Problem Chosen	2024 MCM/ICM Summary Sheet	Team Control Number
D		2425559

Optimizing Great Lakes Water Levels: A Comprehensive Control Model and Analysis

Summary

The Great Lakes are the largest group of freshwater lakes in the world, and their water resources are used for a wide range of purposes. Therefore, managing the inflow and outflow of water to the Great Lakes is critical to many stakeholders. The purpose of this article is to construct a control model for the inflow and outflow of the Great Lakes so that the Great Lakes can be maintained at a water level that satisfies stakeholders as much as possible.

In TASK 1, we constructed **the Great Lakes network flow model** by analyzing the geographical relationship and topographic direction of the Great Lakes, rivers and two major water conservancy projects. In order to meet the needs of various stakeholders, combined with the official water consumption data, we considered the water demand of power generation, ecology and industry, and established a **multi-objective optimization** model. We used *gamultiobj* solver in MATLAB to solve this problem and obtained the optimal water level of the Great Lakes.

In TASK 2, we referred to the **Lake Routing Rules** and established a direct relationship between river flow and lake level. We established a **Model Predictive Control(MPC)** algorithm. Firstly, a prediction model based on water balance was established, and then the optimal water level calculated in Task 1 was taken as the goal, and then differential evolution algorithm was used for rolling optimization. Last but not least, error quantity was introduced for feedback correction. Finally, we get the inflow data and outflow data that can reach the optimal water level.

In TASK 3, we increased and decreased the discharge of the two control dams by 10% respectively for analyzing the sensitivity of the flow control algorithm. After substituting the flow control algorithm, we found that the discharge of each lake had obvious changes, indicating that our algorithm was very sensitive to the discharge of the two control dams. We set up a **mixed benefit scoring model** by obtaining the **benefit scoring curves** and preferences of each stakeholder. Under this scoring model, the stakeholder satisfaction score in 2017 was 80.82 points, while under the new control algorithm, the stakeholder satisfaction score was as high as 94.78 points, which showed a significant improvement, indicating that our control algorithm could better satisfy stakeholders.

In TASK 4, we used **Monte Carlo stochastic simulation** algorithm to simulate the correspondence between rainfall and ice sheet scenarios and water level. Then, we used the two-dimensional **Frank-Copula** function to establish the joint distribution probability and conditional probability between environmental conditions and water level and analyzed the rule and degree of influence of environmental conditions on our algorithm. Finally, we concluded that our flow control algorithm is very sensitive to environmental conditions.

In TASK 5, we only focused on Lake Ontario, and we increased the importance of the Ottawa River and the Port of Montreal in the Lake Ontario area. By recalculating the model, we obtained more stable Lake Ontario water levels and Ottawa River flows, and achieved a stakeholder satisfaction score of 96.27.

Finally, we analyzed the strengths and weaknesses of our model.

Keywords: Network flow model, Multi-objective optimization, Model Predictive Control(MPC), Frank-Copula method

Contents

1 Introduction **3**
- 1.1 Problem Background . . . 3
- 1.2 Our work . . . 3

2 Assumptions and Notations **4**
- 2.1 Assumptions . . . 4
- 2.2 Notations . . . 4

3 Task1: Discover the optimal water levels in lakes **4**
- 3.1 Great Lakes network flow model . . . 4
- 3.2 Multi-objective optimization model . . . 5
  - 3.2.1 Aiming to minimize water shortage in power generation . . . 5
  - 3.2.2 Aiming to minimize ecological water shortage . . . 5
  - 3.2.3 Aiming to minimize industrial water shortages . . . 6
  - 3.2.4 Restrictions . . . 6
- 3.3 Model solving . . . 6

4 Task2: Flow regulation model based on model predictive control **8**
- 4.1 Lake Routing Rules . . . 8
- 4.2 Model building . . . 9
  - 4.2.1 Prediction model based on water balance . . . 9
  - 4.2.2 Optimal control . . . 9
  - 4.2.3 Scroll optimization and feedback correction . . . 9
- 4.3 Model solving . . . 11

5 Task3: Control sensitivity and satisfaction comparison **12**
- 5.1 Sensitivity of the flow control algorithm . . . 12
- 5.2 Mixed benefit scoring model . . . 13
- 5.3 Benefit score surves . . . 14
  - 5.3.1 Navigation score curves . . . 14
  - 5.3.2 Hydrological safety score curve . . . 14
  - 5.3.3 Weather adjustment ability scoring sheet . . . 15
  - 5.3.4 Livability score curve . . . 16
  - 5.3.5 Hydro-power generation score curve . . . 16
- 5.4 Satisfaction comparison . . . 17

6 Task4: Sensitivity of algorithms to environmental conditions **18**
- 6.1 Monte Carlo stochastic simulation . . . 18
- 6.2 Construction of joint distribution of environmental conditions and water level based on two-dimensional Copula function . . . 18
- 6.3 Joint probability and conditional probability . . . 19

7 Task5: Focus only on analysis of Lake Ontario **21**

8 Strengths and Weaknesses **22**

Memo **23**

References **24**

1 Introduction

1.1 Problem Background

The Great Lakes of the United States and Canada are the largest group of freshwater lakes in the world. These lakes serve numerous purposes, from providing habitats for various species to supporting human activities like drinking water supply, commercial shipping, and recreation. Managing the inflow and outflow of water is crucial to prevent the adverse effects of flooding or inadequate water levels for navigation. Water levels are influenced by a complex interplay of factors including weather conditions, precipitation, evaporation, and human-controlled dams and reservoirs. International network Control Modelers asked us to build of a network model for the Great Lakes and connecting river flows from Lake Superior to the Atlantic Ocean. And some other optional considerations or issues might be:

- Determination of the optimal water levels of the Great Lakes at any time of the year, taking into account the various stakeholders' desires.
- Establishment of algorithms to maintain optimal water levels in the lakes from inflow and outflow data for the lakes.
- Understanding of the sensitivity of your control algorithms for the outflow of the two control dams. Given the data for 2017, evaluate whether our new control measures will satisfy various stakeholders or be better than actual recorded water levels for the year.
- Analyze the sensitivity of our algorithm to changes in environmental conditions (e.g. precipitation, winter snow cover, ice jams).
- An extensive analysis focusing on Lake Ontario's stakeholders and influencing factors.

1.2 Our work

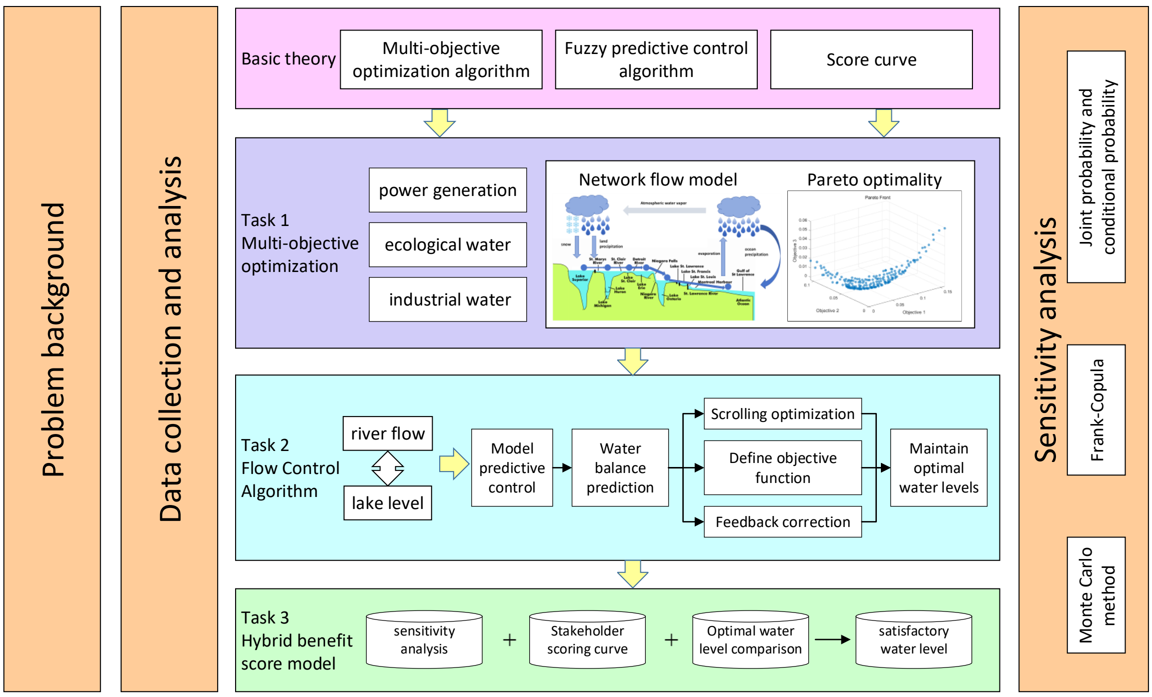


Figure 1: Our work

2 Assumptions and Notations

2.1 Assumptions

- The inflow of the lake is equal to the flow of the upstream river entering the lake.
- The flow of the downstream river leaving the lake is equal to the outflow regulated by the dam.
- There is no hysteresis in the dam's regulation of outflow.
- When considering precipitation and evaporation, the values of rivers are negligible compared to lakes.

2.2 Notations

Notations	Definition
Q	the outflow or river
Q_{in}	the inflow
L_i	the lake level at time i
V	the lake water capacity
W	the water shortage situation
S_i	the stakeholder i's satisfaction
$e(k)$	the deviation value during the kth iteration

Table 1: Notations Table

3 Task1: Discover the optimal water levels in lakes

3.1 Great Lakes network flow model

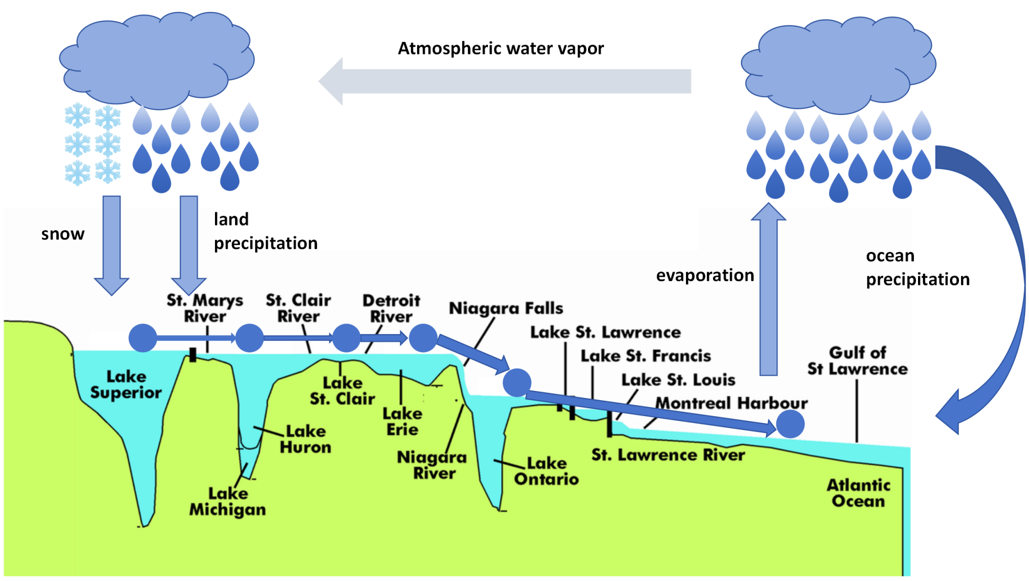


Figure 2: Great Lakes network flow

The sea level base height of the Great Lakes gradually decreases from west to east. There are two primary control mechanisms within the flow of water in the Great Lakes system – Compensating Works of the Soo Locks at Sault Ste. Marie and the Moses-Saunders Dam. Water conservancy projects regulate the water volume of each river, thereby adjusting the flow of rivers and the water levels of lakes. Lakes are connected through rivers and are affected by environmental conditions, forming a water cycle, thus forming a one-way Great Lakes network flow model with variable edge weights.

3.2 Multi-objective optimization model

The appendix lists six types of stakeholders, namely shipping companies, people who manage shipping docks or live near Montreal harbor, environmentalists, property owners on the shores of Lake Ontario, recreational boaters and fishing boats on Lake Ontario, and hydropower generation companies. However, we noticed that there are certain similarities among these stakeholders[1], and we combined the water consumption data given by the official website, so we divided them into three fields, namely power generation, ecology, and industry, so as to better integrate with the hydrological conditions of the Great Lakes. In order to satisfy their interests as much as possible, we construct objective functions for them respectively.

In order to meet the demand for power generation in lakes and river basins and the demand for downstream industry, and domestic water, based on the actual situation of the basin, we use the following goals to build a problem model.

3.2.1 Aiming to minimize water shortage in power generation

Since reservoir power generation is the top priority of the reservoir system, we use the following formula to describe this goal based on the actual conditions of the basin and with full consideration and utilization of water resources[2]:

$$f_1 = \min W_r \tag{1}$$

Where W_r represents the water shortage in power generation in the middle and lower reaches, and satisfies the following formula:

$$W_r = \sum_{t=1}^{T} (w_d - \mathrm{w}_s)\,\Delta t \tag{2}$$

Among them, T is the total calculation period of hydropower station dispatching in the year, Δt represents the tth calculation period, w_d is the power generation water demand, and w_x is the surface water supply.

3.2.2 Aiming to minimize ecological water shortage

For the downstream section of the basin, the ecological water demand of the river is an extremely important part. While hydropower stations can meet the power generation capacity, they should also fully consider the ecological water demand. Here, in order to facilitate the calculation of subsequent algorithms, we convert ecological water demand into ecological water shortage, and use this to construct the objective function. The specific formula is as follows:

$$f_2 = \min W_e \tag{3}$$

Where W_e represents the ecological water shortage of the downstream river, and satisfies the following formula:

$$W_e = \sum_{t=1}^{T} (Q_r - Q_e) \Delta t \tag{4}$$

Where Q_r represents the discharge flow, and Q_e represents the river ecological flow, T is the total calculation period of the hydropower station dispatching year, Δt represents the tth calculation period, Δt then represents the tth calculation period (it is measured in months).

3.2.3 Aiming to minimize industrial water shortages

For some factories downstream of the basin, they need to draw industrial water in the area in actual operations. In order to ensure the water demand supply of the downstream industrial areas, we convert it into the industrial water demand difference and use this to construct the objective function. The specific formula is as follows:

$$f_3 = \min W_I \tag{5}$$

Where W_I represents the industrial water shortage in the middle and lower reaches, and satisfies the following formula:

$$W_I = \sum_{t=1}^{T} (w_g - \mathrm{w}_h) \Delta t \tag{6}$$

Among them, T is the total calculation period of hydropower station dispatching in the year, Δt represents the tth calculation period, w_g is the industrial water demand, and w_h represents the industrial water supply.

3.2.4 Restrictions

Based on the actual conditions of lakes and river basins, we established objective functions that meet different needs, thereby laying the foundation for subsequent model construction. In addition, model construction also needs to meet the following constraints, which are described in detail as water quantity maintains balance constraints:

$$V_{i,t+1} = V_{i,t} + \left(\bar{Q}_{i,t} - Q_{i,t} - Q_{i,ls}\right) * T \tag{7}$$

traffic constraints:

$$Q_{\min\ i,t} < Q_{i,t} < Q_{\max\ i,t} \tag{8}$$

storage capacity constraints:

$$V_{\min} < V_t < V_{\max} \tag{9}$$

Among them, $\bar{Q}_{i,t}$ represents the average inflow of hydropower station i in period t, and $Q_{i,t}$ represents the average inflow of hydropower station i in period t. The power generation reference flow, $Q_{i,ls}$, represents the total flow loss of hydropower station i, and T is the total number of calculation periods. V represents the lake capacity, and its changes and limitations can be approximately reflected and replaced by the water level.

3.3 Model solving

From the above analysis, our mathematical model can be summarized as:

$$F = \min (W_e, W_r, W_I) \tag{10}$$

$$s.t.\begin{cases} V_{i,t+1} = V_{i,t} + \left(\bar{Q}_{i,t} - Q_{i,t} - Q_{i,ls}\right) * T \\ Q_{\min\ i,t} < Q_{i,t} < Q_{\max\ i,t} \\ V_{\min} \ < V_t < V_{\max} \end{cases} \tag{11}$$

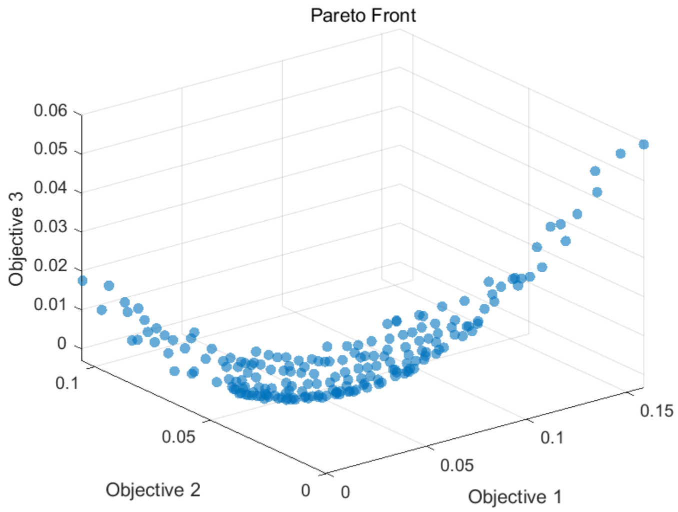


Figure 3: pareto process

We used the gamultiobj solver in MATLAB to solve this problem. The figure above is its pareto front. It can be seen that the three goals are close to 0 at this time, which means that the benefits in power generation, ecology, industry and other fields have reached their maximum value. So we can plot the optimal water levels for the Great Lakes over the course of the year as shown below.

