Educational and Workforce Network for Assessments, Training, Coaching, and the Award of Digital Credentials

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Abstract

There is a fundamental shift occurring within the global workforce, which requires constant learning and acquisition of new skills that are highly valued in the market. The demand for highly skilled workers across several industries and sectors is increasing at a pace far greater than existing traditional and technical colleges can produce. This trend of growing demand for those who have skills that are valued in today and tomorrow's job market has led to an increase for additional education and training options. The online learning and development industry surpassed USD 200 billion in 2019 and is anticipated to grow at over 8% between 2020 and 2026[1]. To penetrate this industry, we have developed the XChainz Platform an artificially intelligent (AI), blockchain-based system that will help people enhance their technical skills that will be required within the world of work of today and tomorrow.

[1] https://www.gminsights.com/industry-analysis/elearning-market-size

Introduction

The future of work on an individual basis will be characterized by high job displacement, frequent job changes, and constant reskilling unlike any time in the history of the world. Organizations are beginning to invest hundreds of millions of dollars to train and retool their workforce. Some small and medium-sized businesses will be unable to train employees and will look to hire only those people with the knowledge, skills, abilities, and other characteristics (KSAOs) companies need to bring value to the marketplace. As a result, individuals will have the responsibility of diversifying their portfolio of verifiable KSAOs during their own time and at their own personal expense. This will be both costly and time-consuming.

Those that do not make reskilling a priority will find that they lack the skills to compete sufficiently in the 21st century workforce. The skills that are valued in the 21st century job market goes behind traditional technical skills and include a combination of both hard and soft skills.

Solution

The XChainz platform is an AI, blockchain-based platform that provides assessment, coaching, training and verifiable certifications for individuals looking to compete within the global workforce of tomorrow. This is accomplished through the development of an artificially intelligent learning platform for the acquisition of hard and soft skills needed in the emerging world of work. Training on the platform will be supplemented by assessments and member coaching services. Algorithms for ability estimation, training difficulty, and assessments are derived and presented in the appendix. Training, assessment, and coaching will be incentivized using the XCZ coin. Certificates of training completion, assessment credentials, coaching, and payments for services will be registered on the XChainz blockchain and the Stellar Blockchain. Stellar is used as the smart contracting mechanism for the awards of an open standards complaint blockchain-based credential, which will be issued via the open-source software, Blockcerts (www.blockcerts.org). As a multi-faceted education platform using the latest innovations in blockchain, artificial intelligence, and scalable database management systems, the platform provides a new level of self-sovereignty and transparency to human capital management practices worldwide.

Assessments

The XChainz skills assessments are used to evaluate and guage professional skills. It is an ideal way to collect data to inform on skills development gaps and needs. Skill gap identification helps students and professionals to understand which skills need to be improved or developed in order for them to meet the demands of today's workforce. Before starting any training, it is important to assess the level of proficiency that each user has by using our skills estimation algorithm. Details on the derivation of this procedure are thoroughly documented in the appendix.

Though a number of different methods can be used in conducting a skills assessment, the XCert platform assessment is structured so the student or professional answers a set of questions to evaluate their skills or abilities. That information is used to determine how they can improve those skills using our adaptive algorithm, which is derived in the appendix and implemented on the platform by the XChainZ team. Once the assessment has been completed and skills need to be further developed is understood, recommendations on actions to take to enhance their performance skills are provided. These recommendations include training and coaching in the specific areas identified in the assessment results needed for skills and capability enhancement.

Training

Convenient and flexible access to learning is one of the key advantages of online learning platforms and within the XCert system, courses can be accessed anytime, anywhere from a mobile device, laptop, tablet or desktop. The platform provides self-learning courses that support professional development and are created by trained, knowledgeable professionals in various disciplines. These training courses are designed to be a component of a comprehensive professional development roadmap and are comprised of various lessons and teaching units to ensure that with each completed lesson, the student has increased their knowledge. The skills of the future include a combination of both hard and soft skills. Hard skills are related to specific technical knowledge and training while soft skills are more aligned to personality traits such as leadership, communication or time management. The online learning courses available will include the following skills areas:

SOFT SKILLS	**HARD SKILLS**
Motivation	Descriptive/Inferential Statistic
Goal Setting	Bayesian Statistics
Meta-Cognition	Machine Learning
Emotional Intelligence	Human Intelligence
Personality	Artificial Intelligence
Leadership	Python Programming
Teamwork	SQL

Flow	Ethical Hacking
Grit	C++
Optimism	Management
Critical Thinking	Organizational Behavior
Creativity	Entrepreneurship
Problem Solving	Human Factors/Performance
	Experimental Design

Although the development of these skills will differ, through the training provided by the platform, students can simultaneously learn and develop both hard and soft skills needed to become competitive in 21st century marketplace. We are attempting to address a market that will be approximately $325 billion in size by 2026.

Coaching

What makes the XChainz platform stand above other similar services, is the offering of both direct and program-based coaching. Direct coaching provides the client with a live 1-on-1 coaching session where the coach and client engage in real-time interactions. This coaching method has been proven to be the most effective way to deliver coaching. The Coaching service will provide a scheduling tool that supports clients in a various global time zones. Today though, due to time or schedule constraints, numerous clients are more comfortable with program-based coaching. The program-based provides pre-designed coaching modules that are available based on the client's availability. Each program consists of a number of lessons and as the client completes each lesson and activities, the next lesson is made available to the client.

Coaching services and offerings will be provided for the following coaching specialty areas:

- Business Coaching: the practice of providing support and occasional advice to an individual or group in order to help them recognize ways in which they can improve the effectiveness of their business. Business coaches often help businesses grow by creating and following a structured, strategic plan to achieve agreed upon goals.
- Personal Development and Career Coaching: the practice of providing a positive, action orientated, future focused approach toward personal development, fulfilling goals, and achieving your own individual definition of a successful life and career.
- Executive Coaching: the practice of assisting top executives, managers, and other identified leaders to perform, learn, stay healthy and balanced, and effectively guide their teams to successfully reach desired goals and exceed individual and corporate expectations.

Within each of these specialty areas coaching will cover a wide-range of in-demand areas. To ensure our coaches can meet and exceed client expectations, each coach will have met the requirements of an industry approved accredited program. Ensuring that each coach has met this high standard of both training and coaching hours completed will increase the credibility of our coaching platform. In addition, this requirement will greatly increase our ability to provide those that seek coaching with the highest quality coaches.

Digital/Micro Credentialing

The XCert platform utilizes new age certifying technologies and provides students and professionals with micro-credentials that certify their areas of competence and specialized learning. Micro-credentials, which are similar to certifications and are also referred to as digital badges; they are earned by completing a course, assessment, or coaching related to a specific topic. These credentials provide an additional benefit not available with standard certifications. They are portable, blockchain-based, standards-compliant certificates issued using Stellar Smart Contracts and the Blockcerts open-source technology. Certificates are cryptographically-signed documentation that a participant has completed the requirements of an offering, however they do not provide details about the criteria required for that certificate. With our micro-credentials, one can access the name and date of the course, the detailed criteria required to earn it, and the competencies that were attained.

Depending on the skill being developed and requirements of the learning, the time required to complete and earn the credential can take anywhere from a few weeks to a year. Regardless of the duration, once all of the requirements have been met, an XChainz branded micro-credential is awarded as official evidence of completion of the necessary work. These types of credentials are now very commonly used for those seeking employment or career advancement opportunities. Blockcerts provides a scalable infrastructure for any organization to easily join the XChainZ network and issue certificates on our platform.

