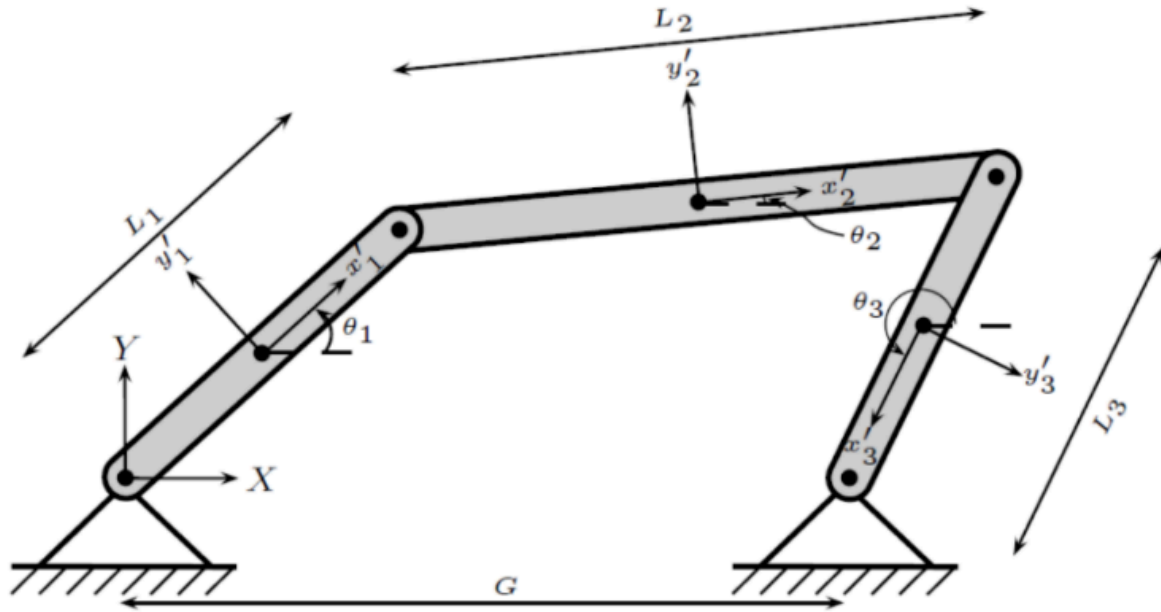


## Assignment 1 - Computational Dynamics

### Assignment 1 - 4 bar linkage

The goal of this assignment is to perform a full kinematic analysis of a four-bar linkage.



**Figure 1:** Four-bar linkage

Use the following lengths:

$$L_1 = 10m; L_2 = 26m; L_3 = 18m; G = 20m$$

### Problem:

#### 1. Identify the number of bodies, joints and degrees of freedom for the mechanism.

The number of bodies (nb) is 3 and there are 4 joints with 1 degree of freedom each (nh = 8).

Therefore there degrees of freedom for the model is  $3 \cdot nb - nh = 1$

The remaining DoF will be governed by a driving constraint  $\Phi^D(q, t) = \phi_1 - \omega t$ ,  $\omega = 1.5 \frac{\text{rad}}{\text{s}}$

#### 2. Setup the kinematic constraints $\Phi^K$ for all the joints

First A and r are defined:

$$A(\phi) = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}, \quad r^p = r + A(\phi)s'^p, \quad q = [x_1 \ y_1 \ \phi_1 \ x_2 \ y_2 \ \phi_2 \ x_3 \ y_3 \ \phi_3]^T$$

The points for the joints are described by:

$$s_1^{p1} = \begin{bmatrix} -\frac{L_1}{2} \\ 0 \end{bmatrix}, \quad s_1^{p2} = \begin{bmatrix} \frac{L_1}{2} \\ 0 \end{bmatrix}, \quad s_2^{p2} = \begin{bmatrix} -\frac{L_2}{2} \\ 0 \end{bmatrix}, \quad s_2^{p3} = \begin{bmatrix} \frac{L_2}{2} \\ 0 \end{bmatrix}, \quad s_3^{p3} = \begin{bmatrix} -\frac{L_3}{2} \\ 0 \end{bmatrix}, \quad s_3^{p4} = \begin{bmatrix} \frac{L_3}{2} \\ 0 \end{bmatrix},$$

so that e.g.  $s_1^{p2}$  describes the local point p2 in body 1.

The coordinates for the  $C$ -vectors are:

$$\mathbf{C}_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \mathbf{C}_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \mathbf{C}_3 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \mathbf{C}_4 = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

And the coordinates for the  $\mathbf{r}$ -vectors are:

$$\mathbf{r}_1 = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}, \quad \mathbf{r}_2 = \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}, \quad \mathbf{r}_3 = \begin{bmatrix} x_3 \\ y_3 \end{bmatrix}$$

The Kinematic constraint equations are as follows:

- $\Phi^{\text{abs1}}(q) = [\mathbf{r}_1 + \mathbf{A}\mathbf{s}'_1{}^{P1} - \mathbf{C}_1] = \begin{bmatrix} x_1 - \frac{L_1}{2} \cos(\phi_1) \\ y_1 - \frac{L_1}{2} \sin(\phi_1) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- $\Phi^{\text{rel1}}(q) = [\mathbf{r}_1 + \mathbf{A}\mathbf{s}'_1{}^{P2} - (\mathbf{r}_2 + \mathbf{A}\mathbf{s}'_2{}^{P2}) - \mathbf{C}_2] = \begin{bmatrix} x_1 + \frac{L_1}{2} \cos(\phi_1) - (x_2 - \frac{L_2}{2} \cos(\phi_2)) \\ y_1 + \frac{L_1}{2} \sin(\phi_1) - (y_2 - \frac{L_2}{2} \sin(\phi_2)) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- $\Phi^{\text{rel2}}(q) = [\mathbf{r}_2 + \mathbf{A}\mathbf{s}'_2{}^{P3} - (\mathbf{r}_3 + \mathbf{A}\mathbf{s}'_3{}^{P3}) - \mathbf{C}_3] = \begin{bmatrix} x_2 + \frac{L_2}{2} \cos(\phi_2) - (x_3 - \frac{L_3}{2} \cos(\phi_3)) \\ y_2 + \frac{L_2}{2} \sin(\phi_2) - (y_3 - \frac{L_3}{2} \sin(\phi_3)) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$
- $\Phi^{\text{abs2}}(q) = [\mathbf{r}_3 + \mathbf{A}\mathbf{s}'_3{}^{P4} - \mathbf{C}_4] = \begin{bmatrix} x_3 + \frac{L_3}{2} \cos(\phi_3) - 20 \\ y_3 + \frac{L_3}{2} \sin(\phi_3) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

As a system of constraints the vectorfunction the above equations become:

$$\Phi^K(q) = \begin{bmatrix} \Phi^{\text{abs1}}(q) \\ \Phi^{\text{rel1}}(q) \\ \Phi^{\text{rel2}}(q) \\ \Phi^{\text{abs2}}(q) \end{bmatrix} = \begin{bmatrix} x_1 - \frac{L_1}{2} \cos(\phi_1) \\ y_1 - \frac{L_1}{2} \sin(\phi_1) \\ x_1 + \frac{L_1}{2} \cos(\phi_1) - (x_2 - \frac{L_2}{2} \cos(\phi_2)) \\ y_1 + \frac{L_1}{2} \sin(\phi_1) - (y_2 - \frac{L_2}{2} \sin(\phi_2)) \\ x_2 + \frac{L_2}{2} \cos(\phi_2) - (x_3 - \frac{L_3}{2} \cos(\phi_3)) \\ y_2 + \frac{L_2}{2} \sin(\phi_2) - (y_3 - \frac{L_3}{2} \sin(\phi_3)) \\ x_3 + \frac{L_3}{2} \cos(\phi_3) - 20 \\ y_3 + \frac{L_3}{2} \sin(\phi_3) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = [\vec{o}]$$

### 3. Setup of the driving constraint $\Phi^D$ that imposes $\phi_1 = \omega t$ , $\omega = 1.5 \frac{\text{rad}}{\text{s}}$

As there is 1 DoF for the system, an absolute driving constraint is added to the system:

$$\Phi^D(q, t) = [\phi_1 - \omega t] = 0$$

### 4. Calculate the constraint jacobian $\Phi_q$

First B is defined:

$$B(\phi) = \begin{bmatrix} -\sin(\phi) & -\cos(\phi) \\ \cos(\phi) & -\sin(\phi) \end{bmatrix}$$

by combining  $\Phi^K$  and  $\Phi^D$  into  $\Phi$  and then taking the partial derivative with respect to  $q$  the constraint jacobian  $\Phi_q$  can be obtained:

$$\Phi_q = \frac{\partial \Phi}{\partial q} = \begin{bmatrix} \frac{\partial \Phi_K}{\partial q} \\ \frac{\partial \Phi_D}{\partial q} \end{bmatrix} = \begin{bmatrix} I_{2 \times 2} & B_1 s_1^{p1} & 0 & 0 & 0 & 0 \\ I_{2 \times 2} & B_1 s_1^{p2} & -I_{2 \times 2} & -B_2 s_2^{p2} & 0 & 0 \\ 0 & 0 & I_{2 \times 2} & B_2 s_2^{p3} & -I_{2 \times 2} & -B_3 s_3^{p3} \\ 0 & 0 & 0 & 0 & I_{2 \times 2} & B_3 s_3^{p4} \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & \frac{L_1}{2} \sin(\phi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{L_1}{2} \cos(\phi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -\frac{L_1}{2} \sin(\phi_1) & -1 & 0 & -\frac{L_2}{2} \sin(\phi_2) & 0 & 0 & 0 \\ 0 & 1 & \frac{L_1}{2} \cos(\phi_1) & 0 & -1 & \frac{L_2}{2} \cos(\phi_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -\frac{L_2}{2} \sin(\phi_2) & -1 & 0 & -\frac{L_3}{2} \sin(\phi_3) \\ 0 & 0 & 0 & 0 & 1 & \frac{L_2}{2} \cos(\phi_2) & 0 & -1 & \frac{L_3}{2} \cos(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\frac{L_3}{2} \sin(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{L_3}{2} \cos(\phi_3) \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

### 5. Setup of the velocity and acceleration equations $\nu$ and $\gamma$

First the equations for the velocities will be setup and then the accelerations.

#### Velocities:

The velocity equation is defined as:

$$\nu = -\Phi_t = \Phi_q \dot{q}, \quad \text{where } \dot{q} = [x_1 \ y_1 \ \dot{\phi}_1 \ x_2 \ y_2 \ \dot{\phi}_2 \ x_3 \ y_3 \ \dot{\phi}_3]^T$$

as only  $\Phi^D$  is a function of time ( $\phi_1 - \omega t$ ) the vector  $-\Phi_t$  becomes:

$$\nu = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ w]^T = \Phi_q \dot{q}$$

By left multiplying with  $\Phi_q^{-1}$  the equation can be rewritten as to obtain  $\dot{q}$ :

$$\dot{q} = \Phi_q^{-1} \nu$$

Which when written out in matrices is:

$$\begin{bmatrix} x_1 \\ y_1 \\ \dot{\phi}_1 \\ x_2 \\ y_2 \\ \dot{\phi}_2 \\ x_3 \\ y_3 \\ \dot{\phi}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \frac{L_1}{2} \sin(\phi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{L_1}{2} \cos(\phi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -\frac{L_1}{2} \sin(\phi_1) & -1 & 0 & -\frac{L_2}{2} \sin(\phi_2) & 0 & 0 & 0 \\ 0 & 1 & \frac{L_1}{2} \cos(\phi_1) & 0 & -1 & \frac{L_2}{2} \cos(\phi_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -\frac{L_2}{2} \sin(\phi_2) & -1 & 0 & -\frac{L_3}{2} \sin(\phi_3) \\ 0 & 0 & 0 & 0 & 1 & \frac{L_2}{2} \cos(\phi_2) & 0 & -1 & \frac{L_3}{2} \cos(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\frac{L_3}{2} \sin(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{L_3}{2} \cos(\phi_3) \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ w \end{bmatrix}$$

for which the velocities can be calculated for each time instance.

#### Accelerations:

The acceleration equations are defined as:

$$\gamma = \Phi_q \ddot{q} = -(\Phi_q \dot{q})_q \dot{q} - 2\Phi_{qt} \dot{q} - \Phi_{tt}, \quad \text{where } \ddot{q} = [\ddot{x}_1 \ \ddot{y}_1 \ \ddot{\phi}_1 \ \ddot{x}_2 \ \ddot{y}_2 \ \ddot{\phi}_2 \ \ddot{x}_3 \ \ddot{y}_3 \ \ddot{\phi}_3]^T$$

As  $\Phi_q$  does not contain  $t$ ,  $\Phi_{qt} = [\vec{0}]$  and as only  $\Phi^D$  is dependant on time:

$$\Phi_{tt} = \frac{\partial^2 \Phi^D}{\partial t^2} = \frac{\partial \Phi_t^D}{\partial t} = [\vec{0}]$$

The expression for  $\gamma$  then becomes:  $\gamma = \Phi_q \ddot{q} = -(\Phi_q \dot{q})_q \dot{q}$ , where when solving for  $\ddot{q}$ :

$$\ddot{\mathbf{q}} = \Phi_{\mathbf{q}}^{-1} \left( -(\Phi_{\mathbf{q}} \dot{\mathbf{q}})_{\mathbf{q}} \dot{\mathbf{q}} \right)$$

Before  $\ddot{\mathbf{q}}$  can be solved for the term  $-(\Phi_{\mathbf{q}} \dot{\mathbf{q}})_{\mathbf{q}} \dot{\mathbf{q}}$  has to be calculated in the order:

1.  $\Phi_{\mathbf{q}} \dot{\mathbf{q}}$
2.  $(\Phi_{\mathbf{q}} \dot{\mathbf{q}})_{\mathbf{q}}$
3.  $-(\Phi_{\mathbf{q}} \dot{\mathbf{q}})_{\mathbf{q}} \dot{\mathbf{q}}$

First:

$$\Phi_{\mathbf{q}} \dot{\mathbf{q}} = \begin{bmatrix} I_{2 \times 2} & B_1 s_1^{p1} & 0 & 0 & 0 & 0 \\ I_{2 \times 2} & B_1 s_1^{p2} & -I_{2 \times 2} & -B_2 s_2^{p2} & 0 & 0 \\ 0 & 0 & I_{2 \times 2} & B_2 s_2^{p3} & -I_{2 \times 2} & -B_3 s_3^{p3} \\ 0 & 0 & 0 & 0 & I_{2 \times 2} & B_3 s_3^{p4} \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{r}}_1 \\ \dot{\phi}_1 \\ \dot{\mathbf{r}}_2 \\ \dot{\phi}_2 \\ \dot{\mathbf{r}}_3 \\ \dot{\phi}_3 \end{bmatrix} = \begin{bmatrix} x_1 + \dot{\phi}_1 \frac{L_1}{2} \sin(\phi_1) \\ y_1 - \dot{\phi}_1 \frac{L_1}{2} \cos(\phi_1) \\ x_1 - \dot{\phi}_1 \frac{L_1}{2} \sin(\phi_1) - x_2 - \dot{\phi}_2 \frac{L_2}{2} \sin(\phi_2) \\ y_1 + \dot{\phi}_1 \frac{L_1}{2} \cos(\phi_1) - y_2 + \dot{\phi}_2 \frac{L_2}{2} \cos(\phi_2) \\ x_2 - \dot{\phi}_2 \frac{L_2}{2} \sin(\phi_2) - x_3 - \dot{\phi}_3 \frac{L_3}{2} \sin(\phi_3) \\ y_2 + \dot{\phi}_2 \frac{L_2}{2} \cos(\phi_2) - y_3 + \dot{\phi}_3 \frac{L_3}{2} \cos(\phi_3) \\ x_3 - \dot{\phi}_3 \frac{L_3}{2} \sin(\phi_3) \\ y_3 + \dot{\phi}_3 \frac{L_3}{2} \cos(\phi_3) \\ \dot{\phi}_1 \end{bmatrix}$$

Then

$$(\Phi_{\mathbf{q}} \dot{\mathbf{q}})_{\mathbf{q}} = \frac{\partial \Phi_{\mathbf{q}} \dot{\mathbf{q}}}{\partial \mathbf{q}} = \begin{bmatrix} 0 & 0 & \dot{\phi}_1 \frac{L_1}{2} \cos(\phi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \dot{\phi}_1 \frac{L_1}{2} \sin(\phi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \dot{\phi}_1 \frac{L_1}{2} \cos(\phi_1) & 0 & 0 & -\dot{\phi}_2 \frac{L_2}{2} \cos(\phi_2) & 0 & 0 & 0 \\ 0 & 0 & \dot{\phi}_1 \frac{L_1}{2} \sin(\phi_1) & 0 & 0 & -\dot{\phi}_2 \frac{L_2}{2} \sin(\phi_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \dot{\phi}_2 \frac{L_2}{2} \cos(\phi_2) & 0 & 0 & -\dot{\phi}_3 \frac{L_3}{2} \cos(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & \dot{\phi}_2 \frac{L_2}{2} \sin(\phi_2) & 0 & 0 & -\dot{\phi}_3 \frac{L_3}{2} \sin(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dot{\phi}_3 \frac{L_3}{2} \cos(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dot{\phi}_3 \frac{L_3}{2} \sin(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and lastly

$$(\Phi_{\mathbf{q}} \dot{\mathbf{q}})_{\mathbf{q}} \dot{\mathbf{q}} = \begin{bmatrix} 0 & 0 & \dot{\phi}_1 \frac{L_1}{2} \cos(\phi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \dot{\phi}_1 \frac{L_1}{2} \sin(\phi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \dot{\phi}_1 \frac{L_1}{2} \cos(\phi_1) & 0 & 0 & -\dot{\phi}_2 \frac{L_2}{2} \cos(\phi_2) & 0 & 0 & 0 \\ 0 & 0 & \dot{\phi}_1 \frac{L_1}{2} \sin(\phi_1) & 0 & 0 & -\dot{\phi}_2 \frac{L_2}{2} \sin(\phi_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \dot{\phi}_2 \frac{L_2}{2} \cos(\phi_2) & 0 & 0 & -\dot{\phi}_3 \frac{L_3}{2} \cos(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & \dot{\phi}_2 \frac{L_2}{2} \sin(\phi_2) & 0 & 0 & -\dot{\phi}_3 \frac{L_3}{2} \sin(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dot{\phi}_3 \frac{L_3}{2} \cos(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \dot{\phi}_3 \frac{L_3}{2} \sin(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{\phi}_1 \\ \dot{x}_2 \\ \dot{y}_2 \\ \dot{\phi}_2 \\ \dot{x}_3 \\ \dot{y}_3 \\ \dot{\phi}_3 \end{bmatrix} = \begin{bmatrix} \dot{\phi}_1^2 \frac{L_1}{2} \cos(\phi_1) \\ \dot{\phi}_1^2 \frac{L_1}{2} \sin(\phi_1) \\ \dot{\phi}_1^2 \frac{L_1}{2} \cos(\phi_1) - \dot{\phi}_2^2 \frac{L_2}{2} \cos(\phi_2) \\ \dot{\phi}_1^2 \frac{L_1}{2} \sin(\phi_1) - \dot{\phi}_2^2 \frac{L_2}{2} \sin(\phi_2) \\ \dot{\phi}_2^2 \frac{L_2}{2} \cos(\phi_2) - \dot{\phi}_3^2 \frac{L_3}{2} \cos(\phi_3) \\ \dot{\phi}_2^2 \frac{L_2}{2} \sin(\phi_2) - \dot{\phi}_3^2 \frac{L_3}{2} \sin(\phi_3) \\ \dot{\phi}_3^2 \frac{L_3}{2} \cos(\phi_3) \\ \dot{\phi}_3^2 \frac{L_3}{2} \sin(\phi_3) \\ 0 \end{bmatrix}$$

Then the expression for  $\gamma$  is:

$$\gamma = -(\Phi_q \dot{q})_q \dot{q} = \begin{bmatrix} -\dot{\phi}_1^2 \frac{L_1}{2} \cos(\phi_1) \\ -\dot{\phi}_1^2 \frac{L_1}{2} \sin(\phi_1) \\ \dot{\phi}_1^2 \frac{L_1}{2} \cos(\phi_1) + \dot{\phi}_2^2 \frac{L_2}{2} \cos(\phi_2) \\ \dot{\phi}_1^2 \frac{L_1}{2} \sin(\phi_1) + \dot{\phi}_2^2 \frac{L_2}{2} \sin(\phi_2) \\ \dot{\phi}_2^2 \frac{L_2}{2} \cos(\phi_2) + \dot{\phi}_3^2 \frac{L_3}{2} \cos(\phi_3) \\ \dot{\phi}_2^2 \frac{L_2}{2} \sin(\phi_2) + \dot{\phi}_3^2 \frac{L_3}{2} \sin(\phi_3) \\ \dot{\phi}_3^2 \frac{L_3}{2} \cos(\phi_3) \\ \dot{\phi}_3^2 \frac{L_3}{2} \sin(\phi_3) \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\phi}_1^2 A_1 s_1'^{p1} \\ \dot{\phi}_1^2 A_1 s_1'^{p2} - \dot{\phi}_2^2 A_2 s_2'^{p2} \\ \dot{\phi}_2^2 A_2 s_2'^{p3} - \dot{\phi}_3^2 A_3 s_3'^{p3} \\ \dot{\phi}_3^2 A_3 s_3'^{p4} \\ 0 \end{bmatrix}$$

With the accelerations solved for:

$$\ddot{q} = \Phi_q^{-1} \left( -(\Phi_q \dot{q})_q \dot{q} \right)$$

and with matrices:

$$\begin{bmatrix} \ddot{x}_1 \\ \ddot{y}_1 \\ \ddot{\phi}_1 \\ \ddot{x}_2 \\ \ddot{y}_2 \\ \ddot{\phi}_2 \\ \ddot{x}_3 \\ \ddot{y}_3 \\ \ddot{\phi}_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & \frac{L_1}{2} \sin(\phi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{L_1}{2} \cos(\phi_1) & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -\frac{L_1}{2} \sin(\phi_1) & -1 & 0 & \frac{L_2}{2} \sin(\phi_2) & 0 & 0 & 0 \\ 0 & 1 & \frac{L_1}{2} \cos(\phi_1) & 0 & -1 & -\frac{L_2}{2} \cos(\phi_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -\frac{L_2}{2} \sin(\phi_2) & -1 & 0 & \frac{L_3}{2} \sin(\phi_3) \\ 0 & 0 & 0 & 0 & 1 & \frac{L_2}{2} \cos(\phi_2) & 0 & -1 & -\frac{L_3}{2} \cos(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -\frac{L_3}{2} \sin(\phi_3) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & \frac{L_3}{2} \cos(\phi_3) \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{-1} \begin{bmatrix} -\dot{\phi}_1^2 \frac{L_1}{2} \cos(\phi_1) \\ -\dot{\phi}_1^2 \frac{L_1}{2} \sin(\phi_1) \\ -\dot{\phi}_1^2 \frac{L_1}{2} \cos(\phi_1) + \dot{\phi}_2^2 \frac{L_2}{2} \cos(\phi_2) \\ -\dot{\phi}_1^2 \frac{L_1}{2} \sin(\phi_1) + \dot{\phi}_2^2 \frac{L_2}{2} \sin(\phi_2) \\ -\dot{\phi}_2^2 \frac{L_2}{2} \cos(\phi_2) + \dot{\phi}_3^2 \frac{L_3}{2} \cos(\phi_3) \\ -\dot{\phi}_2^2 \frac{L_2}{2} \sin(\phi_2) + \dot{\phi}_3^2 \frac{L_3}{2} \sin(\phi_3) \\ -\dot{\phi}_3^2 \frac{L_3}{2} \cos(\phi_3) \\ -\dot{\phi}_3^2 \frac{L_3}{2} \sin(\phi_3) \\ 0 \end{bmatrix}$$

6. Code for the system of equations in python:

7. The motion of the mechanism plotted for 10 seconds:

8. Initial configuration of the mechanism and the trajectories of the revolute joints:

