Solutions – Alpha (v1). If you think there is a mistake, please make a follow-up post to the solution announcement post on Piazza.

UC Berkeley – Computer Science
CS61B: Data Structures

Midterm #2, Spring 2018

This test has 9 questions worth a total of 240 points and is to be completed in 110 minutes. The exam is closed book, except that you are allowed to use two double sided written cheat sheets (can use front and back on both sheets). No calculators or other electronic devices are permitted. Give your answers and show your work in the space provided. **Write the statement out below in the blank provided and sign. You may do this before the exam begins.**

"I have neither given nor received any assistance in the taking of this exam."

Signature: _____

#	Points	#	Points
0	1	6	14
1	24	7	46
2	28	8	45
3	32	9	30
4	0		
5	20		
		TOTAL	240

Name: _____

SID: _____

Three-letter Login ID: _____

Login of Person to Left: _____

Login of Person to Right: _____

Exam Room: _____

Tips:

- There may be partial credit for incomplete answers. Write as much of the solution as you can, but bear in mind that we may deduct points if your answers are much more complicated than necessary.
- There are a lot of problems on this exam. **Work through the ones with which you are comfortable first. Do not get overly captivated by interesting design issues or complex corner cases you're not sure about.**
- Not all information provided in a problem may be useful, and **you may not need all lines.**
- Unless otherwise stated, all given code on this exam should compile. All code has been compiled and executed before printing, but in the unlikely event that we do happen to catch any bugs in the exam, we'll announce a fix. **Unless we specifically give you the option, the correct answer is not 'does not compile.'**
- ○ indicates that only one circle should be filled in.
- ☐ indicates that more than one box may be filled in.
- For answers which involve filling in a ○ or ☐, please fill in the shape completely.
- **Throughout the exam, assume that hash table resizing takes linear time.**

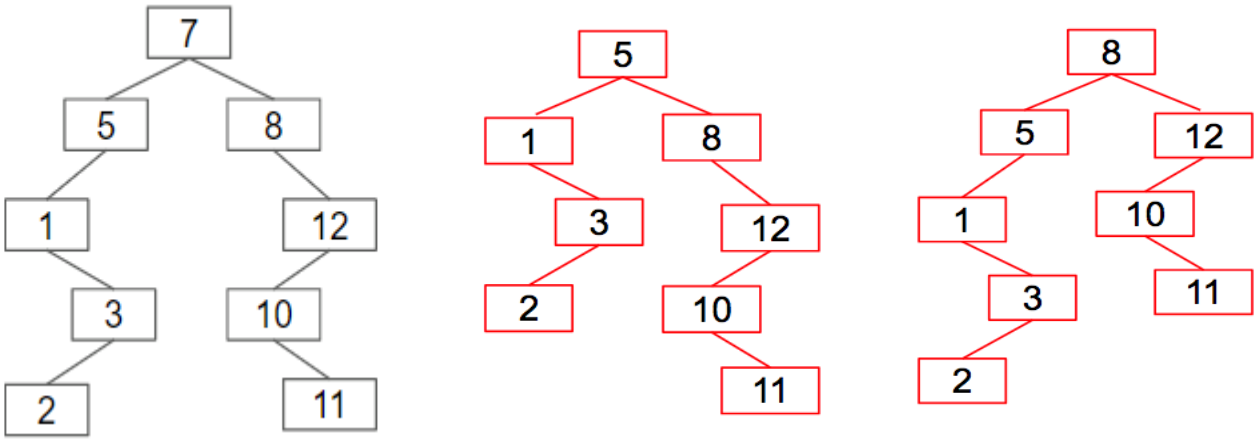
Optional. Mark along the line to show your feelings on the spectrum between ☹ and ☺.

Before exam: [☹_____☺].
After exam: [☹_____☺].

0. So it begins (1 point). Write your name and ID on the front page. Write the exam room. Write the IDs of your neighbors. Write the given statement and sign. Write your login in the corner of every page. Enjoy your free point ☺.

