课后作业

1. 排列

给定一个整数n（$1 \leq n \leq 8$），以及n个各不相同的整数$a_1, a_2, ..., a_n$（$1 \leq a_i \leq 10^9$）。按字典序从小到大输出$a_1, ..., a_n$所有可能的排列。

输入样例:

```
3
4 1 2
```

输出样例:

```
1 2 4
1 4 2
2 1 4
2 4 1
4 1 2
4 2 1
```

2. 电话按键

电话按键上各个数字对应的字母如下:

```
2: A,B,C     5: J,K,L     8: T,U,V
3: D,E,F     6: M,N,O     9: W,X,Y,Z
4: G,H,I     7: P,Q,R,S
```

给定一个由数字$2...9$组成的字符串S（$1 \leq |S| \leq 10$），按字典序输出该字符串对应的所有可能的字母字符串。

输入样例:

```
78
```

输出样例:

```
PT
PU
PV
QT
QU
QV
RT
RU
RV
ST
SU
SV
```

3. 另一个排列生成算法

证明以下程序可以输出 $0 \ldots n-1$ 的所有排列。

```cpp
#include <iostream>
#include <utility>

using namespace std;

void permutation(int n, int depth, int perm[]) {
    if (depth == n) {
        cout << perm[0];
        for (int i = 1; i < n; i++) {
            cout << " " << perm[i];
        }
        cout << endl;
        return;
    }

    for (int i = depth; i < n; i++) {
        swap(perm[depth], perm[i]);
        permutation(n, depth + 1, perm);
        swap(perm[depth], perm[i]);
    }
}

int main() {
    int n;
    cin >> n;

    int a[n];
    for (int i = 0; i < n; i++) {
        a[i] = i;
    }

    permutation(n, 0, a);

    return 0;
}
```

4. 排列数

给定整数P。回答Q（$1 \le Q \le 10^6$）个询问，每个询问包含两个整数n, m（$0 \le m \le n \le 2000$）。求P_n^m，结果对P取模。

输入样例:

```
5 1000000007
0 0
1 0
4 2
10 3
1949 1001
```

输出样例:

```
1
1
12
720
637995377
```

5. (NOIP 2006 – Junior 2) 开心的金明

金明今天很开心,家里购置的新房就要领钥匙了,新房里有一间他自己专用的很宽敞的房间。更让他高兴的是,妈妈昨天对他说:"你的房间需要购买哪些物品,怎么布置,你说了算,只要不超过 N 元钱就行"。今天一早金明就开始做预算,但是他想买的东西太多了,肯定会超过妈妈限定的 N 元。于是,他把每件物品规定了一个重要度,分为 5 等:用整数 $1-5$ 表示,第 5 等最重要。他还从因特网上查到了每件物品的价格(都是整数元)。他希望在不超过 N 元(可以等于 N 元)的前提下,使每件物品的价格与重要度的乘积的总和最大。

设第 j 件物品的价格为 $v[j]$,重要度为 $w[j]$,共选中了 k 件物品,编号依次为 $j_1, j_2, ..., j_k$,则所求的总和为:

$$v[j_1] \times w[j_1] + v[j_2] \times w[j_2] + \cdots + v[j_k] \times w[j_k]。$$

请你帮助金明设计一个满足要求的购物单。

【输入】

输入文件 happy.in 的第 1 行,为 2 个正整数,用一个空格隔开:

$N\ m$

(其中 $N(< 30000)$ 表示总钱数,$m(< 25)$ 为希望购买物品的个数。)

从第 2 行到第 $m+1$ 行,第 j 行给出了编号为 $j-1$ 的物品的基本数据,每行有 2 个非负整数

$v\ p$

(其中 v 表示该物品的价格 $(v \le 10000)$,p 表示该物品的重要度 $(1-5)$)

【输出】

输出文件 happy.out 只有一个正整数,为不超过总钱数的物品的价格与重要度乘积的总和的最大值 (< 100000000)。

【输入样例】

```
1000 5
800 2
400 5
300 5
400 3
200 2
```

【输出样例】

```
3900
```

6. (USACO 2015 US Open – Bronze 3) Trapped in the Haybales

Farmer John has received a shipment of N large hay bales ($1 \leq N \leq 4000$) and placed them at various locations along the road leading to his barn. Unfortunately, he completely forgets that Bessie the cow is out grazing along the road, and she may now be trapped within the bales!

Each bale j has a size S_j and a distinct position P_j giving its location along the one-dimensional road. Bessie the cow starts at some location where there is no hay bale, and can move around freely along the road, even up to the position at which a bale is located, but she cannot cross through this position. As an exception, if she runs in the same direction for D units of distance, she builds up enough speed to break through and permanently eliminate any hay bale of size *strictly* less than D. Of course, after doing this, she might open up more space to allow her to make a run at other hay bales, eliminating them as well.

Bessie can escape to freedom if she can eventually break through either the leftmost or rightmost hay bale. Please compute the total area of the road consisting of real-valued starting positions from which Bessie cannot escape. For example, if Bessie cannot escape if she starts between hay bales at positions 1 and 5, then these encompass an area of size 4 from which she cannot escape.
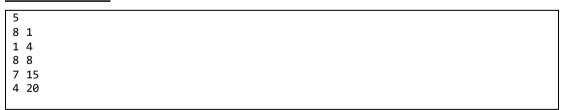
INPUT FORMAT (file trapped.in):

The first line of input contains N. Each of the next N lines describes a bale, and contains two integers giving its size and position, each in the range $1 \ldots 10^9$.

OUTPUT FORMAT (file trapped.out):

Print a single integer, giving the area of the road from which Bessie cannot escape.

SAMPLE INPUT:

```
5
8 1
1 4
8 8
7 15
4 20
```

SAMPLE OUTPUT:

```
14
```

[Problem credits: Brian Dean, 2015]