II. Consider the following system

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{-f\cos x_1 - mLx_2^2 \sin x_1 \cos x_1 + (M+m)g\sin x_1}{L(M + m\sin^2 x_1)}$$

$$y = x_1$$

where m= 0.1, g = 9.81, M = 1.0, L=1 and f is control law. Complete the following tasks.

1. (20%) Based on input-output linearization, design a tracking controller to make the output y(t) track a desired trajectory y_d (t)=cos(2t). Also, show the state response graph (states and time axis) for the two different initial states x=[0.1 0]^T, x=[1.5 0]^T.
2. (30%) Assume f=0 and sampling step=0.01 seconds. Based on Koopman theory, complete the following tasks.
   2.1. Find the observable function. (5 marks)
   2.2. Find Koopman operator using the collected N state transitions. (5 marks)
   2.3. Predict using Koopman operator. Also, show the predicted and actual state response graph (states and time axis) for the two different initial states x=[0.1 0]^T, x=[1.5 0]^T. Assume total time = 5 seconds. (5 marks)
   2.4. Calculate error between the predicted and actual values using RMSE for the two different initial states x=[0.1 0]^T, x=[1.5 0]^T. Assume total time = 5 seconds. (5 marks)
   2.5. Use the different observable function and re-complete 2.2,2.3 and 2.4. Explain the difference between the results of 2.3 and 2.4 and explain why. (10 marks)

Answer:

Percentage of each subproblem solved using an AI model (please fill in the blanks):

☐100%

☐50%

☐20%

☐0%

Assessment:

1. Accuracy.
2. Detail.
3. Clarity.