int main() {
    cin >> n >> Q >> A;
    FOR (i, 1, n + 1) scanf("%d", &w[i]);
    FOR (_, 1, n) {
        int u, v, d; scanf("%d%d%d", &u, &v, &d);
        G[u].push_back({v, d}); G[v].push_back({u, d});
    }
    dfs(1);
    FOR (i, 1, n + 1)
        for (R& x: tr[i]) {
            x.rt->push_back({w[i], x.dep});
            if (x.rt2) x.rt2->push_back({w[i], x.dep});
        }
    FOR (it, pool, pit) {
        it->push_back({-INF, 0});
        sort(it->begin(), it->end(), cmp);
        FOR (i, 1, it->size())
            (*it)[i].s += (*it)[i - 1].s;
    }
    while (Q--) {
        int u; LL a, b; scanf("%d%lld%lld", &u, &a, &b);
        a = (a + ans) % A; b = (b + ans) % A;
        int l = min(a, b), r = max(a, b);
        ans = 0;
        for (R& x: tr[u]) {
            ans += query(*(x.rt), x.dep, l, r);
            if (x.rt2) ans -= query(*(x.rt2), x.dep, l, r);
        }
        printf("%lld\n", ans);
    }
}
```

4.6 Heavy-light Decomposition

```
// clear clk
// usage: hld::predfs(1, 1); hld::dfs(1, 1);
int fa[N], dep[N], idx[N], out[N], ridx[N];
namespace hld {
    int sz[N], son[N], top[N], clk;
    void predfs(int u, int d) {
        dep[u] = d; sz[u] = 1;
        int& maxs = son[u] = -1;
        for (int& v: G[u]) {
            if (v == fa[u]) continue;
            fa[v] = u;
            predfs(v, d + 1);
            sz[u] += sz[v];
            if (maxs == -1 || sz[v] > sz[maxs]) maxs = v;
        }
    }
    void dfs(int u, int tp) {
        top[u] = tp; idx[u] = ++clk; ridx[clk] = u;
        if (son[u] != -1) dfs(son[u], tp);
        for (int& v: G[u])
            if (v != fa[u] && v != son[u]) dfs(v, v);
        out[u] = clk;
    }
    template<typename T>
    int go(int u, int v, T&& f = [](int, int) {}) {
        int uu = top[u], vv = top[v];
        while (uu != vv) {
            if (dep[uu] < dep[vv]) { swap(uu, vv); swap(u, v); }
            f(idx[uu], idx[u]);
            u = fa[uu]; uu = top[u];
        }
        if (dep[u] < dep[v]) swap(u, v);
        // choose one
        // f(idx[v], idx[u]);
        // if (u != v) f(idx[v] + 1, idx[u]);
        return v;
    }
    int up(int u, int d) {
        while (d) {
            if (dep[u] - dep[top[u]] < d) {
                d -= dep[u] - dep[top[u]];
                u = top[u];
            } else return ridx[idx[u] - d];
            u = fa[u]; --d;
        }
        return u;
    }
    int finds(int u, int rt) { // find u in which sub-tree of rt
        while (top[u] != top[rt]) {
            u = top[u];
            if (fa[u] == rt) return u;
            u = fa[u];
        }
        return ridx[idx[rt] + 1];
    }
}
```

4.7 Bipartite Matching

```
struct MaxMatch {
    int n;
    vector<int> G[maxn];
    int vis[maxn], left[maxn], clk;

    void init(int n) {
        this->n = n;
        FOR (i, 0, n + 1) G[i].clear();
        memset(left, -1, sizeof left);
        memset(vis, -1, sizeof vis);
    }

    bool dfs(int u) {
        for (int v: G[u])
            if (vis[v] != clk) {
                vis[v] = clk;
                if (left[v] == -1 || dfs(left[v])) {
                    left[v] = u;
                    return true;
                }
            }
        return false;
    }

    int match() {
        int ret = 0;
        for (clk = 0; clk <= n; ++clk)
            if (dfs(clk)) ++ret;
        return ret;
    }
} MM;

/////////////////////
// max weight: KM
/////////////////////
namespace R {
    const int maxn = 300 + 10;
    int n, m;
    int left[maxn], L[maxn], R[maxn];
    int w[maxn][maxn], slack[maxn];
    bool visL[maxn], visR[maxn];

    bool dfs(int u) {
        visL[u] = true;
        FOR (v, 0, m) {
            if (visR[v]) continue;
            int t = L[u] + R[v] - w[u][v];
            if (t == 0) {
                visR[v] = true;
                if (left[v] == -1 || dfs(left[v])) {
                    left[v] = u;
                    return true;
                }
            } else slack[v] = min(slack[v], t);
        }
        return false;
    }

    int go() {
        memset(left, -1, sizeof left);
        memset(R, 0, sizeof R);
        memset(L, 0, sizeof L);
        FOR (i, 0, n)
            FOR (j, 0, m)
                L[i] = max(L[i], w[i][j]);

        FOR (i, 0, n) {
            memset(slack, 0x3f, sizeof slack);
            while (1) {
                memset(visL, 0, sizeof visL); memset(visR, 0,
                    sizeof visR);
                if (dfs(i)) break;
                int d = 0x3f3f3f3f;
                FOR (j, 0, m) if (!visR[j]) d = min(d, slack[j])
                    ;
                FOR (j, 0, n) if (visL[j]) L[j] -= d;
                FOR (j, 0, m) if (visR[j]) R[j] += d; else slack
                    [j] -= d;
            }
        }
        int ret = 0;
        FOR (i, 0, m) if (left[i] != -1) ret += w[left[i]][i];
        return ret;
    }
}
```

4.8 Virtual Tree

```
void go(vector<int>& V, int& k) {
    int u = V[k]; f[u] = 0;
    dbg(u, k);
    for (auto& e: G[u]) {
        int v = e.to;
        if (v == pa[u][0]) continue;
        while (k + 1 < V.size()) {
            int to = V[k + 1];
            if (in[to] <= out[v]) {
                go(V, ++k);
                if (key[to]) f[u] += w[to];
                else f[u] += min(f[to], (LL)w[to]);
            } else break;
        }
    }
    dbg(u, f[u]);
}
inline bool cmp(int a, int b) { return in[a] < in[b]; }
LL solve(vector<int>& V) {
    static vector<int> a; a.clear();
    for (int& x: V) a.push_back(x);
    sort(a.begin(), a.end(), cmp);
    FOR (i, 1, a.size())
        a.push_back(lca(a[i], a[i - 1]));
    a.push_back(1);
    sort(a.begin(), a.end(), cmp);
    a.erase(unique(a.begin(), a.end()), a.end());
    dbg(a);
    int tmp; go(a, tmp = 0);
    return f[1];
}
```

4.9 Euler Tour

```
int S[N << 1], top;
Edge edges[N << 1];
set<int> G[N];

void DFS(int u) {
    S[top++] = u;
    for (int eid: G[u]) {
        int v = edges[eid].get_other(u);
        G[u].erase(eid);
        G[v].erase(eid);
        DFS(v);
        return;
    }
}
void fleury(int start) {
    int u = start;
    top = 0; path.clear();
    S[top++] = u;
    while (top) {
        u = S[--top];
        if (!G[u].empty())
            DFS(u);
        else path.push_back(u);
    }
}
```

4.10 SCC, 2-SAT

```
int n, m;
vector<int> G[N], rG[N], vs;
int used[N], cmp[N];

void add_edge(int from, int to) {
    G[from].push_back(to);
    rG[to].push_back(from);
}

void dfs(int v) {
    used[v] = true;
    for (int u: G[v]) {
        if (!used[u])
            dfs(u);
    }
    vs.push_back(v);
}

void rdfs(int v, int k) {
    used[v] = true;
    cmp[v] = k;
    for (int u: rG[v])
        if (!used[u])
            rdfs(u, k);
}

int scc() {
    memset(used, 0, sizeof(used));
    vs.clear();
    for (int v = 0; v < n; ++v)
        if (!used[v]) dfs(v);
    memset(used, 0, sizeof(used));
    int k = 0;
    for (int i = (int) vs.size() - 1; i >= 0; --i)
        if (!used[vs[i]]) rdfs(vs[i], k++);
    return k;
}

int main() {
    cin >> n >> m;
    n *= 2;
    for (int i = 0; i < m; ++i) {
        int a, b; cin >> a >> b;
        add_edge(a - 1, (b - 1) ^ 1);
        add_edge(b - 1, (a - 1) ^ 1);
    }
    scc();
    for (int i = 0; i < n; i += 2) {
        if (cmp[i] == cmp[i + 1]) {
            puts("NIE");
            return 0;
        }
    }
    for (int i = 0; i < n; i += 2) {
        if (cmp[i] > cmp[i + 1]) printf("%d\n", i + 1);
        else printf("%d\n", i + 2);
    }
}
```

4.11 Topological Sort

```
vector<int> toporder(int n) {
    vector<int> orders;
    queue<int> q;
    for (int i = 0; i < n; i++)
        if (!deg[i]) {
            q.push(i);
            orders.push_back(i);
        }
    while (!q.empty()) {
        int u = q.front(); q.pop();
        for (int v: G[u])
            if (!--deg[v]) {
                q.push(v);
                orders.push_back(v);
            }
    }
    return orders;
}
```

4.12 General Matching