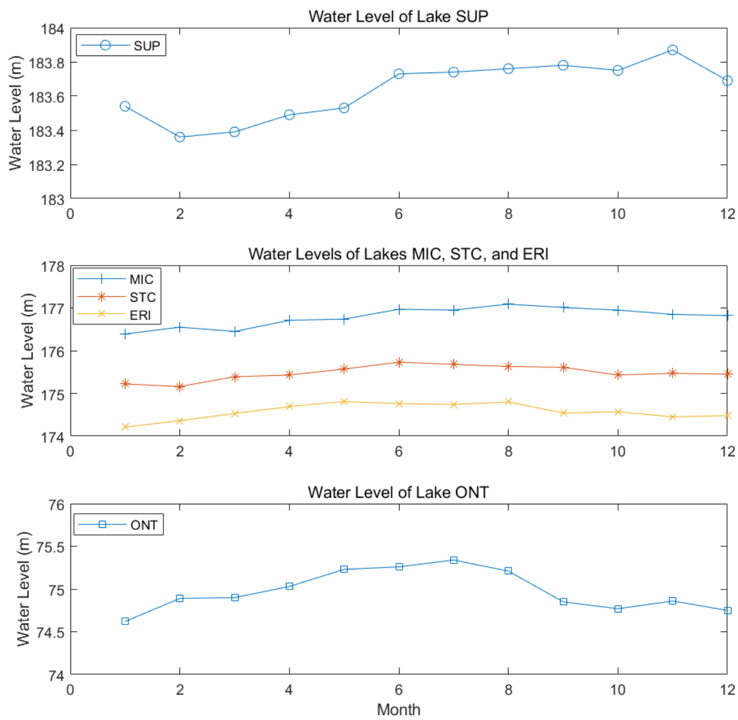


Figure 4: Optimal water levels in the Great Lakes

4 Task2: Flow regulation model based on model predictive control

4.1 Lake Routing Rules

This task requires us to control the inflow and outflow of the lake to achieve the optimal water level. In order to maintain the unity of the dynamic network flow model of the entire Great Lakes, we define the flow of the upstream river of each lake as the inflow of the lake, and the flow of the downstream river of each lake as the outflow of the lake. Taking Lake Erie as an example, its inflow is the flow of the Detroit River, and its outflow is the flow of the Niagara River.

In addition, due to the location of the Ottawa River with Lake Ontario and the St. Lawrence River, it cannot be directly affected by the water level of Lake Ontario, so the Ottawa River is not considered in the flow control algorithm.

Considering the various influences such as channels, dams, and water flow time, it is difficult to define the physical relationship between water flow velocity and lake water level. Therefore, we directly obtain the relationship between water flow velocity and lake water level through the calculation formula of lake outflow and lake water level value in a natural hydraulic state.

A review of the literature shows that each lake and its downstream outflow have corresponding Lake Routing Rules. They are obtained by fitting long-term water level measurement data and downstream flow velocity observation data, intuitively reflecting the relationship between flow velocity and water level:

$$\begin{aligned} St.MarysRiver &: \mathrm{Q} = 824.7 * (\mathrm{SUP} - 181.43)^{1.5} \\ St.ClairRiver &: \mathrm{Q} = 82.2 * ((\mathrm{MHU} + \mathrm{STC})/2 - 166.98)^{1.87} * (\mathrm{MHU} - \mathrm{STC})^{0.36} \\ DetroitRiver &: \mathrm{Q} = 28.8 * (\mathrm{STC} - 164.91)^{2.28} * (\mathrm{STC} - \mathrm{ERI})^{0.305} \\ NiagaraRiver &: \mathrm{Q} = 558.3 * (\mathrm{ERI} - 169.86)^{1.60} \\ St.LawrenceRiver &: \mathrm{Q} = 555.823 * (\mathrm{Oswego} - 0.0014(\mathrm{Year} - 1985) - 69.474)^{1.5} \end{aligned} \tag{12}$$

Among them, Q is the flow rate of the river, SUP represents the water level of Lake Superior, MHU represents the water level of Lake Michigan and Lake Huron, STC represents the water level of Lake St. Clair, ERI represents the water level of Lake Erie, and Oswego represents the water level of Lake Ontario.

Since the changes in lake water level are not dramatic, the relationship between lake water level and water flow velocity can be regarded as locally linear.

Therefore we can get:

$$\begin{aligned} \Delta Q &\approx a * \Delta Level + \beta \\ \Delta Level &= Level_{t+1} - Level_t \\ \Delta Q &= Q_{t+1} - Q_t \end{aligned} \tag{13}$$

Among them, $Level_t$ represents the water level of the lake at time t and Q_t at this time represents the net outflow. From the relationship between the flow rate change and the water level change in this formula, we can figure out how much flow rate needs to be adjusted to change the water level by a certain value.

4.2 Model building

4.2.1 Prediction model based on water balance

During the scheduling and control process of the lake dynamic network flow, we mainly focus on the changes in the lake water level. By observing changes in lake water levels, appropriate inflow and outflow measures are taken for dispatch. Therefore, we established a prediction model based on the water balance principle in the lake dynamic network flow. It can deduce the water level change process in the future based on the current operating water level and the inflow flow, outflow flow, water level and storage capacity relationship in the future period. Choose one month as the sampling interval ΔT.

$$\begin{gathered} V_{i,t+1} = V_{i,t} + (Q_{in,t} - Q_{out,t})\Delta T \\ V_{i,0} = b_i^0 \\ V_{i,T} = b_i T^0 \end{gathered} \tag{14}$$

Among them, i takes 1 to 5 to represent five lakes; $V_{i,t+1}$ is the storage capacity at time t, $V_{i,t+1}$ is $t+1$ is the storage capacity at time; $Q_{in,t}$ is the inbound flow, $Q_{out,t}$ is the outbound flow; ΔT is the unit time length. b_i^0 is the initial storage capacity value, b_{iT}^0 is the final storage capacity value.

4.2.2 Optimal control

In order to best balance the interests of all stakeholders, we use the established water level prediction model to control the inflow and outflow of the lake to maintain the lake's water level at the optimal level.The following objective function formula is established:

$$\min J = [\hat{y}(k+1 \mid k) - L_{op}]^2 \tag{15}$$

Among them, $\hat{y}(k+1 \mid k)$ is the predicted output water level of the system at time k for one time in the future, L_{op} represents the expected optimal water level.

The constraints are the same as Task 1 and also include:

- water quantity maintains balance constraints
- traffic constraints
- storage capacity constraints

4.2.3 Scroll optimization and feedback correction

For the nonlinear system model, that is, the Great Lakes network water level optimization dispatch model in this article, a nonlinear optimization algorithm needs to be used to solve it.The initial water level is set based on lake water level data in previous years, and the optimal dispatch model for the flow water level of the Great Lakes network is solved within the control time domain. The optimal control sequence for the lake outflow in the control time domain is obtained; but in the current period t, only the instructions of the first control sequence are executed. At the same time, the final water level at time t is obtained, that is, the initial water level at time $t+1$. Let $t = t+1$ and re-update the initial value. Repeat the previous process until the entire timeline is optimized.

The model rolling optimization process is as follows:

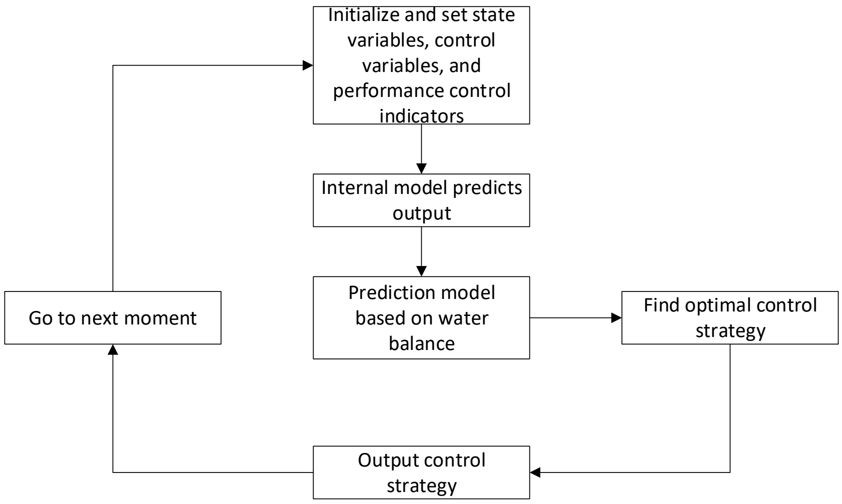


Figure 5: Rolling optimization solution flow chart based on differential evolution algorithm

When building the internal model, we used an approximate mathematical model to simulate the actual controlled Great Lakes network flow optimization dispatch model system, which would lead to mismatch problems between the model and the actual system; in addition, there are precipitation, accumulation of the greater impact caused by external factors such as snow, ice cover and evaporation. Based on the existence of these two perturbations, the system will produce a greater degree of deviation after multiple iterations.

