Assignment 1a

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⚠ All relevant code and files can be found at the below url.

XDGFX/design-optimisation

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https://github.com/XDGFX/design-optimisation/tree/master/Assignment%201a

Initial Investigation

Request 1: fit the greatest volume of liquid into the crate.
Request 2: optimise the can shape to minimise material usage.

Assuming all cans are stacked on top of each other, and as many cans as possible are included, only two factors influence the volume stored:

1. Radius of each can
2. Height of each can

However, they are linked; as each can must store exactly 500ml. It is wasteful for the can to be larger than required, and so each can must have volume of 500 ml (stated that wall size is negligible).

The can material usage is determined by the surface area of the can; the surface area is covered in can material (e.g. aluminium), and it is assumed no material is used elsewhere.

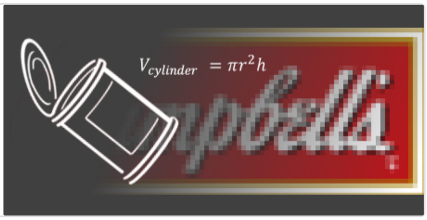
It is also assumed that material usage is constant regardless of where it is used - e.g. a square unit on the top or bottom of the can costs the same as a square unit on the side. This may not be true if there were different thicknesses of material, or processes used to create the shape.

Possibly Useful Notes

Optimal Can Dimensions

The purpose of a food can is to store food. It costs money to manufacture, store, and ship these containers. One would imagine, therefore, that over time a lot of thought has gone into their design and production.

http://datagenetics.com/blog/august12014/index.html



To minimise the surface area (material usage) to volume ratio, the height should be twice the radius.

The best known packings of equal circles in a square

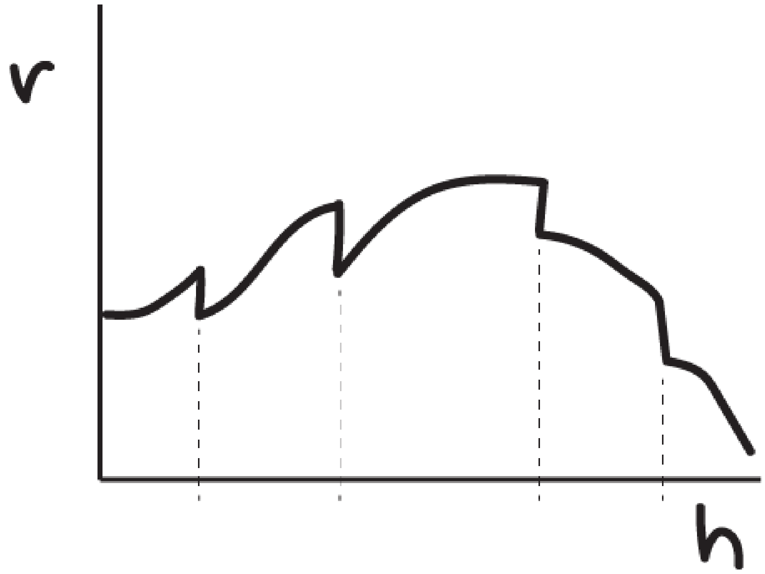
packing of equal circles in a square

http://hydra.nat.uni-magdeburg.de/packing/csq/csq.html

Optimal packing in 2D for circles in a square.

Computational Solution for Stacking Cans (Part iii)

There is some equation which models the volume of oil stored, with relation to radius and height. It may be complex as this is not a continuous equation, as once, e.g. height, reaches a specified value, the number of stacked cans will decrease by a discrete unit value.



The dotted lines indicate critical points in the function

This could likely be accounted for by using different equations for each *segment*, and having acceptable bounds.

Bounds

Assuming **all cans are stacked in an even grid**, the bounds can be found easily with the following equations which must be satisfied.

$$n * h \leq 560$$
$$n * 2r \leq 828.677$$

Where n is the number of cans in that given direction. Assuming cans can fit perfectly into the size given.

Because the box is square, and we are assuming an even grid, the total number of cans per level (in x and z) is interchangeable with the number of cans per side in the x and z dimensions. This will **not be the case if more complex packing arrangements are used**.

We could maximise the radius for each can n, and adjust the height to reach the 500ml volume per can. Alternatively, we could maximise height, and adjust radius.

If we vary radius within a fixed total number of cans on each level, there may be values at which a larger radius allows a smaller height, to a point that more cans can fit in the vertical direction. This will increase the total volume, if each can is still 500ml.

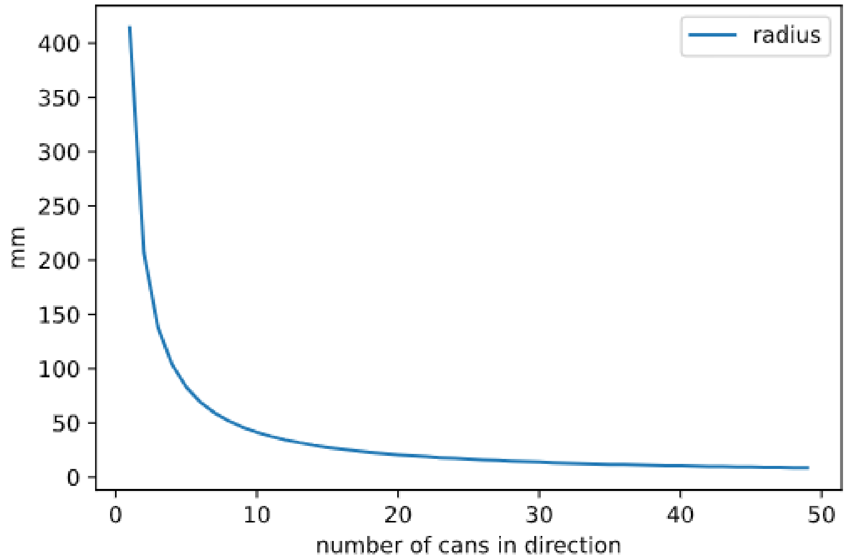
In other words, the greatest possible radius for a given number of cans per vertical level, will result in the smallest possible height per can, and therefore the greatest number of levels.

This can simplify the factors - as we only need to test the maximum possible radius for each n per level. Therefore:

$$r_n = \frac{828.677}{2n}$$

This can be converted into a 'lookup table' for different values of n. This was calculated up to 50 cans for each base dimension, and plotted for easier visualisation.

```
cans_x = range(1, 51)
crate_height = 560
crate_width = 828.677

max_radius = []

for n in cans_x:
    max_radius.append(crate_width / (2 * n))
```



We can then calculate the required height in order to reach 500ml volume

$$v = \pi * r^2 * h$$

$$h = \frac{v}{\pi * r^2}$$

$$h = \frac{500000mm^3}{\pi * r^2}$$

We can therefore calculate h for each value of r previously calculated. This can be used to calculate the maximum number of cans in the y direction, by using the previous formula:

$$n * h \leq 560$$

```
height = []
cans_y = []

for n in cans_x:
    required_height = 500000 / (math.pi * pow(max_radius[n-1], 2))
    height.append(required_height)
    cans_y.append(math.floor(560 / required_height))
```

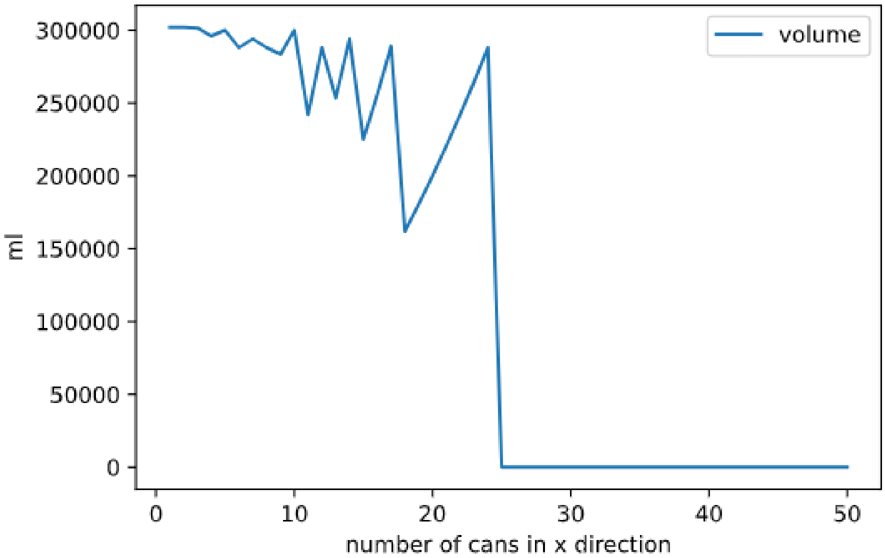
We can find the total cans with:

$$n = {n_x}^2 * n_y$$

And the resultant volume in ml with:

$$V = 500 * n$$

This produces the following graph:



Specifically, the scenarios where there is one can per x direction, and two cans per x direction (four per level) are the most efficient.

Theory 1

This is based on an even rectangular grid for cans.

1. Each can can be represented from the top-view as a square, with a percentage waste area (the corners outside the circle)
2. This waste ratio is always $1 - \pi * 0.5^2 = 0.2146$
3. Therefore, regardless of the number of circles in the square, the waste area is the same; the only waste change is area left on the sides of the square if the circles are not all touching (in this case they are).

4. This leaves waste **only** at the top, if the top level of cans does not touch the ceiling
5. With the largest radius possible, all circle sides are touching all square sides, so top down area is maximised. This also means the height is lowest, meaning this is likely the closest we can get to the top can touching the ceiling.

This explains why the volume gets worse with greater radius, and the oscillatory nature is explained by cans fitting well under the height, vs leaving a large air gap. The radius does not play a part in this case.

This can be converted to an analytical result by disregarding the top down view entirely;

- Stack n measurements of y within a total distance h.
- The value of y is based on the possible heights achieved by cans of radius $r = \frac{828.677}{2n}$, matching a volume $v = 500000mm^3$.

$$500000 = y * \pi * \frac{828.677}{2n}$$

$$y = \frac{1000000n}{\pi * 828.677}$$

$$y = 384.118n$$

We also know that:

$$y * n \leq 560$$

We can instead make both sides equal, as this will just introduce extra space in the top of each can, instead of the top of the crate. This will not affect the total number of cans.

$$y = \frac{560}{n}$$

Analytical Solution for Optimising Material Usage (Part i, ii)

To optimise material usage, surface area must be a minimum.

Therefore, the steps to find the optimal r and h values are as follows:

1. Determine an equation which links r and h based on a constant volume v. E.g. find h in terms of r.
2. Determine an equation which links r to the surface area a.
3. Differentiate this equation with respect to r to find the rate of change of surface area.
4. At $\frac{da}{dr} = 0$ this is a minimum or maximum surface area. These points can be found, and the minimum surface area can be found.

Based on previous working, we already know that:

$$h = \frac{500000mm^3}{\pi * r^2}$$

The surface area of a cylinder is $2\pi r(h + r)$ therefore:

$$a = 2\pi r(\frac{500000}{\pi r^2} + r)$$

$$a = 1000000r^{-1} + 2\pi r^2$$

And:

$$\frac{da}{dr} = -1000000r^{-2} + 4\pi r$$

At a minimum or maximum surface area, $\frac{da}{dr} = 0$.

$$0 = -1000000r^{-2} + 4\pi r$$

$$1000000r^{-2} = 4\pi r$$

$$\frac{1000000}{4\pi} = r^3$$

$$\sqrt[3]{\frac{1000000}{4\pi}} = r$$

$$r = 43.0127$$

Substituting back into the original equation gives:

$$h = \frac{500000}{\pi * 43.0137^2} = 86.0214$$

Combining The Solutions

The assignment states:

> They are now deciding how best to size these cans to minimise the amount of material they need for each can, and also to make their shipping more efficient.

This will be assumed to mean that can material optimisation is the primary objective, and shipping efficiency is secondary. Therefore, setting r and h to the values calculated, we get a maximum of 9.633 or $9\ cans$ in the x and z directions.

