Information Fusion for Online Estimation of the Behavior of Traffic Participants using Belief Function Theory

Tommaso Benciolini, Xuhui Zhang, Dirk Wollherr, Marion Leibold

***Abstract*— Motion planning algorithms for automated vehicles need to assess the intended behavior of other Traffic Participants (TPs), in order to predict the likely future trajectory of TPs and plan the motion consequently. Information resulting from several sources, like sensors, must be gathered and combined into a reliable estimate of the intended behavior of TPs. Such estimate must be sufficiently steady and quantify the inherent uncertainty around the assessment. We present a novel information fusion framework to combine information from different sources into a coherent and reliable estimate. To explicitly account for the uncertainty of estimates, we leverage on the Belief Function Theory [1] and explicitly evaluate and handle possible disagreements between estimates individually provided by the sources. The framework is flexible and can also handle sources that do not discern between some of the considered behaviors and are only capable of assessing the probability of union or clusters of different behaviors. We discuss the strengths of the approach through simulations in SUMO, comparing it with the Interactive Multiple Model [2] algorithm.**

I. INTRODUCTION

The automated driving field has recently seen incredible progress and autonomous vehicles are nowadays capable of dealing with complex situations. Especially in urban automated driving, handling the interaction with traffic participants (TPs) is complicated, as it requires predicting their future motion. Connected and Automated Vehicles (CAVs) alleviate the task, as they share information making other CAVs aware of the future intention. However, although the share of CAVs on roads is expected to quickly increase, still for a significant amount of time traffic will mainly be characterized by human-driven vehicles, whose future motion is unknown. Moreover, also in the long term, automated vehicles will continue to have to deal with the presence of non-communicating TPs, such as cyclists and pedestrians.

In control-based autonomous driving schemes as Model Predictive Control, the future behavior of the automated vehicle is optimized, accounting for the future moves of the other TPs, whose future motion must, therefore, be predicted. Several common approaches to infer the expected future motion of the other TPs rely on running prediction models starting from the current position and velocity of TPs. The design of such prediction models is a challenging task, as they have to consider both the dynamics of TPs and of their internal objective, depending on the driving style and on the maneuver being executed [3]. Tuning the prediction model depending on the maneuver being executed or even the selection of different prediction models depending on the traffic situation requires an online assessment of the behavior of TPs.

The intention driving the behavior of a given TP, be that another vehicle, a cyclist, or a pedestrian, can be estimated based on information collected from onboard sensors, cameras and radars along the road and intelligent infrastructures, that might communicate with automated vehicles. The past trajectory of each TP can be analyzed and candidate behaviors can be ranked evaluating a measure of similarity with respect to properly designed models representing different situations [4]. In [5] a lane change detection mechanism is introduced, relying on phase models of lane changes. However, phase models for lane changes are required, thus the method does not generalize well. In [6], the Interacting Multiple Model (IMM) algorithm is used to estimate the intended trajectory of target vehicles. However, the variability of the estimation is not considered and the estimation might suddenly and repetitively change if none of the considered models perfectly matches the dynamics, making the estimate unreliable. Furthermore, heuristics and other sources, e.g., traffic statistics, can provide a bias useful to categorize the behavior of TPs.

The information provided by the several sources must be combined and gathered in a coherent estimate, that can be passed on to a motion planner accounting for the multiple possible future motions of TPs depending on their probability, as, e.g., in our recent work [7]. One main challenge in combining different behavior estimations lies in handling the inherent uncertainty around the information provided by each source and in their possible discord. Furthermore, the resulting overall estimation of the behavior of TPs should be made as stable and reliable as possible and the reliability of the provided information should be explicitly quantified, to allow motion planners to subsequently take action to address the uncertainty and to avoid aggressive decisions until the estimation is reliable enough. Finally, combining non-homogeneous individual estimations is challenging, for example if some of the sources do not allow to discern between some of the candidate behaviors and to estimate their probability separately, but rather only assign probability to their unions, that is, that either of them will occur. Examples of sensors only capable of assigning belief mass to the union of singletons for the genetic mutation detection application can be found in [8] and for mine detection in [9].

In this work we rely on Belief Function Theory (BFT)

This work is funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) with the project number 490649198.

T. Benciolini, X. Zhang, D. Wollherr, and M. Leibold are with the Chair of Automatic Control Engineering at the Technical University of Munich, Munich, Germany (email: {t.benciolini; xuhui.zhang; dw; marion.leibold}@tum.de).

to combine information resulting from different sources in a reliable way, explicitly accounting for the uncertainty around the estimate. BFT has been utilized to account for the reliability of the information in a safe reinforcement learning framework in our previous works [10], [11], and for grid-based mapping and tracking in our previous works [12], [13]. In [14], BFT is utilized for sensor fusion, through Dempster-Shafer evidence combination rule and an improved evidence combination rule that weighs sensors. However, conflict among observations is not dealt with and thus additional observations always render the result more certain, even when they do not agree with each other. Several conflict detection mechanisms were proposed in [15], which, however, do not consider the uncertainty of the sources, which is to some extent unreasonable. A conflict handling mechanism called Uncertainty Maximization has been presented in [16], transferring part of the belief masses to the uncertainty independently of the degree of conflict, which, depending on the application, is also unreasonable for small conflicts.

