Assignment 1 - Computational Dynamics

Assignment 1 - 4 bar linkage

The goal of this assignment is to perform a full kinematic analysis of a four-bar linkage.

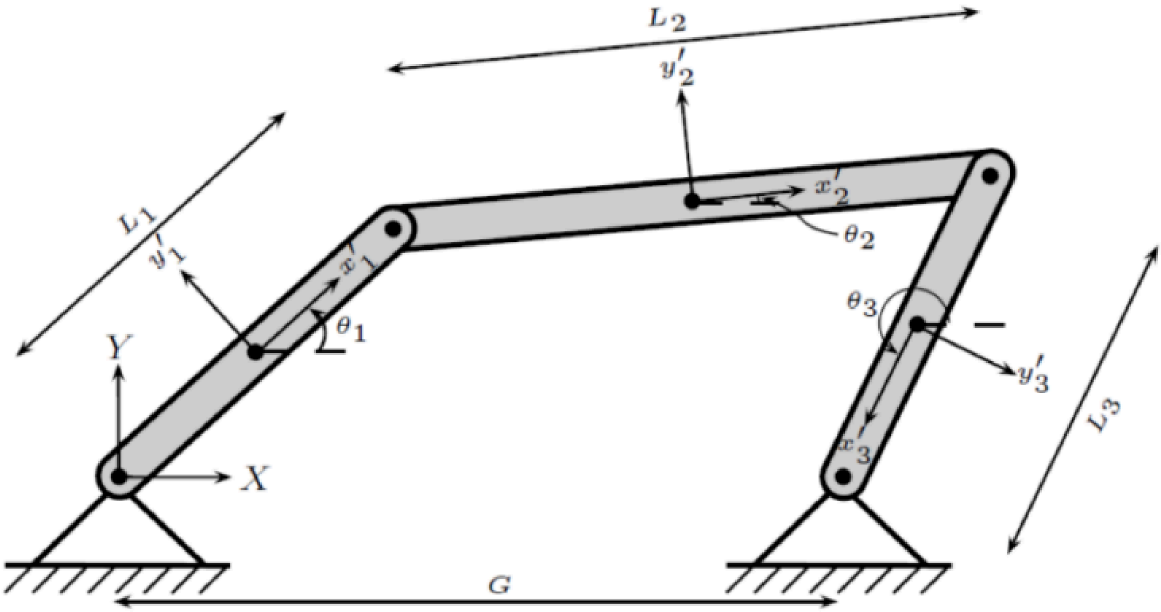


Figure 1: Four-bar linkage

Use the following lengths:

$$L_1 = 10m;\ L_2 = 26m;\ L_3 = 18m;\ G = 20m$$

Problem:

1. Identify the number of bodies, joints and degrees of freedom for the mechanism.

The number of bodies (nb) is 3 and there are 4 joints with 1 degree of freedom each (nh = 8).

Therefore there degrees of freedom for the model is $3 \cdot \text{nb} - \text{nh} = 1$

The remaining DoF will be governed by a driving constraint $\Phi^D(q,t) = \phi_1 - \omega t, \quad \omega = 1.5 \frac{\text{rad}}{\text{s}}$

2. Setup the kinematic constraints Φ^K for all the joints

First A and r are defined:

$$A(\phi) = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}, \quad r^p = r + A(\phi)s'^P, \quad q = \begin{bmatrix} x_1 & y_1 & \phi_1 & x_2 & y_2 & \phi_2 & x_3 & y_3 & \phi_3 \end{bmatrix}^T$$

The points for the joints are described by:

$$s_1'^{\text{p1}} = \begin{bmatrix} -\frac{L_1}{2} \\ 0 \end{bmatrix}, \quad s_1'^{\text{p2}} = \begin{bmatrix} \frac{L_1}{2} \\ 0 \end{bmatrix}, \quad s_2'^{\text{p2}} = \begin{bmatrix} -\frac{L_2}{2} \\ 0 \end{bmatrix}, \quad s_2'^{\text{p3}} = \begin{bmatrix} \frac{L_2}{2} \\ 0 \end{bmatrix}, \quad s_3'^{\text{p3}} = \begin{bmatrix} -\frac{L_3}{2} \\ 0 \end{bmatrix}, \quad s_3'^{\text{p4}} = \begin{bmatrix} \frac{L_3}{2} \\ 0 \end{bmatrix},$$

so that e.g. $s_1'^{\text{p2}}$ describes the local point p2 in body 1.