Using the previous calculations, this finds that a maximum of 7 cans are stacked vertically, resulting in 567 cans total at a volume of $286500mm^3$.

```
    item = 8

    print(cans_x[item])
    print(cans_y[item])
    print(max_radius[item])
    print(height[item])
    print(cans_total[item])
    print(volume[item])

9
7
46.03761111111111
75.09215563384576
567
283500
```

Further Optimisation

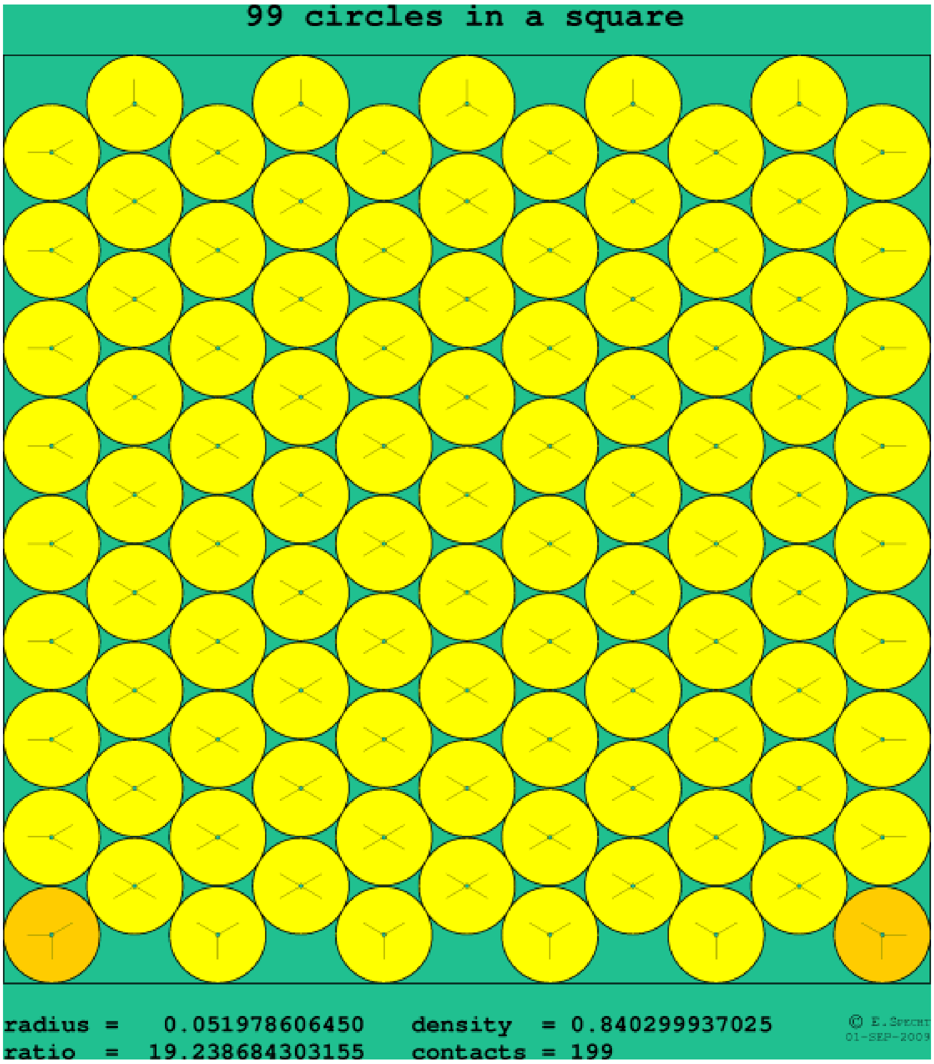
However, the grid pattern is not the most efficient arrangement of cans for each layer.

A table provided at http://hydra.nat.uni-magdeburg.de/packing/csq/csq.html compiles a list of optimal packing arrangements for equal-sized circles in a square. The square is of unit width and height, and the radii of each circle is provided.

Therefore, we can download this data and scale it proportionally to get a side length of 828.677.

```
import pandas as pd
import os

path = os.path.join(os.path.abspath(''), "circle_radius.csv")
circle_radius = pd.read_csv(path)

crate_width = 828.677
can_radius = 43.0127

# Scale all values to side length of crate
circle_radius["radius"] = circle_radius["radius"] * crate_width

# Then find the closest radius larger than or equal to our requirement
suitable_arrangements = circle_radius[circle_radius["radius"] > can_radius]

best_arrangement = [suitable_arrangements["cans"].iloc[-1], suitable_arrangements["radius"].iloc[-1]]
```

This gives the best arrangement of cans as 99 per layer (with a maximum possible radius of 43.0734, which means our smaller cans **will** fit).

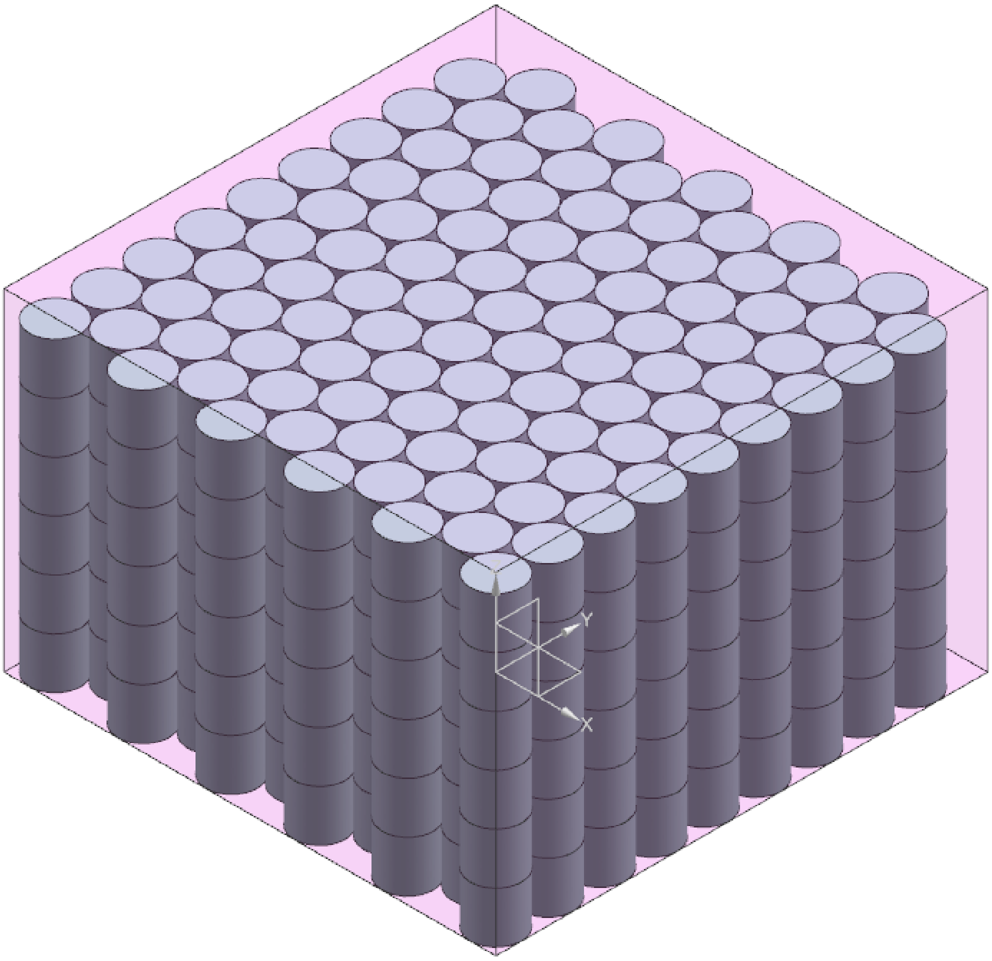


This *almost* uses hexagonal packing, however the circles do not touch the circle above or below them, so it is effectively hexagonal packing which has been reduced in width and increase in height.

Revised Solution

This will give 99 cans per layer, and $\left\lfloor \frac{560}{86.0214} \right\rfloor = 6$ cans stacked vertically, resulting in 594 total cans and a total oil storage volume of $297\ litres$.

This is validated by modelling the cans in 3D, using the optimal arrangement points data, and confirming that they all fit without overlap.



Final Answers

Index	Property	Value (mm or units)
i	Optimal Radius	43.0127
ii	Optimal Height	86.0214
iii	Number of Cans	594

Real World Optimisation

In reality, a balance between material usage and shipping costs would likely need to be made. For example, monetary cost could be used as a measure, and the optimum balance between low material usage and low shipping usage could be found based on current shipping and raw material / processing prices. Alternatively, other costs such as environmental impact could be considered.