1.

```
clc; clear; close all;
```

```
(1)-----------------------------------
```

```
syms x1 x2

eq1 = x2 == 0;                          % dx1/dt = 0
eq2 = -x1 + (x1^3)/6 - x2 == 0;         % dx2/dt = 0

% Solve the system of equations
equilibrium_points = solve([eq1, eq2], [x1, x2]);

disp('(a) Equilibrium Points:');
```

```
(a) Equilibrium Points:
```

```
disp([equilibrium_points.x1, equilibrium_points.x2]);
```

$$\begin{pmatrix} 0 & 0 \\ \sqrt{6} & 0 \\ -\sqrt{6} & 0 \end{pmatrix}$$

```

% Define the system equations
f1 = x2;
f2 = -x1 + (x1^3)/6 - x2;

% Define the candidate Lyapunov function
V = (1/2) * x1^2 + (1/2) * x2^2;

% Compute the Jacobian matrix
J = jacobian([f1, f2], [x1, x2]);
disp('Jacobian Matrix of the system:');
```

```
Jacobian Matrix of the system:
```

```
disp(J);
```

$$\begin{pmatrix} 0 & 1 \\ \frac{{x_1}^2}{2} - 1 & -1 \end{pmatrix}$$

```

% Evaluate the Jacobian matrix at each equilibrium point
disp('Linearized System Matrices at Equilibrium Points:');
```

```
Linearized System Matrices at Equilibrium Points:
```

```
for i = 1:length(equilibrium_points.x1)
```

```
    J_eq = subs(J, [x1, x2], [equilibrium_points.x1(i),
equilibrium_points.x2(i)]);
    fprintf('Jacobian at Equilibrium %d:\n', i);
    disp(vpa(J_eq, 4));  % Display numerical values

    % Compute eigenvalues for stability analysis
    eigenvalues = eig(double(J_eq));
    fprintf('Eigenvalues at Equilibrium %d:\n', i);
    disp(eigenvalues);

    % Stability check
    if all(real(eigenvalues) < 0)
        fprintf('Equilibrium %d is stable.\n\n', i);
    elseif any(real(eigenvalues) > 0)
        fprintf('Equilibrium %d is unstable.\n\n', i);
    else
        fprintf('Equilibrium %d has marginal stability.\n\n', i);
    end
end
```

```
Jacobian at Equilibrium 1:
```

$$\begin{pmatrix} 0 & 1.0 \\ -1.0 & -1.0 \end{pmatrix}$$

```
Eigenvalues at Equilibrium 1:
  -0.5000 + 0.8660i
  -0.5000 - 0.8660i
Equilibrium 1 is stable.
Jacobian at Equilibrium 2:
```

$$\begin{pmatrix} 0 & 1.0 \\ 2.0 & -1.0 \end{pmatrix}$$

```
Eigenvalues at Equilibrium 2:
     1
    -2
Equilibrium 2 is unstable.
Jacobian at Equilibrium 3:
```

$$\begin{pmatrix} 0 & 1.0 \\ 2.0 & -1.0 \end{pmatrix}$$

```
Eigenvalues at Equilibrium 3:
     1
    -2
Equilibrium 3 is unstable.
```

2.

```
clc; clear; close all;
```

```
(2)------------------------------------
```

```
syms x1 x2

eq1 = x1 - x2 == 0;                          % dx1/dt = 0
```

```
eq2 = 0.1*x1 - 2*x2 - x1^2 - 0.1*x1^3 == 0;     % dx2/dt = 0

% Solve the system of equations
equilibrium_points = solve([eq1, eq2], [x1, x2]);

disp('(a) Equilibrium Points:');
```

```
(a) Equilibrium Points:
```

```
disp(equilibrium_points.x1);
```

$$\begin{pmatrix} 0 \\ -\sqrt{6}-5 \\ \sqrt{6}-5 \end{pmatrix}$$

```
disp(equilibrium_points.x2);
```

$$\begin{pmatrix} 0 \\ -\sqrt{6}-5 \\ \sqrt{6}-5 \end{pmatrix}$$

```
% Define the system equations
f1 = -x1 + x2;
f2 = 0.1*x1 - 2*x2 - x1^2 - 0.1*x1^3;

% Compute the Jacobian matrix
J = jacobian([f1, f2], [x1, x2]);
disp('Jacobian Matrix of the system:');
```

```
Jacobian Matrix of the system:
```

```
disp(J);
```

$$\begin{pmatrix} -1 & 1 \\ -\frac{3\,x_1^2}{10}-2\,x_1+\frac{1}{10} & -2 \end{pmatrix}$$

```
% Evaluate the Jacobian matrix at each equilibrium point
disp('Linearized System Matrices at Equilibrium Points:');
```

```
Linearized System Matrices at Equilibrium Points:
```

```
for i = 1:length(equilibrium_points.x1)
    J_eq = subs(J, [x1, x2], [equilibrium_points.x1(i),
equilibrium_points.x2(i)]);
    fprintf('Jacobian at Equilibrium %d:\n', i);
    disp(vpa(J_eq, 4));  % Display numerical values
```

```
    % Compute eigenvalues for stability analysis
    eigenvalues = eig(double(J_eq));
    fprintf('Eigenvalues at Equilibrium %d:\n', i);
    disp(eigenvalues);

    % Stability check
    if all(real(eigenvalues) < 0)
        fprintf('Equilibrium %d is stable.\n\n', i);
    elseif any(real(eigenvalues) > 0)
        fprintf('Equilibrium %d is unstable.\n\n', i);
    else
        fprintf('Equilibrium %d has marginal stability.\n\n', i);
    end
end
```

```
Jacobian at Equilibrium 1:
```

$$\begin{pmatrix} -1.0 & 1.0 \\ 0.1 & -2.0 \end{pmatrix}$$

```
Eigenvalues at Equilibrium 1:
   -0.9084
   -2.0916
Equilibrium 1 is stable.
Jacobian at Equilibrium 2:
```

$$\begin{pmatrix} -1.0 & 1.0 \\ -1.649 & -2.0 \end{pmatrix}$$

```
Eigenvalues at Equilibrium 2:
  -1.5000 + 1.1830i
  -1.5000 - 1.1830i
Equilibrium 2 is stable.
Jacobian at Equilibrium 3:
```

$$\begin{pmatrix} -1.0 & 1.0 \\ 3.249 & -2.0 \end{pmatrix}$$

```
Eigenvalues at Equilibrium 3:
    0.3707
   -3.3707
Equilibrium 3 is unstable.
```