The coordinates for the C-vectors are:

$$\boldsymbol{C_1} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \boldsymbol{C_2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \boldsymbol{C_3} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \boldsymbol{C_4} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

And the coordinates for the $\boldsymbol{r}$-vectors are:

$$\boldsymbol{r_1} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}, \quad \boldsymbol{r_2} = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}, \quad \boldsymbol{r_3} = \begin{bmatrix} x_3 \\ y_3 \end{bmatrix}$$

The Kinematic constraint equations are as follows:

- $\Phi^{\text{abs1}}(q) = \left[\boldsymbol{r_1} + \boldsymbol{A}\boldsymbol{s}_{\boldsymbol{1}}'^{\boldsymbol{P1}} - \boldsymbol{C_1}\right] = \begin{bmatrix} x_1 - \frac{L_1}{2}\cos(\phi_1) \\ y_1 - \frac{L_1}{2}\sin(\phi_1) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- $\Phi^{\text{rel1}}(q) = \left[\boldsymbol{r_1} + \boldsymbol{A}\boldsymbol{s}_{\boldsymbol{1}}'^{\boldsymbol{P2}} - \left(\boldsymbol{r_2} + \boldsymbol{A}\boldsymbol{s}_{\boldsymbol{2}}'^{\boldsymbol{P2}}\right) - \boldsymbol{C_2}\right] = \begin{bmatrix} x_1 + \frac{L_1}{2}\cos(\phi_1) - \left(x_2 - \frac{L_2}{2}\cos(\phi_2)\right) \\ y_1 + \frac{L_1}{2}\sin(\phi_1) - \left(y_2 - \frac{L_2}{2}\sin(\phi_2)\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- $\Phi^{\text{rel2}}(q) = \left[\boldsymbol{r_2} + \boldsymbol{A}\boldsymbol{s}_{\boldsymbol{2}}'^{\boldsymbol{P3}} - \left(\boldsymbol{r_3} + \boldsymbol{A}\boldsymbol{s}_{\boldsymbol{3}}'^{\boldsymbol{P3}}\right) - \boldsymbol{C_3}\right] = \begin{bmatrix} x_2 + \frac{L_2}{2}\cos(\phi_2) - \left(x_3 - \frac{L_3}{2}\cos(\phi_3)\right) \\ y_2 + \frac{L_2}{2}\sin(\phi_2) - \left(y_3 - \frac{L_3}{2}\sin(\phi_3)\right) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- $\Phi^{\text{abs2}}(q) = \left[\boldsymbol{r_3} + \boldsymbol{A}\boldsymbol{s}_{\boldsymbol{3}}'^{\boldsymbol{P4}} - \boldsymbol{C_4}\right] = \begin{bmatrix} x_3 + \frac{L_3}{2}\cos(\phi_3) - 20 \\ y_3 + \frac{L_3}{2}\sin(\phi_3) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

As a system of constraints the vectorfunction the above equations become:

$$\Phi^K(q) = \begin{bmatrix} \Phi^{\text{abs1}}(q) \\ \Phi^{\text{rev1}}(q) \\ \Phi^{\text{rev2}}(q) \\ \Phi^{\text{abs2}}(q) \end{bmatrix} = \begin{bmatrix} x_1 - \frac{L_1}{2}\cos(\phi_1) \\ y_1 - \frac{L_1}{2}\sin(\phi_1) \\ x_1 + \frac{L_1}{2}\cos(\phi_1) - \left(x_2 - \frac{L_2}{2}\cos(\phi_2)\right) \\ y_1 + \frac{L_1}{2}\sin(\phi_1) - \left(y_2 - \frac{L_2}{2}\sin(\phi_2)\right) \\ x_2 + \frac{L_2}{2}\cos(\phi_2) - \left(x_3 - \frac{L_3}{2}\cos(\phi_3)\right) \\ y_2 + \frac{L_2}{2}\sin(\phi_2) - \left(y_3 - \frac{L_3}{2}\sin(\phi_3)\right) \\ x_3 + \frac{L_3}{2}\cos(\phi_3) - 20 \\ y_3 + \frac{L_3}{2}\sin(\phi_3) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = [\vec{o}]$$

