The goal of this assignment is to perform an inverse dynamic analysis of an inverse slider crank.

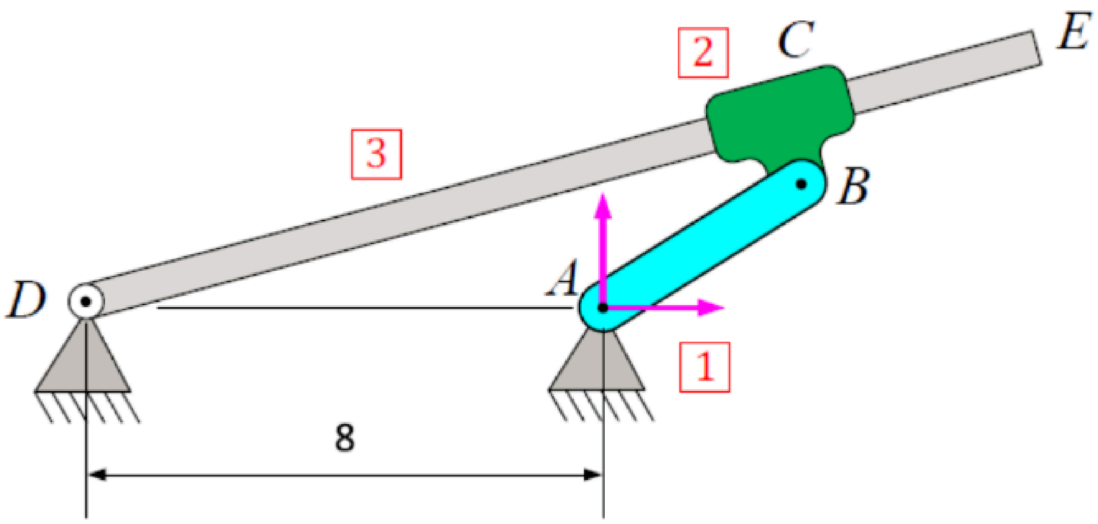


Figure 1: Inverse Slider Crank

Use the following masses:

$$M_1 = 40\,\mathrm{kg}, \quad M_2 = 20\,\mathrm{kg}, \quad M_3 = 160\,\mathrm{kg}$$

In detail, you will need to:

1. Perform a full kinematic analysis like in Assignment 1. (Once again use a driver constraint such that body 1 rotates at $\dot{\phi} = 1.5 \frac{\mathrm{rad}}{\mathrm{s}}$).
2. Determine the maximum value of the acceleration in point E, $|\ddot{r}_E|$.
3. Using inverse dynamics, solve for and plot the torque needed for the driving constraint (motor torque on body 1).
4. Plot the joint reaction forces and torques in the translational joint on body 2. Use the body 2 reference frame as the joint reference frame for the translational joint. Also plot the same reaction forces and torques in the absolute frame.

1.

In this section the kinematic analysis will be performed. This section has some overlap with assignment 1, which has been repeated here.

1.1 DoF for the mechanism

There are 3 bodies in the mechanism with each 3 DoF. There are also 4 joints: 3 revolute joints and 1 translational joint. There is also 1 driver with constant rotational speed.

$\mathrm{DoF} = \mathrm{nb} \cdot 3 - \mathrm{nh} = 3 \cdot 3 - (4 \cdot 2 + 1 \cdot 1) = 0$

1.2 The Joints:

To describe the joints A and r are defined:

$$\boldsymbol{A}(\phi) = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \in \mathbb{R}^{2\text{x}2},\ \boldsymbol{r^p} = \boldsymbol{r} + \boldsymbol{A}(\phi)\boldsymbol{s'^P} \in \mathbb{R}^2$$

$$\boldsymbol{q} = \begin{bmatrix} x_1 & y_1 & \phi_1 & x_2 & y_2 & \phi_2 & x_3 & y_3 & \phi_3 \end{bmatrix}^T \in \mathbb{R}^9$$

Joints A and D

The joints between the bodies 1, 3 and ground at point A and D are both absolute position joints and are described by the following equations:

$$\boldsymbol{\Phi}^{\mathbf{absA}}(\boldsymbol{q}) = \begin{bmatrix} \boldsymbol{r_1} + \boldsymbol{A}\mathbf{s}_{\mathbf{1}}'^{\boldsymbol{PA}} - \boldsymbol{C_1} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{\Phi}^{\mathbf{absD}}(\boldsymbol{q}) = \begin{bmatrix} \boldsymbol{r_3} + \boldsymbol{A}\mathbf{s}_{\mathbf{3}}'^{\boldsymbol{PD}} - \boldsymbol{C_3} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The jacobian entries of the absolute position joint are in general:

$$\boldsymbol{\Phi}^{\text{abs(i)}} = \begin{bmatrix} I_{2\text{x}2} & \boldsymbol{B}_i \mathbf{s}_i'^{P} \end{bmatrix}$$

And for the joints A and D:

$$\boldsymbol{\Phi}^{\text{absA}} = \begin{bmatrix} I_{2\text{x}2} & \boldsymbol{B}_1 \mathbf{s}_1'^{PA} \end{bmatrix}$$

$$\boldsymbol{\Phi}^{\text{absD}} = \begin{bmatrix} I_{2\text{x}2} & \boldsymbol{B}_3 \mathbf{s}_3'^{PD} \end{bmatrix}$$

Joint B

The joint B between body 1 and 2 at point B is a revolute joint is described by the following equations:

$$\boldsymbol{\Phi}^{\mathbf{revB}}(\boldsymbol{q}) = \begin{bmatrix} \boldsymbol{r_1} + \boldsymbol{A}\mathbf{s}_{\mathbf{1}}'^{\boldsymbol{PB}} - \left(\boldsymbol{r_2} + \boldsymbol{A}\mathbf{s}_{\mathbf{2}}'^{\boldsymbol{PB}} \right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The jacobian entries for this joint is

And for the joints A and D:

$$\boldsymbol{\Phi_q}^{\text{revB}} = \begin{bmatrix} I_{2\text{x}2} & \boldsymbol{B}_1 \mathbf{s}_1'^{PB} & -I_{2\text{x}2} & -\boldsymbol{B}_2 \mathbf{s}_2'^{PB} \end{bmatrix}$$

Joint C

The joint C between at point B is a revolute joint are both absolute position joints and are in general described by the following equations:

$$\boldsymbol{\Phi}^{t(i,j)}(\boldsymbol{q}) = \begin{bmatrix} (\boldsymbol{v}_i^{\perp})\boldsymbol{d}_{\text{ij}} \\ (\boldsymbol{v}_i^{\perp})^T \boldsymbol{v}_j \end{bmatrix} = \begin{bmatrix} \boldsymbol{v}_i'^T \boldsymbol{B}_i^T (\boldsymbol{r}_j - \boldsymbol{r}_i) - \boldsymbol{v}_i'^T \boldsymbol{B}_{\text{ij}} \boldsymbol{s}_j'^p - \boldsymbol{v}_i'^T \boldsymbol{R}^T \boldsymbol{s}_i'^P \\ -\boldsymbol{v}_i'^T \boldsymbol{B}_{\text{ij}} \boldsymbol{v}_j' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