We provide a two-step framework to combine the information about behavior of TPs provided by multiple sources in a consistent and reliable estimate using BFT. At first, a modification of the Dempster-Shafer rule is used to gather the information collected by several sources independently during the last sampling time. Sensors estimating the probability of union of events instead of the probabilities of singletons can also be adopted. Furthermore, we design a new conflict handling mechanism taking the reliability of the information provided by each source into account. Then, the information is fused with the estimate from the previous iteration, giving steadiness to the framework. We compare our approach with the IMM algorithm [2] through numerical simulations in SUMO [17].

The remainder of the paper is organized as follows. An overview of the proposed approach is given in Section II. Remarks on BFT and the formal definition of opinions and possible generation mechanisms are given in Section III, whereas the steps of the belief processing mechanism and their properties are outlined in Section IV. Simulations results and conclusive remarks are given in Section V and VI, respectively.

II. Overview of the Approach

The information flow considered in this work is repeated in parallel for each TP separately. The procedure can be divided into three parts, namely generation of opinions, multi-source information fusion and temporal belief distribution propagation. The scheme of information handling is represented in Figure 1. Our focus is combining information with a view at assessing the behavior of TPs, but the framework is not limited to this specific application.

At first, information is collected from each source independently. This step happens outside and independently of our belief processing framework. Based on new data, at each time step k, n_S opinions $\boldsymbol{\omega}_k^1, \dots, \boldsymbol{\omega}_k^{n_S}$ are generated in parallel, one for every source of information, that will be defined more precisely in Section III. The opinion consists not only

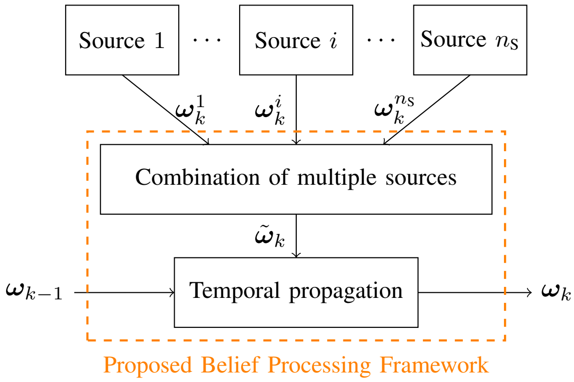


Fig. 1: Scheme of the approach.

of an estimate of the probability of considered outcome, but include an assessment of the reliability of the information provided, taking advantage of the "uncertainty" concept from BFT [18], that represents the epistemic uncertainty regarding the reliability of information. Depending on the sensor or detection mechanism, some individual and separate outcomes might not be distinguishable for a given source of information. In those cases, we allow the source to provide an estimation assigning probability to the union of singletons, rather than, for example, redistributing equally the probability within the singletons involved.

Secondly, independently-generated opinions are combined, gathering the information resulting from the current time step. An agreement between individual estimations provided by different sources is found, based on the last samples available, reporting possible changes in the behavior. The processing of individual estimations is independent of how opinions are generated. Observe that the combined opinion is not just an average of the estimated probabilities from every separated source. Rather, opinions are merged considering the uncertainty of each of them, weighing in more sources that provide more reliability. Furthermore, an estimation on the uncertainty of the combined estimate is also provided, so that several independent but coherent opinions reduce the overall level of uncertainty, whereas possible disagreement is treated through a mechanism outlined in Section IV-A. The combined opinion is labelled $\tilde{\boldsymbol{\omega}}_k$.

Finally, to propagate the estimation over time, the combined opinion gathering information from the current time step $\tilde{\boldsymbol{\omega}}_k$ is fused with the information obtained up to the previous time step, $\boldsymbol{\omega}_{k-1}$, resulting in $\boldsymbol{\omega}_k$, the output of the framework. Differently from the previous combination, here the goal is to give steadiness to the estimate, so that too sudden fluctuations in the estimate, that would make the information unreliable, are attenuated.

In this work, singletons and individual probabilities represent different behaviors of TPs, resulting in different expected future trajectories. As a possible purpose of providing such an estimate, we refer to the motion planning problem using predictive controllers. Therein, an estimation on the probabilities of several candidate future trajectories of TPs allows to optimize with respect to different outcomes depending on their probabilities, as in our previous work [7]. Nevertheless, the framework for information handling is potentially general and applicable for other purposes.

III. Opinions

In this section, we rigorously define the concept of opinion and explain how opinions can be obtained from data. Then, Section IV outlines the mechanism to combine opinions. Depending on the application, different definitions of opinion are possible. The aim of our approach is to combine different sources of information explicitly assessing and handling the inherent uncertainty stemming from incomplete information and from relying on approximated models. Each event or singleton is associated to a possible behavior or maneuver of the TP, determining a (nominal) future trajectory.

In order to quantitatively take the uncertainty into account, rather than the Bayesian probability framework we employ Belief Function Theory [1]. Probabilities assigned to events are called *belief masses*. In the following, we assume that the sources of information provide estimations on the probability of the N outcomes possibly independently of a clear mathematical model. Therefore, the considered estimates of probabilities are in fact subjective probabilities [19].