3. Setup of the driving constraint Φ^D that imposes $\phi_1 = \omega t, \quad \omega = 1.5 \frac{\text{rad}}{\text{s}}$

As there is 1 DoF for the system, an absolute driving constraint is added to the system:

$\Phi^D(q, t) = [\phi_1 - \omega t] = 0$

4. Calculate the constraint jacobian Φ_q

First B is defined:

$B(\phi) = \begin{bmatrix} -\sin(\phi) & -\cos(\phi) \\ \cos(\phi) & -\sin(\phi) \end{bmatrix}$

by combining Φ^K and Φ^D into Φ and then taking the partial derivative with respect to q the constraint jacobian Φ_q can be obtained:

$$\boldsymbol{\Phi_q} = \frac{\partial \Phi}{\partial q} = \begin{bmatrix} \frac{\partial \Phi^K}{\partial q} \\ \frac{\partial \Phi^D}{\partial q} \end{bmatrix} = \begin{bmatrix} I_{2x2} & B_1 s_1'^{p1} & 0 & 0 & 0 & 0 \\ I_{2x2} & B_1 s_1'^{p2} & -I_{2x2} & -B_2 s_2'^{p2} & 0 & 0 \\ 0 & 0 & I_{2x2} & B_2 s_2'^{p3} & -I_{2x2} & -B_3 s_3'^{p3} \\ 0 & 0 & 0 & 0 & I_{2x2} & B_3 s_3'^{p4} \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & \frac{L_1}{2}\sin(\phi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{L_1}{2}\cos(\phi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -\frac{L_1}{2}\sin(\phi_1) & -1 & 0 & -\frac{L_2}{2}\sin(\phi_2) & 0 & 0 & 0 \\ 0 & 1 & \frac{L_1}{2}\cos(\phi_1) & 0 & -1 & \frac{L_2}{2}\cos(\phi_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -\frac{L_2}{2}\sin(\phi_2) & -1 & 0 & -\frac{L_3}{2}\sin(\phi_3) \\ 0 & 0 & 0 & 0 & 1 & \frac{L_2}{2}\cos(\phi_2) & 0 & -1 & \frac{L_3}{2}\cos(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\frac{L_3}{2}\sin(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{L_3}{2}\cos(\phi_3) \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

5. Setup of the velocity and acceleration equations ν and γ

First the equations for the velocities will be setup and then the accelerations.

Velocities:

The velocity equation is defined as:

$\boldsymbol{\nu} = -\boldsymbol{\Phi}_t = \boldsymbol{\Phi_q}\dot{\boldsymbol{q}},\quad$ where $\dot{\boldsymbol{q}} = \begin{bmatrix} \dot{x_1} & \dot{y_1} & \dot{\phi_1} & \dot{x_2} & \dot{y_2} & \dot{\phi_2} & \dot{x_3} & \dot{y_3} & \dot{\phi_3} \end{bmatrix}^T$

as only $\boldsymbol{\Phi}^D$ is a function of time $(\phi_1 - \omega t)$ the vector $-\boldsymbol{\Phi}_t$ becomes:

$\boldsymbol{\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & w \end{bmatrix}^T = \boldsymbol{\Phi_q}\dot{\boldsymbol{q}}$

By left multiplying with ${\boldsymbol{\Phi_q}}^{-1}$ the equation can be rewritten as to obtain $\dot{\boldsymbol{q}}$:

$\dot{\boldsymbol{q}} = {\boldsymbol{\Phi_q}}^{\boldsymbol{-1}}\boldsymbol{\nu}$

Which when written out in matrices is:

$$\begin{bmatrix} \dot{x_1} \\ \dot{y_1} \\ \dot{\phi_1} \\ \dot{x_2} \\ \dot{y_2} \\ \dot{\phi_2} \\ \dot{x_3} \\ \dot{y_3} \\ \dot{\phi_3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \frac{L_1}{2}\sin(\phi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{L_1}{2}\cos(\phi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -\frac{L_1}{2}\sin(\phi_1) & -1 & 0 & -\frac{L_2}{2}\sin(\phi_2) & 0 & 0 & 0 \\ 0 & 1 & \frac{L_1}{2}\cos(\phi_1) & 0 & -1 & \frac{L_2}{2}\cos(\phi_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -\frac{L_2}{2}\sin(\phi_2) & -1 & 0 & -\frac{L_3}{2}\sin(\phi_3) \\ 0 & 0 & 0 & 0 & 1 & \frac{L_2}{2}\cos(\phi_2) & 0 & -1 & \frac{L_3}{2}\cos(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\frac{L_3}{2}\sin(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{L_3}{2}\cos(\phi_3) \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ w \end{bmatrix}$$

for which the velocities can be calculated for each time instance.

Accelerations:

The acceleration equations are defined as:

$\gamma = \boldsymbol{\Phi_q}\ddot{\boldsymbol{q}} = -\left(\boldsymbol{\Phi_q}\dot{\boldsymbol{q}}\right)_{\boldsymbol{q}}\dot{\boldsymbol{q}} - 2\boldsymbol{\Phi}_{\boldsymbol{q}t}\dot{\boldsymbol{q}} - \boldsymbol{\Phi}_{\mathrm{tt}},\quad$ where $\ddot{\boldsymbol{q}} = \begin{bmatrix} \ddot{x_1} & \ddot{y_1} & \ddot{\phi_1} & \ddot{x_2} & \ddot{y_2} & \ddot{\phi_2} & \ddot{x_3} & \ddot{y_3} & \ddot{\phi_3} \end{bmatrix}^T$

As $\boldsymbol{\Phi_q}$ does not contain t, $\boldsymbol{\Phi}_{\boldsymbol{q}\mathrm{t}} = \begin{bmatrix} \vec{0} \end{bmatrix}$ and as only $\boldsymbol{\Phi}^D$ is dependant on time:

$\boldsymbol{\Phi}_{\mathrm{tt}} = \frac{\partial^2 \boldsymbol{\Phi}^D}{\partial^2 t} = \frac{\partial \boldsymbol{\Phi}_t^D}{\partial t} = \begin{bmatrix} \vec{0} \end{bmatrix}$

The expression for $\boldsymbol{\gamma}$ then becomes: $\boldsymbol{\gamma} = \boldsymbol{\Phi_q}\ddot{\boldsymbol{q}} = -\left(\boldsymbol{\Phi_q}\dot{\boldsymbol{q}}\right)_{\boldsymbol{q}}\dot{\boldsymbol{q}}$, where when solving for $\ddot{\boldsymbol{q}}$:

$$\ddot{\boldsymbol{q}} = \boldsymbol{\Phi_q}^{-1}\left(-\left(\boldsymbol{\Phi_q}\dot{\boldsymbol{q}}\right)_{\boldsymbol{q}}\dot{\boldsymbol{q}}\right)$$

Before $\ddot{\boldsymbol{q}}$ can be solved for the term $-\left(\boldsymbol{\Phi_q}\dot{\boldsymbol{q}}\right)_{\boldsymbol{q}}\dot{\boldsymbol{q}}$ has to be calculated in the order:

1. $\boldsymbol{\Phi_q}\dot{\boldsymbol{q}}$
2. $\left(\boldsymbol{\Phi_q}\dot{\boldsymbol{q}}\right)_{\boldsymbol{q}}$
3. $-\left(\boldsymbol{\Phi_q}\dot{\boldsymbol{q}}\right)_{\boldsymbol{q}}\dot{\boldsymbol{q}}$