And for this joint C:

$$\boldsymbol{\Phi}^{\text{C,t}(2,3)}(\boldsymbol{q}) = \begin{bmatrix} (\boldsymbol{v}_2^{\perp})\boldsymbol{d}_{23} \\ (\boldsymbol{v}_2^{\perp})^T \boldsymbol{v}_3 \end{bmatrix} = \begin{bmatrix} \boldsymbol{v}_2'^T \boldsymbol{B}_2^T (\boldsymbol{r}_3 - \boldsymbol{r}_2) - \boldsymbol{v}_2'^T \boldsymbol{B}_{23} \boldsymbol{s}_3'^{\text{pC}} - \boldsymbol{v}_2'^T \boldsymbol{R}^T \boldsymbol{s}_2'^{\text{pC}} \\ -\boldsymbol{v}_2'^T \boldsymbol{B}_{23} \boldsymbol{v}_3' \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

The Constraint jacobian entry for body i and j respectively then becomes:

$$\boldsymbol{\Phi}_{\boldsymbol{q}_i}^{\;t(i,j)} = \begin{bmatrix} -\boldsymbol{v}_i'^T \boldsymbol{B}_i^T & -\boldsymbol{v}_i'^T \boldsymbol{A}_i^T (\boldsymbol{r}_j - \boldsymbol{r}_i) - \boldsymbol{v}_i'^T \boldsymbol{A}_{\text{ij}} \boldsymbol{s}_j'^p \\ 0 & -\boldsymbol{v}_i'^T \boldsymbol{A}_{\text{ij}} \boldsymbol{v}_j' \end{bmatrix}$$

$$\boldsymbol{\Phi}_{\boldsymbol{q}_j}^{\;t(i,j)} = \begin{bmatrix} \boldsymbol{v}_i'^T \boldsymbol{B}_i^T & \boldsymbol{v}_i'^T \boldsymbol{A}_{\text{ij}} \boldsymbol{s}_j'^p \\ 0 & \boldsymbol{v}_i'^T \boldsymbol{A}_{\text{ij}} \boldsymbol{v}_j' \end{bmatrix}$$

The constraint jacobian for the translational joint C then becomes:

$$\boldsymbol{\Phi}_{\boldsymbol{q}}^{\;\text{C,t}(2,3)} = \begin{bmatrix} \boldsymbol{v}_2'^T \boldsymbol{B}_2^T & \boldsymbol{v}_2'^T \boldsymbol{A}_{23} \boldsymbol{s}_3'^{\text{pC}} & -\boldsymbol{v}_2'^T \boldsymbol{B}_2^T & -\boldsymbol{v}_2'^T \boldsymbol{A}_2^T (\boldsymbol{r}_3 - \boldsymbol{r}_2) - \boldsymbol{v}_2'^T \boldsymbol{A}_{23} \boldsymbol{s}_3'^{\text{pC}} \\ 0 & \boldsymbol{v}_2'^T \boldsymbol{A}_{23} \boldsymbol{v}_3' & 0 & -\boldsymbol{v}_2'^T \boldsymbol{A}_{23} \boldsymbol{v}_3' \end{bmatrix}$$

The driving constraint

The driving constraint is defined as:

$$\Phi^D(q,t) = \phi_1 - \omega t, \quad \omega = 1.5 \frac{\text{rad}}{\text{s}}$$

The jacobian entry is

$$\Phi^D(q,t) = I_{1\text{x}1}$$

1.2 The constraint equations $\boldsymbol{\Phi}$

The constraint equations are all the joints and the driver:

$$\boldsymbol{\Phi}(\boldsymbol{q},t) = \begin{bmatrix} \boldsymbol{r_1} + \boldsymbol{A}\mathbf{s}_1'^{\boldsymbol{PA}} - \boldsymbol{C_1} \\ \boldsymbol{r_3} + \boldsymbol{A}\mathbf{s}_3'^{\boldsymbol{PD}} - \boldsymbol{C_3} \\ \boldsymbol{r_1} + \boldsymbol{A}\mathbf{s}_1'^{\boldsymbol{PB}} - \left(\boldsymbol{r_2} + \boldsymbol{A}\mathbf{s}_2'^{\boldsymbol{PB}}\right) - \boldsymbol{C_2} \\ \boldsymbol{v}_2'^T \boldsymbol{B}_2^T (\boldsymbol{r}_3 - \boldsymbol{r}_2) - \boldsymbol{v}_2'^T \boldsymbol{B}_{23} \boldsymbol{s}_3'^{\text{pC}} - \boldsymbol{v}_2'^T \boldsymbol{R}^T \boldsymbol{s}_2'^{\text{pC}} \\ \phi_1 - \omega t \end{bmatrix}$$

1.3 The constraint jacobian $\boldsymbol{\Phi}_{\boldsymbol{q}}$

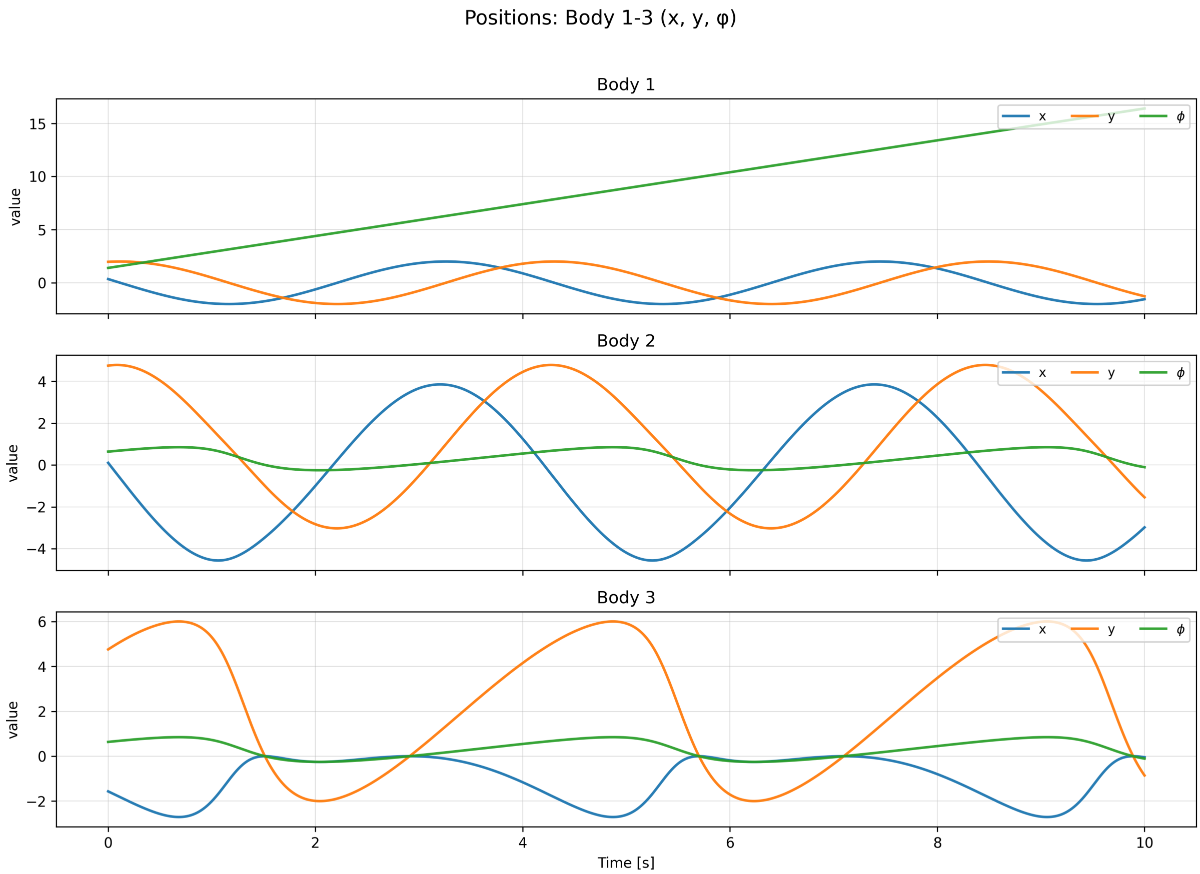
The constraint jacobian can now be assembled using the jacobian entries from each joint:

$$\boldsymbol{\Phi}_{\boldsymbol{q}} = \begin{bmatrix} \boldsymbol{\Phi}^{\text{absA}} & 0_{2\text{x}3} & 0_{2\text{x}3} \\ \boldsymbol{\Phi}^{\text{revB}} & -\boldsymbol{\Phi}^{\text{revB}} & 0_{2\text{x}3} \\ 0_{2\text{x}3} & -\boldsymbol{\Phi}_{\boldsymbol{q}}^{\;\text{C,t}(2,3)} & \boldsymbol{\Phi}_{\boldsymbol{q}}^{\;\text{C,t}(2,3)} \\ 0_{2\text{x}3} & 0_{2\text{x}3} & \boldsymbol{\Phi}^{\text{absD}} \\ [0,0,1] & 0_{1\text{x}3} & 0_{1\text{x}3} \end{bmatrix}$$

1.4 Position analysis

To get the positions for the mechanism at each time instance the constraint jacobian $\boldsymbol{\Phi}_{\boldsymbol{q}}$ is used in conjunction with the constraint equations $\boldsymbol{\Phi}$ and then finding the roots using the newton-rhapson algorithm.

The positions of each body is plotted in the figure below.



1.5 Velocity analysis

The velocity equation is defined as:

$$\boldsymbol{\nu} = -\boldsymbol{\Phi}_t = \boldsymbol{\Phi}_{\boldsymbol{q}}\dot{\boldsymbol{q}}, \quad \text{where } \dot{\boldsymbol{q}} = \begin{bmatrix} \dot{r} \\ \dot{\phi} \end{bmatrix}$$

as only $\boldsymbol{\Phi}^D$ is a function of time $(\phi_1 - \omega t)$ the vector $-\boldsymbol{\Phi}_t$ becomes:

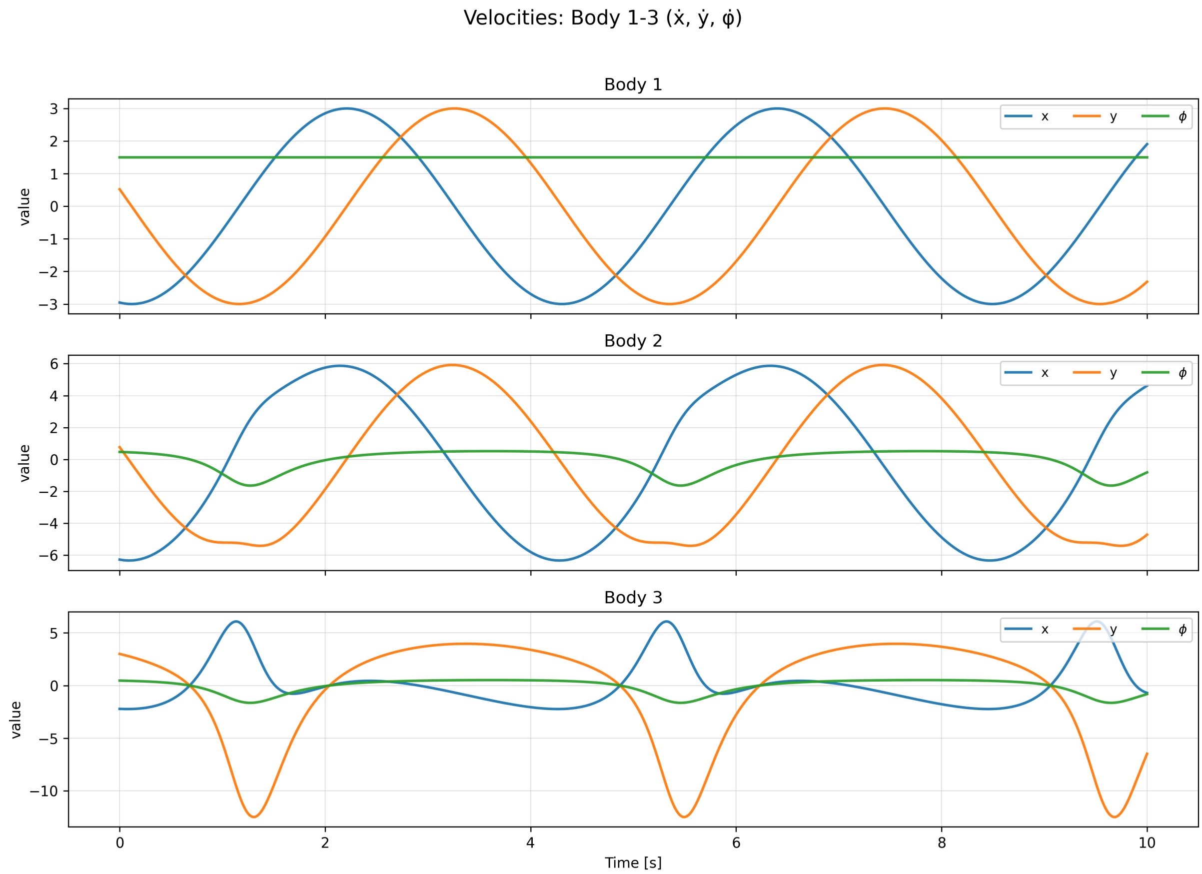
$$\boldsymbol{\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & w \end{bmatrix}^T = \boldsymbol{\Phi}_{\boldsymbol{q}}\dot{\boldsymbol{q}}$$

By left multiplying with ${\boldsymbol{\Phi}_{\boldsymbol{q}}}^{-1}$ the equation can be rewritten as to obtain $\dot{\boldsymbol{q}}$:

$$\dot{\boldsymbol{q}} = {\boldsymbol{\Phi}_{\boldsymbol{q}}}^{-\mathbf{1}}\boldsymbol{\nu}$$

for which the velocities can be calculated for each time instance.

The velocities of each body is plotted in the figure below.