Definition 1: An opinion is the vector

$$\boldsymbol{\omega} = [b_1, \ldots, b_M, \mu]^\top , \quad (1)$$

where scalars $b_1, \ldots, b_M, \mu \geq 0$ are the belief masses. Opinions are 1-norm unit vectors, i.e.,

$$\sum_{i=0}^{M} b_i + \mu = 1. \quad (2)$$

A major difference of BFT compared to Bayesian probability is that probability can be assigned not only to each singleton event, but also to the union of several singletons [20]. Thus, the number of considered belief masses can be larger than the number of singleton events, i.e., $M \geq N$, since, different belief masses can refer to non-mutually-exclusive events. This allows more flexibility in the information handling, considering that some sources of information might not be able to distinguish between different singleton events, and thus only assess the probability of their union. Furthermore, μ is the belief mass assigned to the union of all singletons and is called *uncertainty* [16]. Variable μ gives an additional degree of freedom to quantify the epistemic uncertainty, e.g., assessing the reliability of the information carried by the other belief masses. Uncertainty μ is understood as the belief mass of the whole outcome set, that is, the probability of any of the considered outcomes to occur. Since it represents the gap between the sum of the belief masses of the considered events and 1, the uncertainty is a measure of the inaccuracy of the information, representing the belief mass that cannot be allocated and anyhow further specified yet. Thus, μ is inversely proportional to the subjective confidence in the opinion.

Remark 1: Belief masses $b_1, \ldots, b_N$ are not probabilities, since they do not add up to one and are referred also to unions of singletons. Nevertheless, standard probabilities are obtained for example equally dividing probabilities of unions between all considered events. In doing so, also the belief mass of uncertainty must be equally split among all events.

Example of Opinion Generation

In this section, an example of opinion generation for a TP motion estimation application is given, to clarify how belief masses and uncertainty can be estimated from data. As previously mentioned, the opinion generation takes place upstream of our method and is not part of it, thus this section serves as a pure explanatory example. Any other opinion generation mechanism resulting in opinions in the form (1) is suitable.

We consider a set of candidate behaviors or maneuvers, each determining a (nominal) future trajectory, and propose an opinion generation mechanism based on the lateral y-position of a TP measured by a noisy sensor. For each considered candidate behavior, let $y^i,\ i = 1, \ldots, N$ be the y-position of the nominal trajectory realizing the behavior. Furthermore, for every candidate behavior, we assume that the variance $(\sigma_y^i)^2$ of the y-position is available, quantifying the expected deviation from the nominal trajectory when realizing that behavior, also considering the sensor noise. For example, three behaviors could be turning right, proceeding straight, and turning left at an intersection. At first, the probability of each behavior are estimated comparing the noisy measurements y_k collected at the current time step k with the nominal trajectory y_k^i for maneuver i, accounting for the expected deviation from the nominal trajectory. Precisely, measures of similarities can be obtained from a Gaussian kernel, i.e.,

$$p_k^i = \frac{1}{\sqrt{2\pi}\sigma_y^i} \exp\left\{ -\frac{\left(y_k - y_k^i\right)^2}{2(\sigma_y^i)^2} \right\}, \quad (3)$$

then rescaling masses p_k^i so that they sum to one.

Then, to assess the uncertainty of the probabilities obtained from (3), we consider the perturbation of the distribution of p_k^i over a window of n steps, i.e.,

$$\mu_k = \frac{1}{2(n-1)} \sum_{h=k-n+2}^{k} \|\boldsymbol{p}_h - \boldsymbol{p}_{h-1}\|_1, \quad (4)$$

where $\boldsymbol{p} = \left[p^1, \ldots, p^N\right]^\top$. Expression (4) gives a measure of the variability of the masses distribution over the time window. The summation is normalized with respect to the largest theoretical possible value of variation, ensuring that $\mu_k \in [0, 1]$. Observe that (4) increases in presence of considerable variations in the estimated probability of each considered behavior albeit the relative order between behaviors remains the same. Indeed, since a large perturbations of the mass distributions reflect little reliability of the source in general, (4) serves as a measure of uncertainty as intended in this work. However, since uncertainty is a subjective measure, there is no unique method to quantify it.

Finally, the opinion is obtained as

$$\boldsymbol{\omega}_k = \left[(1-\mu_k)\boldsymbol{p}_k^\top, \mu_k\right]^\top, \tag{5}$$

that is, rescaling the probabilities from (3) given the quantified uncertainty. Depending on the source of information, a similar approach can be used to generate other opinions.

Remark 2: The same procedure applied to the longitudinal velocity of a TP approaching an intersection would not be able to discern between a right and a left turn. Indeed, in both cases, the expected longitudinal speed profile would be the same, as it is reasonable to imagine that a vehicle needs to slow down in a similar way when approaching the intersection. In this case, it is more reasonable to estimate the belief mass of proceeding straight and of a turn, without further specifying the direction of the turn. This is the reason why the the proposed framework admits sources that specify the belief masses of unions of singletons, allowing more flexibility.

Also non-sensor sources can be included, for example using statistics of the recorded traffic. Belief masses can be set proportionally to the statistical frequency of each behavior, and the uncertainty can be determined based on the variance of the recorded data. In this case, the resulting opinion is independent of the current behavior of a TP, but rather constitutes a bias, which can be included in the information fusion.