In order to solve the mismatch between the model and the actual controlled system, We need to introduce the influence function of various external factors on traffic. The first is a rainfall-runoff model, which can be used to estimate the amount of runoff produced by a given rainfall event. We can divide the amount of runoff by the duration of rainfall to get the velocity of flow caused by precipitation.

$$V_p = \frac{P - I_a - \left(\frac{P-I_a}{V}\right)^2}{\triangle t_p} \tag{16}$$

- Q is the outflow water volume,

- P is rainfall,

- I_a is the initial abstract quantity (such as plant interception, evaporation before soil moisture, etc.),

- V is the maximum water storage capacity of the basin.

- $\triangle t_p$ is the time of precipitation.

Then there is the ice jam-runoff model. We introduce Manning's formula[3] and multiply it by the water section area, which is the change in flow velocity caused by ice jam.

$$V_i = \frac{1}{n} R_h^{\frac{2}{3}} S_p^{\frac{2}{3}} S \tag{17}$$

- C is the Chezy coefficient

- n is Manning's roughness coefficient

- S is hydraulic slope

- R_h is the hydraulic radius

We set the iteration error $e(k)$ of the model to:

$$e(k)=\frac{1}{n}R_h^{\frac{2}{3}}S_p^{\frac{2}{3}}S+\frac{P-I_a-\left(\frac{P-I_a}{V}\right)^2}{\triangle t_p} \tag{18}$$

Since the actual measured values at moments t, t+1, ..., t+N_c cannot be known, in order to make the model prediction calculation results replace the real system output values, we correct the model prediction values as follows:

$$\begin{cases} y_p(t+1)=\hat{y}(t+1)+(y(t)-\hat{y}(t)) \\ y_p(t+2)=\hat{y}(t+2)+(y(t+1)-\hat{y}(t+1)) \\ \vdots \\ y_p(t+N_c)=\hat{y}(t+N_c-1)+(y(t+N_c-1)-\hat{y}(t+N_c-1)) \end{cases} \tag{19}$$

From the above formula, it can be seen that the predicted value after considering feedback correction at time $t+1$.

$$y_p(t+1)=\hat{y}(t+1)+e(t) \tag{20}$$

4.3 Model solving

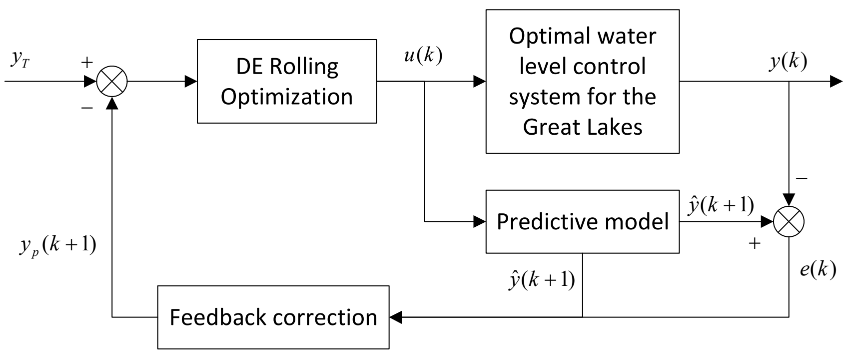


Figure 6: Implementation of model predictive control algorithm for optimal water levels

Combined with what is shown in the figure above, MPC is applied to the optimal water level dispatch of the Great Lakes to make optimal control decisions in the future period. The specific steps are as follows:

Step1: First initialize the relevant parameters of model predictive control: it is known that the water level at time k is input the inlet flow and control sequence at this time: $U_t = [u(t \mid t), u(t+1 \mid t), \ldots, u(t+3 \mid t)]$, set the sampling time to 1 month and the prediction step length to 4 months, and the total scheduling time is 12 months.

Step2: Predict based on the internal model to obtain the outflow volume in the next four months at time k: $\hat{y}(t+1 \mid t), \hat{y}(t+2 \mid t), \ldots, \hat{y}(t+4 \mid t)$.

Step3: Use the differential evolution algorithm to calculate the cost function and solve the optimal control sequence that optimizes the lake water level: $U^* = [u^*(t), u^*(t+1), \ldots, u^*(t+3)]$. Output the first control instruction in the optimal control sequence to schedule the inflow and outflow of the lake.

Step4: Import the flow changes caused by environmental conditions into the MPC internal model prediction model to obtain the initial water level at k+1: $y_p(t+1)=\hat{y}(t+1)+e(t)$to perform water level feedback correction.

Step5: Enter the solution of the optimal control sequence at the next moment.

To achieve optimal water levels, the flow of each river should be as shown in the figure below.

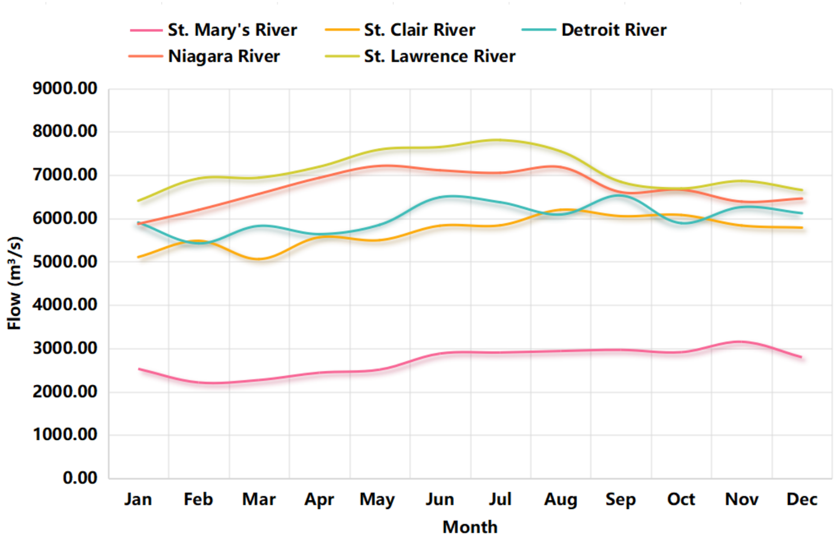


Figure 7: Flow of each river/Controlled outflow

5 Task3: Control sensitivity and satisfaction comparison

5.1 Sensitivity of the flow control algorithm

To study the sensitivity of our control algorithm to outflow from two control dams, we developed a plan of increasing water by 10% and reducing water by 10% inflow at the Compensating Works (Soo Locks) at Sault Ste. Marie and the Moses-Saunders Dam at Cornwall. And we substitute it into our control algorithm so that each lake can reach the optimal water level again.

However, due to the large amount of data, we selected two months for analysis. For the analysis of the Compensating Works (Soo Locks) at Sault Ste. Marie, we chose September for comparison, and the flow of each river is shown in the figure below.

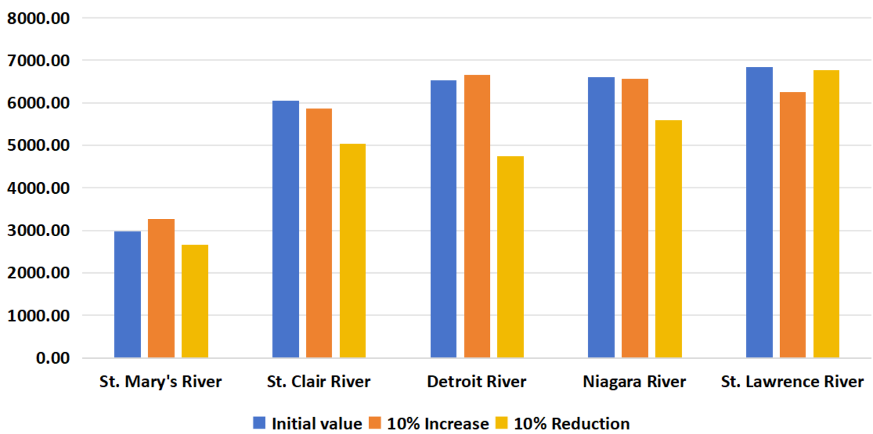


Figure 8: Sensitivity analysis for the Compensating Works (Soo Locks) at Sault Ste. Marie

For the analysis of the Moses-Saunders Dam at Cornwall, we chose March for comparison. The flow of each river is shown in the figure below.

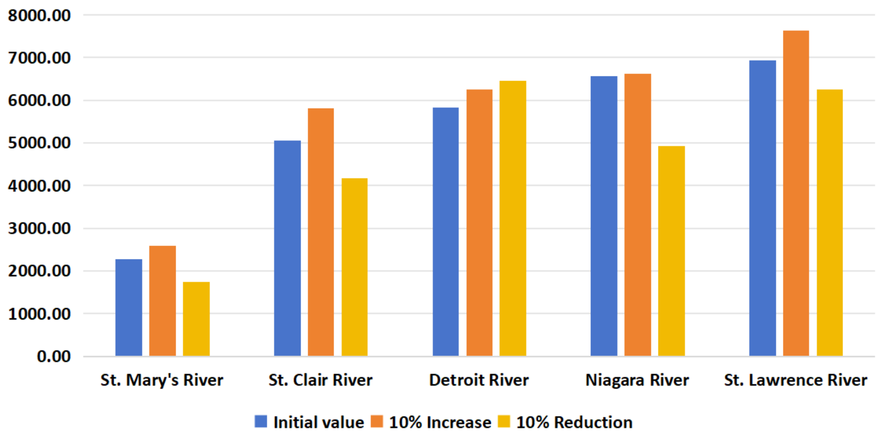


Figure 9: Sensitivity analysis for the Moses-Saunders Dam at Cornwall

As shown in the figure above, the flow changes in the two dams make the flow adjustment very obvious. It can be seen that our control algorithm is very sensitive to the outflow of the two dams.

5.2 Mixed benefit scoring model

For evaluating whether a water level control system and hydrological results are satisfactory, the focus is on not causing significant economic or environmental losses to any sector while providing overall economic and environmental benefits.

Benefit balancing considers the major interests in the system from Lake Ontario downstream to Lawrence River , including the following (in no particular order):

- S_1: Satisfaction about shipping companies, which want high, static water in the St Lawrence River.
- S_2: Satisfaction about people who manage shipping docks or live near Montreal harbor, who want water in the river to be steady and low.
- S_3: Satisfaction about environmentalists, who want seasonal high high-water levels and low low-water levels on Lake Ontario to help maintain the habitat.
- S_4: Satisfaction about property owners on the shores of Lake Ontario, who want mid-level, steady water levels. And satisfaction with recreational boaters and fishing boats on Lake Ontario, who want mid-level, steady water levels;
- S_5: Satisfaction about hydro-power generation companies, which would like more control over water levels to use high-level water as a storage system to maximize flows during high energy usage periods.

Based on the needs of the above six stakeholders, we established Benefit Score Curves to reflect each stakeholder's degree of satisfaction with the current water level control system and hydrological results. For a known annual lake level result, we measure its stakeholder satisfaction S as:

$$S = \sum_{i=1}^{12} \sum_{j=1}^{N} S_j^i \tag{21}$$

Among them, i represents the month, j represents the interested parties, and N represents the total number of interested parties considered.

5.3 Benefit score surves

These curves were initially developed to reflect the relationships between levels or flows and benefits to several of the uses of the system, but some were later modified by trial and error to produce better overall results with respect to the more rigorous performance indicators used in the study evaluation process.The following benefit score curves are all from survey data provided by $PlanD^{+}$ literature, using *IGLD* 1985[4].

5.3.1 Navigation score curves

Shipping companies expect flow rates in the Lawrence River to remain high and stable to facilitate vessel traffic and safety.We measure the flow rate of the Lawrence River by measuring the outflow from Lake Ontario. With the help of the local linear relationship between flow velocity and water level obtained in Task 2, it can be seen that channel navigation expects the water level of Lake Ontario to remain at a high value to provide a sufficiently high river flow velocity, while rejecting excessively high water levels to maintain a stable River flow rate.

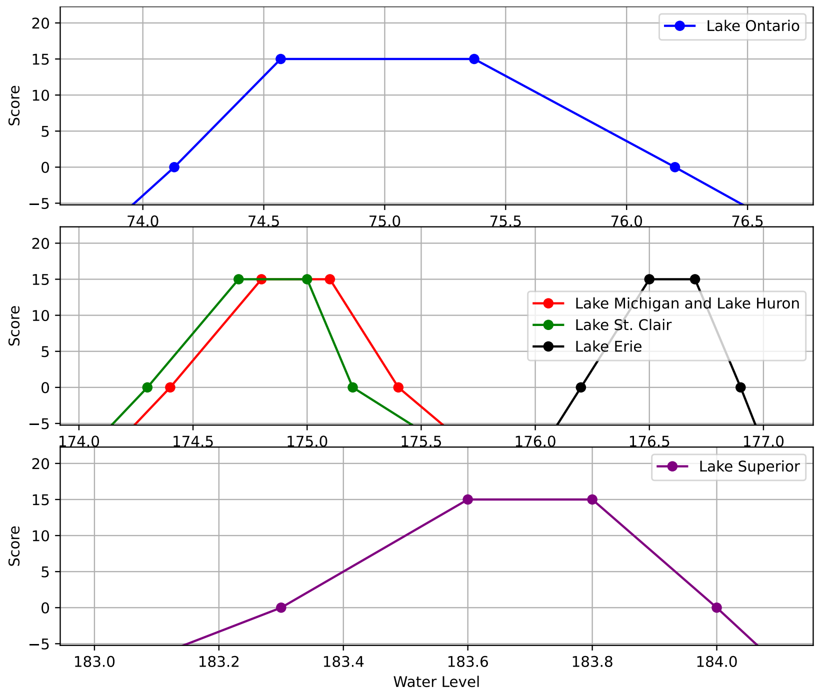


Figure 10: Navigation score curves

From the figure, it can be seen that navigation is most satisfactory with a water level that is moderately high. A water level that is too high will cause the downstream flow velocity to be too high, making it unsuitable for navigation. Lakes with high base levels are more sensitive to changes in water level because equal fluctuations result in larger changes in flow velocity.

5.3.2 Hydrological safety score curve

The managers of the wharf and the residents near Montreal Harbor are more concerned about the safety of the riverside area. Therefore, they prefer that the water level of the lake be maintained at a low level and maintained consistently.This requires us to evaluate the hydrological safety of the lake.

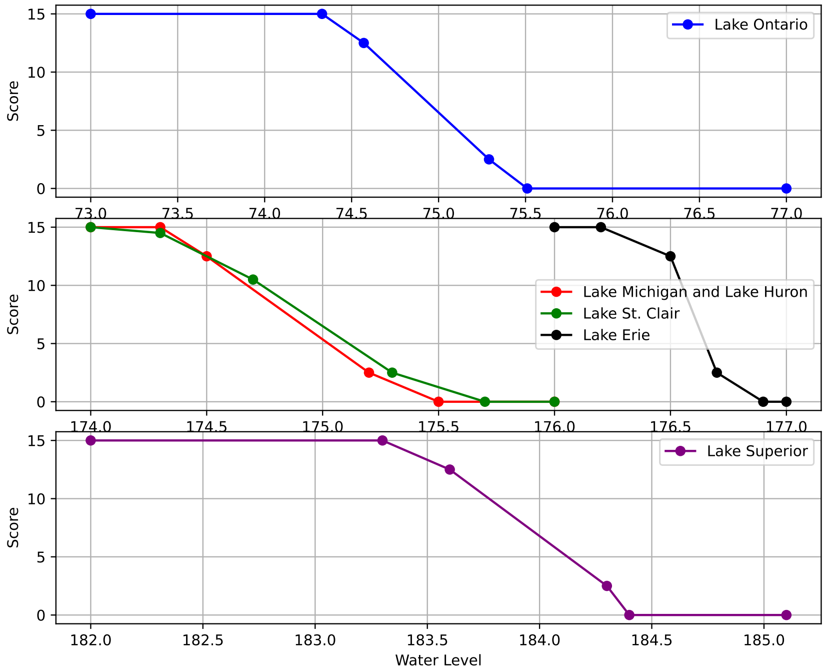


Figure 11: Hydrological safety score curve

From the figure, it can be seen that a lower water level will provide greater safety on the shore. As the water level increases excessively, the level of danger will increase and the satisfaction of shore residents will decrease.

5.3.3 Weather adjustment ability scoring sheet

Environmentalists hope that Lake Ontario will have a stronger ability to regulate seasonal environmental weather to help species thrive and maintain ecological balance. By analyzing the relationship between the water level of Lake Ontario and its surrounding weather, we can adjust a certain climate environment by controlling the water level of the lake:

Lake Ontario Level	Weather Condition
below 74m	very dry
between 74m and 74.5m	dry
between 74.5m and 75m	moderate
between 75m and 75.5m	wet
above 75.5m	very wet

Table 2: Lake Ontario levels and corresponding weather condition

In the spring, environmentalists may advocate for higher water levels to simulate natural spring flooding and provide water and moisture to lakeshores and wetlands. This aids the growth of aquatic plants and the reproduction of fish. In summer, moderate water levels keep the lake's ecosystem in balance. This helps maintain vegetation along the lake's shoreline, providing adequate habitat. In the fall, moderate to lower water levels help slow lakeshore erosion and provide opportunities for aquatic plant seed dispersal. This may also help clean up lake bottom sediments. In winter, environmentalists may support lower water levels to reduce the impact of ice on lake ecology.