As the micro-credentials are specific to the individual who earns it, they can easily be displayed on digital resumes, shared on social media, or as part of an email signature. In addition, the Blockcerts platform solves the problems of credential access and control by providing the individuals with secure storage of the micro-credential, ability to create a virtual skills portfolio, and full control over who can access information. Credentials and payments for training services will be completed on the XChainz blockchain and allows our customers to have total ownership and control of their data and credentials.

Loyalty and Reward Program

Sustainable growth demands customer loyalty and to promote this, we have created a unique XChainz loyalty program that provides XCZ to reward and retain our most valued instructors and students while converting them to powerful brand advocates.

Instructors, assessors, coaches, students and professionals will each earn XCZ rewards to encourage frequent use of the platform for training, assessment, and coaching services. There are rewards for completion of training modules. Those providing training, assessments, and coaching will be rewarded each time their offering is purchased. Rewards will be accumulated over time and follow a tired-reward program which provides a larger amount of XCZ rewards each time a new tier is reached. Providing a higher level of rewards for those that consistently contribute at a higher rate further instills loyalty and strengthens our offerings. This online learning platform enables instructors to design and offer courses that span a variety of different industries and are highly sought after by students.

To provide students and professionals additional motivation to not just purchase an offering, but to complete the offering, a rewards program providing XCZ also exists for them. The standard amount of XCZ awarded is determined by the requirements of the offering and are provided upon completion. The tiered-reward program also applies to the students with the amount of XCZ also increasing as each new tier is reached. Clients can also receive rewards from the referral program which provides XCZ for each referred client that purchases an offering.

Summary (and call to action)

The XChainz Education Platform leverages technology, data, and automation to provide an online learning experience that equips students and professionals with the skills needed to be highly productive and successful.

Masternodes

Masternodes play an important part of the XCZ network. A masternode network is the second layer of the XCZ network that donates processing power to confirm transactions instantly utilizing the SwiftTx technology inherited from PIVX. A masternode then receives a reward for the work performed – one reward per block every 60 seconds. These rewards are directly paid to an XCZ wallet that is linked to the masternode. Using masternodes also ensures the stability and security of the entire network. These nodes serve a special purpose within the network to mix various transaction amounts to increase fungibility and anonymity of transactions.

Block Reward Diagram

The XCZ block reward distribution table reflect the reward amounts for wallets that are open for staking and masternode owners.   With each block a different node is randomly selected and rewarded. The ‘minted’ block rewards are distributed on a sliding percentage scale between masternode owners and staked owners to create a fair distribution of coins.

XCZ Coin Specification

Coin Name:	XChainz Coin
Ticker:	XCZ
Algorithm (POW/POS):	X11/POS
Type/Consensus	Proof of Stake /zPOS
Block Reward:	5 XCZ
Masternode Collateral:	1000 XCZ
Masternode Reward:	80%
Staking (Proof of Stake) Reward:	20%
Block Time:	60 seconds
Total Supply	42,000,000 XZC
Premine:	19,700,000 XCZ
Founders	2,000,000 XCZ (1- year lockup period)
Team Operational Overhead	7,000,000 XCZ
LPC Swap	10,700,000 XCZ

Block Range	**Reward Per Block**	**% Masternode Reward**	**% Staking Reward**	**MN Reward (Coins)**
201 - 4200200	5	80	20	4

The Team & Contact Information

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Appendix A. Training and Assessment Estimation

Proof: Directional Information and $\boldsymbol{\theta}_{max}$

The purpose of this work is to derive explicit formulas for the item information function in any direction and theta maximum for the Multidimensional 3-PL (M3-PL) model. This model is used for the estimation of training difficulty and assessment item difficulty. First, there is a review of the item response model. Next, there is a derivation of the item information in any direction. Finally, a proof is given of theta maximum in any direction for the M3-PL model.

The probability of a correct response for the M3-PL model (Reckase, 1997; Bryant & Davis, 2011) is

$$P_i(\boldsymbol{\theta}_j) = c_i + (1 - c_i)/[1 + \text{Exp}(-L)] \tag{1}$$

where $L = D(\boldsymbol{a}_i' \boldsymbol{\theta}_j + d_i)$, D is equal to a scaling constant 1.7 or 1, $\boldsymbol{a}_i$ is a vector of k discrimination parameters for item i, $[a_{1i}, a_{2i}, \ldots, a_{ki}]'$, k is the number of dimensions, $\boldsymbol{\theta}_j$ is a vector of k ability parameters for person j, $[\theta_{1j}, \theta_{2j}, \ldots, \theta_{kj}]'$, and d_i is a scalar related to difficulty. The probability of an incorrect response is given by $Q_i(\boldsymbol{\theta}_j) = 1 - P_i(\boldsymbol{\theta}_j)$ or

$$Q_i(\boldsymbol{\theta}_j) = (1 - c_i)/[1 + \text{Exp}(L)] \tag{2}$$

The point of steepest slope in the ability space is known as multidimensional discrimination,

$$MDISC_i = \|\boldsymbol{a}_i\| = (\boldsymbol{a}_i' \boldsymbol{a}_i)^{1/2} \tag{3}$$

where || || represents the length of a vector that is computed as the square root of the sum of squared elements of a vector. MDISC is interpreted in the same manner as the discrimination parameter (a_i) in unidimensional IRT. The difficulty of the item is the signed distance from the origin of the multidimensional space to the point of steepest slope. The formula for multidimensional difficulty is given by

$$MDIFF_i = -d_i(\|\boldsymbol{a}_i\|)^{-1} = -d_i / MDISC_i \tag{4}$$

and it is interpreted in the same way as the difficulty parameter (b_i) in unidimensional IRT. Reckase (1985) has shown MDIFF to be equal to the unidimensional measure of difficulty (b_i) when there is only one dimension.

Item Information Function in Any Direction

The IIF in a specific direction for the multidimensional logistic model (Reckase, 1997; Reckase & McKinley, 1991) is

$$I_{i\boldsymbol{u}}(\boldsymbol{\theta}_j) = [\nabla P_i(\boldsymbol{\theta}_j) \bullet \boldsymbol{u}_i]^2 / [P_i(\boldsymbol{\theta}_j) Q_i(\boldsymbol{\theta}_j)] \tag{5}$$

$\nabla P_i(\boldsymbol{\theta}_j) \bullet \boldsymbol{u}_i$ is the directional derivative, $\nabla P_i(\boldsymbol{\theta}_j)$ is the gradient. The vector of directional cosines, $\boldsymbol{u}_i$, is $[a_{1i}/\|\boldsymbol{a}_i\|, a_{2i}/\|\boldsymbol{a}_i\|, \ldots, a_{ki}/\|\boldsymbol{a}_i\|]'$ or $[\cos\alpha_{1i}, \cos\alpha_{2i}, \ldots, \cos\alpha_{ki}]'$, where $\cos\alpha_k$ is the cosine of the angle (α_k) from the axis orthogonal to dimension k. It should be noted that $\|\boldsymbol{u}_i\| = 1$. Next is the derivation of the information function in a specified direction.

The first term in brackets within (5) is the gradient of the function $P_i(\boldsymbol{\theta}_j)$, which can be written as $\nabla P_i(\boldsymbol{\theta}_j)$ =

$$\nabla P_i(\boldsymbol{\theta}_j) = [\partial P_i(\boldsymbol{\theta}_j)/\partial\theta_{1j}, \partial P_i(\boldsymbol{\theta}_j)/\partial\theta_{2j}, \ldots, \partial P_i(\boldsymbol{\theta}_j)/\partial\theta_{kj}]' \tag{6}$$

where the k^{th} term of the vector is the first partial derivative of $P_i(\boldsymbol{\theta}_j)$ with respect to θ_{kj}. The general expression for the derivative of the k^{th} term in the gradient is as follows:

$-(1-c_i)[1+\text{Exp}(-L)]^{-2}\text{Exp}(-L)(-Da_{ki})$

$$= Da_{ki}(1-c_i)\,[1+\mathrm{Exp}(L)]^{-1}[1+\mathrm{Exp}(-L)]^{-1} \quad (7)$$

or substituting expression (2) into (7) yields

$$Da_{ki}\,Q_i(\boldsymbol{\theta}_j)[1+\mathrm{Exp}(-L)]^{-1} \quad (8)$$

With the k elements of the gradient having the general form of (8) and the elements of $\boldsymbol{u}_i$ having the general form of $a_{ki}/\|\boldsymbol{a}_i\|$, the directional derivative of the function $P_i(\boldsymbol{\theta}_j)$ in the direction $\boldsymbol{u}_i$ can be expressed as

$$\nabla P_i(\boldsymbol{\theta}_j) \bullet \boldsymbol{u}_i = D(\boldsymbol{a}_i'\boldsymbol{u}_i)Q_i(\boldsymbol{\theta}_j)[1+\mathrm{Exp}(-L)]^{-1} \quad (9)$$

With the right-hand side of (9) substituted in (5), the IIF for the M3-PL model in a specified direction $\boldsymbol{u}_i$ is

$$I_{i\boldsymbol{u}}(\boldsymbol{\theta}_j) = \{D(\boldsymbol{a}_i'\boldsymbol{u}_i)Q_i(\boldsymbol{\theta}_j)[1+\mathrm{Exp}(-L)]^{-1}\}^2[P_i(\boldsymbol{\theta}_j)Q_i(\boldsymbol{\theta}_j)]^{-1} \quad (10)$$

or with some algebra, the item information in a direction $\boldsymbol{u}$ becomes

$$I_{i\boldsymbol{u}}(\boldsymbol{\theta}_j) = D^2(\boldsymbol{a}_i'\boldsymbol{u}_i)^2Q_i(\boldsymbol{\theta}_j)\{\,P_i(\boldsymbol{\theta}_j)[1+\mathrm{Exp}(-L)]^2\}^{-1} \quad (11)$$

Corollary 1a. If it is assumed that there is no guessing (i.e., $c_i = 0$), the information function in (11) becomes the IIF in a direction for the M2-PL model,

$$I_{i\boldsymbol{u}}(\boldsymbol{\theta}_j) = D^2(\boldsymbol{a}_i'\boldsymbol{u}_i)^2\,P_i(\boldsymbol{\theta}_j)\,Q_i(\boldsymbol{\theta}_j) \quad (12)$$

which is like the formula derived by Reckase and McKinley (1991).

Corollary 1b. If it is assumed that discrimination parameters on all of k dimensions are fixed at 1 and there is no guessing (i.e., $\boldsymbol{a}_i = [1, 1, ..., 1]'$ and $c_i = 0$), then (11) reduces to the M1-PL,

$$I_{i\boldsymbol{u}}(\boldsymbol{\theta}_j) = D^2 k\, P_i(\boldsymbol{\theta}_j)\, Q_i(\boldsymbol{\theta}_j) \qquad (13)$$

where k is equal to the number of dimensions.

Corollary 1c. If there is only one dimension, then (13), (12), and (11) become the item information functions for the unidimensional 1-, 2-, and 3-parameter logistic models, respectively.

$\boldsymbol{\theta}_{\max}$ for the Multidimensional 3-PL Model

The formula for the location of maximum item information or theta maximum in a specified direction is derived by setting the directional derivative of the IIF for the M3-PL model equal to zero,

$$\nabla I_i(\boldsymbol{\theta}_j) \bullet \boldsymbol{u}_i = 0 \qquad (14)$$

where the gradient, $\nabla I_i(\boldsymbol{\theta}_j)$ is

$$\nabla I_i(\boldsymbol{\theta}_j) = [\partial I_i(\boldsymbol{\theta}_j)/\partial\theta_{1j}, \partial I_i(\boldsymbol{\theta}_j)/\partial\theta_{2j}, \ldots, \partial I_i(\boldsymbol{\theta}_j)/\partial\theta_{kj}]'$$

and $\boldsymbol{u}_i$ was defined earlier. The directional information function in (11) is expressed in a different form as

$$I_{i\boldsymbol{u}}(\boldsymbol{\theta}_j)=D^2(\boldsymbol{a}_i'\boldsymbol{u}_i)^2(1-c_i)\text{Exp}(-L)\{[1+c_i\text{Exp}(-L)][1+\text{Exp}(-L)]^2\}^{-1} \qquad (15)$$

where L is the logit, which is equal to $D(\boldsymbol{a}_i'\boldsymbol{\theta}_j + d_i)$. The general form of the kth element of the gradient, $\nabla I_i(\boldsymbol{\theta}_j)$, is derived using the quotient, product, and chain rules of calculus.

$$\partial I_i(\boldsymbol{\theta}_j) / \partial\theta_{kj} = \partial D^2(\boldsymbol{a}_i'\boldsymbol{u}_i)^2(1-c_i)\text{Exp}(-L)\{[1+c_i\text{Exp}(-L)][1+\text{Exp}(-L)]^2\}^{-1} / \partial\theta_{kj}$$

$$= \quad [-Da_{ki}\, D^2(\boldsymbol{a}_i'\boldsymbol{u}_i)^2(1 - c_i)\mathrm{Exp}(-L)\, [1 + c_i\mathrm{Exp}(-L)]\, [1 + \mathrm{Exp}(-L)]^2 -$$

$$\{2[1 + \mathrm{Exp}(-L)]\, \mathrm{Exp}(-L) - Da_{ki}[1+ c_i\mathrm{Exp}(-L)] + [1 + \mathrm{Exp}(-L)]^2 c_i\mathrm{Exp}(-L) - Da_{ki}\}$$

$$D^2(\boldsymbol{a}_i'\boldsymbol{u}_i)^2(1- c_i)\mathrm{Exp}(-L)]\{[1 + c_i\mathrm{Exp}(-L)][1 + \mathrm{Exp}(-L)]^2\}^{-2}$$

$$= [-a_{ki}\, D^3(\boldsymbol{a}_i'\boldsymbol{u}_i)^2\, [\mathrm{Exp}(-L) - c_i\mathrm{Exp}(-L)]\, [1 + c_i\mathrm{Exp}(-L)]\, [1 + \mathrm{Exp}(-L)]^2 \; +$$

$$a_{ki}2\mathrm{Exp}(-L)\, [1 + \mathrm{Exp}(-L)]\, [1+ c_i\mathrm{Exp}(-L)]\, D^3(\boldsymbol{a}_i'\boldsymbol{u}_i)^2[\mathrm{Exp}(-L) - c_i\mathrm{Exp}(-L)] \; +$$

$$a_{ki}c_i\mathrm{Exp}(-L)\, [1 + \mathrm{Exp}(-L)]^2\, D^3(\boldsymbol{a}_i'\boldsymbol{u}_i)^2\, [\mathrm{Exp}(-L) - c_i\mathrm{Exp}(-L)]]\{[1 + c_i\mathrm{Exp}(-L)][1 + \mathrm{Exp}(-L)]^2\}^{-2}$$

$$= [a_{ki}\, \mathrm{D}^3(\boldsymbol{a}_i'\boldsymbol{u}_i)^2\, [\mathrm{Exp}(-L) - c_i\mathrm{Exp}(-L)]\, [1 + \mathrm{Exp}(-L)]^2\{-1 - c_i\mathrm{Exp}(-L) + c_i\mathrm{Exp}(-L)\} + a_{ki}2\mathrm{Exp}(-L)$$

