ELEC-H415

Analysis of a LEO satellite communication link

Communication Channels Project Report

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Contents

1 Introduction

In this project, a Low Earth Orbit (LEO) satellite communication link is simulated and analysed. Take Starlink satellite as example, there are now over 5000 Starlink satellites currently active and orbit around the earth at around 28000 km/h. The geometry and specific parameters [1] are shown below:

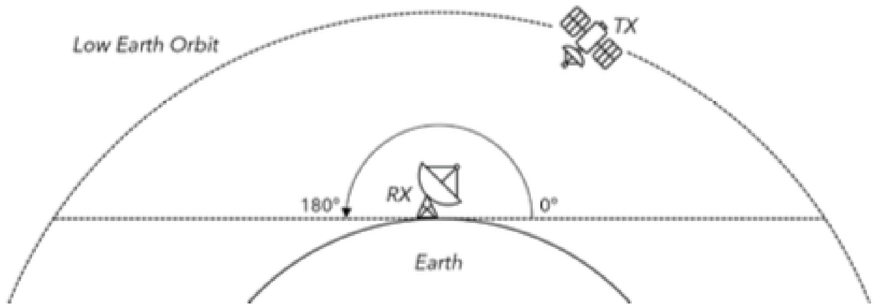

Figure 1.1: Schematic of the satellite scenario. The LEO satellite orbit is circular, and it passes through the zenith. Moreover, the simulations will be done in a vertical plane where the satellite is traveling from 0° elevation to 90° (zenith), then to 180°

Communication parameters:

- Carrier frequency: 26 GHz
- Bandwidth: 40 MHz
- Transmitter EIRP: 40 dBW (10 kW)
- Receiver noise figure: 2 dB
- Receiver antenna temperature: 275 K
- Satellite orbit altitude: 500 km
- Satellite revolution period: 5668 s
- Rain rate exceeded for 0.01% of the time: 30 mm/h

Ray-tracing parameters:

- For every position of the satellite, only the largest multipath component (MPC) reaching the receiver is considered, the others are ignored.
- Only the rays arriving at the receiver with an angle between 0° and 180° are considered, such that the rays reflected off the ground are ignored.

- The relative permittivity of the building walls is 4.
- We only consider the Line-of-Sight (LOS), single reflections off buildings, and diffraction.

2 Part1: LOS scenario

2.1 Description of LOS scenario [1]

Let's first consider that the ground station is equipped with a single lossless isotropic antenna ($G(\theta) = 1$), and that no obstacle is present around the receiver (only the LOS is incident). During one satellite pass, evaluate the following metrics, as a function of the satellite elevation:

- Free space path loss [dB]
- Received power [dBm]
- Signal-to-noise ratio [dB]
- Channel capacity [bit/s]
- Theoretical maximal total transmitted data during one satellite pass [bytes]

2.2 Theoretical foundations

In this part, the easiest LOS scenario is considered. To calculate the metrics, following procedures should be taken:

2.2.1 Free-space path loss

First, one has to convert the elevation angle of the satellite into the distance between satellite to RX via following expression [2]:

$$d = \sqrt{(R_E + h_s)^2 - R_E^2 cos^2 E} - R_E sin E$$

where d is the distance from satellite to RX, R_E is the radius of the earth (6371 km), E is the elevation angle (0°-180°) and h_s is the satellite altitude (500 km). In the MATLAB code this is packed into a function called *angle2dis*.

Then the free-space path loss can be calculated by using:

$$L[linear] = \frac{P_{TX}}{< P_{RX} >} = (\frac{4\pi d}{\lambda})^2$$

where λ is the wavelength. In the MATLAB code this is packed into a function called *PathLoss* under the folder *Part1_LOS*.

2.2.2 Received power

The received power in free-space case can be derived from the Friis formula [3, 4] shown as below:

$$\begin{aligned} P_{RX}(d)[linear] &= P_{TX}G_{TX}(\theta_{TX}, \phi_{TX})G_{RX}(\theta_{RX}, \phi_{RX})(\frac{\lambda}{4\pi d})^2 \\ &= EIRP_{TX} \cdot G_{RX}(\theta_{RX}, \phi_{RX})(\frac{\lambda}{4\pi d})^2 \end{aligned}$$

pay attention that EIRP in here should be converted into linear scale. In the MATLAB code this is packed into a function called *RX_power* under the folder *Part1_LOS*.

2.2.3 Signal-to-noise ratio

In a wireless communication, the noise mainly consists of two parts: input noise add noise added by the receiver. Therefore the SNR can be presented as follows [3]:

$$\begin{aligned} SNR[dB] &= P_{RX}[dBW] - N[dBW] \\ &= P_{RX}[dBW] - F[dB] - 10log_{10}(kTB) \end{aligned}$$

where $F[dB]$ is the receiver noise figure in dB, k is Boltzmann constant which is $1,379 \cdot 10^{-23}[WHz^{-1}K^{-1}]$ and B is the bandwidth in Hz. In the MATLAB code this is packed into a function called *SNR_dB* under the folder *Part1_LOS*.

2.2.4 Channel capacity

The expression to calculate the channel capacity is as follows [**?**]:

$$C[bit/s] = B \cdot log_2(1 + SNR[linear])$$

Therefore this can be easily derived from SNR. In the MATLAB code this is packed into a function called *ChannelCapcity* under the folder *Part1_LOS*.

2.2.5 Theoretical maximal total transmitted data during one satellite pass

In this part, the total transmitted data during one pass can be obtained as [2]:

$$\begin{aligned} Total &= \int_0^{T_{pass}} C(t)dt \\ &= \sum_{n=0}^{N-1} \frac{C_n + Cn + 1}{2}\Delta t_n \end{aligned}$$

where Δt_n is the time interval between two C. However, in this case, the Δt_n should be further derived w.r.t the elevation angle, which could be done in two steps.

Step 1: Convert elevation angle E to the central angle α:

$$\alpha = arcsin(\frac{d}{R_E + h_s} \cdot cosE)$$

Step 2: Calculate Δt_n w.r.t to the central angle α:

$$\Delta t_n = T_{rev}\frac{|\alpha_n - \alpha_{n+1}|}{2\pi}$$

where T_rev is the time that satellite travels through a 180° of orbit with a constant velocity, the value of it is around 94 minutes 28 seconds, 5668 seconds in other words. In MATLAB, both 2 formulas can be applied to calculate the total transmitted bits. If one wants to use integral, then *trapz* function can be applied. In the MATLAB code the integral is applied and this is packed into a function called *TotBitsTX* under the folder *Part1_LOS*.

2.2.6 Rain attenuation

Rain would deteriorate the communication quality as well, it can be calculated as follows [2]:

First calculate the attenuation for 0.01% exceedance as reference:

$$L_r(0.01) = \gamma d_s$$

Then the slant-path through the rain could be:

$$d_s[km] = \frac{h_R}{sinE}$$

where h_R is the rain height equals to $2.4km$ [1].

And the specific attenuation is:

$$\gamma = kR_{0.01}^{\alpha} = 0.15 + 0.003cos^2E \cdot R_{0.01}^{\alpha}$$

where $R_{0.01}$ is the rain rate for 0.01% exceedance in [mm/h] [5] and k, α are the frequency dependent coefficient.

Finally the attenuation can be given as the function of exceedance percentage p:

$$L_r(p) = L_r(0.01)(\frac{p}{0.01})^{-0.65-0.03ln(p)+0.05ln(L_r(0.01))}$$

Pay attention that the formulas have been simplified, such that they approach the ITU recommendation results for elevation angles $E \geq 20$. For smaller elevation angles ($E \leq 20$), consider the rain attenuation constant and equal to the one computed at 20° elevation [1]. In the MATLAB code the integral is applied and this is packed into a function called *RainAttenuation* under the folder *Part1_LOS*.

2.2.7 Transmission with a constant bitrate

In practice the bitrate during the transmission must be lower than the channel capacity in order to demodulate the signal correctly. Therefore in this part a constant bitrate which allows to send the largest amount of data should be find. In the MATLAB code the integral is applied and this is packed into a function called *ConstBitRate* under the folder *Part1_LOS*.

2.3 Result, validation and analysis

2.3.1 Result, validation and analysis for Part 1

The result of Part 1 is shown in Figure 2.1 and Figure 2.2.

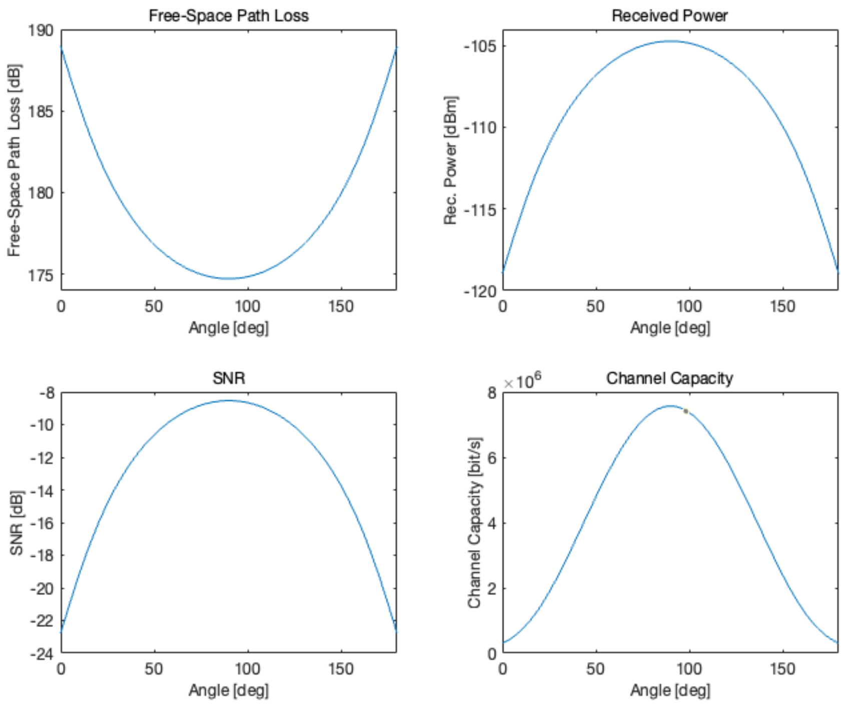

Figure 2.1: Result of Part 1. The top left sub-figure is the relation between free-space path loss and elevation angle. The top right sub-figure is the relation between received power and elevation angle. The bottom left sub-figure is the relation between SNR and elevation angle. The bottom right sub-figure is the relation between channel capacity and elevation angle.

```
TotBits During One Pass = 182763447.1211 Bytes
```

Figure 2.2: Result of Part 1: total transmitted data during one pass. 1.4621e+09 in Bits, 1.8276e+08 in Bytes.

To validate the correctness of the result above, one can first **analysis** the results. The result shows that when the elevation angle is 90 degree, which means that the satellite is right above the RX and the distance is minimal, the path loss reaches its minimal value and P_{RX}, SNR, channel capacity reaches its maximal which makes sense. Because the path loss is negative correlation with the square of distance, and rest of the metrics are positive correlation with the square of distance.

To further **validate** the result, on can take the elevation angle is 90 degree to calculate. In this case the distance d is the altitude of the satellite which is 500 km. The path loss could be:

$$\begin{aligned} L[linear] &= \frac{P_{TX}}{< P_{RX} >} = (\frac{4\pi d}{\lambda})^2 \\ &= (\frac{4\pi \cdot 500 \cdot 10^3 [m]}{0.0115})^2 \\ &= (5.4636 \cdot 10^8)^2 [linear] \\ &= 174.7496 [dB] \end{aligned}$$

which is the same as the result shown in Figure 2.1. Then the received power could be:

$$\begin{aligned}
P_{RX}(d)[linear] &= P_{TX}G_{TX}(\theta_{TX},\phi_{TX})G_{RX}(\theta_{RX},\phi_{RX})(\frac{\lambda}{4\pi d})^2 \\
&= EIRP_{TX}\cdot G_{RX}(\theta_{RX},\phi_{RX})(\frac{\lambda}{4\pi d})^2 \\
&= 10\cdot 10^3[W]\cdot 1\cdot(\frac{0.0115}{4\pi\cdot 500\cdot 10^3[m]})^2 \\
&= 3.3499\cdot 10^{-14}[W] = -104.7497[dBm]
\end{aligned}$$

which is the same as the result shown in Figure 2.1. Then the SNR could be:

$$\begin{aligned}
SNR[dB] &= P_{RX}[dBW] - N[dBW] \\
&= P_{RX}[dBW] - F[dB] - 10log_{10}(kTB) \\
&= -104.7497[dBm] - 30 - 2[dB] - 10log_{10}(1,379\cdot 10^{-23}[WHz^{-1}K^{-1}]\cdot 275K\cdot 40\cdot 10^6 Hz) \\
&= -8.5593[dB]
\end{aligned}$$

which is the same as the result shown in Figure 2.1. Finally the channel capacity could be:

$$\begin{aligned}
C[bit/s] &= B\cdot log_2(1+SNR[linear]) \\
&= 40\cdot 10^6\cdot log_2(1+10^{(-8.5593[dB]/10)}) \\
&= 7.5278\cdot 10^6[bit/s]
\end{aligned}$$

which is the same as the result shown in Figure 2.1. The last part could be total transmitted bits, which is difficult to validate the result. However, one can verify if the sum of variable Δt_n is correct. Therefore, first the range of α should be calculated:

$$\begin{aligned}
\alpha_{begin} &= arcsin(\frac{d}{R_E+h_s}\cdot cosE) \\
&= arcsin(\frac{2.5731\cdot 10^6}{6371\cdot 10^3+500\cdot 10^3}\cdot cos0) \\
&= 21.99deg
\end{aligned}$$

$$\begin{aligned}
\alpha_{end} &= arcsin(\frac{d}{R_E+h_s}\cdot cosE) \\
&= arcsin(\frac{2.5731\cdot 10^6}{6371\cdot 10^3+500\cdot 10^3}\cdot cos0) \\
&= -21.99[deg]
\end{aligned}$$

therefore

$$\Delta\alpha_{begin2end} = \alpha_{begin} - \alpha_{end} = 43.98[deg] = 0.7677[rad]$$

Finally since

$$\begin{aligned}
\sum \Delta t_n &= \sum T_{rev}\frac{|\alpha_n - \alpha_{n+1}|}{2\pi} \\
&= \frac{T_{rev}}{2\pi}\cdot\sum\Delta\alpha_n \\
&= \frac{T_{rev}}{2\pi}\cdot\Delta\alpha_{begin2end} \\
&= \frac{5668[sec]}{2\pi}\cdot 0.7677[rad] = 692.5347[sec]
\end{aligned}$$

According to the MATLAB simulation the sum of Δt_n is 692.5314s which is almost the same result. Therefore the result of the total transmitted bits should be correct as well.

2.3.2 Result, validation and analysis for Bonus 1.1

The result for bonus 1.1 is shown in Figure 2.3.

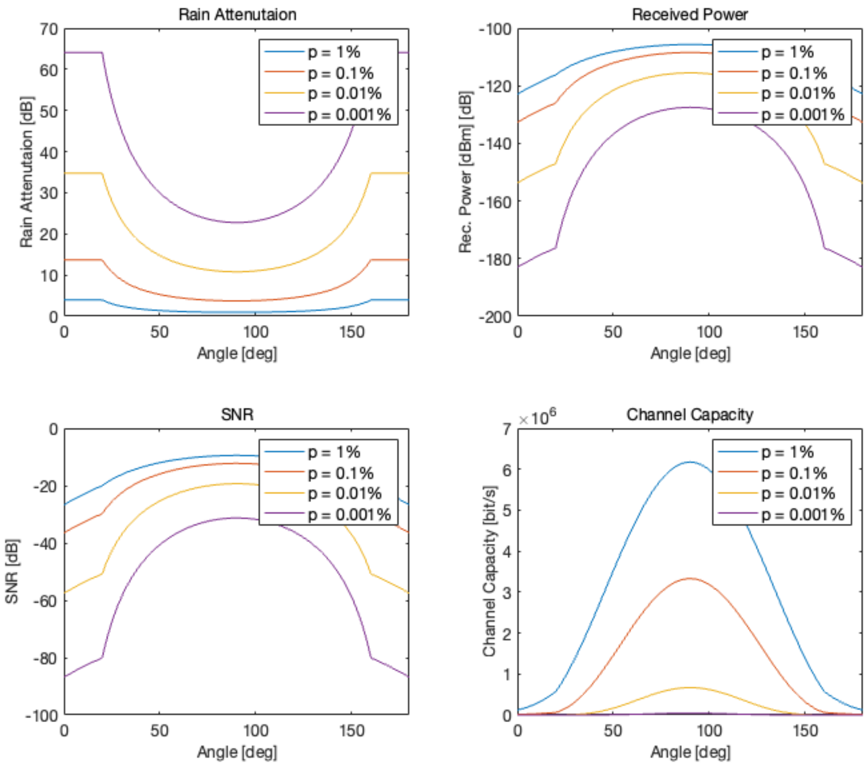

Figure 2.3: Result of Bonus 1.1. The top left sub-figure is the relation between rain attenuation and elevation angle with different exceedance. The top right sub-figure is the relation between received power and elevation angle with different exceedance. The bottom left sub-figure is the relation between SNR and elevation angle with different exceedance. The bottom right sub-figure is the relation between channel capacity and elevation angle with different exceedance.

What worth mentioning is that exceedance $p\%$ means in $p\%$ of the time within a year, the signal attenuation exceeds a certain value. Therefore a lower p will lead to a higher attenuation. Moreover, since one assumes that when the angle between satellite and ground is smaller than 20 degree, the value remains at 20 degree, which leads to the flat region in rain attenuation sub-figure. Also, it makes sense that the heavier the rain, the worse the communication quality (i.e. worse P_{RX}, SNR and channel capacity).

To further validate it, one can use the formula mentioned in section 2.2.6 and assume 90 degree of elevation angle, 0.001% of the p:

$$\begin{aligned} L_r(0.01) &= \gamma d_s \\ &= 0.15 \cdot 30^{0.15/0.15} \cdot 2.4 = 10.8000 \end{aligned}$$

Therefore

$$\begin{aligned} L_r(p) &= L_r(0.01)(\frac{p}{0.01})^{-0.65-0.03ln(p)+0.05ln(L_r(0.01))} \\ &= 10.8000 \cdot (\frac{0.001}{0.01})^{-0.65-0.03ln(0.001)+0.05ln(4.5)} \\ &= 22.7622[dB] \end{aligned}$$

which is the same as MATLAB simulation. For the rest of the results, the previous functions in section 2.3.1 are used (already validated), therefore no further validation for them here.

2.3.3 Result, validation and analysis for Bonus 1.2

The result for bonus 1.2 is shown in Figure 2.4.

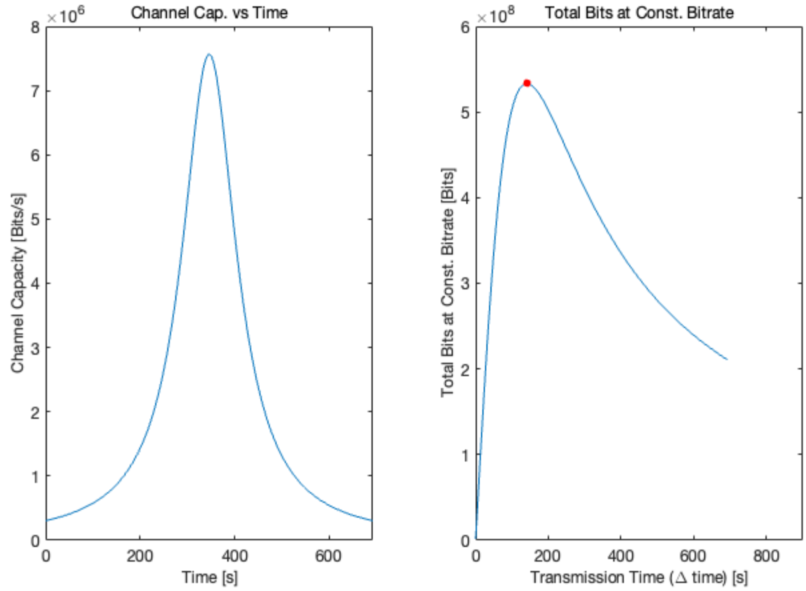

Figure 2.4: Result of Bonus 1.2. The left sub-figure is the relation between channel capacity and time without rain. The right sub-figure is the relation between total bits transmitted and transmission time without rain (i.e. the interval between the start of the transmission and end of the transmission)

The maximal transmitted bits and their corresponding elevation angle are shown in Figure 2.5.

```
========================== Constant Bits ==========================
Max TotBits without Rain = 66633712.8412 Bytes at Angle 41.8
Max TotBits with p 1 = 47444205.5005 Bytes at Angle 48.4
Max TotBits with p 0.1 = 20490718.3552 Bytes at Angle 56.3
Max TotBits with p 0.01 = 3113330.7711 Bytes at Angle 64
Max TotBits with p 0.001 = 158497.5202 Bytes at Angle 69.1
```

Figure 2.5: Result of Bonus 1.2: total transmitted data during one pass. When no rain: max total bits is $5.33067 \cdot 10^8$ *Bits* obtained at elevation angle 41.8 degree. When exceedance is 1: max total bits is $3.7955 \cdot 10^8$ *Bits* obtained at elevation angle 48.4 degree. When exceedance is 0.1: max total bits is $1.6393 \cdot 10^8$ *Bits* obtained at elevation angle 56.3 degree. When exceedance is 0.01: max total bits is $2.4907 \cdot 10^7$ *Bits* obtained at elevation angle 64 degree. When exceedance is 0.001: max total bits is $1.2680 \cdot 10^6$ *Bits* obtained at elevation angle 69.1 degree.

3 Part2: Beamforming

3.1 Description of beamforming [1]

Let's consider now that the ground station is equipped with a horizontal Uniform Linear Array (ULA), which can have two types of elementary unit:

- An isotropic antenna with $U(\theta) = U_0$
- A patch antenna (microstrip) with $U(\theta) = U_0 \cdot sin^2\theta$

The antennas in the array are spaced by a distance $\lambda/2$. When doing beamforming, it is assumed that the ULA is always perfectly pointing to the satellite position. The schematic is shown in Figure 3.1.

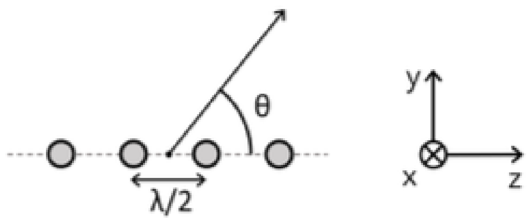

Figure 3.1: Antenna array

3.2 Theoretical foundations

In this simulation, a far-field environment can be assumed (since it is the communication between ground and satellite), which means that the angle θ and the distance between antenna to satellite r can be assumed to be unchanged for the amplitude term. The formula [6] can be thus written into:

$$\begin{aligned}
\vec{E_T} &= j\frac{Z_0 I_0}{2\pi}\frac{\cos\left(\frac{\pi}{2}\cos\theta_0\right)}{\sin\theta_0}\frac{e^{-j\beta r_0}}{r_0}\vec{1}_\theta \\
&+ j\frac{Z_0 I_1 e^{-j\delta_1}}{2\pi}\frac{\cos\left(\frac{\pi}{2}\cos\theta_1\right)}{\sin\theta_1}\frac{e^{-j\beta r_1}}{r_1}\vec{1}_\theta \\
&+ \dots \\
&+ j\frac{Z_0 I_{N-1} e^{-j\delta_{N-1}}}{2\pi}\frac{\cos\left(\frac{\pi}{2}\cos\theta_{N-1}\right)}{\sin\theta_{N-1}}\frac{e^{-j\beta r_{N-1}}}{r_{N-1}}\vec{1}_\theta \\
&= \left(I_0 + I_1 e^{-j\delta_1}e^{j\beta d\cos\theta} + \dots + I_{N-1}e^{-j\delta_{N-1}}e^{j(N-1)\beta d\cos\theta}\right) j\frac{Z_0}{2\pi}\frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta}\frac{e^{-j\beta r}}{r}\vec{1}_\theta
\end{aligned}$$

The term in the parentheses is denoted as array factor (AF) and the term left is the $\vec{E}$ for one antenna. Rewritten AF [6] into:

$$\begin{aligned} AF &= 1 + e^{-j\delta}e^{j\beta d\cos\theta_m} + \ldots + e^{-j(N-1)\delta}e^{j(N-1)\beta d\cos\theta_m} \\ &= \sum_{n=1}^{N} e^{j(n-1)(\beta d\cos\theta_m - \delta)} \\ &= \sum_{n=1}^{N} e^{j(n-1)\psi} = \frac{1-e^{jN\psi}}{1-e^{j\psi}} \end{aligned}$$

where $\psi = \beta d\cos\theta_m - \delta$ is the term that **controls the directivity of the antenna**. Therefore, this simulation is done by first choose the pointing angle θ_m to determine the phase shift of the ULA ψ, the use the following formula [2] to calculate the gain pattern:

$$G(\theta) = \frac{U_{Array}(\theta)}{\frac{1}{4\pi}\int_\Omega U_{Array}(\theta)\, d\Omega}$$

where,

$$U_{Array}(\theta) = |AF(\theta)|^2 \cdot U_{single}(\theta)$$

and

$$\int_\Omega U(\theta)\, d\Omega = 2\pi \int_0^\pi U(\theta)\sin\theta\, d\theta$$

3.3 Result, validation and analysis

3.3.1 Result, validation and analysis for Part 2

In this part, first the gain patterns (from 0° to 180°) for the pointing angles 90°, 45°, 30° and 0° for antenna number 4, 8 and 16 is drew. The result is shown in Figure 3.2.

Then the received power, channel capacity and theoretical maximal total transmitted data during one satellite pass is calculated again as in Part 1 for each uniform linear array (ULA). The result for isotropic ULA is shown in Figure 3.5 , the result for patch ULA is shown in Figure 3.6

The result of the gain pattern can be done as follows. One can know that the maximal gain of the ULA can be obtained when the direction of the ULA is perfectly match the direction of the satellite, which means $\psi = 0$. Therefore one can have:

$$AF = \sum_{n=1}^{N} e^{j(n-1)\psi} = \sum_{n=1}^{N} 1 = N$$

and if the same amount of power is injected, then at maximal gain,

$$|I_n| = \sqrt{\frac{2}{R}\cdot P_n} = \sqrt{\frac{P_{total}}{N}\cdot\frac{2}{R}} \propto \frac{1}{\sqrt{N}}$$

Therefore

$$G_{Array}(\theta) \propto N \cdot G_{single}(\theta)$$

By looking into the Figure 3.2, one can see that the maximal gain obtained is the number of antennas in the array times the gain for the single antenna, which is consistent with the validation.

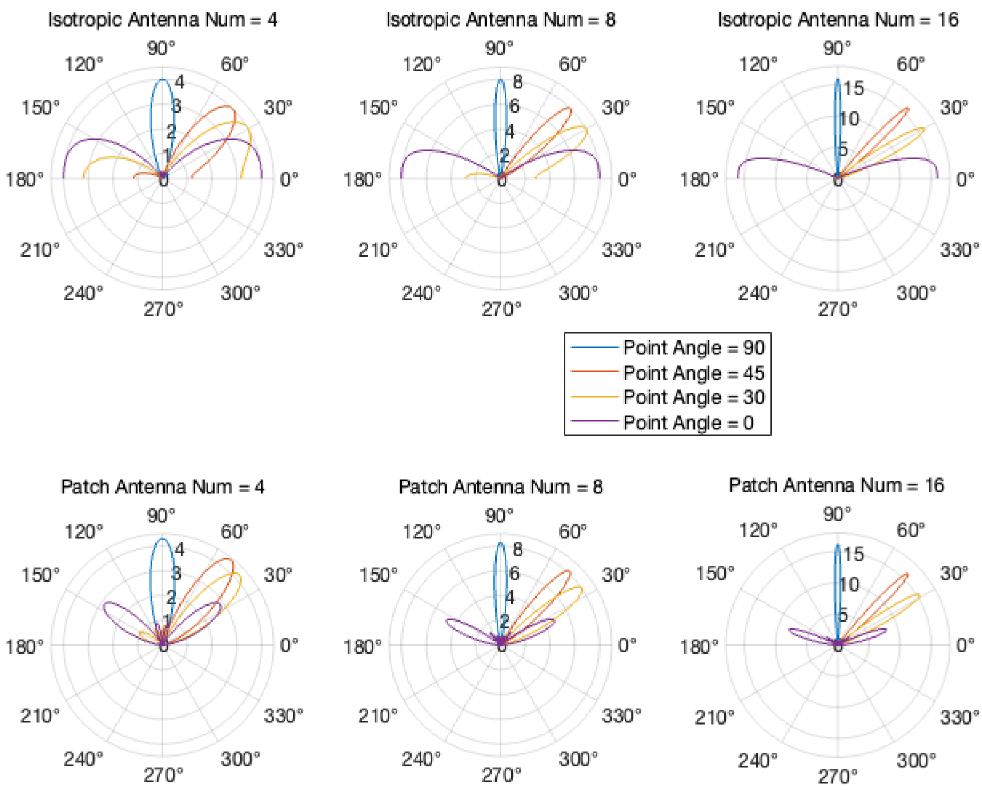

Figure 3.2: Antenna array

To validate the correctness of the isotropic ULA metrics, one can compare the result obtained with the result in previous chapter, which is Figure 2.1. The shape of the curve is almost the same. For the Figure 3.6, one can see that when the elevation angle is small, the channel capacity is almost 0, this can be explained by using the gain pattern.

From the gain pattern of the patch ULA, it is obviously that unlike isotropic antenna, when the pointing angle for patch ULA is 0, its gain is almost 0 as well. However for an isotropic ULA, it can still get some data even for the small elevation angle. This can lead to a trade-off between two antenna arrays.

- If one want to receive data even at very small elevation angle, then isotropic ULA is better.
- If one want to receive data in a wider elevation angle range with a relatively stable SNR, but does not care the SNR at small elevation angle that much, then patch ULA is better.

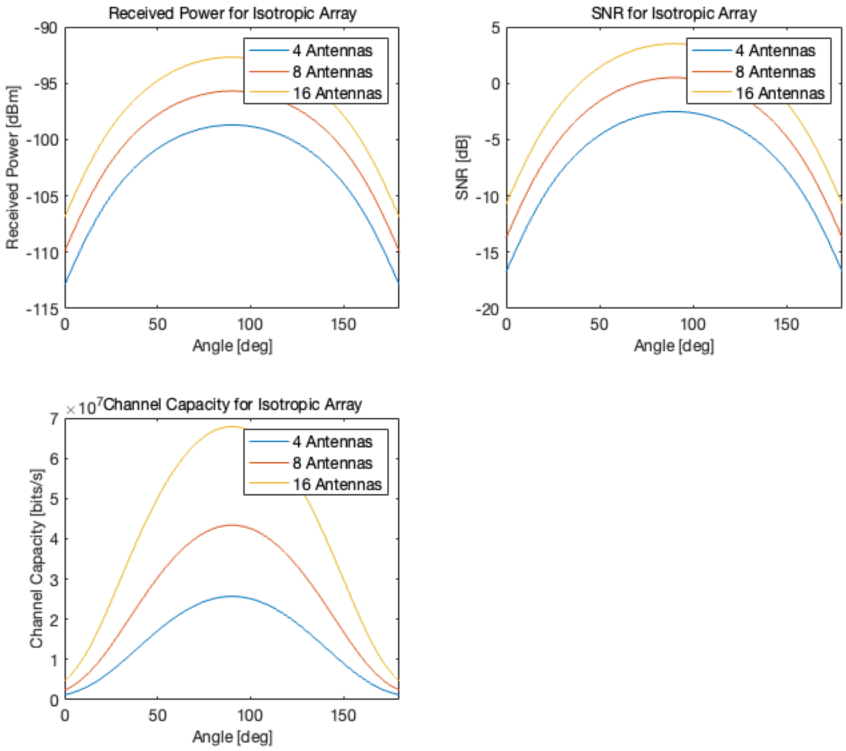

Figure 3.3: Result for isotropic ULA. The top left sub-figure is the relation between received power and elevation angle. The top right sub-figure is the relation between SNR and elevation angle. The bottom left sub-figure is the relation between channel capacity and elevation angle.

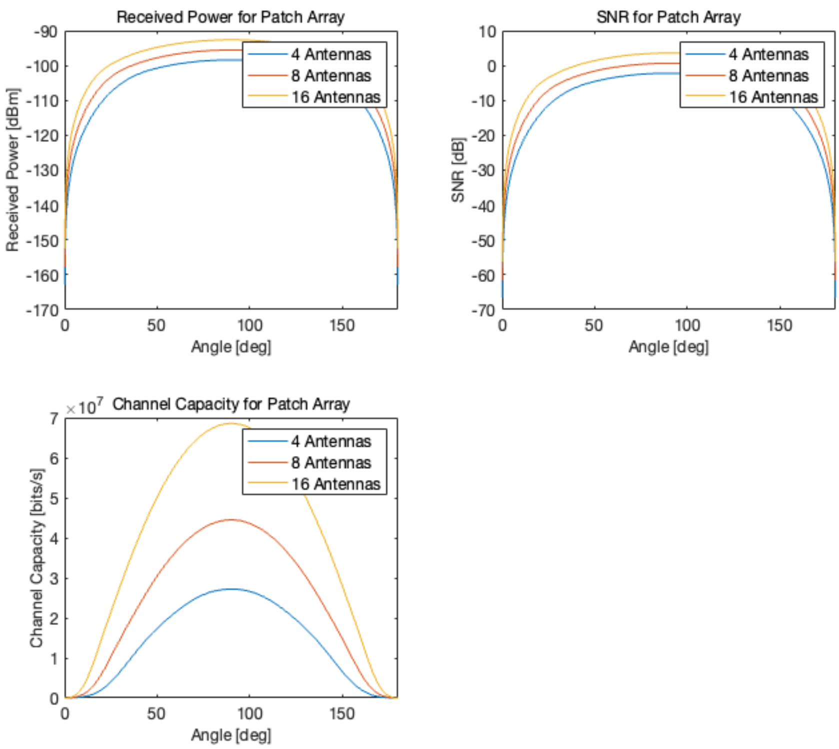

Figure 3.4: Result for patch ULA. The configuration of the plot is the same as Figure 3.5

The total transmitted bits during one pass is
For isotropic ULA:

- Number of elements is 4 -> Bits is 6.637709936983514e+08 bytes
- Number of elements is 8 -> Bits is 1.198373711369801e+09 bytes
- Number of elements is 16 -> Bits is 2.052878623414760e+09 bytes

For patch ULA:

- Number of elements is 4 -> Bits is 5.214495829074546e+08 bytes
- Number of elements is 8 -> Bits is 9.679340331992280e+08 bytes
- Number of elements is 16 -> Bits is 1.703613991155562e+09 bytes

3.3.2 Result, validation and analysis for Bonus 2.1

In this section, the RX ULA on the ground can no longer perfectly track the satellite and there will be a de-pointing angle from 0.2 degree to 2 degree. If the number of antennas in the ULA increases, the gain is more likely to point to a particular direction with higher gain. In this case a small de-pointing angle between ULA and satellite will cause a significant drop of gain, thus lower total transmitted bits. The relation between de-pointing angle and total transmitted bits is shown in Figure 3.5 and Figure 3.6.

For isotropic ULA:

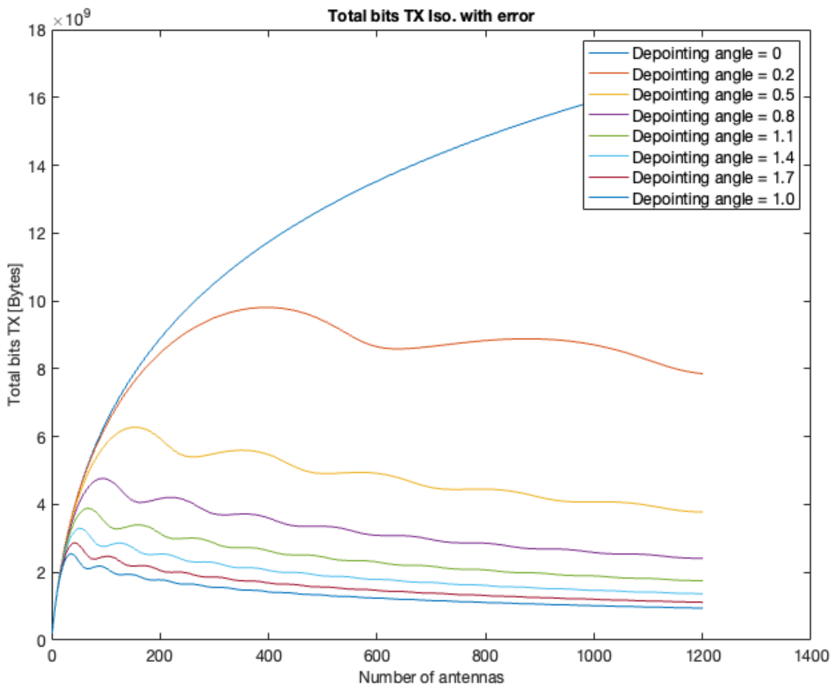

Figure 3.5: Result for isotropic ULA.

In order to make sure the calculation makes sense, the total transmitted bits for 0 degree of de-pointing angle (perfect tracking) is also added in both figures. One can see that when perfect tracking, the more antennas used, the higher the total transmitted bits, which makes sense because in perfect point case, one can always get maximal gain for the ULA. However when the de-pointing angle increases, it is obvious that the total transmitted bits drop after a certain amount of antennas. And the higher the de-pointing angle, the earlier the total transmitted bits start to drop.

What is interesting is that one can also find some 'waves' when the de-pointing angle is no longer 0 degree. This is because the existence of the side lobes. When the number of antennas is large, the pointing may falls into the side lobes, which may cause an increase of the total transmitted bits.

For patch ULA:

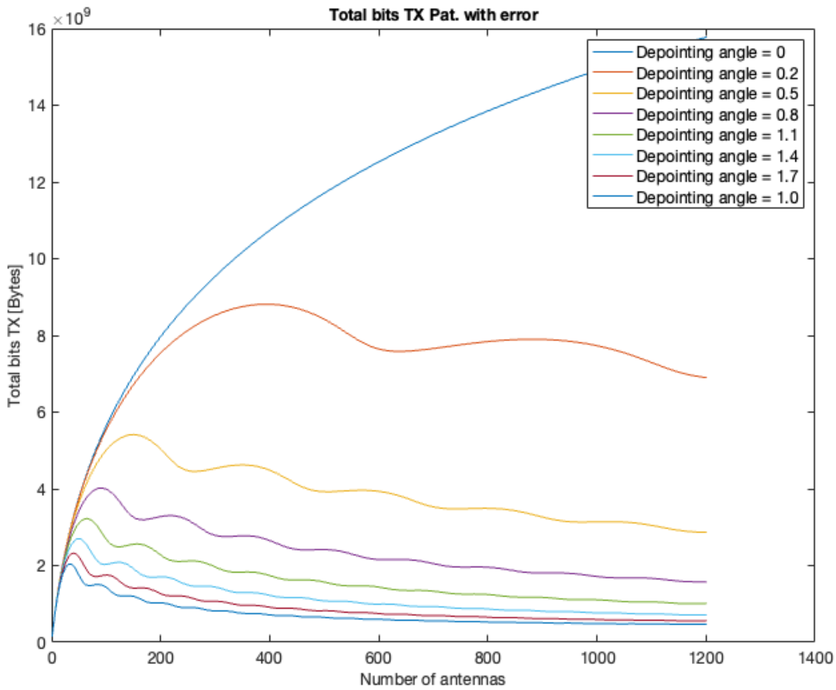

Figure 3.6: Result for patch ULA.

The result for the optimal number of antennas in the linear array given a pointing error from 0.2° to 2° is shown as Table below. The corresponding MATLAB screenshot is shown as 5.1 in Appendix.

Depointing Angle	**Antenna Type**	**Number of Antennas**	**Total Bits (Bytes)**
0	Isotropic	1201	1.6733016810e10
	Pat	1201	1.5774211092e10
0.2	Isotropic	396	9.8100336538e9
	Patch	396	8.8127095898e9
0.5	Isotropic	156	6.2754763273e9
	Patch	151	5.4137106668e9
0.8	Isotropic	96	4.7690108344e9
	Patch	91	4.0219444599e9
1.1	Isotropic	66	3.8875162095e9
	Patch	66	3.2247400115e9
1.4	Isotropic	51	3.2971658218e9
	Patch	51	2.7006574398e9
1.7	Isotropic	41	2.8689028319e9
	Patch	41	2.3274339375e9
2.0	Isotropic	36	2.5469939833e9
	Patch	36	2.0392755597e9

4 Part3: Ray tracing

4.1 Description of ray tracing [1]

Consider that two buildings are next to the ground station, as depicted in Figure 4.1. What is the largest MPC received by the ground station for every elevation angle of the satellite? Does it change depending on the type of antenna? Considering only the strongest MPC.

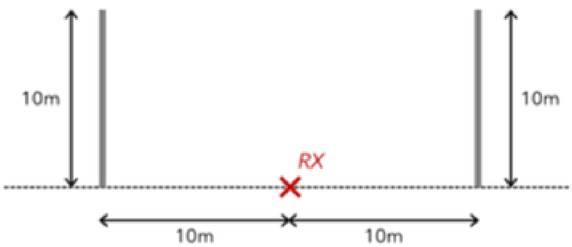

Figure 4.1: Geometry of ray tracing

4.2 Theoretical foundations

In this part the ray tracing is performed. There are mainly 3 different types of ray-tracing exist in this case: LOS, reflection and diffraction.

For the LOS, the following formula can be used:

$$P_{RX} = \left(\frac{\lambda}{4\pi}\right)^2 G_{\mathrm{RX}} G_{\mathrm{TX}} P_{\mathrm{TX}} \left|\frac{e^{-j\beta d}}{d}\right|^2$$

For the reflection, the following formula can be used:

$$P_{RX} = \left(\frac{\lambda}{4\pi}\right)^2 G_{\mathrm{RX}} G_{\mathrm{TX}} P_{\mathrm{TX}} \left|\Gamma_{\perp}\frac{e^{-j\beta d}}{d}\right|^2$$

where

$$\Gamma_{\perp} = \frac{\cos\theta_i - \sqrt{\varepsilon_r}\sqrt{1-\frac{1}{\varepsilon_r}\sin^2\theta_i}}{\cos\theta_i + \sqrt{\varepsilon_r}\sqrt{1-\frac{1}{\varepsilon_r}\sin^2\theta_i}}$$

is the reflection coefficient.

For the diffraction, the knife-edge model can be applied by using the approximation proposed by ITU:

$$|F(\nu)|^2[dB] \simeq -6.9 - 20\log\left(\sqrt{(\nu-0.1)^2+1} + \nu - 0.1\right)$$

4.3 Result, validation and analysis

4.3.1 Result, validation and analysis for Part 3

The result of 4, 8 and 16 ULA ray tracing is shown in Figure 4.2 and Figure 4.3

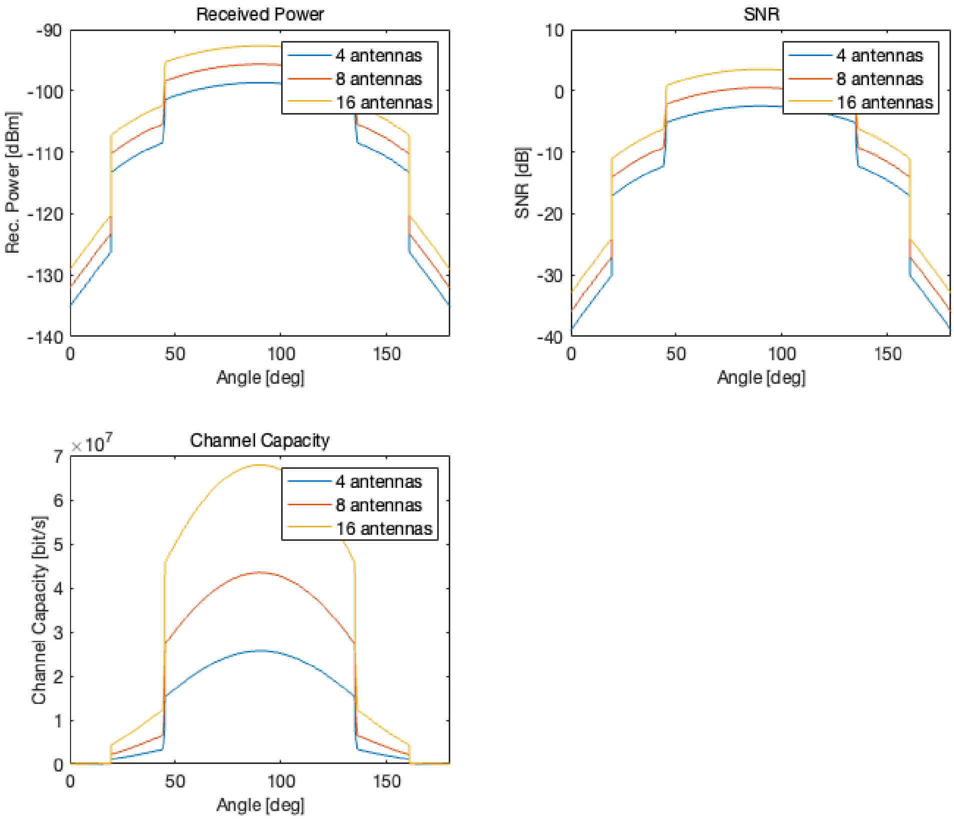

Figure 4.2: Ray tracing for isotropic ULA

From the figures above, one can see that when diffraction applies (when the elevation angle is really small), the channel capacity is almost 0, which means that the communication almost stops. This is because the high frequency and relatively low SNR, which will make a very large attenuation for diffraction case. When the elevation angle gets a little bit larger, the channel capacity suddenly gets larger. And after 45 degree, the LOS applies, which leads to another sudden rise in both P_{RX}, SNR and channel capacity. Moreover, by looking into the figures, when there is no de-pointing, the more antenna used in the array, the better the quality.

The validation can then be applied, since both ULA use same set of algorithm, only the validation of isotropic antenna with 4 antennas is done here: First of all, one can verify that between 0 degree to 90 degree:

- LOS happens when elevation angle is between 45 degree to 90 degree.
- Reflection and diffraction happen when elevation angle is between 19.4712 degree ($arcsin(\frac{10}{30})$) to 45 degree.
- Diffraction happens when elevation angle is between 0 degree to 19.4712 degree.

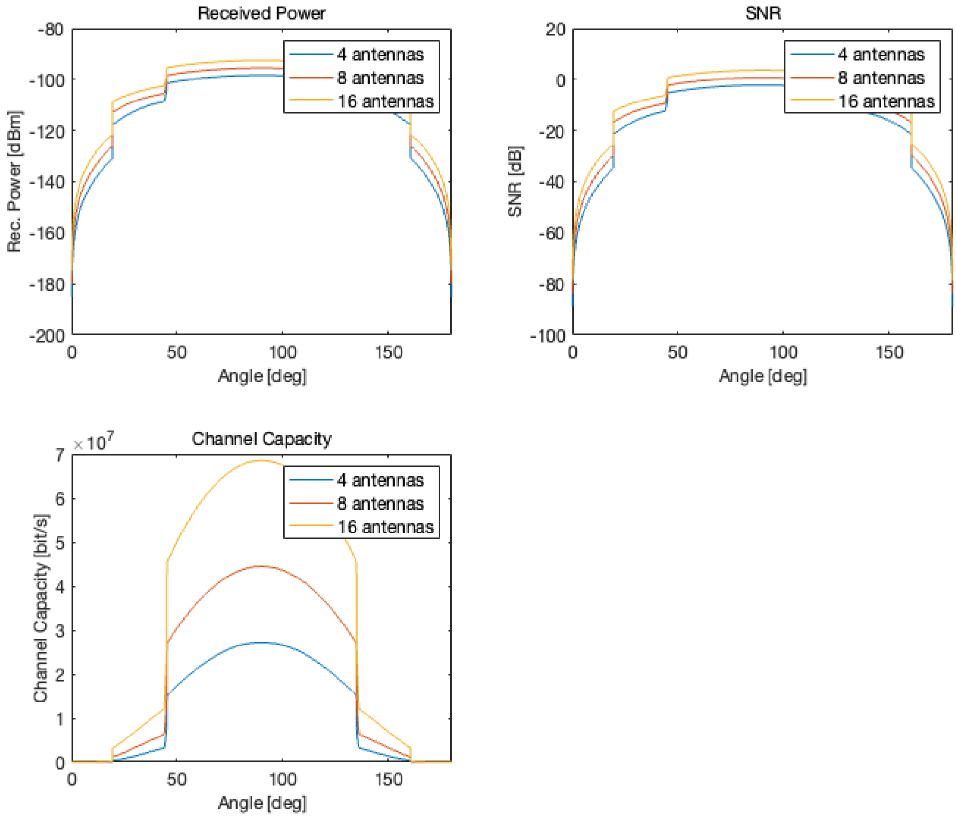

Figure 4.3: Ray tracing for isotropic ULA

1. LOS case: Assume 90 degree of elevation angle:

$$
\begin{aligned}
P_{RX} &= \left(\frac{\lambda}{4\pi}\right)^2 G_{\text{RX}} G_{\text{TX}} P_{\text{TX}} \left|\frac{e^{-j\beta d}}{d}\right|^2 \\
&= \left(\frac{0.0115}{4\pi}\right)^2 \cdot 4 \cdot 10000 \cdot \left|\frac{e^{-j544.9197 \cdot 500 \cdot 10^3}}{500 \cdot 10^3}\right|^2 \\
&= 1.3400 \cdot 10^{-13} = -98.7290[dBm]
\end{aligned}
$$

which is the same as MATLAB simulation (**-98.7290 [dBm]**).

2. Reflection case: Assume 40 degree of elevation angle: When the elevation angle is 40 degree, the x-y position of satellite is $[5.6786e+05, 4.7649e+05]$. The reflection coefficient is then:

$$
\begin{aligned}
\Gamma_{\perp} &= \frac{\cos\theta_i - \sqrt{\varepsilon_r}\sqrt{1 - \frac{1}{\varepsilon_r}\sin^2\theta_i}}{\cos\theta_i + \sqrt{\varepsilon_r}\sqrt{1 - \frac{1}{\varepsilon_r}\sin^2\theta_i}} \\
&= \frac{cos 40^\circ - \sqrt{4}\sqrt{1 - \frac{1}{4} sin^2 40^\circ}}{cos 40^\circ + \sqrt{4}\sqrt{1 - \frac{1}{4} sin^2 40^\circ}} = -0.4240
\end{aligned}
$$

Therefore

$$
\begin{aligned}
P_{RXREF} &= \left(\frac{\lambda}{4\pi}\right)^2 G_{\mathrm{RX}} G_{\mathrm{TX}} P_{\mathrm{TX}} \left|\Gamma_\perp \frac{e^{-j\beta d}}{d}\right|^2 \\
&= \left(\frac{0.0115}{4\pi}\right)^2 \cdot 4 \cdot 10000 \cdot \left|-0.4240 \cdot \frac{e^{-j544.9197 \cdot 7.4129 \cdot 10^5}}{7.4129 \cdot 10^5}\right|^2 \\
&= 1.0960 \cdot 10^{-14} = -109.6021[dBm]
\end{aligned}
$$

Meanwhile, the diffraction could happen as well.

$$
\begin{aligned}
\nu &= \sqrt{\frac{2}{\pi}\beta \Delta r} = \sqrt{\frac{2}{\pi}\beta(s_{satellite-point} + s_{RX-point} - d_{LOS})} \\
&= \sqrt{\frac{2}{\pi} \cdot 544.9197 \cdot (10\sqrt{2} + 7.4127 \cdot 10^5 - 7.4129 \cdot 10^5)} \\
&= 4.3201
\end{aligned}
$$

$$
\begin{aligned}
|F(\nu)|^2[dB] &\simeq -6.9 - 20\log\left(\sqrt{(\nu - 0.1)^2 + 1} + \nu - 0.1\right) \\
&= -6.9 - 20\log\left(\sqrt{(4.3201 - 0.1)^2 + 1} + 4.3201 - 0.1\right) \\
&= 0.0535[linear]
\end{aligned}
$$

Therefore

$$
\begin{aligned}
P_{RXDIF} &= F(\nu) \cdot \left(\frac{\lambda}{4\pi}\right)^2 G_{\mathrm{RX}} G_{\mathrm{TX}} P_{\mathrm{TX}} \left|\frac{e^{-j\beta d}}{d}\right|^2 \\
&= 0.0535 \cdot \left(\frac{0.0115}{4\pi}\right)^2 \cdot 4 \cdot 10000 \cdot \left|\frac{e^{-j544.9197 \cdot 7.4129 \cdot 10^5}}{7.4129 \cdot 10^5}\right|^2 \\
&= 3.2614 \cdot 10^{-15} = -114.8659[dBm] \leq P_{RXREF}
\end{aligned}
$$

which is the same as MATLAB simulation (**-109.6021 [dBm]**).

3. Diffraction case: Assume 10 degree of elevation angle: When the elevation angle is 10 degree, the x-y position of satellite is $[1.6688e+06, 2.9426e+05]$. The reflection coefficient is then:

$$
\begin{aligned}
\nu &= \sqrt{\frac{2}{\pi}\beta \Delta r} = \sqrt{\frac{2}{\pi}\beta(s_{satellite-point} + s_{RX-point} - d_{LOS})} \\
&= \sqrt{\frac{2}{\pi} \cdot 544.9197 \cdot (10\sqrt{2} + 1.69456 \cdot 10^6 - 1.69457 \cdot 10^6)} \\
&= 29.7867
\end{aligned}
$$

$$
\begin{aligned}
|F(\nu)|^2[dB] &\simeq -6.9 - 20\log\left(\sqrt{(\nu - 0.1)^2 + 1} + \nu - 0.1\right) \\
&= -6.9 - 20\log\left(\sqrt{(29.7867 - 0.1)^2 + 1} + 29.7867 - 0.1\right) \\
&= 0.0076[linear]
\end{aligned}
$$

Therefore

$$
\begin{aligned}
P_{RX} &= F(\nu)\left(\frac{\lambda}{4\pi}\right)^2 G_{\mathrm{RX}} G_{\mathrm{TX}} P_{\mathrm{TX}} \left|\frac{e^{-j\beta d}}{d}\right|^2 \\
&= 0.0076 \cdot \left(\frac{0.0115}{4\pi}\right)^2 \cdot 4 \cdot 10000 \cdot \left|\frac{e^{-j544.9197 \cdot 1.69457 \cdot 10^6}}{1.69457 \cdot 10^6}\right|^2 \\
&= 8.8757 \cdot 10^{-17} = -130.5180[dBm]
\end{aligned}
$$

which is the same as MATLAB simulation (**-130.5180 [dBm]**).
The corresponding result for validation is shown in Figure 4.4.

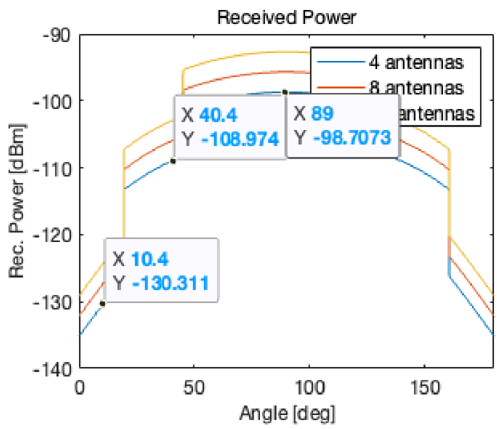

Figure 4.4: Validation of isotropic ULA

And the total transmitted bits during one pass is:

Antenna Type	Number of Antennas	Total Bits (Bytes)
Isotropic	4	0.3851e9
Pat	4	0.3875e9
Isotropic	8	0.6749e9
Patch	8	0.6711e9
Isotropic	16	1.1106e9
Patch	16	1.1014e9

4.3.2 Result for Bonus 3.1

In practice, due to the high frequency of satellite transmitted signal, it would be difficult for it to penetrate through the buildings and improve the data rate.

To improve the data rate, one can add another relay node at the rooftop to receive the signal when transmission is blocked by the building and then forward the data to the RX.

5 Appendix

```
Iso: For depointing angle is 0. Max total bits TX get when 1201 antennas, bits in total is 16733016810.2172 bytes.
Pat: For depointing angle is 0. Max total bits TX get when 1201 antennas, bits in total is 15774211092.4801 bytes.
==========================================================================================================
Iso: For depointing angle is 0.2. Max total bits TX get when 396 antennas, bits in total is 9810303653.7969 bytes.
Pat: For depointing angle is 0.2. Max total bits TX get when 396 antennas, bits in total is 8812709589.815 bytes.
==========================================================================================================
Iso: For depointing angle is 0.5. Max total bits TX get when 156 antennas, bits in total is 6275476327.3401 bytes.
Pat: For depointing angle is 0.5. Max total bits TX get when 151 antennas, bits in total is 5413170666.7817 bytes.
==========================================================================================================
Iso: For depointing angle is 0.8. Max total bits TX get when 96 antennas, bits in total is 4769010834.3804 bytes.
Pat: For depointing angle is 0.8. Max total bits TX get when 91 antennas, bits in total is 4021944459.8992 bytes.
==========================================================================================================
Iso: For depointing angle is 1.1. Max total bits TX get when 66 antennas, bits in total is 3887516209.4994 bytes.
Pat: For depointing angle is 1.1. Max total bits TX get when 66 antennas, bits in total is 3224740011.526 bytes.
==========================================================================================================
Iso: For depointing angle is 1.4. Max total bits TX get when 51 antennas, bits in total is 3297165821.8437 bytes.
Pat: For depointing angle is 1.4. Max total bits TX get when 51 antennas, bits in total is 2700657439.7952 bytes.
==========================================================================================================
Iso: For depointing angle is 1.7. Max total bits TX get when 41 antennas, bits in total is 2868902831.8937 bytes.
Pat: For depointing angle is 1.7. Max total bits TX get when 41 antennas, bits in total is 2327433937.459 bytes.
==========================================================================================================
Iso: For depointing angle is 2. Max total bits TX get when 36 antennas, bits in total is 2546993983.3255 bytes.
Pat: For depointing angle is 2. Max total bits TX get when 36 antennas, bits in total is 2039279559.6938 bytes.
==========================================================================================================
```

Figure 5.1: Optimal number of antennas in the linear array given a pointing error from 0.2° to 2°

Bibliography

[1] Analysis of a LEO satellite communication link

[2] Communication Channels Project Presentation

[3] Propagation models for mobile communications

[4] The wireless communications channel

[5] Long-range communications

[6] Base station antennas