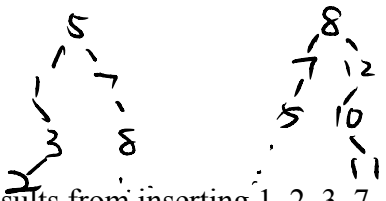
1. Tree time.

a) **(4 points).** Suppose we have the BST shown below. Give a valid tree that results from deleting "7" using the procedure from class (a.k.a. Hibbard deletion). Draw your answer to the right of the given tree in the box. Two possible answers; either is correct.



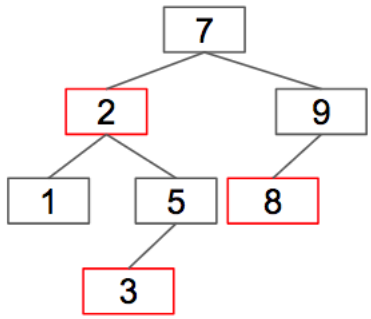
b) **(4 points).** Give an example of a rotation operation on the original tree from 1a (on the left) that would increase the height. **<u>You do not need to draw the tree</u>**, just write the operation, e.g. "rotateRight(11)".

rotateLeft(7) or rotateRight(7)

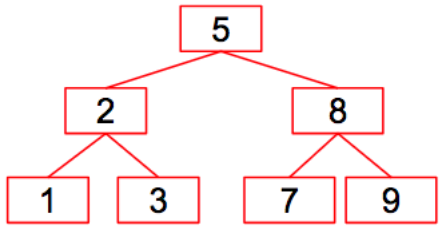


c) **(4 points).** Draw the 2-3 tree that results from inserting 1, 2, 3, 7, 8, 9, 5 in that order.

d) **(3 points).** Draw the LLRB that results from inserting 1, 2, 3, 7, 8 9, 5 in that order. Write the word "red" next to any red link.



e) **(3 points).** Draw a valid BST of minimum height containing the keys 1, 2, 3, 7, 8 9, 5.



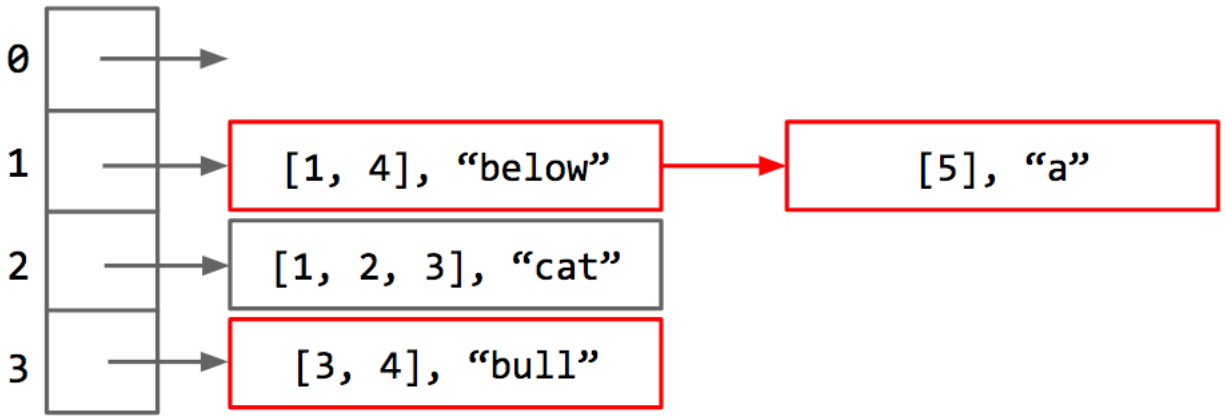
f) (6 points). Under what conditions is a **complete** BST containing N items **unique**? By "unique" we mean the BST is the only complete BST that contains exactly those N items. By "complete" we mean the same concept that was required for a tree to be considered a heap (precise definition not repeated here). Reminder: We never allow duplicates in a BST.

N can be any value (that's non-negative of course).
We know that there is at least one complete BST containing a specific set of N items, let's call it T. We also know that there is only one way to arrange the N nodes to form a complete BST (e.g. a linear chain of N nodes is not a complete BST).
Now we show that the arrangement of values in T is unique. Suppose we take two different values x and y in T; without loss of generality, assume x < y (the same argument applies for y > x, and we know x != y because all items are unique). If we try to swap the places of x and y, we would obtain a tree where y is to the absolute left of x, which would violate the property of BST's where all items to the absolute left of a node are less than or equal to the item in that node.

