

## 1 WQU and Path Compression

Assume we have eight sets, represented by integers 1 through 8, that start off as completely disjoint sets. Draw the WQU Tree after the series of union() and find() operations with path compression. Write down the result of find() operations. Break ties by choosing the smaller integer to be the root.

union(2, 3); ✓

union(1, 6); ✓

union(5, 7); ✓

union(8, 4); ✓

union(7, 2); ✓

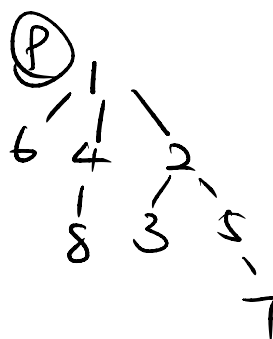
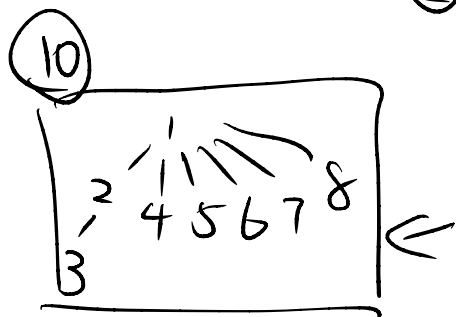
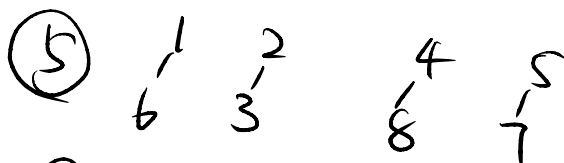
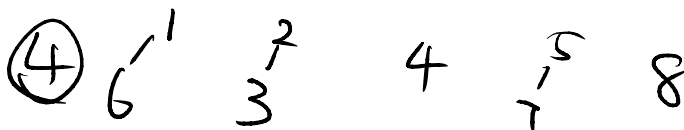
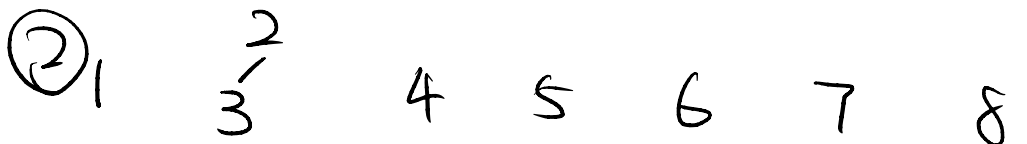
find(3); ✓

union(6, 4); ✓

union(6, 3); ✓

find(7);

find(8);



## 2 Is This a BST?

The following method `isBSTBad` is supposed to check if a given binary tree is a BST, though for some binary trees, it is returning the wrong answer. Think about an example of a binary tree for which `isBSTBad` fails. Then, write `isBSTGood` so that it returns the correct answer for any binary tree. The `TreeNode` class is defined as follows:

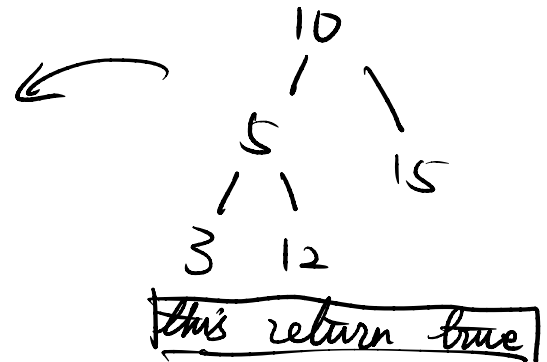
```
class TreeNode {
    int val;
    TreeNode left;
    TreeNode right;
}
```

**Hint:** You will find `Integer.MIN_VALUE` and `Integer.MAX_VALUE` helpful when writing `isBSTGood`.

```
public static boolean isBSTBad(TreeNode T) {
    if (T == null) {
        return true;
    } else if (T.left != null && T.left.val > T.val) {
        return false;
    } else if (T.right != null && T.right.val < T.val) {
        return false;
    } else {
        return isBSTBad(T.left) && isBSTBad(T.right);
    }
}
```

```
public static boolean isBSTGood(TreeNode T) {
    return isBSTHelper(
    );
}
```

```
public static boolean isBSTHelper(
    ) {
```