```
// O(n^3)
vector<int> G[N];
int fa[N], mt[N], pre[N], mk[N];
int lca_clk, lca_mk[N];
pair<int, int> ce[N];
void connect(int u, int v) {
    mt[u] = v;
    mt[v] = u;
}
int find(int x) { return x == fa[x] ? x : fa[x] = find(fa[x]); }
void flip(int s, int u) {
    if (s == u) return;
    if (mk[u] == 2) {
        int v1 = ce[u].first, v2 = ce[u].second;
        flip(mt[u], v1);
        flip(s, v2);
        connect(v1, v2);
    } else {
        flip(s, pre[mt[u]]);
        connect(pre[mt[u]], mt[u]);
    }
}
int get_lca(int u, int v) {
    lca_clk++;
    for (u = find(u), v = find(v); ; u = find(pre[u]), v = find(
        pre[v])) {
        if (u && lca_mk[u] == lca_clk) return u;
        lca_mk[u] = lca_clk;
        if (v && lca_mk[v] == lca_clk) return v;
        lca_mk[v] = lca_clk;
    }
}
void access(int u, int p, const pair<int, int>& c, vector<int>&
    q) {
    for (u = find(u); u != p; u = find(pre[u])) {
        if (mk[u] == 2) {
            ce[u] = c;
            q.push_back(u);
        }
        fa[find(u)] = find(p);
    }
}
bool aug(int s) {
    fill(mk, mk + n + 1, 0);
    fill(pre, pre + n + 1, 0);
    iota(fa, fa + n + 1, 0);
        vector<int> q = {s};
        mk[s] = 1;
    int t = 0;
    for (int t = 0; t < (int) q.size(); ++t) {
        // q size can be changed
        int u = q[t];
        for (int &v: G[u]) {
            if (find(v) == find(u)) continue;
            if (!mk[v] && !mt[v]) {
                flip(s, u);
                connect(u, v);
                return true;
            } else if (!mk[v]) {
                int w = mt[v];
                mk[v] = 2; mk[w] = 1;
                pre[w] = v; pre[v] = u;
                q.push_back(w);
            } else if (mk[find(v)] == 1) {
                int p = get_lca(u, v);
                access(u, p, {u, v}, q);
                access(v, p, {v, u}, q);
            }
        }
    }
    return false;
}

int match() {
    fill(mt + 1, mt + n + 1, 0);
    lca_clk = 0;
    int ans = 0;
    FOR (i, 1, n + 1)
        if (!mt[i]) ans += aug(i);
    return ans;
}
```

4.13 Tarjan

```
 // articulation points
// note that the graph might be disconnected
int dfn[N], low[N], clk;
void init() { clk = 0; memset(dfn, 0, sizeof dfn); }
void tarjan(int u, int fa) {
    low[u] = dfn[u] = ++clk;
    int cc = fa != -1;
    for (int& v: G[u]) {
        if (v == fa) continue;
        if (!dfn[v]) {
            tarjan(v, u);
            low[u] = min(low[u], low[v]);
            cc += low[v] >= dfn[u];
        } else low[u] = min(low[u], dfn[v]);
    }
    if (cc > 1) // ...
}

// bridge
// note that the graph might have multiple edges or be
    disconnected
int dfn[N], low[N], clk;
void init() { memset(dfn, 0, sizeof dfn); clk = 0; }
void tarjan(int u, int fa) {
    low[u] = dfn[u] = ++clk;
    int _fst = 0;
    for (E& e: G[u]) {
        int v = e.to; if (v == fa && ++_fst == 1) continue;
        if (!dfn[v]) {
            tarjan(v, u);
            if (low[v] > dfn[u]) // ...
            low[u] = min(low[u], low[v]);
        } else low[u] = min(low[u], dfn[v]);
    }
}

// scc
int low[N], dfn[N], clk, B, bl[N];
vector<int> bcc[N];
void init() { B = clk = 0; memset(dfn, 0, sizeof dfn); }
void tarjan(int u) {
    static int st[N], p;
    static bool in[N];
    dfn[u] = low[u] = ++clk;
    st[p++] = u; in[u] = true;
    for (int& v: G[u]) {
        if (!dfn[v]) {
            tarjan(v);
            low[u] = min(low[u], low[v]);
        } else if (in[v]) low[u] = min(low[u], dfn[v]);
    }
    if (dfn[u] == low[u]) {
        while (1) {
            int x = st[--p]; in[x] = false;
            bl[x] = B; bcc[B].push_back(x);
            if (x == u) break;
        }
        ++B;
    }
}
```

4.14 Bi-connected Components, Block-cut Tree

```
// Array size should be 2 * N
// Single edge also counts as bi-connected comp
// Use |V| <= |E| to filter
struct E { int to, nxt; } e[N];
int hd[N], ecnt;
void addedge(int u, int v) {
    e[ecnt] = {v, hd[u]};
    hd[u] = ecnt++;
}
int low[N], dfn[N], clk, B, bno[N];
vector<int> bc[N], be[N];
bool vise[N];
void init() {
    memset(vise, 0, sizeof vise);
    memset(hd, -1, sizeof hd);
    memset(dfn, 0, sizeof dfn);
    memset(bno, -1, sizeof bno);
    B = clk = ecnt = 0;
}

void tarjan(int u, int feid) {
    static int st[N], p;
    static auto add = [&](int x) {
        if (bno[x] != B) { bno[x] = B; bc[B].push_back(x); }
    };
    low[u] = dfn[u] = ++clk;
    for (int i = hd[u]; ~i; i = e[i].nxt) {
        if ((feid ^ i) == 1) continue;
        if (!vise[i]) { st[p++] = i; vise[i] = vise[i ^ 1] =
              true; }
        int v = e[i].to;
        if (!dfn[v]) {
            tarjan(v, i);
            low[u] = min(low[u], low[v]);
            if (low[v] >= dfn[u]) {
                bc[B].clear(); be[B].clear();
                while (1) {
                    int eid = st[--p];
                    add(e[eid].to); add(e[eid ^ 1].to);
                    be[B].push_back(eid);
                    if ((eid ^ i) <= 1) break;
                }
                ++B;
            }
        } else low[u] = min(low[u], dfn[v]);
    }
}

////////////////////////
// block-cut tree
// cactus -> block-cut tree
// N >= |E| * 2
////////////////////////


vector<int> G[N];
int nn;

struct E { int to, nxt; };
namespace C {
    E e[N * 2];
    int hd[N], ecnt;
    void addedge(int u, int v) {
        e[ecnt] = {v, hd[u]};
        hd[u] = ecnt++;
    }
    int idx[N], clk, fa[N];
    bool ring[N];
    void init() { ecnt = 0; memset(hd, -1, sizeof hd); clk = 0;
         }
    void dfs(int u, int feid) {
        idx[u] = ++clk;
        for (int i = hd[u]; ~i; i = e[i].nxt) {
            if ((i ^ feid) == 1) continue;
            int v = e[i].to;
            if (!idx[v]) {
                fa[v] = u; ring[u] = false;
                dfs(v, i);
                if (!ring[u]) { G[u].push_back(v); G[v].
                       push_back(u); }
            } else if (idx[v] < idx[u]) {
                ++nn;
                G[nn].push_back(v); G[v].push_back(nn); // put
                       the root of the cycle in the front
                for (int x = u; x != v; x = fa[x]) {
                    ring[x] = true;
                    G[nn].push_back(x); G[x].push_back(nn);
                }
                ring[v] = true;
            }
        }
    }
}
```

4.15 Minimum Directed Spanning Tree

```
// edges will be modified
vector<E> edges;
int in[N], id[N], pre[N], vis[N];
// a copy of n is needed
LL zl_tree(int rt, int n) {
    LL ans = 0;
    int v, _n = n;
    while (1) {
        fill(in, in + n, INF);
        for (E &e: edges) {
            if (e.u != e.v && e.w < in[e.v]) {
                pre[e.v] = e.u;
                in[e.v] = e.w;
            }
        }
        FOR (i, 0, n) if (i != rt && in[i] == INF) return -1;
        int tn = 0;
        fill(id, id + _n, -1); fill(vis, vis + _n, -1);
        in[rt] = 0;
        FOR (i, 0, n) {
            ans += in[v = i];
            while (vis[v] != i && id[v] == -1 && v != rt) {
                vis[v] = i; v = pre[v];
            }
            if (v != rt && id[v] == -1) {
                for (int u = pre[v]; u != v; u = pre[u]) id[u] =
                      tn;
                id[v] = tn++;
            }
        }
        if (tn == 0) break;
        FOR (i, 0, n) if (id[i] == -1) id[i] = tn++;
        for (int i = 0; i < (int) edges.size(); ) {
            auto &e = edges[i];
            v = e.v;
            e.u = id[e.u]; e.v = id[e.v];
            if (e.u != e.v) { e.w -= in[v]; i++; }
            else { swap(e, edges.back()); edges.pop_back(); }
        }
        n = tn; rt = id[rt];
    }
    return ans;
}
```

4.16 Cycles

```
// refer to cheatsheet for elaboration
LL cycle4() {
    LL ans = 0;
    iota(kth, kth + n + 1, 0);
    sort(kth, kth + n, [&](int x, int y) { return deg[x] < deg[y
         ]; });
    FOR (i, 1, n + 1) rk[kth[i]] = i;
    FOR (u, 1, n + 1)
        for (int v: G[u])
            if (rk[v] > rk[u]) key[u].push_back(v);
    FOR (u, 1, n + 1) {
        for (int v: G[u])
            for (int w: key[v])
                if (rk[w] > rk[u]) ans += cnt[w]++;
        for (int v: G[u])
            for (int w: key[v])
                if (rk[w] > rk[u]) --cnt[w];
    }
    return ans;
}
int cycle3() {
    int ans = 0;
    for (E &e: edges) { deg[e.u]++; deg[e.v]++; }
    for (E &e: edges) {
        if (deg[e.u] < deg[e.v] || (deg[e.u] == deg[e.v] && e.u
             < e.v))
            G[e.u].push_back(e.v);
        else G[e.v].push_back(e.u);
    }
    FOR (x, 1, n + 1) {
        for (int y: G[x]) p[y] = x;
        for (int y: G[x]) for (int z: G[y]) if (p[z] == x) ans
             ++;
    }
    return ans;
}
```

4.17 Dominator Tree

```
vector<int> G[N], rG[N];
vector<int> dt[N];

namespace tl{
    int fa[N], idx[N], clk, ridx[N];
    int c[N], best[N], semi[N], idom[N];
    void init(int n) {
        clk = 0;
        fill(c, c + n + 1, -1);
        FOR (i, 1, n + 1) dt[i].clear();
        FOR (i, 1, n + 1) semi[i] = best[i] = i;
        fill(idx, idx + n + 1, 0);
    }
    void dfs(int u) {
        idx[u] = ++clk; ridx[clk] = u;
        for (int& v: G[u]) if (!idx[v]) { fa[v] = u; dfs(v); }
    }
    int fix(int x) {
        if (c[x] == -1) return x;
        int &f = c[x], rt = fix(f);
        if (idx[semi[best[x]]] > idx[semi[best[f]]]) best[x] =
             best[f];
        return f = rt;
    }
    void go(int rt) {
        dfs(rt);
        FORD (i, clk, 1) {
            int x = ridx[i], mn = clk + 1;
            for (int& u: rG[x]) {
                if (!idx[u]) continue;  // reaching all might
                    not be possible
                fix(u); mn = min(mn, idx[semi[best[u]]]);
            }
            c[x] = fa[x];
            dt[semi[x] = ridx[mn]].push_back(x);
            x = ridx[i - 1];
            for (int& u: dt[x]) {
                fix(u);
                if (semi[best[u]] != x) idom[u] = best[u];
                else idom[u] = x;
            }
            dt[x].clear();
        }

        FOR (i, 2, clk + 1) {
            int u = ridx[i];
            if (idom[u] != semi[u]) idom[u] = idom[idom[u]];
            dt[idom[u]].push_back(u);
        }
    }
}
```

4.18 Global Minimum Cut

```
struct StoerWanger {
    LL n, vis[N];
    LL dist[N];
    LL g[N][N];

    void init(int nn, LL w[N][N]) {
        n = nn;
        FOR (i, 1, n + 1) FOR (j, 1, n + 1)
            g[i][j] = w[i][j];
        memset(dist, 0, sizeof(dist));
    }

    LL min_cut_phase(int clk, int &x, int &y) {
        int t;
        vis[t = 1] = clk;
        FOR (i, 1, n + 1) if (vis[i] != clk)
            dist[i] = g[1][i];
        FOR (i, 1, n) {
            x = t; t = 0;
            FOR (j, 1, n + 1)
                if (vis[j] != clk && (!t || dist[j] > dist[t]))
                    t = j;
            vis[t] = clk;
            FOR (j, 1, n + 1) if (vis[j] != clk)
                dist[j] += g[t][j];
        }
        y = t;
        return dist[t];
    }

    void merge(int x, int y) {
        if (x > y) swap(x, y);
        FOR (i, 1, n + 1)
            if (i != x && i != y) {
                g[i][x] += g[i][y];
                g[x][i] += g[i][y];
            }
        if (y == n) return;
        FOR (i, 1, n) if (i != y) {
            swap(g[i][y], g[i][n]);
            swap(g[y][i], g[n][i]);
        }
    }

    LL go() {
        LL ret = INF;
        memset(vis, 0, sizeof vis);
        for (int i = 1, x, y; n > 1; ++i, --n) {
            ret = min(ret, min_cut_phase(i, x, y));
            merge(x, y);
        }
        return ret;
    }
} sw;
```

5 Geometry

5.1 2D Basics

```
int sgn(LD x) { return fabs(x) < eps ? 0 : (x > 0 ? 1 : -1); }
struct L;
struct P;
typedef P V;
struct P {
    LD x, y;
    explicit P(LD x = 0, LD y = 0): x(x), y(y) {}
    explicit P(const L& l);
};
struct L {
    P s, t;
    L() {}
    L(P s, P t): s(s), t(t) {}
};

P operator + (const P& a, const P& b) { return P(a.x + b.x, a.y
     + b.y); }
P operator - (const P& a, const P& b) { return P(a.x - b.x, a.y
     - b.y); }
P operator * (const P& a, LD k) { return P(a.x * k, a.y * k); }
P operator / (const P& a, LD k) { return P(a.x / k, a.y / k); }
inline bool operator < (const P& a, const P& b) {
    return sgn(a.x - b.x) < 0 || (sgn(a.x - b.x) == 0 && sgn(a.y
         - b.y) < 0);
}
bool operator == (const P& a, const P& b) { return !sgn(a.x - b.
     x) && !sgn(a.y - b.y); }
P::P(const L& l) { *this = l.t - l.s; }
ostream &operator << (ostream &os, const P &p) {
    return (os << "(" << p.x << "," << p.y << ")");
}
istream &operator >> (istream &is, P &p) {
    return (is >> p.x >> p.y);
}

LD dist(const P& p) { return sqrt(p.x * p.x + p.y * p.y); }
LD dot(const V& a, const V& b) { return a.x * b.x + a.y * b.y; }
LD det(const V& a, const V& b) { return a.x * b.y - a.y * b.x; }
LD cross(const P& s, const P& t, const P& o = P()) { return det(
     s - o, t - o); }
```

5.2 Polar angle sort

```
int quad(P p) {
    int x = sgn(p.x), y = sgn(p.y);
    if (x > 0 && y >= 0) return 1;
    if (x <= 0 && y > 0) return 2;
    if (x < 0 && y <= 0) return 3;
    if (x >= 0 && y < 0) return 4;
    assert(0);
}

struct cmp_angle {
    P p;
    bool operator () (const P& a, const P& b) {
        int qa = quad(a - p), qb = quad(b - p);
        if (qa != qb) return qa < qb; // compare quad
        int d = sgn(cross(a, b, p));
        if (d) return d > 0;
        return dist(a - p) < dist(b - p);
    }
};
```

5.3 Segments, lines

```
bool parallel(const L& a, const L& b) {
    return !sgn(det(P(a), P(b)));
}
bool l_eq(const L& a, const L& b) {
    return parallel(a, b) && parallel(L(a.s, b.t), L(b.s, a.t));
}
// counter-clockwise r radius
P rotation(const P& p, const LD& r) { return P(p.x * cos(r) - p.
     y * sin(r), p.x * sin(r) + p.y * cos(r)); }
P RotateCCW90(const P& p) { return P(-p.y, p.x); }
P RotateCW90(const P& p) { return P(p.y, -p.x); }
V normal(const V& v) { return V(-v.y, v.x) / dist(v); }
// inclusive: <=0; exclusive: <0
bool p_on_seg(const P& p, const L& seg) {
    P a = seg.s, b = seg.t;
    return !sgn(det(p - a, b - a)) && sgn(dot(p - a, p - b)) <=
         0;
}
LD dist_to_line(const P& p, const L& l) {
    return fabs(cross(l.s, l.t, p)) / dist(l);
}
LD dist_to_seg(const P& p, const L& l) {
    if (l.s == l.t) return dist(p - l);
    V vs = p - l.s, vt = p - l.t;
    if (sgn(dot(l, vs)) < 0) return dist(vs);
    else if (sgn(dot(l, vt)) > 0) return dist(vt);
    else return dist_to_line(p, l);
}

// make sure they have intersection in advance
P l_intersection(const L& a, const L& b) {
    LD s1 = det(P(a), b.s - a.s), s2 = det(P(a), b.t - a.s);
    return (b.s * s2 - b.t * s1) / (s2 - s1);
}
LD angle(const V& a, const V& b) {
    LD r = asin(fabs(det(a, b)) / dist(a) / dist(b));
    if (sgn(dot(a, b)) < 0) r = PI - r;
    return r;
}
// 1: proper; 2: improper
int s_l_cross(const L& seg, const L& line) {
    int d1 = sgn(cross(line.s, line.t, seg.s));
    int d2 = sgn(cross(line.s, line.t, seg.t));
    if ((d1 ^ d2) == -2) return 1; // proper
    if (d1 == 0 || d2 == 0) return 2;
    return 0;
}
// 1: proper; 2: improper
int s_cross(const L& a, const L& b, P& p) {
    int d1 = sgn(cross(a.t, b.s, a.s)), d2 = sgn(cross(a.t, b.t,
         a.s));
    int d3 = sgn(cross(b.t, a.s, b.s)), d4 = sgn(cross(b.t, a.t,
         b.s));
    if ((d1 ^ d2) == -2 && (d3 ^ d4) == -2) { p = l_intersection
        (a, b); return 1; }
    if (!d1 && p_on_seg(b.s, a)) { p = b.s; return 2; }
    if (!d2 && p_on_seg(b.t, a)) { p = b.t; return 2; }
    if (!d3 && p_on_seg(a.s, b)) { p = a.s; return 2; }
    if (!d4 && p_on_seg(a.t, b)) { p = a.t; return 2; }
    return 0;
}
```

5.4 Polygons

```
typedef vector<P> S;

