Tutorial. Sensitivity analysis - Case 0

Files

From Brightspace, download and decompress the file `sa.zip`.

- Place the folder `sa` in the folder `MBPS\mbps\tutorials`.

The sensitivity of a model output with respect to a model parameter is defined as:

$$S = \frac{\partial y}{\partial p}$$

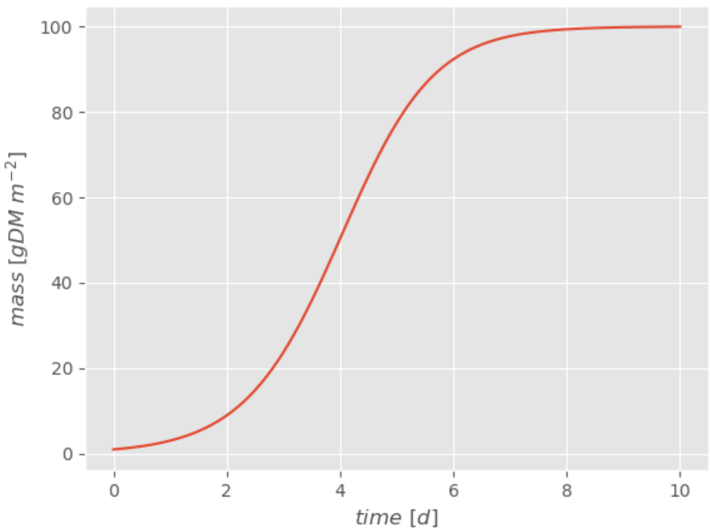
In System Dynamics, a model output is typically a vector (a function of time, $y(t)$). Therefore, the sensitivity is also a vector $S(t)$.

1. Logistic growth – Sensitivity (S)

The logistic growth model is described by the differential equation:

$$\frac{dm}{dt} = r\,m\left(1 - \frac{m}{K}\right)$$

with model parameters for relative growth rate (r) and maximum carrying capacity (K). Therefore, there are two sensitivity curves: ${}^{\partial m}/_{\partial r}$ and ${}^{\partial m}/_{\partial K}$. For a given set of model parameters and initial conditions, the model simulation shows the following curve:



Reflect and discuss

Draw by hand and discuss with your team the expected change in the logistic growth model behaviour when increasing and decreasing $r \pm 5\%$ (draw m vs. t). Don't worry about the values, just the general behaviour. At which point in time do you expect the largest change with respect to the reference model?

Do the same for the change in $K \pm 5\%$ (draw m vs. t). At which point in time do you expect the largest change with respect to the reference model?

What are the units of each sensitivity curve S?

Based on your expected logistic growth curves, draw and discuss with your team the expected behaviour of $S(t)$ when increasing r and K by 5%. When do you expect the largest sensitivity values (S)?

Exercise 1. Sensitivity, S(t)

You will calculate and analyse the sensitivity (S) when changing the values of each model parameter by $\pm 5\%$. Make sure you always return the modified parameter to its reference value for every subsequent calculation. This is called a local, one-at-a-time sensitivity analysis.

Open the file `t_log_growth_sa.py`. Fill in the required code indicated with the comment #TODO. You can use the suggested method or develop your own code implementation. Make four plots:

1. $m\ vs.\ t$, when changing $r \pm 5\%$ (three lines in the same plot, including the reference).
2. $m\ vs.\ t$, when changing K$\pm 5\%$ (three lines in the same plot, including the reference)
3. $S\ vs\ t$, when changing $r \pm 5\%$ (two lines in the same plot, including the reference).
4. $S\ vs\ t$, when changing $K \pm 5\%$ (two lines in the same plot, including the reference).

Reflect and discuss

Do the simulation results match your expectations? If not, explain the mismatch.

Notice that the peak values for the sensitivity to K are ca. 100 times larger than to r. Does this mean that the impact of K is 100 times larger?

What are the units of each sensitivity curve? Can they be compared?

2. Logistic growth – Normalized sensitivity (NS)

The order of magnitude of the parameter values, due to their physical meaning, has a direct impact on the calculated sensitivity (S).