First:

$$\boldsymbol{\Phi_q}\dot{\boldsymbol{q}} = \begin{bmatrix} I_{\text{2x2}} & B_1 {s_1'}^{\text{p1}} & 0 & 0 & 0 & 0 \\ I_{\text{2x2}} & B_1 {s_1'}^{\text{p2}} & -I_{\text{2x2}} & -B_2 {s_2'}^{\text{p2}} & 0 & 0 \\ 0 & 0 & I_{\text{2x2}} & B_2 {s_2'}^{\text{p3}} & -I_{\text{2x2}} & -B_3 {s_3'}^{\text{p3}} \\ 0 & 0 & 0 & 0 & I_{\text{2x2}} & B_3 {s_3'}^{\text{p4}} \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\boldsymbol{r}}_{\boldsymbol{1}} \\ \dot{\phi}_1 \\ \dot{\boldsymbol{r}}_{\boldsymbol{2}} \\ \dot{\phi}_2 \\ \dot{\boldsymbol{r}}_{\boldsymbol{3}} \\ \dot{\phi}_3 \end{bmatrix} = \begin{bmatrix} \dot{x}_1 + \dot{\phi}_1 \frac{L_1}{2}\sin(\phi_1) \\ \dot{y}_1 - \dot{\phi}_1 \frac{L_1}{2}\cos(\phi_1) \\ \dot{x}_1 - \dot{\phi}_1 \frac{L_1}{2}\sin(\phi_1) - \dot{x}_2 - \dot{\phi}_2 \frac{L_2}{2}\sin(\phi_2) \\ \dot{y}_1 + \dot{\phi}_1 \frac{L_1}{2}\cos(\phi_1) - \dot{y}_2 + \dot{\phi}_2 \frac{L_2}{2}\cos(\phi_2) \\ \dot{x}_2 - \dot{\phi}_2 \frac{L_2}{2}\sin(\phi_2) - \dot{x}_3 - \dot{\phi}_3 \frac{L_3}{2}\sin(\phi_3) \\ \dot{y}_2 + \dot{\phi}_2 \frac{L_2}{2}\cos(\phi_2) - \dot{y}_3 + \dot{\phi}_3 \frac{L_3}{2}\cos(\phi_3) \\ \dot{x}_3 - \dot{\phi}_3 \frac{L_3}{2}\sin(\phi_3) \\ \dot{y}_3 + \dot{\phi}_3 \frac{L_3}{2}\cos(\phi_3) \\ \dot{\phi}_1 \end{bmatrix}$$

Then

$$\left(\boldsymbol{\Phi_q}\dot{\boldsymbol{q}}\right)_{\boldsymbol{q}} = \frac{\partial \boldsymbol{\Phi_q}\dot{\boldsymbol{q}}}{\partial \boldsymbol{q}} = \begin{bmatrix} 0 & 0 & \dot{\phi}_1 \frac{L_1}{2}\cos(\phi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \dot{\phi}_1 \frac{L_1}{2}\sin(\phi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\dot{\phi}_1 \frac{L_1}{2}\cos(\phi_1) & 0 & 0 & \dot{\phi}_2 \frac{L_2}{2}\cos(\phi_2) & 0 & 0 & 0 \\ 0 & 0 & -\dot{\phi}_1 \frac{L_1}{2}\sin(\phi_1) & 0 & 0 & \dot{\phi}_2 \frac{L_2}{2}\sin(\phi_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\dot{\phi}_2 \frac{L_2}{2}\cos(\phi_2) & 0 & 0 & \dot{\phi}_3 \frac{L_3}{2}\cos(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & -\dot{\phi}_2 \frac{L_2}{2}\sin(\phi_2) & 0 & 0 & \dot{\phi}_3 \frac{L_3}{2}\sin(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\dot{\phi}_3 \frac{L_3}{2}\cos(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\dot{\phi}_3 \frac{L_3}{2}\sin(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and lastly

$$\left(\boldsymbol{\Phi_q}\dot{\boldsymbol{q}}\right)_{\boldsymbol{q}}\dot{\boldsymbol{q}} =$$