// 0 = outside, 1 = inside, -1 = on border
int inside(const S& s, const P& p) {
    int cnt = 0;
    FOR (i, 0, s.size()) {
        P a = s[i], b = s[nxt(i)];
        if (p_on_seg(p, L(a, b))) return -1;
        if (sgn(a.y - b.y) <= 0) swap(a, b);
        if (sgn(p.y - a.y) > 0) continue;
        if (sgn(p.y - b.y) <= 0) continue;
        cnt += sgn(cross(b, a, p)) > 0;
    }
    return bool(cnt & 1);
}
// can be negative
LD polygon_area(const S& s) {
    LD ret = 0;
    FOR (i, 1, (LL)s.size() - 1)
        ret += cross(s[i], s[i + 1], s[0]);
    return ret / 2;
}
// duplicate points are not allowed
// s is subject to change
const int MAX_N = 1000;
S convex_hull(S& s) {
//    assert(s.size() >= 3);
    sort(s.begin(), s.end());
    S ret(MAX_N * 2);
    int sz = 0;
    FOR (i, 0, s.size()) {
        while (sz > 1 && sgn(cross(ret[sz - 1], s[i], ret[sz -
             2])) < 0) --sz;
        ret[sz++] = s[i];
    }
    int k = sz;
    FORD (i, (LL)s.size() - 2, -1) {
        while (sz > k && sgn(cross(ret[sz - 1], s[i], ret[sz -
             2])) < 0) --sz;
        ret[sz++] = s[i];
    }
    ret.resize(sz - (s.size() > 1));
    return ret;
}
// centroid
P ComputeCentroid(const vector<P> &p) {
    P c(0, 0);
    LD scale = 6.0 * polygon_area(p);
    for (unsigned i = 0; i < p.size(); i++) {
        unsigned j = (i + 1) % p.size();
        c = c + (p[i] + p[j]) * (p[i].x * p[j].y - p[j].x * p[i
             ].y);
    }
    return c / scale;
}
// Rotating Calipers, find convex hull first
LD rotatingCalipers(vector<P>& qs) {
    int n = qs.size();
    if (n == 2)
        return dist(qs[0] - qs[1]);
    int i = 0, j = 0;
    FOR (k, 0, n) {
        if (!(qs[i] < qs[k])) i = k;
        if (qs[j] < qs[k]) j = k;
    }
    LD res = 0;
    int si = i, sj = j;
    while (i != sj || j != si) {
        res = max(res, dist(qs[i] - qs[j]));
        if (sgn(cross(qs[(i+1)%n] - qs[i], qs[(j+1)%n] - qs[j]))
              < 0)
            i = (i + 1) % n;
        else j = (j + 1) % n;
    }
    return res;
}
```

5.5 Half-plane intersection

```
struct LV {
    P p, v; LD ang;
    LV() {}
    LV(P s, P t): p(s), v(t - s) { ang = atan2(v.y, v.x); }
};  // ⊠⊠⊠⊠⊠⊠⊠
bool operator < (const LV &a, const LV& b) { return a.ang < b.
     ang; }
bool on_left(const LV& l, const P& p) { return sgn(cross(l.v, p
     - l.p)) >= 0; }
P l_intersection(const LV& a, const LV& b) {
    P u = a.p - b.p; LD t = cross(b.v, u) / cross(a.v, b.v);
    return a.p + a.v * t;
}

S half_plane_intersection(vector<LV>& L) {
    int n = L.size(), fi, la;
    sort(L.begin(), L.end());
    vector<P> p(n); vector<LV> q(n);
    q[fi = la = 0] = L[0];
    FOR (i, 1, n) {
        while (fi < la && !on_left(L[i], p[la - 1])) la--;
        while (fi < la && !on_left(L[i], p[fi])) fi++;
        q[++la] = L[i];
        if (sgn(cross(q[la].v, q[la - 1].v)) == 0) {
            la--;
            if (on_left(q[la], L[i].p)) q[la] = L[i];
        }
        if (fi < la) p[la - 1] = l_intersection(q[la - 1], q[la
             ]);
    }
    while (fi < la && !on_left(q[fi], p[la - 1])) la--;
    if (la - fi <= 1) return vector<P>();
    p[la] = l_intersection(q[la], q[fi]);
    return vector<P>(p.begin() + fi, p.begin() + la + 1);
}

S convex_intersection(const vector<P> &v1, const vector<P> &v2)
     {
    vector<LV> h; int n = v1.size(), m = v2.size();
    FOR (i, 0, n) h.push_back(LV(v1[i], v1[(i + 1) % n]));
    FOR (i, 0, m) h.push_back(LV(v2[i], v2[(i + 1) % m]));
    return half_plane_intersection(h);
}
```

5.6 Circles

```
struct C {
    P p; LD r;
    C(LD x = 0, LD y = 0, LD r = 0): p(x, y), r(r) {}
    C(P p, LD r): p(p), r(r) {}
};

P compute_circle_center(P a, P b, P c) {
    b = (a + b) / 2;
    c = (a + c) / 2;
    return l_intersection({b, b + RotateCW90(a - b)}, {c , c +
         RotateCW90(a - c)});
}

// intersections are clockwise subject to center
vector<P> c_l_intersection(const L& l, const C& c) {
    vector<P> ret;
    P b(l), a = l.s - c.p;
    LD x = dot(b, b), y = dot(a, b), z = dot(a, a) - c.r * c.r;
    LD D = y * y - x * z;
    if (sgn(D) < 0) return ret;
    ret.push_back(c.p + a + b * (-y + sqrt(D + eps)) / x);
    if (sgn(D) > 0) ret.push_back(c.p + a + b * (-y - sqrt(D)) /
          x);
    return ret;
}

vector<P> c_c_intersection(C a, C b) {
    vector<P> ret;
    LD d = dist(a.p - b.p);
    if (sgn(d) == 0 || sgn(d - (a.r + b.r)) > 0 || sgn(d + min(a
         .r, b.r) - max(a.r, b.r)) < 0)
        return ret;
    LD x = (d * d - b.r * b.r + a.r * a.r) / (2 * d);
    LD y = sqrt(a.r * a.r - x * x);
    P v = (b.p - a.p) / d;
    ret.push_back(a.p + v * x + RotateCCW90(v) * y);
    if (sgn(y) > 0) ret.push_back(a.p + v * x - RotateCCW90(v) *
          y);
    return ret;
}
// 1: inside, 2: internally tangent
// 3: intersect, 4: ext tangent 5: outside
int c_c_relation(const C& a, const C& v) {
    LD d = dist(a.p - v.p);
    if (sgn(d - a.r - v.r) > 0) return 5;
    if (sgn(d - a.r - v.r) == 0) return 4;
    LD l = fabs(a.r - v.r);
    if (sgn(d - l) > 0) return 3;
    if (sgn(d - l) == 0) return 2;
    if (sgn(d - l) < 0) return 1;
}

// circle triangle intersection
// abs might be needed
LD sector_area(const P& a, const P& b, LD r) {
    LD th = atan2(a.y, a.x) - atan2(b.y, b.x);
    while (th <= 0) th += 2 * PI;
    while (th > 2 * PI) th -= 2 * PI;
    th = min(th, 2 * PI - th);
    return r * r * th / 2;
}
LD c_tri_area(P a, P b, P center, LD r) {
    a = a - center; b = b - center;
    int ina = sgn(dist(a) - r) < 0, inb = sgn(dist(b) - r) < 0;
    // dbg(a, b, ina, inb);
    if (ina && inb) {
        return fabs(cross(a, b)) / 2;
    } else {
        auto p = c_l_intersection(L(a, b), C(0, 0, r));
        if (ina ^ inb) {
            auto cr = p_on_seg(p[0], L(a, b)) ? p[0] : p[1];
            if (ina) return sector_area(b, cr, r) + fabs(cross(a
                 , cr)) / 2;
            else return sector_area(a, cr, r) + fabs(cross(b, cr
                 )) / 2;
        } else {
            if ((int) p.size() == 2 && p_on_seg(p[0], L(a, b)))
                 {
                if (dist(p[0] - a) > dist(p[1] - a)) swap(p[0],
                     p[1]);
                return sector_area(a, p[0], r) + sector_area(p
                     [1], b, r)
                   + fabs(cross(p[0], p[1])) / 2;
            } else return sector_area(a, b, r);
        }
    }
}
typedef vector<P> S;
LD c_poly_area(S poly, const C& c) {
    LD ret = 0; int n = poly.size();
    FOR (i, 0, n) {
        int t = sgn(cross(poly[i] - c.p, poly[(i + 1) % n] - c.p
             ));
        if (t) ret += t * c_tri_area(poly[i], poly[(i + 1) % n],
             c.p, c.r);
    }
    return ret;
}
```

5.7 Circle Union

```
// version 1
// union O(n^3 log n)
struct CV {
    LD yl, yr, ym; C o; int type;
    CV() {}
    CV(LD yl, LD yr, LD ym, C c, int t)
        : yl(yl), yr(yr), ym(ym), type(t), o(c) {}
};
pair<LD, LD> c_point_eval(const C& c, LD x) {
    LD d = fabs(c.p.x - x), h = rt(sq(c.r) - sq(d));
    return {c.p.y - h, c.p.y + h};
}
```

```
pair<CV, CV> pairwise_curves(const C& c, LD xl, LD xr) {
    LD yl1, yl2, yr1, yr2, ym1, ym2;
    tie(yl1, yl2) = c_point_eval(c, xl);
    tie(ym1, ym2) = c_point_eval(c, (xl + xr) / 2);
    tie(yr1, yr2) = c_point_eval(c, xr);
    return {CV(yl1, yr1, ym1, c, 1), CV(yl2, yr2, ym2, c, -1)};
}
bool operator < (const CV& a, const CV& b) { return a.ym < b.ym;
      }
LD cv_area(const CV& v, LD xl, LD xr) {
    LD l = rt(sq(xr - xl) + sq(v.yr - v.yl));
    LD d = rt(sq(v.o.r) - sq(l / 2));
    LD ang = atan(l / d / 2);
    return ang * sq(v.o.r) - d * l / 2;
}
LD circle_union(const vector<C>& cs) {
    int n = cs.size();
    vector<LD> xs;
    FOR (i, 0, n) {
        xs.push_back(cs[i].p.x - cs[i].r);
        xs.push_back(cs[i].p.x);
        xs.push_back(cs[i].p.x + cs[i].r);
        FOR (j, i + 1, n) {
            auto pts = c_c_intersection(cs[i], cs[j]);
            for (auto& p: pts) xs.push_back(p.x);
        }
    }
    sort(xs.begin(), xs.end());
    xs.erase(unique(xs.begin(), xs.end(), [](LD x, LD y) {
          return sgn(x - y) == 0; }), xs.end());
    LD ans = 0;
    FOR (i, 0, (int) xs.size() - 1) {
        LD xl = xs[i], xr = xs[i + 1];
        vector<CV> intv;
        FOR (k, 0, n) {
            auto& c = cs[k];
            if (sgn(c.p.x - c.r - xl) <= 0 && sgn(c.p.x + c.r -
                  xr) >= 0) {
                auto t = pairwise_curves(c, xl, xr);
                intv.push_back(t.first); intv.push_back(t.second
                      );
            }
        }
        sort(intv.begin(), intv.end());

        vector<LD> areas(intv.size());
        FOR (i, 0, intv.size()) areas[i] = cv_area(intv[i], xl,
              xr);

        int cc = 0;
        FOR (i, 0, intv.size()) {
            if (cc > 0) {
                ans += (intv[i].yl - intv[i - 1].yl + intv[i].yr
                      - intv[i - 1].yr) * (xr - xl) / 2;
                ans += intv[i - 1].type * areas[i - 1];
                ans -= intv[i].type * areas[i];
            }
            cc += intv[i].type;
        }
    }
    return ans;
}