2. Hash Tables.

a) **(5 points).** Draw the hash table that is created by the following code. Assume that `XList` is a list of integers, and the hash code of an `XList` is the sum of the digits in the list. Assume that `XLists` are considered equal only if they have the same length and the same values in the same order. Assume that `FourBucketHashMaps` use external chaining and that new items are added to the end of each bucket. Assume `FourBucketHashMaps` always have four buckets and never resize. The result of the first `put` is provided for you. Represent lists with square bracket notation as in the example given.

```
FourBucketHashMap<XList, String> fbhm = new FourBucketHashMap<>();
fbhm.put(XList.of(1, 2, 3), "cat");
fbhm.put(XList.of(1, 4), "riding");
fbhm.put(XList.of(5), "a");
fbhm.put(XList.of(3, 4), "bull");
fbhm.put(XList.of(1, 4), "below");
```



b) **(4.5 points).** Next to the calls to `get`, write the return value of the `get` call. Assume that `get` returns `null` if the item cannot be found.

```
FourBucketHashMap<XList, String> fbhm = new FourBucketHashMap<>();
XList firstList = XList.of(1, 2, 3);
fbhm.put(firstList, "cat");
fbhm.get(XList.of(1, 2, 3));      __cat_____________
firstList.addLast(0); // list is now [1, 2, 3, 0]
fbhm.get(firstList);              __cat_____________
fbhm.get(XList.of(1, 2, 3));      __null____________
```

⇒ does not change hash value.

c) **(10.5 points).** Next to the calls to `get`, write the return value(s) of the `get` call. Assume that `get` returns `null` if the item cannot be found.

```
FourBucketHashMap<XList, String> fbhm = new FourBucketHashMap<>();
XList firstList = XList.of(1, 2, 3);
fbhm.put(firstList, "cat");
firstList.addLast(1); // list is now [1, 2, 3, 1]
fbhm.get(firstList);              ___null_____________
fbhm.get(XList.of(1, 2, 3));      ___null_____________
fbhm.get(XList.of(1, 2, 3, 1));   ___null_____________
fbhm.get(XList.of(3, 4));         ___null_____________
fbhm.put(firstList, "dog");
fbhm.get(firstList);              ___dog______________
fbhm.get(XList.of(1, 2, 3));      ___null_____________
fbhm.get(XList.of(1, 2, 3, 1));   ___dog______________
```

⇒ changes hash value. null.

d) **(4 points).** What are the best and worst case `get` and `put` runtimes for `FourBucketHashMap` as a function of N, the number of items in the HashMap? Don't assume anything about the distribution of keys.

`.get` best case: Θ(1)
`.get` worst case: Θ(N)
`.put` best case: Θ(1)
`.put` worst case Θ(N)

e) **(4 points).** If we modify `FourBucketHashMap` so that it triples the number of buckets when the load factor exceeds 0.7 instead of always having four buckets, what are the best and worst case runtimes in terms of N? Don't assume anything about the distribution of keys.

`.get` best case: Θ(1)
`.get` worst case: Θ(N)
`.put` best case: Θ(1)
`.put` worst case Θ(N)

As noted on the front page, throughout the exam you should assume that a single resize operation on any hash map takes linear time.

please note M/N is finite

3. Weighted Quick Union.

a) **(10 points).** Define a "fully connected" `DisjointSets` object as one in which `connected` returns true for any arguments, due to prior calls to `union`. Suppose we have a fully connected `DisjointSets` object with **6 items**. Give the best and worst case height for the two implementations below. The height is the number of links from the root to the deepest leaf, i.e. a tree with 1 element has a height of 0. **Give your answer as an exact value**. Assume Heighted Quick Union is like Weighted Quick Union, except uses height instead of weight to determine which subtree is the new root.

