Lecture 14: Access Methods & Operators

Pace & Lecture content

- We are slowing down:
  - In response to feedback from some that they like this pace better!
  - Due to **great** questions! *Makes us seriously happy!*
  - The details are more **fun!** *Makes one of us seriously happy.*

- We may cut some topics listed (maybe not).
  - We have a lot of (we think) good material ☹ but…
  - We'd prefer depth and happiness to breadth.

- Please refresh lectures before (changes are minor)
  - Cannot tell you how much time we spend tweaking… it's sad really…

Project #2 Hint

- You may want to do *Trigger activity* for project 2.
  - We've noticed those who do it have less trouble with project!
  - Seems like we're good here ☺ Exciting for us!
  - Definitely use piazza actively: students have been giving great answers
    - Hats are back ordered! (well not really)

Today's Lecture

1. B+ Trees

2. Nested Loop Joins

1. B+ Trees

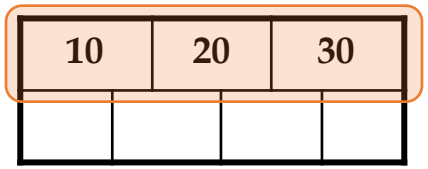
What you will learn about in this section

1. B+ Trees: Basics
2. B+ Trees: Design & Cost
3. Clustered Indexes

B+ Trees

- Search trees
  - B does not mean binary!
- Idea in B Trees:
  - make 1 node = 1 physical page
  - Balanced, height adjusted tree (not the B either)
- Idea in B+ Trees:
  - Make leaves into a linked list (for range queries)

B+ Tree Basics

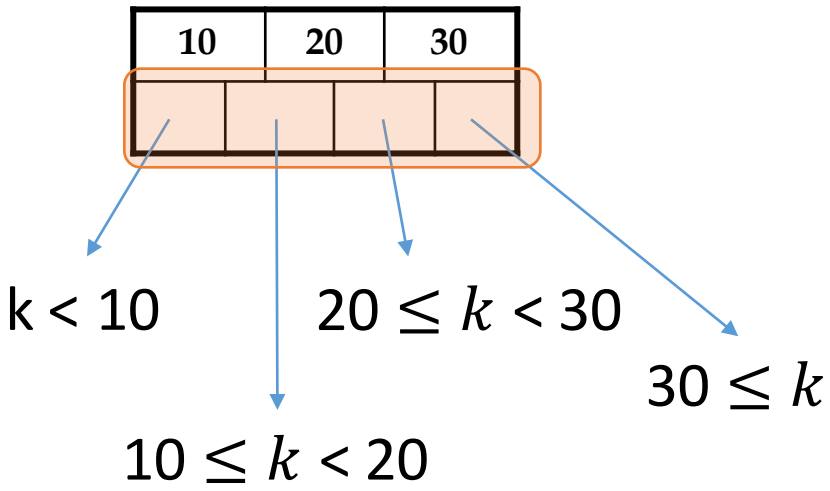


Parameter ***d*** = the degree

Each *non-leaf ("interior")* ***node*** has ≥ d and ≤ 2d ***keys****

except for root node, which can have between **1 and 2d keys*

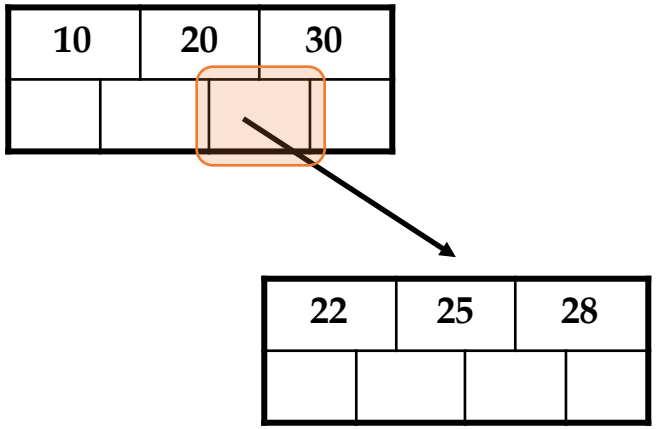
B+ Tree Basics



The *n* keys in a node define *n+1* ranges

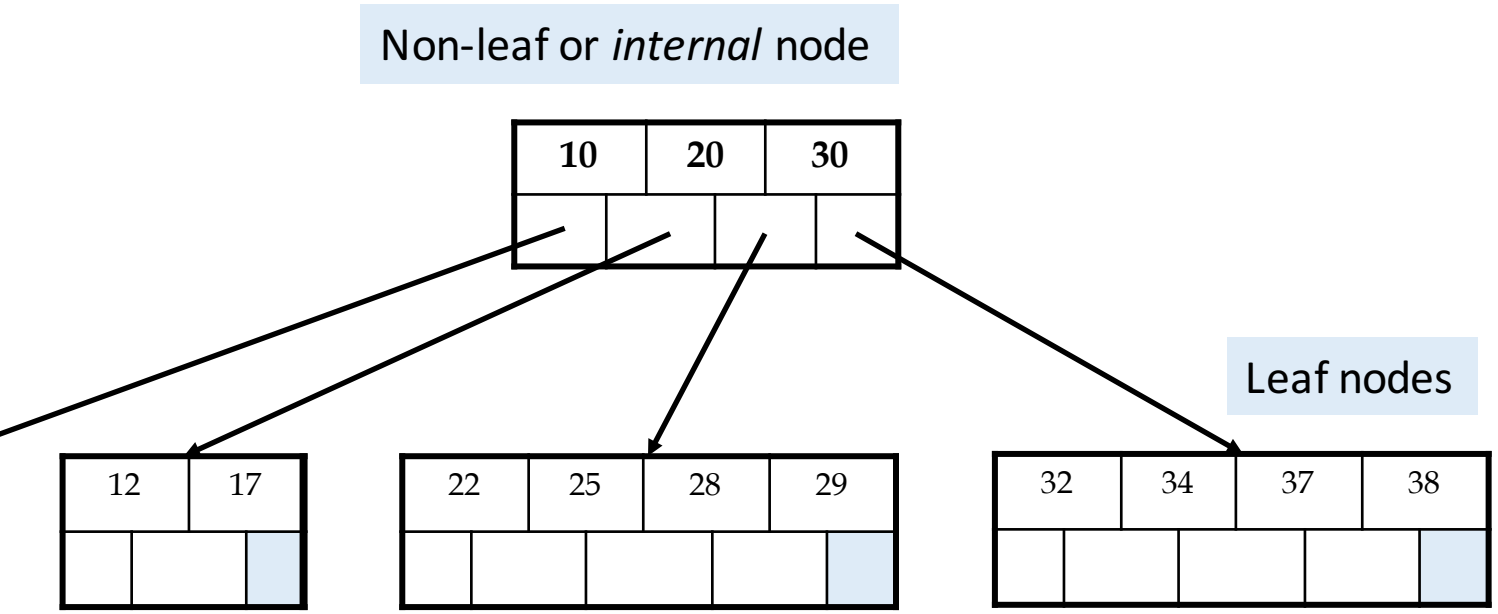
B+ Tree Basics

Non-leaf or *internal* node



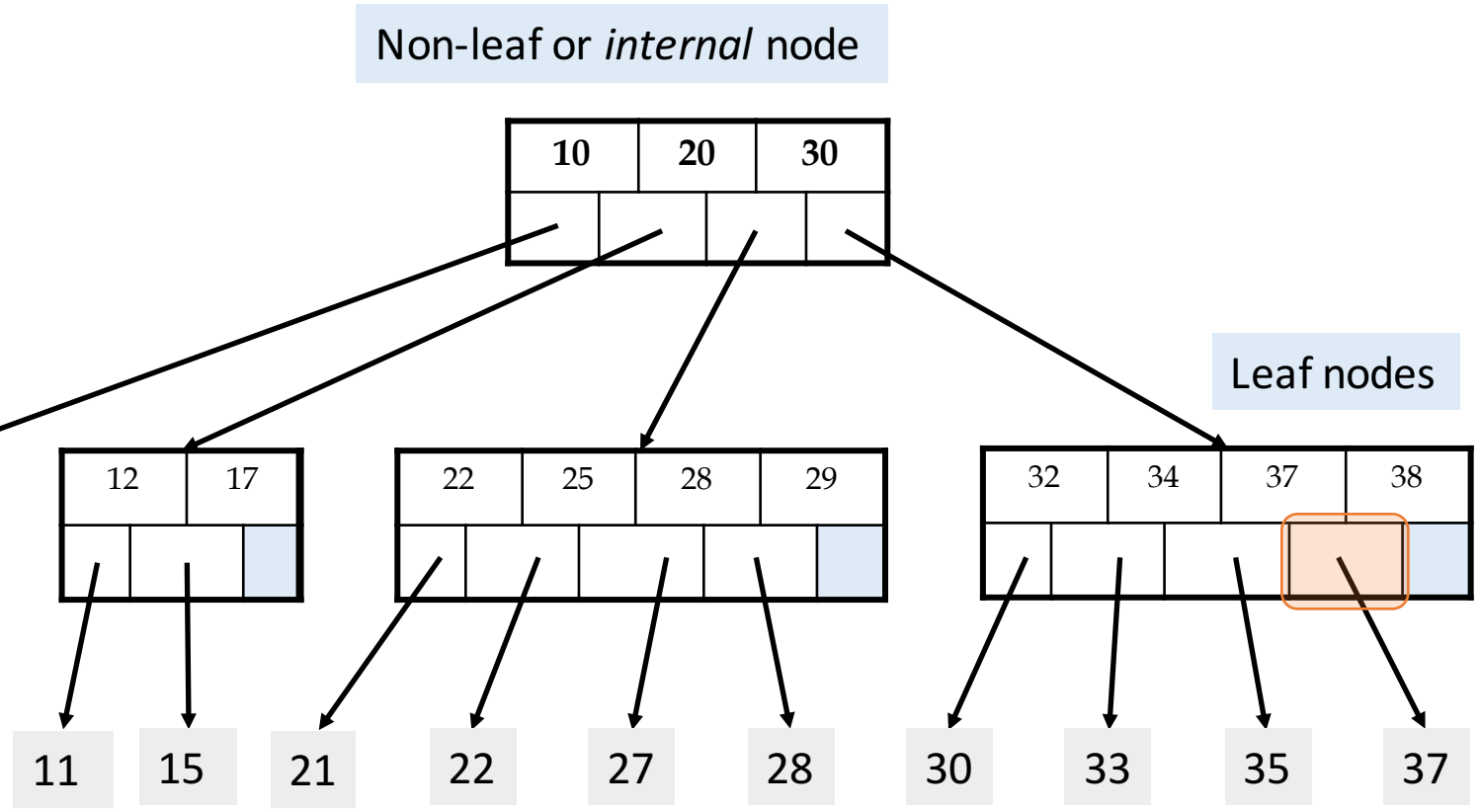
For each range, in a *non-leaf* node, there is a **pointer** to another node with keys in that range

B+ Tree Basics



Leaf nodes also have between *d* and *2d* keys, and are different in that:

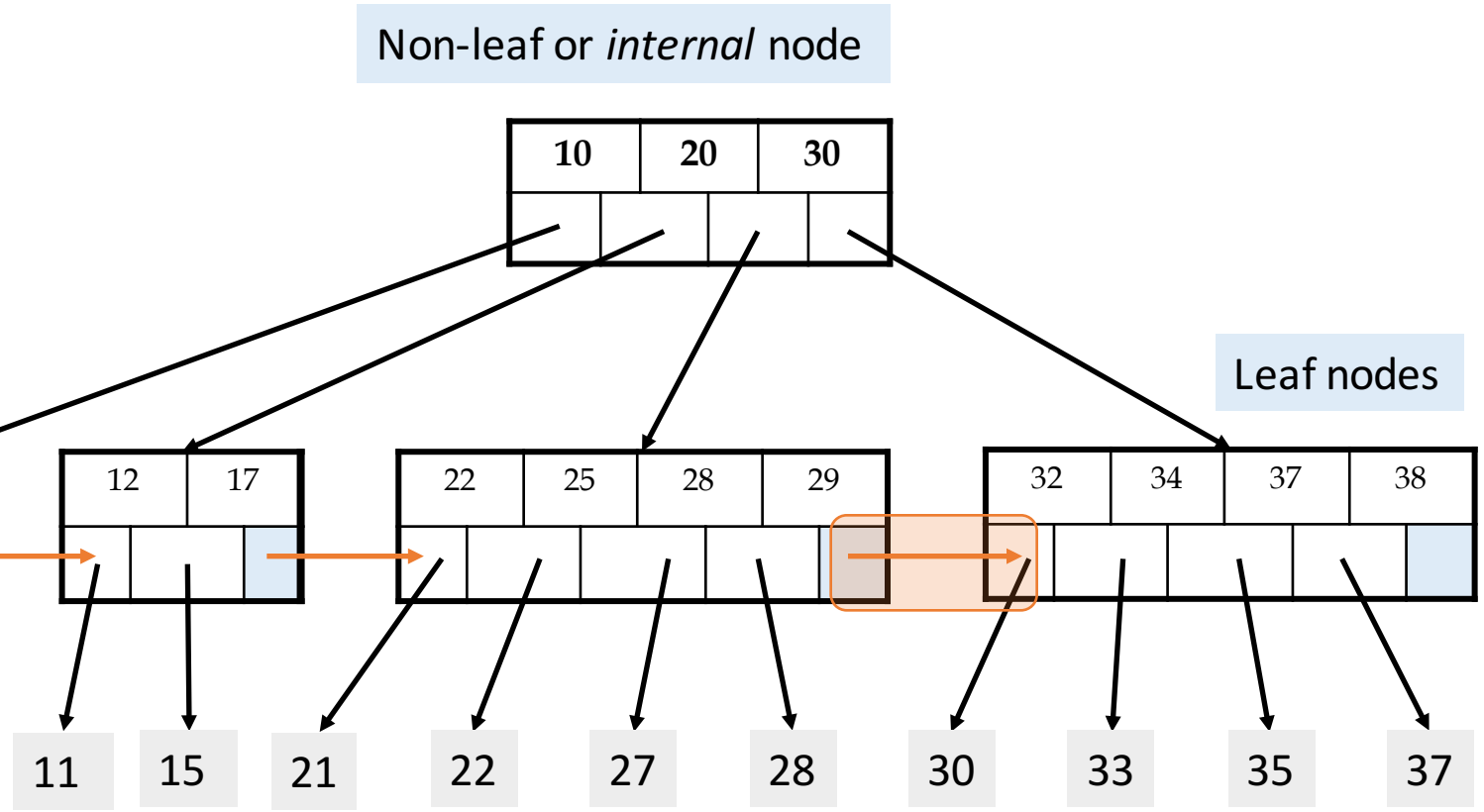
B+ Tree Basics



Leaf nodes also have between *d* and *2d* keys, and are different in that:

Their key slots contain pointers to data records

B+ Tree Basics

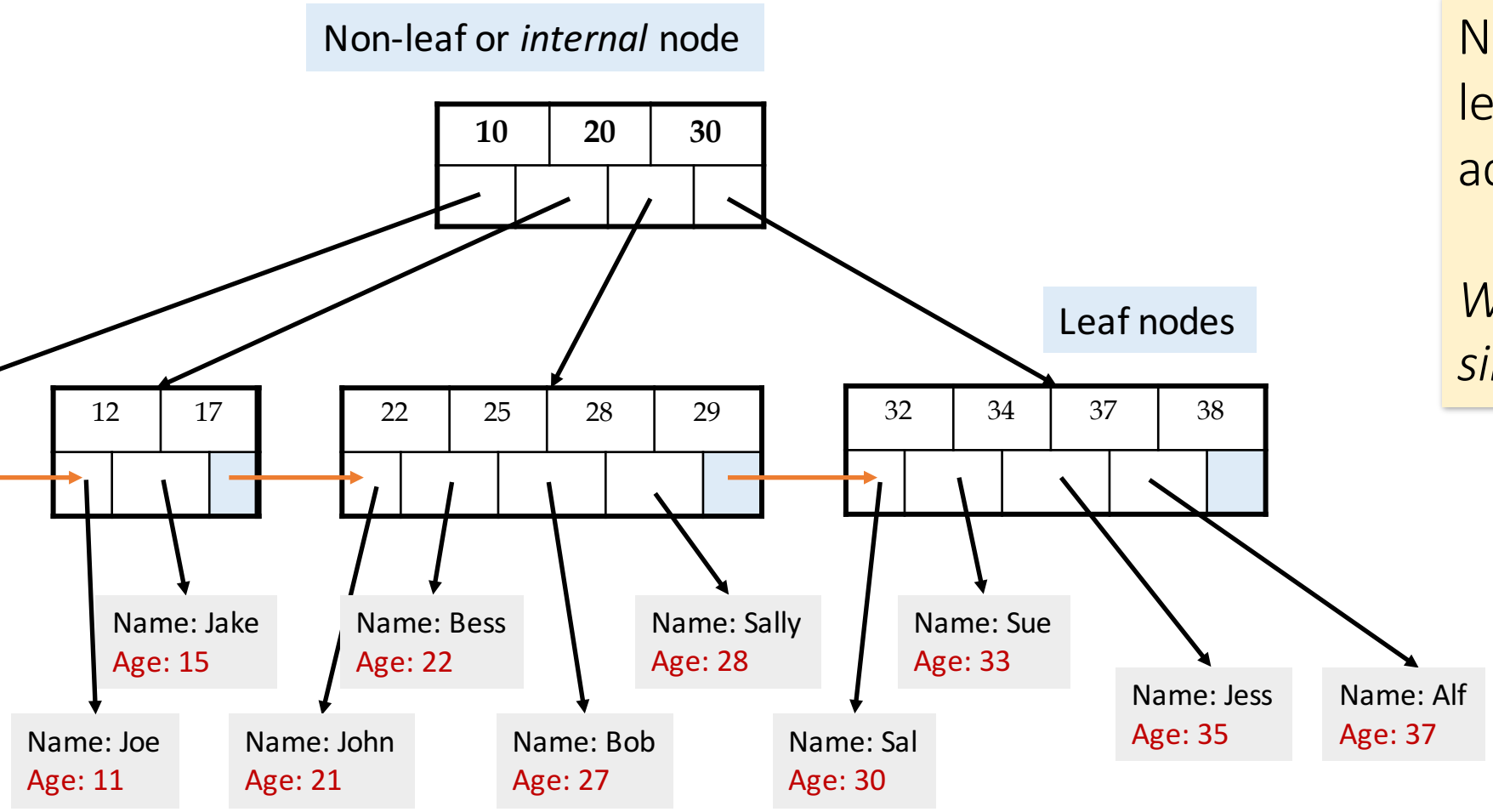


Leaf nodes also have between *d* and *2d* keys, and are different in that:

Their key slots contain pointers to data records

They contain a pointer to the next leaf node as well, ***for faster sequential traversal***

B+ Tree Basics



Note that the pointers at the leaf level will be to the actual data records (rows).

We might truncate these for simpler display (as before)…

Some finer points of B+ Trees

Searching a B+ Tree

- For exact key values:
  - Start at the root
  - Proceed down, to the leaf

```
SELECT name
FROM   people
WHERE  age = 25
```

- For range queries:
  - As above
  - *Then sequential traversal*

```
SELECT name
FROM   people
WHERE  20 <= age
  AND  age <= 30
```

B+ Tree Exact Search Animation

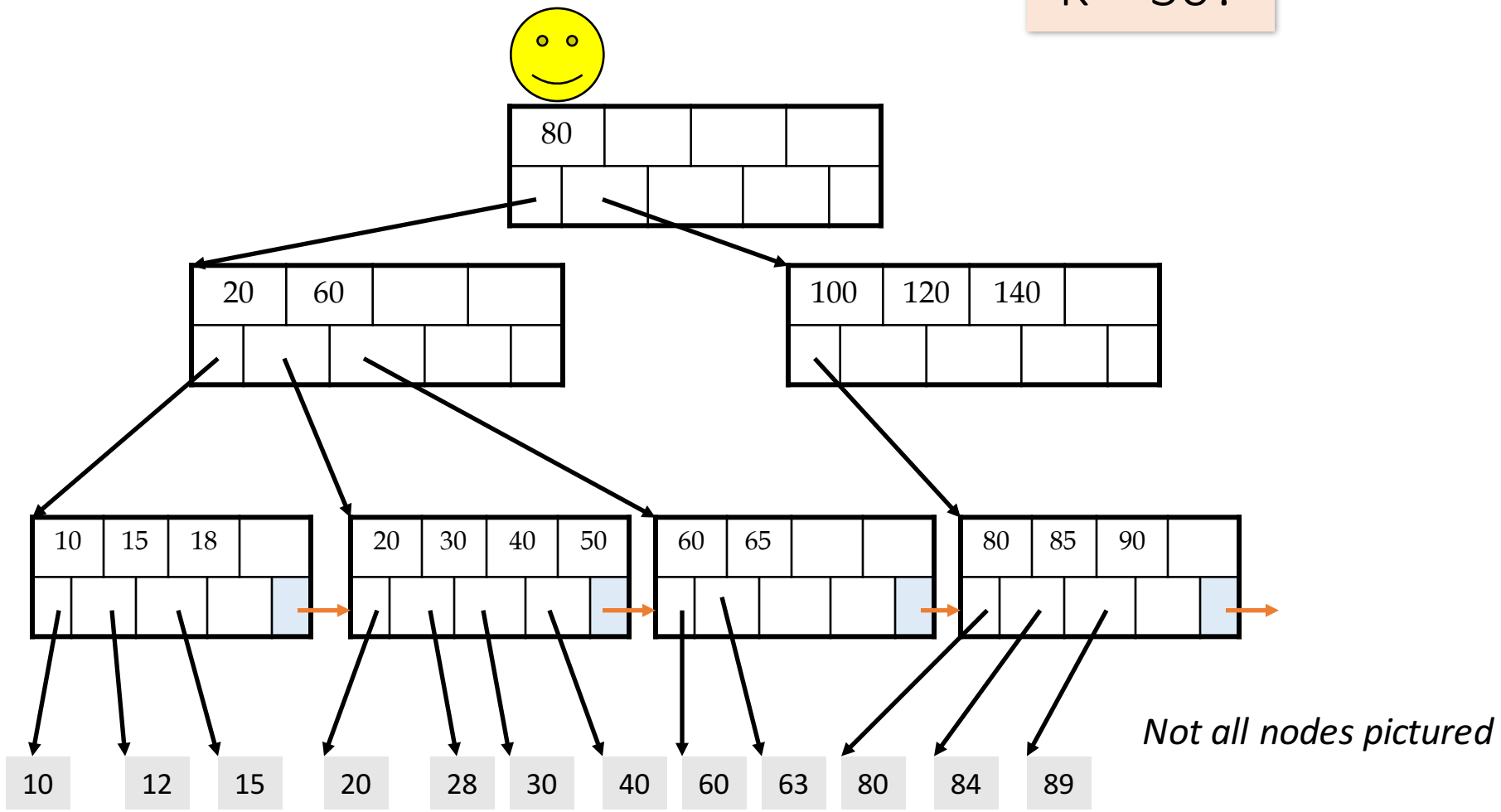
K = 30?