Remark 3: Independently of the opinion generation mechanism, the choice of the set of candidate behaviors plays a crucial role, as previously discussed in [7]. However, should a TP execute a maneuver that does not belong in the set of considered behaviors, the uncertainty component would increase, signalizing that none of the considered modes can be reliably trusted.

IV. Combination of Opinions

In this section, we outline the two steps composing the proposed belief processing framework, consisting of the combination of information from multiple sources and of the temporal propagation.

A. Combination of Multiple Sources

Here we introduce the first opinion combination mechanism. The aim is to gather the whole information collected at the current iteration, that is, merge opinions $\boldsymbol{\omega}^1, \ldots, \boldsymbol{\omega}_{\mathrm{S}}^n$, obtained from each source independently, in the combination $\overline{\boldsymbol{\omega}} = \left[\overline{b}_1, \ldots, \overline{b}_N, \overline{\mu}\right]^\top$. Bearing in mind that the scope of such information fusion is feeding an accommodation algorithm to account for several outcomes depending on their probabilities, e.g., [7], the combined opinion $\overline{\boldsymbol{\omega}}$ only comprises N beliefs for mutually-exclusive singleton events and assessment of the overall uncertainty. However, if of interest, straightforward adaptions allow to maintain belief masses of unions also in the remaining steps of the algorithm.

To combine the most recent opinions generated by different sources of information, we use a revised version of the Dempster's rule of combination [20] to allow the combined opinion to only consist of singletons and of the union of all singletons. Opinions are iteratively combined two at a time. We assume possibly heterogeneous sources of information, thus the opinions might, in general, consider different unions of singletons. Given two opinions $\boldsymbol{\omega}^{\mathrm{A}}$ and $\boldsymbol{\omega}^{\mathrm{B}}$, the new belief masses for all singletons $i = 1, \ldots, N$ and the overall uncertainty are obtained as:

$$\overline{b}_i = \frac{\sum_{j\cap h = i} b_j^{\mathrm{A}} b_h^{\mathrm{B}}}{1 - \sum_{(j\cap h)\not\subset \mathbb{I}} b_j^{\mathrm{A}} b_h^{\mathrm{B}}} \quad \forall i = 1, \ldots, N \tag{6a}$$

$$\overline{\mu} = \frac{\mu^{\mathrm{A}}\mu^{\mathrm{B}}}{1 - \sum_{(j\cap h)\not\subset \mathbb{I}} b_j^{\mathrm{A}} b_h^{\mathrm{B}}}, \tag{6b}$$

where $\mathbb{I} = \{1, \ldots, N, \cup_{i=1}^N i\}$ is the set comprising each singleton and the union of all singletons. The belief mass of each singleton is obtained summing contributions from every possible combination that makes the singleton possible, and then belief masses are normalized leaving out the belief mass of combinations that do not uniquely identify a singleton. For some unrealistic degenerate cases, the denominator in (6) could be zero: in such cases, the combined opinion $\overline{\boldsymbol{\omega}}$ is set as the completely uncertain opinion, i.e., $\overline{\mu} = 1$ and all other belief masses equal to zero, since further specifications of the belief masses are not possible.

The procedure is commutative with respect to the order of the opinions and satisfies the following property.

Theorem 1: The combined uncertainty $\overline{\mu}$ obtained from (6) cannot increase with respect to the two individual uncertainties μ^{A} and μ^{B}.

Proof: The proof is given in Appendix A. ■

As a result, the more opinions are considered, the smaller the uncertainty of the combination will be, following the idea that more independent sources make the information more reliable. However, this effect might also be undesirable, if the sources of information contradict one another. For this reason, we add a conflict detection mechanism, that reassigns part of the belief mass to the uncertainty of the combined opinion $\overline{\boldsymbol{\omega}}$ depending on the possible conflict among opinions.

Inspired by [15], [16], we define the conflict between two opinions as

$$C^{\mathrm{A,B}} = \frac{1}{2}\left\| \frac{\boldsymbol{b}^{\mathrm{A}}}{\|\boldsymbol{b}^{\mathrm{A}}\|_1} - \frac{\boldsymbol{b}^{\mathrm{B}}}{\|\boldsymbol{b}^{\mathrm{B}}\|_1} \right\|_1 \sqrt{(1-\mu^{\mathrm{A}})(1-\mu^{\mathrm{B}})}, \tag{7}$$

where $\boldsymbol{b}^{\mathrm{A}}$ and $\boldsymbol{b}^{\mathrm{B}}$ contain all belief masses of the opinions but the uncertainty component, i.e., $\boldsymbol{\omega} = \left[\boldsymbol{b}^\top, \mu\right]^\top$. $C^{\mathrm{A,B}}$ is designed to increase if the considered opinions assign differently the belief masses while having high confidence (small uncertainty). Consistency in the ratio of belief masses rather than values matters in the comparison of belief distribution, whereas uncertainty does not play a role, since it only scales belief masses. This is why belief masses are normalized before taking the difference in (7).