1.6 Acceleration analysis

The acceleration equations are defined as:

$$\gamma = \mathbf{\Phi}_{\boldsymbol{q}}\ddot{\boldsymbol{q}} = -\left(\mathbf{\Phi}_{\boldsymbol{q}}\dot{\boldsymbol{q}}\right)_{\boldsymbol{q}}\dot{\boldsymbol{q}} - 2\mathbf{\Phi}_{\boldsymbol{q}t}\dot{\boldsymbol{q}} - \mathbf{\Phi}_{\text{tt}}, \quad \text{where } \ddot{\boldsymbol{q}} = \begin{bmatrix} \ddot{r} \\ \ddot{\phi} \end{bmatrix}$$

As $\mathbf{\Phi}_{\boldsymbol{q}}$ does not contain t, $\mathbf{\Phi}_{\boldsymbol{q}\text{t}} = \left[\vec{\mathbf{0}}\right]$ and as only $\mathbf{\Phi}^D$ is dependant on time:

$$\mathbf{\Phi}_{\text{tt}} = \frac{\partial^2 \mathbf{\Phi}^D}{\partial^2 t} = \frac{\partial \mathbf{\Phi}_t^D}{\partial t} = \left[\vec{0}\right]$$

The absolute position joints and relative rotation joint has been described in assignment 1 so the only new joint type is the translational joint for which γ is:

$$\gamma^{t(i,j)} = -\begin{bmatrix} \boldsymbol{v}_i'^T\left[\boldsymbol{B}_{\text{ij}}\boldsymbol{s}_j'^p\left(\dot{\boldsymbol{\phi}}_{\boldsymbol{j}} - \dot{\boldsymbol{\phi}}_{\boldsymbol{i}}\right)^2 - \boldsymbol{B}_i^T\left(\dot{\boldsymbol{r}}_{\boldsymbol{j}} - \dot{\boldsymbol{r}}_{\boldsymbol{i}}\right)\dot{\boldsymbol{\phi}}_i^2 - 2\boldsymbol{A}_i^T\left(\dot{\boldsymbol{r}}_{\boldsymbol{j}} - \dot{\boldsymbol{r}}_{\boldsymbol{i}}\right)\dot{\boldsymbol{\phi}}_i\right] \\ 0 \end{bmatrix}$$

Please note that this is from Haug's book 2nd edition below equation 3.3.14 where the parenthesis error is fixed.

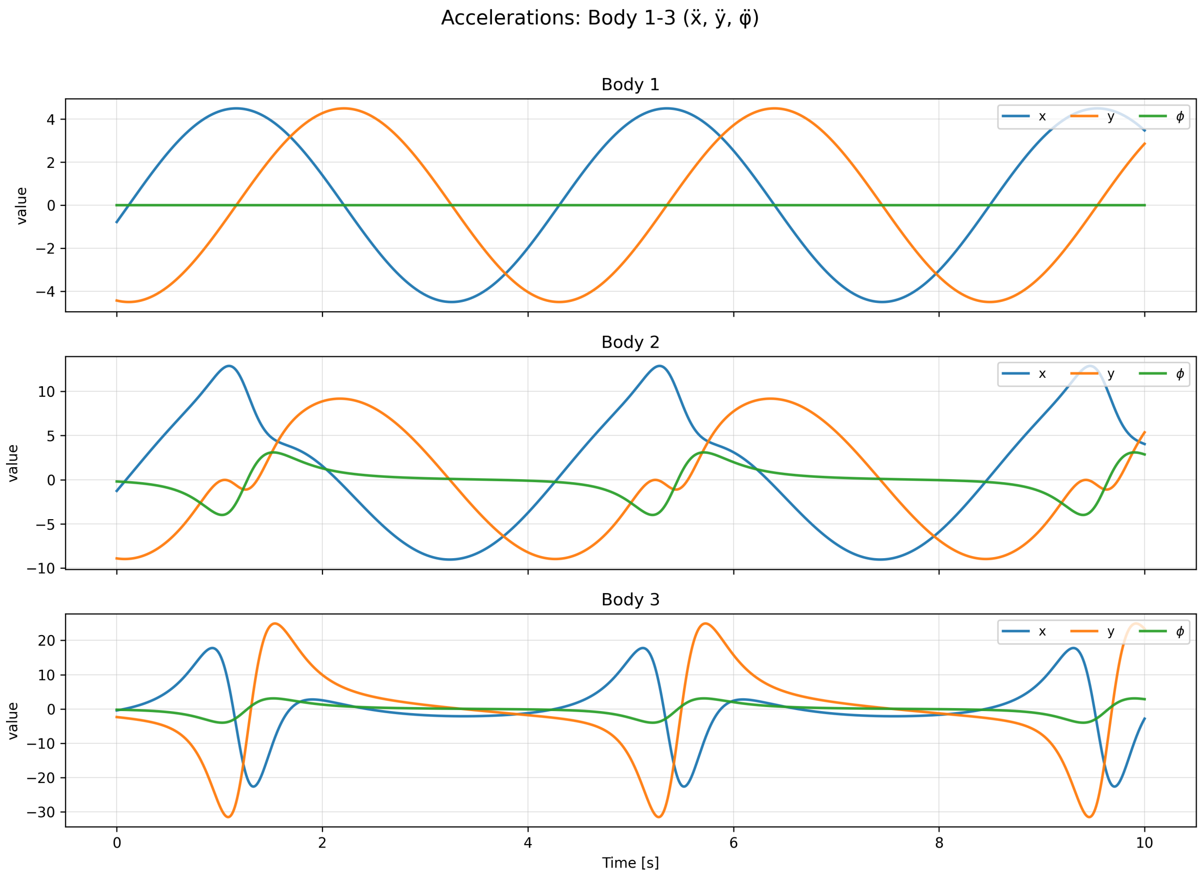
The expression for γ then becomes: $\gamma = \mathbf{\Phi}_{\boldsymbol{q}}\ddot{\boldsymbol{q}} = -\left(\mathbf{\Phi}_{\boldsymbol{q}}\dot{\boldsymbol{q}}\right)_{\boldsymbol{q}}\dot{\boldsymbol{q}}$, where when solving for $\ddot{\boldsymbol{q}}$:

$$\ddot{\boldsymbol{q}} = \mathbf{\Phi}_{\boldsymbol{q}}^{-1}\left(-\left(\mathbf{\Phi}_{\boldsymbol{q}}\dot{\boldsymbol{q}}\right)_{\boldsymbol{q}}\dot{\boldsymbol{q}}\right)$$

γ for this system then becomes:

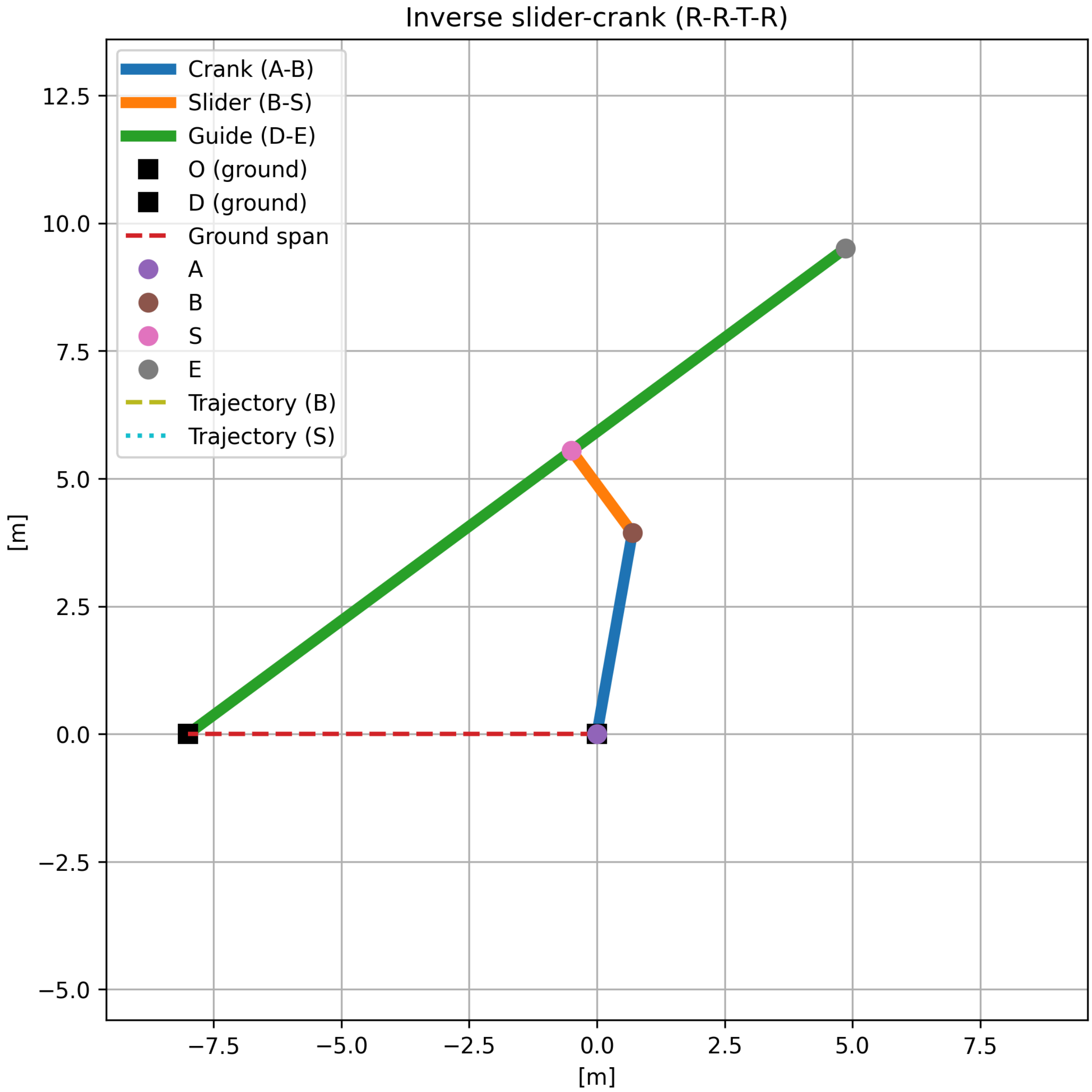
$$\gamma = \begin{bmatrix} \dot{\phi}_1^2 \boldsymbol{A}_1 \boldsymbol{s}'^A \\ \dot{\phi}_1^2 \boldsymbol{A}_1 \boldsymbol{s}'^B - \dot{\phi}_2^2 \boldsymbol{A}_2 \boldsymbol{s}'^B \\ -\boldsymbol{v}_2'^T \left[\boldsymbol{B}_{23} \boldsymbol{s}_3'^p \left(\dot{\boldsymbol{\phi}}_{\boldsymbol{3}} - \dot{\boldsymbol{\phi}}_{\boldsymbol{2}} \right)^2 - \boldsymbol{B}_2^T (\dot{\boldsymbol{r}}_{\boldsymbol{3}} - \dot{\boldsymbol{r}}_{\boldsymbol{2}}) \dot{\boldsymbol{\phi}}_2^2 - 2\boldsymbol{A}_2^T (\dot{\boldsymbol{r}}_{\boldsymbol{3}} - \dot{\boldsymbol{r}}_{\boldsymbol{2}}) \dot{\boldsymbol{\phi}}_2 \right] \\ 0 \\ \dot{\phi}_3^2 \boldsymbol{A}_3 \boldsymbol{s}'^D \\ 0 \end{bmatrix}$$

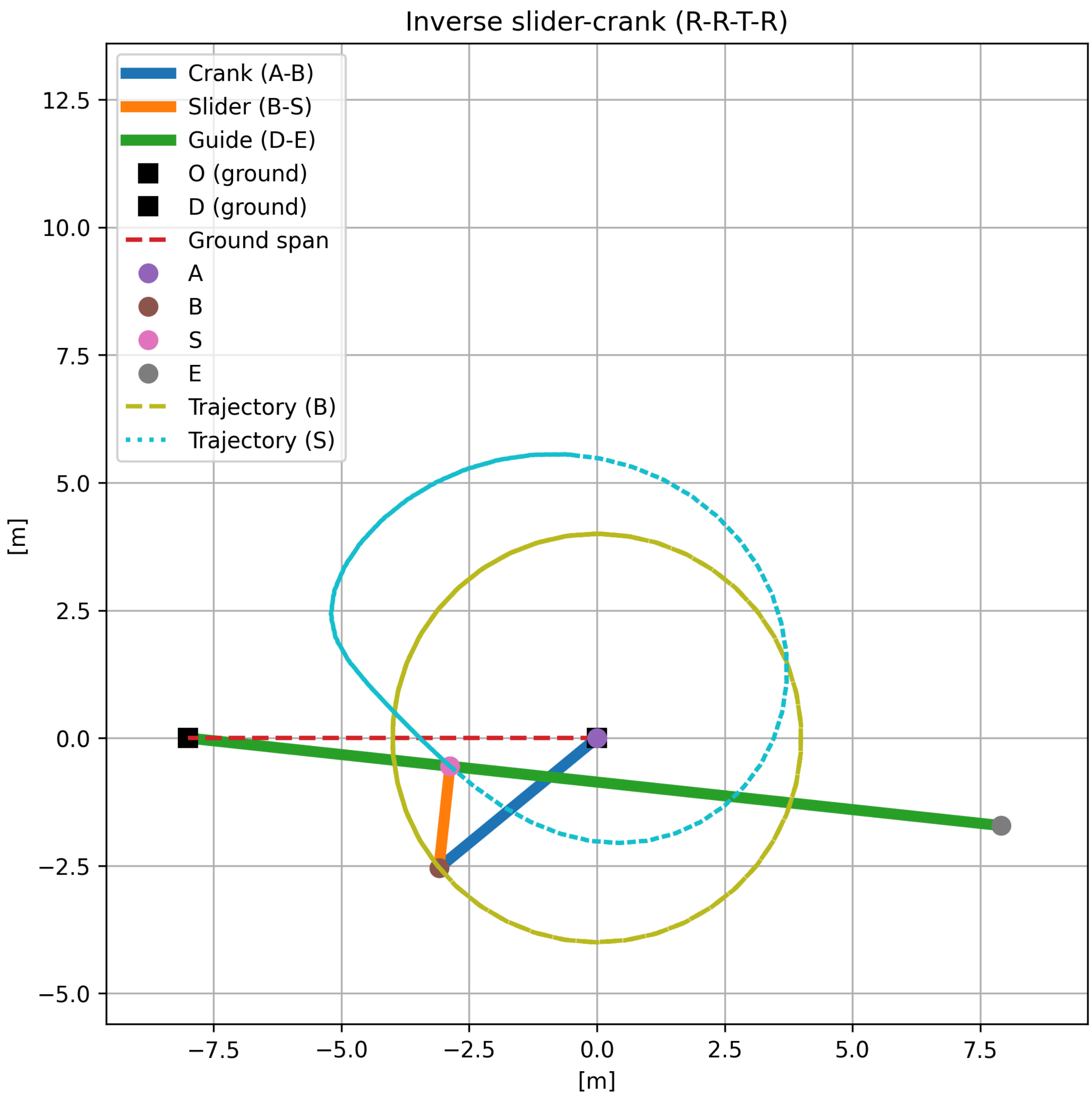
The accelerations of each body is plotted in the figure below.



1.7 The mechanism and trajectory plotted

The trajectories of the mechanism can be seen in the figure.


Inverse slider-crank (R-R-T-R)
Crank (A-B)
Slider (B-S)
Guide (D-E)
O (ground)
D (ground)
Ground span
A
B
S
E
Trajectory (B)
Trajectory (S)
[m]
[m]
12.5
10.0
7.5
5.0
2.5
0.0
−2.5
−5.0
−7.5
−5.0
−2.5
0.0
2.5
5.0
7.5



2. Accelerations at point E

In this section the accelerations at point E will be calculated.

2.1 Describing the accelerations at point E

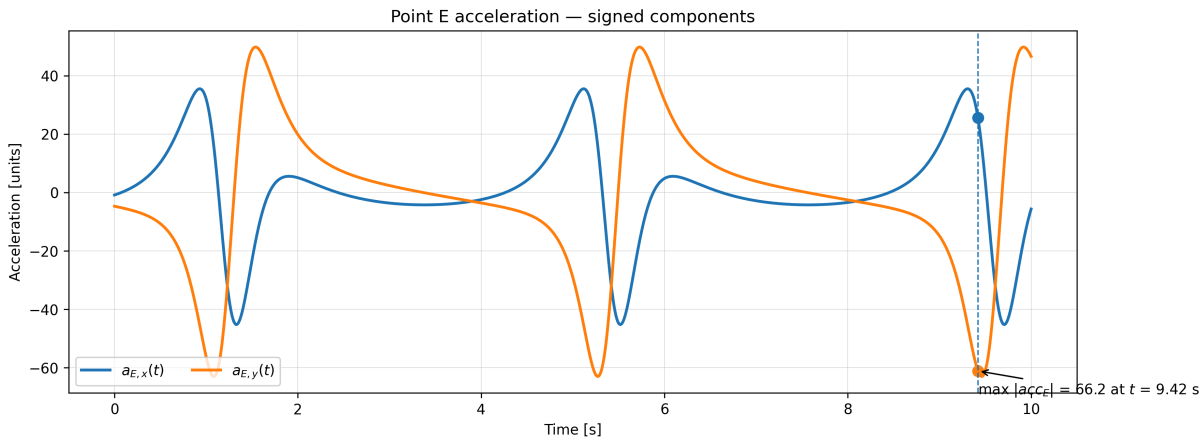
The accelerations at point E can be calculated by using the following equation:

$$\ddot{\boldsymbol{r}}^E = \ddot{\boldsymbol{r}}_3 + \ddot{\phi}_3 \boldsymbol{B}_3 \boldsymbol{s}'^E - \dot{\phi}_3^2 \boldsymbol{A}_3 \boldsymbol{s}'^E$$

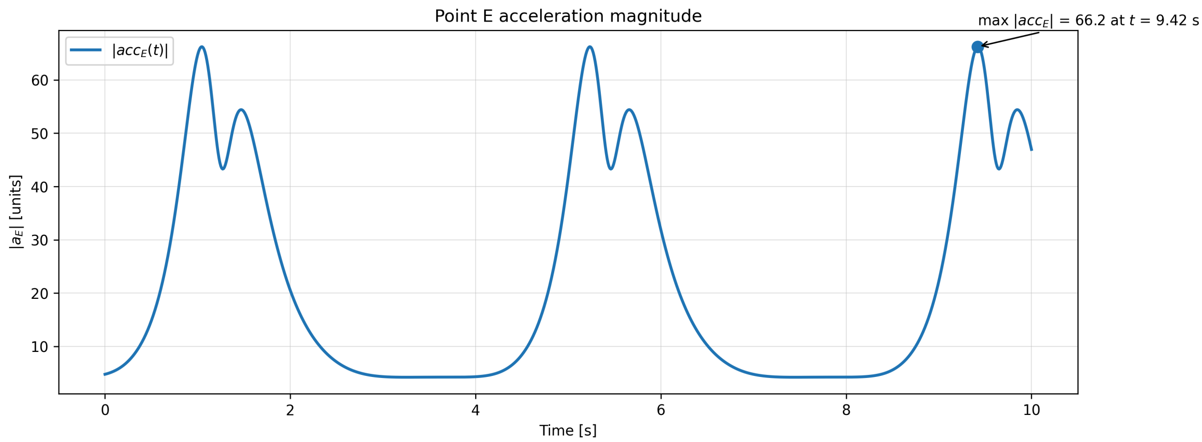
And as $\ddot{\boldsymbol{r}}_3, \ddot{\phi}_3, \dot{\phi}_3, \phi_3, \boldsymbol{s}'^E$ are calculated prior in section 1 then all the values of the variables to each timestep are known.

2.2 Plots of the accelerations at point E

The figure below shows each component of accelerations at point E.



The Figure below shows the magnitude of the accelerations at point E, where it is notable that the accelerations of about $66\frac{m}{s^2}$ is approximately 6.7 times larger than the acceleration due to gravitational forces. It happens approximately when the body 2 is closer to point D which makes sense as the prescribed movement of the driver will cause a larger acceleration at E when the mechanical advantage is higher.



3. Driver torque using inverse Dynamics

In this section the driver torque will be solved for using inverse dynamics and then plotted.

3.1 Equations of motion using DAE's

By using equation 6.3.18 in Haug's book (2nd ed.) the matrix form of the equation is:

$$\begin{bmatrix} \boldsymbol{M} & \boldsymbol{\Phi_q}^T \\ \boldsymbol{\Phi_q} & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \ddot{q} \\ \boldsymbol{\lambda} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Q}^A \\ \gamma \end{bmatrix}$$

where $\boldsymbol{M}$ is the positive definite mass matrix of the system, $\boldsymbol{\Phi_q}$ is the constraint jacobian, $\boldsymbol{\lambda}$ is the Lagrange multiplier (vector) for the system, $\boldsymbol{Q}^A$ is the generalized applied forces and γ used in the acceleration equation.

for the system the mass matrix will be a diagonal matrix of sizes $3 \cdot \text{nb} \times 3 \cdot \text{nb} = 9 \times 9$. For a single body it would be

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_1 & 0 \\ 0 & 0 & J' \end{bmatrix}$$

Where m_1 and J' describe the mass of the element and the local inertia of the body.

For the generalized applied forces: $\boldsymbol{Q}^A$ the only forces acting on the bodies will be gravity:

$$\boldsymbol{Q}^A = [0 \ \ -m_1 \cdot g \ \ 0 \ \ 0 \ \ -m_2 \cdot g \ \ 0 \ \ \ldots \ \ 0 \ \ -m_{\text{nb}} \cdot g \ \ 0]^T$$

3.2 Solving the equations of motion

To be able to solve the equations of motion the masses and moments of the bodies must be calculated.

In the description the masses are:

$$m_1 = 40 \text{ kg}, \ m_2 = 20 \text{ kg}, \ m_3 = 160 \text{ kg}$$

The bodies 1 and 3 are assumed to be approximately thin rods and body 2 a rectangle using the formulas in table 6.11 in Haug's book (2nd edition).

$$J'_{\text{thin rod}} = \frac{m}{12} l^2, \ J'_{\text{rectangle}} = \frac{1}{12} m(a^2 + b^2)$$

Body 2 is assumed per the hint in the description to have the width and length of 2 meters.

$$J'_1 = \frac{40 \text{ kg}}{12} \cdot (4m)^2 = 53.33 \text{ kg} \cdot m^2$$

$$J'_3 = \frac{160 \text{ kg}}{12} \cdot (16m)^2 = 3413 \text{ kg} \cdot m^2$$

$$J'_2 = \frac{1}{12} 20 \text{ kg} \cdot \left((2m)^2 + (2m)^2\right) = 13.33 \text{ kg} \cdot m^2$$

3.3 Solving reaction forces and torques

As the accelerations, constraint jacobian and $\boldsymbol{\gamma}$ is found in section 1 it is possible to solve for the Langrange multiplier $\boldsymbol{\lambda}$ using equation 6.3.16:

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + {\boldsymbol{\Phi}_{\boldsymbol{q}}}^T \boldsymbol{\lambda} = \boldsymbol{Q}^A \Leftrightarrow \boldsymbol{\lambda} = {\boldsymbol{\Phi}_{\boldsymbol{q}}}^{T^{-1}} \left(\boldsymbol{Q}^A - \boldsymbol{M}\ddot{\boldsymbol{q}}\right)$$

By also using equation 6.6.8 and 6.6.9 the reaction forces and the driver torque can be calculated:

$$\boldsymbol{F}_i''^k = -\boldsymbol{C}_i^T \boldsymbol{A}_i^T \boldsymbol{\Phi}_{r_i}^{k^T} \boldsymbol{\lambda}^k$$

$$\boldsymbol{T}_i''^k = \left(\boldsymbol{s}_i'^{P^T} \boldsymbol{B}_i^T \boldsymbol{\Phi}_{r_i}^{k^T} - \boldsymbol{\Phi}_{\phi_i}^{k^T}\right) \boldsymbol{\lambda}^k$$

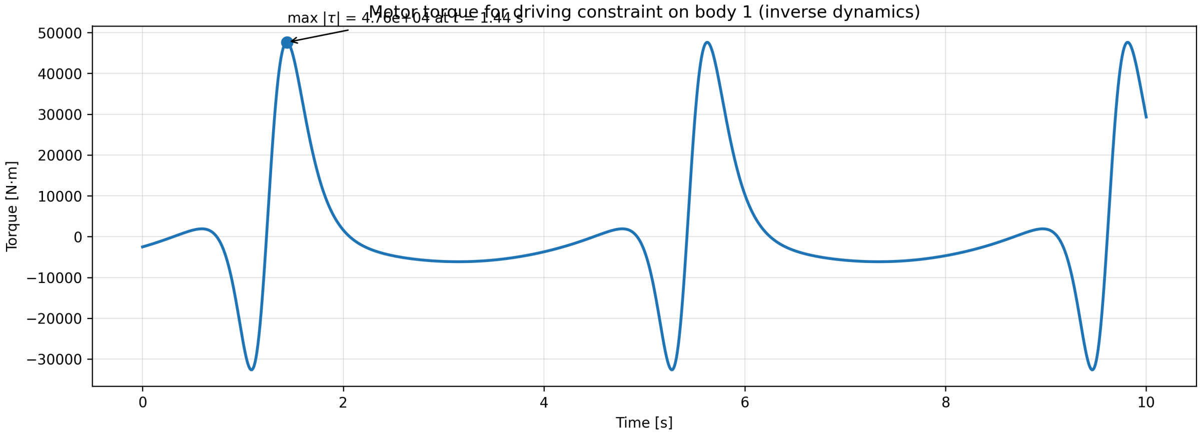
In this system with the driver at A:

$$\boldsymbol{T}_1''^A = \left(\boldsymbol{s}_1'^{A^T} \boldsymbol{B}_1^T \boldsymbol{\Phi}_{r_1}^{\text{driver}^T} - \boldsymbol{\Phi}_{\phi_1}^{\text{driver}^T}\right) \boldsymbol{\lambda}^{\text{driver}}$$

Where ${\boldsymbol{s}_1'}^{A^T} = [-2, 0]$, $\boldsymbol{B}_1^T = \begin{bmatrix} -\sin(\phi_1) & -\cos(\phi_1) \\ \cos(\phi_1) & -\sin(\phi_1) \end{bmatrix}$, $\boldsymbol{\Phi}_{r_1}^{\text{driver}} = [0, 0]$, $\boldsymbol{\Phi}_{\phi_1}^{\text{driver}} = [1]$

Therefore the $\boldsymbol{F}_i''^A = 0$ as it is multiplied by a zero-vector and $\boldsymbol{T}_1''^A = -\boldsymbol{\lambda}^{\text{driver}}$

The driver torque is plotted in the figure below:



4.

In this section the reaction forces of the translation joint on body 2 will be calculated

4.1

The equations for the reaction forces and torques are:

$$\boldsymbol{F}_i''^k = -\boldsymbol{C}_i^T \boldsymbol{A}_i^T \boldsymbol{\Phi}_{r_i}^{k^T} \boldsymbol{\lambda}^k$$

$$\boldsymbol{T}_i''^k = \left(\boldsymbol{s}_i'^{P^T} \boldsymbol{B}_i^T \boldsymbol{\Phi}_{r_i}^{k^T} - \boldsymbol{\Phi}_{\phi_i}^{k^T}\right) \boldsymbol{\lambda}^k$$

Which is in this system:

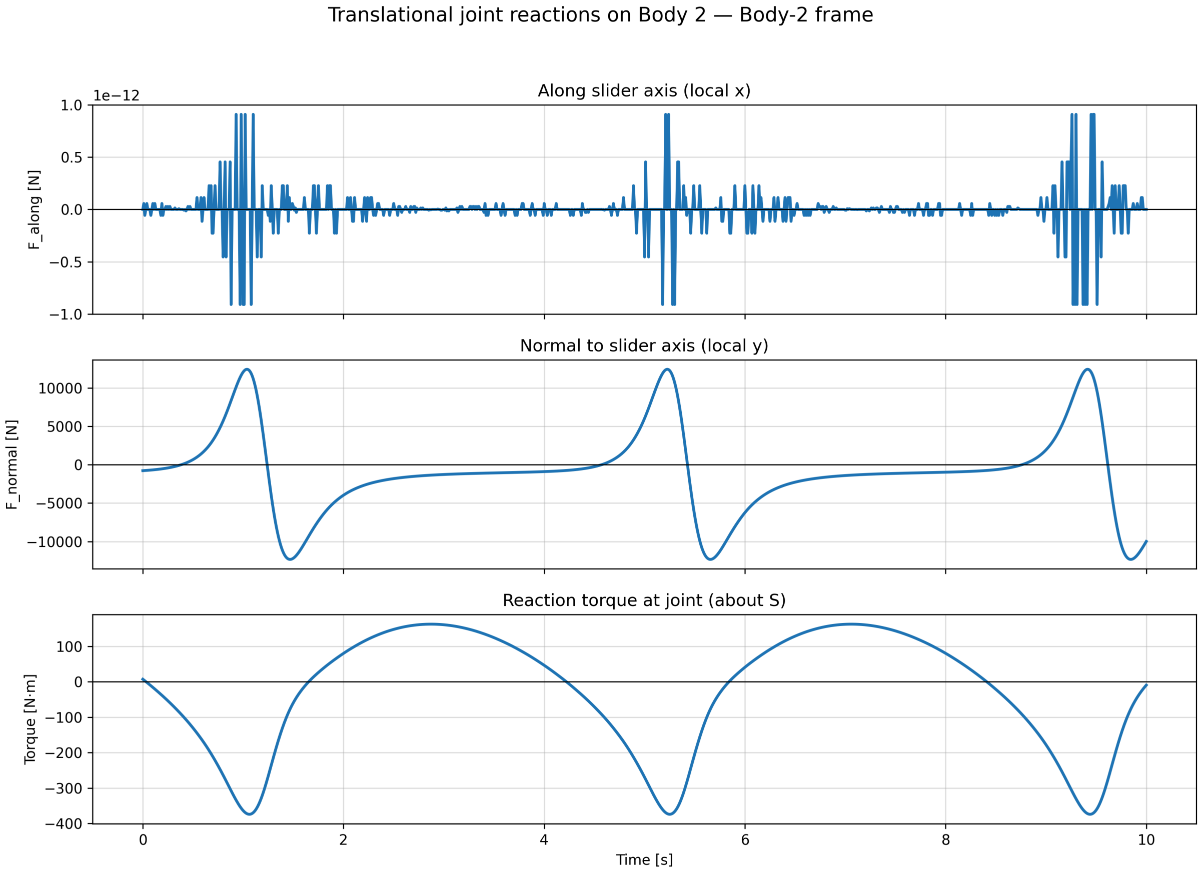
$$\boldsymbol{F}_2''^C = -\boldsymbol{C}_2^T \boldsymbol{A}_2^T \boldsymbol{\Phi}_{r_2}^{\text{trans}^T} \boldsymbol{\lambda}^{\text{trans}}$$

$$\boldsymbol{T}_2''^C = \left(\boldsymbol{s}_2'^{C^T} \boldsymbol{B}_2^T \boldsymbol{\Phi}_{r_2}^{\text{trans}^T} - \boldsymbol{\Phi}_{\phi_2}^{\text{trans}^T}\right) \boldsymbol{\lambda}^{\text{trans}}$$

where $\boldsymbol{s}_2^C = [0, 1], \boldsymbol{v}_2 = [1, 0], \boldsymbol{\Phi}_{r_2}^{\text{trans}} = \begin{bmatrix} -\boldsymbol{v}_2'^T \boldsymbol{B}_2^T \\ 0 \end{bmatrix}, \boldsymbol{\Phi}_{\phi_2}^{\text{trans}} = \begin{bmatrix} -\boldsymbol{v}_2'^T \boldsymbol{A}_2^T (\boldsymbol{r}_3 - \boldsymbol{r}_2) - \boldsymbol{v}_2'^T \boldsymbol{A}_{23} \boldsymbol{s}_3'^{\text{pC}} \\ -\boldsymbol{v}_2'^T \boldsymbol{A}_{23} \boldsymbol{v}_3' \end{bmatrix}$

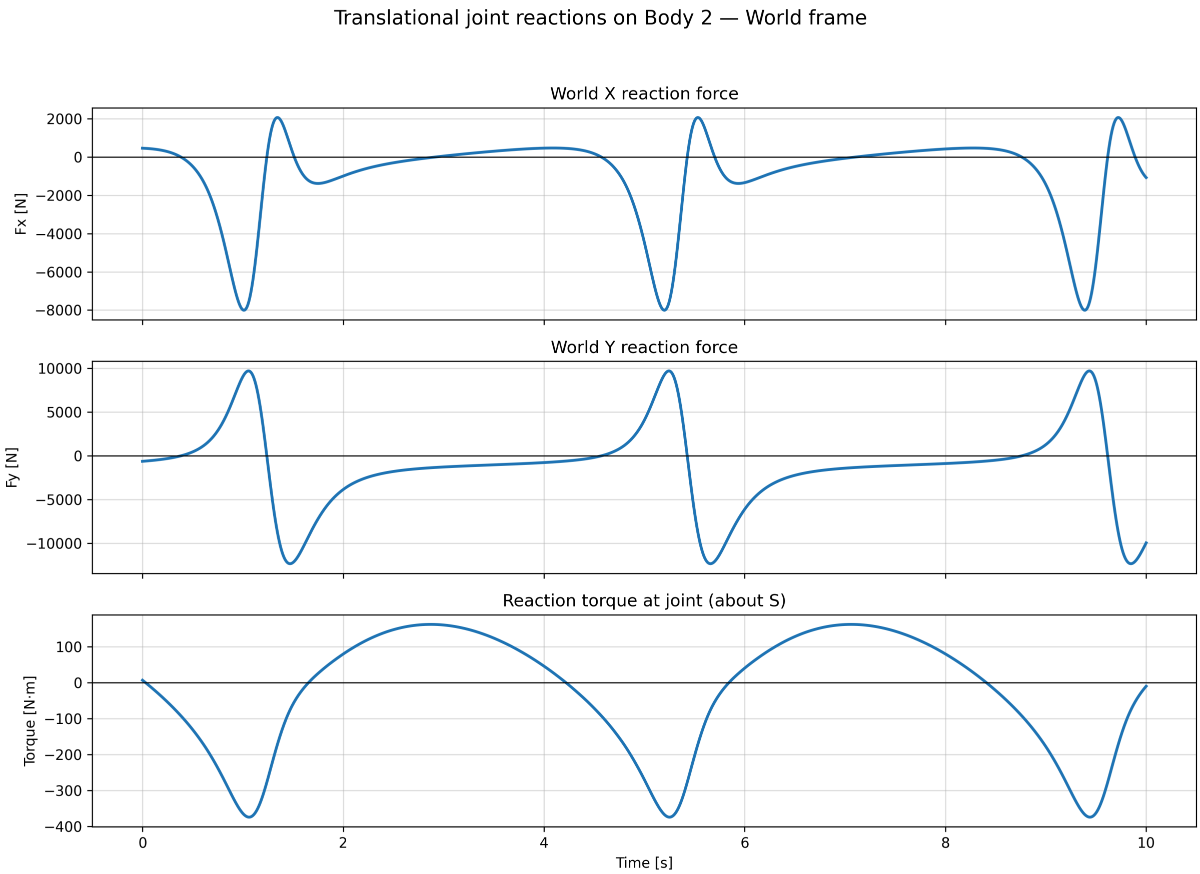
For the reaction forces and torques in the global from the

The joint reaction forces at the translational joint in body 2 is plotted in the figure below:



It can be seen from the plot above that the local x-components are in the order e-12 which is approximately 0 as the computations are numerical.

The joint reaction forces at the translational joint in the world frame is plotted in the figure below:


Translational joint reactions on Body 2 — World frame
World X reaction force
Fx [N]
2000
0
−2000
−4000
−6000
−8000
World Y reaction force
Fy [N]
10000
5000
0
−5000
−10000
Reaction torque at joint (about S)
Torque [N·m]
100
0
−100
−200
−300
−400
0
2
4
6
8
10
Time [s]