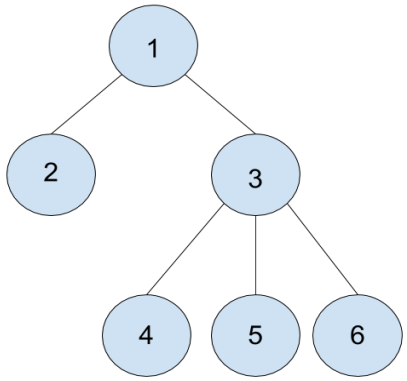
	Best Case Height	Worst Case Height
Weighted Quick Union	1	2
Heighted Quick Union	1	2

b) **(8 points).** Suppose we have a Weighted Quick Union object of height H. Give a general formula for the minimum number of objects in a tree of height H as a function of H. Your answer must be exact (e.g. not big theta).

2^H

c) **(6 points).** Draw a Quick Union tree that would be **possible for Heighted Quick Union**, but **impossible** for Weighted Quick Union. If no such tree exists, simply write "none exists."

Example tree shown below. In general, the heights of the two input trees should be the same, but the heavier tree should be attached to the lighter one.



d) **(8 points).** Create a set for storing `SimpleOomage` objects Assume that `hashCode()` for `SimpleOomage` is the perfect hashcode you were expected to write in HW3, where hash code values are unique and always between 0 and 140,607, inclusive.

```
public class SimpleOomageSet {
  private WeightedQuickUnionUF wq = new WeightedQuickUnionUF(140609);
  public void add(T item) {
      union(item.hashCode(), 140608);
  }
  public boolean contains(T item) {
      return connected(item.hashCode(), 140608);
  }
} // reminder: WeightedQuickUnionUF methods are union() and connected()
```

4. PNH (0 points). This 1996 simulation video game by Maxis had a hidden feature introduced secretly by a programmer, where on certain dates of the year, "muscleboys in swim trunks" would appear by the hundreds and hug and kiss each other.

SimCopter

5. Multiset. The `Multiset` interface is a generalization of the idea of a set, where items can occur multiple times.

```
public interface Multiset<T> {
    public void add(T item);          // adds item.
    public boolean contains(T item); // true if item occurs at least once.
    public int multiplicity(T item); // number of times item occurs.
}
```

For example, if we call `add(5)`, `add(5)`, `add(10)`, `add(15)`, `add(5)`, then the resulting Multiset contains {5, 5, 10, 15, 5}. In this case, `multiplicity(5)` will return `3`.

a) **(10 points).** A 61B student suggests that **one way to implement `Multiset` is to <u>modify a BST</u>** so that it is instead a "Trinary Search Tree", where the left branch is all items less than the current item, the middle branch is all items equal to the current item, and the right branch is all items greater than the current item. The multiplicity is then simply the number of times that an item appears in the tree. Implement the `add` method below.

```
public class TriSTMultiset<T extends Comparable<T>> implements Multiset<T> {
    private class Node {
        private T item;
        private Node left, middle, right;
        public Node(T i) { item = i; }
    }
    Node root = null;
    public void add(T item) {
        root = add(item, root);
    }
    private Node add(T item, Node p) {
        if (p == null) { return new Node(item); }
        int cmp = item.compareTo(p.item);
        if (cmp < 0) {
            p.left = add(item, p.left);
        } else if (cmp > 0) {
            p.right = add(item, p.right);
        } else {
            p.middle = add(item, p.middle);
        }
        return p;
    } ...
```

b) **(6 points).** Let X be an item with multiplicity M, and let N be the number of nodes in the tree. Give an Omega bound for the best case runtime of any possible implementation of `multiplicity(X)` for a `TriSTMultiset`. Give the best possible bound you can.

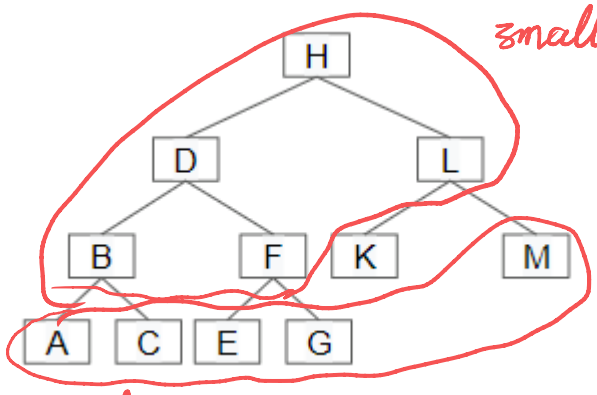
$\Omega(M)$ ⇒ at least traverse each node once.

