Computation Dynamics Q – Assignment Q1

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Q1.1 – USE OF RIGID BODY TRANSFORMATION MATRICES

Let two rigid bodies, 1 and 2, with body-fixed reference frames, $\mathbb{B}_1$ and $\mathbb{B}_2$ respectively, be defined in an inertial reference frame $\mathbb{I}$. Furthermore, let w and x be two points on Body 1 with relative positions as seen from $\mathbb{B}_1$ to be defined as,

$$^{\mathbb{B}_1}l(w,x) = [1.0 \quad -0.1 \quad 0.4]^\top \ , \tag{1}$$

and y and z be two points on Body 2 with relative position as seen from $\mathbb{B}_2$ to be defined as,

$$^{\mathbb{B}_2}l(y,z) = [1.0 \quad 0 \quad 0.2]^\top \ , \tag{2}$$

with point x and y coinciding and connected through some hinge H – see figure 1.

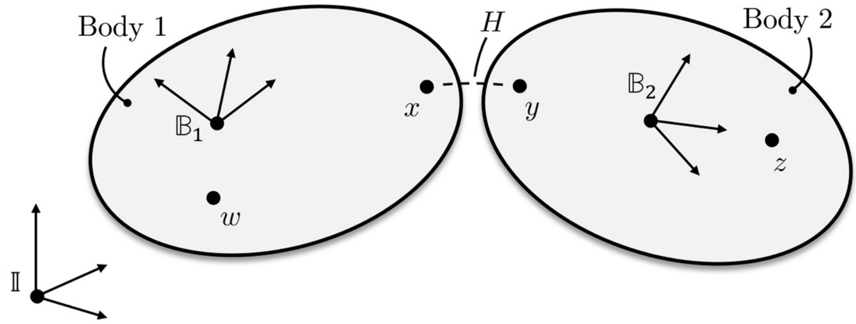

Figure 1

The velocity of w with respect to $\mathbb{I}$ as seen from $\mathbb{B}_1$ is,

$$^{\mathbb{B}_1}\mathcal{V}(\mathbb{I},w) \triangleq \begin{bmatrix} ^{\mathbb{B}_1}\omega(\mathbb{I},w) \\ ^{\mathbb{B}_1}\nu(\mathbb{I},w) \end{bmatrix} = [1.2 \quad 0.7 \quad 0 \quad 0.3 \quad 0 \quad 2.1]^\top \ , \tag{3}$$

and the orientation of $\mathbb{B}_1$ relative to $\mathbb{I}$ and $\mathbb{B}_2$ relative to $\mathbb{B}_1$ can be respectively described with the quaternions[1],

$$\begin{aligned} ^{\mathbb{I}}q_{(\mathbb{B}_1)} &= \left[\frac{1}{2\sqrt{3}} \quad \frac{1}{2\sqrt{3}} \quad \frac{1}{2\sqrt{3}} \quad \frac{\sqrt{3}}{2}\right]^\top , \\ ^{(\mathbb{B}_1)}q_{(\mathbb{B}_2)} &= \left[\frac{1}{2\sqrt{3}} \quad -\frac{1}{2\sqrt{3}} \quad \frac{1}{2\sqrt{3}} \quad \frac{\sqrt{3}}{2}\right]^\top . \end{aligned} \tag{4}$$

Q1.1.1 – Single transformation

Use a homogenous transformation and a rigid-body transformation matrix to determine $^{\mathbb{I}}\mathcal{V}(\mathbb{I},z)$ if H is a rigid connection, i.e. the bodies are fixed to each other.

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[1] All quaternions going forward will be defined as in Equation B.12 of [1].

Q1.1.2 – Chaining

Use rigid-body transformation matrices to determine ${}^{\mathbb{I}}\mathcal{V}(\mathbb{I}, z)$ if H is a spherical joint, i.e. only rotation is allowed, and the angular velocity of $\mathbb{B}_2$ with respect to $\mathbb{B}_1$ as seen from $\mathbb{B}_1$ is,

$$ {}^{\mathbb{B}_1}\omega(\mathbb{B}_1, \mathbb{B}_2) = [3.2 \quad 0 \quad 1.2]^\top \ . \tag{5}$$

Q1.2 – SINGLE RIGID BODY DYNAMICS IN BODY FRAME REFERENCE

Let a body with a body-fixed centroidal reference frame, $\mathbb{C}$, be defined in an inertial-fixed reference frame, $\mathbb{I}$, with a point z on the body to be defined from $\mathbb{C}$ as seen from $\mathbb{C}$ as,

$$ {}^{\mathbb{C}}l(\mathbb{C}, z) = [0.5 \quad 1.0 \quad 0]^\top \ , \tag{6}$$

and a body-fixed spatial force, ${}^{\mathbb{C}}\mathfrak{f}(z) \in \mathbb{R}^6$, acting on z as seen from $\mathbb{C}$ be defined as,

$$ {}^{\mathbb{C}}\mathfrak{f}(z) \triangleq \begin{bmatrix} {}^{\mathbb{C}}N(z) \\ {}^{\mathbb{C}}F(z) \end{bmatrix} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 50]^\top \ , \tag{7}$$

with the rotational component, ${}^{\mathbb{C}}N(z) \in \mathbb{R}^3$, and the linear component, ${}^{\mathbb{C}}F(z) \in \mathbb{R}^3$ - see Figure 2.

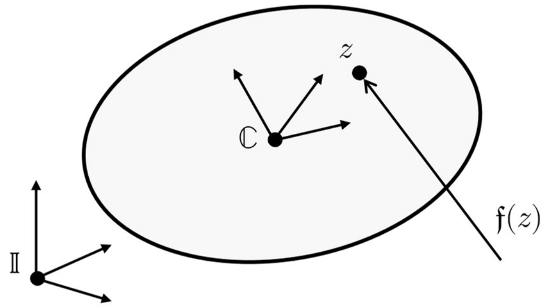

Figure 2

The body has a mass $m = 100.000$ kg and a rotational moment of inertia, ${}^{\mathbb{C}}\mathscr{I}(\mathbb{C}) \in \mathbb{R}^{3\times3}$, at $\mathbb{C}$ and as seen from $\mathbb{C}$ as,

$$ {}^{\mathbb{C}}\mathscr{I}(\mathbb{C}) = \begin{bmatrix} 500 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 500 \end{bmatrix} \ . \tag{8}$$

Q1.2.1 – Equation of motion

Write the equation of motion represented in z as seen from $\mathbb{C}$ for the body using quaternions, ${}^{\mathbb{I}}q_{\mathbb{C}}$, for the orientation and the position of $\mathbb{C}$, ${}^{\mathbb{I}}l(\mathbb{I}, \mathbb{C})$, as the generalized position for the body.

Hints:

- Since the force doesn't act on the centroidal point of the body and is seen from the body frame, use a derivation of the equation of motion that is derived at an arbitrary point in the body frame.
- The equation of motion should take a state vector (generalized position and velocity concatenated) and return the derivative of the state vector (generalized velocity and acceleration concatenated). For the derivative of ${}^{\mathbb{I}}q_{\mathbb{C}}$ you can use the same code as in Exercise Q1.2.

Q1.2.2 – Simulation

Simulate the motion of the body by integration the equation of motion for 5 sec using the initial generalized velocities,

$$\beta_{\mathbb{C}} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]^{\top} \,. \tag{9}$$

The initial position and orientation of z should be such that $\mathbb{C}$ is aligned and coincides with $\mathbb{I}$. Plot the trajectory of both $\mathbb{C}$ and z in $\mathbb{I}$ in an (x, y)-plot and a (x, z)-plot as seen from $\mathbb{I}$. The final motion in a 3D plot should look like the depiction in Figure 3 (optional plot).

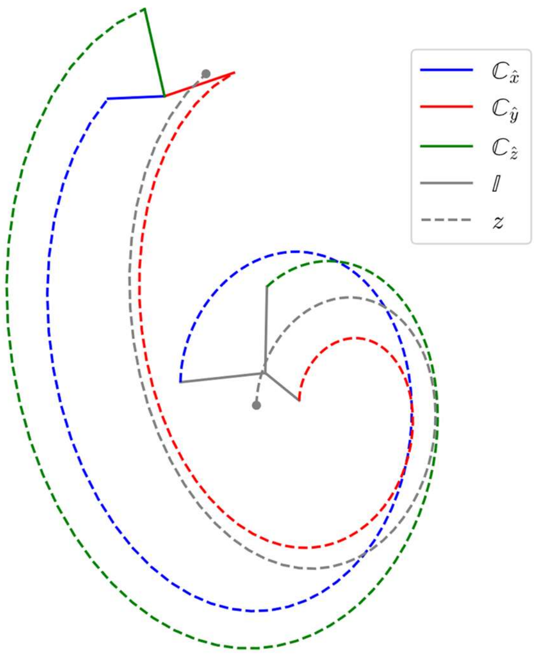

Figure 3

Hints:

- The integration should return orientation, position and velocity through time of z where the position and velocities are in $\mathbb{I}$ but seen from $\mathbb{C}$. They must be seen from $\mathbb{I}$ to plot it in that frame. Use the orientation to represent the position of z in $\mathbb{I}$ and as seen from $\mathbb{I}$ and then afterwards use that to determine $\mathbb{C}$ in $\mathbb{I}$ as seen from $\mathbb{I}$.

- A quaternion, q, representing the orientation between two frames that are aligned, as in the initial condition, is $q = [0 \quad 0 \quad 0 \quad 1]^\top$.
- Because the equation of motion is represented in z, the initial position should not be zero, since that would make $\mathbb{I}$ and z coincide and not $\mathbb{I}$ and $\mathbb{C}$.

NOTE ON ASSIGNMENT FORMAT

Two formats of the hand-in is accepted:

(1) A pdf with your explanations, plots, and/or code snippets with the runnable code as an attachment as either a Live-Script / Jupyter Notebook or raw script(s).
(2) A detailed Live-Script / Jupyter Notebook with inline comments, explanations and plots with a print of the Live-Script / Jupyter Notebook as a pdf attachment.

The above assumes that you are using either Matlab or Python, but other languages are also welcome.

Follow each subtask with a brief interpretation of the result.

REFERENCES

[1] Jain, A. (2011). *Robot and Multibody Dynamics*. Springer science & Business Media.