课后作业

1. 拆分 I

给定正整数N（$1 \leq N \leq 100$）。将N拆分成不同的正整数之和，求拆分的方法数。

输入样例 1：

```
5
```

输出样例 1：

```
3
```

可以将5拆分成$5, 1+4, 2+3$。

输入样例 2：

```
7
```

输出样例 2：

```
5
```

可以将7拆分成$7, 1+6, 2+5, 3+4, 1+2+4$。

输入样例 3：

```
100
```

输出样例 3：

```
444793
```

2. 差值最大公约数

给定N（$2 \leq N \leq 10^5$）个各不相同的整数$a_1, a_2, \dots, a_N$（$-10^9 \leq a_i \leq 10^9$）。求最大正整数d，使得对于任意$1 \leq i < j \leq N$，均有$d \mid a_i - a_j$。

输入样例：

```
4
1 9 3 5
```

输出样例：

```
2
```

3. (USACO 2019 January – Silver 1) Grass Planting

It's the time of year for Farmer John to plant grass in all of his fields. The entire farm consists of N fields ($1 \leq N \leq 10^5$), conveniently numbered $1 \ldots N$ and conveniently connected by $N-1$ bidirectional pathways in such a way that every field can reach every other field via some collection of pathways.

Farmer John can potentially plant a different type of grass in each field, but he wants to minimize the number of grass types he uses in total, since the more types of grass he uses, the more expense he incurs.

Unfortunately, his cows have grown rather snobbish about their selection of grass on the farm. If the same grass type is planted in two adjacent fields (directly connected by a pathway) or even two nearly-adjacent fields (both directly connected to a common field with pathways), then the cows will complain about lack of variety in their dining options. The last thing Farmer John needs is complaining cows, given how much mischief they have been known to create when dissatisfied.

Please help Farmer John determine the minimum number of types of grass he needs for his entire farm.

INPUT FORMAT (file planting.in):

The first line of input contains N. Each of the remaining $N-1$ lines describes a pathway in terms of the two fields it connects.

OUTPUT FORMAT (file planting.out):

Print the minimum number of types of grass that Farmer John needs to use.

SAMPLE INPUT:

```
4
1 2
4 3
2 3
```

SAMPLE OUTPUT:

```
3
```

In this simple example, there are 4 fields all connected in a linear fashion. A minimum of three grass types are needed. For example, Farmer John could plant the fields with grass types A, B, and C as A - B - C - A.

Problem credits: Dhruv Rohatgi

4. (USACO 2016 December - Silver 3) Moocast

Farmer John's N cows ($1 \leq N \leq 200$) want to organize an emergency "moo-cast" system for broadcasting important messages among themselves.

Instead of mooing at each-other over long distances, the cows decide to equip themselves with walkie-talkies, one for each cow. These walkie-talkies each have a limited transmission radius -- a walkie-talkie of power P can only transmit to other cows up to a distance of P away (note that cow A might be able to transmit to cow B even if cow B cannot transmit back, due to cow A's power being larger than that of cow B). Fortunately, cows can relay messages to one-another along a path consisting of several hops, so it is not necessary for every cow to be able to transmit directly to every other cow.

Due to the asymmetrical nature of the walkie-talkie transmission, broadcasts from some cows may be more effective than from other cows in their ability to reach large numbers of recipients (taking relaying into account). Please help the cows determine the maximum number of cows that can be reached by a broadcast originating from a single cow.

INPUT FORMAT (file moocast.in):

The first line of input contains N.

The next N lines each contain the x and y coordinates of a single cow (integers in the range $0 \ldots 25{,}000$) followed by p, the power of the walkie-talkie held by this cow.

OUTPUT FORMAT (file moocast.out):

Write a single line of output containing the maximum number of cows a broadcast from a single cow can reach. The originating cow is included in this number.

SAMPLE INPUT:

```
4
1 3 5
5 4 3
7 2 1
6 1 1
```

SAMPLE OUTPUT:

```
3
```

In the example above, a broadcast from cow 1 can reach 3 total cows, including cow 1.

Problem credits: Brian Dean

5. (USACO 2016 US Open - Silver 3) Closing the Farm

Farmer John and his cows are planning to leave town for a long vacation, and so FJ wants to temporarily close down his farm to save money in the meantime.

The farm consists of N barns connected with M bidirectional paths between some pairs of barns ($1 \leq N, M \leq 3000$). To shut the farm down, FJ plans to close one barn at a time. When a barn closes, all paths adjacent to that barn also close, and can no longer be used.

FJ is interested in knowing at each point in time (initially, and after each closing) whether his farm is "fully connected" -- meaning that it is possible to travel from any open barn to any other open barn along an appropriate series of paths. Since FJ's farm is initially in somewhat in a state of disrepair, it may not even start out fully connected.

INPUT FORMAT (file closing.in):

The first line of input contains N and M. The next M lines each describe a path in terms of the pair of barns it connects (barns are conveniently numbered $1 \ldots N$). The final N lines give a permutation of $1 \ldots N$ describing the order in which the barns will be closed.

OUTPUT FORMAT (file closing.out):

The output consists of N lines, each containing "YES" or "NO". The first line indicates whether the initial farm is fully connected, and line $i+1$ indicates whether the farm is fully connected after the ith closing.

SAMPLE INPUT:

```
4 3
1 2
2 3
3 4
3
4
1
2
```

SAMPLE OUTPUT:

```
YES
NO
YES
YES
```

Problem credits: Yang Liu