$$[1 + \mathrm{Exp}(-L)]\, [1+ c_i\mathrm{Exp}(-L)]\, \mathrm{D}^3(\boldsymbol{a}_i'\boldsymbol{u}_i)^2[\mathrm{Exp}(-L) - c_i\mathrm{Exp}(-L)]]\, \{[1 + c_i\mathrm{Exp}(-L)][1 + \mathrm{Exp}(-L)]^2\}^{-2}$$

$$= [a_{ki}\mathrm{D}^3(\boldsymbol{a}_i'\boldsymbol{u}_i)^2\, [\mathrm{Exp}(-L) - c_i\mathrm{Exp}(-L)]\, [1 + \mathrm{Exp}(-L)]^2\{-1\} + a_{ki}2\mathrm{Exp}(-L)\, [1 + \mathrm{Exp}(-L)]\, [1+$$

$$c_i\mathrm{Exp}(-L)]\, D^3(\boldsymbol{a}_i'\boldsymbol{u}_i)^2[\mathrm{Exp}(-L) - c_i\mathrm{Exp}(-L)]]\{[1 + c_i\mathrm{Exp}(-L)][1 + \mathrm{Exp}(-L)]^2\}^{-2}$$

$$= D^3a_{ki}(\boldsymbol{a}_i'\boldsymbol{u}_i)^2\, [\mathrm{Exp}(-L) - c_i\mathrm{Exp}(-L)]\, [1 + \mathrm{Exp}(-L)]\, [\{-1\}[1 + \mathrm{Exp}(-L)] + 2\mathrm{Exp}(-L)$$

$$[1+ c_i\mathrm{Exp}(-L)]\,]\{[1 + c_i\mathrm{Exp}(-L)][1 + \mathrm{Exp}(-L)]^2\}^{-2}$$

$$= D^3a_{ki}(\boldsymbol{a}_i'\boldsymbol{u}_i)^2\, [\mathrm{Exp}(-L) - c_i\mathrm{Exp}(-L)]\, [1 + \mathrm{Exp}(-L)]\, [\{-1\}[1 + \mathrm{Exp}(-L)] + 2\mathrm{Exp}(-L)$$

$$[1 + c_i\mathrm{Exp}(-L)]\,]\{\, [1 + c_i\mathrm{Exp}(-L)]^2[1 + \mathrm{Exp}(-L)]^4\}^{-1}$$

$$= D^3a_{ki}(\boldsymbol{a}_i'\boldsymbol{u}_i)^2\, [\mathrm{Exp}(-L) - c_i\mathrm{Exp}(-L)]\, [\{-1\}[1 + \mathrm{Exp}(-L)] + 2\mathrm{Exp}(-L)$$

$$[1 + c_i\mathrm{Exp}(-L)]\,]\{[1 + c_i\mathrm{Exp}(-L)]^2[1 + \mathrm{Exp}(-L)]^3\}^{-1}$$

$= -D^3 a_{ki}(\boldsymbol{a}_i'\boldsymbol{u}_i)^2[\text{Exp}(-L) - c_i\text{Exp}(-L)]\,[1 + \text{Exp}(-L)]\{[1 + c_i\text{Exp}(-L)]^2[1 + \text{Exp}(-L)]^3\}^{-1} +$

$D^3 a_{ki}(\boldsymbol{a}_i'\boldsymbol{u}_i)^2\,[\text{Exp}(-L) - c_i\text{Exp}(-L)]\;2\text{Exp}(-L)\,[1 + c_i\text{Exp}(-L)]\{[1 + c_i\text{Exp}(-L)]^2[1 + \text{Exp}(-L)]^3\}^{-1}$

$= -D^3 a_{ki}(\boldsymbol{a}_i'\boldsymbol{u}_i)^2\,[\text{Exp}(-L) - c_i\text{Exp}(-L)]\{[1 + c_i\text{Exp}(-L)]^2[1 + \text{Exp}(-L)]^2\}^{-1} +$

$D^3 a_{ki}(\boldsymbol{a}_i'\boldsymbol{u}_i)^2\,[\text{Exp}(-L) - c_i\text{Exp}(-L)]\;2\text{Exp}(-L)\{[1 + c_i\text{Exp}(-L)]\,[1 + \text{Exp}(-L)]^3\}^{-1}$

(16)

From (16), the k^{th} element of the gradient $\nabla I_i(\boldsymbol{\theta}_j)$ can be expressed as

$D^3 a_{ki}(\boldsymbol{a}_\text{i}'\boldsymbol{u}_i)^2[\text{Exp}(-L)-c_i\text{Exp}(-L)]\{[1+c_i\text{Exp}(-L)][1+\text{Exp}(-L)]^2\}^{-1}\{-1[1+c_i\text{Exp}(-L)]^{-1}+2\text{Exp}(-L)[1+\text{Exp}(-L)]^{-1}\}$

(17)

With the k elements of the gradient, $\nabla I_i(\boldsymbol{\theta}_j)$, having the general form of (17) and the elements of $\boldsymbol{u}_i$ having the general form of $a_{ki}/||\boldsymbol{a}_i||$, the directional derivative of the IIF, $I_i(\boldsymbol{\theta}_j)$, in the direction $\boldsymbol{u}_i$ is expressed as

$\nabla I_i(\boldsymbol{\theta}_j)\bullet \boldsymbol{u}_i=[D^3(\boldsymbol{a}_i'\boldsymbol{u}_i)^3[\text{Exp}(-L)-c_i\text{Exp}(-L)]\{[1+c_i\text{Exp}(-L)]\,[1+\text{Exp}(-L)]^2\}^{-1}]\{-1[1+c_i\text{Exp}(-L)]^{-1}+2\text{Exp}(-L)[1+\text{Exp}(-L)]^{-1}\}=0$

(18)

From (18), item information is maximized when the following condition is satisfied:

$$-1[1+c_i\text{Exp}(-L)]^{-1}+2\text{Exp}(-L)[1+\text{Exp}(-L)]^{-1}=0 \qquad (19)$$

or

$$P_i(\theta_j)=.5\ \text{Exp}(L) \qquad (20)$$

Expression (20) is a sufficient condition for maximizing item information for the unidimensional and multidimensional 1-, 2-, and 3-PL models. In the unidimensional 1-PL model, information is maximized when $P_i(\theta_j)=Q_i(\theta_j)=.5$. When $P_i(\theta_j)$ is equal to .5, then the natural logarithm of the odds of getting the item correct is 0, i.e., $\ln[P_i(\theta_j)/Q_i(\theta_j)]=0$, which implies that L in (20) is 0 and $\mathrm{Exp}(L)=1$. This leaves the well-known condition for maximizing information in the 1- and 2-PL cases, which is $P_i(\theta_j)=Q_i(\theta_j)=.5$. For the 3-PL unidimensional and multidimensional models, the right-hand side of (20) and the probability of a correct response are not equal to .5 when information is maximized, but the equality is satisfied at a different value, which is primarily a function of guessing. The expression in (20) can also be written as