$$\begin{bmatrix} 0 & 0 & \dot{\phi}_1 \frac{L_1}{2}\cos(\phi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \dot{\phi}_1 \frac{L_1}{2}\sin(\phi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\dot{\phi}_1 \frac{L_1}{2}\cos(\phi_1) & 0 & 0 & \dot{\phi}_2 \frac{L_2}{2}\cos(\phi_2) & 0 & 0 & 0 \\ 0 & 0 & -\dot{\phi}_1 \frac{L_1}{2}\sin(\phi_1) & 0 & 0 & \dot{\phi}_2 \frac{L_2}{2}\sin(\phi_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\dot{\phi}_2 \frac{L_2}{2}\cos(\phi_2) & 0 & 0 & \dot{\phi}_3 \frac{L_3}{2}\cos(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & -\dot{\phi}_2 \frac{L_2}{2}\sin(\phi_2) & 0 & 0 & \dot{\phi}_3 \frac{L_3}{2}\sin(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\dot{\phi}_3 \frac{L_3}{2}\cos(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\dot{\phi}_3 \frac{L_3}{2}\sin(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{\phi}_1 \\ \dot{x}_2 \\ \dot{y}_2 \\ \dot{\phi}_2 \\ \dot{x}_3 \\ \dot{y}_3 \\ \dot{\phi}_3 \end{bmatrix} = \begin{bmatrix} {\dot{\phi}_1}^2 \frac{L_1}{2}\cos(\phi_1) \\ {\dot{\phi}_1}^2 \frac{L_1}{2}\sin(\phi_1) \\ -{\dot{\phi}_1}^2 \frac{L_1}{2}\cos(\phi_1) + {\dot{\phi}_2}^2 \frac{L_2}{2}\cos(\phi_2) \\ -{\dot{\phi}_1}^2 \frac{L_1}{2}\sin(\phi_1) + {\dot{\phi}_2}^2 \frac{L_2}{2}\sin(\phi_2) \\ -{\dot{\phi}_2}^2 \frac{L_2}{2}\cos(\phi_2) + {\dot{\phi}_3}^2 \frac{L_3}{2}\cos(\phi_3) \\ -{\dot{\phi}_2}^2 \frac{L_2}{2}\sin(\phi_2) + {\dot{\phi}_3}^2 \frac{L_3}{2}\sin(\phi_3) \\ -{\dot{\phi}_3}^2 \frac{L_3}{2}\cos(\phi_3) \\ -{\dot{\phi}_3}^2 \frac{L_3}{2}\sin(\phi_3) \\ 0 \end{bmatrix}$$

Then the expression for $\boldsymbol{\gamma}$ is:

$$\boldsymbol{\gamma} = -\left(\boldsymbol{\Phi_q}\dot{\boldsymbol{q}}\right)_{\boldsymbol{q}}\dot{\boldsymbol{q}} = \begin{bmatrix} -\dot{\phi_1}^2\frac{L_1}{2}\cos(\phi_1) \\ -\dot{\phi_1}^2\frac{L_1}{2}\sin(\phi_1) \\ \dot{\phi_1}^2\frac{L_1}{2}\cos(\phi_1) - \dot{\phi_2}^2\frac{L_2}{2}\cos(\phi_2) \\ \dot{\phi_1}^2\frac{L_1}{2}\sin(\phi_1) - \dot{\phi_2}^2\frac{L_2}{2}\sin(\phi_2) \\ \dot{\phi_2}^2\frac{L_2}{2}\cos(\phi_2) - \dot{\phi_3}^2\frac{L_3}{2}\cos(\phi_3) \\ \dot{\phi_2}^2\frac{L_2}{2}\sin(\phi_2) - \dot{\phi_3}^2\frac{L_3}{2}\sin(\phi_3) \\ \dot{\phi_3}^2\frac{L_3}{2}\cos(\phi_3) \\ \dot{\phi_3}^2\frac{L_3}{2}\sin(\phi_3) \\ 0 \end{bmatrix}$$

With the accelerations solved for:

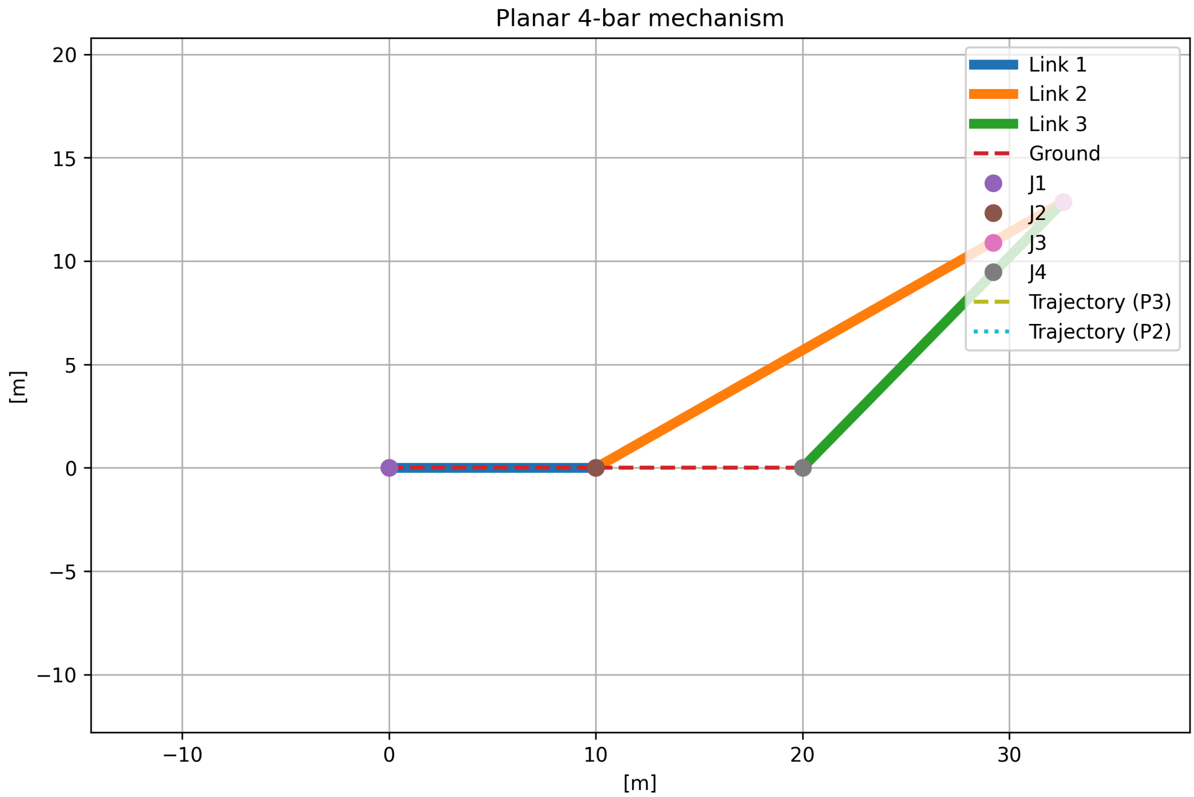
$$\ddot{\boldsymbol{q}} = \boldsymbol{\Phi_q}^{-1}\left(-\left(\boldsymbol{\Phi_q}\dot{\boldsymbol{q}}\right)_{\boldsymbol{q}}\dot{\boldsymbol{q}}\right)$$

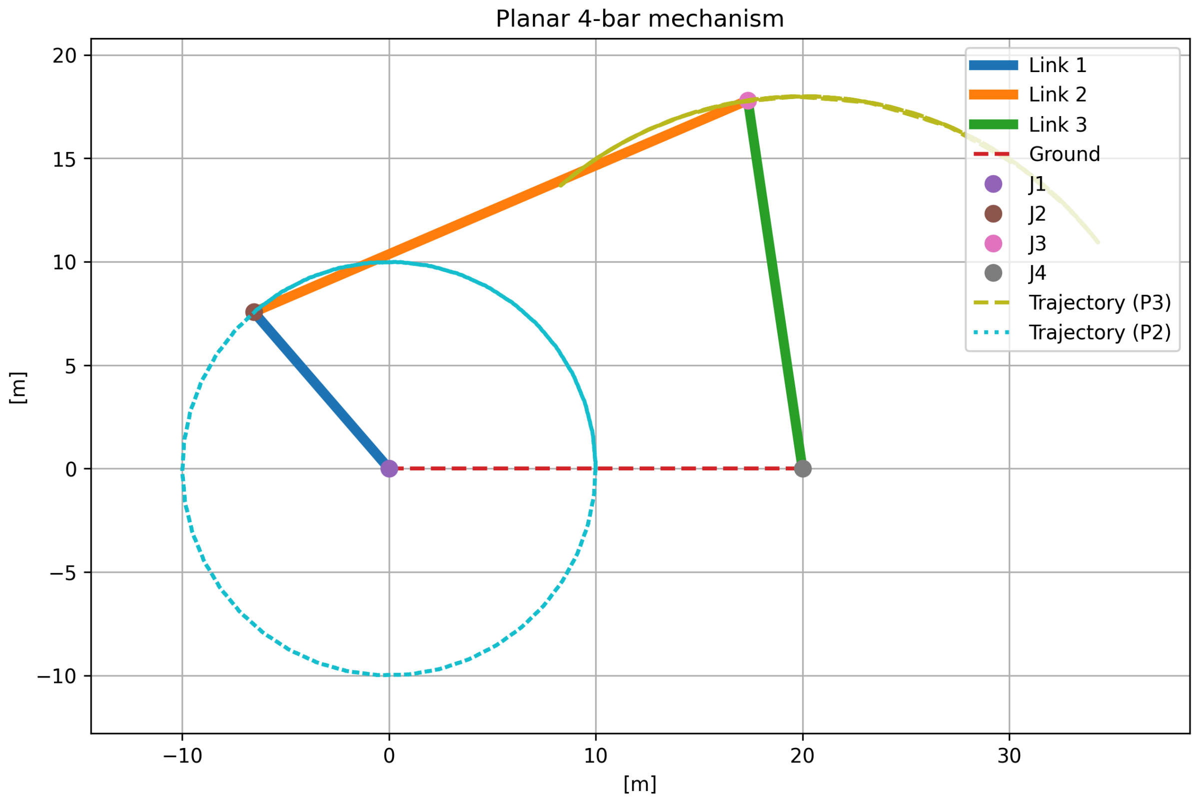
and with matrices:

$$\begin{bmatrix} \ddot{x_1} \\ \ddot{y_1} \\ \ddot{\phi_1} \\ \ddot{x_2} \\ \ddot{y_2} \\ \ddot{\phi_2} \\ \ddot{x_3} \\ \ddot{y_3} \\ \ddot{\phi_3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \frac{L_1}{2}\sin(\phi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{L_1}{2}\cos(\phi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -\frac{L_1}{2}\sin(\phi_1) & -1 & 0 & \frac{L_2}{2}\sin(\phi_2) & 0 & 0 & 0 \\ 0 & 1 & \frac{L_1}{2}\cos(\phi_1) & 0 & -1 & -\frac{L_2}{2}\cos(\phi_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -\frac{L_2}{2}\sin(\phi_2) & -1 & 0 & \frac{L_3}{2}\sin(\phi_3) \\ 0 & 0 & 0 & 0 & 1 & \frac{L_2}{2}\cos(\phi_2) & 0 & -1 & -\frac{L_3}{2}\cos(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\frac{L_3}{2}\sin(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{L_3}{2}\cos(\phi_3) \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -\dot{\phi_1}^2\frac{L_1}{2}\cos(\phi_1) \\ -\dot{\phi_1}^2\frac{L_1}{2}\sin(\phi_1) \\ \dot{\phi_1}^2\frac{L_1}{2}\cos(\phi_1) - \dot{\phi_2}^2\frac{L_2}{2}\cos(\phi_2) \\ \dot{\phi_1}^2\frac{L_1}{2}\sin(\phi_1) - \dot{\phi_2}^2\frac{L_2}{2}\sin(\phi_2) \\ \dot{\phi_2}^2\frac{L_2}{2}\cos(\phi_2) - \dot{\phi_3}^2\frac{L_3}{2}\cos(\phi_3) \\ \dot{\phi_2}^2\frac{L_2}{2}\sin(\phi_2) - \dot{\phi_3}^2\frac{L_3}{2}\sin(\phi_3) \\ \dot{\phi_3}^2\frac{L_3}{2}\cos(\phi_3) \\ \dot{\phi_3}^2\frac{L_3}{2}\sin(\phi_3) \\ 0 \end{bmatrix}$$

6. Code for the system of equations in python:

7. The motion of the mechanism plotted for 10 seconds:





8. Initial configuration of the mechanism and the trajectories of the revolute joints: