大数据科学与应用技术

Lecture：焦在滨（ Zaibin JIAO ）
Email：jiaozaibin@mail.xjtu.edu.cn

OUTLINES

- Data Cleaning
- Data Transformation
- Data Description
- Feature Selection
- Feature Extraction

Where are Data from?

https://www.sohu.com/a/373148792_120491688

Why Data Pre-processing

Real data are notoriously dirty!

The biggest challenge in many data mining projects

- Incomplete（缺失）
  Occupancy = " "
- Noisy（噪声）
  Salary = "-100"
- Inconsistent （不一致）
  Age = "42" vs. Birthday = "01/09/1985"
- Redundant（重复）
  Too much data or too many features for analytical analysis
- Others
  Data types
  Imbalanced datasets

Missing Data（数据缺失）

- Data are not always available.
  - One or more attributes of a sample may have empty values.
  - Many data mining algorithms cannot handle missing values directly.
  - May cause significant troubles.

- Possible Reasons
  - Equipment malfunction
  - Data not provided
  - Not Applicable (N/A)
- Different Types
  - Missing completely at random
  - Missing conditionally at random
  - Not missing at random

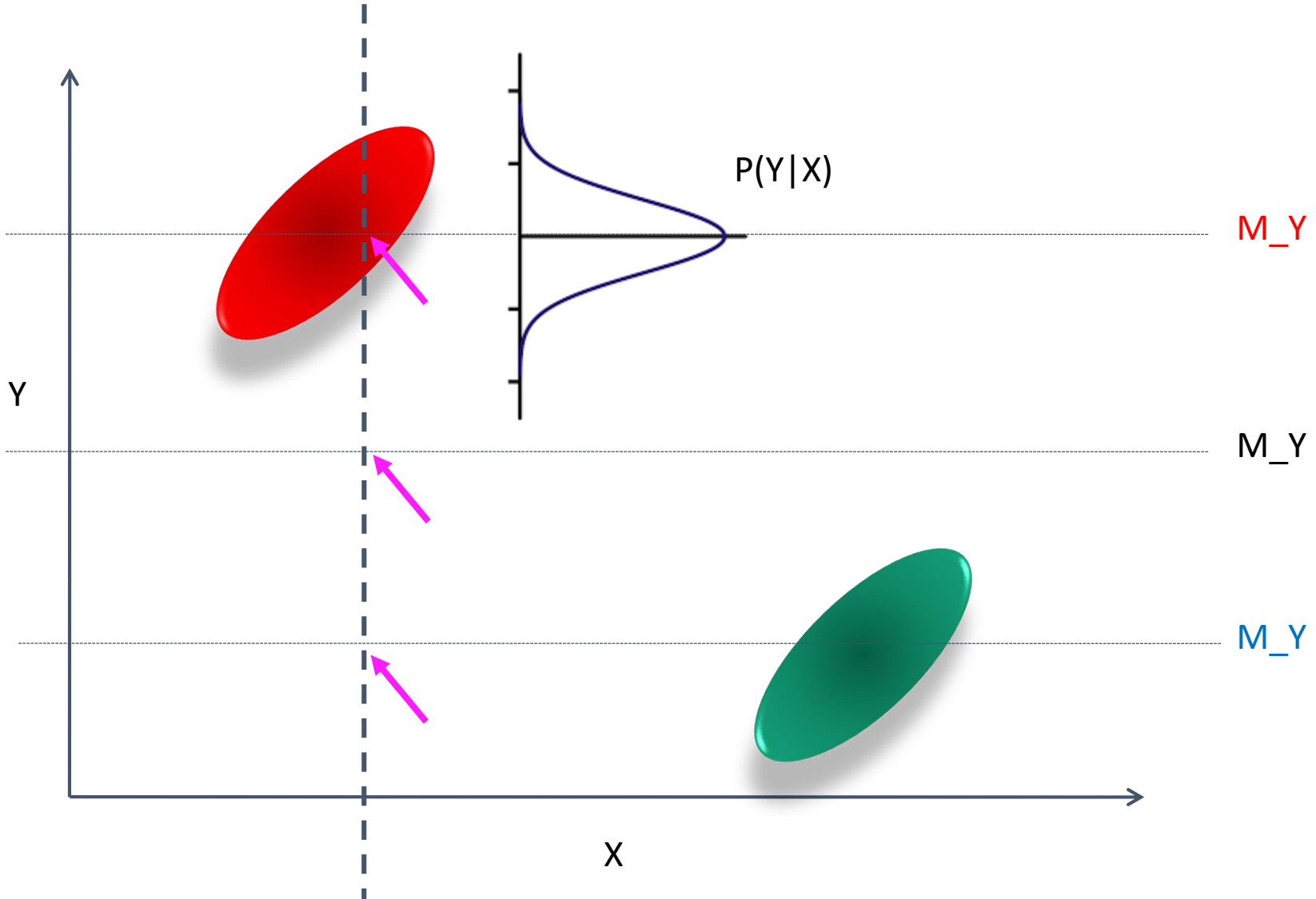
How to handle missing data?

- Ignore
  - Remove samples/attributes with missing values
  - The easiest and most straightforward way
  - Work well with low missing rates

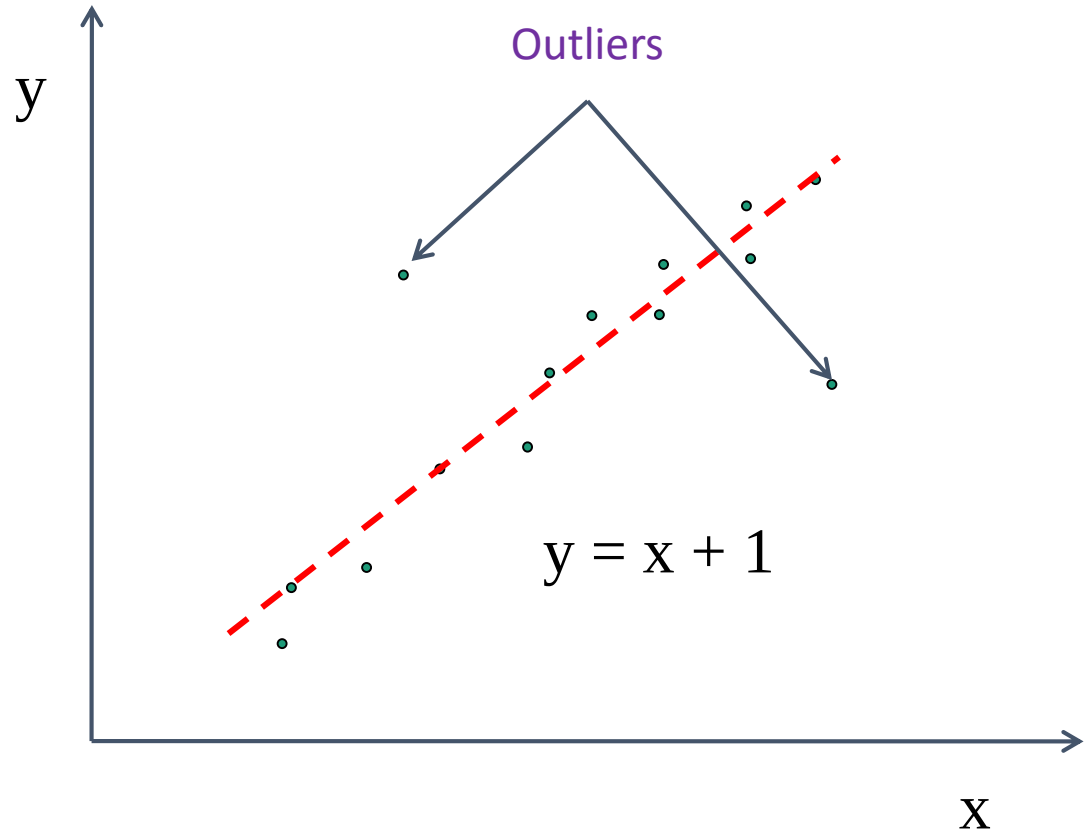
- Fill in the missing values manually
  - Recollect the data
  - Domain knowledge
  - Tedious/Infeasible
- More art than science

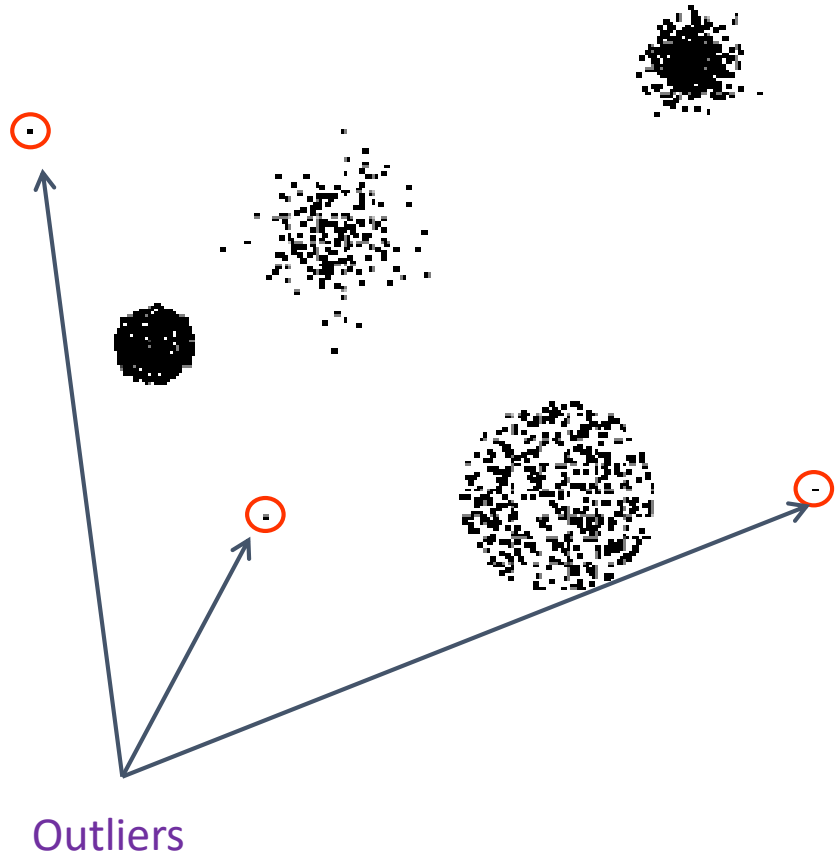
- Fill in the missing values automatically
  - A global constant
  - The mean or median
  - Most probable values

Missing Data --- An Example



Outliers（离群点）

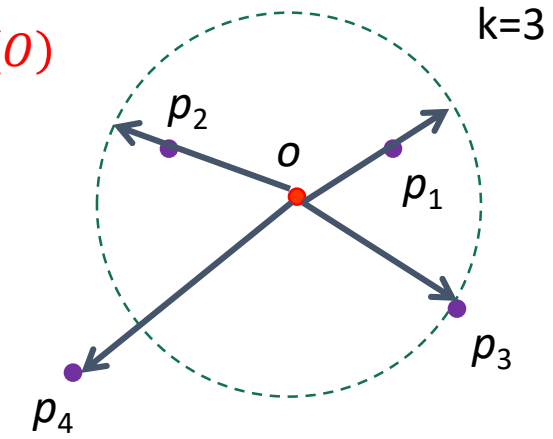




Anomaly（异常点） vs. Outlier（离群点）

Local Outlier Factor

$distance_k(O)$

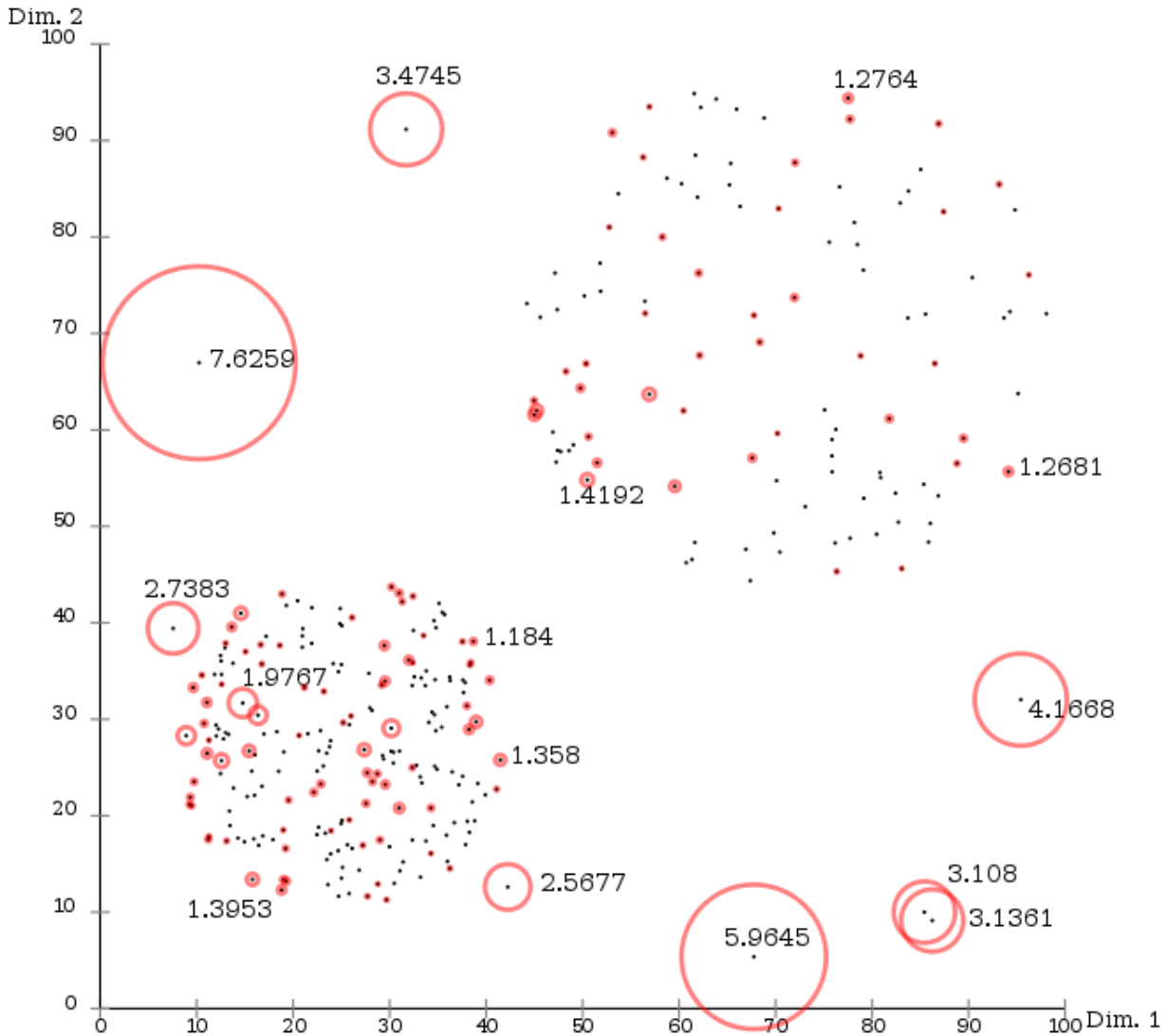


$$distance_k(A,B) = max\{distance_k(B), d(A,B)\}$$

$$lrd(A) = 1/\left(\frac{\sum_{B\in N_k(A)} distance_k(A,B)}{|N_k(A)|}\right)$$

$$LOF_k(A) = \frac{\sum_{B\in N_k(A)} \frac{lrd(B)}{lrd(A)}}{|N_k(A)|} = \frac{\sum_{B\in N_k(A)} lrd(B)}{|N_k(A)|} / lrd(A)$$

Local Outlier Factor



Similarity and Dissimilarity

- Similarity（相似度）
  - Numerical measure of how alike two data objects are.
  - Is higher when objects are more alike.
  - Often falls in the range [0,1]
- Dissimilarity（相异度）
  - Numerical measure of how different are two data objects
  - Lower when objects are more alike
  - Minimum dissimilarity is often 0
  - Upper limit varies
- Proximity refers to a similarity or dissimilarity

Attribute Types

- Continuous
  - Real values: Temperature, Height, Weight …
- Discrete
  - Integer values: Number of people …
- Ordinal
  - Rankings: {Average, Good, Best}, {Low, Medium, High} …
- Nominal
  - Symbols: {Teacher, Worker, Salesman}, {Red, Green, Blue} …
- String
  - Text: “Xi’an Jiaotong University”, “No. 28, Xianning West Road” …

Similarity/Dissimilarity for Simple Attributes

p and *q* are the attribute values for two data objects.

Attribute Type	Dissimilarity	Similarity
Nominal	$d = \begin{cases} 0 & \text{if } p = q \\ 1 & \text{if } p \neq q \end{cases}$	$s = \begin{cases} 1 & \text{if } p = q \\ 0 & \text{if } p \neq q \end{cases}$
Ordinal	$d = \frac{\lvert p-q \rvert}{n-1}$ (values mapped to integers 0 to $n-1$, where n is the number of values)	$s = 1 - \frac{\lvert p-q \rvert}{n-1}$
Interval or Ratio	$d = \lvert p - q \rvert$	$s = -d$, $s = \frac{1}{1+d}$ or $s = 1 - \frac{d - min\_d}{max\_d - min\_d}$

Common Properties of a Distance

- Distances, such as the Euclidean distance, have some well known properties.

1. $d(p, q) \geq 0$ for all p and q and $d(p, q) = 0$ only if $p = q$. (Positive definiteness)
2. $d(p, q) = d(q, p)$ for all p and q. (Symmetry)
3. $d(p, r) \leq d(p, q) + d(q, r)$ for all points p, q, and r. (Triangle Inequality)

where $d(p, q)$ is the distance (dissimilarity) between points (data objects), p and q.

- A distance that satisfies these properties is a **metric** （度量）

Common Properties of a Similarity

- Similarities, also have some well known properties.

1. *s(p, q) = 1* (or maximum similarity) only if *p = q*.

2. *s(p, q) = s(q, p)* for all *p* and *q*. (Symmetry)

where *s(p, q)* is the similarity between points (data objects), *p* and *q*.

Euclidean Distance

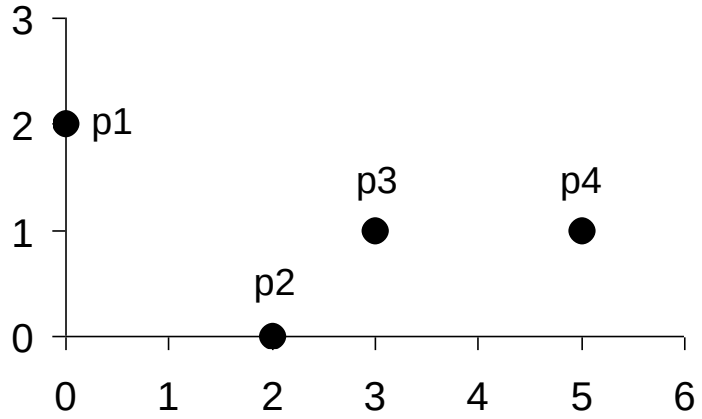
- Euclidean Distance

$$dist = \sqrt{\sum_{k=1}^{n} (p_k - q_k)^2}$$

Where n is the number of dimensions (attributes) and p_k and q_k are, respectively, the kth attributes (components) or data objects p and q.

- Standardization is necessary, if scales differ.

Euclidean Distance



point	x	y
p1	0	2
p2	2	0
p3	3	1
p4	5	1

	p1	p2	p3	p4
p1	0	2.828	3.162	5.099
p2	2.828	0	1.414	3.162
p3	3.162	1.414	0	2
p4	5.099	3.162	2	0

Distance Matrix

Minkowski Distance

- Minkowski Distance is a generalization of Euclidean Distance

$$dist = \left(\sum_{k=1}^{n} | p_k - q_k |^r \right)^{\frac{1}{r}}$$

Where *r* is a parameter, *n* is the number of dimensions (attributes) and p_k and q_k are, respectively, the kth attributes (components) or data objects *p* and *q*.

Minkowski Distance

- $r = 1$. City block (Manhattan, taxicab, L_1 norm) distance.
  - A common example of this is the Hamming distance, which is just the number of bits that are different between two binary vectors
- $r = 2$. Euclidean distance
- $r \to \infty$. "supremum" (L_{max} norm, L_∞ norm) distance.
  - This is the maximum difference between any component of the vectors
- Do not confuse *r* with *n*, i.e., all these distances are defined for all numbers of dimensions.

Minkowski Distance

point	x	y
p1	0	2
p2	2	0
p3	3	1
p4	5	1

L1	p1	p2	p3	p4
p1	0	4	4	6
p2	4	0	2	4
p3	4	2	0	2
p4	6	4	2	0

L2	p1	p2	p3	p4
p1	0	2.828	3.162	5.099
p2	2.828	0	1.414	3.162
p3	3.162	1.414	0	2
p4	5.099	3.162	2	0

L_∞	p1	p2	p3	p4
p1	0	2	3	5
p2	2	0	1	3
p3	3	1	0	2
p4	5	3	2	0

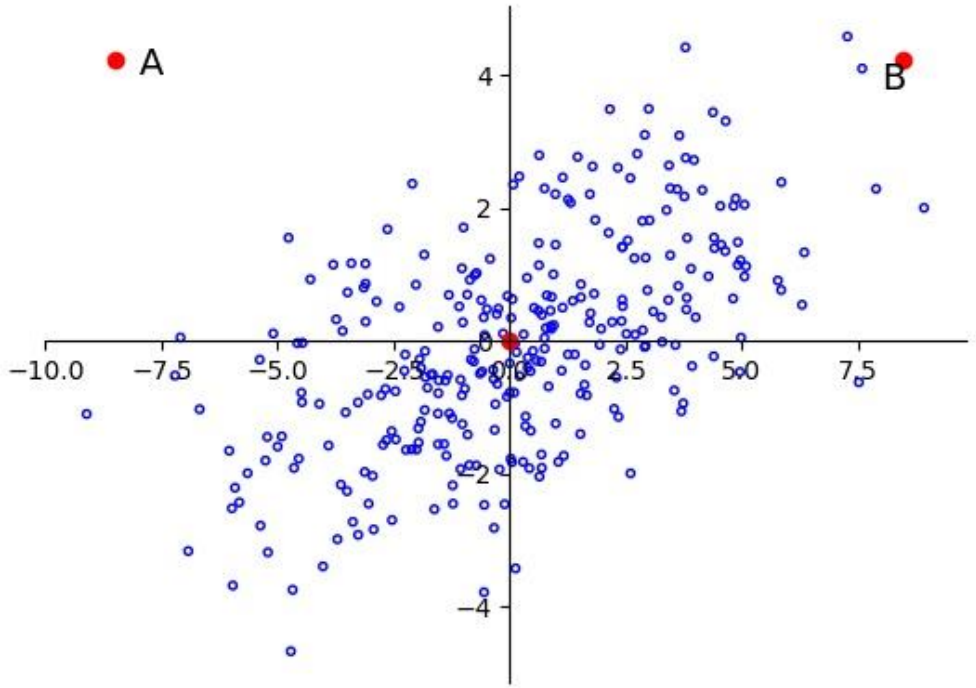
Distance Matrix

Mahalanobis Distance

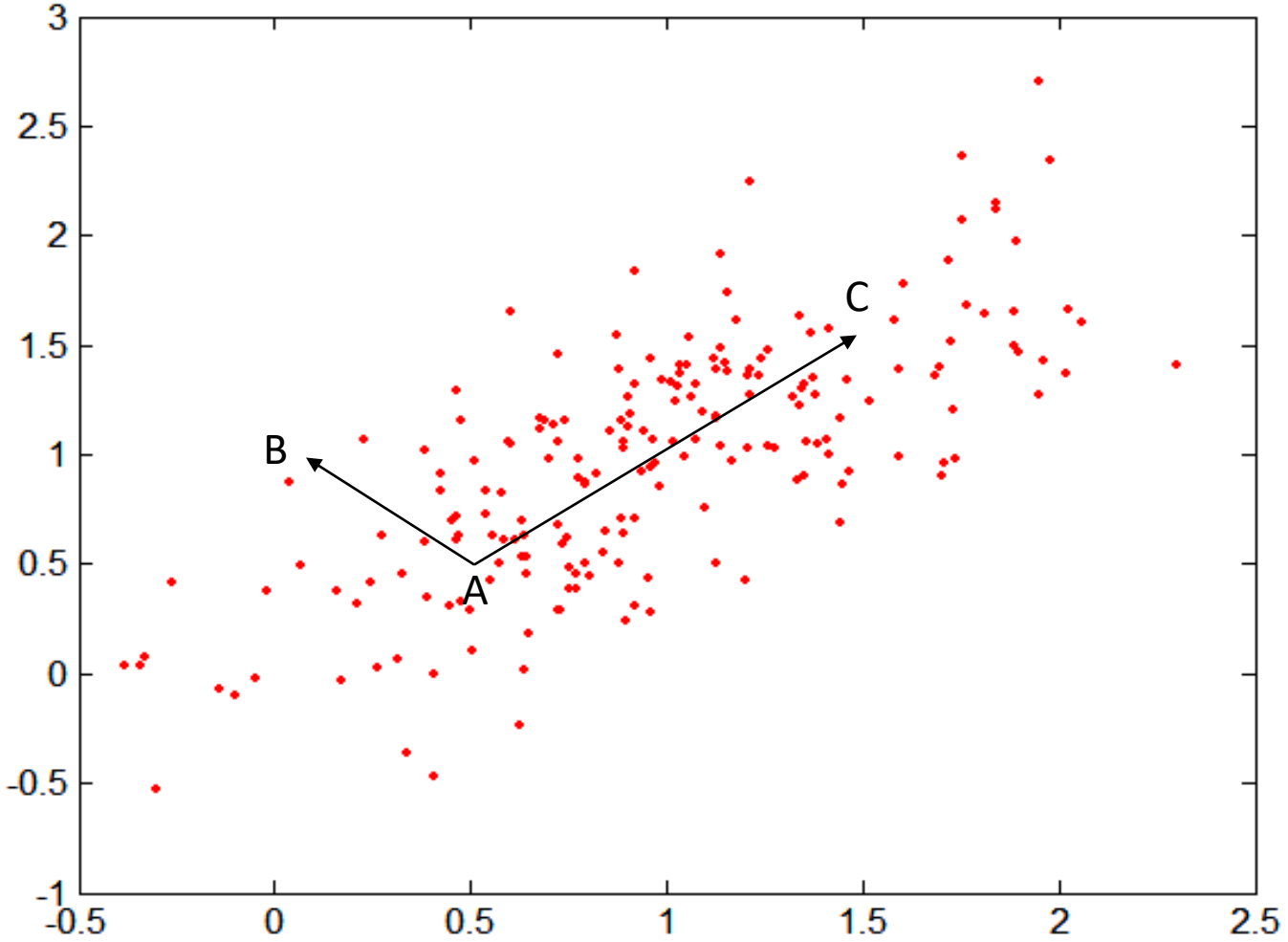
$$mahalanobis(p,q) = \sqrt{(p-q)\Sigma^{-1}(p-q)^T}$$

Σ is the covariance matrix of the input data X

$$\Sigma_{j,k} = \frac{1}{n-1}\sum_{i=1}^{n}(X_{ij} - \overline{X}_j)(X_{ik} - \overline{X}_k)$$



Mahalanobis Distance



Covariance Matrix:

$$\Sigma = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.3 \end{bmatrix}$$

A: (0.5, 0.5)

B: (0, 1)

C: (1.5, 1.5)

Duplicate Data（重复数据）

Customer (source 1)

CID	*Name*	*Street*	*City*	*Sex*
11	Kristen Smith	2 Hurley Pl	South Fork, MN 48503	0
24	Christian Smith	Hurley St 2	S Fork MN	1

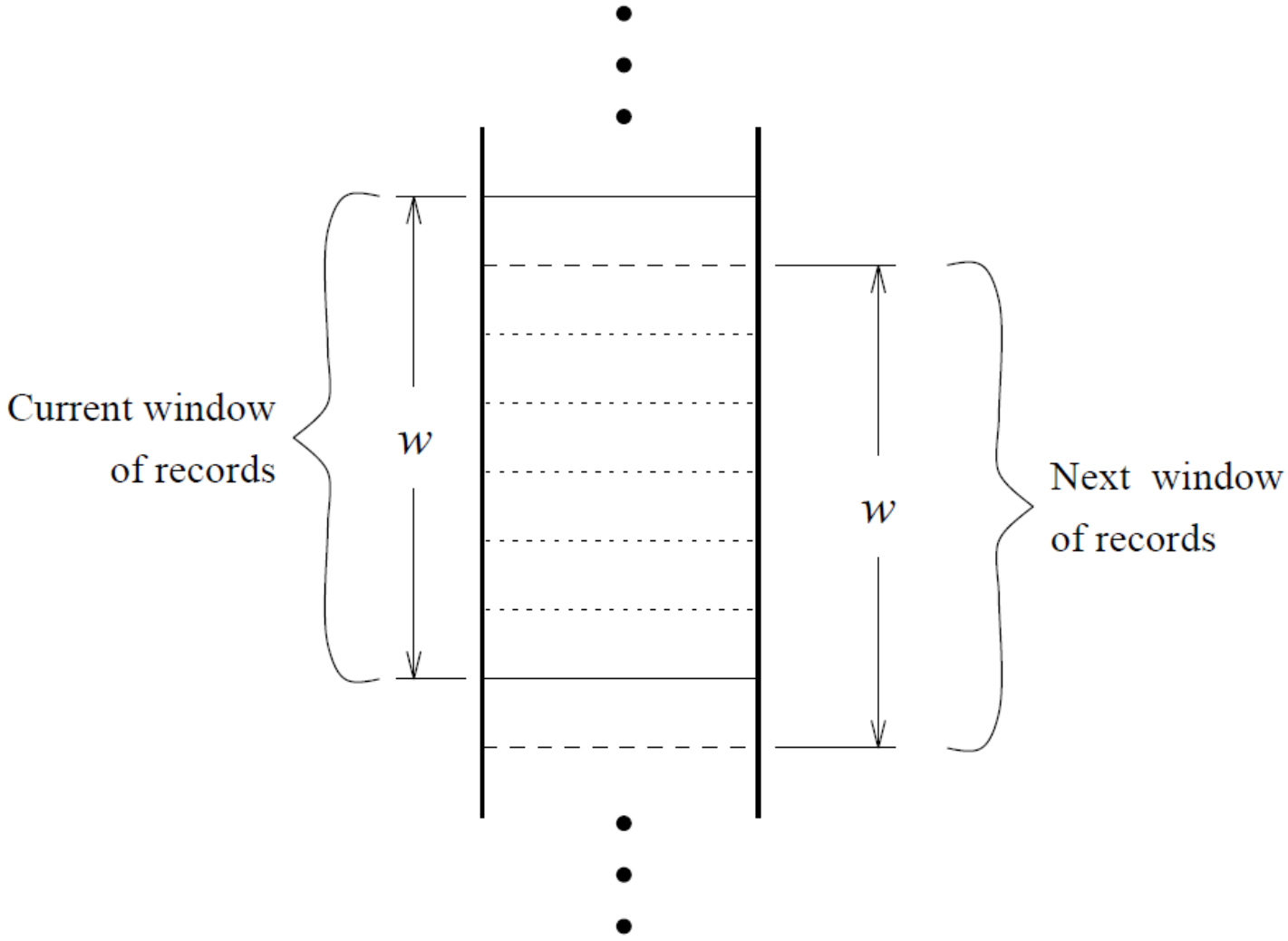
Client (source 2)

Cno	*LastName*	*FirstName*	*Gender*	*Address*	*Phone/Fax*
24	Smith	Christoph	M	23 Harley St, Chicago IL, 60633-2394	333-222-6542 / 333-222-6599
493	Smith	Kris L.	F	2 Hurley Place, South Fork MN, 48503-5998	444-555-6666

Customers (integrated target with cleaned data)

No	*LName*	*FName*	*Gender*	*Street*	*City*	*State*	*ZIP*	*Phone*	*Fax*	*CID*	*Cno*
1	Smith	Kristen L.	F	2 Hurley Place	South Fork	MN	48503-5998	444-555-6666		11	493
2	Smith	Christian	M	2 Hurley Place	South Fork	MN	48503-5998			24	
3	Smith	Christoph	M	23 Harley Street	Chicago	IL	60633-2394	333-222-6542	333-222-6599		24

Duplicate Data



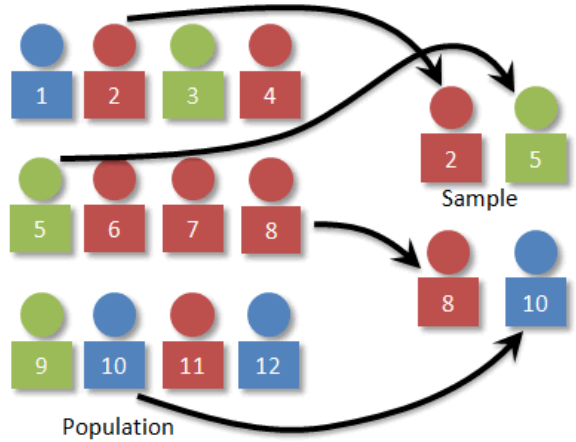
Duplicate Data

First	Last	Address	ID	Key
Sal	Stolfo	123 First Street	45678987	STLSAL123FRST456
Sal	Stolfo	123 First Street	45678986	STLSAL123FRST456
Sal	Stolpho	123 First Street	45688987	STLSAL123FRST456
Sal	Stiles	123 Forest Street	45654321	STLSAL123FRST456

```
Given two records, r1 and r2.
IF the last name of r1 equals the last name of r2,
       AND the first names differ slightly,
       AND the address of r1 equals the address of r2
THEN
       r1 is equivalent to r2.
```

Data Transformation

- Now we have an error free dataset.
- Still needs to be **standardized**.
- **Type Conversion**
- **Normalization**
- **Sampling**



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- String
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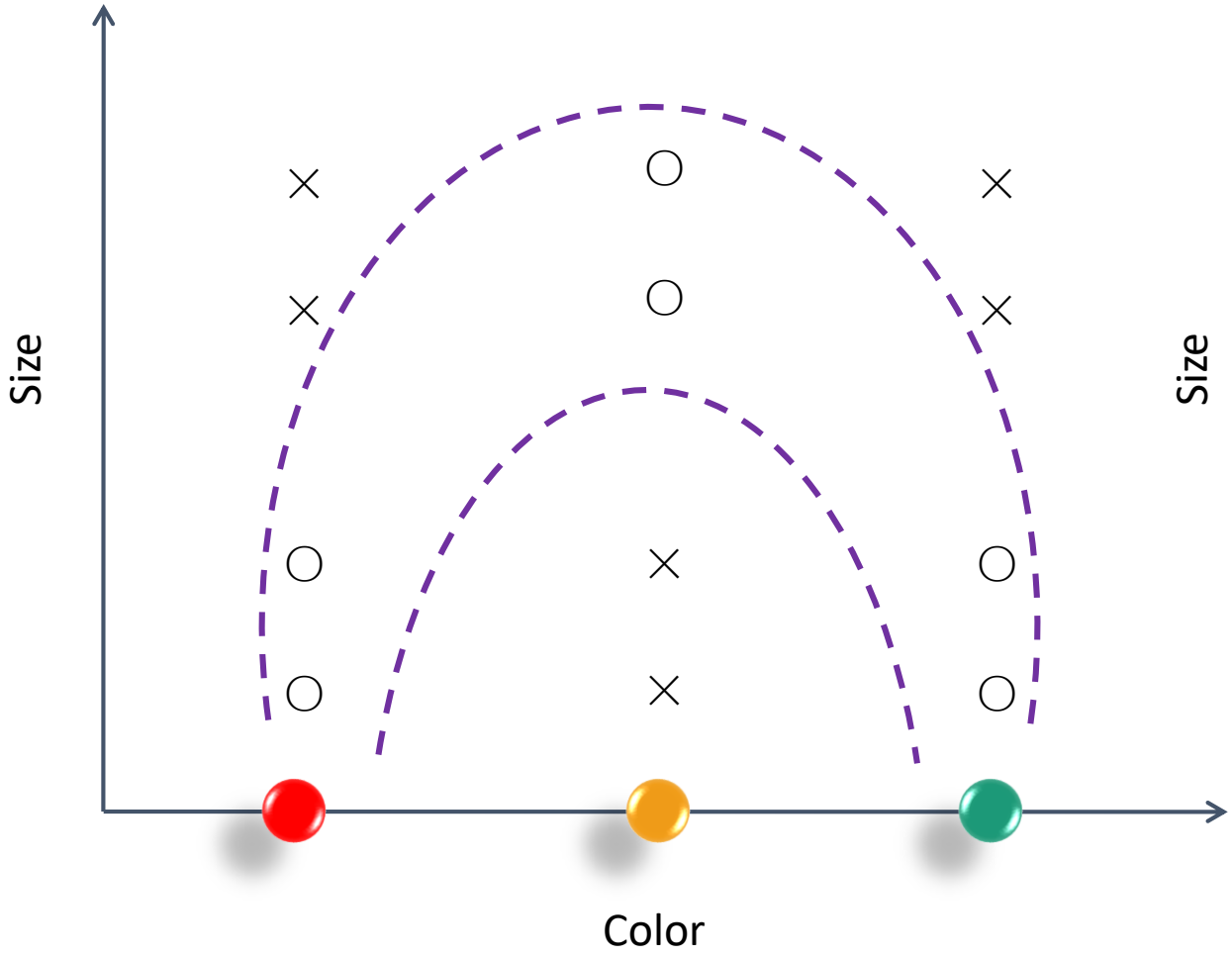
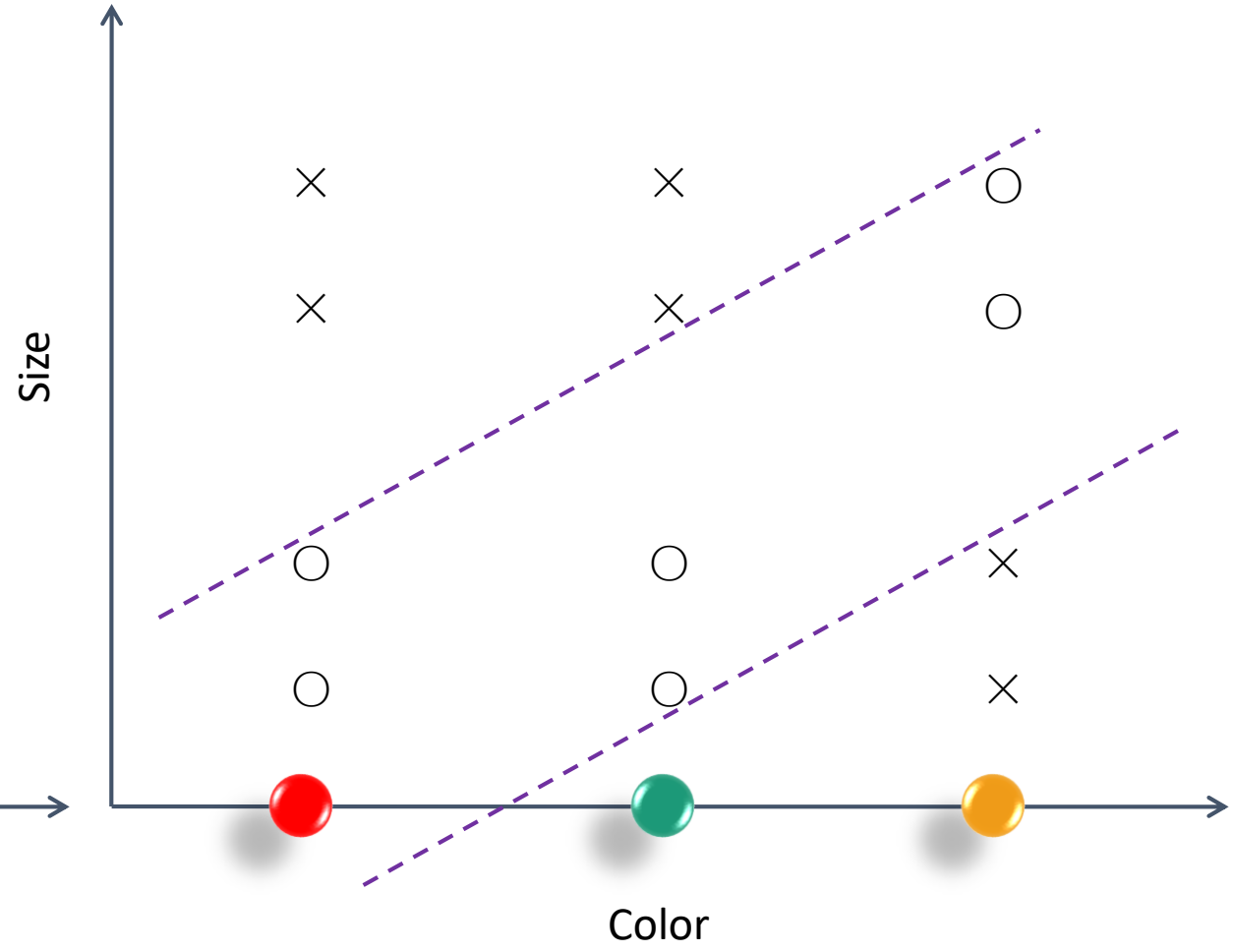
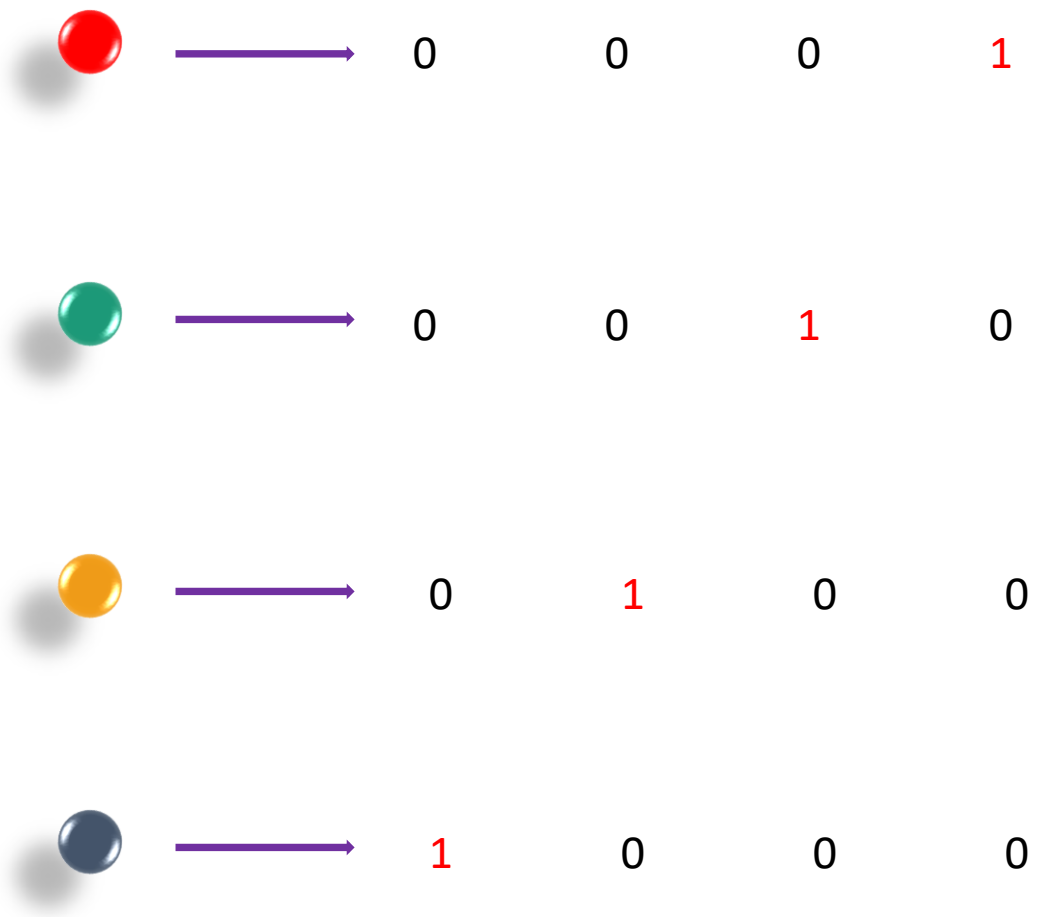
Type Conversion





Type Conversion



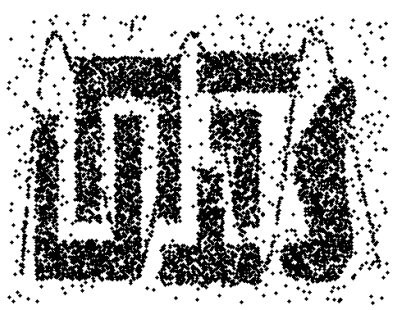
Sampling

- A database/data warehouse may store terabytes of data.
- Processing limits: CPU, Memory, I/O …
- Sampling is applied to reduce the time complexity.
- In statistics, sampling is applied often because obtaining the entire set of data is expensive.
- Aggregation（聚合）
  - Change of scale:
    - Cities → States; Days → Months
  - More stable and less variability
- Sampling can be also used to adjust the class distributions.
  - Imbalanced dataset

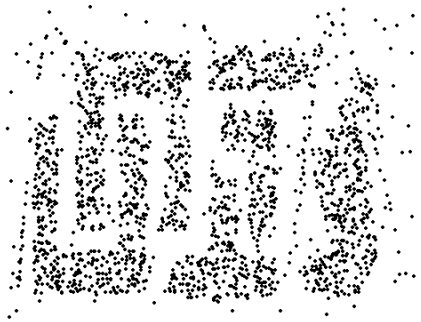
Sampling

- The key principle for effective sampling is the following:
  - using a sample will work almost as well as using the entire data sets, if the sample is representative
  - A sample is representative if it has approximately the same property (of interest) as the original set of data

- Types of Sampling
  - ➢ Simple random sampling
    - without replacement
    - with replacement
  - ➢ Stratified Sampling



8000 points

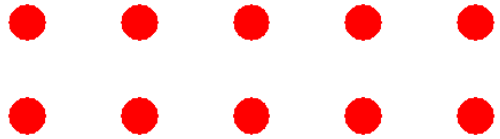


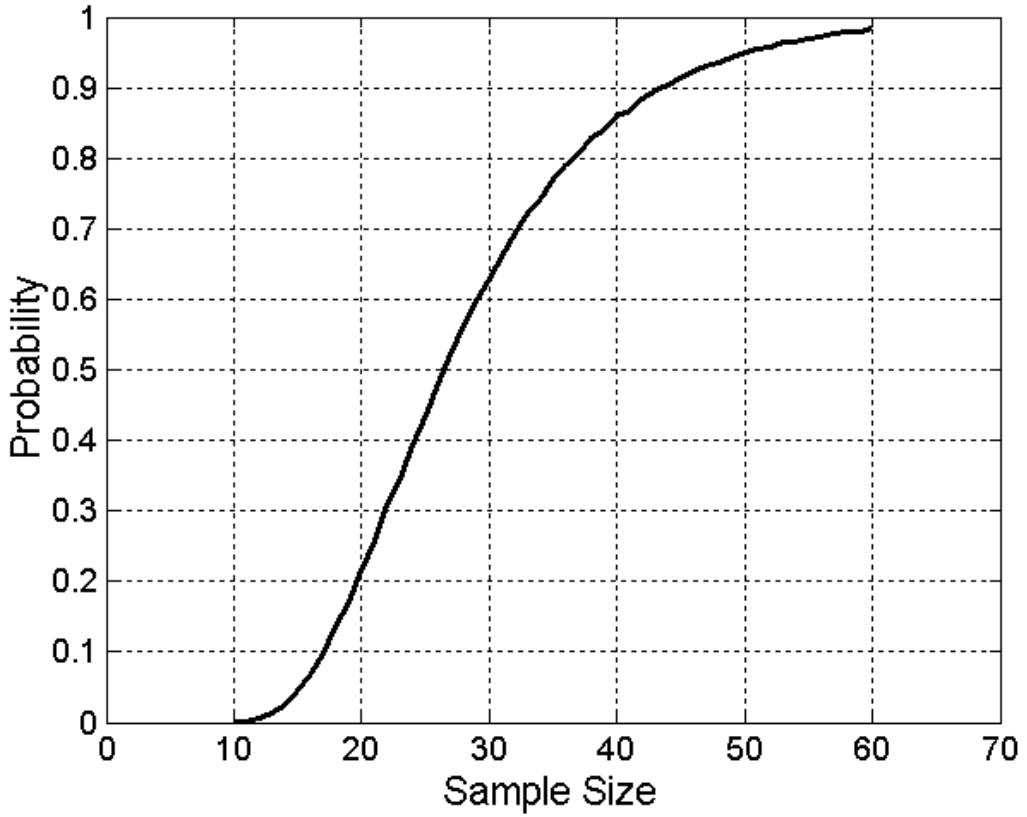
2000 Points

500 Points

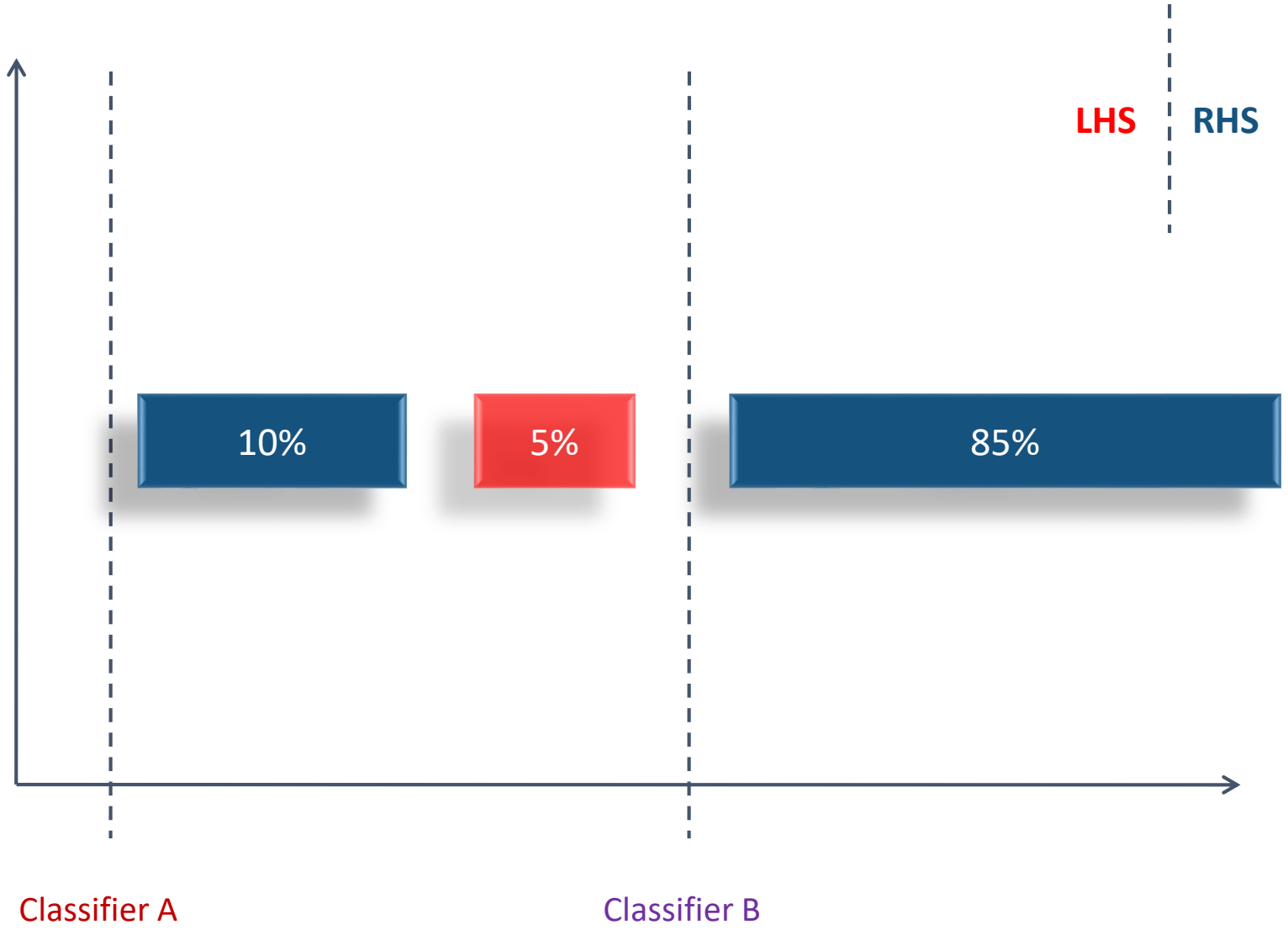
Sample Size

- What sample size is necessary to get at least one object from each of 10 groups.





Imbalanced Datasets



Imbalanced Datasets

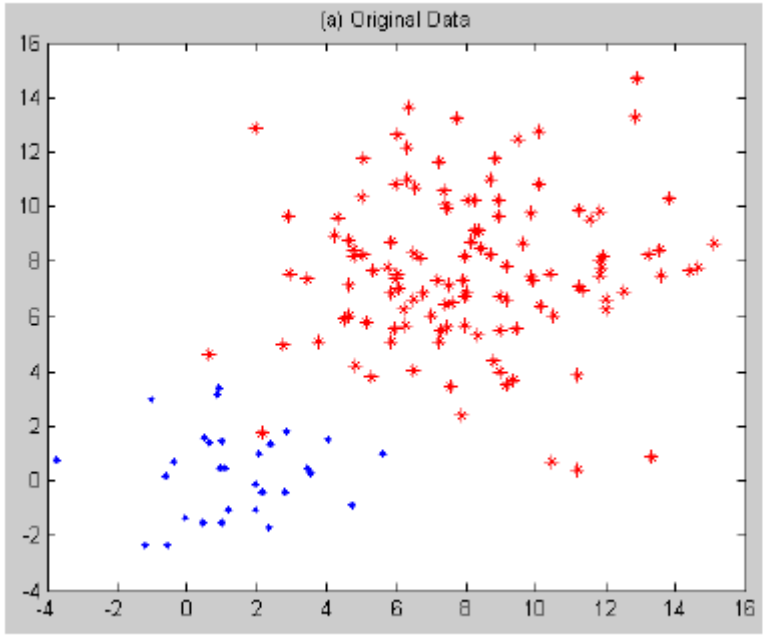
$$G - mean = (Acc^{+} \times Acc^{-})^{1/2}$$

$$where \;\; Acc^{+} = \frac{TP}{TP + FN}; \quad Acc^{-} = \frac{TN}{TN + FP}$$

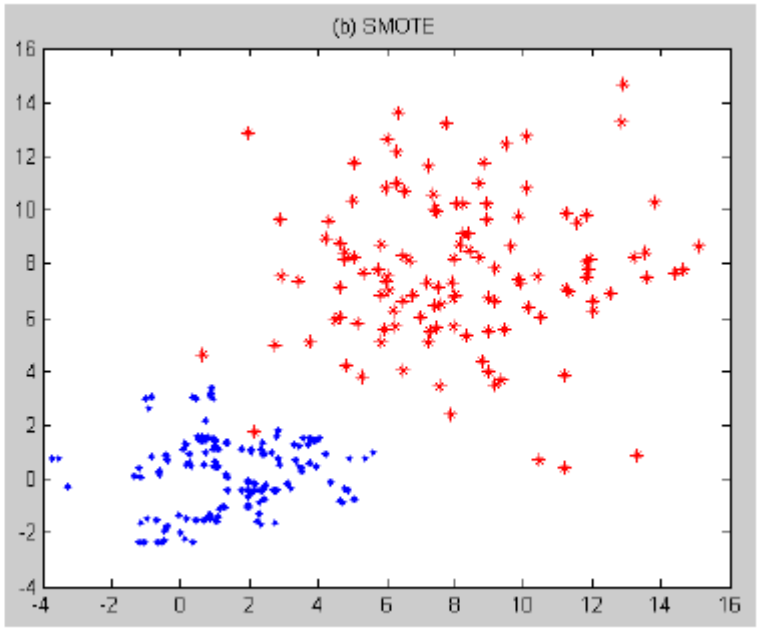
$$F - measure = \frac{2 \times Precision \times Recall}{Precision + Recall}$$

$$where \quad \underset{\text{查准率}}{Precision} = \frac{TP}{TP + FP}; \quad \underset{\text{查全率}}{Recall} = \frac{TP}{TP + FN} = Acc^{+}$$

Over-Sampling



(a)



(b)

SMOTE

Normalization

The height of someone can be 1.7 or 170 or 1700 …

Min-max normalization---线性函数归一化:

$$v' = \frac{v - min}{max - min}(new\_max - new\_min) + new\_min$$

- Let income range \$12,000 to \$98,000 be normalized to [0.0, 1.0]. Then \$73,600 is mapped to

$$\frac{73{,}600 - 12{,}000}{98{,}000 - 12{,}000}(1.0 - 0) + 0 = 0.716$$

Z-score normalization--- 0均值归一化 (μ: mean, σ: standard deviation) :

$$v' = \frac{v - \mu}{\sigma}$$

- Let μ = 54,000, σ = 16,000. Then $\frac{73{,}600 - 54{,}000}{16{,}000} = 1.225$

Data Description

Mean

- Arithmetic mean

$$\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = \frac{1}{n}(x_1 + \cdots + x_n)$$

Median $P(X \leq m) = P(X \geq m) = \int_{-\infty}^{m} f(x)dx = \frac{1}{2}$

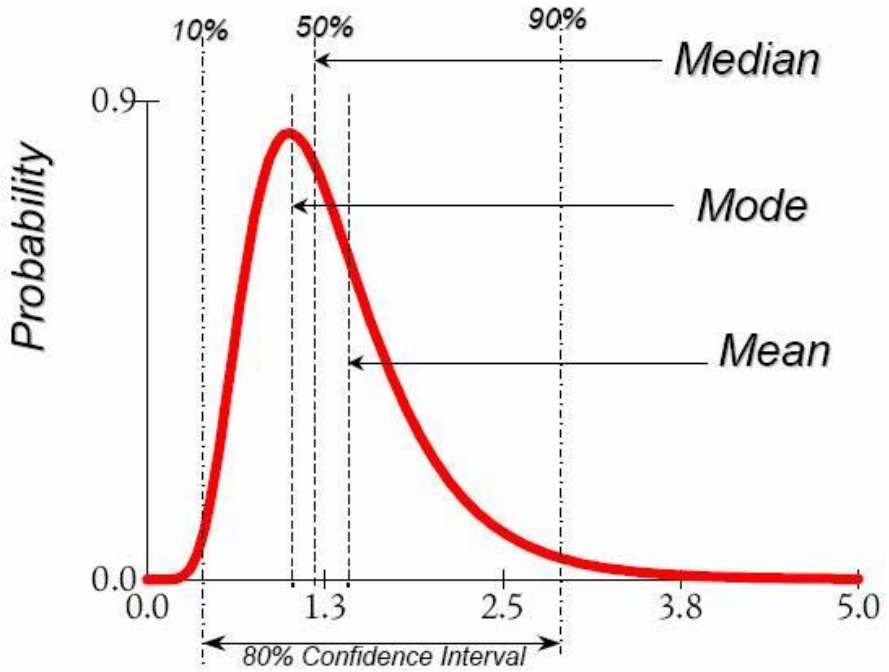
Mode

- The most frequently occurring value in a list
- Can be used with non-numerical data.

Variance

- Degree of diversity $Var(X) = E[(X - \mu)^2]$

$$Var(X) = \int (x - \mu)^2 f(x)dx$$



Data Description

Pearson's product moment correlation coefficient（皮尔逊积矩相关系数）

$$r_{A,B} = \frac{\sum (A - \overline{A})(B - \overline{B})}{(n-1)\sigma_A \sigma_B} = \frac{\sum (AB) - n\overline{A}\overline{B}}{(n-1)\sigma_A \sigma_B}$$

- If $r_{A, B} > 0$, A and B are positively correlated.
- If $r_{A, B} = 0$, no linear correlation between A and B.
- If $r_{A, B} < 0$, A and B are negatively correlated.

Data Description

- Pearson's chi-square (χ^2) test

$$\chi^2 = \sum \frac{(Observed - Expected)^2}{Expected}$$

	Play chess	Not play chess	Sum (row)
Like science fiction	250 (90)	200 (360)	450
Not like science fiction	50 (210)	1000 (840)	1050
Sum (col.)	300	1200	1500

$$\chi^2 = \frac{(250-90)^2}{90} + \frac{(50-210)^2}{210} + \frac{(200-360)^2}{360} + \frac{(1000-840)^2}{840} = 507.93$$

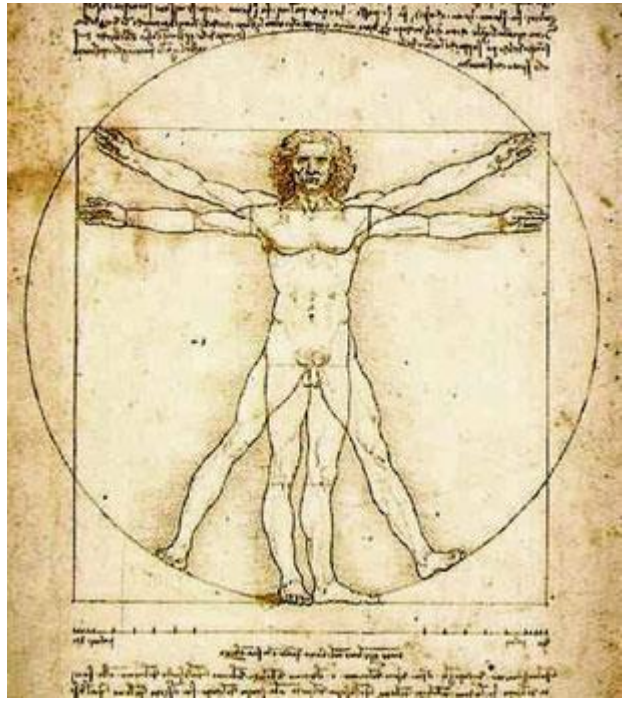
Feature Selection

ID

Weight

Age

Gender



Noisy?

Height

Irrelevant?

Education

Duplicate?

Income

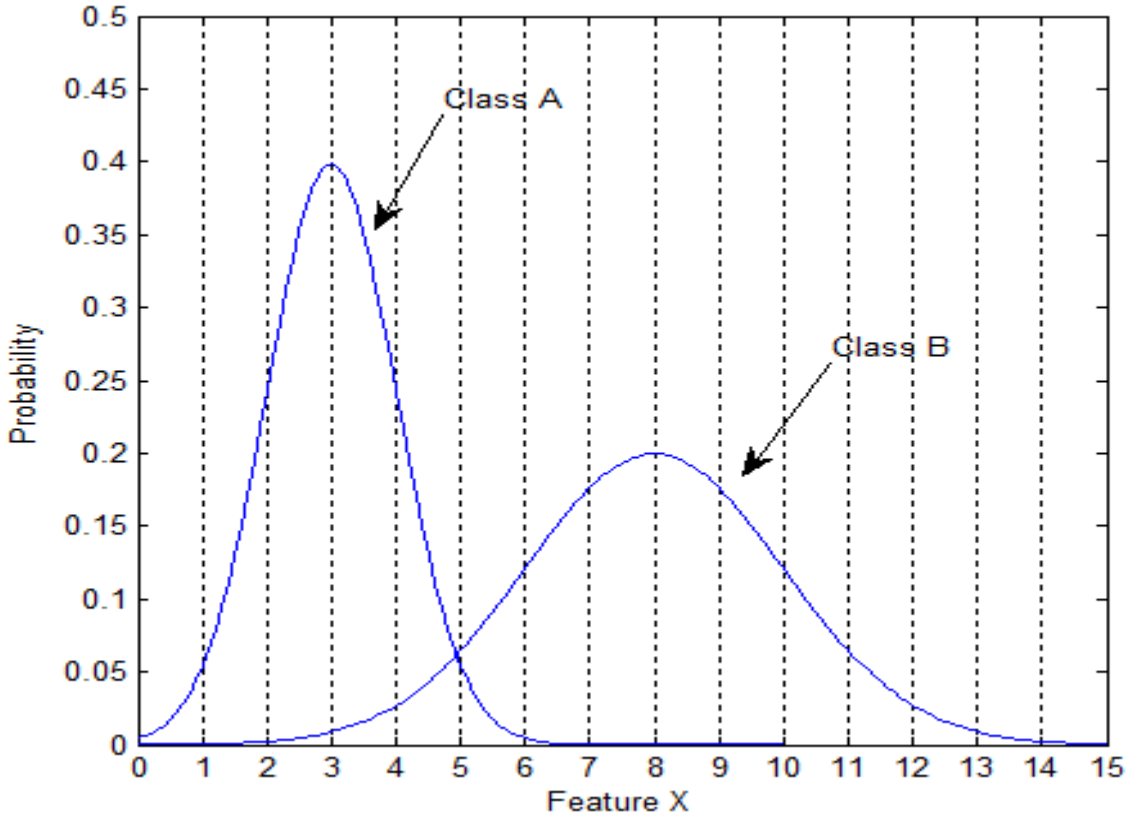
Address

Complexity?

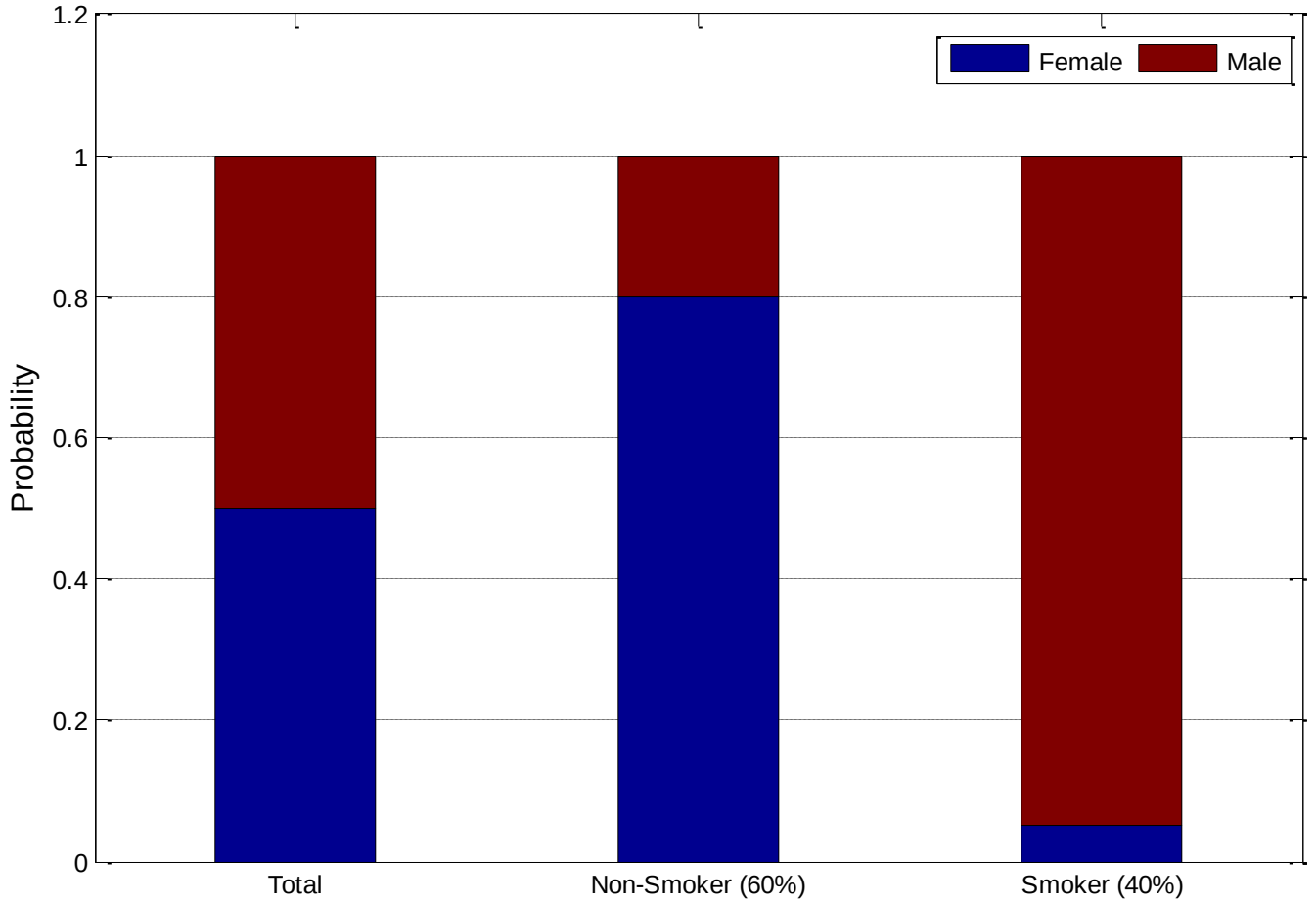
Location

Occupation

Class Distributions

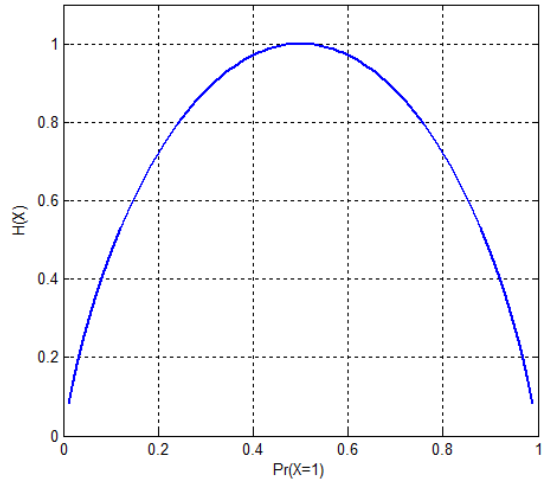


Class Distributions



Entropy（熵）

$$H(X) = -\sum_{i=1}^{n} p(x_i) \log_b p(x_i)$$



$$H(S) = -0.5 \cdot \log_2 0.5 - 0.5 \cdot \log_2 0.5 = 1.0$$

$$X: \{a = \text{"Non-Smoker"};\quad b = \text{"Smoker"}\}$$

$$H(S \mid X = a) = -0.8 \cdot \log_2 0.8 - 0.2 \cdot \log_2 0.2 = 0.7219$$

$$H(S \mid X = b) = -0.05 \cdot \log_2 0.05 - 0.95 \cdot \log_2 0.95 = 0.2864$$

$$H(S \mid X) = 0.6 \cdot H(S \mid X = a) + 0.4 \cdot H(S \mid X = b) = 0.5477$$

$$Gain(S, X) = H(S) - H(S \mid X) = 0.4523$$

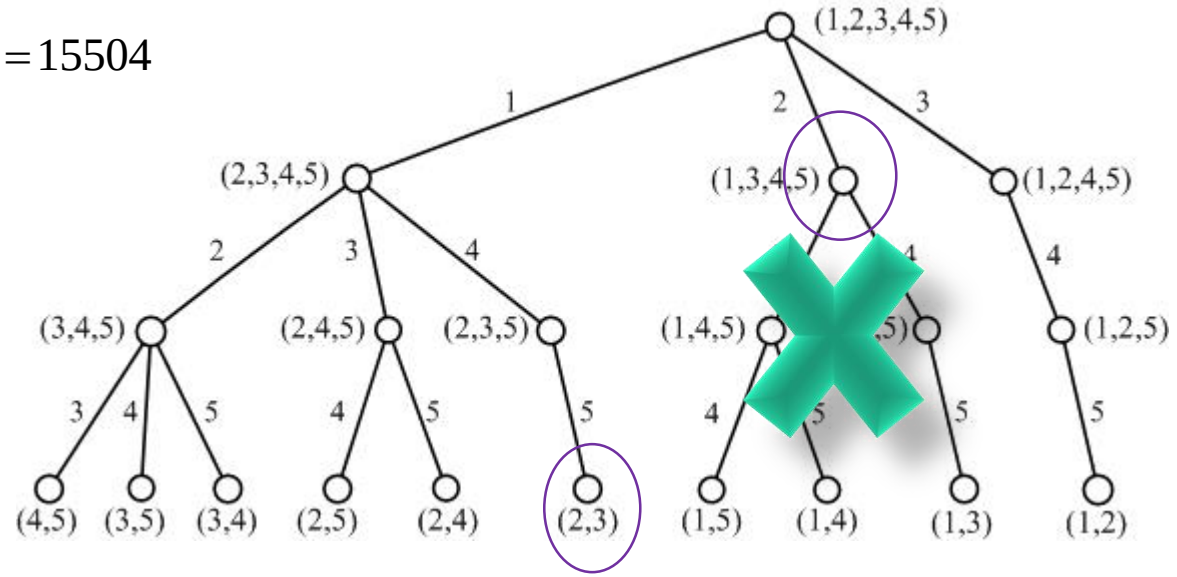
Information Gain（信息增益）

Feature Subset Search

- Exhaustive
  - All possible combinations

$$C_{10}^{3} = \frac{10!}{(10-3)!3!} = 120 \qquad C_{20}^{5} = \frac{20!}{(20-5)!5!} = 15504$$

- Branch and Bound（分支定界）



$$S_1 \supset S_2 \supset S_3 \quad \Rightarrow \quad J(S_1) > J(S_2) > J(S_3)$$

Feature Subset Search

- Top K Individual Features

$$J(X_k) = \{J(x_1), \cdots, J(x_k)\},\ J(x_1) > J(x_2) > \ldots > J(x_k)$$

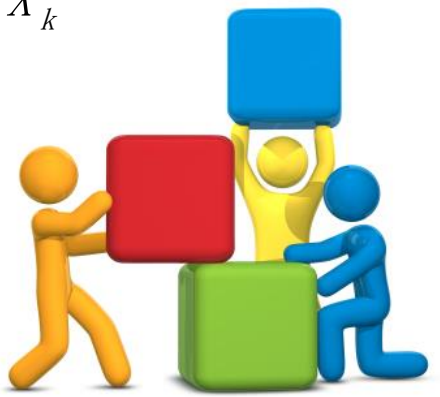
- Sequential Forward Selection

$$J(X_k + x_1) > J(X_k + x_2) > \cdots > J(X_k + x_{D-k}),\ x_i \notin X_k$$

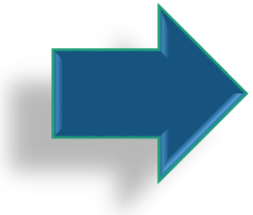
- Sequential Backward Selection

$$J(X_k - x_1) > J(X_k - x_2) > \cdots > J(X_k - x_k),\ x_i \in X_k$$

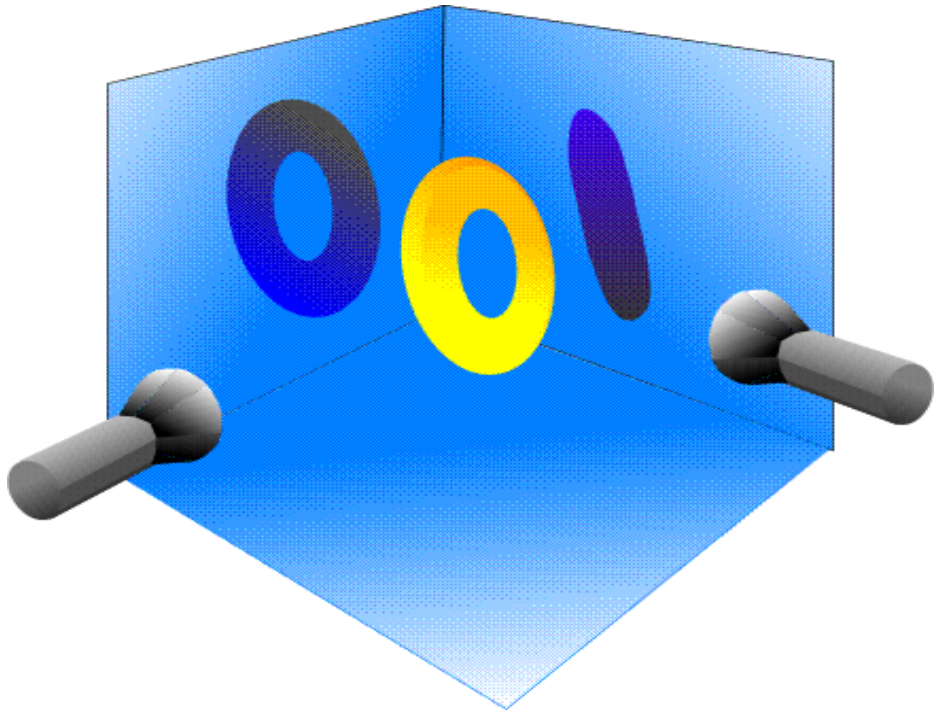
- Optimization Algorithms
  - Simulated Annealing
  - Tabu Search
  - Genetic Algorithms



Feature Extraction

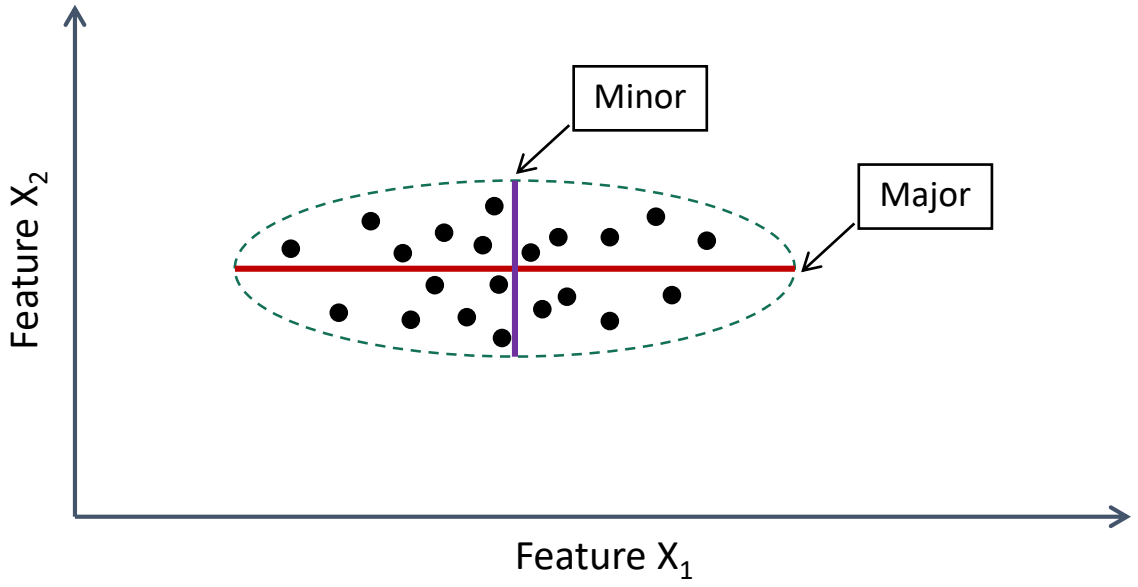


Principal Component Analysis （PCA）
主成分分析---数据规约

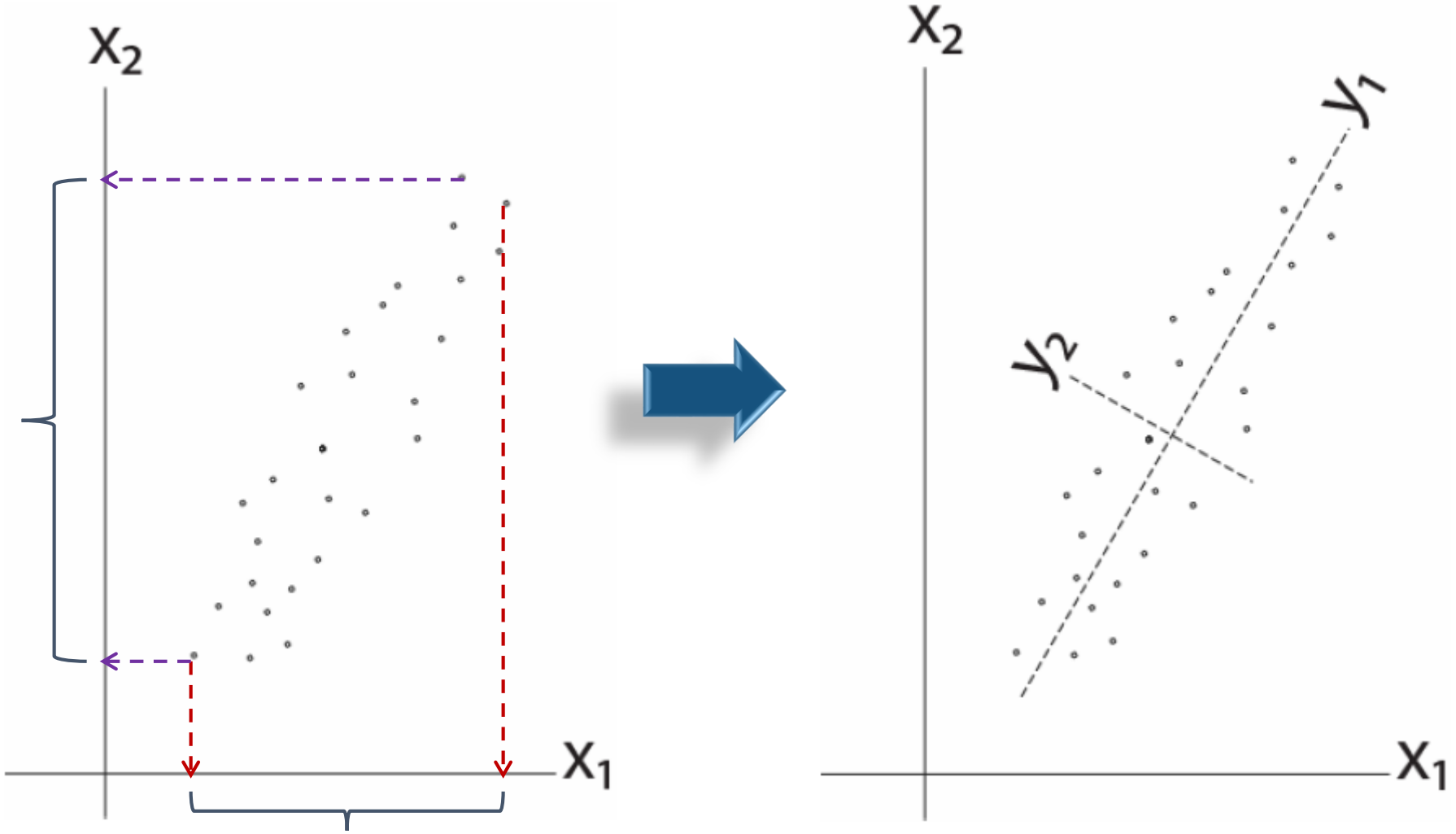


2D Example

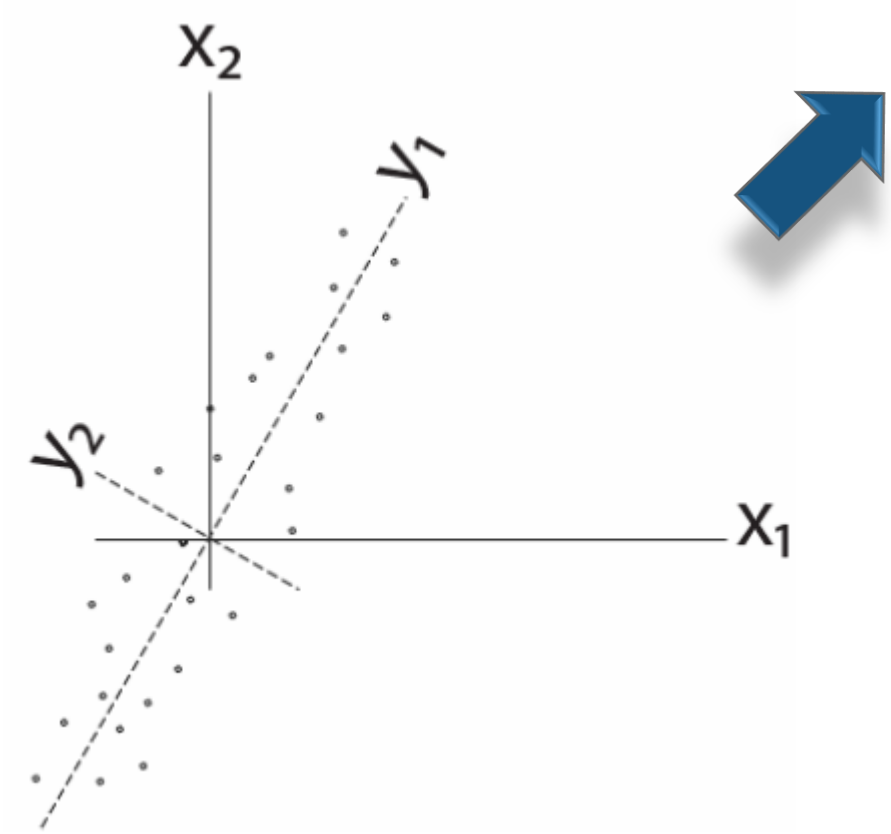
- Data: Gaussian Distribution
- Variance: Information
- Ellipse: Major Axis vs. Minor Axis
- Select the attribute corresponding to the Major Axis.

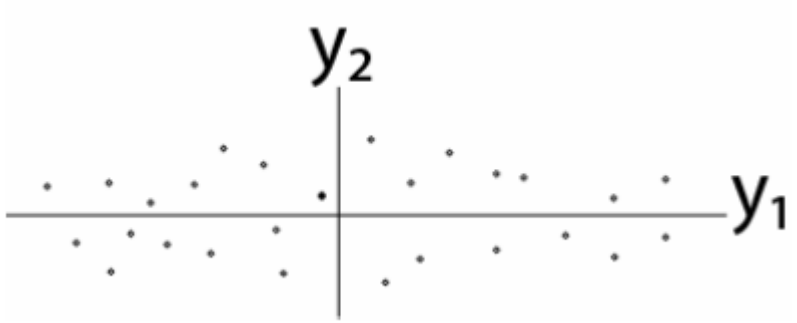


2D Example



2D Example





$$S(X) = \frac{1}{n-1} XX^T$$

Remove correlation

$$S(Y) = \frac{1}{n-1} YY^T$$

Some Math...

Goal: S(Y) has nonzero diagonal entries and all off-diagonal elements are zero.

$$Y = PX \Rightarrow S(Y) = \frac{1}{n-1} YY^T \Rightarrow YY^T = (PX)(PX)^T = PXX^TP^T.$$

$$XX^T = QDQ^T$$

Eigen decomposition

$$(n-1)S(Y) = PXX^TP^T = PQDQ^TP^T = (PQ)D(PQ)^T$$

$$P = Q^T$$

A Different View

$$J(e) = \sum_{k=1}^{n} \|x_k' - x_k\|^2$$

$$= \sum_{k=1}^{n} \|\alpha_k e - x_k\|^2$$

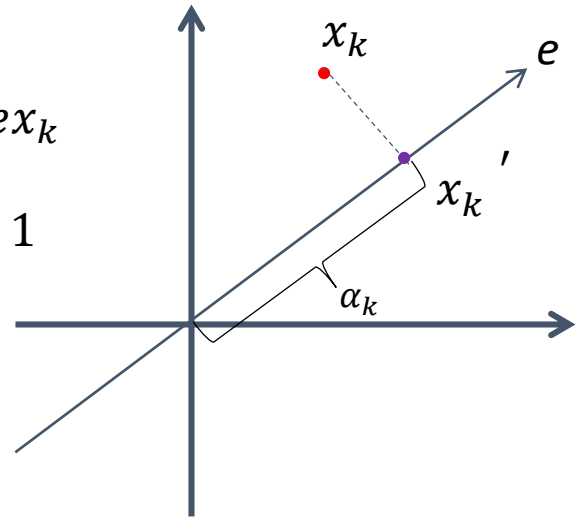
$$= \sum_{k=1}^{n} {\alpha_k}^2 \|e\|^2 - 2\sum_{k=1}^{n} \alpha_k e x_k + \sum_{k=1}^{n} \|x_k\|^2$$

$$= -\sum_{k=1}^{n} {\alpha_k}^2 + \sum_{k=1}^{n} \|x_k\|^2$$

$$= -\sum_{k=1}^{n} e x_k x_k^T e^T + \sum_{k=1}^{n} \|x_k\|^2$$

$\alpha_k = e x_k$

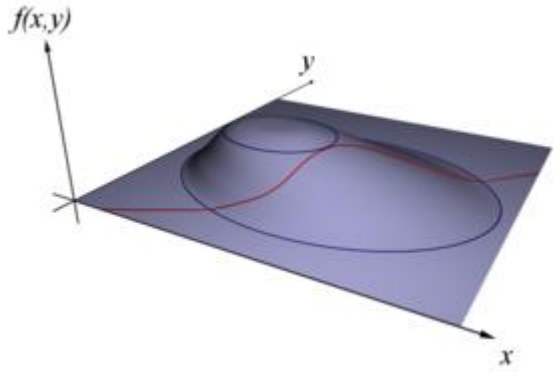
$\|e\| = 1$

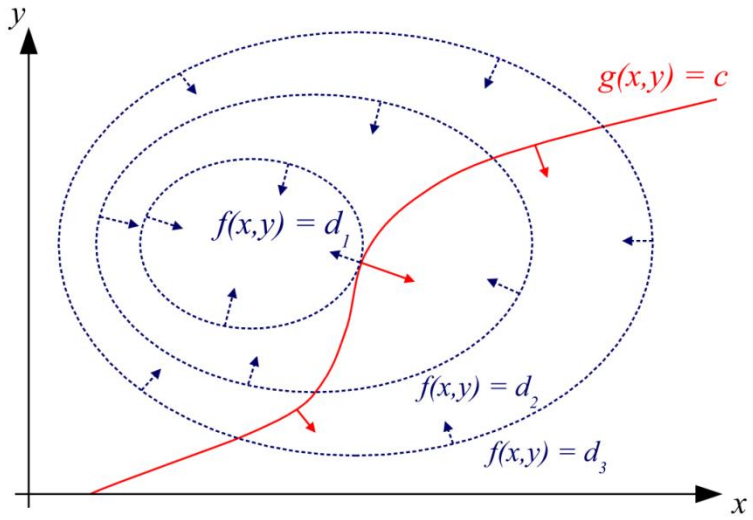


$$\max_e e S e^t$$
$$s.t. \ \|e\| = 1$$

$$S = \sum_{k=1}^{n} x_k x_k^t$$

Lagrange Multipliers





$$\max_{x,y} f(x,y) = 3xy \quad s.t.\ 2x + y = 8$$

$$F(x,y,\lambda) = 3xy - \lambda(2x + y - 8)$$

$$F_x = 3y - 2\lambda$$
$$F_y = 3x - \lambda$$
$$F_\lambda = -(2x + y - 8)$$

$$\lambda = 6$$
$$x = \frac{\lambda}{3} = 2$$
$$y = \frac{2}{3}\lambda = 4$$
$$f(2,4) = 3 \cdot 2 \cdot 4 = 24$$

More Math...

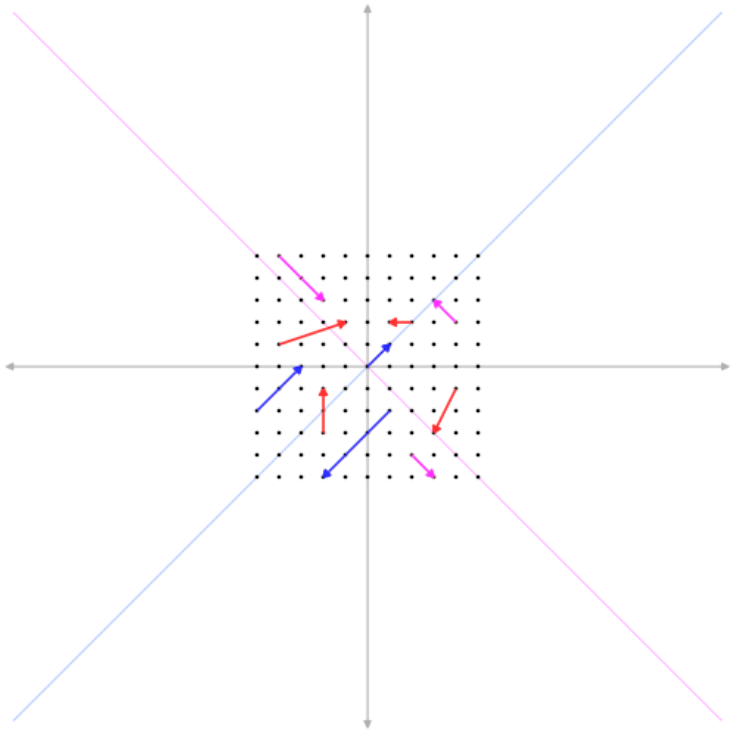
$$u = eSe^t - \lambda(ee^t - 1)$$

$$\frac{\partial u}{\partial e} = 2Se - 2\lambda e$$

$Se = \lambda e$ ← eigenvector

↑ eigenvalue

$$eSe^t = \lambda ee^t = \lambda$$



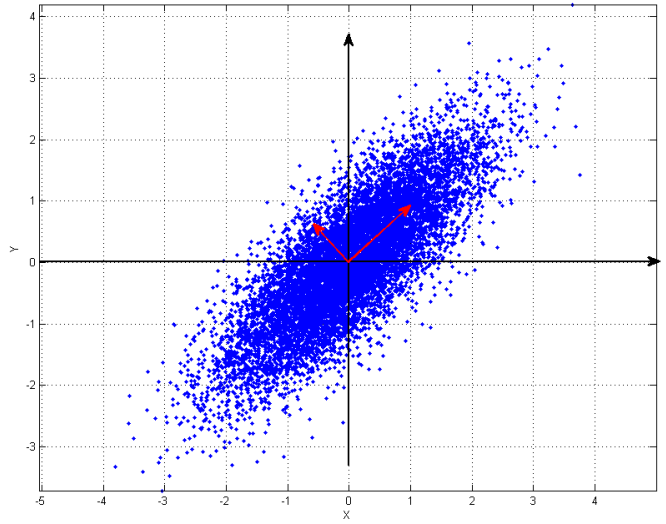
PCA

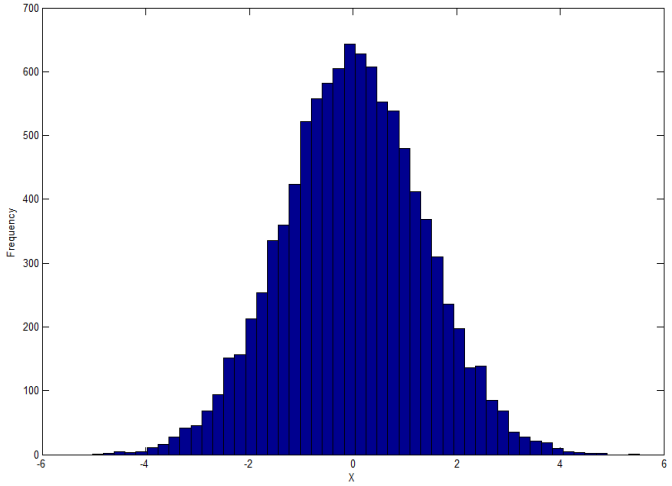
To project the original data to the eigenvectors of S with the largest eigenvalues.

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

PCA Examples





$$s = \begin{bmatrix} 1 & 0.8 \\ 0.8 & 1 \end{bmatrix};$$

$x = mvnrnd(zeros(10000,2), s);$

$plot(x(:,1), x(:,2), '.');$

$axis\ equal;$

$grid\ on;$

$[V, D] = eig(s);$

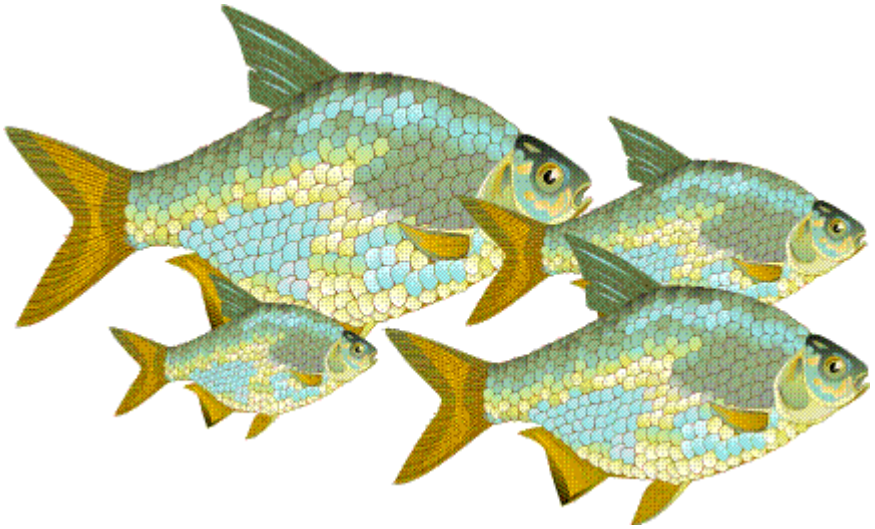
$$V = \begin{bmatrix} -0.7071 & 0.7071 \\ 0.7071 & 0.7071 \end{bmatrix};$$

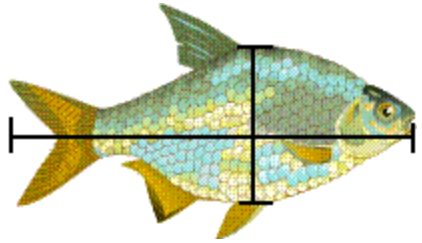
$$D = \begin{bmatrix} 0.2 & 0 \\ 0 & 1.8 \end{bmatrix};$$

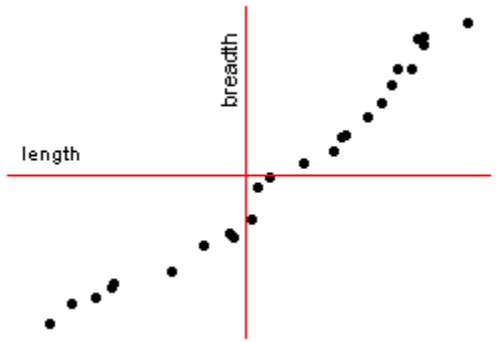
$newx = x * V(:,2);$

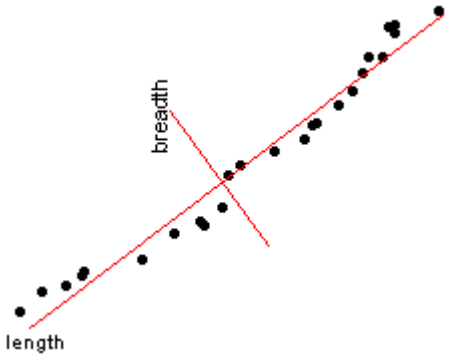
$hist(newx, 50);$

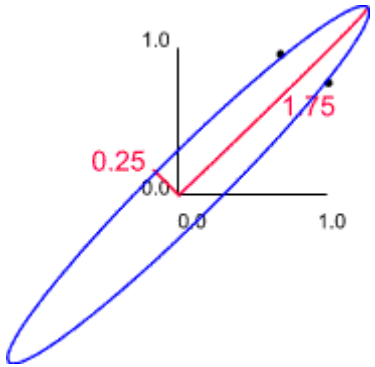
Fishes



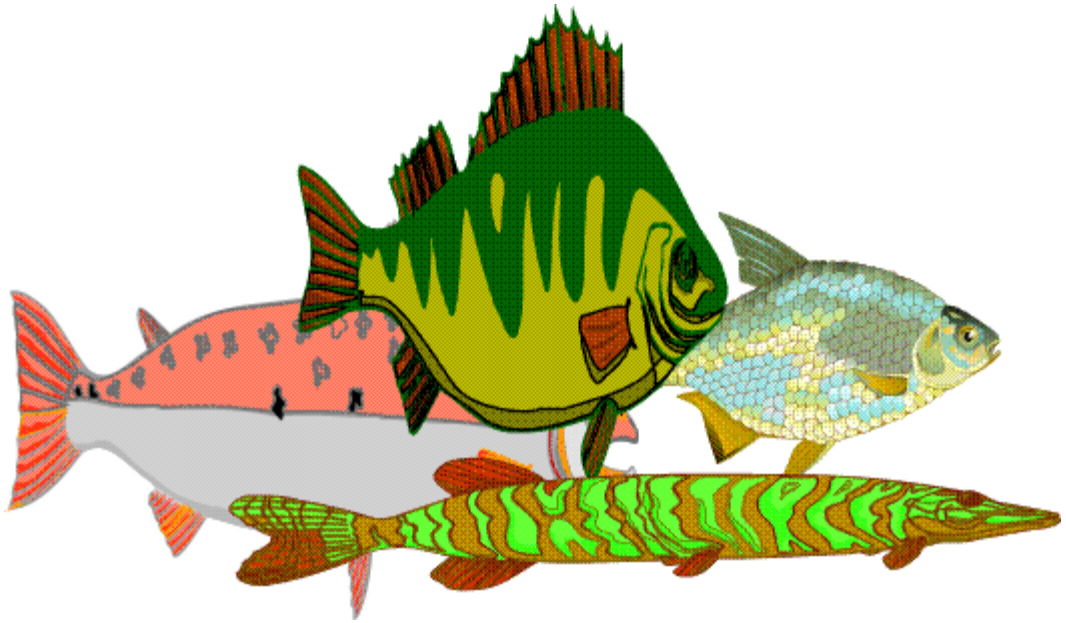


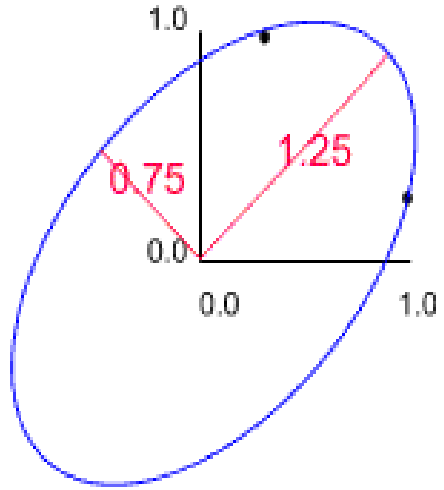




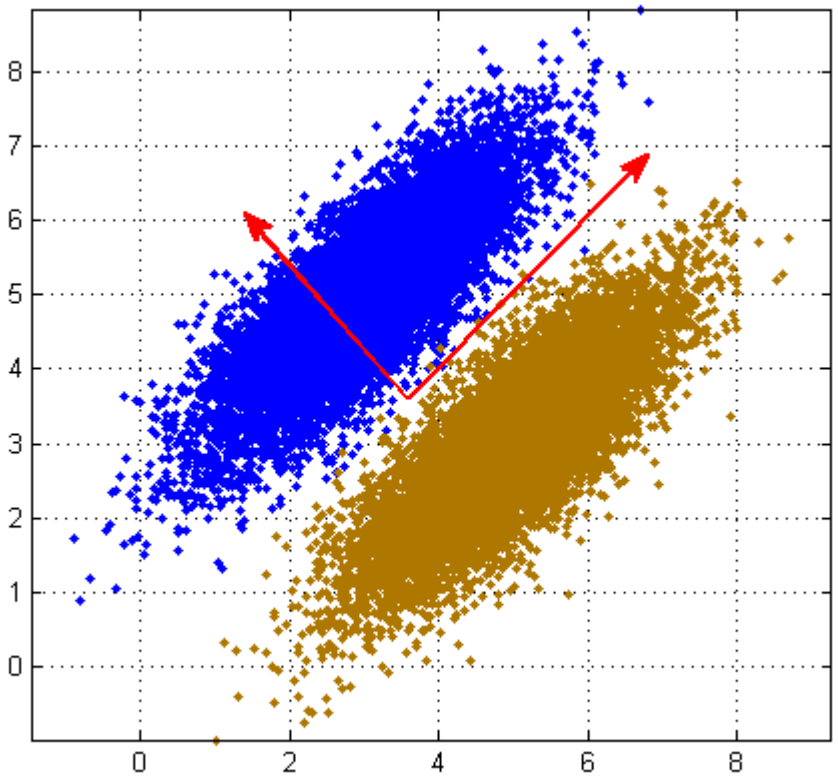


Fishes





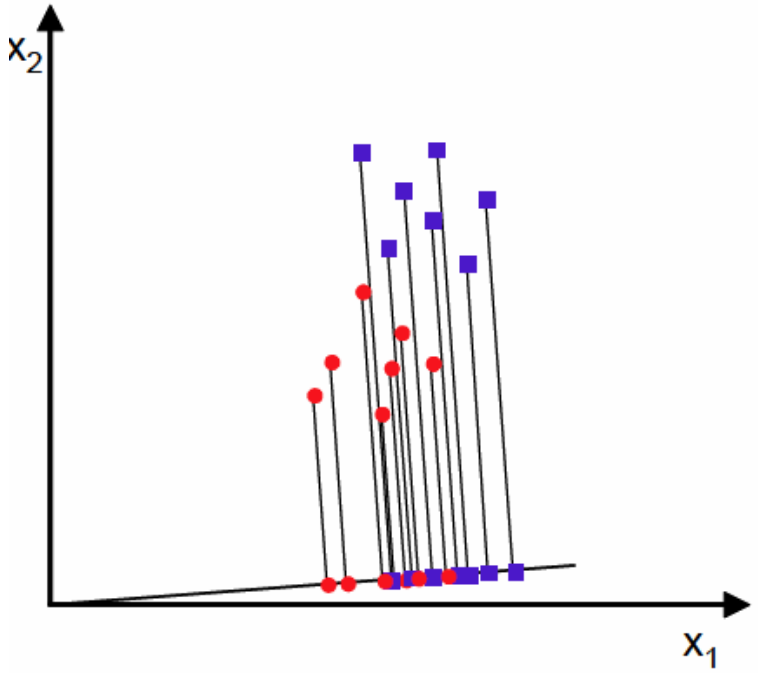
Further & More

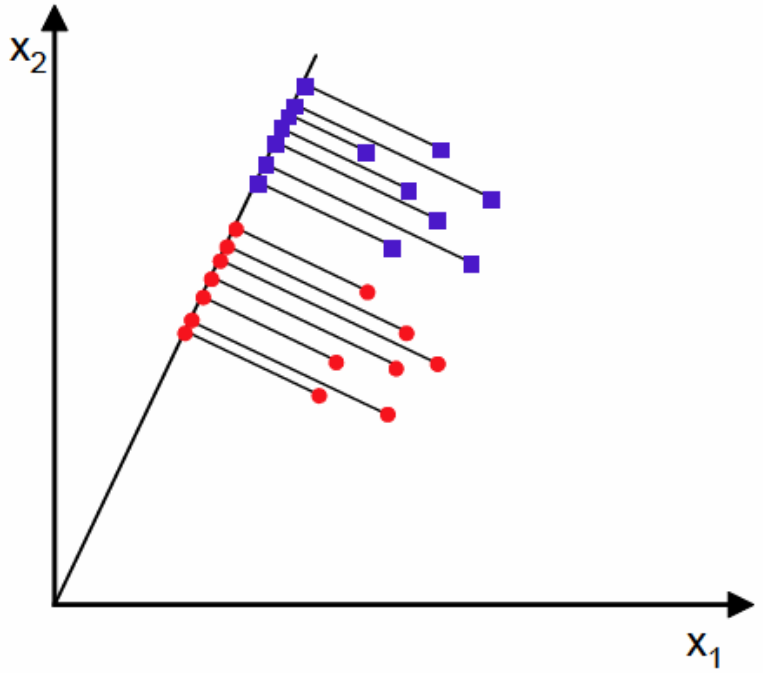


Now, let’s consider class information ...

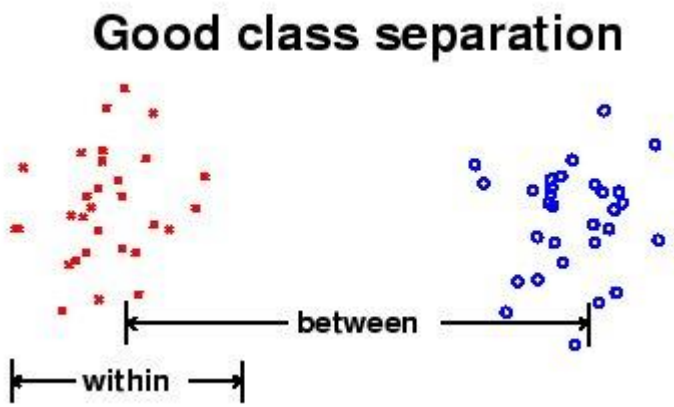
Linear Discriminant Analysis（LDA）
线性判别分析（降维）

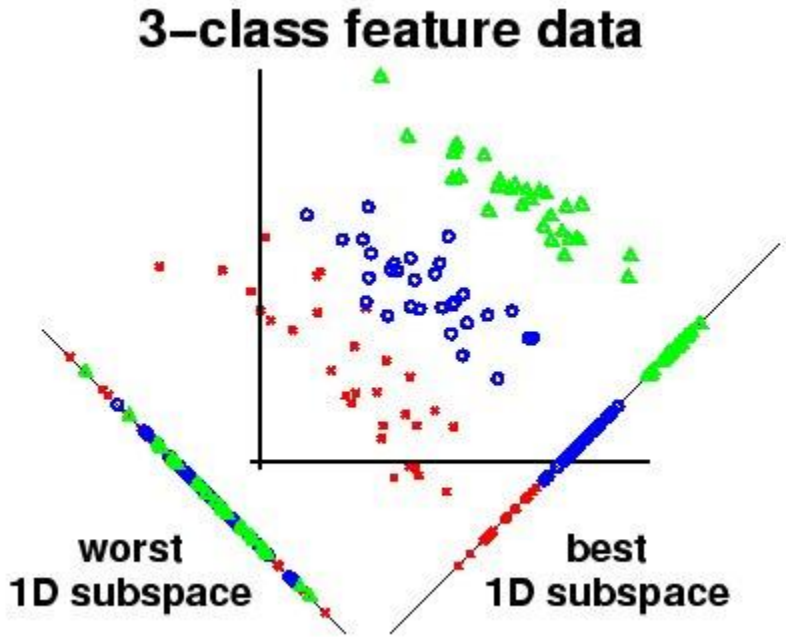
- The objective of LDA is to perform dimension reduction while preserving as much of the class discriminatory (类的区分信息)information as possible.
- Given a set of d-D vectors x_1, x_2, ..., x_n of which N_1 belong to ω_1, and N_2 to ω_2. Of all the possible projection lines $y=w^Tx$, find the one that maximizes the separability.



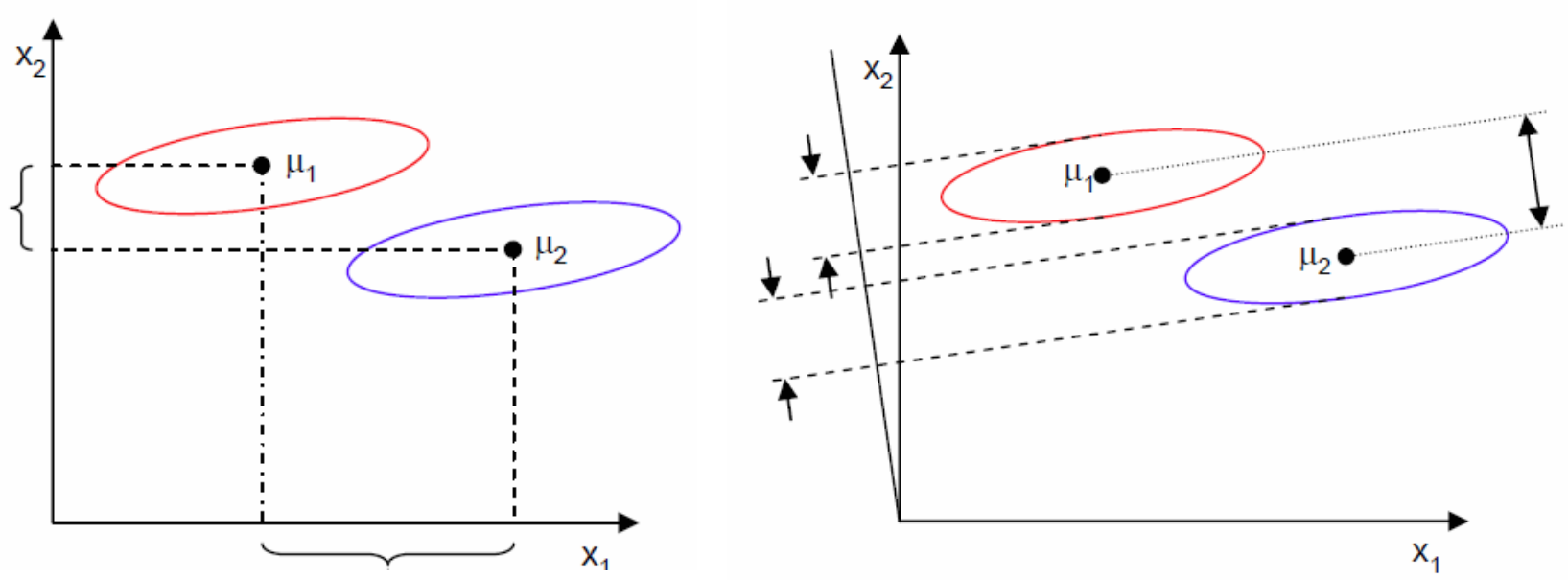


Measure of Separability





Fisher Criterion



$$J = \frac{\left|\mu_1 - \mu_2\right|^2}{S_1^2 + S_2^2}$$

⟸ Maximize the distance between classes

⟸ Minimize the scatter within each class

Some Math…

$$\mu_i = \frac{1}{N_i}\sum_{x\in\omega_i} x \quad \text{and} \quad \tilde{\mu}_i = \frac{1}{N_i}\sum_{y\in\omega_i} y = \frac{1}{N_i}\sum_{x\in\omega_i} w^T x = w^T\mu_i$$

between-class scatter

$$(\tilde{\mu}_1 - \tilde{\mu}_2)^2 = (w^T\mu_1 - w^T\mu_2)^2 = w^T\underbrace{(\mu_1-\mu_2)(\mu_1-\mu_2)^T}_{S_B}w = w^T S_B w$$

$$S_i = \sum_{x\in\omega_i}(x-\mu_i)(x-\mu_i)^T$$

$$S_1 + S_2 = S_W$$

within-class scatter

$$J(w) = \frac{\left|\tilde{\mu}_1 - \tilde{\mu}_2\right|^2}{\tilde{s}_1^2 + \tilde{s}_2^2} \Rightarrow J(w) = \frac{w^T S_B w}{w^T S_W w}$$

$$\tilde{s}_i^2 = \sum_{y\in\omega_i}(y-\tilde{\mu}_i)^2 = \sum_{x\in\omega_i}(w^T x - w^T\mu_i)^2 = \sum_{x\in\omega_i} w^T(x-\mu_i)(x-\mu_i)^T w = w^T S_i w$$

$$\tilde{s}_1^2 + \tilde{s}_2^2 = w^T S_W w$$

Some Math…

$$J(w) = \frac{w^T S_B w}{w^T S_W w}$$ ← generalized Rayleigh quotient

$$\frac{d}{dw}[J(w)] = \frac{d}{dw}\left[\frac{w^T S_B w}{w^T S_W w}\right] = 0 \Rightarrow$$

$$\Rightarrow \left[w^T S_W w\right]\frac{d\left[w^T S_B w\right]}{dw} - \left[w^T S_B w\right]\frac{d\left[w^T S_W w\right]}{dw} = 0 \Rightarrow$$

$$\Rightarrow \left[w^T S_W w\right]2S_B w - \left[w^T S_B w\right]2S_W w = 0$$

$$\frac{\left[w^T S_W w\right]}{\left[w^T S_W w\right]}S_B w - \frac{\left[w^T S_B w\right]}{\left[w^T S_W w\right]}S_W w = 0 \Rightarrow$$

$$\Rightarrow S_B w - J S_W w = 0 \Rightarrow$$

$$\Rightarrow S_W^{-1} S_B w - J w = 0$$

Some Math...

$$S_W^{-1} S_B w = Jw$$ ← eigenvector problem!

$$S_B w = (\mu_1 - \mu_2)(\mu_1 - \mu_2)^T w = (\mu_1 - \mu_2) R$$

$$R = (\mu_1 - \mu_2)^T w$$ ← scalar

$$Jw = S_w^{-1}(S_B w) = S_w^{-1}(\mu_1 - \mu_2) R$$

$$w = \frac{R}{J} S_w^{-1}(\mu_1 - \mu_2)$$

$$w^* = \underset{w}{\text{argmax}} \left\{ \frac{w^T S_B w}{w^T S_W w} \right\} = S_W^{-1}(\mu_1 - \mu_2)$$

LDA Example

- Dataset
  - $C1 = [(1,2); (2,3); (3,3); (4,5); (5,5)]$
  - $C2 = [(1,0); (2,1); (3,1); (3,2); (5,3); (6,5)]$
- Covariance of [c1; c2]
  - $\mathrm{Z} = \begin{bmatrix} 2.7636 & 2.2545 \\ 2.2545 & 3.0182 \end{bmatrix}$
- Eigenvectors and Eigenvalues of Z
  - $V = \begin{bmatrix} -0.7268 & 0.6869 \\ 0.6869 & 0.7268 \end{bmatrix}$
  - $D = \begin{bmatrix} 0.6328 & 0 \\ 0 & 5.1490 \end{bmatrix}$
- The direction of PCA projection: $[0.6869, 0.7268]^T$

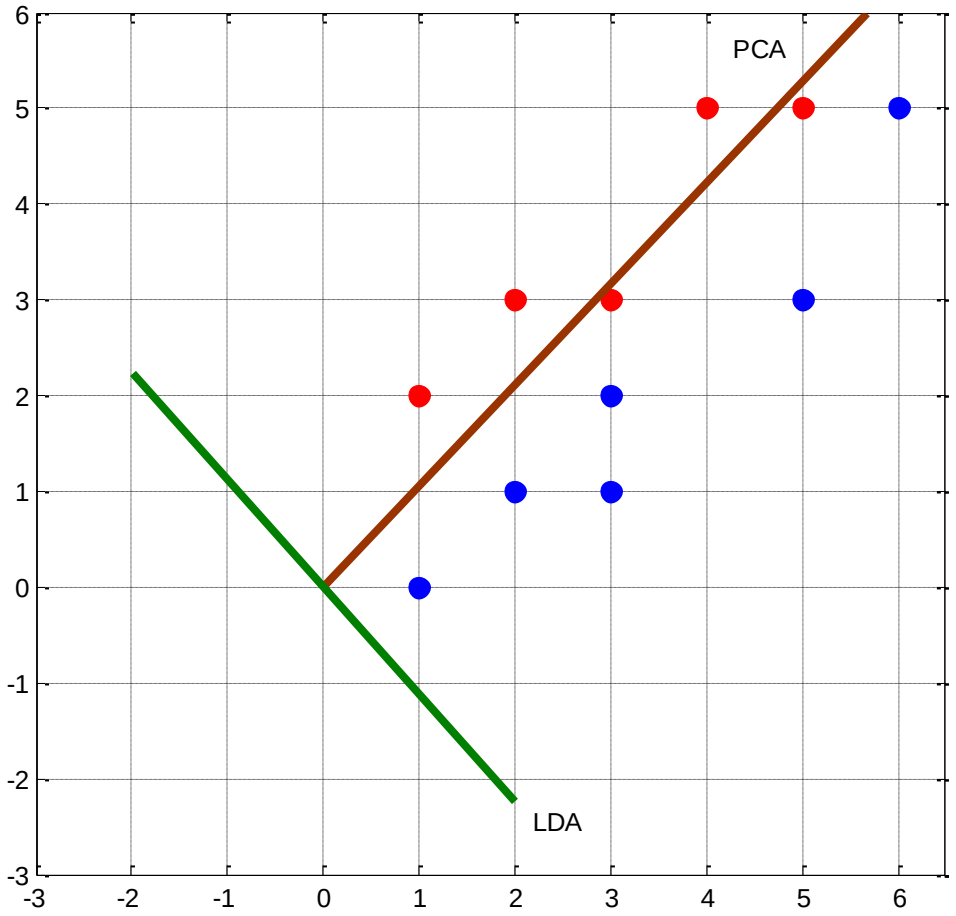
LDA Example

- The mean of each class
  - $\mu_1 = mean(c1) = [3.0, 3.6]^T$
  - $\mu_2 = mean(c2) = [3.3, 2.0]^T$
- $S_B = (\mu_1 - \mu_2)(\mu_1 - \mu_2)^T = \begin{bmatrix} 0.11 & -0.53 \\ -0.53 & 2.56 \end{bmatrix}$
- The scatter of each class
  - $S_1 = 4 \times cov(c1) = \begin{bmatrix} 10 & 8 \\ 8 & 7.2 \end{bmatrix}$
  - $S_2 = 5 \times cov(c2) = \begin{bmatrix} 17.3 & 16 \\ 16 & 16 \end{bmatrix}$
- $S_w = S_1 + S_2 = \begin{bmatrix} 27.3 & 24 \\ 24 & 23.2 \end{bmatrix}$

LDA Example

- $S_w{}^{-1}S_B = \begin{bmatrix} 0.26 & -1.27 \\ -0.30 & 1.42 \end{bmatrix}$
- Eigenvectors and Eigenvalues of $S_w{}^{-1}S_B$
  - $V = \begin{bmatrix} -0.98 & 0.67 \\ -0.20 & -0.75 \end{bmatrix}$
  - $D = \begin{bmatrix} 0 & 0 \\ 0 & 1.69 \end{bmatrix}$
- The direction of LDA projection: $[0.6656, -0.7463]^T$
- Alternatively
  - $S_w{}^{-1}(\mu_1 - \mu_2)^T = [-0.7936, 0.8899]^T$
  - After normalization: $[-0.6656, 0.7463]^T$

LDA Example



C-Class LDA

- **Fisher's LDA generalizes very gracefully for C-class problems**
  - Instead of one projection **y**, we will now seek (C-1) projections $[y_1,y_2,\ldots,y_{C-1}]$ by means of (C-1) projection vectors w_i, which can be arranged by columns into a projection matrix $W=[w_1|w_2|\ldots|w_{C-1}]$:

$$y_i = w_i^T x \Rightarrow y = W^T x$$

- **Derivation**
  - The generalization of the within-class scatter is

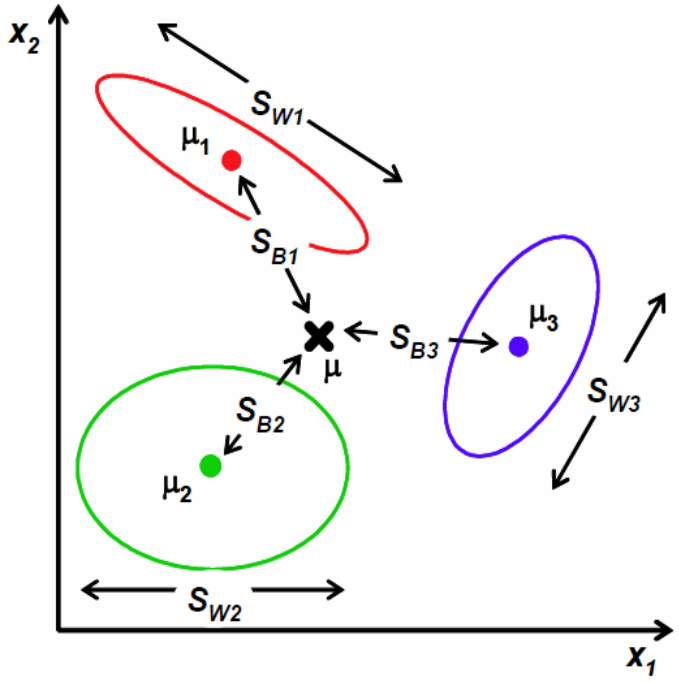
$$S_W = \sum_{i=1}^{C} S_i$$

$$\text{where } S_i = \sum_{x\in\omega_i} (x-\mu_i)(x-\mu_i)^T \text{ and } \mu_i = \frac{1}{N_i}\sum_{x\in\omega_i} x$$

  - The generalization for the between-class scatter is

$$S_B = \sum_{i=1}^{C} N_i(\mu_i-\mu)(\mu_i-\mu)^T$$

$$\text{where } \mu = \frac{1}{N}\sum_{\forall x} x = \frac{1}{N}\sum_{x\in\omega_i} N_i\mu_i$$



C-Class LDA

- For C-class LDA with C=2, S_B is defined as:

$$S_B = N_1(\mu_1 - \mu)(\mu_1 - \mu)^T + N_2(\mu_2 - \mu)(\mu_2 - \mu)^T$$

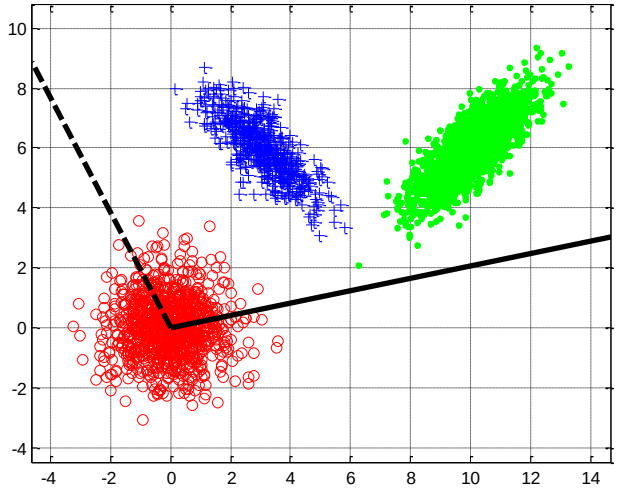
$$= N_1\left(\mu_1 - \frac{N_1\mu_1 + N_2\mu_2}{N}\right)\left(\mu_1 - \frac{N_1\mu_1 + N_2\mu_2}{N}\right)^T + N_2\left(\mu_2 - \frac{N_1\mu_1 + N_2\mu_2}{N}\right)\left(\mu_2 - \frac{N_1\mu_1 + N_2\mu_2}{N}\right)^T$$

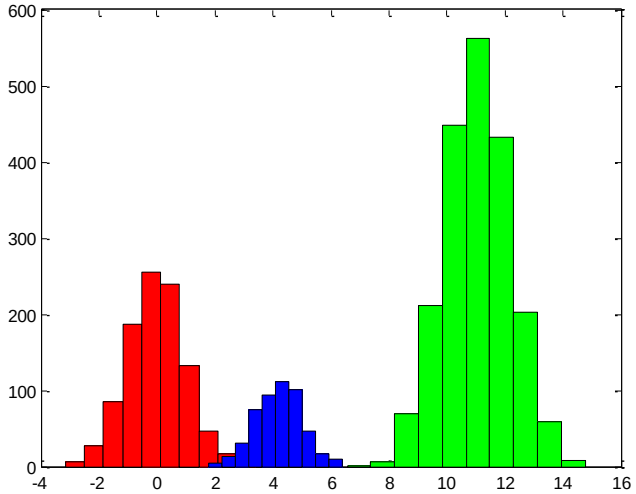
$$= N_1\left(\frac{N_2\mu_1 - N_2\mu_2}{N}\right)\left(\frac{N_2\mu_1 - N_2\mu_2}{N}\right)^T + N_2\left(\frac{N_1\mu_2 - N_1\mu_1}{N}\right)\left(\frac{N_1\mu_2 - N_1\mu_1}{N}\right)^T$$

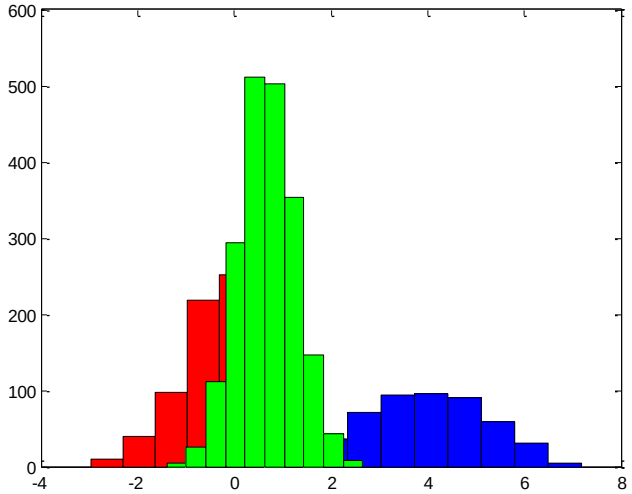
$$= \frac{N_1N_2^2}{N^2}(\mu_1 - \mu_2)(\mu_1 - \mu_2)^T + \frac{N_1^2N_2}{N^2}(\mu_2 - \mu_1)(\mu_2 - \mu_1)^T$$

$$= \frac{N_1N_2}{N}(\mu_1 - \mu_2)(\mu_1 - \mu_2)^T$$

C-Class LDA



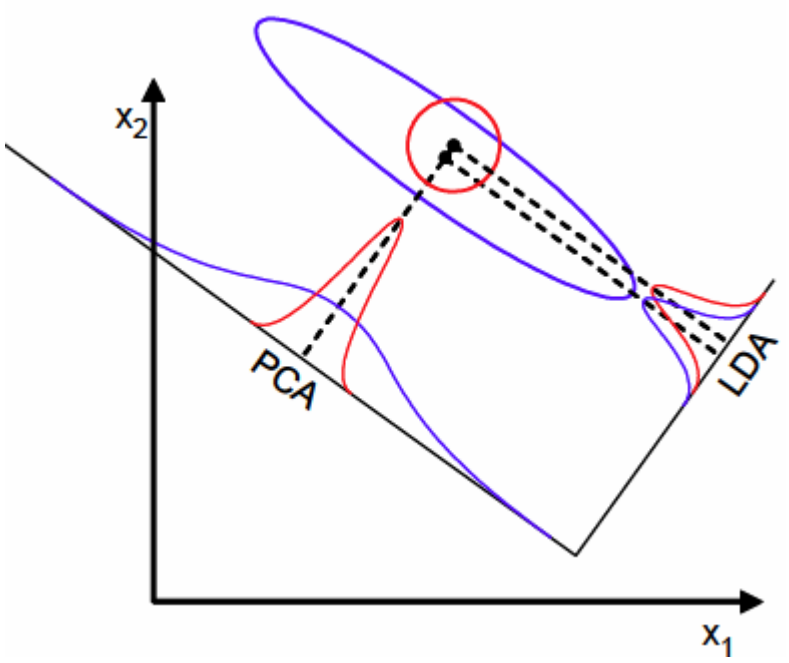


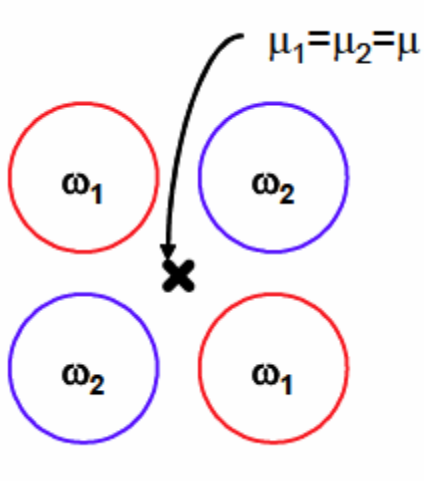


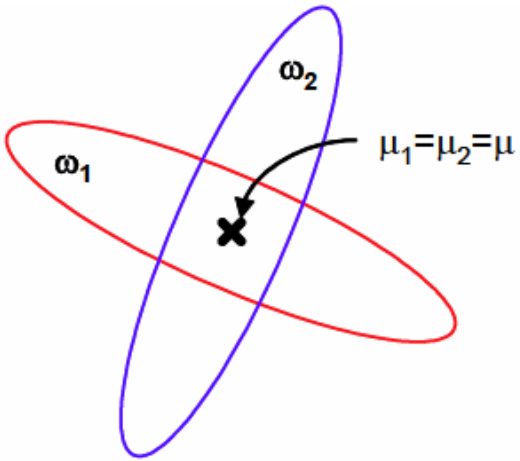

```
N1=1000; N2=500; N3=2000;
X1=mvnrnd([0,0], [1,0;0,1], N1);
X2=mvnrnd([3,6], [1,-0.8;-0.8,1], N2);
X3=mvnrnd([10,6], [1,0.8;0.8,1], N3);
S1=(N1-1)*cov(X1); S2=(N2-1)*cov(X2); S3=(N3-1)*cov(X3);
Sw=S1+S2+S3;
M1=mean(X1); M2=mean(X2); M3=mean(X3);
Mu=(N1*M1+N2*M2+N3*M3)/(N1+N2+N3);
Sb=N1*(M1-Mu)'*(M1-Mu)+N2*(M2-Mu)'*(M2-Mu) +N3*(M3-Mu)'*(M3-Mu);
J=inv(Sw)*Sb;  [V,D]=eig(J);
```

Measure of Separability

- LDA produces at most C-1 projections
  - S_B is a matrix with rank C-1 or less.
- S_W may be singular.
- LDA does not work well when ...







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ALSO
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DISK
TARGET
APPLIED
SENSOR
MAY
SHARED
DEFINITION
CURRENT
MOVING
WITHIN
ARCHIVES
MANAGE
EXAMPLES
EVERY
PRACTITIONERS
CAPTURE
TECHNOLOGIES
INTERNET
BUSINESS
SETS
GENOMICS
DISTRIBUTED
NEEDED
TIME
ONE
LARGE
MANAGEMENT
TERABYTES
INFORMATION
SOFTWARE
PETABYTES
INCLUDE
ABILITY
PERFORMANCE
STORE
SYSTEMS
TOLERABLE
SIZE
RESEARCH
STORAGE
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COST
PARALLEL
SAN
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SEARCH
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USE
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PROCESS