c) **(6 points).** Rather than building an entirely new data structure from scratch, we might consider implementing `Multiset` using delegation or extension with an existing data structure from the `java.util` library. Which is the better choice: delegation or extension? If delegation, what class should you delegate to? If extension, what class should you extend? **If applicable, provide generic types.** Fill in **one of the bubbles** and the corresponding blank below. There may be multiple reasonable answers.

keys ↑ times

○ Delegation to an instance of the `java.util.`TreeMap<T, Integer> class is better.
○ Extending the `java.util.`_____ class is better.

6. Min Heaps (14 points). Consider the min heap below, where each letter ***represents*** some value in the tree. For each question, indicate which letter(s) correspond to the specified value. **Assume each value in the tree is unique.**



smallest five

largest five

a) increasing order.

Smallest value: ☐A ☐B ☐C ☐D ☐E ☐F ☐G ☐H ☐K ☐L ☐M

Median value: ☐A ☐B ☐C ☐D ☐E ☐F ☐G ☐H ☐K ☐L ☐M

Largest value: ☐A ☐B ☐C ☐D ☐E ☐F ☐G ☐H ☐K ☐L ☐M

b) **(2 points).** Assuming values are inserted into the heap in **decreasing order**, indicate all letters which could represent the following value:

Smallest value: ☐A ☐B ☐C ☐D ☐E ☐F ☐G ☐H ☐K ☐L ☐M

c) **(6 points).** Assuming values are inserted into the heap in an **unknown order**, indicate all letters which could represent the following values:

Median value: ☐A ☐B ☐C ☐D ☐E ☐F ☐G ☐H ☐K ☐L ☐M

Largest value: ☐A ☐B ☐C ☐D ☐E ☐F ☐G ☐H ☐K ☐L ☐M

only D & H have more than four values that are larger than them ⇒ not possible.

7. Iteration.

a) **(12 points).** Fill in the `toList` method. It takes as input an `Iterable<T>`, where `T` is a generic type argument, and returns a `List<T>`. If any items in the `iterable` are null, it should throw an `IllegalArgumentException`. You should use for-each notation (e.g. `for (x : blah)`). Do not use `.next` and `.hasNext` explicitly.

```
public class IterableUtils {
    public static <T> List<T> toList(Iterable<T> iterable) {
        List<T> r = new ArrayList<T>();

        for (T t: iterable) {
            if (t == null) {
                throw new IllegalArgumentException();
            }
            r.add(t);
        }
        return r;
    }
} // assume any classes you need from java.util have been imported
```

b) **(8 points).** The `ReverseOddDigitIterator` implements `Iterable<Integer>`, and its job is to iterate through the odd digits of an integer in reverse order. **For example, the code below will print out 77531.**

```
ReverseOddDigitIterator rodi = new ReverseOddDigitIterator(12345770);
for (int i : rodi) {
    System.out.print(i);
}
```

Write a JUnit test that verifies that `ReverseOddDigitIterator` works correctly using your `toList` method from part a. If you did not complete part a, you can still do this problem. Use the `List.of` method, e.g. `List.of(3, 4, 5)` returns a list containing 3 then 4 then 5.

```
import org.junit.Test;
import static org.junit.Assert.*;
public class TestRODI {
  @Test
  public void testRODI() {
      ReverseOddDigitIterator odi = new ReverseOddDigitIterator(12345770);
      List<Integer> expected = List.of(7, 7, 5, 3, 1);
      List<Integer> actual = IterableUtils.toList(odi);
      assertEquals(expected, actual);
  }
} // assume any classes you need from java.util have been imported
```

c) **(18 points).** Fill in the implementation of the `ReverseOddDigitIterator` class below.

```
public class ReverseOddDigitIterator implements Iterable<Integer>,
                                     Iterator<Integer> {

    private int value;
    public ReverseOddDigitIterator(int v) {
        value = v;
    }
    public boolean hasNext() {
        if (value == 0) {
            return false;                          // hint: this class should
        }                                          // be implemented
        if (value % 2 == 1) {                      // so that the example
            return true;                           // code that prints
        } else {                                   // 77531 on the previous
            value = value / 10;                    // page works.
            return hasNext();
        }
    }

    public Integer next() {
        int d = value % 10;
        value = value / 10;
        return d;
    }
    public Iterator<Integer> iterator() {
        return this;
    }
} // assume any classes you need from java.util have been imported
```

a combination of Iterable & Iterator

jump all even number

d) **(8 points).** If you didn't complete part c, assume it is completed and compiles. **For each of the following, which file (if any) will fail to compile as a <u>direct result</u> of the removal?** By "direct result", we mean the compilation failure is not caused by one if its dependencies failing to compile.