To remove the effect of different orders of magnitude, we use the normalized sensitivity (NS):

$$NS = \frac{\partial y}{\partial p} \cdot \frac{p_{ref}}{y_{ref}}$$

This normalized sensitivity is also a vector (a function of time, $NS(t)$).

Reflect and discuss

What are the units of each normalized sensitivity curve (NS)?

The calculation of NS requires dividing by vector y_{ref}. Can you think of possible problems resulting from this division?

Exercise 2. Normalized sensitivity (NS)

You will calculate the normalized sensitivities (NS) when changing the values of each model parameter by $\pm 5\%$.

Continue working in the file `t_log_growth_sa.py`. Code the calculation of NS for each of the S vectors. Make one plot with 4 lines:

1. Two lines for $NS\ vs.\ t$, when changing $r \pm 5\%$.
2. Two lines for $NS\ vs.\ t$, when changing $K \pm 5\%$.

Reflect and discuss

In this case, the values for NS are always positive. What does this mean?

Do you observe any differences between the two NS lines for r? Describe and explain.

Do you observe any differences between the two NS lines for K? Describe and explain.

This calculation of NS uses vector m, which is low at the start and high at the end. What impact do you think this could have on the values of NS between r and K?

Can the sensitivities always be compared fairly when using y_{ref}? What situations produce biased results?

3. Logistic growth – Normalized sensitivity (NS) with mean y

The calculation of the normalized sensitivity may result in bias numerical errors. You will calculate the normalized sensitivities (NS) based on the mean value of the model output ($\bar{y}_{ref}$):

$$NS = \frac{\partial y}{\partial p} \cdot \frac{p_{ref}}{\bar{y}_{ref}}$$

Notice that $\bar{y}_{ref}$ is a scalar (a single value, as opposed to a vector $y_{ref}(t)$).

Reflect and discuss

What are the units of each normalized sensitivity curve (NS) with this new definition based on y_{ref}?

Exercise 3. Normalized sensitivity with mean output

Continue working in the file `t_log_growth_sa.py`. Modify the calculation of NS (or add a new one) for each of the S vectors, using the mean value of $\bar{y}_{ref}$. Make one plot with 4 lines:

1. Two lines for $NS\ vs. t$, when changing $r \pm 5\%$.
2. Two lines for $NS\ vs. t$, when changing $K \pm 5\%$.

Reflect and discuss

What differences do you observe with respect to the previous exercise. Explain.

Challenge

So far, we have ignored the effect of a “hidden model parameter”, the initial condition m_0. Modify the code to include the sensitivity and normalized sensitivity curves with respect to m_0.

4. Impact of reference values

The normalized sensitivity (NS) using the mean value of the reference model output (y_{ref}) provides a fair comparison to assess the impact of model parameters.

From the previous exercises, you may have concluded that r has a slightly larger impact than K in the logistic growth model.

Reflect and discuss

Do you think r has a larger impact than K for all the possible reference values (r_{ref}, K_{ref})?

Exercise 4. Normalized sensitivity for different reference parameters

From Brightspace, download the text file `ns.txt`. Copy-paste the function into the class Module, located in the file `MBPS\mbps\classes\module.py`. Indent the function to make sure it belongs to the class.

Open the Jupyter notebook file `logisticgrowth_ns.ipynb`. You can run Jupyter via:

- The Anaconda navigator
- Opening a new miniconda command window/terminal, and typing `jupyter notebook`.

Run the script to generate interactive plots for the model output, and its sensitivities to parameters normalized to the mean reference output, and the chosen reference parameters.

Use the sliders to select different reference parameter values.

Reflect and discuss

Looking at the NS curves, try to predict how the m curve will change when you move each slider slightly to the left or right.

Is the NS to r always larger than to K? What combination of reference values inverts the resulting NS values (for at least part of the time)?

Exercise 5. Normalized sensitivity for different reference parameters and model outputs

Open the Jupyter notebook file `lotkavolterra_ns.ipynb`. Run the script to generate interactive plots.

Reflect and discuss

Looking at the NS curves, try to predict how the prey and predator populations change at times 100, 150, and 200 days, when you move each slider slightly to the left or right.