// version 2 (k-cover, O(n^2 log n))
inline LD angle(const P &p) { return atan2(p.y, p.x); }

// Points on circle
// p is coordinates relative to c
struct CP {
  P p;
  LD a;
  int t;
  CP() {}
  CP(P p, LD a, int t) : p(p), a(a), t(t) {}
};
bool operator<(const CP &u, const CP &v) { return u.a < v.a; }
LD cv_area(LD r, const CP &q1, const CP &q2) {
  return (r * r * (q2.a - q1.a) - cross(q1.p, q2.p)) / 2;
}

LD ans[N];
void circle_union(const vector<C> &cs) {
  int n = cs.size();
  FOR(i, 0, n) {
    // same circle, only the first one counts
    bool ok = true;
    FOR(j, 0, i)
    if (sgn(cs[i].r - cs[j].r) == 0 && cs[i].p == cs[j].p) {
      ok = false;
      break;
    }
    if (!ok)
      continue;
    auto &c = cs[i];
    vector<CP> ev;
    int belong_to = 0;
    P bound = c.p + P(-c.r, 0);
    ev.emplace_back(bound, -PI, 0);
    ev.emplace_back(bound, PI, 0);
    FOR(j, 0, n) {
      if (i == j)
        continue;
      if (c_c_relation(c, cs[j]) <= 2) {
        if (sgn(cs[j].r - c.r) >= 0) // totally covered
          belong_to++;
        continue;
      }
      auto its = c_c_intersection(c, cs[j]);
      if (its.size() == 2) {
        P p = its[1] - c.p, q = its[0] - c.p;
        LD a = angle(p), b = angle(q);
        if (sgn(a - b) > 0) {
          ev.emplace_back(p, a, 1);
          ev.emplace_back(bound, PI, -1);
          ev.emplace_back(bound, -PI, 1);
          ev.emplace_back(q, b, -1);
        } else {
          ev.emplace_back(p, a, 1);
          ev.emplace_back(q, b, -1);
        }
      }
    }
    sort(ev.begin(), ev.end());
    int cc = ev[0].t;
    FOR(j, 1, ev.size()) {
      int t = cc + belong_to;
      ans[t] += cross(ev[j - 1].p + c.p, ev[j].p + c.p) / 2;
      ans[t] += cv_area(c.r, ev[j - 1], ev[j]);
      cc += ev[j].t;
    }
  }
}
```

5.8 Minimum Covering Circle

```
P compute_circle_center(P a, P b) { return (a + b) / 2; }
bool p_in_circle(const P& p, const C& c) {
    return sgn(dist(p - c.p) - c.r) <= 0;
}
C min_circle_cover(const vector<P> &in) {
    vector<P> a(in.begin(), in.end());
    dbg(a.size());
    random_shuffle(a.begin(), a.end());
    P c = a[0]; LD r = 0; int n = a.size();
    FOR (i, 1, n) if (!p_in_circle(a[i], {c, r})) {
        c = a[i]; r = 0;
        FOR (j, 0, i) if (!p_in_circle(a[j], {c, r})) {
            c = compute_circle_center(a[i], a[j]);
            r = dist(a[j] - c);
            FOR (k, 0, j) if (!p_in_circle(a[k], {c, r})) {
                c = compute_circle_center(a[i], a[j], a[k]);
                r = dist(a[k] - c);
            }
        }
    }
    return {c, r};
}
```

5.9 Circle Inversion

```
C inv(C c, const P& o) {
    LD d = dist(c.p - o);
    assert(sgn(d) != 0);
    LD a = 1 / (d - c.r);
    LD b = 1 / (d + c.r);
    c.r = (a - b) / 2 * R2;
    c.p = o + (c.p - o) * ((a + b) * R2 / 2 / d);
    return c;
}
```

5.10 3D Basics

```
struct P;
struct L;
typedef P V;
struct P {
    LD x, y, z;
    explicit P(LD x = 0, LD y = 0, LD z = 0): x(x), y(y), z(z)
          {}
    explicit P(const L& l);
};
struct L {
    P s, t;
    L() {}
    L(P s, P t): s(s), t(t) {}
};
struct F {
    P a, b, c;
    F() {}
    F(P a, P b, P c): a(a), b(b), c(c) {}
};
P operator + (const P& a, const P& b) { }
P operator - (const P& a, const P& b) { }
P operator * (const P& a, LD k) { }
P operator / (const P& a, LD k) { }
inline int operator < (const P& a, const P& b) {
    return sgn(a.x - b.x) < 0 || (sgn(a.x - b.x) == 0 && (sgn(a.
          y - b.y) < 0 ||
                                   (sgn(a.y - b.y) == 0 && sgn(a.
                                        z - b.z) < 0)));
}
bool operator == (const P& a, const P& b) { return !sgn(a.x - b.
      x) && !sgn(a.y - b.y) && !sgn(a.z - b.z); }
P::P(const L& l) { *this = l.t - l.s; }
ostream &operator << (ostream &os, const P &p) {
    return (os << "(" << p.x << "," << p.y << "," << p.z << ")")
          ;
}
istream &operator >> (istream &is, P &p) {
    return (is >> p.x >> p.y >> p.z);
}
LD dist2(const P& p) { return p.x * p.x + p.y * p.y + p.z * p.z;
      }
LD dist(const P& p) { return sqrt(dist2(p)); }
LD dot(const V& a, const V& b) { return a.x * b.x + a.y * b.y +
      a.z * b.z; }
P cross(const P& v, const P& w) {
    return P(v.y * w.z - v.z * w.y, v.z * w.x - v.x * w.z, v.x *
          w.y - v.y * w.x);
}
LD mix(const V& a, const V& b, const V& c) { return dot(a, cross
      (b, c)); }
// counter-clockwise r radius
// axis = 0 around axis x
// axis = 1 around axis y
// axis = 2 around axis z
P rotation(const P& p, const LD& r, int axis = 0) {
    if (axis == 0)
        return P(p.x, p.y * cos(r) - p.z * sin(r), p.y * sin(r)
              + p.z * cos(r));
    else if (axis == 1)
        return P(p.z * cos(r) - p.x * sin(r), p.y, p.z * sin(r)
             + p.x * cos(r));
    else if (axis == 2)
        return P(p.x * cos(r) - p.y * sin(r), p.x * sin(r) + p.y
              * cos(r), p.z);
}
// n is normal vector
// this is clockwise
P rotation(const P& p, const LD& r, const P& n) {
    LD c = cos(r), s = sin(r), x = n.x, y = n.y, z = n.z;
    return P((x * x * (1 - c) + c) * p.x + (x * y * (1 - c) + z
         * s) * p.y + (x * z * (1 - c) - y * s) * p.z,
             (x * y * (1 - c) - z * s) * p.x + (y * y * (1 - c)
                  + c) * p.y + (y * z * (1 - c) + x * s) * p.z,
             (x * z * (1 - c) + y * s) * p.x + (y * z * (1 - c)
                  - x * s) * p.y + (z * z * (1 - c) + c) * p.z)
                  ;
}
```

5.11 3D Line, Face

```
// <= 0 inproper, < 0 proper
bool p_on_seg(const P& p, const L& seg) {
    P a = seg.s, b = seg.t;
    return !sgn(dist2(cross(p - a, b - a))) && sgn(dot(p - a, p
         - b)) <= 0;
}
LD dist_to_line(const P& p, const L& l) {
    return dist(cross(l.s - p, l.t - p)) / dist(l);
}
LD dist_to_seg(const P& p, const L& l) {
    if (l.s == l.t) return dist(p - l.s);
    V vs = p - l.s, vt = p - l.t;
    if (sgn(dot(l, vs)) < 0) return dist(vs);
    else if (sgn(dot(l, vt)) > 0) return dist(vt);
    else return dist_to_line(p, l);
}

P norm(const F& f) { return cross(f.a - f.b, f.b - f.c); }
int p_on_plane(const F& f, const P& p) { return sgn(dot(norm(f),
      p - f.a)) == 0; }
// if two points are on the opposite side of a line
// return 0 if points is on the line
// makes no sense if points and line are not coplanar
int opposite_side(const P& u, const P& v, const L& l) {
        return sgn(dot(cross(P(l), u - l.s), cross(P(l), v - l.s
             ))) < 0;
}

bool parallel(const L& a, const L& b) { return !sgn(dist2(cross(
     P(a), P(b)))); }
int s_intersect(const L& u, const L& v) {
    return p_on_plane(F(u.s, u.t, v.s), v.t) &&
           opposite_side(u.s, u.t, v) &&
           opposite_side(v.s, v.t, u);
}
```

5.12 3D Convex