Please consult Piazza 1.C for details

Suppose we remove "`implements Iterable<Integer>`", which file will **fail** to compile?
○ `IterableUtils` ○ `TestRODI` ○ `ReverseOddDigitIterator` ○ None

Suppose we remove `implements Iterator<Integer>`, which file will **fail** to compile?
○ `IterableUtils` ○ `TestRODI` ○ `ReverseOddDigitIterator` ○ None

Suppose we remove the `hasNext` method, which file will **fail** to compile?
○ `IterableUtils` ○ `TestRODI` ○ `ReverseOddDigitIterator` ○ None

Suppose we remove the `iterator` method, which file will **fail** to compile?
○ `IterableUtils` ○ `TestRODI` ○ `ReverseOddDigitIterator` ○ None

8. Asymptotics

a) **(12 points).** Give the runtime of the following functions in Θ notation. Your answer should be a function of N that is as simple as possible with no unnecessary leading constants or lower order terms. **Don't spend too much time on these**!

_Θ(N^6)____

```
public static void g1(int N) {
    for (int i = 0; i < N*N*N; i += 1) {
        for (int j = 0; j < N*N*N; j += 1) {
            System.out.print("fyhe");
        }
    }
}
```

_Θ(2^N)_____

same as 2^{N+1}

```
public static void g2(int N) {
    for (int i = 0; i < N; i += 1) {
        int numJ = Math.pow(2, i + 1) - 1; // <-- constant time!
        for (int j = 0; j < numJ; j += 1) {
            System.out.print("fhet");
        }
    }
}
```

$2^1 + 2^2 + \cdots + 2^N \Rightarrow 2^N$

_ Θ(N)_____

```
public static void g3(int N) {
    for (int i = 2; i < N; i *= i) {}
    for (int i = 2; i < N; i++) {}
}
```

✓

b) **(4 points).** Suppose we have an algorithm with a runtime that is Θ(N^2 log N) in all cases. Which of these statements are definitely true about the runtime, definitely false, or there is not enough information (NEI)?

O(N^2 log N)	○ True	○ False	○ NEI
Ω(N^2 log N)	○ True	○ False	○ NEI
O(N^3)	○ True	○ False	○ NEI
Θ(N^2 $\log_4$ N)	○ True	○ False	○ NEI

c) **(5 points).** Suppose we have an algorithm with a runtime that is O(N^3) in all cases.

There exists some inputs for which the runtime is Θ(N^2)	○ True	○ False	○ NEI
There exists some inputs for which the runtime is Θ(N^3)	○ True	○ False	○ NEI
There exists some inputs for which the runtime is Θ(N^4)	○ True	○ False	○ NEI
The worst case runtime is O(N^3)	○ True	○ False	○ NEI
The worst case runtime has order of growth N^3	○ True	○ False	○ NEI

maybe Θ(N^2) ⇒ than order of N^2.

d) **(12 points).** Give the best and worst case runtime of the following functions in Θ notation. Your answer should be as simple as possible with no unnecessary leading constants or lower order terms. **Don't spend too much time on these**! Assume K(N) runs in constant time and returns a boolean.

```
public static void g4(int N) {
    if (N == 0) { return; }
    g4(N - 1);
    if (k(N)) { g4(N - 1); }
}
```

k(N) always false 0 1 2

Best case: _Θ(N)_____ → k(N) always true ⇒ 1 3 7.
Worst case: _Θ(2^N)_____→

```
public static void g5(int N) {
    if (N == 0) { return; }
    g5(N / 2);
    if (k(N)) { g5(N / 2); }
}
```

0 1 2 4 8.
1 3 7 15 31.