Then, drawing from [15], [16], belief masses from (6), i.e., $\overline{\boldsymbol{\omega}} = \left[\overline{\boldsymbol{b}}^\top, \overline{\mu}\right]^\top$, are redistributed as follows

$$\tilde{\boldsymbol{b}} = \left(\prod_{i,j} \left(1 - C^{i,j}\right)\right)^{\frac{1}{n_S}} \overline{\boldsymbol{b}} \tag{8a}$$

$$\tilde{\mu} = 1 - \|\tilde{\boldsymbol{b}}\|_1. \tag{8b}$$

The resulting opinion $\tilde{\boldsymbol{\omega}} = \left[\tilde{\boldsymbol{b}}^\top, \tilde{\mu}\right]^\top$ gathers the whole information collected by different sources in the last sampling time.

B. Temporal Propagation

After combining the information collected through different sources in the current time step, the combined opinion is mixed with the information collected up to the previous step. The aim of this second combination is the temporal propagation of the information, allowing the detection of patterns that are revealed over multiple steps. At the same time, providing a stable and reliable estimate preventing large fluctuations between consecutive time steps is of primary concern. Therefore, we adopt the Weighted Belief Fusion (WBF) operator [16].

Opinion $\tilde{\boldsymbol{\omega}}_k$ gathering information from the current time step and obtained through the procedure in Section IV-A is combined with the overall estimate $\boldsymbol{\omega}_{k-1}$ from the previous time step, originating the overall estimate for the current time step $\boldsymbol{\omega}_k = \left[\boldsymbol{b}_k^\top, \mu_k\right]^\top$, with

$$\boldsymbol{b}_k = \frac{\tilde{\boldsymbol{b}}_k(1-\tilde{\mu}_k)\mu_{k-1} + \boldsymbol{b}_{k-1}(1-\mu_{k-1})\tilde{\mu}_k}{\tilde{\mu}_k + \mu_{k-1} - 2\tilde{\mu}_k\mu_{k-1}} \tag{9a}$$

$$\mu_k = \frac{(2-\tilde{\mu}_k - \mu_{k-1})\tilde{\mu}_k\mu_{k-1}}{\tilde{\mu}_k + \mu_{k-1} - 2\tilde{\mu}_k\mu_{k-1}}. \tag{9b}$$

Observe that from (9), if one of the two opinions is certain ($\mu = 0$), then the result coincides with that one, that is the other one is neglected and uncertainty is also set to zero. If both opinions are certain but indicating different beliefs, then $\boldsymbol{\omega}_k$ is set to the completely uncertain opinion, to signal that the information is completely unreliable.

In general, the resulting opinion is obtained as a weighted average of the two resulting opinion, accounting for each of them depending on its uncertainty. Moreover, in this case uncertainty does not necessarily decrease and for example the combination of two identical opinions results in the same opinion as an output, leaving the level of uncertainty unchanged. Indeed, WBF can be adopted under assumption of dependent sources, thus additional sources do not necessarily result in additional evidence [16].

V. Simulation Results

In this section we showcase the belief processing framework through numerical simulations in SUMO. We consider two scenarios involving processing information provided by different sources to estimate the intended maneuver executed by a vehicle approaching an intersection and a pedestrian. We consider both the case in which sources provide coherent individual estimations and the case with conflict among sources. We compare our BFT-based framework with the IMM algorithm. The IMM algorithm consists of one Kalman Filter for every every candidate behavior, and estimates from each filter and combined depending on the estimated probability, accounting for possible switches between behaviors between consecutive time steps.

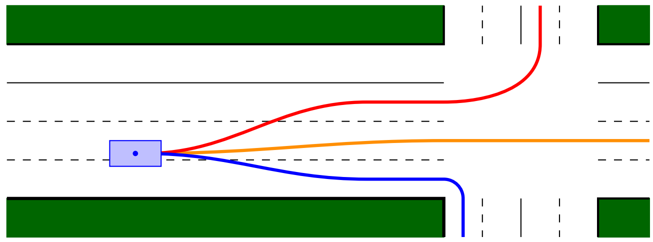


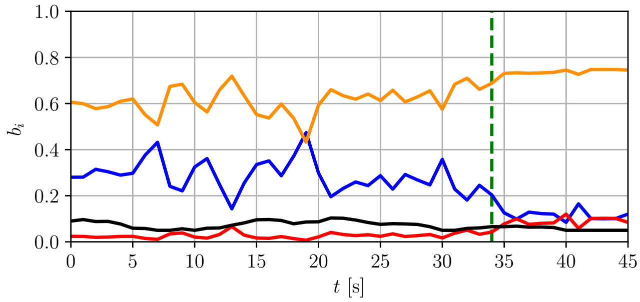
Fig. 2: Candidate behaviors of the vehicle.

In the simulations, the BFT-based framework combines opinions generated by three sources: a sensor for the lateral position applying the procedure presented in Section III; a sensor for the longitudinal velocity also applying the procedure from Section III; a constant opinion representing traffic statistics. The IMM framework is implemented along the lines of [7, Section IV-B], but considering as measurements the lateral position and the longitudinal velocity. The motion of both the vehicle and the pedestrian is realized using the TraCI library in SUMO and noisy measurements are used in both estimation frameworks.

