Assignment #4 MOGA Assignment

1. You can refer to the paper: NSGA-II (A Fast and Elitist Multiobjective Genetic Algorithm) for more details of the MOGA by Deb.
2. Table I lists the benchmark functions.
3. Solve three problems of your choice from Table I, including at least one with dimension 10 (i.e., ZDT4, ZDT6) and one with dimension 30 (i.e., ZDT1, ZDT2, ZDT3). A maximum of 25000 function evaluations (FEs) can only be used. (每人可任選3個problem進行最佳化，但至少需要包含dimension為10 (i.e., ZDT4, ZDT6)及dimension為30 (i.e., ZDT1, ZDT2, ZDT3)之問題各一個，並討論其執行結果)
4. Submit a report of no less than 6 pages, including experimental results, analysis, and discussions together with your source code before the deadline. Submitting late work will be penalized by a deduction of 10% per day of the total mark awarded for each assignment. (作業繳交包含程式碼與書面報告，逾時成績逐日扣-10%)

TABLE I
TEST PROBLEMS USED IN THIS STUDY

Problem	n	Variable bounds	Objective functions	Optimal solutions	Comments
SCH	1	$[-10^3, 10^3]$	$f_1(x) = x^2$ $f_2(x) = (x-2)^2$	$x \in [0, 2]$	convex
FON	3	$[-4, 4]$	$f_1(\mathbf{x}) = 1 - \exp\left(-\sum_{i=1}^{3}\left(x_i - \frac{1}{\sqrt{3}}\right)^2\right)$ $f_2(\mathbf{x}) = 1 - \exp\left(-\sum_{i=1}^{3}\left(x_i + \frac{1}{\sqrt{3}}\right)^2\right)$	$x_1 = x_2 = x_3$ $\in [-1/\sqrt{3}, 1/\sqrt{3}]$	nonconvex
POL	2	$[-\pi, \pi]$	$f_1(\mathbf{x}) = \left[1 + (A_1 - B_1)^2 + (A_2 - B_2)^2\right]$ $f_2(\mathbf{x}) = \left[(x_1+3)^2 + (x_2+1)^2\right]$ $A_1 = 0.5\sin 1 - 2\cos 1 + \sin 2 - 1.5\cos 2$ $A_2 = 1.5\sin 1 - \cos 1 + 2\sin 2 - 0.5\cos 2$ $B_1 = 0.5\sin x_1 - 2\cos x_1 + \sin x_2 - 1.5\cos x_2$ $B_2 = 1.5\sin x_1 - \cos x_1 + 2\sin x_2 - 0.5\cos x_2$	(refer [1])	nonconvex, disconnected
KUR	3	$[-5, 5]$	$f_1(\mathbf{x}) = \sum_{i=1}^{n-1}\left(-10\exp\left(-0.2\sqrt{x_i^2 + x_{i+1}^2}\right)\right)$ $f_2(\mathbf{x}) = \sum_{i=1}^{n}\left(\lvert x_i\rvert^{0.8} + 5\sin x_i^3\right)$	(refer [1])	nonconvex
ZDT1	30	$[0, 1]$	$f_1(\mathbf{x}) = x_1$ $f_2(\mathbf{x}) = g(\mathbf{x})\left[1 - \sqrt{x_1/g(\mathbf{x})}\right]$ $g(\mathbf{x}) = 1 + 9\left(\sum_{i=2}^{n} x_i\right)/(n-1)$	$x_1 \in [0, 1]$ $x_i = 0,$ $i = 2, \ldots, n$	convex
ZDT2	30	$[0, 1]$	$f_1(\mathbf{x}) = x_1$ $f_2(\mathbf{x}) = g(\mathbf{x})\left[1 - (x_1/g(\mathbf{x}))^2\right]$ $g(\mathbf{x}) = 1 + 9\left(\sum_{i=2}^{n} x_i\right)/(n-1)$	$x_1 \in [0, 1]$ $x_i = 0,$ $i = 2, \ldots, n$	nonconvex
ZDT3	30	$[0, 1]$	$f_1(\mathbf{x}) = x_1$ $f_2(\mathbf{x}) = g(\mathbf{x})\left[1 - \sqrt{x_1/g(\mathbf{x})} - \frac{x_1}{g(\mathbf{x})}\sin(10\pi x_1)\right]$ $g(\mathbf{x}) = 1 + 9\left(\sum_{i=2}^{n} x_i\right)/(n-1)$	$x_1 \in [0, 1]$ $x_i = 0,$ $i = 2, \ldots, n$	convex, disconnected
ZDT4	10	$x_1 \in [0, 1]$ $x_i \in [-5, 5],$ $i = 2, \ldots, n$	$f_1(\mathbf{x}) = x_1$ $f_2(\mathbf{x}) = g(\mathbf{x})\left[1 - \sqrt{x_1/g(\mathbf{x})}\right]$ $g(\mathbf{x}) = 1 + 10(n-1) + \sum_{i=2}^{n}\left[x_i^2 - 10\cos(4\pi x_i)\right]$	$x_1 \in [0, 1]$ $x_i = 0,$ $i = 2, \ldots, n$	nonconvex
ZDT6	10	$[0, 1]$	$f_1(\mathbf{x}) = 1 - \exp(-4x_1)\sin^6(6\pi x_1)$ $f_2(\mathbf{x}) = g(\mathbf{x})\left[1 - (f_1(\mathbf{x})/g(\mathbf{x}))^2\right]$ $g(\mathbf{x}) = 1 + 9\left[\left(\sum_{i=2}^{n} x_i\right)/(n-1)\right]^{0.25}$	$x_1 \in [0, 1]$ $x_i = 0,$ $i = 2, \ldots, n$	nonconvex, nonuniformly spaced

All objective functions are to be minimized.

Reference:
https://pymoo.org/problems/multi/zdt.html