Best case: _Θ(log N)_____ → k(N) always false
Worst case: _Θ(N)_____→ k(N) always true

e) **(6 points).** Give the best and worst case runtime of the code below in terms of N, the length of x. Assume `HashSet61Bs` are implemented exactly like hash tables from class, where we used external chaining to resolve collisions, and resize when the load factor becomes too large. Assume `resize()` is implemented without any sort of traversal of the linked lists that make up the external chains.

```
public Set<Planet> uniques(ArrayList<Planet> x) {
    HashSet61B<Planet> items = new HashSet61B<>();
    for (int i = 0; i < x.size(); i += 1) {
        items.add(x.get(i));
    }
    return items;
}
```

all in the same bucket

evenly distributed

Best case runtime for `uniques`: _ Θ(N)__ Worst case runtime for `uniques`: _ Θ(N^2) _

f) **(6 points).** Consider the same code from part b, but suppose that instead of `Planets`, `x` is a list of `Strings`. Suppose that the list contains N strings, each of which is of length N. Give the best and worst case runtime.

Best case runtime for `uniques`: _ Θ(N)___ Worst case runtime for `uniques`: _ Θ(N^3)__

checks of strings require linear time

9. **(30 points).** Imagine that we have a list of every commercial airline flight that has ever been taken, stored as an `ArrayList<Flight>`. Each `Flight` object stores a flight start time, a flight ending time, and a number of passengers. These values are all stored as `int`s.

The trick we use to store a flight start time (or end time) as an `int`, rather than as some sort of `Time` object, is to store the number of minutes that had elapsed in the Pacific Time Zone since midnight on January 1st, 1914, which was the first day of commercial air travel.

For example, a flight taking off at 2:02 PM on March 6th, 1917 and landing at 3:03 PM the same day carrying 30 passengers would have takeoff time 1,671,243, landing time 1,671,304, and number of passengers 30.

Give an algorithm for finding the largest number of people that have ever been in flight at once.

Your algorithm must run in N log N time, where N is the number of total commercial flights ever taken. Your algorithm must not have a runtime that is explicitly dependent on the number of minutes since January 1st, 1914, i.e. you can't just consider each minute since that day and count the number of passengers from each minute and return the max.

Your algorithm may use any data structures discussed in the course (e.g. arrays, `ArrayDeque`, `LinkedListDeque`, `ArrayList`, `LinkedList`, `WeightedQuickUnion`, `TreeMap`, `HashMap`, `TreeSet`, `HashSet`, `HeapMinPQ`, `QuadTree`, etc.)

a. List any data structures needed by your algorithm, including the type stored in the data structure (if applicable). If you use a data structure that requires a `compareTo` or `compare` method, describe **briefly** how the objects are compared. Do not include the provided `ArrayList<Flight>` in your list of data structures. Please list concrete implementations, not abstract data types.

Canonically, use a HeapMinPQ<Flight> sorted by start times, and HeapMinPQ<Flight> sorted by end times. Any data structure that can provide an N log N ordered retrieval will work.

Alternatively, any unordered data structure that can be sorted in N log N and then retrieved in O(N log N) time will work. This includes sorting the input ArrayList<Flight> (in which case only one additional data structure was needed).

A note on BSTs and BST-variants: due to the clarification that flights can have duplicate start/end times, any implementation using data structures that did not support duplicates required additional handling of duplicate cases (partial credit was given if not done).

b. Briefly describe your algorithm in plain English. Be as concise and clear as possible.

First, insert all flights into the two PQs. Set the current tally to zero. Peek at the top of both PQs. Remove the smaller one. If it came from the start PQ, then add the number of passengers to the tally. If the tally is larger than it has ever been, record that as best. If it came from the end PQ, subtract from the tally. Return best.

Remove ⇒ log N.
total iterate ⇒ N } ⇒ N log N.

Here's an example of a sorted input and PQ alternate solution. Sort the input ArrayList by start time using an N log N sort. Initialize a PQ sorted by end time, and leave it empty. Then, iterate through the sorted input list, adding flights into the PQ when seen by the input list iterator, and removing from the PQ in a similar manner to above. Again, track current tally and best.