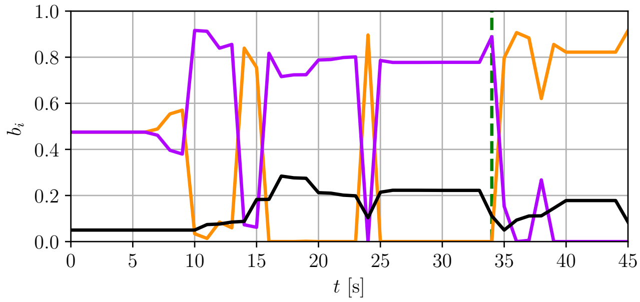
A. Uncertain Behavior of a Vehicle

In the first simulation, a vehicle approaches an intersection on a three-lane road and the candidate intended behaviors are: A) right turn, B) proceed straight, C) left turn, see Figure 2. The vehicle exhibits an ambiguous behavior as it approaches the intersection, possibly simulating a distracted driver. The vehicle stays on the center lane, as to proceed straight, but not at the center, rather proceeds irregularly and a little to the right of the lane, close to the edge. Thus, the opinion based on the lateral position regards going straight as the most likely maneuver, although the belief mass of a right turn maneuver is non-negligible, see Figure 3a. Furthermore, the vehicle slows down, therefore the opinion based on the longitudinal velocity tends to give a higher belief mass to the turn maneuver, see Figure 3b. This behavior is the union of the left and of the right turn maneuvers, that are not distinguishable considering the longitudinal velocity. Finally, the BFT framework includes a constant bias $\boldsymbol{\omega} = [b_{\text{right}}, b_{\text{straight}}, b_{\text{left}}, \mu]^\top = [0.18, 0.32, 0.17, 0.33]^\top$.

The result of the estimation for the two frameworks is presented in Figure 4a and Figure 4b, respectively. While the vehicle is still approaching the intersection and showing an ambiguous behavior, the opinions produced by the three sources are conflicting. The combination provided by our BFT-based framework generally assigns the relative highest belief to the proceed straight maneuver, which is altogether

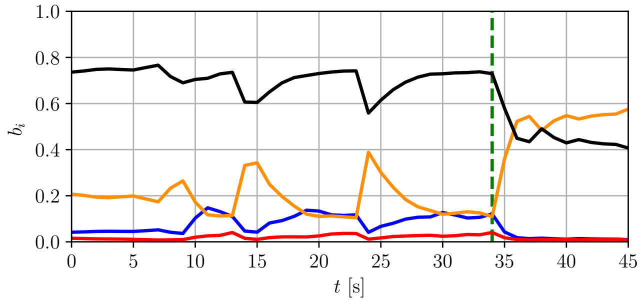


(a) Opinion generated from lateral position measurements: right turn (blue), proceed straight (orange), left turn (red), uncertainty (black).

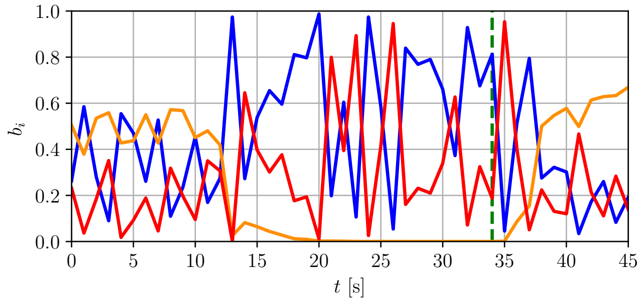


(b) Opinion generated from longitudinal velocity measurements: turn (purple), proceed straight (orange), uncertainty (black).

Fig. 3: Opinions generated by different sources. The vertical line in green represents the moment in which the vehicle reaches the intersection and proceeds straight.



(a) Combined opinion generated by the BFT-based framework.



(b) Probabilities estimated by the IMM.

Fig. 4: Probability estimate produced by the two frameworks: right turn (blue), proceed straight (orange), left turn (red), uncertainty (black). The vertical line in green represents the moment in which the vehicle reaches the intersection and proceeds straight.

to be considered the most likely. However, the uncertainty is very high, quantitatively expressing that a reliable estimate cannot be extracted by the current data. Therefore, a motion planner based on the BFT estimate might make use of the uncertainty information to use caution while the intention of this TP is not clear and the prediction could not be reliable. However, once the vehicle actually reaches the intersection, the conflict between sources is resolved and the belief mass of the proceed straight maneuver gradually increases and the uncertainty decreases. Conversely, the IMM estimation is extremely noisy and shows large and repetitive fluctuations even between consecutive time steps, since none of the models can consistently explain the data. Moreover, confidence oscillates between right turn and left turn, whereas the proceed straight maneuver only emerges as dominant when the vehicle reaches the intersection. As a matter of fact, the IMM considers the dynamics of the model, thus the irregular behavior is repeatedly interpreted as the beginning of a turning maneuver. The IMM estimation is not reliable enough to be used for prediction of TP future trajectories.