$$2\mathrm{Exp}(-L)=[P_i(\theta_j)]^{-1} \tag{21}$$

The probability of a correct response can also be written as

$$P_i(\theta_j)=[\mathrm{Exp}(L)+c_i][\mathrm{Exp}(L)+1]^{-1} \tag{22}$$

With the right-hand side of (22) substituted into (21),

$$2\mathrm{Exp}(-L)=[\mathrm{Exp}(L)+1][\mathrm{Exp}(L)+c_i]^{-1} \tag{23}$$

To solve for L, expression (23) is written as

$$2+2c_i\mathrm{Exp}(-L)=\mathrm{Exp}(L)+1 \tag{24}$$

After multiplying both sides by Exp(L),

$$2\mathrm{Exp}(L)+2c_i=\mathrm{Exp}(2L)+\mathrm{Exp}(L) \tag{25}$$

which, after subtracting 2Exp(L) from both sides, is equivalent to

$$2c_i=\mathrm{Exp}(2L)-\mathrm{Exp}(L) \tag{26}$$

After multiplying both sides by 4, expression (26) be comes

$$8c_i=4\text{Exp}(2L)–4\text{Exp}(L) \qquad (27)$$

When 1 is added to both sides, then (27) becomes

$$8c_i+1=4\text{Exp}(2L)–4\text{Exp}(L)+1 \qquad (28)$$

By way of the binomial theorem, the equality of (28) can be written as

$$8c_i+1=[2\text{Exp}(L)–1]^2 \qquad (29)$$

which after taking the square root of both sides becomes

$$(8c_i+1)^{1/2}=2\ \text{Exp}(L)–1 \qquad (30)$$

Solving for L in (30) gives

$$L= \ln\{.5[1+(8c_i+1)^{1/2}]\} \qquad (31)$$

which becomes

$$D(\boldsymbol{a}_i{}'\theta_j+d_i)=\ln\{.5[1+(8c_i+1)^{1/2}]\} \qquad (32)$$

Now the objective is to solve for the vector $\boldsymbol{\theta}$ that maximizes the information function of the M3-PL model in the direction $\boldsymbol{u}_i$. To that end,

$$\boldsymbol{a}_i{}'\theta=\ln\{.5\ [1+(8c_i+1)^{1/2}\ ]\}D^{-1}–d_i \qquad (33)$$

Because $\boldsymbol{u}_i{}' = \boldsymbol{a}_i{}'/\|\boldsymbol{a}_i\|$, both sides are divided by $\|\boldsymbol{a}_i\|$ or $MDISC_i$ that results in

$$\boldsymbol{u}_i{}'\theta=\ln\{.5\ [1+(8c_i+1)^{1/2}]\}(D\|\boldsymbol{a}_i\|)^{-1}– d_i(\|\boldsymbol{a}_i\|)^{-1} \qquad (34)$$

Now both sides are pre-multiplied by $\boldsymbol{u}_{\mathrm{i}}$,

$$\boldsymbol{u}_i\boldsymbol{u}_i{}'\theta=\boldsymbol{u}_i[\ln\{.5[1+(8c_i+1)^{1/2}]\}(D\|\boldsymbol{a}_i\|)^{-1}–d_i(\|\boldsymbol{a}_i\|)^{-1}] \qquad (35)$$

Because the associative law holds for multiplication of matrices, the left-hand side of (35) can be written as $(\boldsymbol{u}_i\boldsymbol{u}_i{}')\boldsymbol{\theta}$. The product of $(\boldsymbol{u}_i\ \boldsymbol{u}_i{}')$ is a k by k matrix, $\boldsymbol{U}$, which has a few properties that should be noted: (1) It is symmetric, thus $\boldsymbol{U}=\boldsymbol{U}'$, (2) The determinant of the matrix $\boldsymbol{U}$ is zero (0), thus it is singular, (3) $\boldsymbol{U}^2=\boldsymbol{U}'\boldsymbol{U}=\boldsymbol{UU}=\boldsymbol{U}$, thus $\boldsymbol{U}$ is idempotent, (4) The main diagonal elements of $\boldsymbol{U}$ are $\cos^2\alpha_{ki}$, thus the trace of $\boldsymbol{U}$ [i.e., tr($\boldsymbol{U}$)] is equal to one (1), and 5. The rank of an idempotent matrix is equal to its trace, thus the rank of $\boldsymbol{U}$ is equal to 1. With $\mathbf{U}$ substituted for $\boldsymbol{u}_i\,\boldsymbol{u}_i'$, expression (35) is written as

$$\boldsymbol{U}\theta=\boldsymbol{u}_i[\ln\{.5[1+(8c_i+1)^{1/2}]\}(D\|\boldsymbol{a}_i\|)^{-1}–d_i(\|\boldsymbol{a}_i\|)^{-1}] \tag{36}$$

Pre-multiply both sides by $\boldsymbol{U}$ yields

$$\boldsymbol{UU}\theta=\boldsymbol{Uu}_i[\ln\{.5[1+(8c_i+1)^{1/2}]\}(D\|\boldsymbol{a}_i\|)^{-1}–d_i(\|\boldsymbol{a}_i\|)^{-1}] \tag{37}$$

By the third property mentioned above for $\mathbf{U}$, (37) can be written as

$$\boldsymbol{U}\theta=\boldsymbol{Uu}_i[\ln\{.5[1+(8c_i+1)^{1/2}]\}(D\|\boldsymbol{a}_i\|)^{-1}–d_i(\|\boldsymbol{a}_i\|)^{-1}] \tag{38}$$

Expression (38) is written as

$$\boldsymbol{U}\theta–\boldsymbol{Uu}_i[\ln\{.5[1+(8c_i+1)^{1/2}]\}(D\|\boldsymbol{a}_i\|)^{-1}–d_i(\|\boldsymbol{a}_i\|)^{-1}]=\mathbf{0} \tag{39}$$

where $\mathbf{0}$ is the same size as $\boldsymbol{u}_i$ or by the distributive law for matrices,

$$\boldsymbol{U}\{\theta–\boldsymbol{u}_i[\ln\{.5[1+(8c_i+1)^{1/2}]\}(D\|\boldsymbol{a}_i\|)^{-1}– d_i(\|\boldsymbol{a}_i\|)^{-1}]\}=\mathbf{0} \tag{40}$$

Solutions that satisfy (40) are when $\boldsymbol{U}$ is equal to a null matrix ($\boldsymbol{O}$) and when

$$\boldsymbol{\theta}=\boldsymbol{u}_i[\ln\{.5[1+(8c_i+1)^{1/2}]\}(D\|\boldsymbol{a}_i\|)^{-1}–d_i(\|\boldsymbol{a}_i\|)^{-1}] \tag{41}$$

Therefore, theta maximum in a specified direction for the M3-PL model is

$$\boldsymbol{\theta}_{max}=\boldsymbol{u}_i[\ln\{.5[1+(8c_i+1)^{1/2}]\}(D\|\boldsymbol{a}_i\|)^{-1}– d_i(\|\boldsymbol{a}_i\|)^{-1}] \tag{42}$$

or substituting (3) and (4) into (42)

$$\boldsymbol{\theta}_{max}=\boldsymbol{u}_i[\ln\{.5[1+(8c_i +1)^{1/2}]\}(D\cdot MDISC_i)^{-1}+MDIFF_i] \tag{43}$$

Several corollaries follow from this result in (43).

Corollary 2a. The location on the k^{th} dimension where information is maximized is

$$\theta_{max\,k}=[\ln\{.5\,[1+(8c_i+1)^{1/2}]\}(D{\cdot}MDISC_i)^{-1}+MDIFF_i]\cos\alpha_{ki} \tag{44}$$

Corollary 2b. If it is assumed that there is no guessing, i.e., c_i=0, expression (43) reduces to

$$\boldsymbol{\theta}_{max}=MDIFF_i\boldsymbol{u}_i \tag{45}$$

Expression (45) is the same as that implied by Reckase and McKinley (1991) for the location of maximum item information for the multidimensional 2-PL model.

Corollary 2c. If it is assumed that there is only one dimension, then the expression in (43) reduces to the well-known formula for theta maximum derived by Birnbaum (1968),

$$\theta_{max}=\ln\{.5\,[1+(8c_i+1)^{1/2}]\}(Da_i)^{-1}+b_i \tag{46}$$

IIFs and formulas for location of maximum item information for the multidimensional 1-, 2-, and 3-PL models are listed in Table 2.

Appendix B. Ability Estimation Algorithm

The purpose of this appendix is to provide a proof of the ability estimation algorithm used to estimate student proficiency as s/he navigates through different training modules and. assessments. With advances in computer technology, Item Response Theory (IRT) has moved from a theoretical discussion to a practical framework for designing cognitive assessments. It has been shown that greater measurement efficiency can be obtained with fewer items and in less test administration time when using IRT as compared to Classical Test Theory (CTT), especially when computer-adaptive testing is involved (Hambleton, Swaminathan, & Rogers, 1991; Segall, 1996; Weiss, 1982). The improvement in efficiency can be attributed to the development of several aspects of IRT, i.e., item parameter estimation (Bock & Aitken, 1981), item selection (Birnbaum, 1968; Bryant, 2005; van der Linden, 1999), and latent trait (θ) estimation (Bock & Mislevy, 1982). For the purpose of this appendix, attention is focused on trait estimation.

Several ability estimation methods are available, which include maximum likelihood (ML), maximum a posteriori (MAP), and expected a posteriori (EAP) (Bock & Aiken, 1981; Embretson & Reise, 2000; Hambleton et al., 1991; Kim & Nicewander, 1993). Of the approaches mentioned above, MAP and EAP are Bayesian methods that incorporate information about the prior ability distribution in order to better approximate the posterior distribution of the latent trait (Bock & Mislevy, 1982). One advantage of using prior information is that an estimate of ⎝ exists when examinees have either all correct or all incorrect responses; this advantage is invaluable in the early stages of a computer-adaptive test (Segall, 1996; Weiss, 1982). Despite the benefits of Bayesian methods over ML, there are distinctions between MAP and EAP that should be noted.

First, the EAP estimate is relatively easy to compute, i.e., it does not require the first and second partial derivatives of the likelihood function that are employed to compute a solution as in MAP and ML approaches. Second, "the EAP estimator has minimum mean square error over the population of ability and, in terms of average accuracy, cannot be improved upon" (Bock & Mislevy, 1982, p. 439). Third, the method is non-iterative, which permits fast calculations of the provisional ability estimate in computer-adaptive testing. These advantages have been demonstrated in empirical research.

In a work investigating ability estimation procedures, Kim and Nicewander (1993) found evidence in support of the minimum standard error of EAP estimates relative to other methods, which included MAP, ML, and weighted likelihood estimation. Using the generalized partial credit model, Wang and Wang (2001) also demonstrated the superiority of EAP over other trait estimation methods in terms of minimum standard error. However, in light of the benefits of EAP in the unidimensional context, little research has thoroughly developed EAP estimation procedures and estimates of the covariance matrix of the posterior distribution of the latent traits for multidimensional IRT (MIRT) models.

In a multidimensional context, Carlson (1987) created a joint ML method of estimating MIRT item parameters and multiple latent traits, $\theta = [\theta_1, \theta_2, \ldots, \theta_k]$. Segall (1996) developed a MAP ability estimation approach for calculating θ while also deriving an item selection algorithm based on the covariance matrix of the posterior distribution of latent traits. Notwithstanding progress made in MIRT (van der Linden, 1999), the potential advantages of EAP estimation of θ have not been thoroughly investigated, perhaps due to the fact that formulas for θ based on EAP estimation and the covariance matrix of the posterior distribution of θ have not been fully developed.

Thus, the purpose of this study is to present EAP formulas for estimating θ and θ_{comp} along with the appropriate standard errors. First, there is a brief presentation of the item response model and the underlying assumptions. Second, the multidimensional extension of EAP ability estimation (θ_{EAP}) is provided, and the formula for computing the covariance matrix of the posterior distribution of latent traits is shown. Third, formulas for the latent trait composite (θ_{comp}) and the standard error of the composite are given. Finally, a numerical example of both θ_{EAP} and θ_{comp} are provided along with the associated standard errors.

Item Response Model

Let a set of k latent traits be represented by the vector $\theta = [\theta_1, \theta_2, \ldots, \theta_k]'$, and assume that each of the k traits influences performance on one or more items. The response function for item i is given by the Multidimensional 3-Parameter Logistic model (M3PL; Reckase, 1997),

$$P_i(u_i = 1 \mid \theta) = c_i + (1 - c_i)\,\{1 + \exp[-D(\boldsymbol{a}_i'\theta + d_i)]\}^{-1}, \tag{1}$$

where u_i is the dichotomous response to item i ($u_i = 1$, if the item is correct, and $u_i = 0$, otherwise), D is equal to a scaling constant 1.7, $\boldsymbol{a}_i$ is a vector of k discrimination parameters for item i, $[a_{1i}, a_{2i}, \ldots, a_{ki}]'$, and d_i is a scalar related to item difficulty. The probability of an incorrect response is $Q_i(\theta) = 1 - P_i(\theta)$. Under the assumption of local independence, the likelihood of a set of responses, $\boldsymbol{u} = [u_1, u_2, \ldots, u_n]$, given θ is

$$L(\boldsymbol{u} \mid \theta) = L(u_1, u_2, \ldots, u_n \mid \theta_1, \theta_2, \ldots, \theta_k) = \prod^{n} P_i(\theta)^{ui}\, Q_i(\theta)^{1-ui}. \tag{2}$$

Under the assumption of monotonicity, increases in one or more of the *k* latent traits result in a greater chance of getting an item correct.

EAP Estimation of $\boldsymbol{\theta}$*.*

The estimate of the latent trait vector, $\boldsymbol{\theta}$, using the Bayesian estimation procedure is an extension of the method proposed by Bock and Aitken (1981). In the unidimensional context, a prior density, $f(\theta)$, is specified and is used to compute the posterior density, $f(\theta \mid \boldsymbol{u})$, which is proportional to the product of the prior density and the likelihood function, $L(\boldsymbol{u} \mid \theta)$. In the multidimensional case, the posterior density of $\boldsymbol{\theta}$ is

$$f(\boldsymbol{\theta} \mid \boldsymbol{u}) = L(\boldsymbol{u} \mid \boldsymbol{\theta})\ f(\boldsymbol{\theta}) / f(\boldsymbol{u}), \tag{3}$$

where $f(\boldsymbol{\theta})$ is the prior density given $\boldsymbol{\theta}$, $L(\boldsymbol{u} \mid \boldsymbol{\theta})$ is provided in Equation 2, and $f(\mathbf{u})$ is the marginal probability of $\boldsymbol{u}$. The prior density can be empirically derived from the data or can be considered as multivariate normal with a mean of $\boldsymbol{\mu}$ and covariance of $\boldsymbol{\Sigma}$; i.e.,

$$f(\boldsymbol{\theta}) = (2\pi)^{-k/2}\, |\boldsymbol{\Sigma}|^{-1/2} \exp[-.5(\boldsymbol{\theta} - \boldsymbol{\mu})'\, \boldsymbol{\Sigma}^{-1}(\boldsymbol{\theta} - \boldsymbol{\mu})]\ , \tag{4}$$

where $\pi \approx 3.1459$ and $|\boldsymbol{\Sigma}|$ is the determinant of $\boldsymbol{\Sigma}$.

The EAP estimate of the latent trait vector, $\boldsymbol{\theta}_{\text{EAP}}$, is given by

$$E(\boldsymbol{\theta} | u_1, \ldots, u_n) = \boldsymbol{\theta}_{\text{EAP}} = \frac{\int \ldots \int \boldsymbol{\theta}\, L(\boldsymbol{u} \mid \boldsymbol{\theta})\, f(\boldsymbol{\theta})\, d\theta_1 \ldots d\theta_k}{\int \ldots \int L(\boldsymbol{u} \mid \boldsymbol{\theta})\, f(\boldsymbol{\theta})\, d\theta_1 \ldots d\theta_k}\ . \tag{5}$$

Consistent with the meaning of θ_{EAP} in the unidimensional case, $\boldsymbol{\theta}_{\text{EAP}}$ is the mean (vector) of the posterior multivariate distribution. The covariance matrix of the posterior distribution is

$$C(\theta \mid u_1, \ldots, u_n) = C = \frac{\int \ldots \int (\theta - \theta_{\text{EAP}})(\theta - \theta_{\text{EAP}})' \, L(\boldsymbol{u} \mid \theta) \, f(\theta) \, d\theta_1 \ldots d\theta_k}{\int \ldots \int L(\boldsymbol{u} \mid \theta) \, f(\theta) \, d\theta_1 \ldots d\theta_k} . \tag{6}$$

It should be noted that θ_{EAP} in Equation 5 is a k x 1 vector. The covariance matrix of the posterior distribution, $\boldsymbol{C}$, in Equation 6 is a k x k matrix. The variance of the k^{th} latent trait is the k^{th} element in the main diagonal of $\boldsymbol{C}$, and the corresponding standard error for each of the latent traits is computed as the square root of each of the respective elements in the main diagonal of $\boldsymbol{C}$.

Under the assumption of multivariate normality, Equation 5 can be reasonably approximated using the Gauss-Hermite quadrature nodes (θ_q) and weights, W(θ_q), that correspond to probabilities at the quadrature nodes across the discrete multivariate distribution. The approximation of Equation 5 is

$$\theta_{\text{EAP}} \approx \frac{\sum_{x=1}^{q} \theta_q \, L(\boldsymbol{u} \mid \theta_q) \, \mathrm{W}(\theta_q)}{\sum_{x=1}^{q} L(\boldsymbol{u} \mid \theta_q) \, \mathrm{W}(\theta_q)} , \tag{7}$$

where θ_q is one of the finite quadrature nodes in the discrete k-dimensional multivariate distribution, $L(\boldsymbol{u} \mid \theta_q)$ is the likelihood of the response pattern given θ_q, and W(θ_q) is the weight, usually the prior density of the multivariate normal distribution, $f(\theta_q)$. An additional requirement is that the weights are normed so that the sum of W(θ_q) across the discrete multivariate distribution is one. The numerator in Equation 7 is the sum of the products of the quadratures, likelihoods, and weights at the quadratures across the discrete multivariate distribution, and the denominator is the sum of the products of the likelihoods and weights of the quadratures across the discrete multivariate distribution.

The covariance matrix in Equation 6 is approximated by the following expression:

$$\boldsymbol{C} \approx \frac{\sum_{x=1}^{q} (\theta_q - \theta_{\mathrm{EAP}})(\theta_q - \theta_{\mathrm{EAP}})' \, L(\boldsymbol{u} \mid \theta_q) \, \mathrm{W}(\theta_q)}{\sum_{x=1}^{q} L(\boldsymbol{u} \mid \theta_q) \, \mathrm{W}(\theta_q)}, \tag{8}$$

where θ_{EAP} is given by Equation 7.

Estimation of θ_{comp}.

In educational and employment settings, decisions are often made on the basis of a composite of scores on a battery of subtests. The *Standards for Educational and Psychological Testing* (American Educational Research Association, American Psychological Association, & National Council on Measurement in Education, 1999) recommend that when composites are used in decision-making, information related to the standard errors of the composite and subtests should be documented (Standard 2.1). Thus, it is necessary that latent trait composites have the associated standard errors. Formulas for both the composite and the standard error are presented next.

Assume that k subtest scores correspond to k latent traits, and the subtests are weighted by a vector, $\mathbf{v} = [v_1, \ldots, v_k]'$, to create a composite score. This composite corresponds to a composite of latent traits, θ_{comp}. Also, a requirement is made that $\mathbf{v}_{k \times 1}$ is a unit vector, i.e., $\mathbf{v}'\,\mathbf{v} = 1$. The composite of the latent traits using the EAP estimate of θ is

$$\theta_{\mathrm{comp}} = \mathbf{v}'\,\theta_{\mathrm{EAP}}. \tag{9}$$

The variance of θ_{comp} is

$$\mathrm{Var}(\theta_{\mathrm{comp}}) = \mathbf{v}'\,\boldsymbol{C}\,\mathbf{v}\,, \tag{10}$$

and the standard error of the EAP composite of latent traits is

$$\mathrm{SE}(\theta_{\mathrm{comp}}) = (\mathbf{v}'\, \boldsymbol{C}\, \mathbf{v})^{1/2}. \tag{11}$$

Now that formulas for computing θ_{EAP} and θ_{comp} have been presented with the associated variance and standard errors of estimation, a numerical example is presented that demonstrates the use of the formulas.

Numerical Example of Estimating θ_{EAP} and θ_{comp}.

The item response model used for this numerical example is the M3PL model in Equation 1. The simulated test contains 20 items; each item measures two latent traits, $\theta = [\theta_1, \theta_2]'$. The prior multivariate distribution is assumed to be normal with $\mu = [0, 0]'$ and a $\Sigma = \mathbf{I}$. The identity matrix, **I,** is a 2 x 2 matrix with ones in the main diagonal and zeros in the off-diagonals. The lower and upper bounds of the discrete multivariate distribution are –4 and +4, respectively, on each trait with an equally spaced step size of .5 (i.e., the first quadrature node is [–4.0, –4.0], the second node is [–4.0, –3.5], the third is [–4.0, –3.0], … , etc). The weighting vector used to create the composite is $\mathbf{v} = [.7071, .7071]'$, which is a measurement direction of approximately 45 degrees from the first latent trait and 45 degrees from the second latent trait. The parameters for the 20 items are listed in Table 1. The simulated examinee used in this example has a true latent trait vector of $\theta_{\mathrm{true}} = [.90, .40]'$, and a true latent composite of $\theta_{\mathrm{true\ comp}} = \mathbf{v}'\, \theta_{\mathrm{true}} = .919$. The author wrote a program using the Python programming language to estimate θ_{EAP} and θ_{comp}.

The results of the simulation are provided in Table 2. The simulated responses of the examinee to the 20-item test are given in Column 4 of Table 2. The provisional estimates of

$\boldsymbol{\theta}_{EAP}$ and θ_{comp} were computed after each item with the associated standard errors; $\boldsymbol{\theta}_{EAP}$ is in Columns 5 and 6 with the standard error for each latent trait in parentheses. θ_{comp} is provided in Column 7 with the standard error of the latent composite in parentheses. For each item, the multidimensional extension of the point of maximum item information ($\boldsymbol{\theta}_{max}$; Bryant, 2005) is given in Columns 2 and 3.

For the total test, $\boldsymbol{\theta}_{EAP}$ and θ_{comp} are estimated reasonably well given the response pattern and item parameters with $\boldsymbol{\theta}_{EAP}$ = [.90, .40]' and θ_{comp} = .922; these estimates are nearly identical to the true values. Also, ability estimates and standard errors exist for the first nine correct responses; these estimates do not exist using the ML approach. Thus, EAP estimation may have utility in situations when all correct or all incorrect responses are present in the early stages of a multidimensional, computer-adaptive test.

In Table 2, observe that as more items are administered, both the standard errors associated with $\boldsymbol{\theta}_{EAP}$ and θ_{comp} decrease. For example, θ_{comp} has standard errors of .48, .35, and .29 after the administration of 10, 14, and 17 items, respectively. It appears that the standard errors can be managed with the increase in the number of items, thus making it possible to employ the standard error as a stopping rule in an adaptive testing context. For θ_{comp}, if a .5 standard error criterion were used as a stopping rule, testing in this simulated example would have ended after 10 items were administered. If the commonly used .3 standard error criterion were used, then testing would terminate after 17 items. For $\boldsymbol{\theta}_{EAP}$, if a .45 standard error on each latent trait were used, then testing would terminate after 17 items; the estimate of the trait vector using the .45 stopping rule is $\boldsymbol{\theta}_{EAP}$ = [.923 , .446], which is not far from the true ability vector, $\boldsymbol{\theta}_{true}$ = [.90, .40]'.

The purpose of this appendix was to present the formulas for estimating θ_{EAP} and θ_{comp}. Formulas for approximating the standard errors of the latent trait vector and the composite were given. A numerical example using the formulas was provided. The main results suggest that the formulas may have utility in estimating ability in a multidimensional adaptive testing context. So, what are some of the specific implications of this research?

Because formulas for EAP estimation of multiple traits are now provided, a comparison of the different estimation procedures in a multidimensional context may begin within the context of the XCert platform. Given the results of other studies from the unidimensional perspective, it is expected that some of the advantages of EAP in the unidimensional context will extend to the multidimensional setting. Does EAP estimation in a multidimensional setting produce scores with the minimum mean square error across the population of ability for which the prior distribution is specified? Future research could perhaps speak to this issue.

The results of this study also showed that θ_{EAP}, θ_{comp}, and the standard errors associated with each estimate exist when all correct responses are present. This is an invaluable advantage in a computer-adaptive setting where estimates of ability in the initial stages are used to select the next item. Once again, ML is unable to produce ability estimates when all correct or all incorrect responses are present, thus making it a poor choice as an ability estimator in a multidimensional setting. However, with the incorporation of prior information, EAP estimates of a latent trait vector may be improved dramatically in the selection of the next item in an adaptive testing environment.

Evidence in this study also suggests that the standard error of θ_{EAP} and θ_{comp} may be used as a stopping criterion in a computer-adaptive test. By specifying the amount of error that a decision-maker is willing to accept in scores, the testing session may be terminated once an acceptable amount of error associated with an ability estimate is reached. Future research could perhaps focus on the similarity and differences of using a minimum error criterion, a fixed length stopping criterion, or a fixed time limit criterion in a multidimensional, computer-adaptive test.

The *Standards for Educational and Psychological Testing* (American Educational Research Association, American Psychological Association, & National Council on Measurement in Education, 1999) state that the standard error associated with both composite and subtest scores should be reported. This study provides standard errors for both the latent traits and the composite of the latent traits and allows researchers and practitioners to meet the requirement with respect to EAP estimation. In particular, an advantage of being able to compute the standard error associated with the composite is that if the weighting vector used to create the latent composite changes, then the standard error of θ_{comp} will change. In a CTT context, when combining subtest scores to create a composite, the standard error does not change as a function of the weighting of the composite. IRT definitely provides a benefit in this respect.

In summary, the estimation of θ_{EAP} and θ_{comp} along with the standard errors may prove useful in advancing MIRT within this learning context. First, the formulas are relatively easy to compute as compared to the multidimensional extensions of ML and MAP ability estimation procedures. Second, EAP estimates and standard errors exist for all correct and all incorrect response patterns. Third, the error of the ability estimates may have utility as a termination criterion in computer-based assessments. Finally, the formulas, used in conjunction with each other, provide researchers and practitioners the necessary tools to meet professional testing standards.

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Table 1.

Simulated Item Parameters for the Multidimensional 3-Parameter Logistic Model

Item	a_{i1}	a_{i2}	c_i	$MDIFF_i$	Item	a_{i1}	a_{i2}	c_i	$MDIFF_i$
1	1.5	.5	.10	-1.0	11	1.5	.5	.10	.85
2	.5	1.5	.10	-.7	12	1.5	.5	.10	.9
3	1.5	.5	.10	-.5	13	1.5	.5	.10	.9
4	.5	1.5	.10	-.4	14	1.5	.5	.10	1.0
5	1.5	.5	.10	0	15	.5	1.5	.10	1.0
6	.5	1.5	.10	0	16	.5	1.5	.10	1.5
7	.5	1.5	.10	.4	17	1.5	.5	.10	1.5
8	.5	1.5	.10	.5	18	.5	1.5	.10	1.5
9	.5	1.5	.10	.5	19	.5	1.5	.10	2.0
10	.5	1.5	.10	.6	20	1.5	.5	.10	2.0

Note: a_{i1} is the discrimination parameter for θ_1, a_{i2} is the discrimination parameter for θ_2, $MDIFF_i$ is the multidimensional difficulty, c_i is the pseudo-guessing parameter, and d_i is –$MDIFF_i\,[(a_{i1})^2 + (a_{i2})^2]^{1/2}$.

Table 2.

Simulated Results With True θ = [.9, .4] and Prior Distribution (μ = [0, 0], Σ = **I**)

	θ_{max}			θ_{EAP}		
Item	θ_{1max}	θ_{2max}	Response	θ_1 (*SE*)	θ_2(*SE*)	θ_{comp}(*SE*)
1	-.893	-.298	1	.244 (.880)	.081 (.987)	.230 (.890)
2	-.202	-.608	1	.291 (.886)	.349 (.864)	.453 (.808)
3	-.419	-.140	1	.488 (.811)	.381 (.869)	.615 (.757)
4	-.108	-.234	1	.505 (.821)	.561 (.798)	.754 (.715)
5	.056	.019	1	.719 (.760)	.580 (.810)	.918 (.682)
6	.019	.056	1	.727 (.770)	.757 (.756)	1.05 (.653)
7	.145	.435	1	.739 (.783)	.959 (.711)	1.20 (.631)
8	.177	.530	1	.751 (.793)	1.11 (.676)	1.32 (.614)
9	.177	.530	1	.761 (.800)	1.22 (.651)	1.40 (.601)
10	.208	.625	0	.711 (.764)	.712 (.495)	1.01 (.482)
11	.862	.287	1	1.05 (.712)	.646 (.499)	1.20 (.459)
12	.909	.303	1	1.29 (.655)	.601 (.502)	1.34 (.433)
13	.909	.303	1	1.45 (.614)	.572 (.504)	1.43 (.415)
14	1.00	.335	0	1.01 (.489)	.652 (.487)	1.17 (.348)
15	.335	1.00	0	1.04 (.485)	.467 (.421)	1.07 (.317)
16	.493	1.48	0	1.05 (.484)	.415 (.401)	1.04 (.309)
17	1.48	.493	0	.923 (.441)	.446 (.398)	.968 (.290)
18	.493	1.48	0	.932 (.441)	.406 (.384)	.946 (.284)
19	.651	1.95	0	.934 (.441)	.395 (.381)	.940 (.283)
20	1.95	.651	0	.900 (.428)	.403 (.379)	.922 (.277)

I is a 2 x 2 identity matrix with ones on the main diagonal and zeros on the off-diagonal. The weight of the composite is **v** = [.7071, .7071].