1.

Consider

$$\dot{x}_1 = -x_1 + x_2^2$$
$$\dot{x}_2 = -x_2$$

Analyze the nonlinear systems using Lyapunov's theorem.

(a) Find a Lyapunov function.

(b) Plot a Lyapunov function using MATLAB

(c) Plot state trajectories of a dynamical system near the origin using MATLAB

(d) Show the origin is stable using Lyapunov's direct method.

(e) Show the origin is asymptotically stable using Lyapunov's direct method.

(f) Is it global asymptotically stable?

2.

Consider

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -x_1 - x_2 - (2x_2 + x_1)(1 - x_2^2)$$

Analyze the nonlinear systems using Lyapunov's theorem.

(d) Find a Lyapunov function.

(e) Plot a Lyapunov function using MATLAB

(f) Plot state trajectories of a dynamical system near the origin using MATLAB

(b) Show the origin is stable using Lyapunov's direct method.

(c) Show the origin is asymptotically stable using Lyapunov's direct method.