The IMM does not explicitly quantify probability uncertainty, rather model mismatches result in large estimation variance for the state estimate, which, depending on the application, might be impractical. Moreover, different sensors are included in the IMM framework as different outputs of the model. Thus, the handling of the degree of

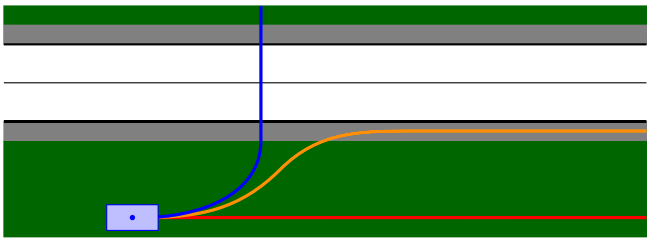
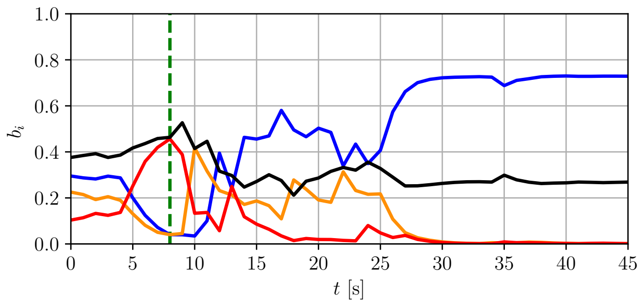


Fig. 5: Candidate behaviors of the pedestrian.

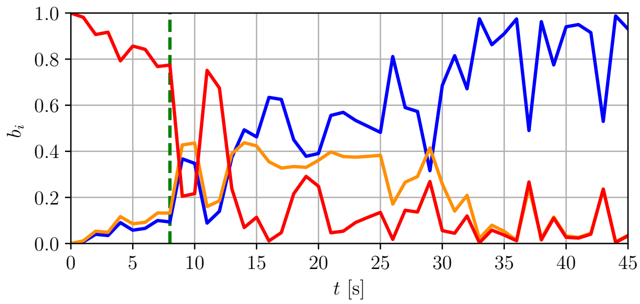
uncertainty of different sources in the IMM is restricted to specifying different statistical properties of the disturbances in the underlying models. However, such statistical properties can not be straightforwardly adapted online depending on the data collected over time, thus the IMM framework is less suited to cope with sources with different reliability. Furthermore, conflict between information provided by each sensor is not explicitly addressed and might reflect in large fluctuations in the estimate yielded at consecutive time steps.

B. *Clear Behavior of a Pedestrian*

In the second simulation, the two frameworks are compared in estimating the intended behavior of a pedestrian, initially located on the grass in the vicinity of the road. The three candidate behaviors are: A) remain on the grass, B) move to the sidewalk, C) jaywalk, see Figure 5. For



(a) Combined opinion generated by the BFT-based framework.



(b) Probabilities estimated by the IMM.

Fig. 6: Probability estimate produced by the two frameworks: grass (red), sidewalk (orange), jaywalking (blue), uncertainty (black). The vertical line in green represents the moment in which the pedestrian starts moving toward the road.

this simulation, both the BFT-based framework and the IMM collect measurements of the lateral velocity, rather than of the lateral position. Furthermore, the bias is set to $\boldsymbol{\omega} = [b_{\text{grass}}, b_{\text{sidewalk}}, b_{\text{jaywalk}}, \mu]^\top = [0.36, 0.22, 0.1, 0.32]^\top$, to discourage the combined BFT opinion from getting to the conclusion that the pedestrian is indeed attempting to cross the road outside of the crosswalk, as normally is not the case.

The results of the estimation for the two frameworks are presented in Figure 6a and Figure 6b, respectively. Despite discouraged by the bias opinion, the BFT framework quickly recognizes that the pedestrian wants to jaywalk. The two sensor-based opinions generate coherent estimates, thus the uncertainty of the combination is mainly due to partially contradicting measurements, because of the noise, and due to conflicts with the bias opinion. The latter, however, has a large uncertainty, being based on statistics. Therefore, the conflict handling mechanism introduced in Section IV-A does not severely affect the estimation and the uncertainty of the combined estimate is considerably smaller than in the previous simulation. Finally, the IMM also promptly recognizes that the pedestrian will jaywalk. However, due to the lack of the uncertainty component in the estimate, model mismatches and noisy measurement result in considerably large fluctuations in the estimated probabilities, requiring a post-processing of the estimate to smooth the results.

VI. Conclusions

We presented a novel information fusion framework to combine information resulting from multiple sources in a coherent and steady estimation of the behavior of TPs. By leveraging on BFT, an explicit quantification of the uncertainty of the estimates is provided, representing the reliability of the information. At first, opinions provided by independent sources during the last time step are combined, evaluating and handling possible conflicts. Then, the information is propagated over time, giving steadiness to the estimate, which is suitable to be used as a basis for a motion planning algorithm. Sources that cannot discern between all considered individual behaviors, but rather assess the probability of unions of singletons can also be included. We discussed the advantages through numerical simulations in SUMO and compared the performances with the IMM algorithm.

The proposed framework allows great flexibility and is well suited to combine information from possibly inconsistent sources. The provided estimate incorporates a quantitative, though subjective, measure of reliability of the information contained, which motion planning algorithms can take advantage from.