30 < 80

30 in [20,60)

30 in [30,40)

To the data!



Not all nodes pictured

B+ Tree Range Search Animation

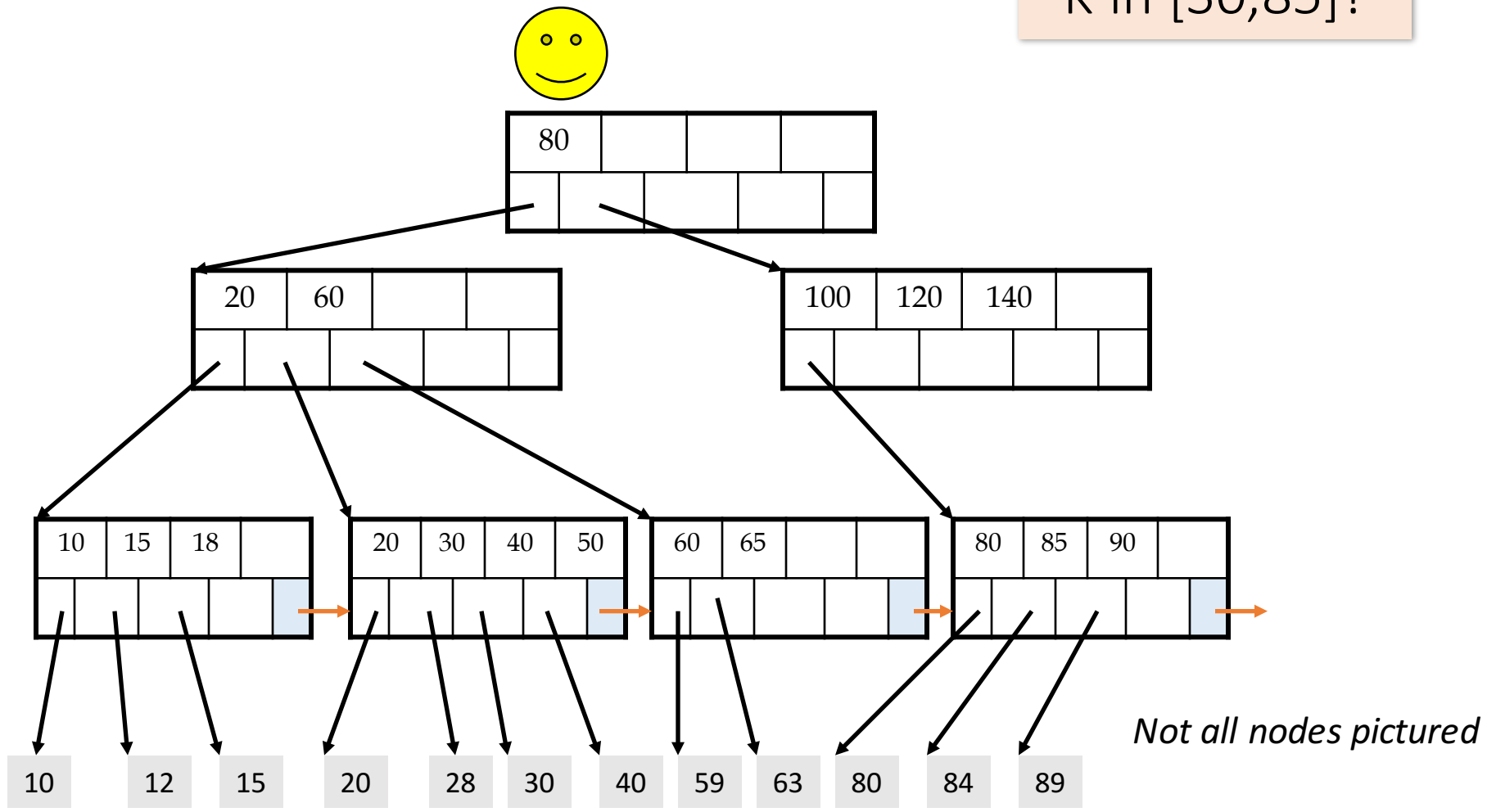
K in [30,85]?

30 < 80

30 in [20,60)

30 in [30,40)

To the data!



B+ Tree Design

- How large is ***d***?

- Example:
  - Key size = 4 bytes
  - Pointer size = 8 bytes
  - Block size = 4096 bytes

NB: Oracle allows 64K = 2^16 byte blocks
→ d <= 2730

- We want each *node* to fit on a single *block/page*
  - 2d x 4 + (2d+1) x 8 <= 4096 → ***d <= 170***

B+ Tree: High Fanout = Smaller & Lower IO

- As compared to e.g. binary search trees, B+ Trees have **high *fanout*** (***between d+1 and 2d+1***)
- This means that the **depth of the tree is small** → getting to any element requires very few IO operations!
  - Also can often store most or all of the B+ Tree in main memory!
- A TiB = 2^{40} Bytes. What is the height of a B+ Tree (with fill-factor = 1) that indexes it (with 64K pages)?
  - $(2*2730 + 1)^h = 2^{40}$ → ***h = 4***

The **fanout** is defined as the number of pointers to child nodes coming out of a node

Note that fanout is dynamic- we'll often assume it's constant just to come up with approximate eqns!

The known universe contains ~10^{80} particles… what is the height of a B+ Tree that indexes these?

B+ Trees in Practice

- Typical order: d=100. Typical fill-factor: 67%.
  - average fanout = 133
- Typical capacities:
  - Height 4: 133^4 = 312,900,700 records
  - Height 3: 133^3 = 2,352,637 records
- Top levels of tree sit *in the buffer pool*:
  - Level 1 = 1 page = 8 Kbytes
  - Level 2 = 133 pages = 1 Mbyte
  - Level 3 = 17,689 pages = 133 MBytes

Fill-factor is the percent of available slots in the B+ Tree that are filled; is usually < 1 to leave slack for (quicker) insertions

Typically, only pay for one IO!

Simple Cost Model for Search

NOTE: This has been tweaked slightly since presentation in lecture- read carefully!

- Let:
  - ***f*** = fanout, which is **in [d+1, 2d+1]** ***(we'll assume it's constant for our cost model…)***
  - ***N*** = the total number of *pages* we need to index
  - ***F*** = fill-factor (usually ~= 2/3)

- Our B+ Tree needs to have room to index ***N / F*** pages!
  - We have the fill factor in order to leave some open slots for faster insersions

- What height (*h*) does our B+ Tree need to be?
  - h=1 → Just the root node- room to index f pages
  - h=2 → f leaf nodes- room to index f^2 pages
  - h=3 → f^2 leaf nodes- room to index f^3 pages
  - …
  - h → f^{h-1} leaf nodes- room to index f^h pages!

→ We need a B+ Tree of height h = $\left\lceil \log_f \frac{N}{F} \right\rceil$!

Simple Cost Model for Search

NOTE: This has been tweaked slightly since presentation in lecture- read carefully!

- Note that if we have ***B*** available buffer pages, by the same logic:
  - We can store L_B levels of the B+ Tree in memory
  - where ***L_B is the number of levels such that the sum of all the levels' nodes fit in the buffer:***
    - $B \geq 1 + f + \cdots + f^{L_B - 1} = \sum_{l=0}^{L_B - 1} f^l$

- In summary: to do exact search:
  - We read in one page per level of the tree
  - However, levels that we can fit in buffer are free!
  - Finally we read in the actual record

IO Cost: $\left\lceil \log_f \frac{N}{F} \right\rceil - L_B + 1$

where $B \geq \sum_{l=0}^{L_B - 1} f^l$

Simple Cost Model for Search

NOTE: This has been tweaked slightly since presentation in lecture- read carefully!

- To do range search, we just follow the horizontal pointers

- The IO cost is that of loading additional leaf nodes we need to access + the IO cost of loading each ***page*** of the results- we phrase this as "Cost(OUT)"

IO Cost: $\left\lceil \log_f \frac{N}{F} \right\rceil - L_B + Cost(OUT)$

where $B \geq \sum_{l=0}^{L_B - 1} f^l$

Fast Insertions & Self-Balancing

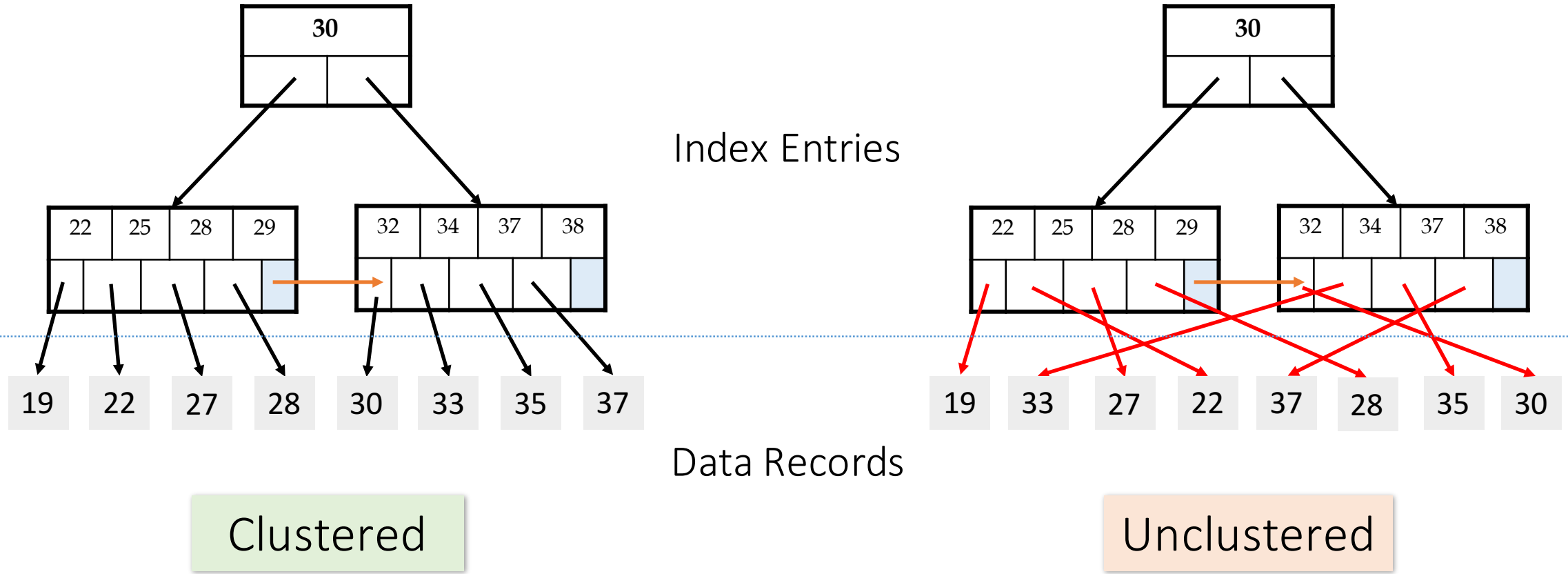
- We won't go into specifics of B+ Tree insertion algorithm, but has several attractive qualities:
  - **~ Same cost as exact search**
  - ***Self-balancing:*** B+ Tree remains **balanced** (with respect to height) even after insert

B+ Trees also (relatively) fast for single insertions!
However, can become bottleneck if many insertions (if fill-factor slack is used up…)

Clustered Indexes

An index is ***clustered*** if the underlying data is ordered in the same way as the index's data entries.

Clustered vs. Unclustered Index



Clustered vs. Unclustered Index

- Recall that for a disk with block access, **sequential IO is much faster than random IO**
- For exact search, no difference between clustered / unclustered
- For range search over R values: difference between **1 random IO + R sequential IO**, and **R random IO**:
  - A random IO costs ~ 10ms (sequential much much faster)
  - For R = 100,000 records- **difference between ~10ms and ~17min!**

Summary *[From Lecture 13 too…]*

- We covered an algorithm + some optimizations for sorting larger-than-memory files efficiently
  - An ***IO aware*** algorithm!

- We create **indexes** over tables in order to support ***fast (exact and range) search*** and ***insertion*** over ***multiple search keys***

- **B+ Trees** are one index data structure which support very fast exact and range search & insertion via ***high fanout***
  - ***Clustered vs. unclustered*** makes a big difference for range queries too

2. Nested Loop Joins

What you will learn about in this section

1. RECAP: Joins
2. Nested Loop Join (NLJ)
3. Block Nested Loop Join (BNLJ)
4. Index Nested Loop Join (INLJ)

RECAP: Joins

Joins: Example

$\mathbf{R} \bowtie \boldsymbol{S}$

```
SELECT R.A,B,C,D
FROM   R, S
WHERE  R.A = S.A
```

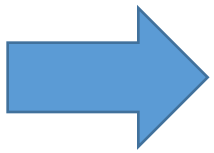
Example: Returns all pairs of tuples $\mathrm{r} \in R, s \in S$ *such that* $r.A = s.A$

R

A	B	C
1	0	1
2	3	4
2	5	2
3	1	1

S

A	D
3	7
2	2
2	3



A	B	C	D
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Joins: Example

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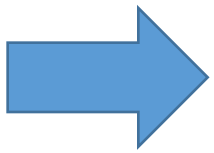

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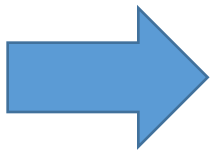

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2	3	4	2
2	3	4	3
2	5	2	2

Joins: Example

$\mathbf{R} \bowtie \boldsymbol{S}$

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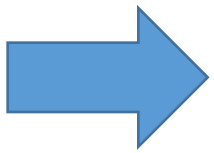

A	B	C	D
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Joins: Example

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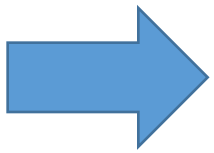

A	B	C	D
2	3	4	2
2	3	4	3
2	5	2	2
2	5	2	3
3	1	1	7

Semantically: A Subset of the Cross Product

$\mathbf{R} \bowtie \boldsymbol{S}$

```
SELECT R.A,B,C,D
FROM   R, S
WHERE  R.A = S.A
```

Example: Returns all pairs of tuples $r \in R, s \in S$ *such that* $r.A = s.A$

R

A	B	C
1	0	1
2	3	4
2	5	2
3	1	1

×

S

A	D
3	7
2	2
2	3

→ Cross Product → ... → Filter by conditions (r.A = s.A)

A	B	C	D
2	3	4	2
2	3	4	3
2	5	2	2
2	5	2	3
3	1	1	7

Can we actually implement a join in this way?

Notes

- We write $R \bowtie S$ to mean *join R and S by returning all tuple pairs where* ***all shared attributes*** *are equal*
- We write $R \bowtie S$ **on** A to mean *join R and S by returning all tuple pairs where* ***attribute(s) A*** *are equal*
- For simplicity, we'll consider joins on **two tables** and with **equality constraints** ("equijoins")

However joins *can* merge > 2 tables, and some algorithms do support non-equality constraints!

Nested Loop Joins

Notes

- We are again considering "IO aware" algorithms: ***care about disk IO***

- Given a relation R, let:
  - T(R) = # of tuples in R
  - P(R) = # of pages in R

Recall that we read / write entire pages with disk IO

- Note also that we omit ceilings in calculations… good exercise to put back in!

Nested Loop Join (NLJ)

Compute $R \bowtie S \; on \; A$:

```
for r in R:
  for s in S:
    if r[A] == s[A]:
      yield (r,s)
```

Nested Loop Join (NLJ)

```
Compute R ⋈ S on A:
  for r in R:
    for s in S:
      if r[A] == s[A]:
        yield (r,s)
```

Cost:

P(R)

1. **Loop over the tuples in R**

Note that our IO cost is based on the number of ***pages*** loaded, not the number of tuples!

Nested Loop Join (NLJ)

```
Compute R ⋈ S on A:
  for r in R:
    for s in S:
      if r[A] == s[A]:
        yield (r,s)
```

Cost:

P(R) + T(R)*P(S)

1. Loop over the tuples in R
2. **For every tuple in R, loop over all the tuples in S**

Have to read ***all of S*** from disk for ***every tuple in R!***

Nested Loop Join (NLJ)

```
Compute R ⋈ S on A:
  for r in R:
    for s in S:
      if r[A] == s[A]:
        yield (r,s)
```

Cost:

P(R) + T(R)*P(S)

1. Loop over the tuples in R
2. For every tuple in R, loop over all the tuples in S
3. **Check against join conditions**

Note that NLJ can handle things other than equality constraints… just check in the *if* statement!

Nested Loop Join (NLJ)

```
Compute R ⋈ S on A:
  for r in R:
    for s in S:
      if r[A] == s[A]:
        yield (r,s)
```

What would *OUT* be if our join condition is trivial (*if TRUE)?*

OUT could be bigger than P(R)*P(S)… but usually not that bad

Cost:

P(R) + T(R)*P(S) + OUT

1. Loop over the tuples in R
2. For every tuple in R, loop over all the tuples in S
3. Check against join conditions
4. **Write out (to page, then when page full, to disk)**

Nested Loop Join (NLJ)

```
Compute R ⋈ S on A:
  for r in R:
    for s in S:
      if r[A] == s[A]:
        yield (r,s)
```

Cost:

P(R) + T(R)*P(S) + OUT

What if R ("outer") and S ("inner") switched?

P(***S***) + T(***S***)*P(***R***) + OUT

Outer vs. inner selection makes a huge difference- DBMS needs to know which relation is smaller!

IO-Aware Approach

Block Nested Loop Join (BNLJ)

Given ***B+1*** pages of memory

```
Compute R ⋈ S on A:
  for each B-1 pages pr of R:
    for page ps of S:
      for each tuple r in pr:
        for each tuple s in ps:
          if r[A] == s[A]:
            yield (r,s)
```

Cost:

$$P(R)$$

1. **Load in B-1 pages of R at a time (leaving 1 page each free for S & output)**

Note: There could be some speedup here due to the fact that we're reading in multiple pages sequentially however we'll ignore this here!

Block Nested Loop Join (BNLJ)

Given ***B+1*** pages of memory

```
Compute R ⋈ S on A:
  for each B-1 pages pr of R:
    for page ps of S:
      for each tuple r in pr:
        for each tuple s in ps:
          if r[A] == s[A]:
            yield (r,s)
```

Cost:

$$P(R) + \frac{P(R)}{B-1} P(S)$$

1. Load in B-1 pages of R at a time (leaving 1 page each free for S & output)

2. **For each (B-1)-page segment of R, load each page of S**

Note: Faster to iterate over the *smaller* relation first!

Block Nested Loop Join (BNLJ)

Given ***B+1*** pages of memory

```
Compute R ⋈ S on A:
  for each B-1 pages pr of R:
    for page ps of S:
      for each tuple r in pr:
        for each tuple s in ps:
          if r[A] == s[A]:
            yield (r,s)
```

Cost:

$$P(R) + \frac{P(R)}{B-1} P(S)$$

1. Load in B-1 pages of R at a time (leaving 1 page each free for S & output)
2. For each (B-1)-page segment of R, load each page of S
3. **Check against the join conditions**

BNLJ can also handle non-equality constraints

Block Nested Loop Join (BNLJ)

Given ***B+1*** pages of memory

```
Compute R ⋈ S on A:
  for each B-1 pages pr of R:
    for page ps of S:
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          if r[A] == s[A]:
            yield (r,s)
```

Cost:

$$P(R) + \frac{P(R)}{B-1} P(S) + \text{OUT}$$

1. Load in B-1 pages of R at a time (leaving 1 page each free for S & output)
2. For each (B-1)-page segment of R, load each page of S
3. Check against the join conditions
4. **Write out**

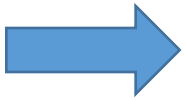
Again, ***OUT*** could be bigger than P(R)*P(S)… but usually not that bad

BNLJ vs. NLJ: Benefits of IO Aware

- In BNLJ, by loading larger chunks of R, we minimize the number of full *disk reads* of S
  - We only read all of S from disk for ***every (B-1)-page segment of R***!
  - Still the full cross-product, but more done only *in memory*

NLJ

P(R) + T(R)*P(S) + OUT



BNLJ

$$P(R) + \frac{P(R)}{B-1} P(S) + \text{OUT}$$

BNLJ is faster by roughly $\frac{(B-1)T(R)}{P(R)}$!

BNLJ vs. NLJ: Benefits of IO Aware

- Example:
  - R: 500 pages
  - S: 1000 pages
  - 100 tuples / page
  - We have 12 pages of memory (B = 11)

Ignoring OUT here…

- NLJ: Cost = 500 + **50,000*1000** = **50 Million IOs ~= <u>140 hours</u>**

- BNLJ: Cost = 500 + $\frac{500*1000}{10}$ = **50 *Thousand* IOs ~= <u>0.14 hours</u>**

A very real difference from a small change in the algorithm!

Smarter than Cross-Products

Smarter than Cross-Products: From Quadratic to Nearly Linear

- All joins that compute the ***full cross-product*** have some **quadratic** term
  - For example we saw:

NLJ $P(R) + T(R)P(S) + OUT$

BNLJ $P(R) + \frac{P(R)}{B-1} P(S) + OUT$

- Now we'll see some (nearly) linear joins:
  - ~ O(P(R) + P(S) + ***OUT***), where again ***OUT*** could be quadratic but is usually better

We get this gain by ***taking advantage of structure***- moving to equality constraints ("equijoin") only!

Index Nested Loop Join (INLJ)

```
Compute R ⋈ S on A:
  Given index idx on S.A:
    for r in R:
      s in idx(r[A]):
        yield r,s
```

Cost:

P(R) + T(R)*L + OUT

where L is the IO cost to access all the distinct values in the index; assuming these fit on one page, $\mathrm{L} \sim 3$ is good est.

→ We can use an **index** (e.g. B+ Tree) to ***avoid doing the full cross-product!***

Lecture 15: Joins- A Cage Match

Announcements

1. There was a bug on one sub problem of the midterm
    1. Precedence order for NOT (see Piazza!)
    2. We gave everyone 5 bonus points: *this disadvantages no one!*
    3. cookies!

2. PS #3 & Project 3 out soon! The last of each! ☺
    1. We want to give you time for Project 3 (it's more learn by doing)
    2. And we want to grade PS#3 to give feedback (exam material)
        1. I'm worried you won't have enough time for relational algebra, which I consider important… so we built some new materials for it (next topic).

Today's Lecture

1. Sort-Merge Join (SMJ)
2. Hash Join (HJ)
3. The Cage Match: SMJ vs. HJ

1. Sort-Merge Join (SMJ)

What you will learn about in this section

1. Sort-Merge Join
2. “Backup” & Total Cost
3. Optimizations
4. ACTIVITY: Sequential Flooding

Sort Merge Join (SMJ): Basic Procedure

To compute $R \bowtie S \; on \; A$:

1. Sort R, S on A using ***external merge sort***
2. ***Scan*** sorted files and "merge"
3. *[May need to "backup"- see next subsection]*

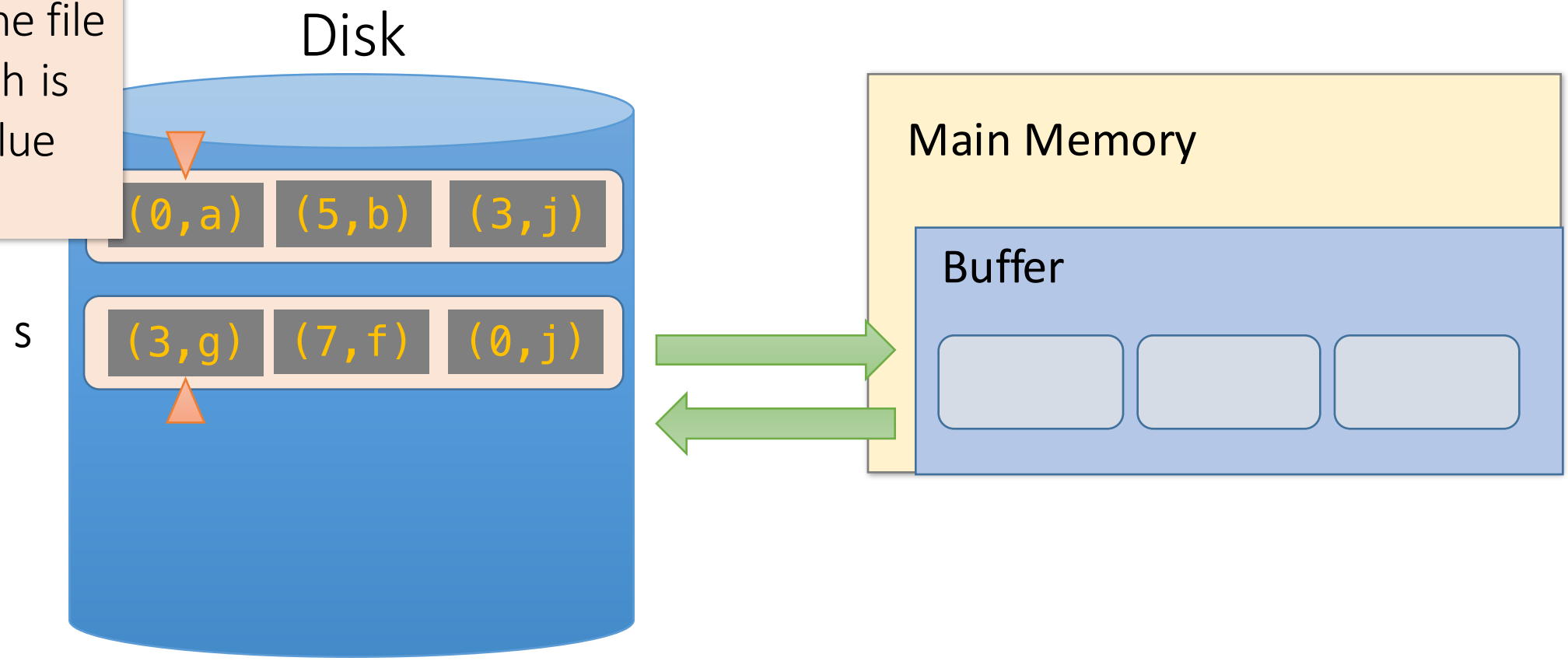
Note that we are only considering equality join conditions here

Note that if R, S are already sorted on A, SMJ will be awesome!

SMJ Example: $R \bowtie S$ *on* A with 3 page buffer

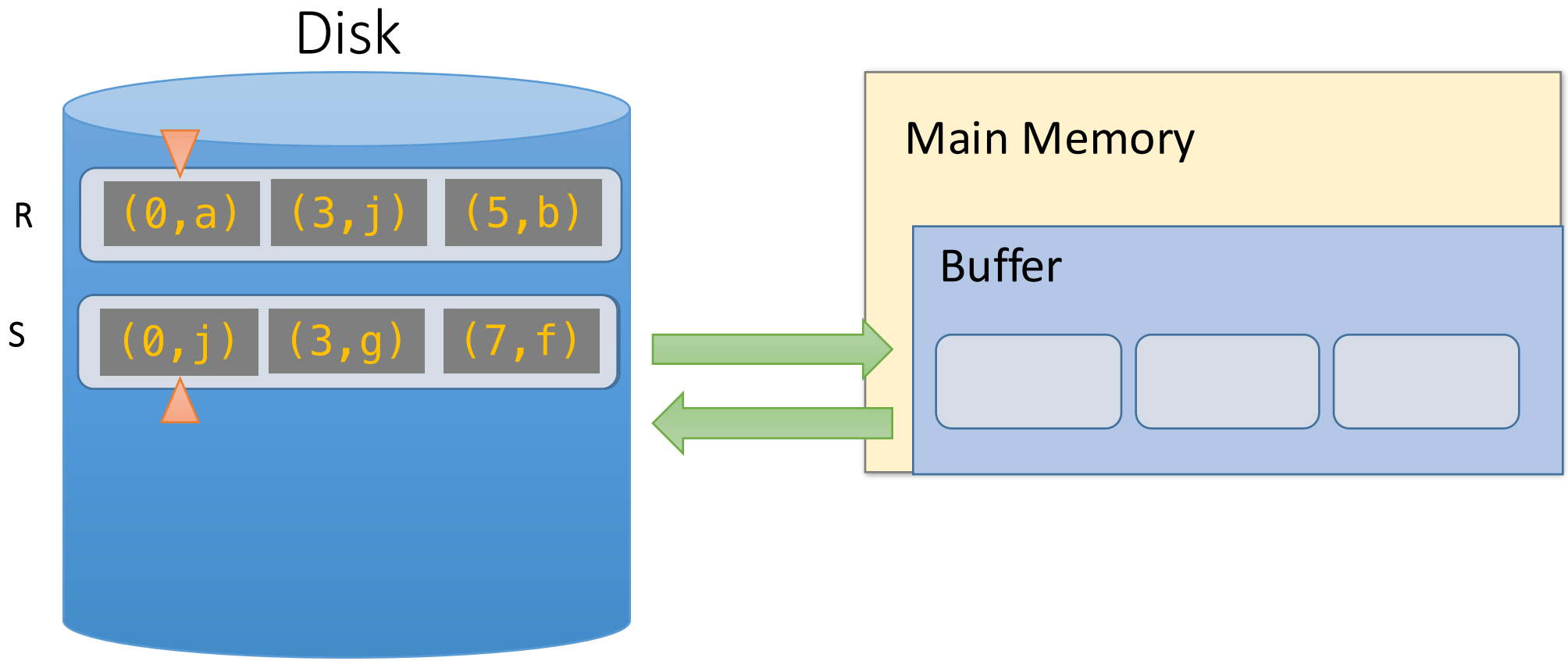
- For simplicity: Let each page be ***one tuple***, and let the first value be A

We show the file HEAD, which is the next value to be read!



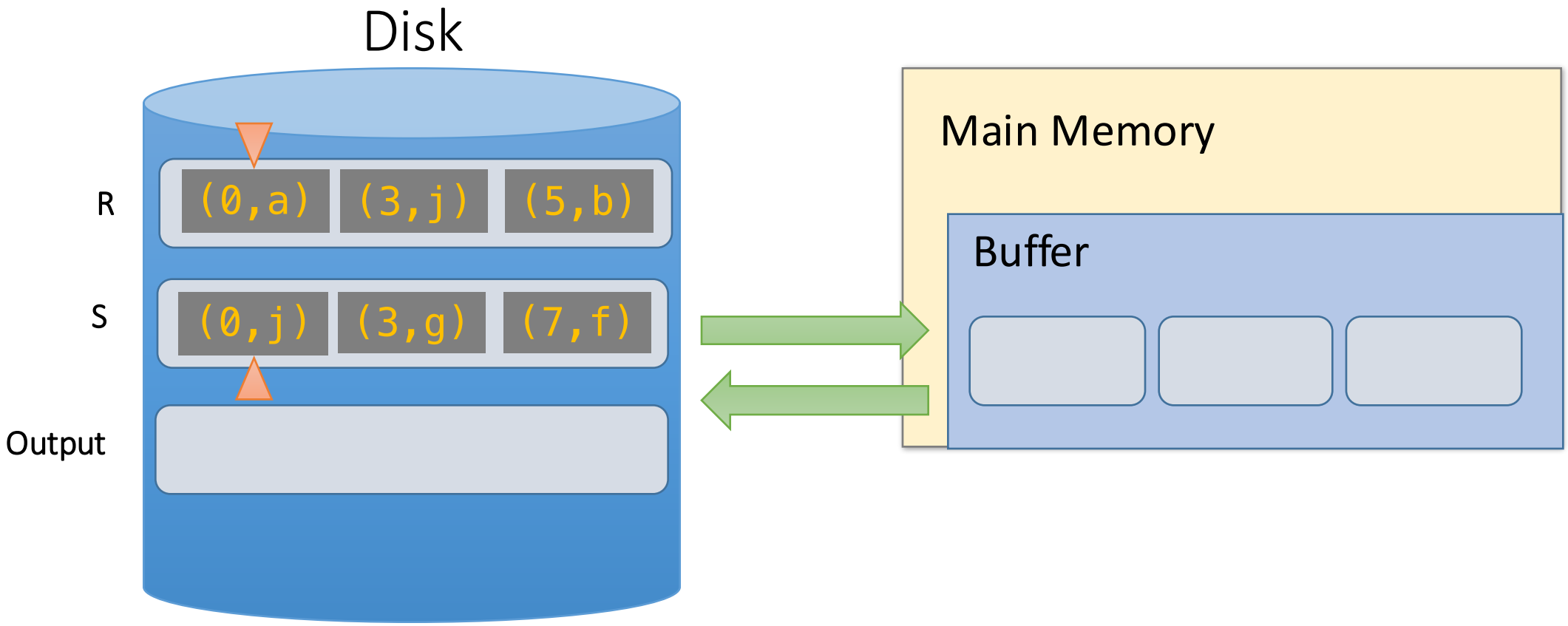
SMJ Example: $R \bowtie S$ on A with 3 page buffer

1. Sort the relations R, S on the join key (first value)



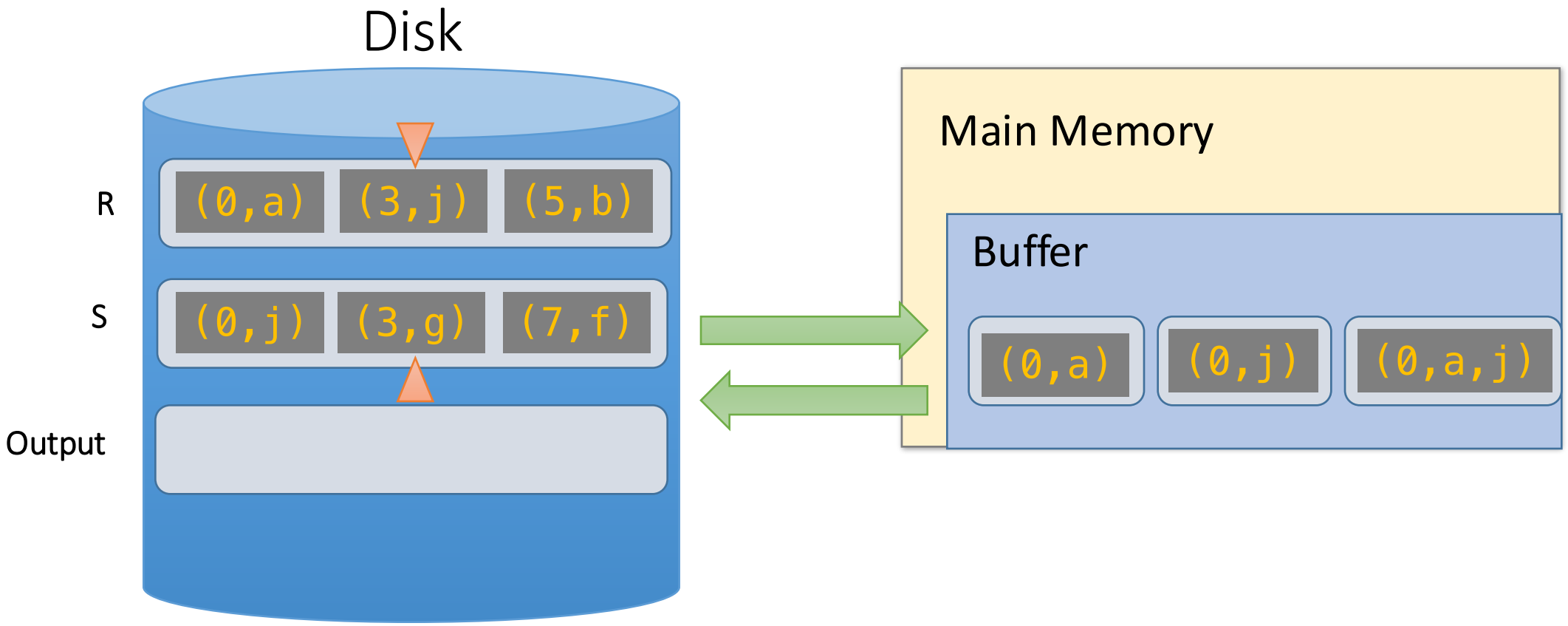
SMJ Example: $R \bowtie S$ on A with 3 page buffer

2. Scan and “merge” on join key!



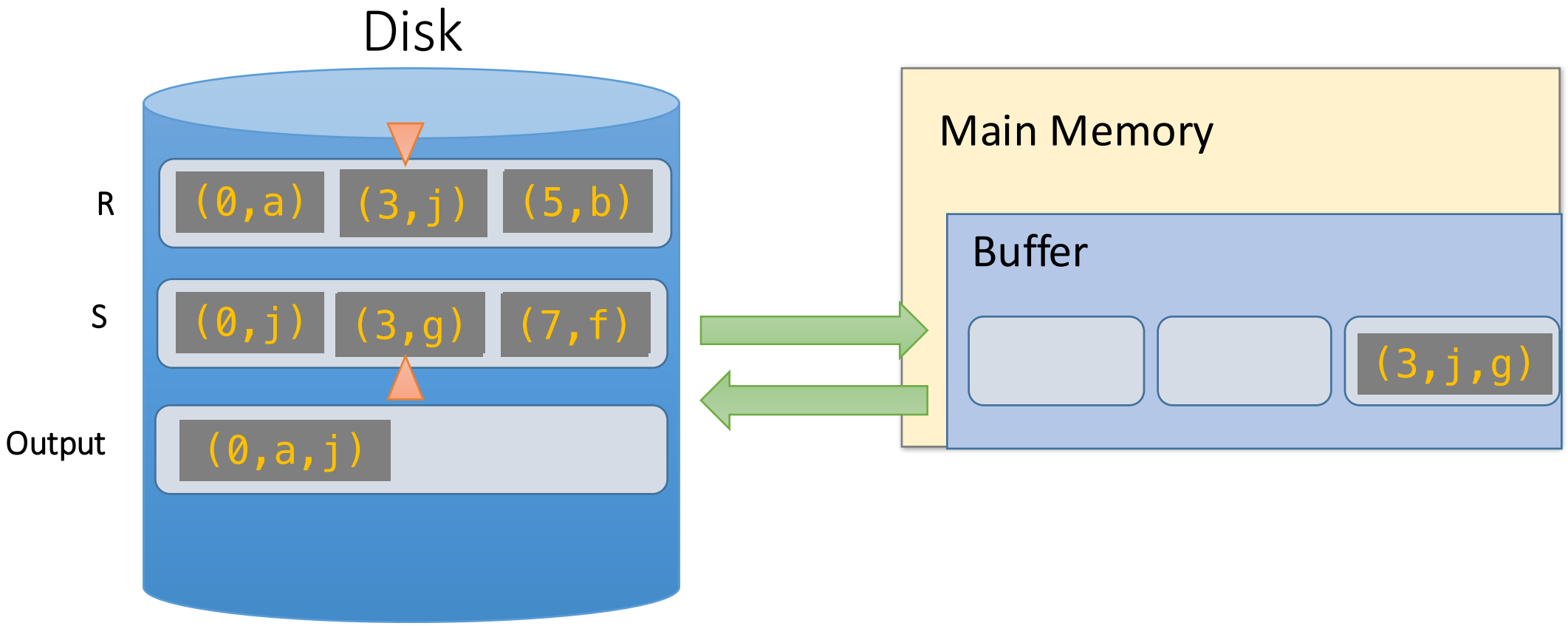
SMJ Example: $R \bowtie S$ on A with 3 page buffer

2. Scan and “merge” on join key!



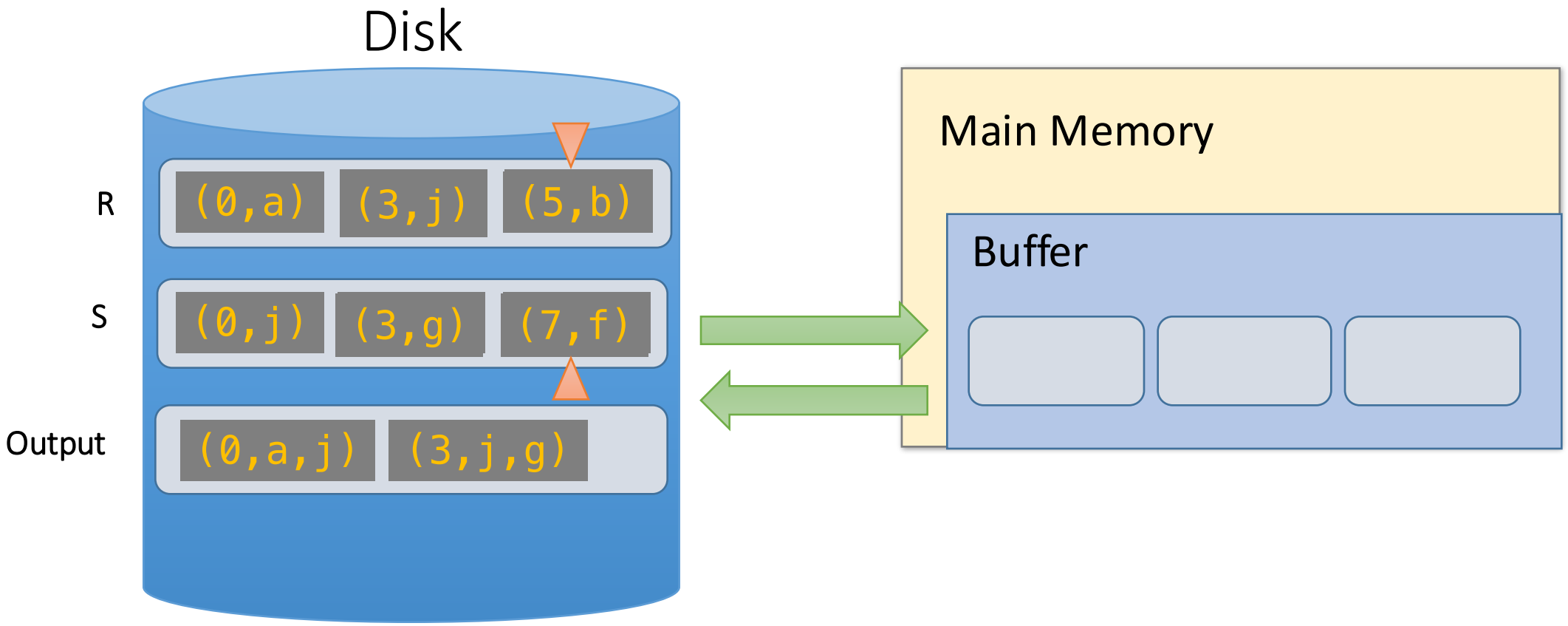
SMJ Example: $R \bowtie S$ *on* A with 3 page buffer

2. Scan and “merge” on join key!



SMJ Example: $R \bowtie S$ on A with 3 page buffer

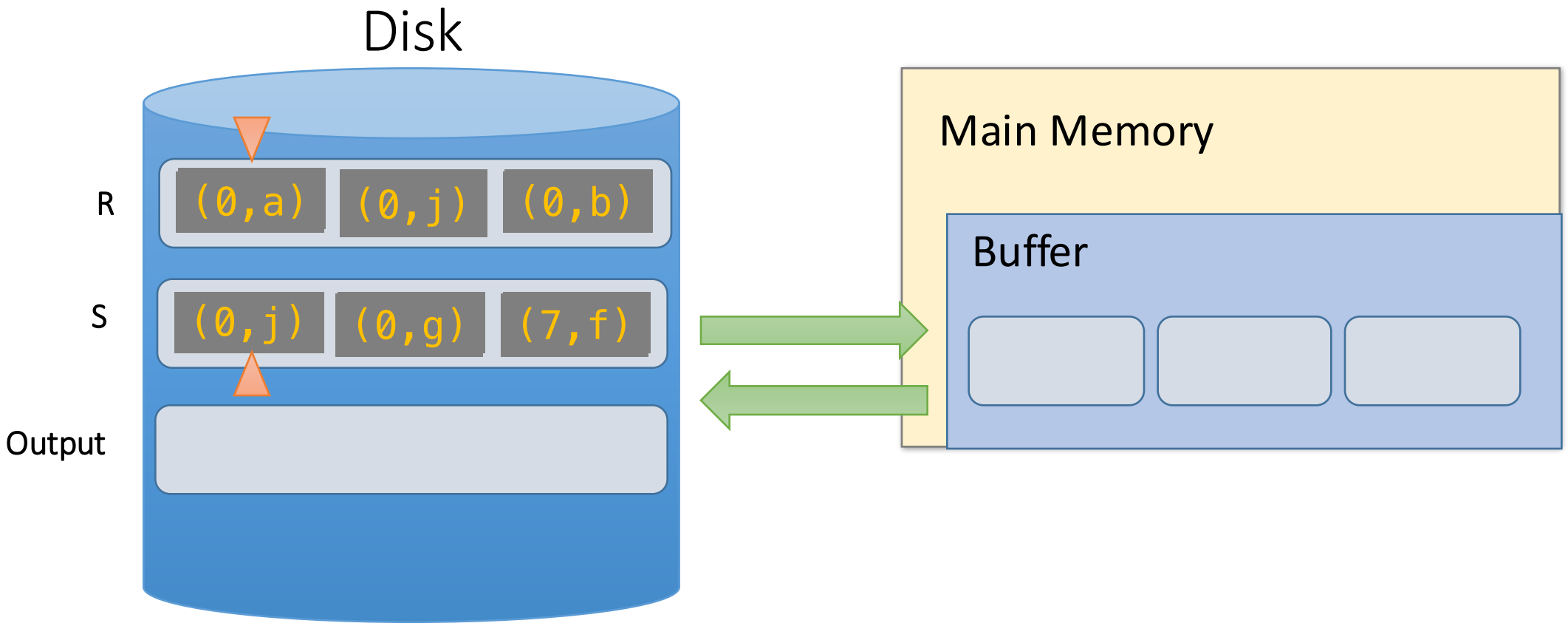
2. Done!



What happens with duplicate join keys?

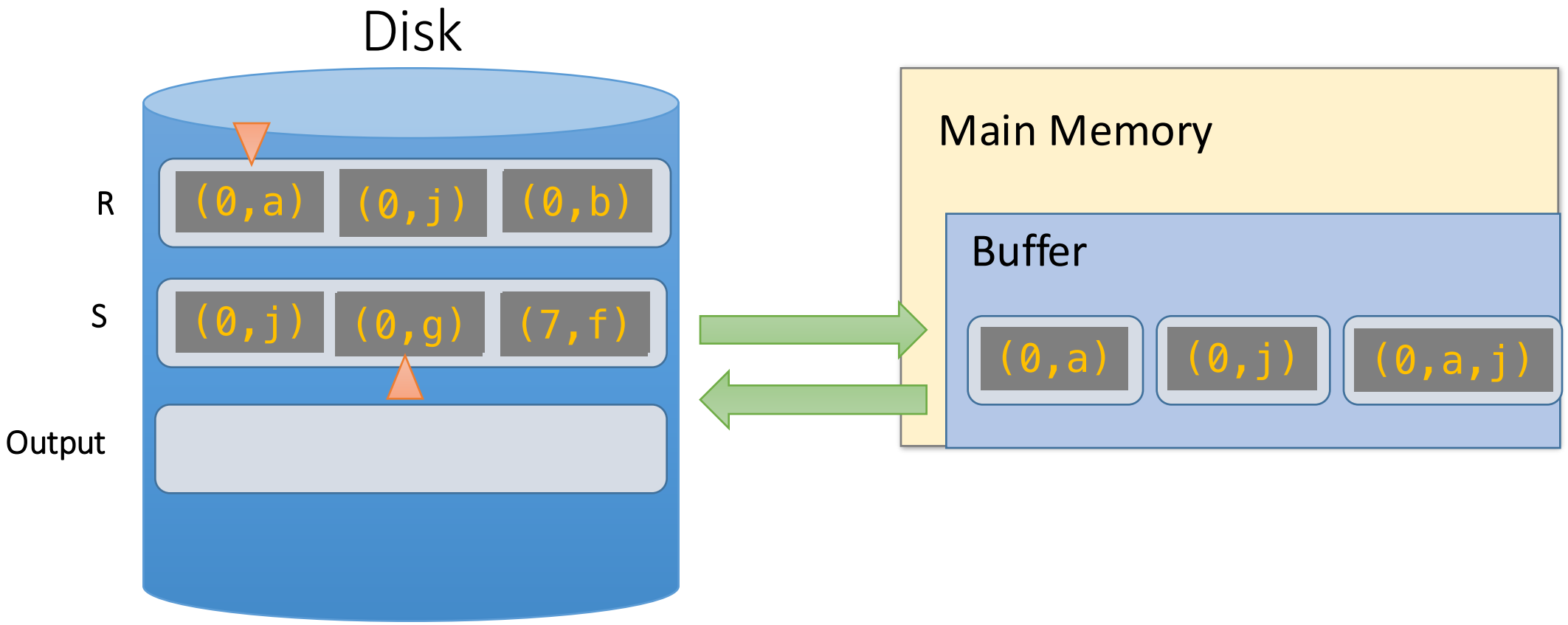
Multiple tuples with Same Join Key: “Backup”

1. Start with sorted relations, and begin scan / merge…



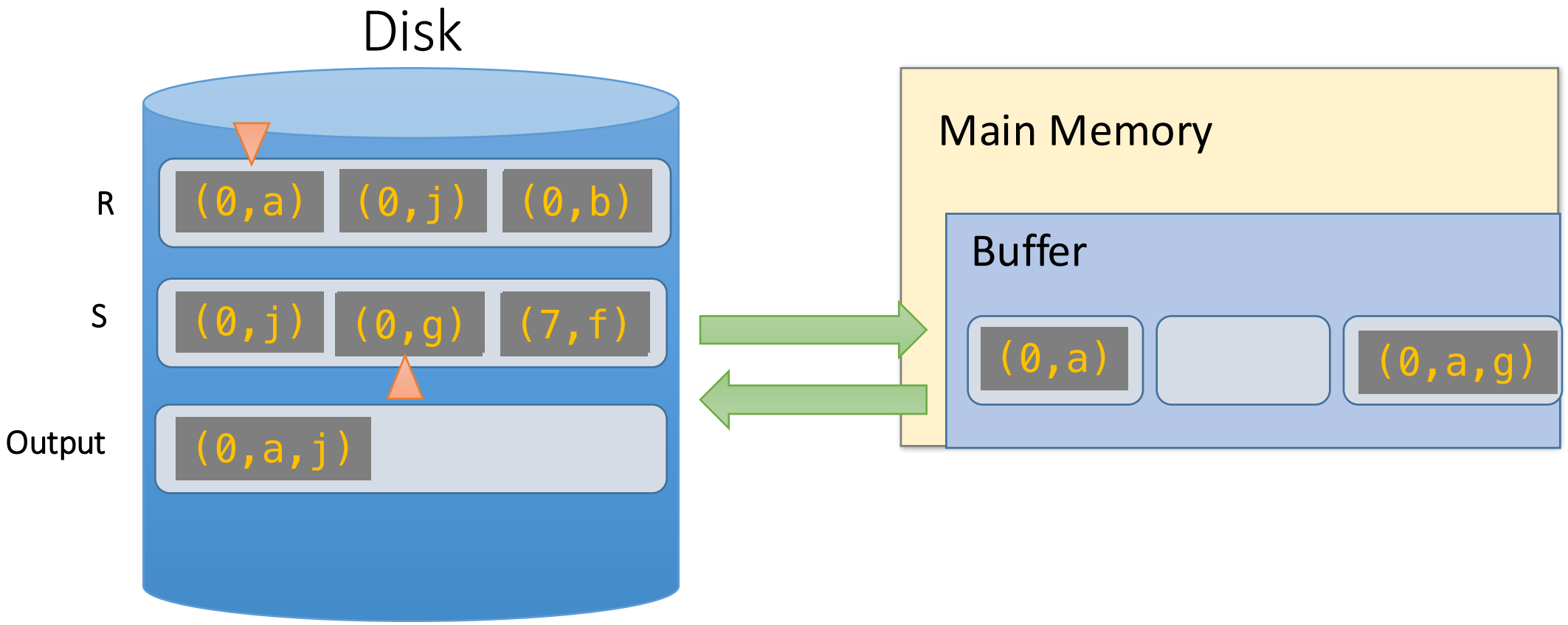
Multiple tuples with Same Join Key: “Backup”

1. Start with sorted relations, and begin scan / merge…



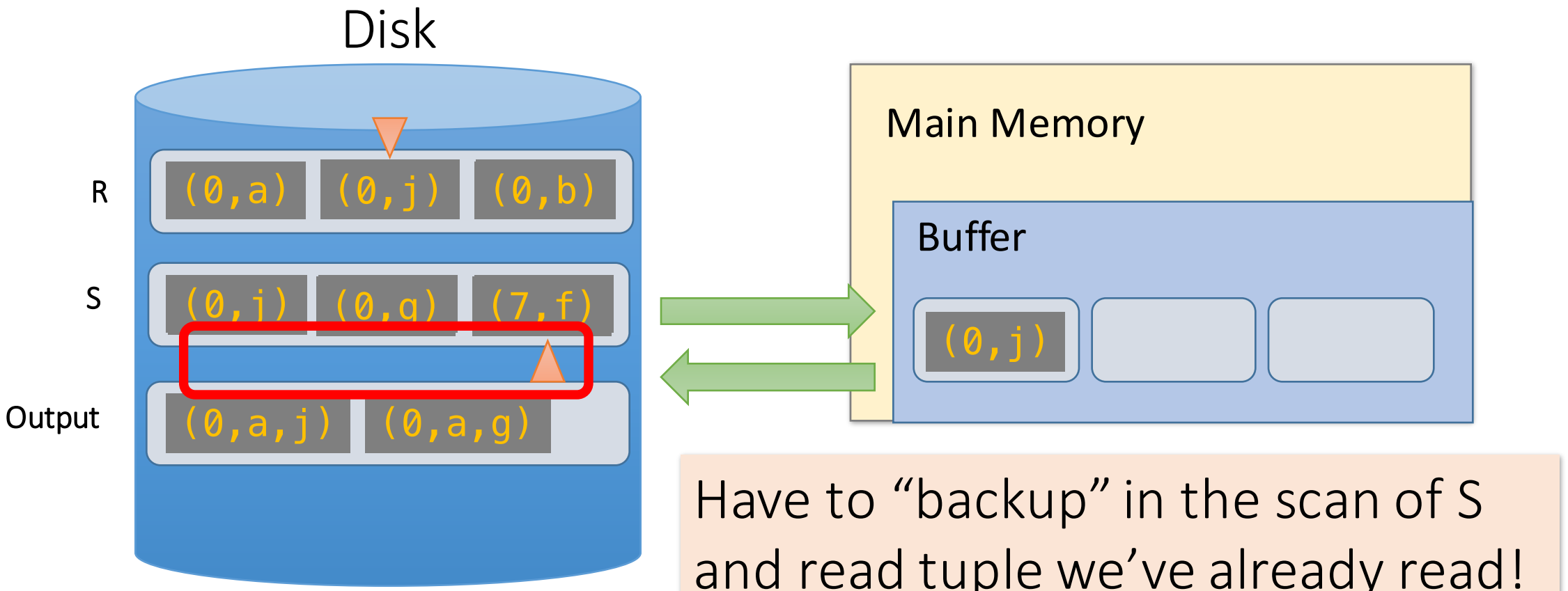
Multiple tuples with Same Join Key: “Backup”

1. Start with sorted relations, and begin scan / merge…



Multiple tuples with Same Join Key: "Backup"

1. Start with sorted relations, and begin scan / merge…



Have to "backup" in the scan of S and read tuple we've already read!

Backup

- At best, no backup → scan takes **P(R) + P(S)** reads
  - For ex: if no duplicate values in join attribute
- At worst (e.g. full backup each time), scan could take **P(R) * P(S)** reads!
  - For ex: if *all* duplicate values in join attribute, i.e. all tuples in R and S have the same value for the join attribute
  - Roughly: For each page of R, we'll have to *back up* and read each page of S…
- Often not that bad however, plus we can:
  - Leave more data in buffer (for larger buffers)
  - Can "zig-zag" (see animation)

SMJ: Total cost

- Cost of SMJ is **cost of sorting** R and S…
- Plus the **cost of scanning**: ~P(R)+P(S)
  - Because of *backup*: in worst case P(R)*P(S); but this would be very unlikely
- Plus the **cost of writing out**: ~P(R)+P(S) but in worst case T(R)*T(S)

~ Sort(P(R)) + Sort(P(S)) + P(R) + P(S) + OUT

Recall: Sort(N) $\approx 2N\left(\left\lceil \log_B \frac{\boldsymbol{N}}{\boldsymbol{2(B+1)}} \right\rceil + 1\right)$

Note: this is using repacking, where we estimate that we can create initial runs of length ~2(B+1)

SMJ vs. BNLJ: Steel Cage Match

- If we have 100 buffer pages, P(R) = 1000 pages and P(S) = 500 pages:
  - Sort both in two passes: 2 * 2 * 1000 + 2 * 2 * 500 = **6,000 IOs**
  - Merge phase 1000 + 500 = 1,500 IOs
  - **= 7,500 IOs + OUT**

What is BNLJ?

- 500 + 1000*$\left\lceil \frac{500}{98} \right\rceil$ = **6,550 IOs + OUT**

- But, if we have 35 buffer pages?
  - Sort Merge has same behavior (still 2 passes)
  - BNLJ? ***15,500 IOs + OUT!***

SMJ is ~ linear vs. BNLJ is quadratic…

A Simple Optimization: Merges Merged!

Given ***B+1*** buffer pages

- SMJ is composed of a ***sort phase*** and a ***merge phase***

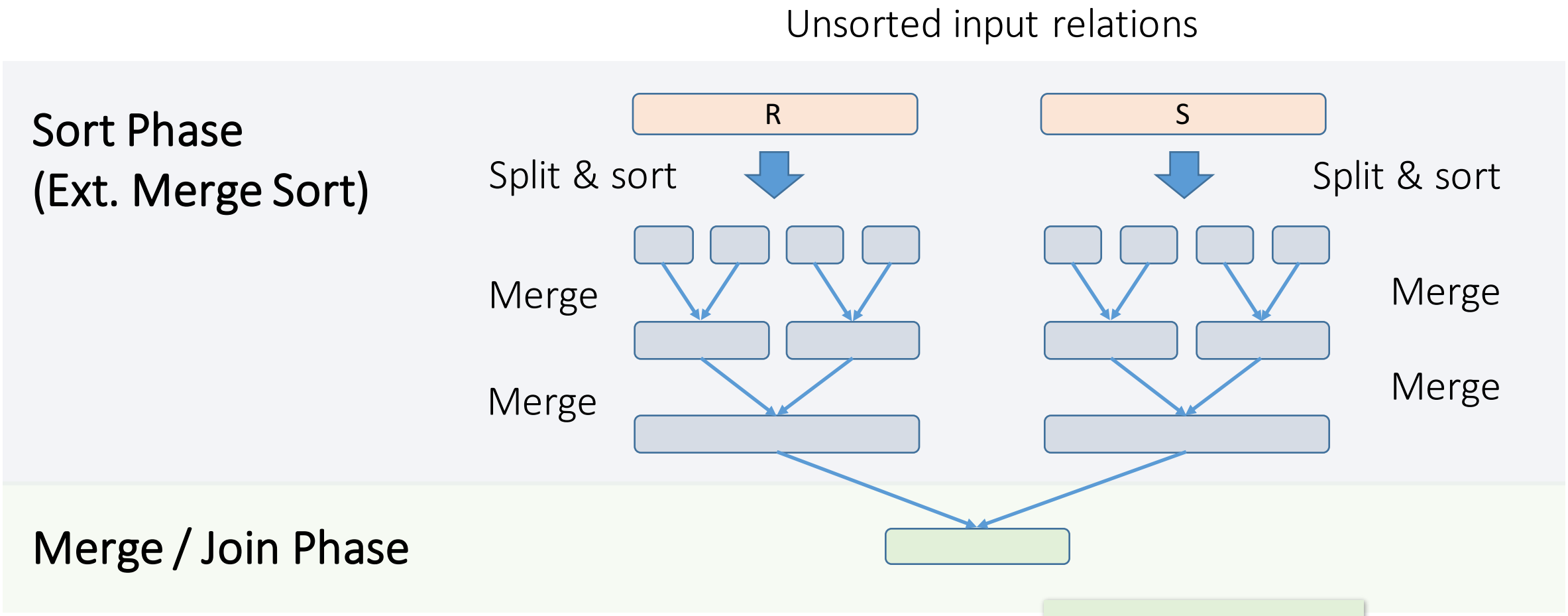
- During the ***sort phase***, run passes of external merge sort on R and S
  - Suppose at some point, R and S have <= ***B*** (sorted) runs in total

  - We could do two merges (for each of R & S) at this point, complete the sort phase, and start the merge phase…

  - OR, we could combine them: do **one** B-way merge and complete the join!

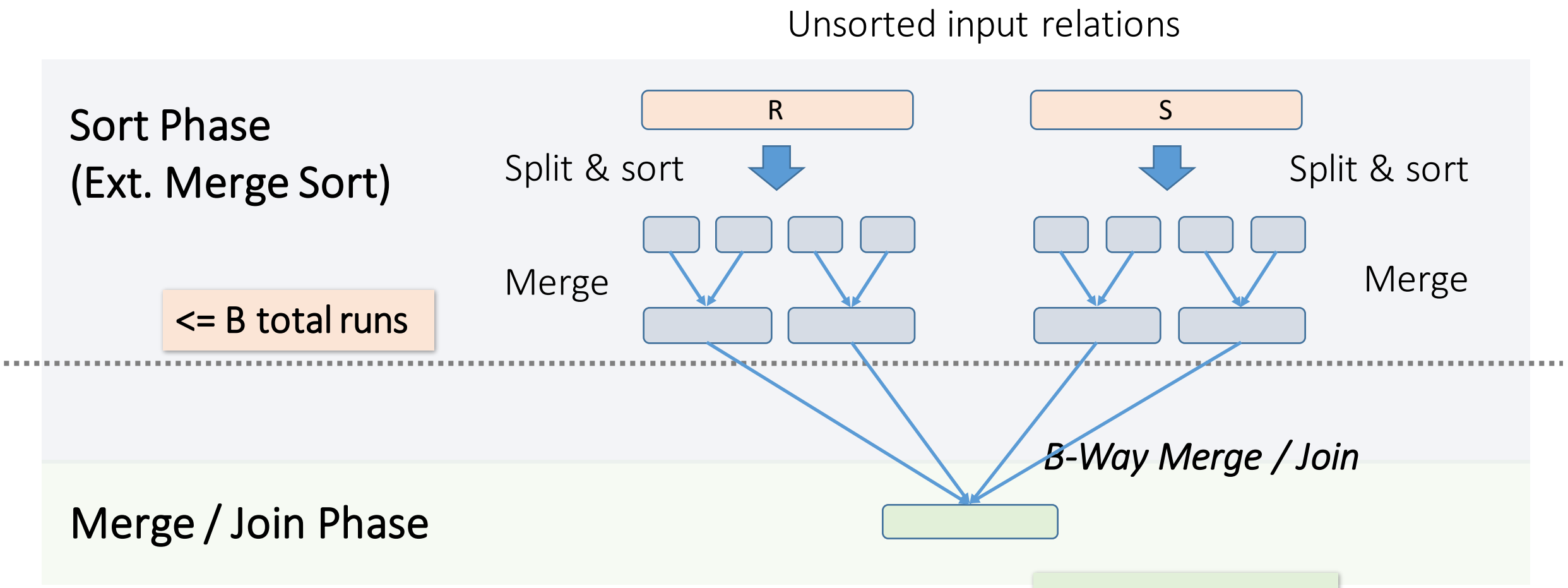
Un-Optimized SMJ

Given ***B+1*** buffer pages



Simple SMJ Optimization

Given ***B+1*** buffer pages



Simple SMJ Optimization

Given ***B+1*** buffer pages

- Now, on this last pass, we only do P(R) + P(S) IOs to complete the join!

- If we can initially split R and S into **B total runs each of length approx. <= 2(B+1)**, *assuming repacking lets us create initial runs of ~2(B+1)*- then we only need ***3(P(R) + P(S)) + OUT*** for SMJ!
  - 2 R/W per page to sort runs in memory, 1 R per page to B-way merge / join!

- How much memory for this to happen?
  - $\frac{P(R)+P(S)}{B} \leq 2(B+1) \Rightarrow \sim \mathrm{P(R)} + \mathrm{P(S)} \leq 2B^2$
  - **Thus, $\mathbf{max\{P(R), P(S)\}} \leq \boldsymbol{B^2}$ is an approximate sufficient condition**

If the larger of R,S has <= B^2 pages, then SMJ costs 3(P(R)+P(S)) + OUT!

Takeaway points from SMJ

If input already sorted on join key, skip the sorts.

- SMJ is basically linear.
- Nasty but unlikely case: Many duplicate join keys.

SMJ needs to sort **both** relations

- If max { P(R), P(S) } < B^2 then cost is 3(P(R)+P(S)) + OUT

Activity-15.ipynb

4. Hash Join (HJ)

What you will learn about in this section

1. Hash Join

2. Memory requirements

Recall: Hashing

- **Magic of hashing**:
  - A hash function h_B maps into [0,B-1]
  - And maps nearly uniformly

- A hash **collision** is when x != y but $h_B(x) = h_B(y)$
  - Note however that it will **never** occur that x = y but $h_B(x)$!= $h_B(y)$

- We hash on an attribute A, so our has function is $h_B(t)$ has the form $h_B(t.A)$.
  - **Collisions** may be more frequent.

Recall: Mad Hash Collisions

Say something here to justify this slide's existence? [TODO]

Hash Join: High-level procedure

To compute $\mathrm{R} \bowtie S \; on \; A$:

Note again that we are only considering equality constraints here

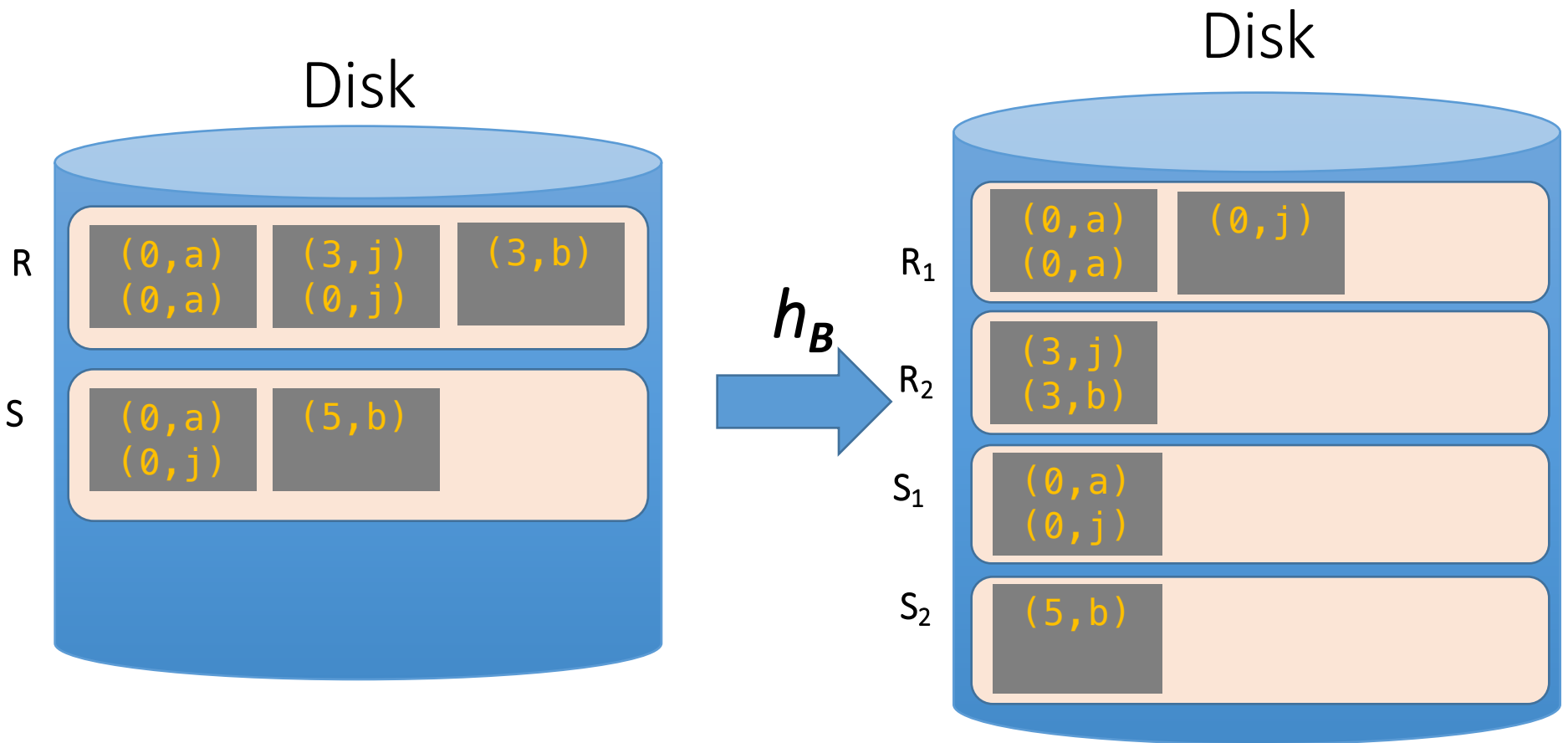
1. **Partition Phase:** Using one (shared) hash function $\boldsymbol{h_B}$, partition R *and* S into ***B*** buckets

2. **Matching Phase:** Take pairs of buckets whose tuples have the same values for ***h***, and join these
   1. Use BNLJ here; or hash again → either way, operating on small partitions so fast!

We ***decompose*** the problem using $\boldsymbol{h_B}$, then complete the join

Hash Join: High-level procedure

1. Partition Phase: Using one (shared) hash function h_B, partition R *and* S into ***B*** buckets

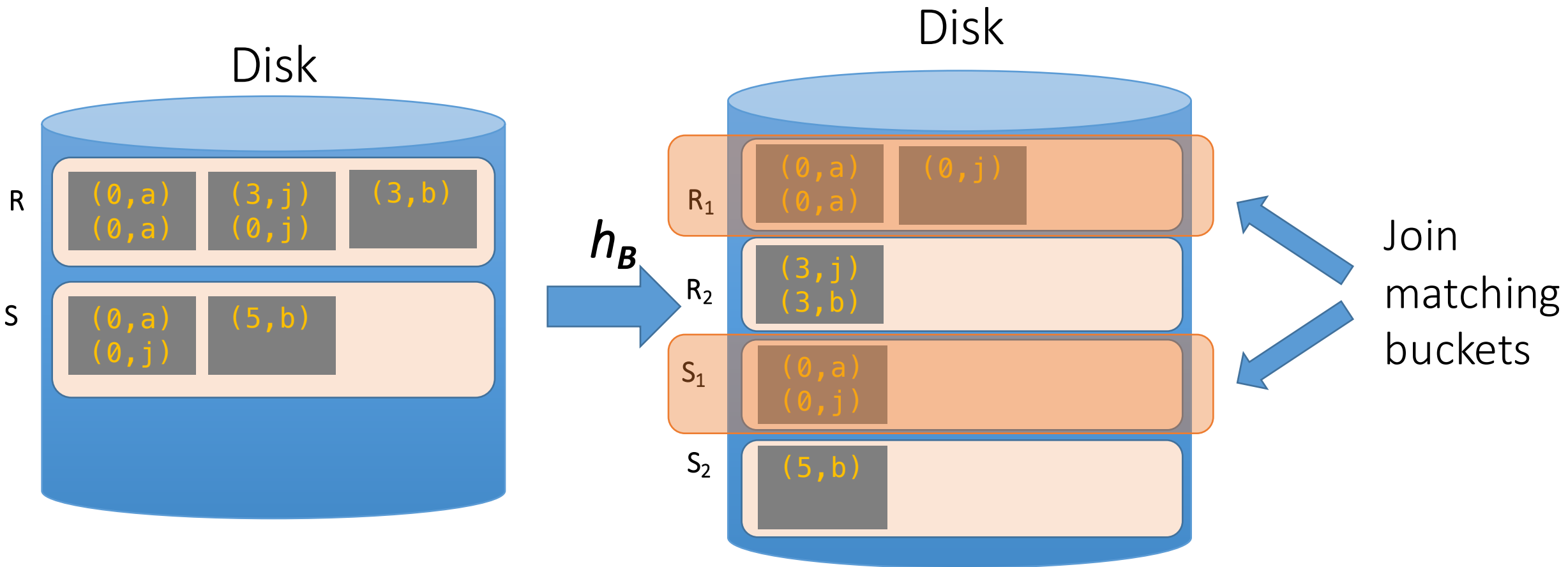


Note our new convention: pages each have two tuples (one per row)

More detail in a second…

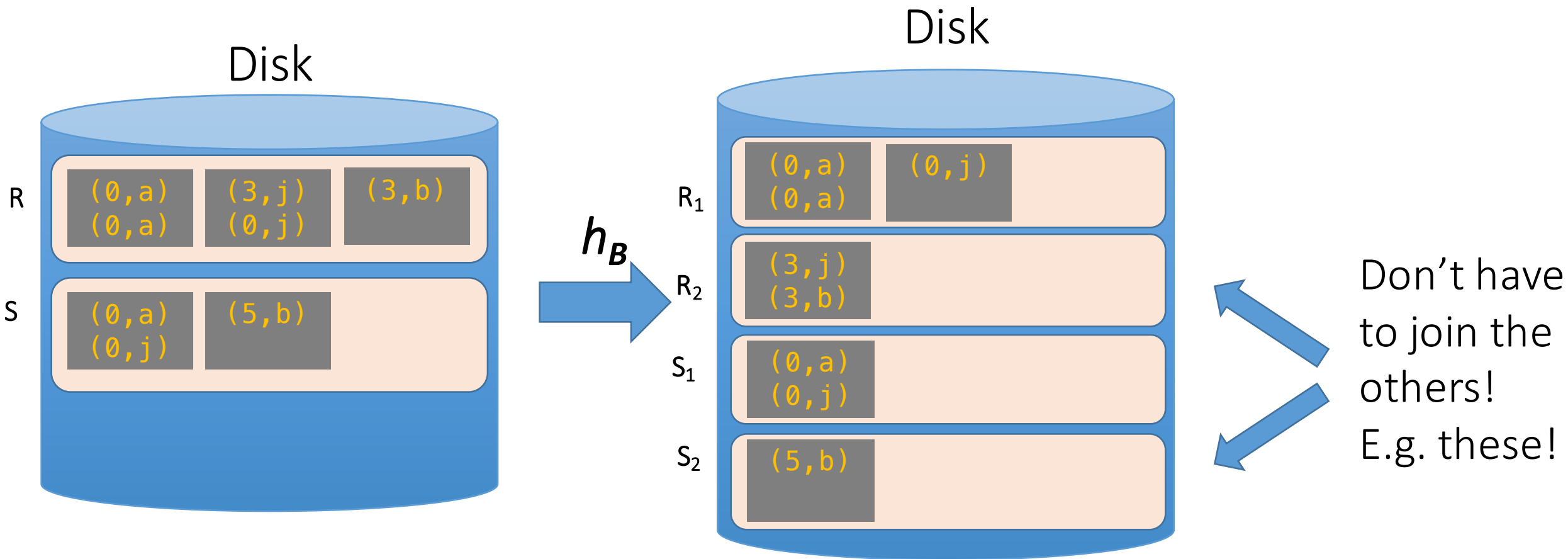
Hash Join: High-level procedure

2. Matching Phase: Take pairs of buckets whose tuples have the same values for h_B, and join these



Hash Join: High-level procedure

2. Matching Phase: Take pairs of buckets whose tuples have the same values for h_B, and join these



Hash Join Phase 1: Partitioning

Goal: For each relation, partition relation into **buckets** such that if $h_B(t.A) = h_B(t'.A)$ they are in the same bucket

Given B+1 buffer pages, we partition into B buckets:

- We use B buffer pages for output (one for each bucket), and 1 for input
  - The "dual" of sorting.
  - For each tuple t in input, copy to buffer page for $h_B(t.A)$
  - When page fills up, flush to disk.

How big are the resulting buckets?

Given *B+1* buffer pages

- Given **N input pages, we partition into B buckets**:
  - → Ideally our buckets are each of size **~ N/B pages**

- What happens if there are **hash collisions**?
  - Buckets could be > N/B
  - **We'll do several passes…**

- What happens if there are **duplicate join keys?**
  - Nothing we can do here… could have some **skew** in size of the buckets

How big *do we want* the resulting buckets?

Given ***B+1*** buffer pages

- Ideally, our buckets would be of size $\leq \boldsymbol{B-1}$ **pages**
  - ***1*** for input page, ***1*** for output page, ***B-1*** for each bucket

- Recall: If we want to join a bucket from R and one from S, we can do BNLJ **in linear time** if for *one of them (wlog say R),* $\boldsymbol{P(R) \leq B-1}$!
  - And more generally, being able to fit bucket in memory is advantageous

Recall for BNLJ:

$$\mathrm{P}(R) + \frac{P(R)P(S)}{B-1}$$

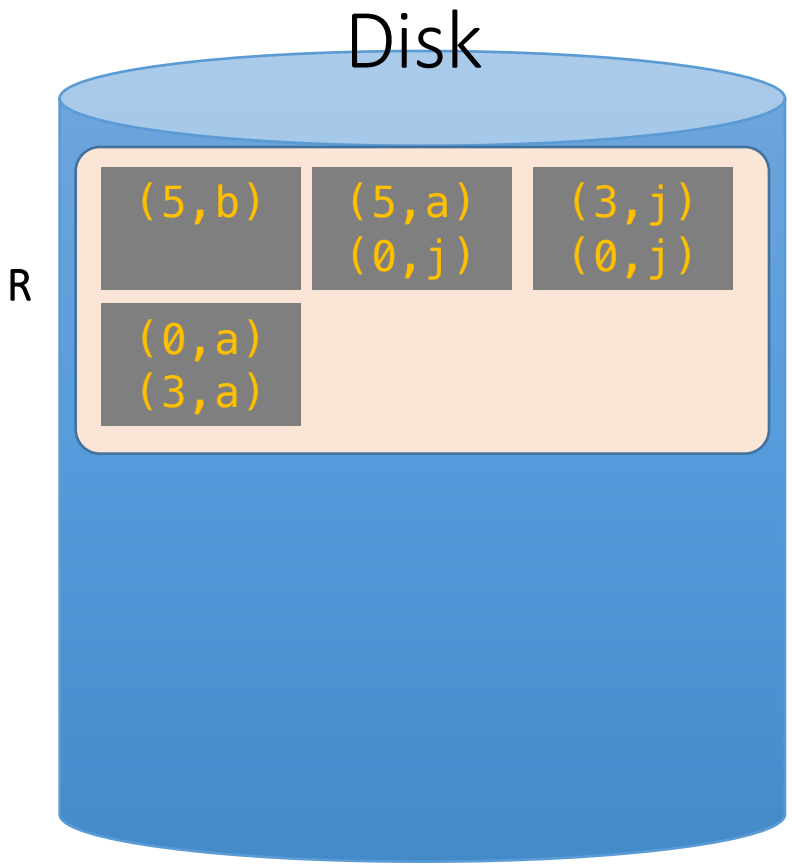
- We can keep partitioning buckets that are > B-1 pages, until they are $\leq \boldsymbol{B-1}$ **pages**
  - Using a new hash key which will split them…

We'll call each of these a "pass" again…

Hash Join Phase 1: Partitioning

Given ***B+1 = 3*** buffer pages

We partition into ***B = 2*** buckets **using hash function h_2** so that we can have one buffer page for each partition (and one for input)



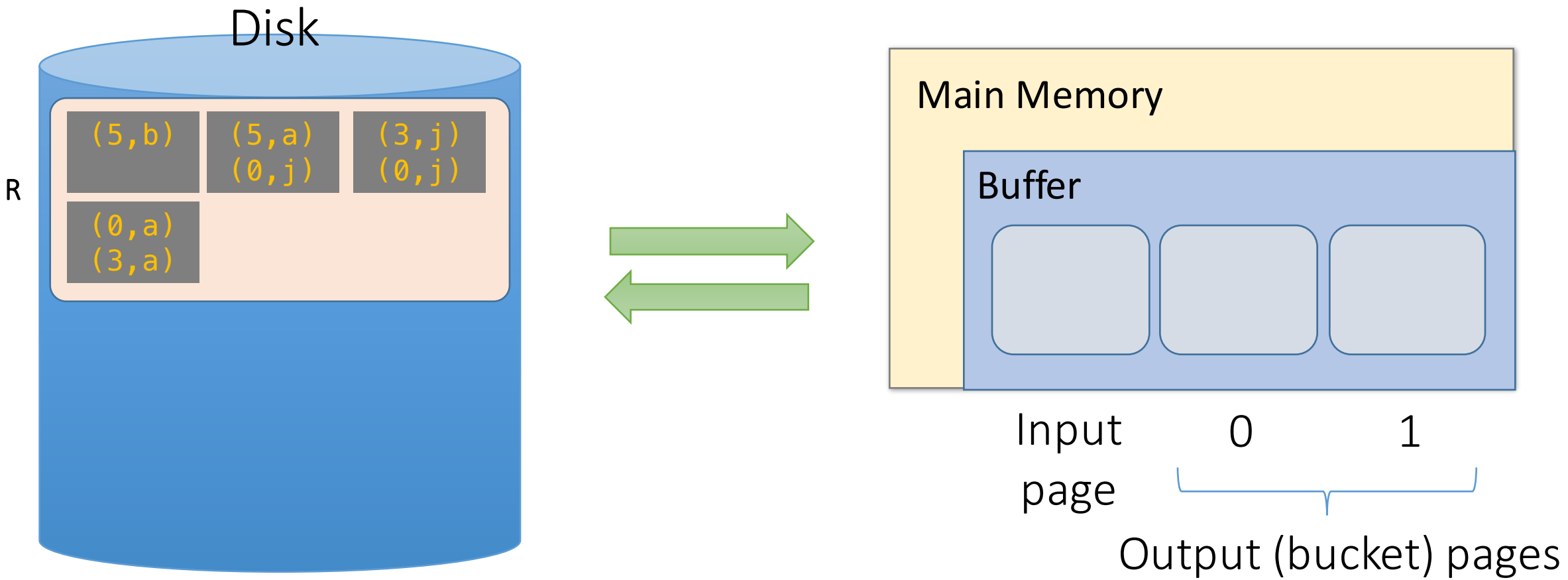
For simplicity, we'll look at partitioning one of the two relations- we just do the same for the other relation!

Recall: our goal will be to get ***B = 2 buckets*** of size <= ***B-1*** → ***1 page each***

Hash Join Phase 1: Partitioning

Given *B+1 = 3* buffer pages

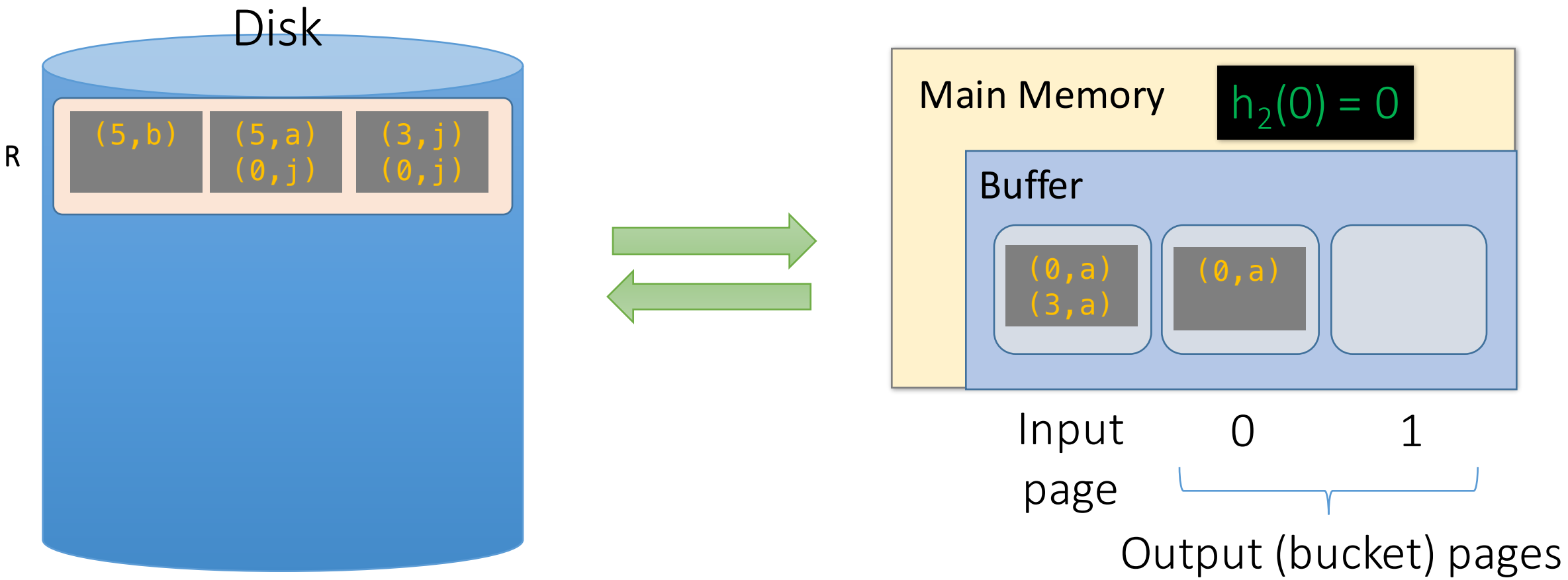
1. We read pages from R into the “input” page of the buffer…



Hash Join Phase 1: Partitioning

Given *B+1 = 3* buffer pages

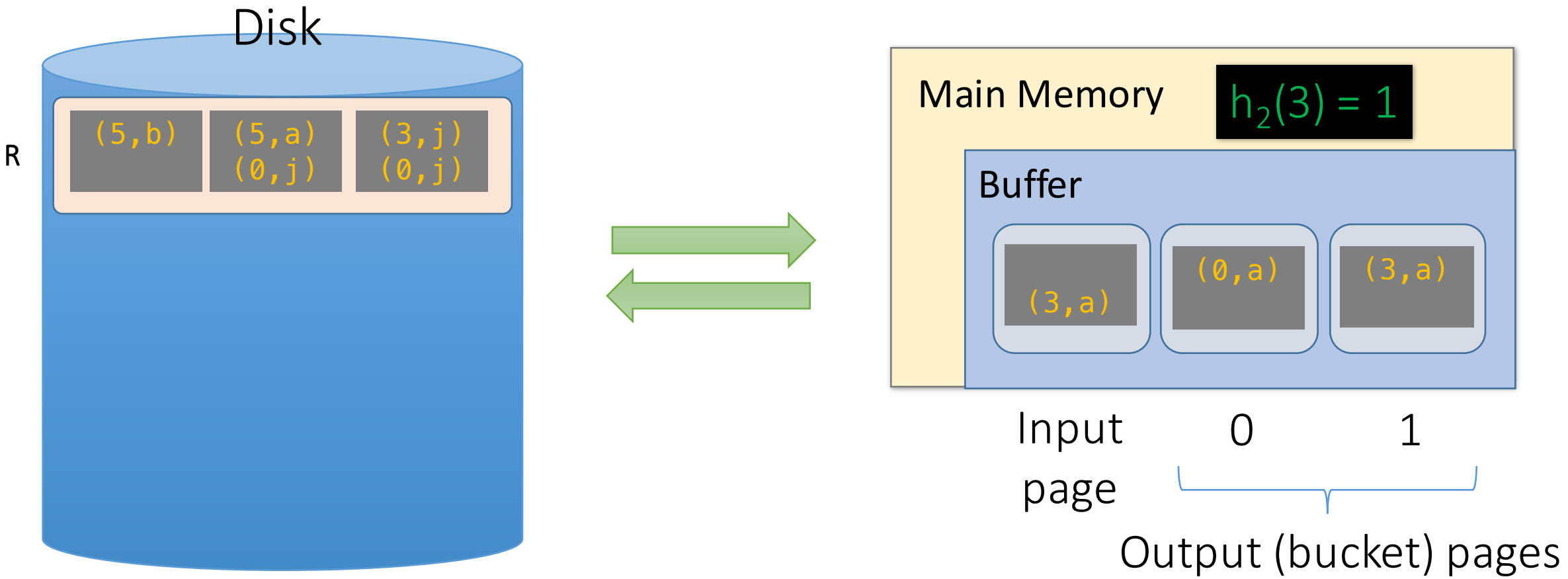
2. Then we use **hash function h_2** to sort into the buckets, which each have one page in the buffer



Hash Join Phase 1: Partitioning

Given *B+1 = 3* buffer pages

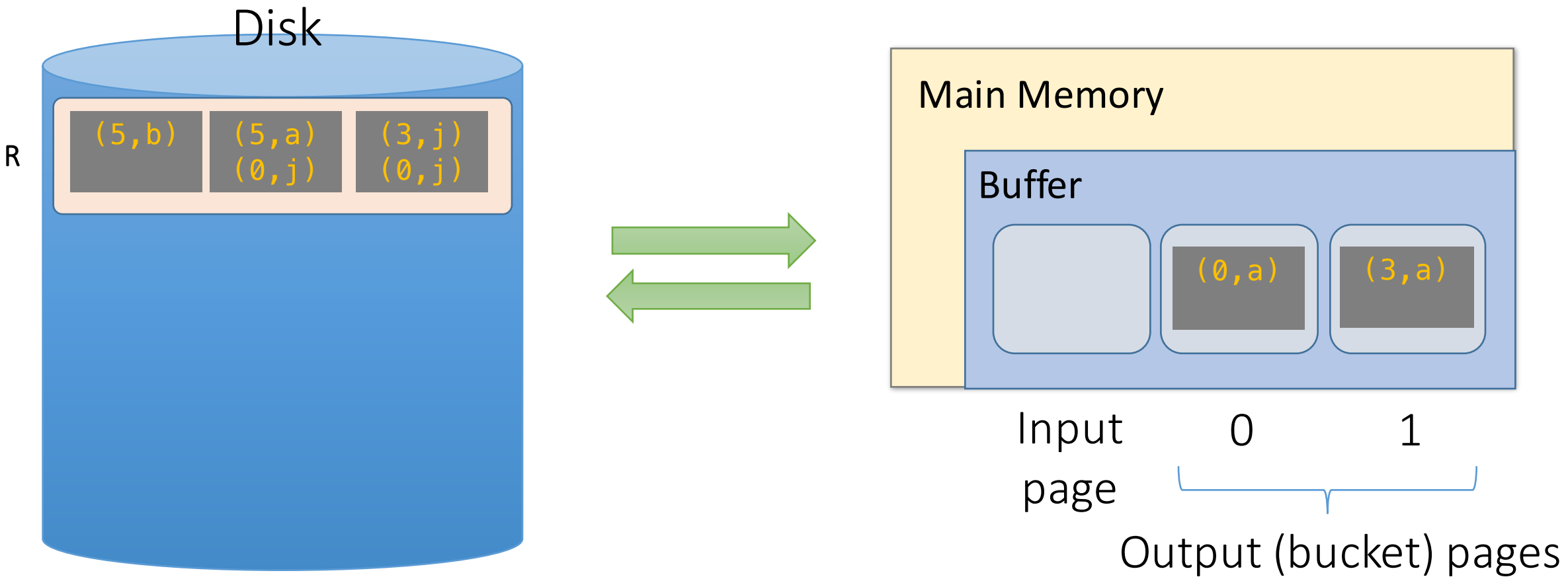
2. Then we use **hash function h_2** to sort into the buckets, which each have one page in the buffer



Hash Join Phase 1: Partitioning

Given *B+1 = 3* buffer pages

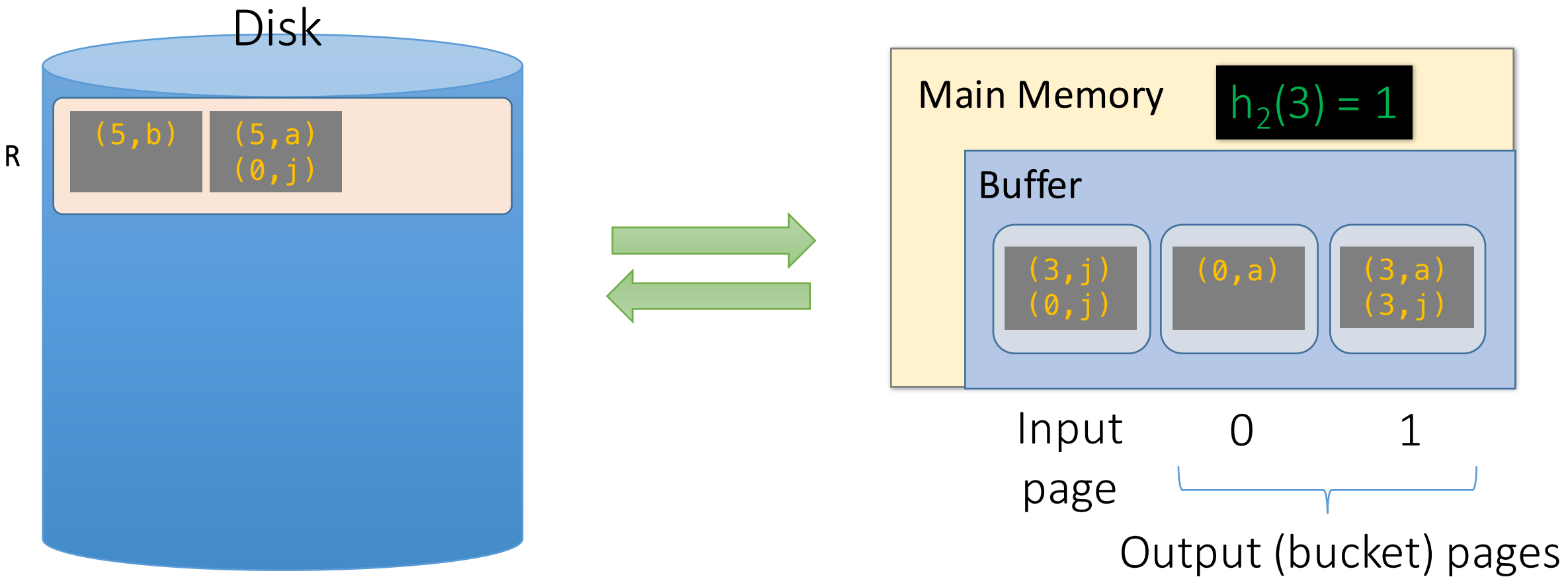
3. We repeat until the buffer bucket pages are full…



Hash Join Phase 1: Partitioning

Given *B+1 = 3* buffer pages

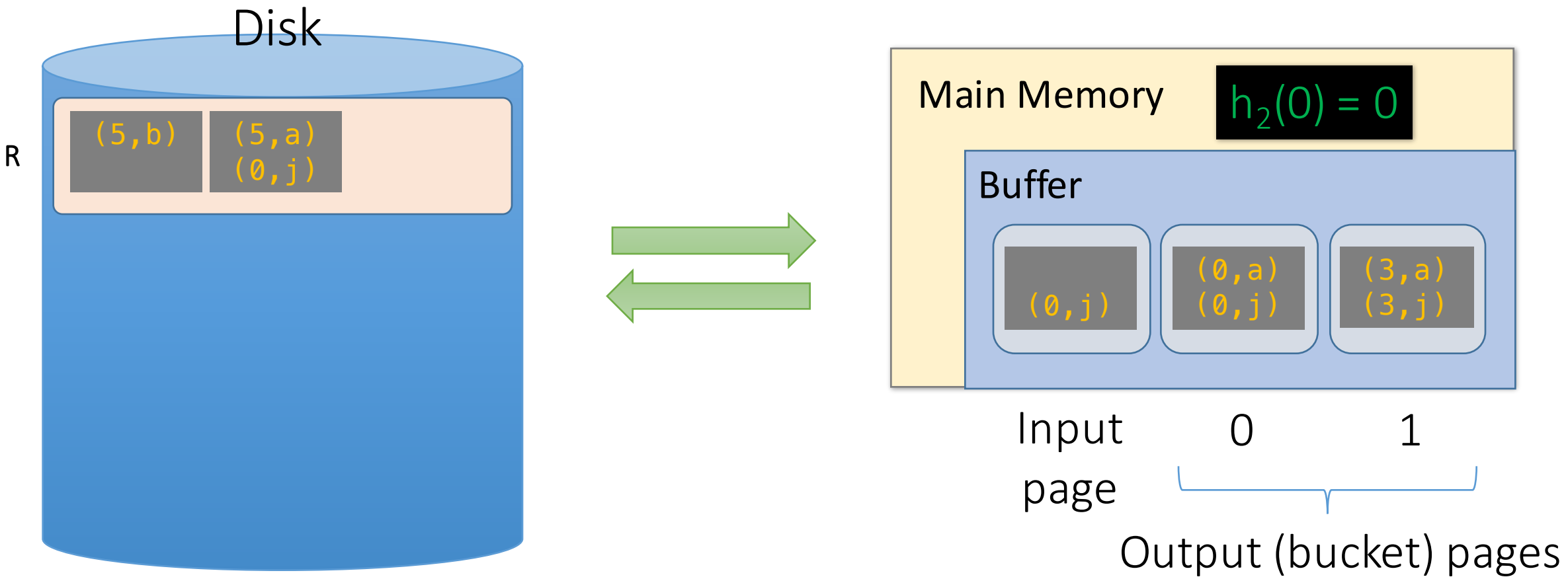
3. We repeat until the buffer bucket pages are full…



Hash Join Phase 1: Partitioning

Given *B+1 = 3* buffer pages

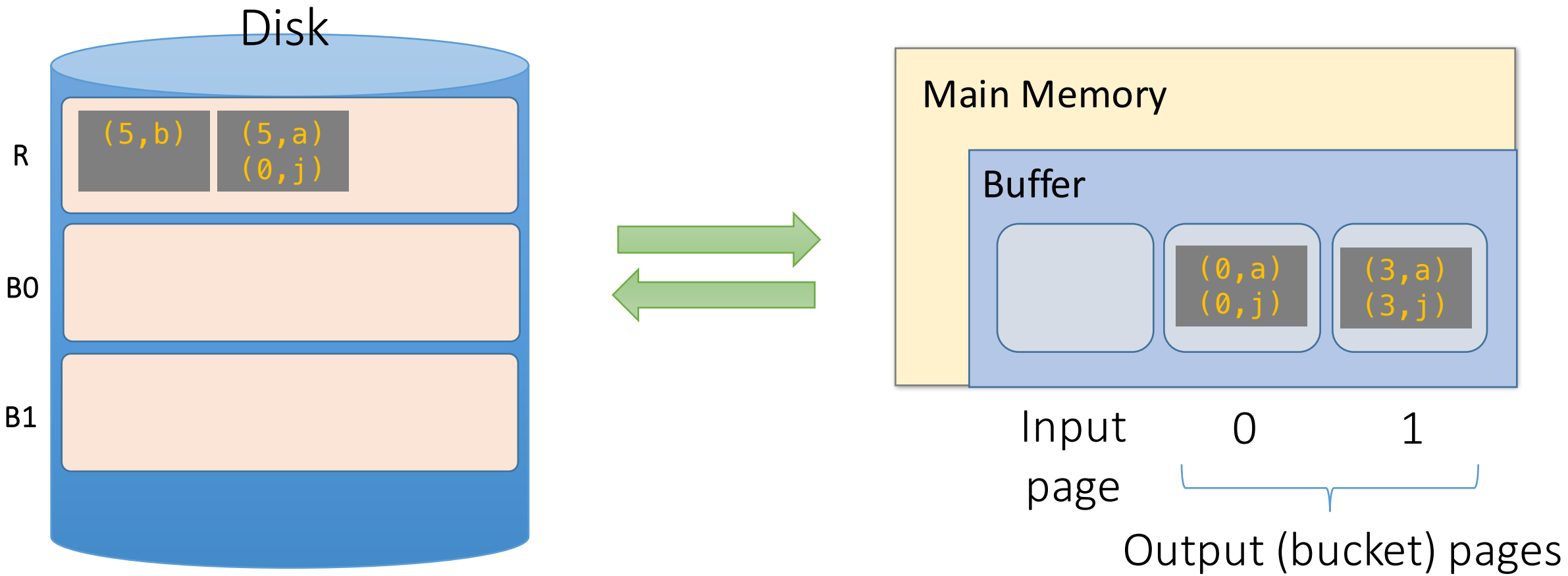
3. We repeat until the buffer bucket pages are full…



Hash Join Phase 1: Partitioning

Given *B+1 = 3* buffer pages

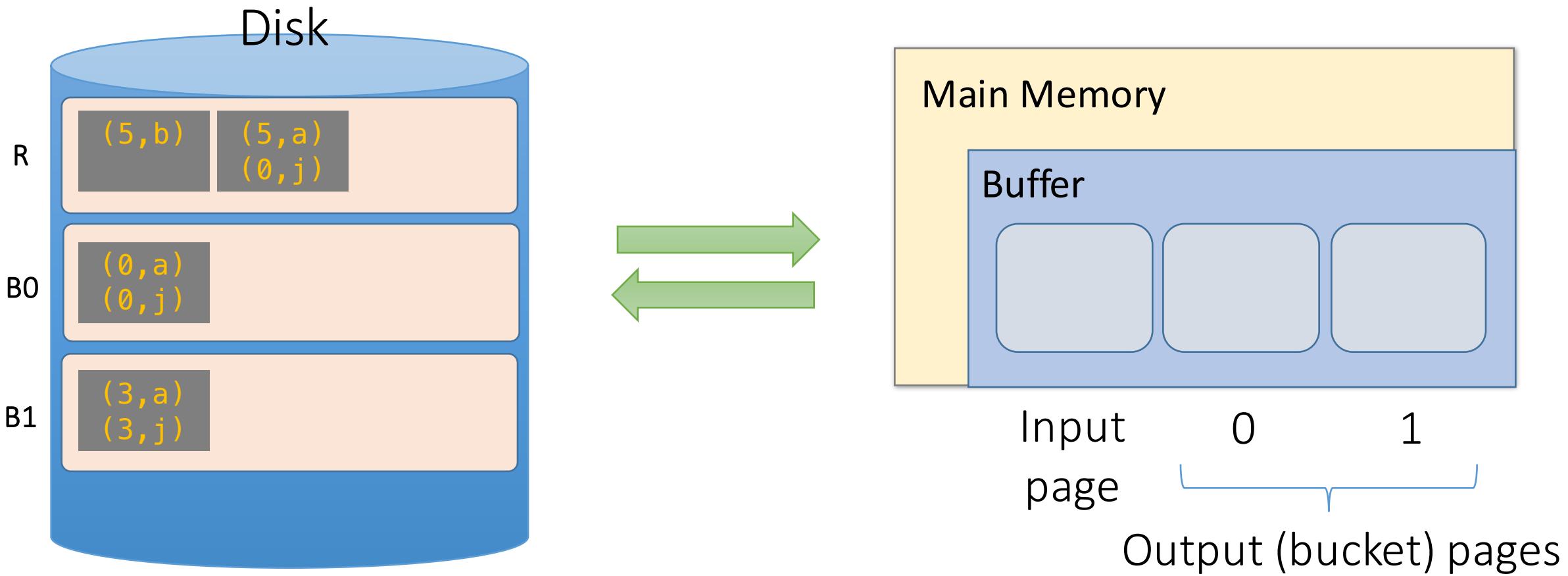
3. We repeat until the buffer bucket pages are full… then flush to disk



Hash Join Phase 1: Partitioning

Given *B+1 = 3* buffer pages

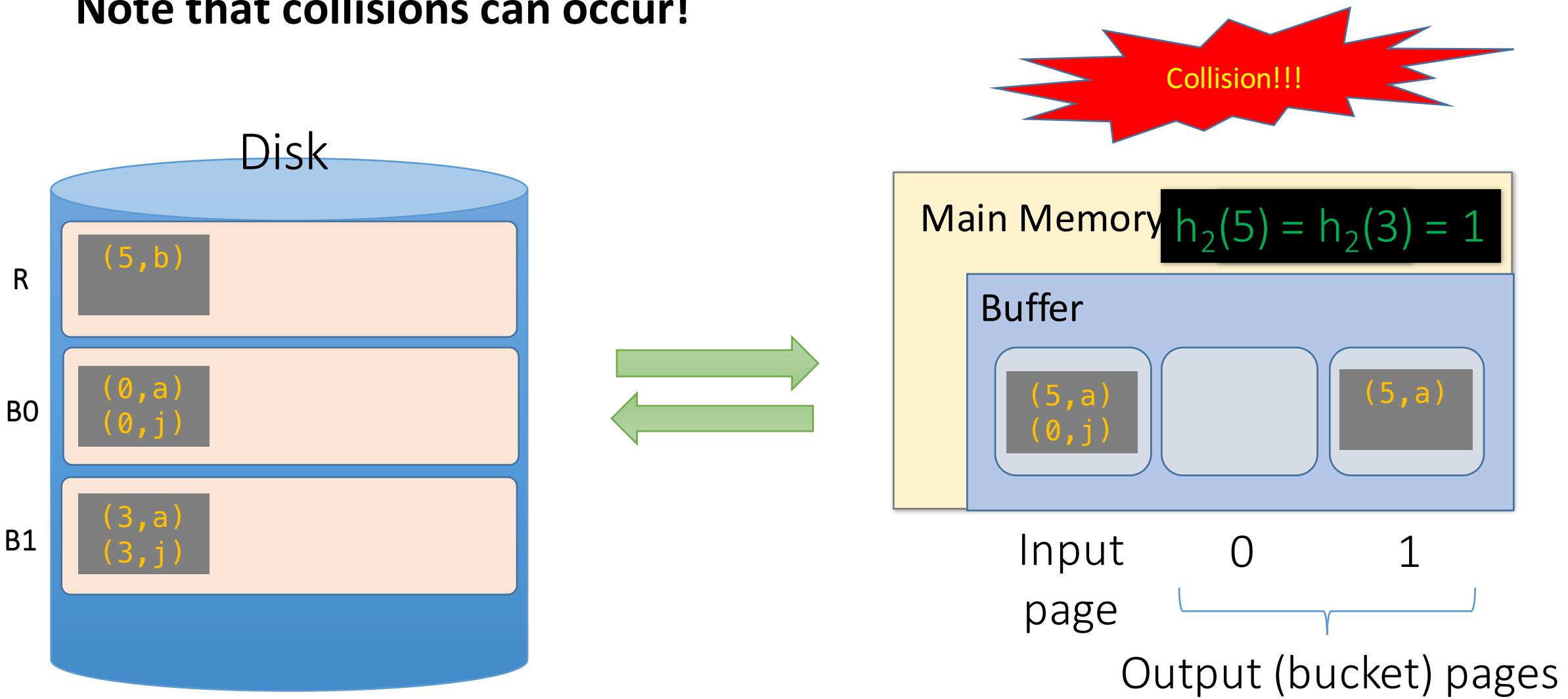
3. We repeat until the buffer bucket pages are full… then flush to disk



Hash Join Phase 1: Partitioning

Given *B+1 = 3* buffer pages

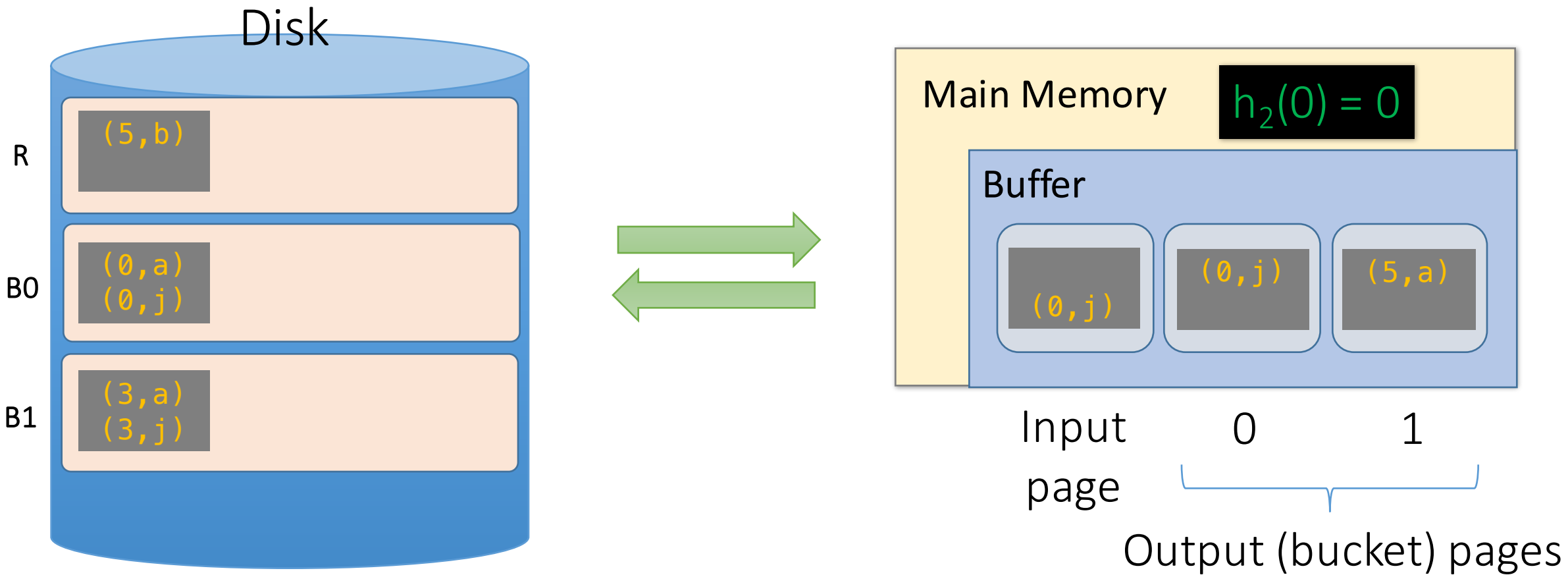
Note that collisions can occur!



Hash Join Phase 1: Partitioning

Given *B+1 = 3* buffer pages

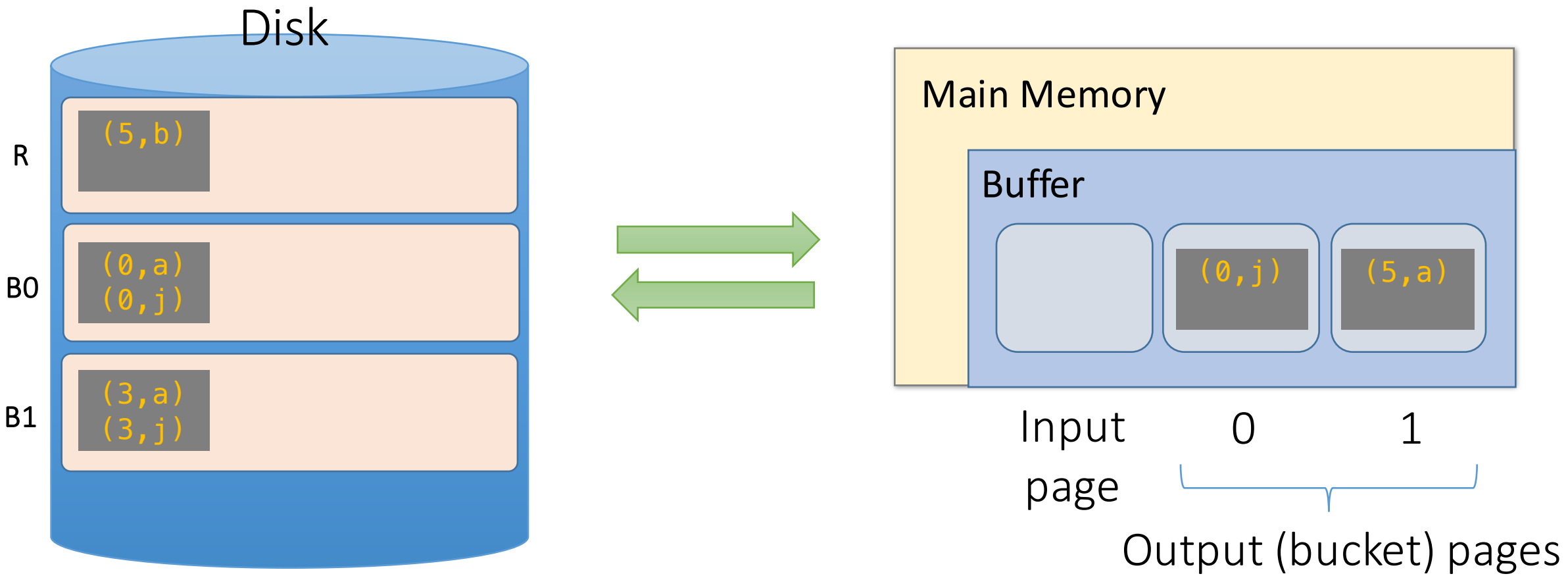
Finish this pass…



Hash Join Phase 1: Partitioning

Given *B+1 = 3* buffer pages

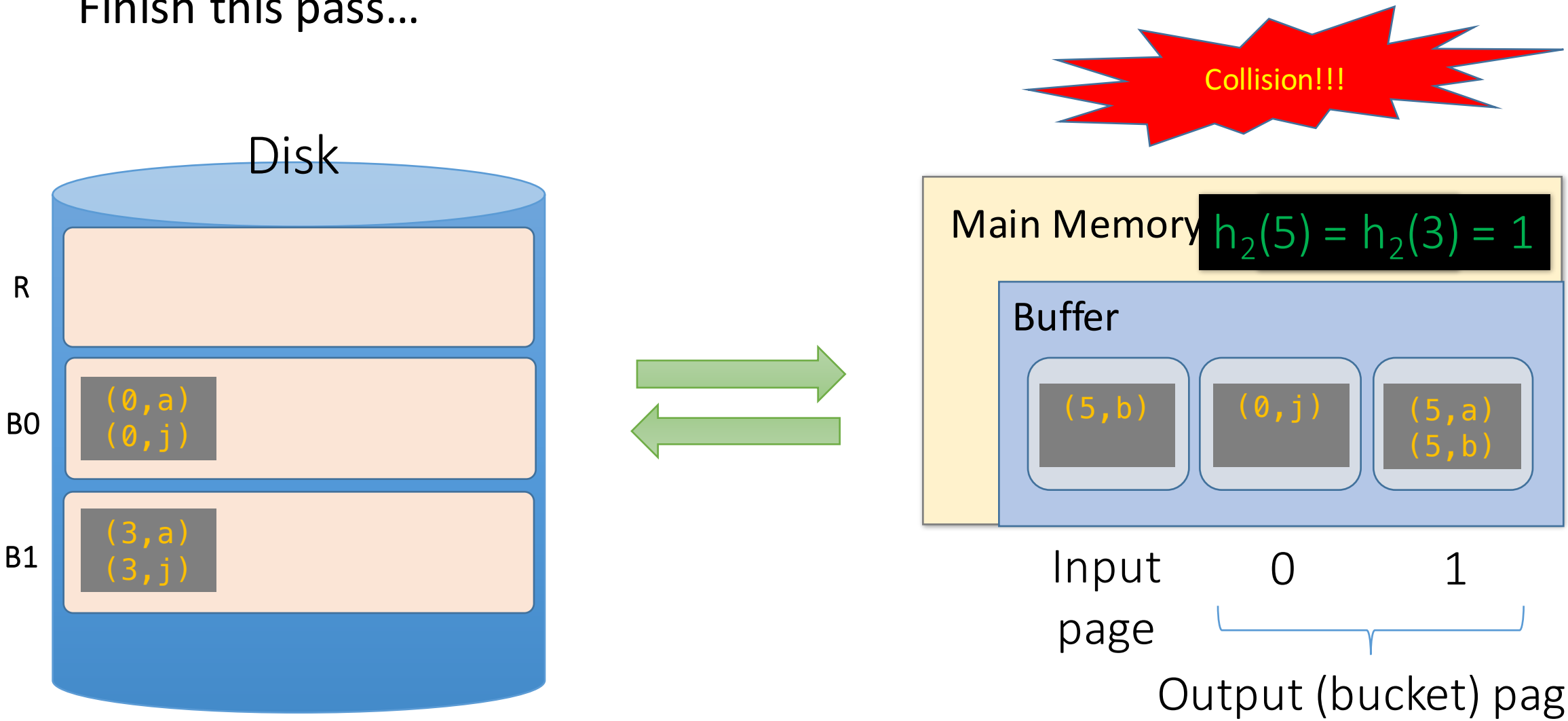
Finish this pass…



Hash Join Phase 1: Partitioning

Given *B+1 = 3* buffer pages

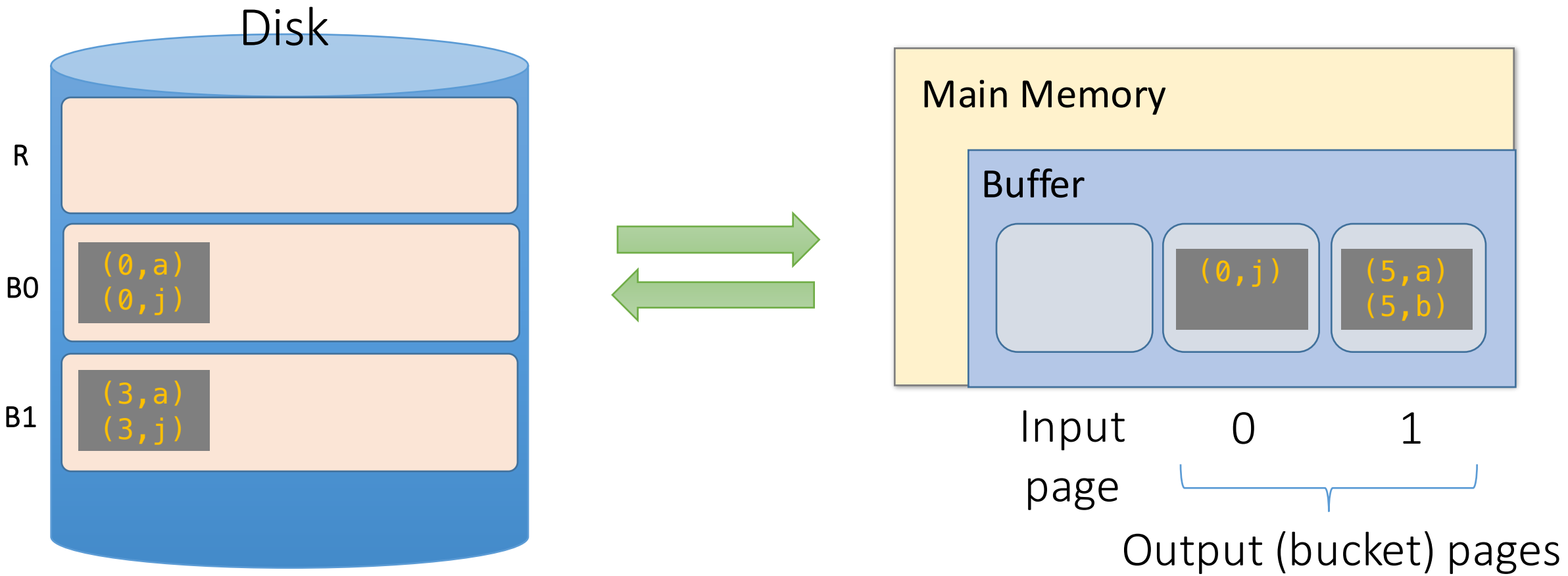
Finish this pass…



Hash Join Phase 1: Partitioning

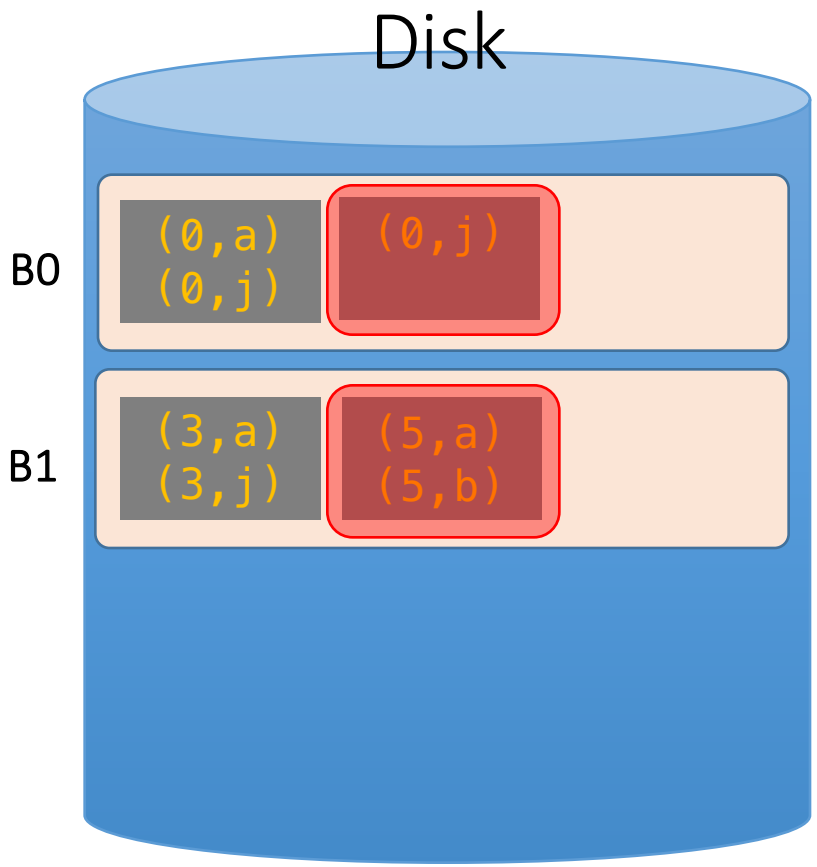
Given *B+1 = 3* buffer pages

Finish this pass…



Hash Join Phase 1: Partitioning

Given ***B+1 = 3*** buffer pages



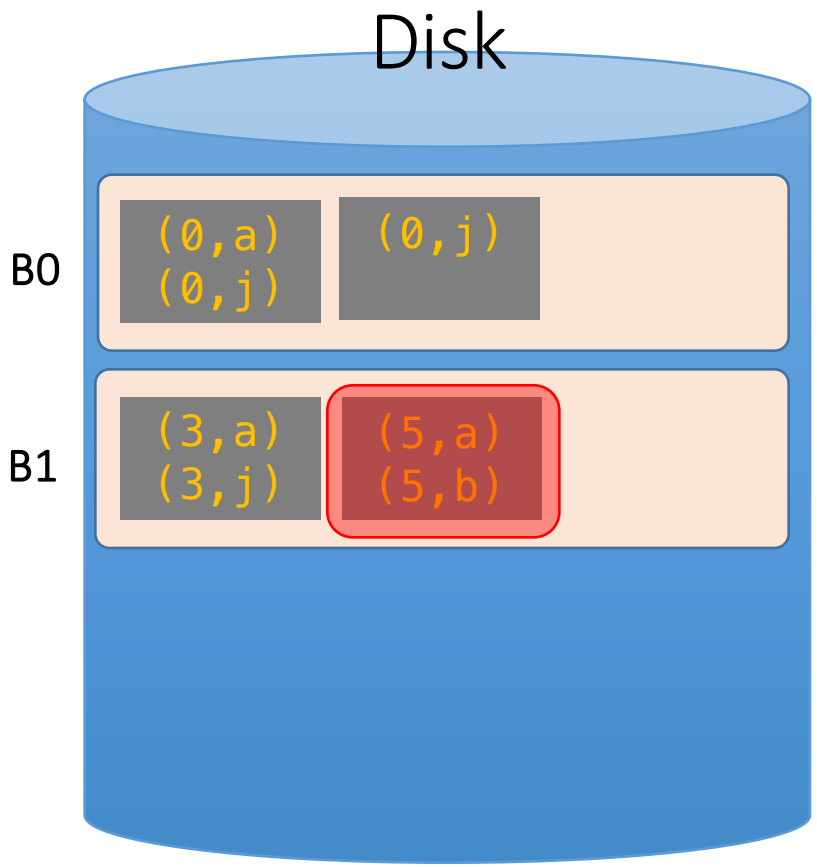
We wanted buckets of size ***B-1 = 1…***
however we got larger ones due to:

(1) Duplicate join keys

(2) Hash collisions

Hash Join Phase 1: Partitioning

Given ***B+1 = 3*** buffer pages



To take care of larger buckets caused by (2) hash collisions, we can just do another pass!

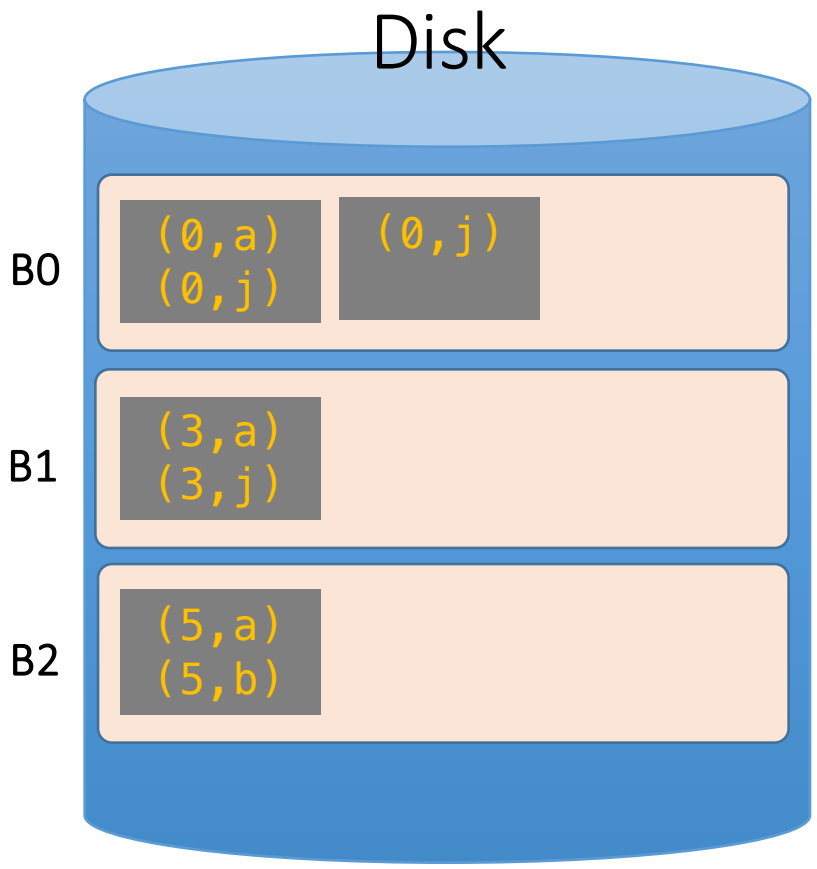
What hash function should we use?

Do another pass with a different hash function, h'_2, ideally such that:

$$h'_2(3) \mathrel{!}= h'_2(5)$$

Hash Join Phase 1: Partitioning

Given ***B+1 = 3*** buffer pages



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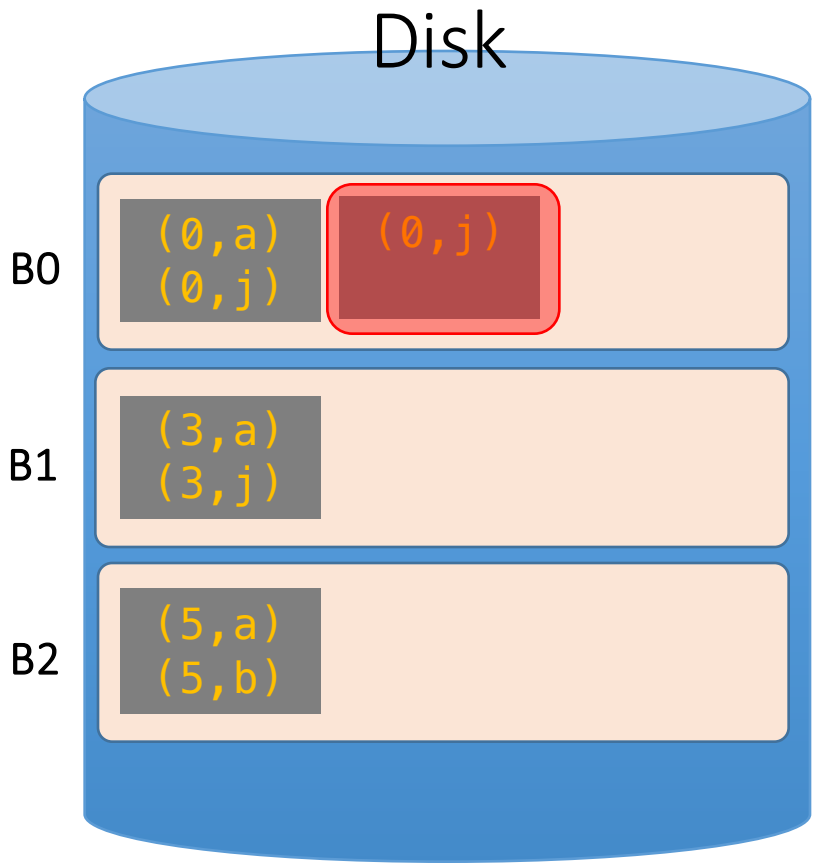
Hash Join Phase 1: Partitioning

Given ***B+1 = 3*** buffer pages



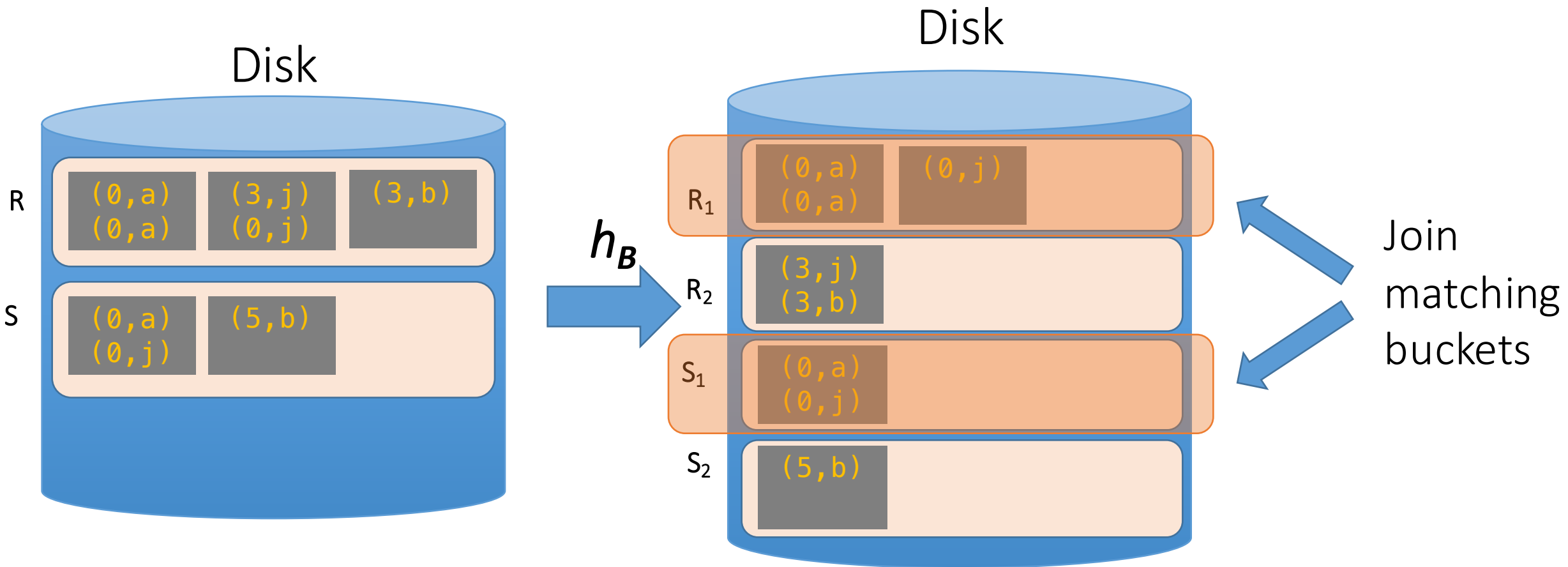
What about duplicate join keys? Unfortunately this is a problem… but usually not a huge one.

We call this unevenness in the bucket size **skew**

Now that we have partitioned R and S…

Hash Join Phase 2: Matching

- Now, we just join pairs of buckets from R and S that have the same hash value to complete the join!



Hash Join Phase 2: Matching

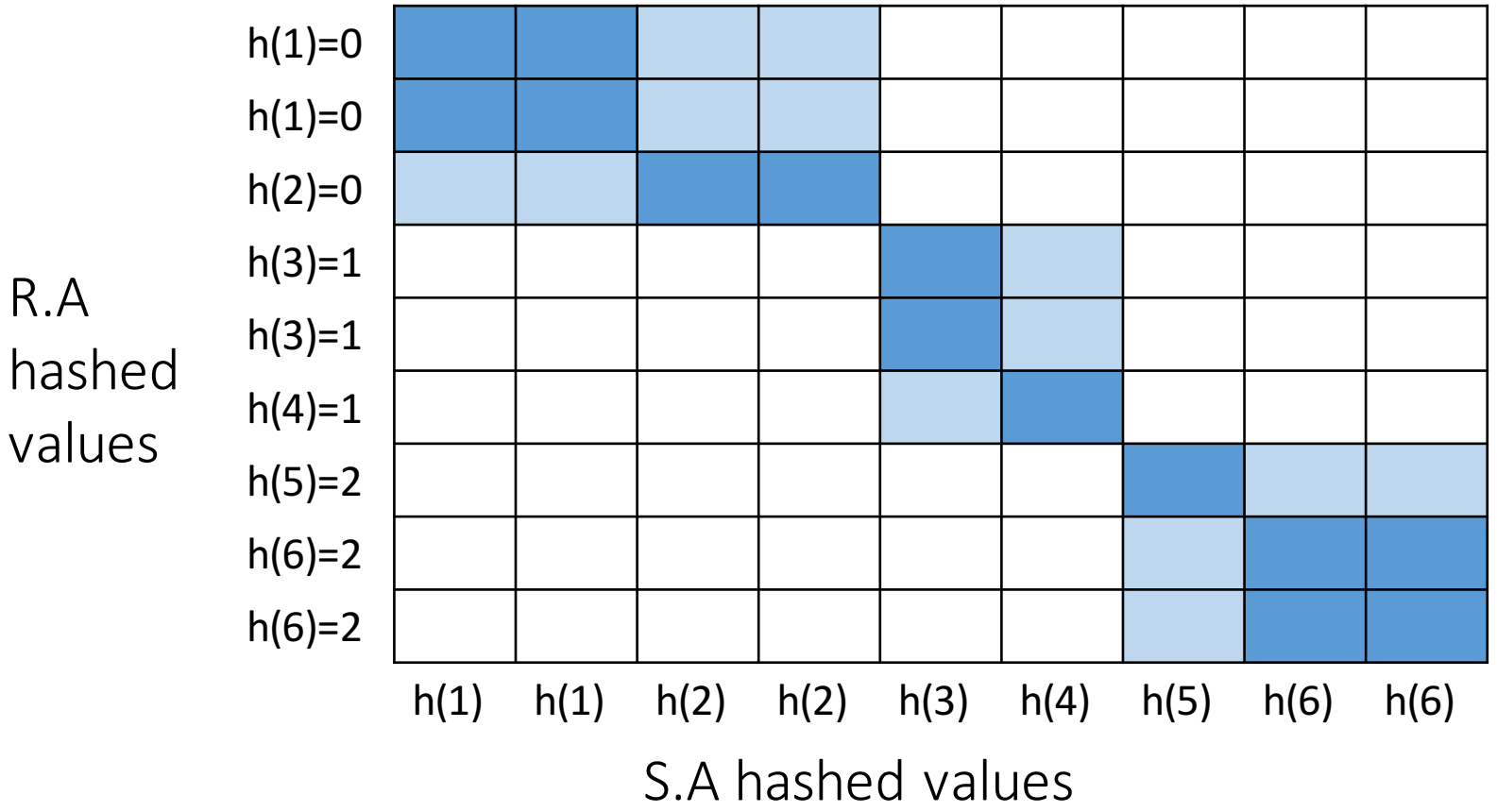
- Note that since x = y → h(x) = h(y), we only need to consider pairs of buckets (one from R, one from S) that have the same hash function value

- If our buckets are $\sim \boldsymbol{B-1}$ **pages,** can join each such pair using BNLJ ***in linear time***; recall (with P(R) = B-1):

BNLJ Cost: $\mathrm{P}(R) + \frac{P(R)P(S)}{B-1} = P(R) + \frac{(B-1)P(S)}{B-1}$ = P(R) + P(S)

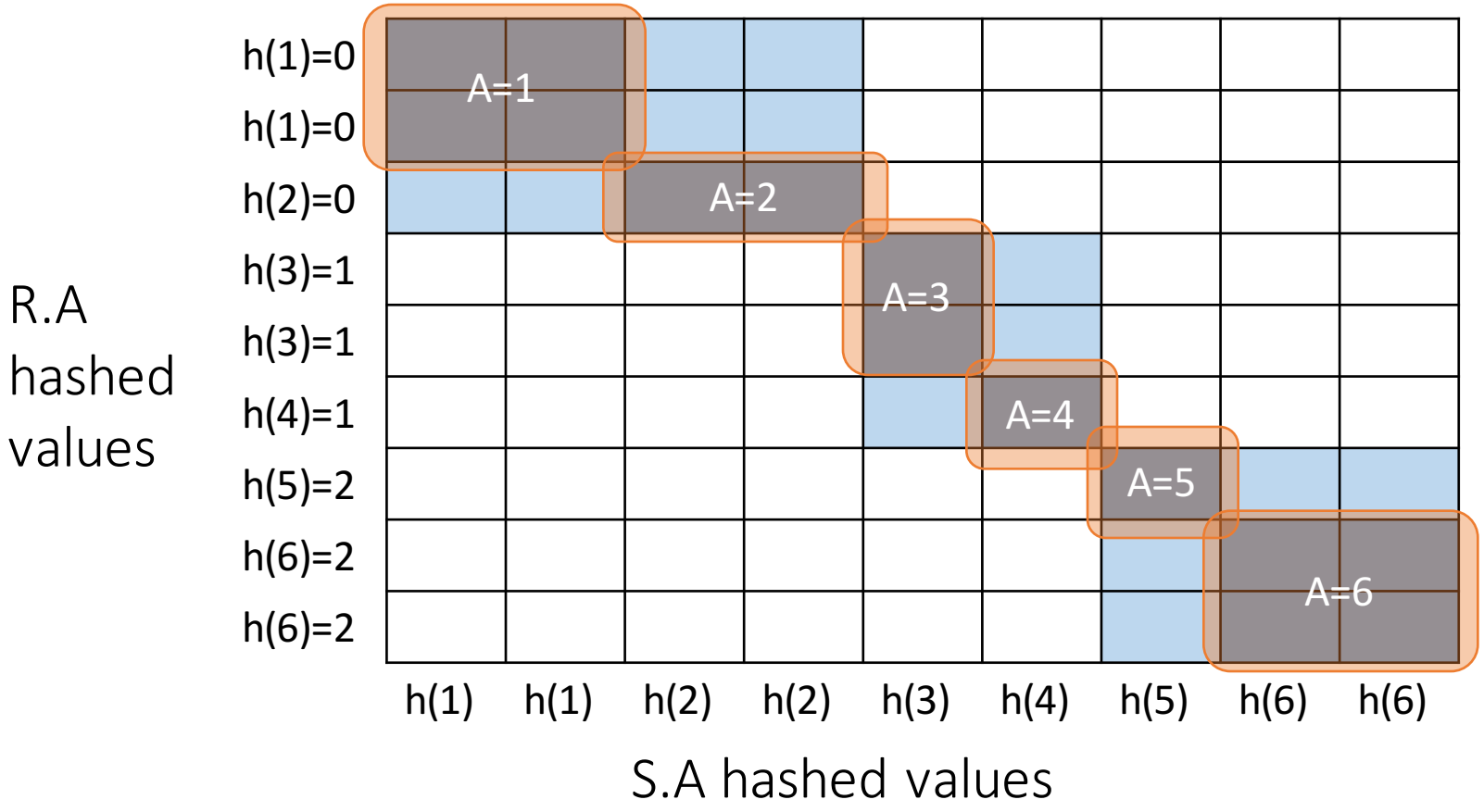
Joining the pairs of buckets is linear!
(As long as smaller bucket <= B-1 pages)

Hash Join Phase 2: Matching



$\mathrm{R} \bowtie S \; on \; A$

Hash Join Phase 2: Matching

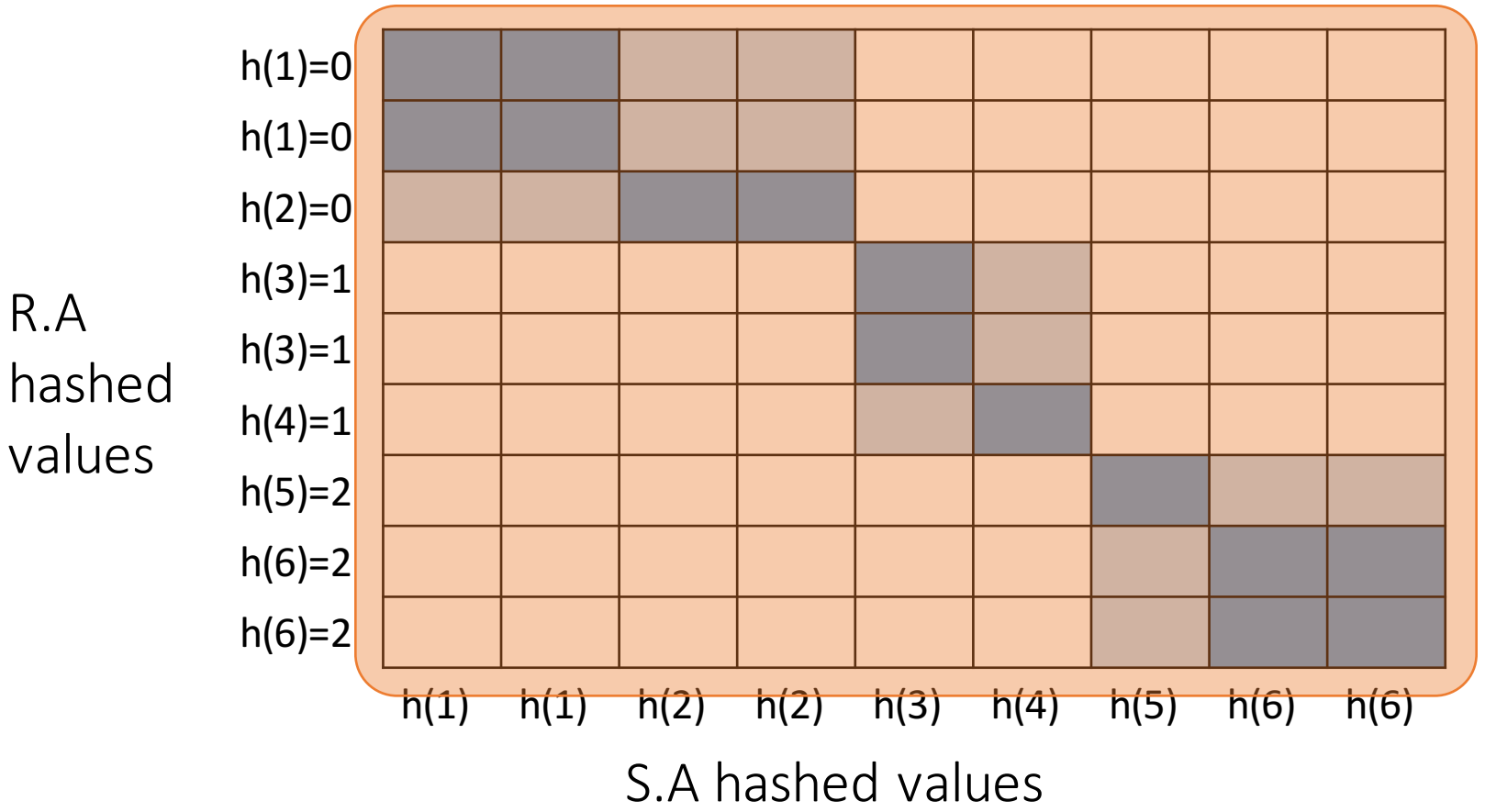


$R \bowtie S \; on \; A$

To perform the join, we ideally just need to explore the dark blue regions

= the tuples with same values of the join key A

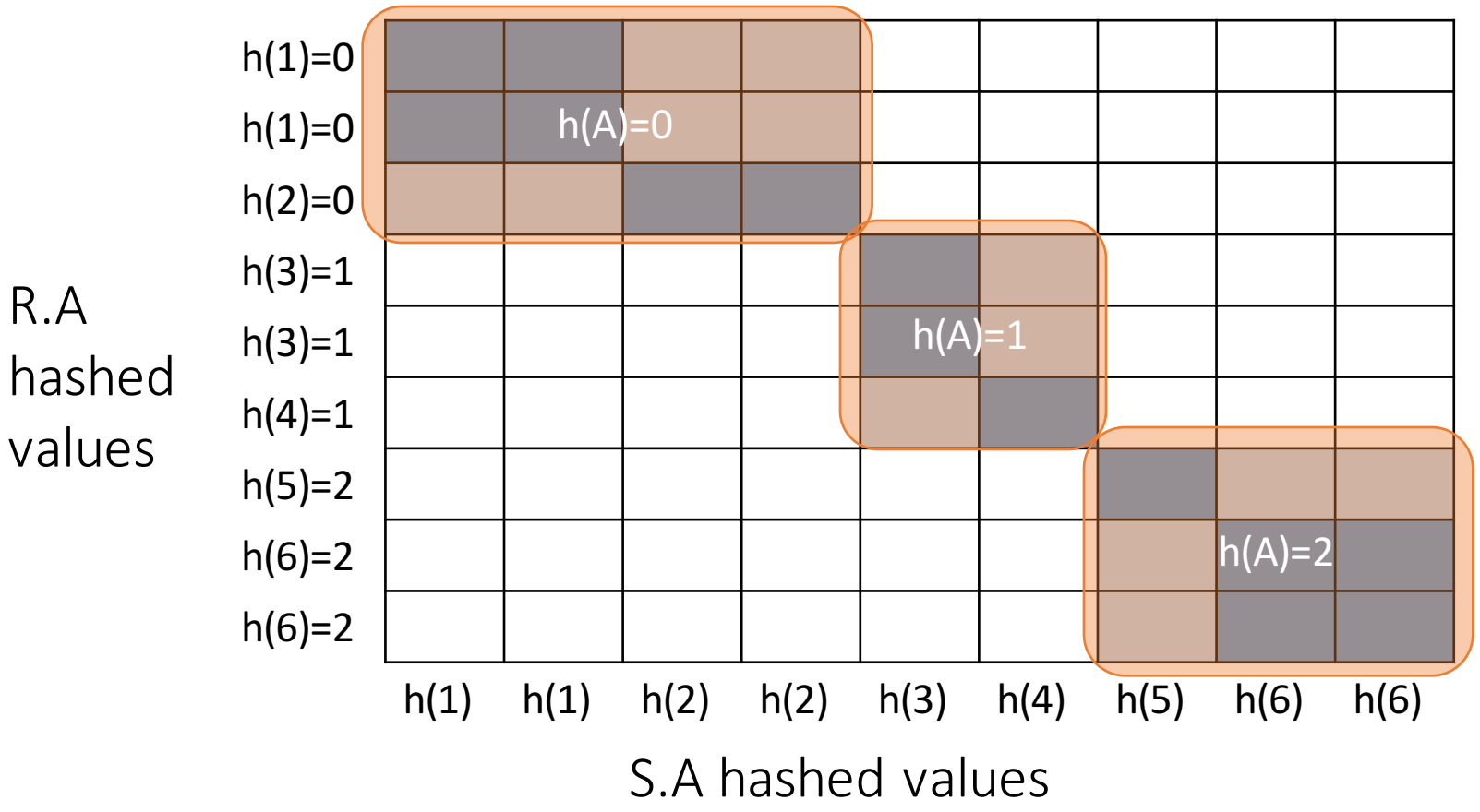
Hash Join Phase 2: Matching



$R \bowtie S \; on \; A$

With a join algorithm like BNLJ that doesn't take advantage of equijoin structure, we'd have to explore this ***whole grid!***

Hash Join Phase 2: Matching



$R \bowtie S \; on \; A$

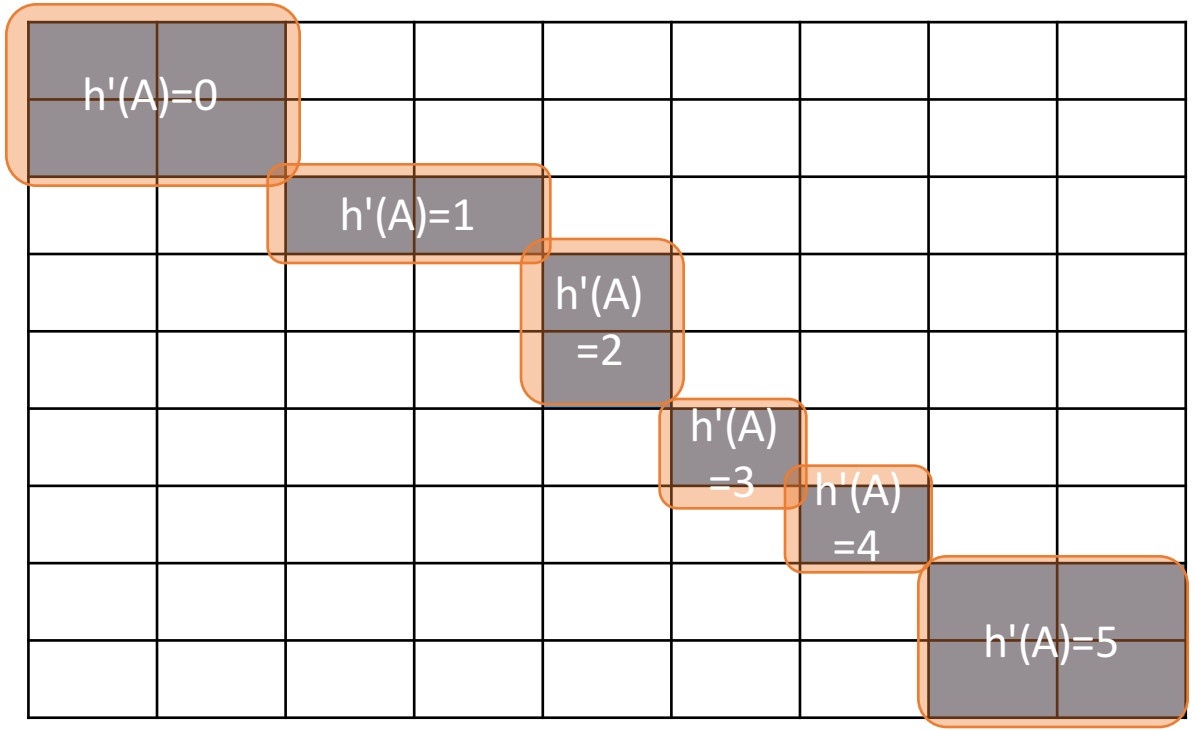
With HJ, we only explore the ***blue*** regions

= the tuples with same values of ***h(A)!***

We can apply BNLJ to each of these regions

Hash Join Phase 2: Matching

R.A hashed values



S.A hashed values

$R \bowtie S \; on \; A$

An alternative to applying BNLJ:

We could also hash again, and keep doing passes in memory to reduce further!

How much memory do we need for HJ?

- Given B+1 buffer pages

+ WLOG: Assume P(R) <= P(S)

- Suppose (reasonably) that we can partition into B buckets in 2 passes:
  - For R, we get B buckets of size ~P(R)/B
  - To join these buckets in linear time, we need these buckets to fit in B-1 pages, so we have:

$$B - 1 \geq \frac{P(R)}{B} \Rightarrow \sim \boldsymbol{B^2} \geq \boldsymbol{P(R)}$$

Quadratic relationship between ***smaller relation's*** size & memory!

Hash Join Summary

- *Given enough buffer pages as on previous slide…*
  - **Partitioning** requires reading + writing each page of R,S
    - → 2(P(R)+P(S)) IOs
  - **Matching** (with BNLJ) requires reading each page of R,S
    - → P(R) + P(S) IOs
  - **Writing out results** could be as bad as P(R)*P(S)… but probably closer to P(R)+P(S)

HJ takes **~3(P(R)+P(S)) + *OUT*** IOs!

3. The Cage Match

Sort-Merge v. Hash Join

- ***Given enough memory***, both SMJ and HJ have performance:

$$\sim 3(P(R)+P(S)) + OUT$$

- ***"Enough" memory =***
  - SMJ: $B^2 > \max\{P(R), P(S)\}$
  - HJ: $B^2 > \min\{P(R), P(S)\}$

Hash Join superior if relation sizes ***differ greatly***. Why?

Further Comparisons of Hash and Sort Joins

- Hash Joins are highly parallelizable.

- Sort-Merge less sensitive to data skew and result is sorted

Summary

- Saw IO-aware join algorithms
  - Massive difference
- Memory sizes key in hash versus sort join
  - Hash Join = Little dog (depends on smaller relation)
- Skew is also a major factor