So we can get the weather changes expected by environmentalists throughout the year[5], which also reflects their satisfaction with lake water levels and the lake's ability to regulate climate:

spring	summer	autumn	winter
very wet/wet	wet	moderate	dry

Table 3: Weather changes environmentalists hope for

If the water level curve meets the requirements of weather changes in the current season, 6 points will be scored, otherwise, no points will be scored. That is:

$$\begin{cases} S_3 = 6, & if\, meet\, the\, weather \\ S_3 = 0, & otherwise \end{cases} \tag{22}$$

5.3.4 Livability score curve

Property owners as well as recreational boaters and fishing boats on Lake Ontario would prefer that Lake Ontario maintain a moderate water level. At the same time, individual residents and boats should be more responsive to changes in lake water levels than to water level demands for navigation[6].

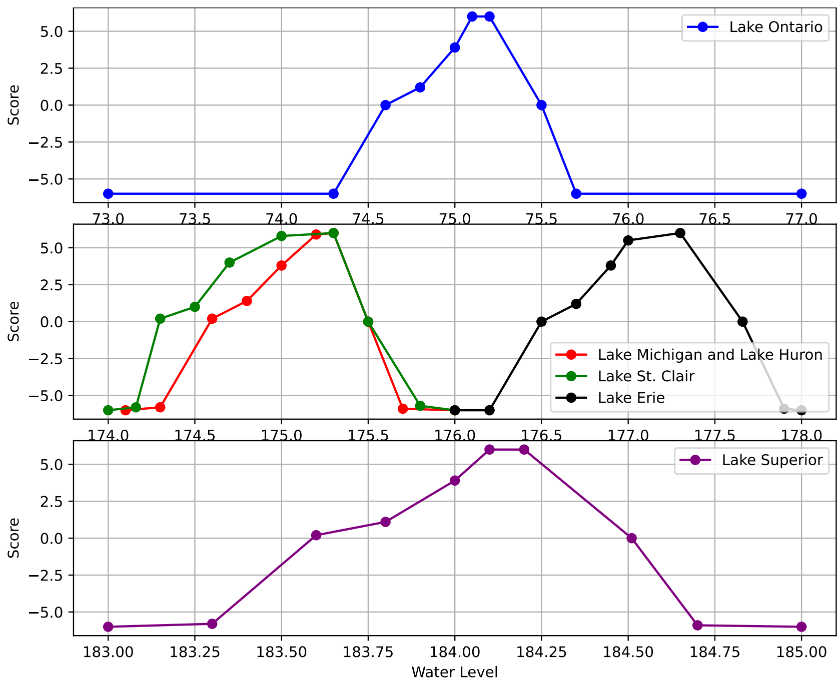


Figure 12: Livability score curve

Owner and boater satisfaction with water levels follows similar trends as navigation, but the changes are more subtle.

5.3.5 Hydro-power generation score curve

Hydroelectric companies want to regulate Lake Ontario to high water levels to maximize the flow of electricity.

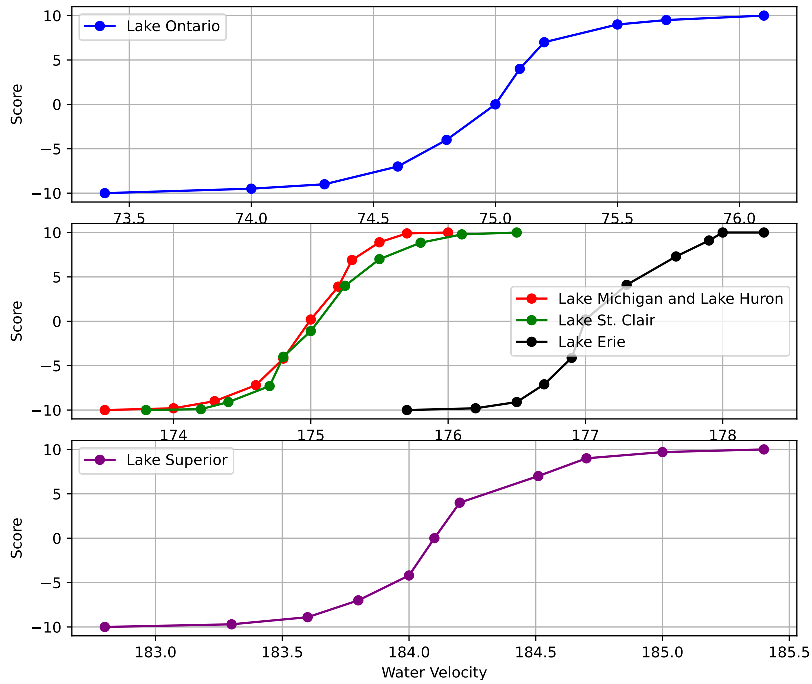


Figure 13: Hydro-power generation score curve

We can see that higher water levels mean higher outflows, which will be more popular with hydropower formulas. However, as the water level continues to rise, it will become more difficult to control the flow, so the upward trend in satisfaction will gradually slow down.

5.4 Satisfaction comparison

We put the optimal water level we calculated and the actual recorded water level in the past 2017 into each scoring curve and table to calculate the satisfaction score. We can get the satisfaction score under our new rules and the original actual recorded water level satisfaction score for the five major lakes:

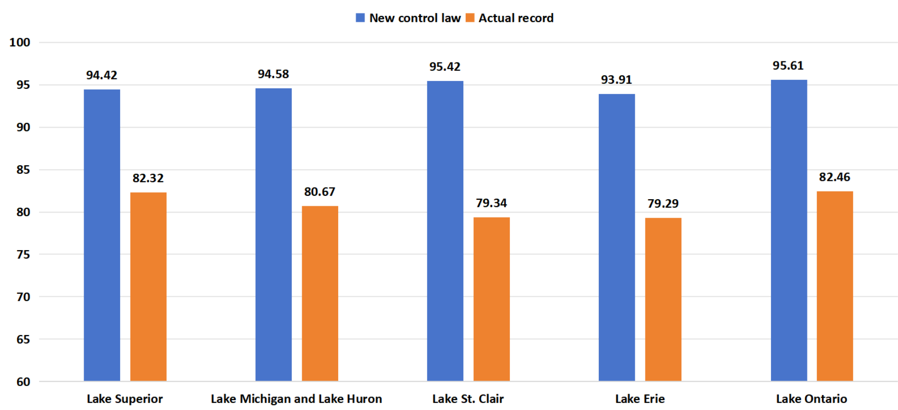


Figure 14: Satisfaction comparison

It can be seen that the optimal water level under the new rules performs better and is more stable in the Great Lakes scores. The maximum is 16 points higher, which is equivalent to a gap where the interests of two stakeholders are not considered, and the advantage is at least 12 points.

6 Task4: Sensitivity of algorithms to environmental conditions

6.1 Monte Carlo stochastic simulation

This chapter considers the impact of environmental conditions such as precipitation, snow, and ice jams on the flow control algorithm, conducts a random sampling of environmental factors, establishes a calculation model based on the Monte Carlo stochastic simulation method for the impact of environmental factors, and calculates the flow control algorithm under different environmental conditions. In order to quantify the influence of risk factors, a two-dimensional Copula function[7] is used to analyze the correlation between influencing factors and algorithms to express the sensitivity of uncertainty factors.

It is worth noting that since the result of snow accumulation can be divided into melting into water or freezing, it is similar to the effect of precipitation and ice jams on water levels, and its delay is negligible in the time dimension of one month. Therefore, this section only discusses the effects of precipitation and ice jams on water levels.

By generating a large number of possible scenarios, Monte Carlo simulation can be used to evaluate the impact of uncertainty factors on the maximum water level under wide changes, and is suitable for handling complex nonlinear systems and evaluating the probability of extreme situations[8].

This article uses round-out sampling to generate random numbers. When the inverse function of the probability density function of a random variable is difficult to obtain through calculation, the random number of a given probability distribution can be obtained by the selective sampling method. The general steps of the round-out sampling method are as follows:

Assume that the probability density function of random variable X is f(x) and there is a pair of real numbers a,b(a< b)

a) Suppose there is c such that f(x)$\leq$ c, x$\in$ (a,b),

b) Use a pseudo-random number generator to generate a pair of pseudo-random numbers distributed in [0,1] , r_1 and r_2 . Let y=a+(b-a)r_2 , substitute y into the probability density function to get D=f(y).

c) Compare r_2 with D/C, if $r_2 \leq D/C$, then x=y; otherwise, discard $r_1 and r_2$ and return to step b) until the conditions are met.