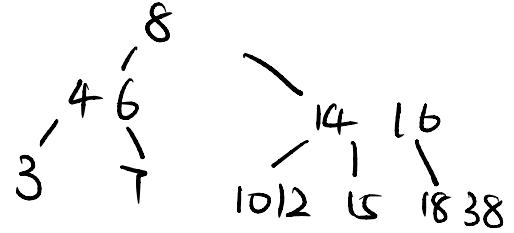
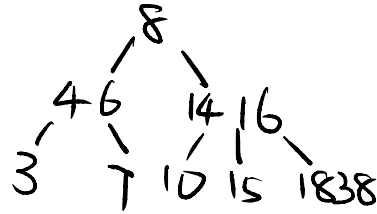
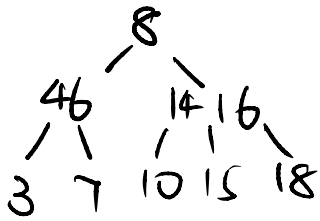
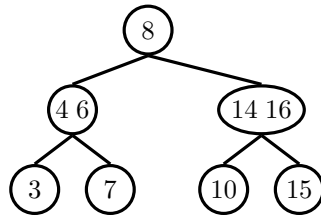


see discussion repository.

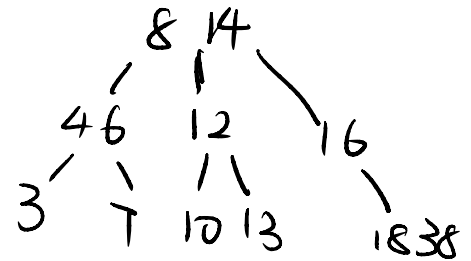
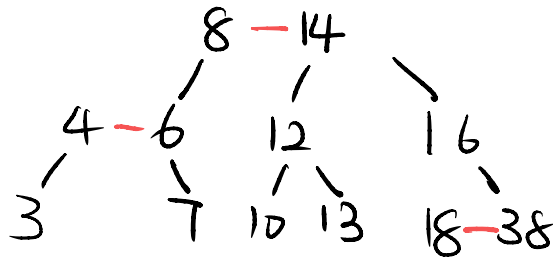
```
}
```

## 3 2-3 Trees and LLRB's

3.1 Draw what the following 2-3 tree would look like after inserting 18, 38, 12, and 13.



3.2 Now, convert the resulting 2-3 tree to a left-leaning red-black tree.



3.3 *Extra:* If a 2-3 tree has depth  $H$  (that is, the leaves are at distance  $H$  from the root), what is the maximum number of comparisons done in the corresponding red-black tree to find whether a certain key is present in the tree?

$2H$

## 4 Hashing

- 4.1 Here are three potential implementations of the Integer's hashCode() function. Categorize each as either a valid or an invalid hash function. If it is invalid, explain why. If it is valid, point out a flaw or disadvantage.

```
1 public int hashCode() {
2     return -1;
3 }
```

valid as for same int, it returns same hashCode(), but a terrible one, 100% collision

```
1 public int hashCode() {
2     return intValue() * intValue();
3 }
```

valid

but for numbers that have the same abs, it returns the same hash number. better to just return intValue()

```
1 public int hashCode() {
2     return super.hashCode();
3 }
```

Invalid, as same ints may not have the same hash number (it returns the address of the object.)

- 4.2 Extra, but highly recommended: For each of the following questions, answer Always, Sometimes, or Never.

- (a) When you modify a key that has been inserted into a HashMap will you be able to retrieve that entry again? Explain.

Sometimes  $\Rightarrow$  if the change of the key modifies the hashCode of the key, we may not retrieve it.

- (b) When you modify a value that has been inserted into a HashMap will you be able to retrieve that entry again? Explain.

Always  $\Rightarrow$  values do not matter in a hash table, it is key that decide how we find a value.