I. If there is a Lyapunov function satisfying asymptotic stability over domain D, and set $\Omega c = \{x \in R^n \mid V(x) \leq c\}$ is bounded and contained in D, all trajectories starting in Ωc remains there and converges to 0 at $t \rightarrow \infty$, where Ωc is the region of attraction.

Based on the above description, please design your own non-linear systems with local asymptotic stability (non-linear system should be designed by yourself), where the designed system must satisfy that at least one initial value of the state is in the set D, but the final trajectory of the initial value of the state diverges to infinity. Also, complete the following tasks.

1. (30%) For the designed 2-order system,
   - 1.1 find a Lyapunov function.
     - 1.1.1 Find the set D. (3 marks)
     - 1.1.2 Find two initial states in the set D, one of which will eventually converge to an equilibrium point, and the other will diverge to infinity. Also, show the state response graph (states and time axis) for the two different initial states. (6 marks)
     - 1.1.3 Find the set Ωc. (2 marks)
     - 1.1.4 Plot the 3D Lyapunov function using MATLAB and show the two initial state response trajectories on this graph. (2 marks)
   - 1.2 find another different Lyapunov function.
     - 1.2.1 Find the set D. (3 marks)
     - 1.2.2 Find two initial states in the set D, one of which will eventually converge to an equilibrium point, and the other will diverge to infinity. Also, show the state response graph (states and time axis) for the two different initial states. (6 marks)
     - 1.2.3 Find the set Ωc. (2 marks)
     - 1.2.4 Plot the 3D Lyapunov function using MATLAB and show the two initial state response trajectories on this graph. (2 marks)
   - 1.3 Compare and explain the difference according to the Ωc and D results in 1.1 and 1.2. (4 marks)
2. (20%) For the designed 3-order system,
   - 2.1 find a Lyapunov function.
     - 2.1.1 Find the set D. (2 marks)
     - 2.1.2 Find two initial states in the set D, one of which will eventually converge to an equilibrium point, and the other will diverge to infinity. Also, show the state response graph (states and time axis) for the two different initial states. (6 marks)

2.1.3 Find the set Ωc. (2 marks)

2.2 find another different Lyapunov function.

2.2.1 Find the set D. (2 marks)

2.2.2 Find two initial states in the set D, one of which will eventually converge to an equilibrium point, and the other will diverge to infinity. Also, show the state response graph (states and time axis) for the two different initial states. (6 marks)

2.2.3 Find the set Ωc. (2 marks)

Answer:

Using artificial intelligence models(Please fill in the blanks):

☐100%

☐50%

☐20%

☐0%

Assessment:

1. Selection of the designed non-linear system (the non-linear system should not be copied from the literature).
2. Accuracy.
3. Detail.
4. Clarity.

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