6.2 Construction of joint distribution of environmental conditions and water level based on two-dimensional Copula function

We can call the Copula function "connection function" or "dependence function". Each function has its own marginal distribution[9]. The Copula function connects them together and combines them into a unique joint distribution to reflect the changing relationship between them. The defined interval is on [0,1]. The expression is as follows:

$$H(x_1,x_2,...,x_n) = C(F_1(x_1),F_2(x_2),...,F_n(x_n)) \tag{23}$$

For the parameter estimation of the two-dimensional Copula function, the correlation index method is relatively simple and convenient, and the most commonly used is the correlation index method. However, the correlation index method also has certain conditions for use, that is, the parameter θ is related to the Kendall rank correlation coefficient τ or to the Spearman There must be a clear system expression between the rank correlation coefficient ρ, such as Gumbel Copula and Frank Copula.

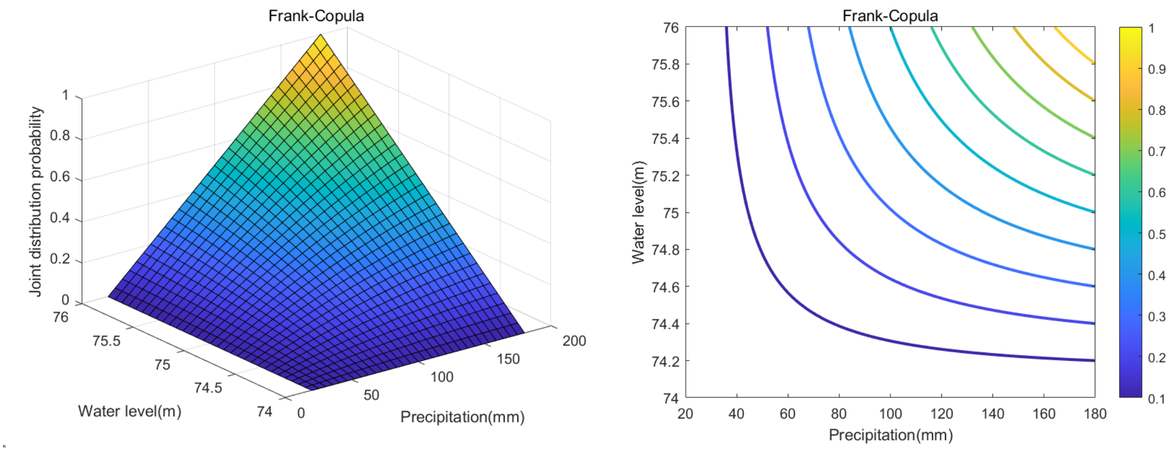
We calculated and compared the three indicators of RMSE, AIC and BIC to find the Copula function with the best fitting degree. The parameter values and goodness-of-fit evaluation values are shown in the table below. It can be seen from the table that the RMSE, AIC and BIC index of the Frank Copula function are the smallest, so the Frank Copula function is selected to build the two-dimensional joint distribution model[10].

Type	Parameter	RMSE	AIC	BIC
Gumbel	8.38	0.04	-130.67	-136.54
Clayton	10.09	0.03	-145.68	-130.53
Frank	15.51	0.02	-151.77	-144.06

Table 4: Goodness of fit evaluation value

6.3 Joint probability and conditional probability

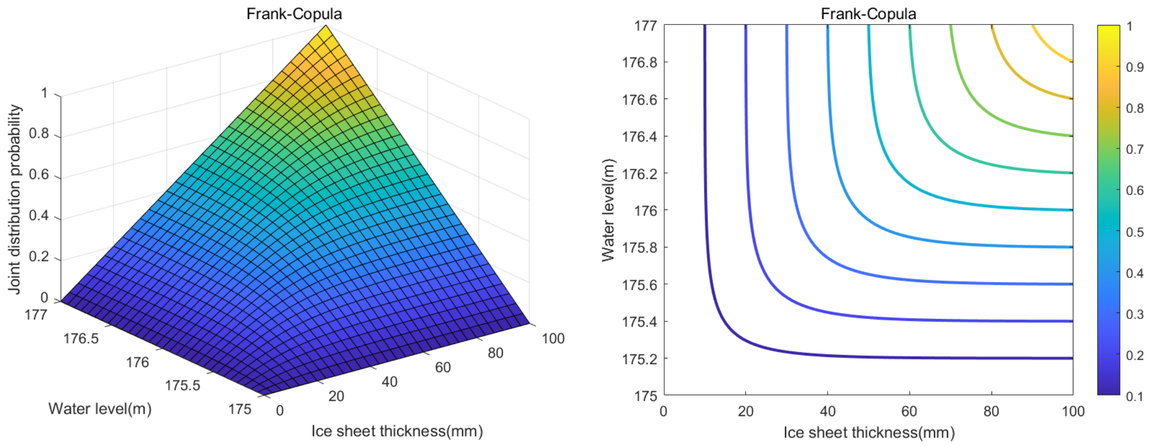
Through the simulation of the above random algorithm and the solution of the joint probability distribution. We obtained the joint probability distribution and joint probability value of precipitation and ice cover indicators and water level indicators.



(a) precipitation probability (b) precipitation contours

Figure 15: Joint probability and contours of precipitation and water level

In Figure 15(a), the joint probability three-dimensional diagram visually displays the joint probability distribution and joint probability value of the precipitation indicator and water level indicator. As the precipitation index and water level index values increase, the joint probability value increases.



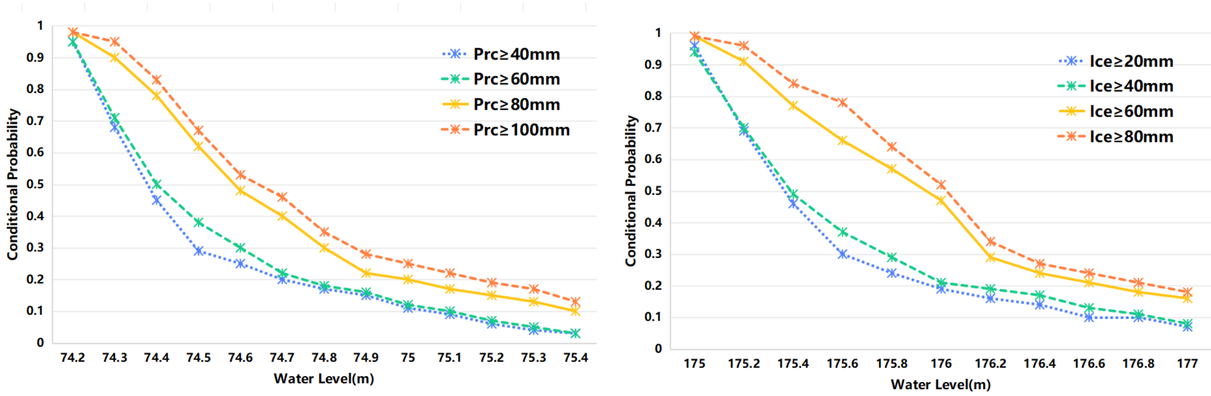
(a) ice probability (b) ice cover contours

Figure 16: Joint probability and contours of ice cover and water level

In the joint probability contour map Figure 15(b), you can more clearly see the probability values of different indicator value combinations. The greater the values of the precipitation indicator and water level indicator, the greater the joint probability value. When the value of the precipitation indicator is constant, the water level indicator The larger the value, the greater the probability value. The contour density of the probability value contour changes from large to small after the precipitation value is 100mm, which also shows that the water level index changes from large to small with the precipitation index after this point; similarly, when the water level index value is constant, the joint probability changes with the It increases with the increase of precipitation index value. The same analysis goes for ice cover.

In addition to obtaining the law of the influence of precipitation and ice cover on water level changes through joint probability, we can also intuitively obtain the degree of influence of precipitation and ice cover on water level through the conditional probability relationship.

We calculated the probability that the water level index is greater than or equal to a certain value under the conditions that the precipitation/ice cover index is greater than or equal to 40mm, 60mm, 80mm and 100mm respectively. The specific values of the water level are selected as 74.2m, 74.3m, ... and increase by 0.1m to 75.4m in sequence.



(a) conditional probability of precipitation (b) conditional probability of ice cover

Figure 17: Conditional probability of precipitation/ice cover and water level

In Figure 17(a), the four curves all reflect that when the precipitation index value is constant, the conditional probability that the water level index is greater than or equal to a certain value decreases as the water level index value increases, that is, the less likely it is to occur; overall observation For the four probability curves, the greater the given precipitation index value, the greater the overall conditional probability, that is, the greater the probability of water level occurrence. Especially when the precipitation index value is greater than or equal to 80mm, compared with greater than or equal to 60mm, the curve has a significant upward shift and an increase. bigger. For example, when precipitation $\geq$ 60mm, the conditional probability of water level $\geq$ 74.4m is 0.38, and when precipitation $\geq$ 80mm, the conditional probability of water level $\geq$ 74.4m is 0.62. Therefore, it shows that the degree of influence of water level by precipitation is proportional to the given condition value of precipitation index. The greater the condition value, the greater the degree of influence.