Appendix
Proof of Theorem 1

Proof: We show that $\overline{\mu} \leq \mu^{\text{B}}$, likewise it is obtained that $\overline{\mu} \leq \mu^{\text{A}}$. Since $b_1^{\text{B}}, \ldots, b_M^{\text{B}} \geq 0$ and $\sum_{i=0}^{M} b_i^{\text{B}} \leq 1$, it holds that

$$\sum_{(j \cap h) \not\subset \mathbb{I}} b_j^{\text{A}} b_h^{\text{B}} = \sum_{j=0}^{M} b_j^{\text{A}} \sum_{\substack{h=1 \\ (j \cap h) \not\subset \mathbb{I}}}^{M} b_h^{\text{B}} \leq \sum_{j=0}^{M} b_j^{\text{A}} \sum_{h=1}^{M} b_h^{\text{B}} \leq \sum_{j=0}^{M} b_j^{\text{A}}. \tag{10}$$

Then, since from (2) $\mu^{\text{A}} = 1 - \sum_{j=0}^{M} b_j^{\text{A}}$, (6b) yields

$$\overline{\mu} = \frac{\mu^{\text{A}} \mu^{\text{B}}}{1 - \sum\limits_{(j \cap h) \not\subset \mathbb{I}} b_j^{\text{A}} b_h^{\text{B}}} = \frac{1 - \sum\limits_{j=0}^{M} b_j^{\text{A}}}{1 - \sum\limits_{(j \cap h) \not\subset \mathbb{I}} b_j^{\text{A}} b_h^{\text{B}}} \mu^{\text{B}} \leq \mu^{\text{B}}, \tag{11}$$

that is, the uncertainty of the combined opinion is upper bounded by the uncertainty of each individual opinion. ■

References

[1] G. Shafer, *A Mathematical Theory of Evidence*. Princeton University Press, 1976.
[2] A. F. Genovese, "The Interacting Multiple Model Algorithm for Accurate State Estimation of Maneuvering Targets," *JOHNS HOPKINS APL TECHNICAL DIGEST*, 2001.
[3] S. Lefevre, D. Vasquez, and C. Laugier, "A survey on motion prediction and risk assessment for intelligent vehicles," *Robomech Journal*, 2014.
[4] Q. Tran and J. Firl, "Online maneuver recognition and multimodal trajectory prediction for intersection assistance using non-parametric regression," in *IEEE IV*, 2014.
[5] H. Woo, Y. Ji, H. Kono, Y. Tamura, Y. Kuroda, T. Sugano, Y. Yamamoto, A. Yamashita, and H. Asama, "Lane-Change Detection Based on Vehicle-Trajectory Prediction," *IEEE Robotics and Automation Letters*, 2017.

[6] A. Carvalho, Y. Gao, S. Lefevre, and F. Borrelli, "Stochastic predictive control of autonomous vehicles in uncertain environments," in *12th International Symposium on Advanced Vehicle Control*, 2014.

[7] T. Benciolini, D. Wollherr, and M. Leibold, "Non-Conservative Trajectory Planning for Automated Vehicles by Estimating Intentions of Dynamic Obstacles," *IEEE Transactions on Intelligent Vehicles*, 2023.

[8] A. Jøsang, "Belief Mosaics of Subjective Opinions," in *International Conference on Information Fusion (FUSION)*, 2019.

[9] N. Milisavljevic and I. Bloch, "Sensor fusion in anti-personnel mine detection using a two-level belief function model," *IEEE Transactions on Systems, Man, and Cybernetics, Part C*, 2003.

[10] Z. Zhou, O. S. Oguz, M. Leibold, and M. Buss, "A General Framework to Increase Safety of Learning Algorithms for Dynamical Systems Based on Region of Attraction Estimation," *IEEE Transactions on Robotics*, 2020.

[11] ——, "Learning a Low-Dimensional Representation of a Safe Region for Safe Reinforcement Learning on Dynamical Systems," *IEEE Transactions on Neural Networks and Learning Systems*, 2021.

[12] G. Tanzmeister, J. Thomas, D. Wollherr, and M. Buss, "Grid-based mapping and tracking in dynamic environments using a uniform evidential environment representation," in *IEEE International Conference on Robotics and Automation (ICRA)*, 2014.

[13] G. Tanzmeister and D. Wollherr, "Evidential Grid-Based Tracking and Mapping," *IEEE Transactions on Intelligent Transportation Systems*, 2017.

[14] H. Wu, M. Siegel, R. Stiefelhagen, and J. Yang, "Sensor fusion using Dempster-Shafer theory [for context-aware HCI]," in *IEEE Instrumentation and Measurement Technology Conference*, 2002.

[15] A. Martin, "Conflict Management in Information Fusion with Belief Functions," in *Information Quality in Information Fusion and Decision Making*, ser. Information Fusion and Data Science, É. Bossé and G. L. Rogova, Eds. Springer International Publishing, 2019.

[16] A. Jøsang, "Categories of Belief Fusion," *Journal of Advances in Information Fusion*, 2019.

[17] M. Behrisch, L. Bieker, J. Erdmann, and D. Krajzewicz, "SUMO – Simulation of Urban MObility: An Overview," in *The Third International Conference on Advances in System Simulation (SIMUL)*, 2011.

[18] T. Reineking, "Belief Functions: Theory and Algorithms," Ph.D. dissertation, Universität Bremen, 2014.

[19] D. Kahneman and A. Tversky, "Subjective probability: A judgment of representativeness," *Cognitive Psychology*, 1972.

[20] R. P. Srivastava, "An introduction to evidential reasoning for decision making under uncertainty: Bayesian and belief function perspectives," *International Journal of Accounting Information Systems*, 2011.