```
struct FT {
    int a, b, c;
    FT() { }
    FT(int a, int b, int c) : a(a), b(b), c(c) { }
};

bool p_on_line(const P& p, const L& l) {
    return !sgn(dist2(cross(p - l.s, P(l))));
}

vector<F> convex_hull(vector<P> &p) {
    sort(p.begin(), p.end());
    p.erase(unique(p.begin(), p.end()), p.end());
    random_shuffle(p.begin(), p.end());
    vector<FT> face;
    FOR (i, 2, p.size()) {
        if (p_on_line(p[i], L(p[0], p[1]))) continue;
        swap(p[i], p[2]);
        FOR (j, i + 1, p.size())
            if (sgn(mix(p[1] - p[0], p[2] - p[1], p[j] - p[0])))
                  {
                swap(p[j], p[3]);
                face.emplace_back(0, 1, 2);
                face.emplace_back(0, 2, 1);
                goto found;
            }
    }
found:
    vector<vector<int>> mk(p.size(), vector<int>(p.size()));
    FOR (v, 3, p.size()) {
        vector<FT> tmp;
        FOR (i, 0, face.size()) {
            int a = face[i].a, b = face[i].b, c = face[i].c;
            if (sgn(mix(p[a] - p[v], p[b] - p[v], p[c] - p[v]))
                 < 0) {
                mk[a][b] = mk[b][a] = v;
                mk[b][c] = mk[c][b] = v;
                mk[c][a] = mk[a][c] = v;
            } else tmp.push_back(face[i]);
        }
        face = tmp;
        FOR (i, 0, tmp.size()) {
            int a = face[i].a, b = face[i].b, c = face[i].c;
            if (mk[a][b] == v) face.emplace_back(b, a, v);
            if (mk[b][c] == v) face.emplace_back(c, b, v);
            if (mk[c][a] == v) face.emplace_back(a, c, v);
        }
    }
    vector<F> out;
    FOR (i, 0, face.size())
        out.emplace_back(p[face[i].a], p[face[i].b], p[face[i].c
             ]);
    return out;
}
```

6 String

6.1 Aho-Corasick Automation

```
const int N = 1e6 + 100, M = 26;
int mp(char ch) { return ch - 'a'; }
struct ACA {
    int ch[N][M], danger[N], fail[N];
    int sz;
    void init() {
        sz = 1;
        memset(ch[0], 0, sizeof ch[0]);
        memset(danger, 0, sizeof danger);
    }
    void insert(const string &s, int m) {
        int n = s.size(); int u = 0, c;
        FOR (i, 0, n) {
            c = mp(s[i]);
            if (!ch[u][c]) {
                memset(ch[sz], 0, sizeof ch[sz]);
                danger[sz] = 0; ch[u][c] = sz++;
            }
            u = ch[u][c];
        }
        danger[u] |= 1 << m;
    }
    void build() {
        queue<int> Q;
        fail[0] = 0;
        for (int c = 0, u; c < M; c++) {
            u = ch[0][c];
            if (u) { Q.push(u); fail[u] = 0; }
        }
        while (!Q.empty()) {
            int r = Q.front(); Q.pop();
            danger[r] |= danger[fail[r]];
            for (int c = 0, u; c < M; c++) {
                u = ch[r][c];
                if (!u) {
                    ch[r][c] = ch[fail[r]][c];
                    continue;
                }
                fail[u] = ch[fail[r]][c];
                Q.push(u);
            }
        }
    }
} ac;

char s[N];
int main() {
    int n; scanf("%d", &n);
    ac.init();
    while (n--) {
        scanf("%s", s);
        ac.insert(s, 0);
    }
    ac.build();
    scanf("%s", s);
    int u = 0; n = strlen(s);
    FOR (i, 0, n) {
        u = ac.ch[u][mp(s[i])];
        if (ac.danger[u]) {
            puts("YES");
            return 0;
        }
    }
    puts("NO");
    return 0;
}
```

6.2 Hash

```
const int p1 = 1e9 + 7, p2 = 1e9 + 9;
ULL xp1[N], xp2[N], xp[N];
void init_xp() {
    xp1[0] = xp2[0] = xp[0] = 1;
    for (int i = 1; i < N; ++i) {
        xp1[i] = xp1[i - 1] * x % p1;
        xp2[i] = xp2[i - 1] * x % p2;
        xp[i] = xp[i - 1] * x;
    }
}
struct String {
    char s[N];
    int length, subsize;
    bool sorted;
    ULL h[N], hl[N];
    ULL hash() {
        length = strlen(s);
        ULL res1 = 0, res2 = 0;
        h[length] = 0;  // ATTENTION!
        for (int j = length - 1; j >= 0; --j) {
        #ifdef ENABLE_DOUBLE_HASH
            res1 = (res1 * x + s[j]) % p1;
            res2 = (res2 * x + s[j]) % p2;
            h[j] = (res1 << 32) | res2;
        #else
            res1 = res1 * x + s[j];
            h[j] = res1;
        #endif
            // printf("%llu\n", h[j]);
        }
        return h[0];
    }
    // hash of [left, right)
    ULL get_substring_hash(int left, int right) const {
        int len = right - left;
    #ifdef ENABLE_DOUBLE_HASH
        // get hash of s[left...right-1]
        unsigned int mask32 = ~(0u);
        ULL left1 = h[left] >> 32, right1 = h[right] >> 32;
        ULL left2 = h[left] & mask32, right2 = h[right] & mask32
           ;
        return (((left1 - right1 * xp1[len] % p1 + p1) % p1) <<
           32) |
            (((left2 - right2 * xp2[len] % p2 + p2) % p2));
    #else
        return h[left] - h[right] * xp[len];
    #endif
    }
    void get_all_subs_hash(int sublen) {
        subsize = length - sublen + 1;
        for (int i = 0; i < subsize; ++i)
            hl[i] = get_substring_hash(i, i + sublen);
        sorted = 0;
    }
    void sort_substring_hash() {
        sort(hl, hl + subsize);
        sorted = 1;
    }
    bool match(ULL key) const {
        if (!sorted) assert (0);
        if (!subsize) return false;
        return binary_search(hl, hl + subsize, key);
    }
    void init(const char *t) {
        length = strlen(t);
        strcpy(s, t);
    }
};
int LCP(const String &a, const String &b, int ai, int bi) {
    // Find LCP of a[ai...] and b[bi...]
    int l = 0, r = min(a.length - ai, b.length - bi);
    while (l < r) {
        int mid = (l + r + 1) / 2;
        if (a.get_substring_hash(ai, ai + mid) == b.
           get_substring_hash(bi, bi + mid))
            l = mid;
        else r = mid - 1;
    }
    return l;
}
```

6.3 KMP

```
void get_pi(int a[], char s[], int n) {
    int j = a[0] = 0;
    FOR (i, 1, n) {
        while (j && s[i] != s[j]) j = a[j - 1];
        a[i] = j += s[i] == s[j];
    }
}
void get_z(int a[], char s[], int n) {
    int l = 0, r = 0; a[0] = n;
    FOR (i, 1, n) {
        a[i] = i > r ? 0 : min(r - i + 1, a[i - l]);
        while (i + a[i] < n && s[a[i]] == s[i + a[i]]) ++a[i];
        if (i + a[i] - 1 > r) { l = i; r = i + a[i] - 1; }
    }
}
```

6.4 Manacher

```
int RL[N];
void manacher(int* a, int n) { // "abc" => "#a#b#a#"
    int r = 0, p = 0;
    FOR (i, 0, n) {
        if (i < r) RL[i] = min(RL[2 * p - i], r - i);
        else RL[i] = 1;
        while (i - RL[i] >= 0 && i + RL[i] < n && a[i - RL[i]]
              == a[i + RL[i]])
            RL[i]++;
        if (RL[i] + i - 1 > r) { r = RL[i] + i - 1; p = i; }
    }
    FOR (i, 0, n) --RL[i];
}
```

6.5 Palindrome Automation

```
// num: the number of palindrome suffixes of the prefix
      represented by the node
// cnt: the number of occurrences in string (should update to
      father before using)
namespace pam {
    int t[N][26], fa[N], len[N], rs[N], cnt[N], num[N];
    int sz, n, last;
    int _new(int l) {
        memset(t[sz], 0, sizeof t[0]);
        len[sz] = l; cnt[sz] = num[sz] = 0;
        return sz++;
    }
    void init() {
        rs[n = sz = 0] = -1;
        last = _new(0);
        fa[last] = _new(-1);
    }
    int get_fa(int x) {
        while (rs[n - 1 - len[x]] != rs[n]) x = fa[x];
        return x;
    }
    void ins(int ch) {
        rs[++n] = ch;
        int p = get_fa(last);
        if (!t[p][ch]) {
            int np = _new(len[p] + 2);
            num[np] = num[fa[np] = t[get_fa(fa[p])][ch]] + 1;
            t[p][ch] = np;
        }
        ++cnt[last = t[p][ch]];
    }
}
```

6.6 Suffix Array

```
struct SuffixArray {
    const int L;
    vector<vector<int> > P;
    vector<pair<pair<int, int>, int> > M;
    int s[N], sa[N], rank[N], height[N];
    // s: raw string
    // sa[i]=k: s[k...L-1] ranks i (0 based)
    // rank[i]=k: the rank of s[i...L-1] is k (0 based)
    // height[i] = lcp(sa[i-1], sa[i])
    SuffixArray(const string &raw_s) : L(raw_s.length()), P(1,
         vector<int>(L, 0)), M(L) {
        for (int i = 0; i < L; i++)
            P[0][i] = this->s[i] = int(raw_s[i]);
        for (int skip = 1, level = 1; skip < L; skip *= 2, level
           ++) {
            P.push_back(vector<int>(L, 0));
            for (int i = 0; i < L; i++)
                M[i] = make_pair(make_pair(P[level - 1][i], i +
                     skip < L ? P[level - 1][i + skip] : -1000)
                     , i);
            sort(M.begin(), M.end());
            for (int i = 0; i < L; i++)
                P[level][M[i].second] = (i > 0 && M[i].first ==
                     M[i - 1].first) ? P[level][M[i - 1].second
                     ] : i;
        }
        for (unsigned i = 0; i < P.back().size(); ++i) {
            rank[i] = P.back()[i];
            sa[rank[i]] = i;
        }
    }
    // This is a traditional way to calculate LCP
    void getHeight() {
        memset(height, 0, sizeof height);
        int k = 0;
        for (int i = 0; i < L; ++i) {
            if (rank[i] == 0) continue;
            if (k) k--;
            int j = sa[rank[i] - 1];
            while (i + k < L && j + k < L && s[i + k] == s[j + k
                 ]) ++k;
            height[rank[i]] = k;
        }
        rmq_init(height, L);
    }
    int f[N][Nlog];
    inline int highbit(int x) {
        return 31 - __builtin_clz(x);
    }
    int rmq_query(int x, int y) {
        int p = highbit(y - x + 1);
        return min(f[x][p], f[y - (1 << p) + 1][p]);
    }
    // arr has to be 0 based
    void rmq_init(int *arr, int length) {
        for (int x = 0; x <= highbit(length); ++x)
            for (int i = 0; i <= length - (1 << x); ++i) {
                if (!x) f[i][x] = arr[i];
                else f[i][x] = min(f[i][x - 1], f[i + (1 << (x -
                     1))][x - 1]);
            }
    }
    #ifdef NEW
    // returns the length of the longest common prefix of s[i...
         L-1] and s[j...L-1]
    int LongestCommonPrefix(int i, int j) {
        int len = 0;
        if (i == j) return L - i;
        for (int k = (int) P.size() - 1; k >= 0 && i < L && j <
             L; k--) {
            if (P[k][i] == P[k][j]) {
                i += 1 << k;
                j += 1 << k;
                len += 1 << k;
            }
        }
        return len;
    }
    #else
    int LongestCommonPrefix(int i, int j) {
        // getHeight() must be called first
        if (i == j) return L - i;
        if (i > j) swap(i, j);
        return rmq_query(i + 1, j);
    }
    #endif
    int checkNonOverlappingSubstring(int K) {
        // check if there is two non-overlapping identical
             substring of length K
        int minsa = 0, maxsa = 0;
        for (int i = 0; i < L; ++i) {
            if (height[i] < K) {
                minsa = sa[i]; maxsa = sa[i];
            } else {
                minsa = min(minsa, sa[i]);
                maxsa = max(maxsa, sa[i]);
                if (maxsa - minsa >= K) return 1;
            }
        }
        return 0;
    }
    int checkBelongToDifferentSubstring(int K, int split) {
        int minsa = 0, maxsa = 0;
        for (int i = 0; i < L; ++i) {
            if (height[i] < K) {
                minsa = sa[i]; maxsa = sa[i];
            } else {
                minsa = min(minsa, sa[i]);
                maxsa = max(maxsa, sa[i]);
                if (maxsa > split && minsa < split) return 1;
            }
        }
        return 0;
    }
} *S;
int main() {
    int sp = s.length();
    s += "*" + t;
    S = new SuffixArray(s);
    S->getHeight();
    int left = 0, right = sp;
    while (left < right) {
        // ...
        if (S->checkBelongToDifferentSubstring(mid, sp))
            // ...
    }
    printf("%d\n", left);
}

/////////////////////////////////////
// rk [0..n-1] -> [1..n], sa/ht [1..n]
// s[i] > 0 && s[n] = 0
// b: normally as bucket
// c: normally as bucket1
// d: normally as bucket2
// f: normally as cntbuf
/////////////////////////////////////
template<size_t size>
struct SuffixArray {
    bool t[size << 1];
    int b[size], c[size];
    int sa[size], rk[size], ht[size];
    inline bool isLMS(const int i, const bool *t) { return i > 0
          && t[i] && !t[i - 1]; }
    template<class T>
    inline void inducedSort(T s, int *sa, const int n, const int
          M, const int bs,
                            bool *t, int *b, int *f, int *p) {
        fill(b, b + M, 0); fill(sa, sa + n, -1);
        FOR (i, 0, n) b[s[i]]++;
        f[0] = b[0];
        FOR (i, 1, M) f[i] = f[i - 1] + b[i];
        FORD (i, bs - 1, -1) sa[--f[s[p[i]]]] = p[i];
        FOR (i, 1, M) f[i] = f[i - 1] + b[i - 1];
        FOR (i, 0, n) if (sa[i] > 0 && !t[sa[i] - 1]) sa[f[s[sa[
              i] - 1]]++] = sa[i] - 1;
        f[0] = b[0];
        FOR (i, 1, M) f[i] = f[i - 1] + b[i];
        FORD (i, n - 1, -1) if (sa[i] > 0 && t[sa[i] - 1]) sa[--
              f[s[sa[i] - 1]]] = sa[i] - 1;
    }
    template<class T>
    inline void sais(T s, int *sa, int n, bool *t, int *b, int *
          c, int M) {
        int i, j, bs = 0, cnt = 0, p = -1, x, *r = b + M;
        t[n - 1] = 1;
        FORD (i, n - 2, -1) t[i] = s[i] < s[i + 1] || (s[i] == s
              [i + 1] && t[i + 1]);
        FOR (i, 1, n) if (t[i] && !t[i - 1]) c[bs++] = i;
        inducedSort(s, sa, n, M, bs, t, b, r, c);
        for (i = bs = 0; i < n; i++) if (isLMS(sa[i], t)) sa[bs
              ++] = sa[i];
        FOR (i, bs, n) sa[i] = -1;
        FOR (i, 0, bs) {
            x = sa[i];
            for (j = 0; j < n; j++) {
                if (p == -1 || s[x + j] != s[p + j] || t[x + j]
                      != t[p + j]) { cnt++, p = x; break; }
                else if (j > 0 && (isLMS(x + j, t) || isLMS(p +
                      j, t))) break;
            }
            x = (~x & 1 ? x >> 1 : x - 1 >> 1), sa[bs + x] = cnt
                  - 1;
        }
        for (i = j = n - 1; i >= bs; i--) if (sa[i] >= 0) sa[j
              --] = sa[i];
        int *s1 = sa + n - bs, *d = c + bs;
        if (cnt < bs) sais(s1, sa, bs, t + n, b, c + bs, cnt);
        else FOR (i, 0, bs) sa[s1[i]] = i;
        FOR (i, 0, bs) d[i] = c[sa[i]];
        inducedSort(s, sa, n, M, bs, t, b, r, d);
    }
    template<typename T>
    inline void getHeight(T s, const int n, const int *sa) {
        for (int i = 0, k = 0; i < n; i++) {
            if (rk[i] == 0) k = 0;
            else {
                if (k > 0) k--;
                int j = sa[rk[i] - 1];
                while (i + k < n && j + k < n && s[i + k] == s[j
                      + k]) k++;
            }
            ht[rk[i]] = k;
        }
    }
    template<class T>
    inline void init(T s, int n, int M) {
        sais(s, sa, ++n, t, b, c, M);
        for (int i = 1; i < n; i++) rk[sa[i]] = i;
        getHeight(s, n, sa);
    }
};
SuffixArray<N> sa;
int main() {
    int n = s.length();
    sa.init(s, n, 128);
    FOR (i, 1, n + 1) printf("%d%c", sa.sa[i] + 1, i == _i - 1 ?
           '\n' : ' ');
    FOR (i, 2, n + 1) printf("%d%c", sa.ht[i], i == _i - 1 ? '\n
          ' : ' ');
}
```

6.7 Suffix Automation

```
namespace sam {
    const int M = N << 1;
    int t[M][26], len[M] = {-1}, fa[M], sz = 2, last = 1;
    void init() { memset(t, 0, (sz + 10) * sizeof t[0]); sz = 2;
           last = 1; }
    void ins(int ch) {
        int p = last, np = last = sz++;
        len[np] = len[p] + 1;
        for (; p && !t[p][ch]; p = fa[p]) t[p][ch] = np;
        if (!p) { fa[np] = 1; return; }
        int q = t[p][ch];
        if (len[p] + 1 == len[q]) fa[np] = q;
        else {
            int nq = sz++; len[nq] = len[p] + 1;
            memcpy(t[nq], t[q], sizeof t[0]);
            fa[nq] = fa[q];
            fa[np] = fa[q] = nq;
            for (; t[p][ch] == q; p = fa[p]) t[p][ch] = nq;
        }
    }
    int c[M] = {1}, a[M];
    void rsort() {
        FOR (i, 1, sz) c[i] = 0;
        FOR (i, 1, sz) c[len[i]]++;
        FOR (i, 1, sz) c[i] += c[i - 1];
        FOR (i, 1, sz) a[--c[len[i]]] = i;
    }
}
// really-generalized sam
int t[M][26], len[M] = {-1}, fa[M], sz = 2, last = 1;
LL cnt[M][2];
void ins(int ch, int id) {
    int p = last, np = 0, nq = 0, q = -1;
    if (!t[p][ch]) {
        np = sz++;
        len[np] = len[p] + 1;
        for (; p && !t[p][ch]; p = fa[p]) t[p][ch] = np;
    }
    if (!p) fa[np] = 1;
    else {
        q = t[p][ch];
        if (len[p] + 1 == len[q]) fa[np] = q;
        else {
            nq = sz++; len[nq] = len[p] + 1;
            memcpy(t[nq], t[q], sizeof t[0]);
            fa[nq] = fa[q];
            fa[np] = fa[q] = nq;
            for (; t[p][ch] == q; p = fa[p]) t[p][ch] = nq;
        }
    }
    last = np ? np : nq ? nq : q;
    cnt[last][id] = 1;
}
// lexicographical order
// rsort2 is not topo sort
void ins(int ch, int pp) {
    int p = last, np = last = sz++;
    len[np] = len[p] + 1; one[np] = pos[np] = pp;
    for (; p && !t[p][ch]; p = fa[p]) t[p][ch] = np;
    if (!p) { fa[np] = 1; return; }
    int q = t[p][ch];
    if (len[q] == len[p] + 1) fa[np] = q;
    else {
        int nq = sz++; len[nq] = len[p] + 1; one[nq] = one[q];
        memcpy(t[nq], t[q], sizeof t[0]);
        fa[nq] = fa[q];
        fa[q] = fa[np] = nq;
        for (; p && t[p][ch] == q; p = fa[p]) t[p][ch] = nq;
    }
}
// lexicographical order
// generalized sam
int up[M], c[256] = {2}, a[M];
void rsort2() {
    FOR (i, 1, 256) c[i] = 0;
    FOR (i, 2, sz) up[i] = s[one[i] + len[fa[i]]];
    FOR (i, 2, sz) c[up[i]]++;
    FOR (i, 1, 256) c[i] += c[i - 1];
    FOR (i, 2, sz) a[--c[up[i]]] = i;
    FOR (i, 2, sz) G[fa[a[i]]].push_back(a[i]);
}

int t[M][26], len[M] = {0}, fa[M], sz = 2, last = 1;
char* one[M];
void ins(int ch, char* pp) {
    int p = last, np = 0, nq = 0, q = -1;
    if (!t[p][ch]) {
        np = sz++; one[np] = pp;
        len[np] = len[p] + 1;
        for (; p && !t[p][ch]; p = fa[p]) t[p][ch] = np;
    }
    if (!p) fa[np] = 1;
    else {
        q = t[p][ch];
        if (len[p] + 1 == len[q]) fa[np] = q;
        else {
            nq = sz++; len[nq] = len[p] + 1; one[nq] = one[q];
            memcpy(t[nq], t[q], sizeof t[0]);
            fa[nq] = fa[q];
            fa[np] = fa[q] = nq;
            for (; t[p][ch] == q; p = fa[p]) t[p][ch] = nq;
        }
    }
    last = np ? np : nq ? nq : q;
}
int up[M], c[256] = {2}, aa[M];
vector<int> G[M];
void rsort() {
    FOR (i, 1, 256) c[i] = 0;
    FOR (i, 2, sz) up[i] = *(one[i] + len[fa[i]]);
    FOR (i, 2, sz) c[up[i]]++;
    FOR (i, 1, 256) c[i] += c[i - 1];
    FOR (i, 2, sz) aa[--c[up[i]]] = i;
    FOR (i, 2, sz) G[fa[aa[i]]].push_back(aa[i]);
}
// match
int u = 1, l = 0;
FOR (i, 0, strlen(s)) {
    int ch = s[i] - 'a';
    while (u && !t[u][ch]) { u = fa[u]; l = len[u]; }
    ++l; u = t[u][ch];
    if (!u) u = 1;
    if (l) // do something...
}
// substring state
int get_state(int l, int r) {
    int u = rpos[r], s = r - l + 1;
    FORD (i, SP - 1, -1) if (len[pa[u][i]] >= s) u = pa[u][i];
    return u;
}

// LCT-SAM
namespace lct_sam {
    extern struct P *const null;
    const int M = N;
    struct P {
        P *fa, *ls, *rs;
        int last;

        bool has_fa() { return fa->ls == this || fa->rs == this;
               }
        bool d() { return fa->ls == this; }
        P*& c(bool x) { return x ? ls : rs; }
        P* up() { return this; }
        void down() {
            if (ls != null) ls->last = last;
            if (rs != null) rs->last = last;
        }
        void all_down() { if (has_fa()) fa->all_down(); down();
              }
    } *const null = new P{0, 0, 0, 0}, pool[M], *pit = pool;
    P* G[N];
    int t[M][26], len[M] = {-1}, fa[M], sz = 2, last = 1;

    void rot(P* o) {
        bool dd = o->d();
        P *f = o->fa, *t = o->c(!dd);
        if (f->has_fa()) f->fa->c(f->d()) = o; o->fa = f->fa;
        if (t != null) t->fa = f; f->c(dd) = t;
        o->c(!dd) = f->up(); f->fa = o;
    }
    void splay(P* o) {
        o->all_down();
        while (o->has_fa()) {
            if (o->fa->has_fa())
                rot(o->d() ^ o->fa->d() ? o : o->fa);
            rot(o);
        }
        o->up();
    }
    void access(int last, P* u, P* v = null) {
        if (u == null) { v->last = last; return; }
        splay(u);
        P *t = u;
        while (t->ls != null) t = t->ls;
        int L = len[fa[t - pool]] + 1, R = len[u - pool];

        if (u->last) bit::add(u->last - R + 2, u->last - L + 2,
              1);
        else bit::add(1, 1, R - L + 1);
        bit::add(last - R + 2, last - L + 2, -1);

        u->rs = v;
        access(last, u->up()->fa, u);
    }
    void insert(P* u, P* v, P* t) {
        if (v != null) { splay(v); v->rs = null; }
        splay(u);
        u->fa = t; t->fa = v;
    }

    void ins(int ch, int pp) {
        int p = last, np = last = sz++;
        len[np] = len[p] + 1;
        for (; p && !t[p][ch]; p = fa[p]) t[p][ch] = np;
        if (!p) fa[np] = 1;
        else {
            int q = t[p][ch];
            if (len[p] + 1 == len[q]) { fa[np] = q; G[np]->fa =
                 G[q]; }
            else {
                int nq = sz++; len[nq] = len[p] + 1;
                memcpy(t[nq], t[q], sizeof t[0]);
                insert(G[q], G[fa[q]], G[nq]);
                G[nq]->last = G[q]->last;
                fa[nq] = fa[q];
                fa[np] = fa[q] = nq;
                G[np]->fa = G[nq];
                for (; t[p][ch] == q; p = fa[p]) t[p][ch] = nq;
            }
        }
        access(pp + 1, G[np]);
    }

    void init() {
        ++pit;
        FOR (i, 1, N) {
            G[i] = pit++;
            G[i]->ls = G[i]->rs = G[i]->fa = null;
        }
        G[1] = null;
    }
}
```

7 Miscellaneous

7.1 Date

```
// Routines for performing computations on dates.  In these
// routines, months are exprsesed as integers from 1 to 12, days
// are expressed as integers from 1 to 31, and
// years are expressed as 4-digit integers.
string dayOfWeek[] = {"Mo", "Tu", "We", "Th", "Fr", "Sa", "Su"};
// converts Gregorian date to integer (Julian day number)
int DateToInt (int m, int d, int y){
  return
    1461 * (y + 4800 + (m - 14) / 12) / 4 +
    367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
    3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
    d - 32075;
}
// converts integer (Julian day number) to Gregorian date: month
     /day/year
void IntToDate (int jd, int &m, int &d, int &y){
  int x, n, i, j;
  x = jd + 68569;
  n = 4 * x / 146097;
  x -= (146097 * n + 3) / 4;
  i = (4000 * (x + 1)) / 1461001;
  x -= 1461 * i / 4 - 31;
  j = 80 * x / 2447;
  d = x - 2447 * j / 80;
  x = j / 11;
  m = j + 2 - 12 * x;
  y = 100 * (n - 49) + i + x;
}
// converts integer (Julian day number) to day of week
string IntToDay (int jd){
  return dayOfWeek[jd % 7];
}
```

7.2 Subset Enumeration

```
// all proper subset
for (int s = (S - 1) & S; s; s = (s - 1) & S) {
    // ...
}

// subset of length k
template<typename T>
void subset(int k, int n, T&& f) {
    int t = (1 << k) - 1;
    while (t < 1 << n) {
        f(t);
        int x = t & -t, y = t + x;
        t = ((t & ~y) / x >> 1) | y;
    }
}
```

7.3 Digit DP

```
LL dfs(LL base, LL pos, LL len, LL s, bool limit) {
    if (pos == -1) return s ? base : 1;
    if (!limit && dp[base][pos][len][s] != -1) return dp[base][
          pos][len][s];
    LL ret = 0;
    LL ed = limit ? a[pos] : base - 1;
    FOR (i, 0, ed + 1) {
        tmp[pos] = i;
        if (len == pos)
            ret += dfs(base, pos - 1, len - (i == 0), s, limit
                  && i == a[pos]);
        else if (s &&pos < (len + 1) / 2)
            ret += dfs(base, pos - 1, len, tmp[len - pos] == i,
                  limit && i == a[pos]);
        else
            ret += dfs(base, pos - 1, len, s, limit && i == a[
                  pos]);
    }
    if (!limit) dp[base][pos][len][s] = ret;
    return ret;
}

LL solve(LL x, LL base) {
    LL sz = 0;
    while (x) {
        a[sz++] = x % base;
        x /= base;
    }
    return dfs(base, sz - 1, sz - 1, 1, true);
}
```

7.4 Simulated Annealing

```
// Minimum Circle Cover
using LD = double;
const int N = 1E4 + 100;
int x[N], y[N], n;
LD eval(LD xx, LD yy) {
    LD r = 0;
    FOR (i, 0, n)
        r = max(r, sqrt(pow(xx - x[i], 2) + pow(yy - y[i], 2)));
    return r;
}
mt19937 mt(time(0));
auto rd = bind(uniform_real_distribution<LD>(-1, 1), mt);
int main() {
    int X, Y;
    while (cin >> X >> Y >> n) {
        FOR (i, 0, n) scanf("%d%d", &x[i], &y[i]);
        pair<LD, LD> ans;
        LD M = 1e9;
        FOR (_, 0, 100) {
            LD cur_x = X / 2.0, cur_y = Y / 2.0, T = max(X, Y);
            while (T > 1e-3) {
                LD best_ans = eval(cur_x, cur_y);
                LD best_x = cur_x, best_y = cur_y;
                FOR (___, 0, 20) {
                    LD nxt_x = cur_x + rd() * T, nxt_y = cur_y +
                          rd() * T;
                    LD nxt_ans = eval(nxt_x, nxt_y);
                    if (nxt_ans < best_ans) {
                        best_x = nxt_x; best_y = nxt_y;
                        best_ans = nxt_ans;
                    }
                }
                cur_x = best_x; cur_y = best_y;
                T *= .9;
            }
```

```
if (eval(cur_x, cur_y) < M) {
    ans = {cur_x, cur_y}; M = eval(cur_x, cur_y);
}
```

```
}
printf("(%.1f,%.1f).\n%.1f\n", ans.first, ans.second,
      eval(ans.first, ans.second));
```

```
    }
}
```