For ice jams, it can be seen that the joint distribution and conditional probabilities between it and water levels are similar to those between precipitation and water levels. Therefore, with the help of relevant analysis methods, it can be concluded that ice jams also have a certain degree of impact on water levels.

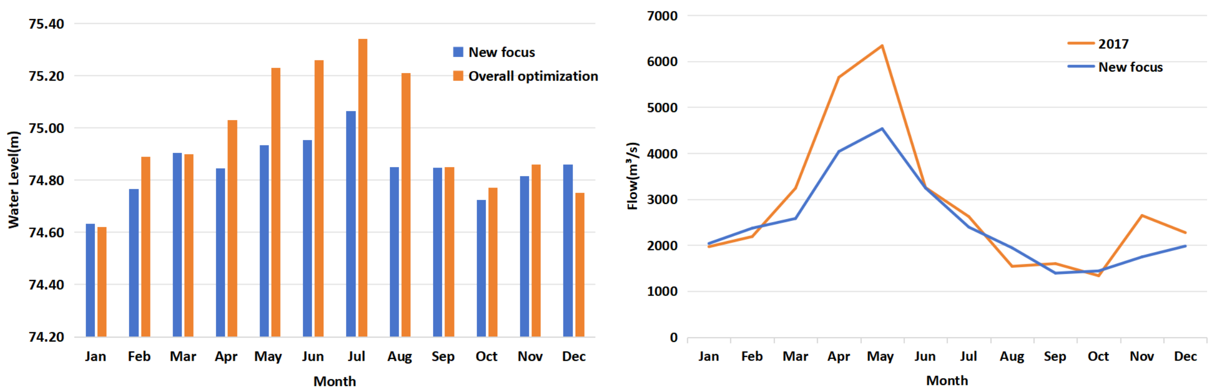
In summary, environmental conditions, including precipitation, snow, ice jams, etc., are closely related to the water level of the lake, which means that our control algorithm is very sensitive to environmental conditions.

7 Task5: Focus only on analysis of Lake Ontario

While we focus on Lake Ontario, we also need to consider the Ottawa River and the Port of Montreal.

For the Ottawa River, since it has 50 major dams and hydroelectric power stations and is connected to 13 large reservoirs, its power generation needs cannot be ignored. So based on Task 2, we need to add the power generation demand of the Ottawa River to the objective function. In addition, because these reservoirs store a large portion of spring runoff, reducing the risk of flooding for the Port of Montreal downstream, our evaluation criteria for the Livability score curve will also need to change.

By recalculating the model, we can obtain the optimal water levels and flows for the new Lake Ontario region (including the Ottawa River).



(a) Comparison of optimal water levels in Lake Ontario (b) Comparison of the flow of the Ottawa River

Figure 18: New optimal water level and flow

For Lake Ontario, the new optimal water level has become more stable than the original, and at the same time, the peak water level during the rainy season has also been reduced. This is because after taking the Ottawa River into account, Lake Ontario's water storage capacity for power generation is much smaller in comparison. Therefore, it is more focused on domestic and ecological water needs, and it is easier to satisfy stakeholders such as shipping companies, residents, and fishing boats. Pursuit: The water level remains moderate and stable.

For the Ottawa River, compared with 2017, the new flow reduces its peak in the rainy season, reducing the risk of flooding, but the flow rate remains at a high level throughout the year, which is conducive to power generation by its power stations.

We re-scored stakeholder satisfaction for the Lake Ontario region.

Table 5: Satisfaction comparison

Type	Overall optimization	New focus
Satisfaction score	95.61	96.27

It can be found that when focusing only on the Lake Ontario area, the stakeholder satisfaction scores have not improved much. Combined with the water level and flow data above, it can be inferred that this is because the power generation has decreased, but the satisfaction of the remaining stakeholders The degree has increased, making the overall satisfaction score improved compared with the original one, but the difference is not big.

8 Strengths and Weaknesses

- Strengths:

1.The model presented in this paper offers an overarching perspective on lake water level prediction and flow regulation, considering various objectives. This ensures that the strategy meets the diverse needs of stakeholders, enhancing decision effectiveness.

2.The model combines multiple operations research algorithms, improving the accuracy and robustness of prediction and regulation, for better determination of optimal water levels for the Great Lakes.

3.Prioritizing stakeholder satisfaction, the performance of control algorithms is evaluated through benefit score curves and sensitivity analysis.

- Weaknessess:

1. Complexity and Computational Intensity: The described model may exhibit high complexity and computational intensity, particularly when determining optimal water levels for multiple lakes simultaneously.

2.Despite considering factors like precipitation and ice dams, the model may still be sensitive to changes in other environmental conditions.

Overall, the model in this paper demonstrates strengths in a comprehensive approach, integration of advanced technologies, and stakeholder-centric design. However, challenges related to complexity, computational intensity, and sensitivity to environmental changes must be carefully considered and addressed to enhance its practical effectiveness.

Memo

Dear IJC leaders,

I am excited to present our comprehensive control model for managing the inflow and outflow of the Great Lakes, addressing the crucial task of maintaining optimal water levels to satisfy diverse stakeholders. The Great Lakes, being the largest group of freshwater lakes globally, play a vital role in various sectors, necessitating effective water resource management.

The purpose of this memo is to outline the construction and application of a control model to regulate the inflow and outflow of the Great Lakes, ensuring stakeholders' satisfaction. In addition to the model tasks discussed earlier, we have incorporated historic hydrologic, water level, and water use data to enhance the robustness of our approach.

First and foremost, we developed the Great Lakes Network Flow Model, we incorporated Historic Hydrologic Data, including evaporation, precipitation, and runoff for all Lakes. By incorporating Historic Water Use Data, such as withdrawals, diversions, and consumption, This comprehensive dataset allowed us to refine our multi-objective optimization model, ensuring a more accurate representation of stakeholder needs.

Moving on, we implemented the Lake Routing Rules and established a direct relationship between river flow and lake level, we integrated Historic Water Levels and Hydrologic Data. This addition enriched our Model Predictive Control (MPC) algorithm, providing a more nuanced understanding of the factors influencing water levels.

In addition, involved adjusting the discharge of control dams to analyze the sensitivity of the flow control algorithm. We gained valuable insights into the algorithm's response to variations in water use, further strengthening our mixed benefit scoring model. It achieved a high score of 94.78.

As our fourth task, we utilized the Monte Carlo stochastic simulation algorithm to assess environmental conditions, and We used the Frank-Copula function to verify the sensitivity of environmental conditions to the algorithm. We integrated Historic Hydrologic Data. This addition facilitated a more comprehensive analysis of the algorithm's sensitivity to factors like precipitation, winter snow accumulation, and ice jams.

Lastly, we focused exclusively on Lake Ontario, emphasizing the Ottawa River and the Port of Montreal. Here, we utilized Historic Water Levels and Hydrologic Data to refine our model, ensuring a more accurate representation of past conditions. Recalculating the model with this historical context led to more stable water levels and achieved a remarkable stakeholder satisfaction score of 96.27.

In conclusion, our control model offers a robust solution for managing the Great Lakes' water levels, providing a scientific basis for decision-making. The integration of historic hydrologic, water level, and water use data enhances the accuracy and applicability of our model.

I look forward to discussing our model further and exploring its alignment with the strategic goals of our organization.

Thank you for considering our work. Please feel free to reach out for any additional information or clarification.

Yours Sincerely,

Team #2425559

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Appendices

Algorithm 1 Monte Carlo Simulation

Input: Uncertain parameters $\theta_1, \theta_2, \ldots, \theta_n$
Output: Highest water level $H_{\max}$
function MONTECARLO($\theta_1, \theta_2, \ldots, \theta_n$)
    $N \leftarrow$ Number of Monte Carlo samples
    $H_{\max} \leftarrow 0$
    for $i \leftarrow 1$ to N **do**
        Sample $\theta_1^*, \theta_2^*, \ldots, \theta_n^*$ from probability distributions
        Simulate water level H_i based on sampled parameters
        if $H_i > H_{\max}$ **then**
            $H_{\max} \leftarrow H_i$
        end if
    end for
    return $H_{\max}$
end function

Algorithm 2 Frank-Copula Algorithm

Input: Marginal distributions $F_1, F_2, \ldots, F_n$, Copula parameter θ
Output: Joint distribution F_{joint}
function FRANKCOPULA($F_1, F_2, \ldots, F_n, \theta$)
    Generate random samples $u_1, u_2, \ldots, u_n$ from uniform distribution
    Transform u_i to v_i using inverse transform method with marginal distribution F_i
    Calculate copula function $C_\theta(v_1, v_2, \ldots, v_n) = \exp\left(-\frac{1}{\theta}\left(\sum_{i=1}^{n}\left(\frac{\exp(-\theta v_i)-1}{\exp(-\theta)-1}\right)\right)\right)$
    return Joint distribution F_{joint}
end function