# The Relational Model

# Today's Lecture

1. The Relational Model & Relational Algebra
2. Plan Optimization
3. Relational Algebra Pt. II *[Optional: may skip]*

# 1. The Relational Model & Relational Algebra

# What you will learn about in this section

1. The Relational Model
2. Relational Algebra: Basic Operators
3. Execution
4. ACTIVITY: From SQL to RA & Back

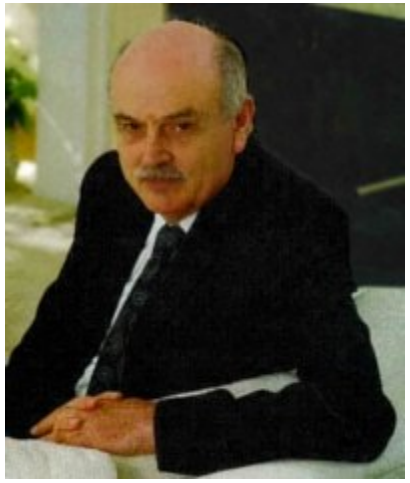
# Motivation

The Relational model is **precise**, **implementable**, and we can operate on it (query/update, etc.)

Database maps internally into this *procedural language*.

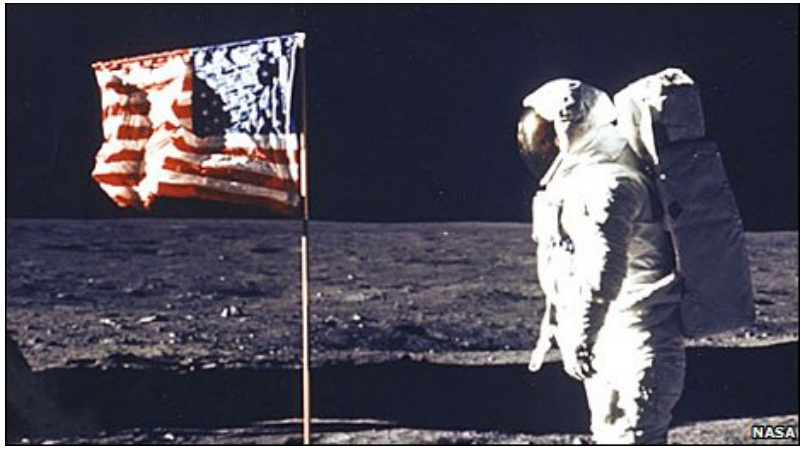
# A Little History

- Relational model due to Edgar “Ted” Codd, a mathematician at IBM in 1970
  - A Relational Model of Data for Large Shared Data Banks". *Communications of the ACM* **13** (6): 377–387



Won Turing award 1981

- IBM didn’t want to use relational model (take money from IMS)
  - *Apparently used in the moon landing…*



# The Relational Model: Schemata

- Relational Schema:


Relation name

*String, float, int, etc.* are the **domains** of the attributes

Attributes

# The Relational Model: Data

An **attribute** (or **column**) is a typed data entry present in each tuple in the relation

**Student**

| sid | name | gpa |
|---|---|---|
| 001 | Bob | 3.2 |
| 002 | Joe | 2.8 |
| 003 | Mary | 3.8 |
| 004 | Alice | 3.5 |

The number of attributes is the **arity** of the relation

# The Relational Model: Data

**Student**

| sid | name | gpa |
|---|---|---|
| 001 | Bob | 3.2 |
| 002 | Joe | 2.8 |
| 003 | Mary | 3.8 |
| 004 | Alice | 3.5 |

The number of tuples is the **cardinality** of the relation

A **tuple** or **row** (or *record)* is a single entry in the table having the attributes specified by the schema

# The Relational Model: Data

**Student**

| sid | name | gpa |
|---|---|---|
| 001 | Bob | 3.2 |
| 002 | Joe | 2.8 |
| 003 | Mary | 3.8 |
| 004 | Alice | 3.5 |

Recall: In practice DBMSs relax the set requirement, and use multisets.

A **relational instance** is a ***set*** of tuples all conforming to the same *schema*

# To Reiterate

- A *relational schema* describes the data that is contained in a *relational instance*

Let $R(f_1:Dom_1,\ldots,f_m:Dom_m)$ be a *relational schema* then, an *instance* of R is a subset of $Dom_1 \times Dom_2 \times \ldots \times Dom_n$

In this way, a *relational schema* R is a **total function from attribute *names* to types**

# One More Time

- A *relational schema* describes the data that is contained in a *relational instance*

A relation R of arity *t* is a function:
$R : Dom_1 \times \ldots \times Dom_t \rightarrow \{0,1\}$

*I.e. returns whether or not a tuple of matching types is a member of it*

Then, the schema is simply the *signature* of the function

Note here that order matters, attribute name doesn't…
We'll (mostly) work with the other model (last slide) in which **attribute name matters, order doesn't!**

# A relational database

- A *relational database schema* is a set of relational schemata, one for each relation

- A *relational database instance* is a set of relational instances, one for each relation

Two conventions:
1. We call relational database instances as simply ***databases***
2. We assume all instances are valid, i.e., satisfy the *domain constraints*

# Remember the CMS

- *Relation DB Schema*
  - Students(sid: *string*, name: *string*, gpa: *float*)
  - Courses(cid: *string*, cname: *string*, credits: *int*)
  - Enrolled(sid: *string,* cid*: string,* grade*: string*)

*Note that the schemas impose effective domain / type constraints, i.e. Gpa can't be "Apple"*

Relation Instances

| Sid | Name | Gpa |
|---|---|---|
| 101 | Bob | 3.2 |
| 123 | Mary | 3.8 |

Students

| sid | cid | Grade |
|---|---|---|
| 123 | 564 | A |

Enrolled

| cid | cname | credits |
|---|---|---|
| 564 | 564-2 | 4 |
| 308 | 417 | 2 |

Courses

# 2nd Part of the Model: Querying

```
SELECT S.name
FROM Students S
WHERE S.gpa > 3.5;
```

We don't tell the system *how* or *where* to get the data- **just what we want**, **i.e., Querying is *declarative***

*"Find names of all students with GPA > 3.5"*

To make this happen, we need to translate the *declarative* query into a series of operators… we'll see this next!

Actually, I showed how to do this translation for a much richer language!

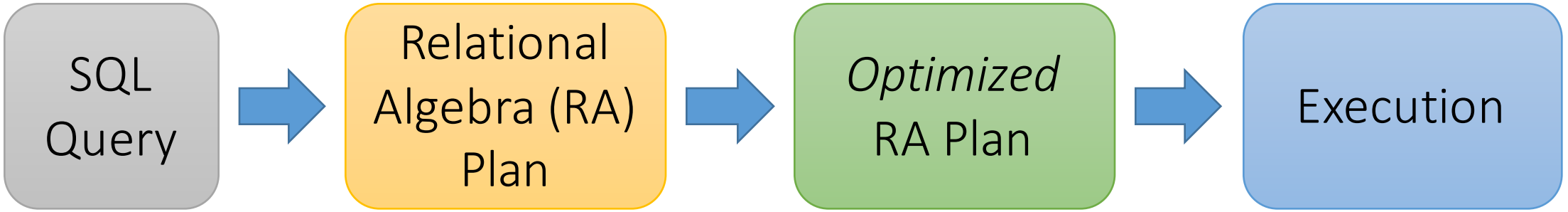
# Virtues of the model

- Physical independence (logical too), Declarative
- Simple, elegant clean: Everything is a relation
- Why did it take multiple years?
  - Doubted it could be done *efficiently*.

# Relational Algebra

# RDBMS Architecture

## How does a SQL engine work ?




Declarative query (from user)

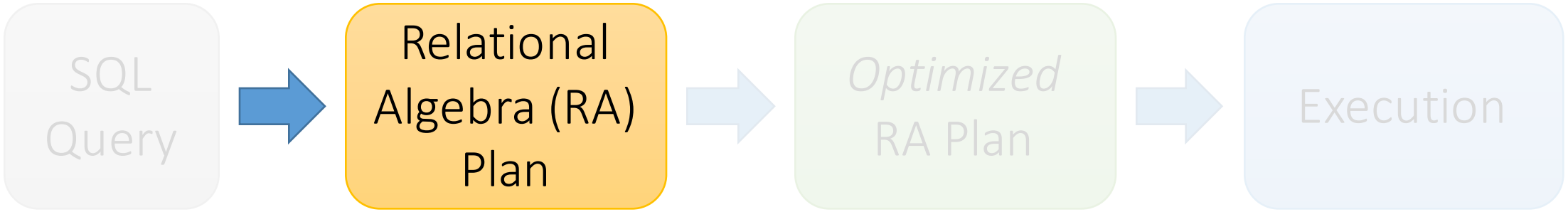
Translate to relational algebra expresson

*Find logically equivalent- but more efficient- RA expression*

Execute each operator of the optimized plan!

# RDBMS Architecture

## How does a SQL engine work ?




Relational Algebra allows us to translate declarative (SQL) queries into precise and optimizable expressions!

# Relational Algebra (RA)

- Five **basic** operators:
  1. Selection: $\sigma$
  2. Projection: $\Pi$
  3. Cartesian Product: $\times$
  4. Union: $\cup$
  5. Difference: -
- Derived or auxiliary operators:
  - Intersection, complement
  - Joins (natural, equi-join, theta join, semi-join)
  - Renaming: $\rho$
  - Division

We'll look at these first!

*And also at one example of a derived operator (natural join) and a special operator (renaming)*

# Keep in mind: RA operates on sets!

- RDBMSs use *multisets*, however in relational algebra formalism we will consider **sets!**

- Also: we will consider the ***named perspective***, where every attribute must have a unique name
  - →attribute order does not matter…

Now on to the basic RA operators…

# 1. Selection ($\sigma$)

- Returns all tuples which satisfy a condition
- Notation: $\sigma_c(R)$
- Examples
  - $\sigma_{Salary > 40000}$ (Employee)
  - $\sigma_{name = \text{“Smith”}}$ (Employee)
- The condition c can be =, <, ≤, >, ≥, <>

```
Students(sid,sname,gpa)
```

*SQL:*

```
SELECT *
FROM Students
WHERE gpa > 3.5;
```

*RA:*

$$\sigma_{gpa > 3.5}(Students)$$

Another example:

| SSN | Name | Salary |
|---|---|---|
| 1234545 | John | 200000 |
| 5423341 | Smith | 600000 |
| 4352342 | Fred | 500000 |

$\sigma_{Salary > 40000}$ (Employee)

| SSN | Name | Salary |
|---|---|---|
| 5423341 | Smith | 600000 |
| 4352342 | Fred | 500000 |

# 2. Projection ($\Pi$)

- Eliminates columns, then removes duplicates
- Notation: $\Pi_{A1,\ldots,An}(R)$
- Example: project social-security number and names:
  - $\Pi_{SSN,\ Name}$ (Employee)
  - Output schema: Answer(SSN, Name)

```
Students(sid,sname,gpa)
```

*SQL:*

```
SELECT DISTINCT
   sname,
   gpa
FROM Students;
```

*RA:*

$$\Pi_{sname,gpa}(Students)$$

Another example:

| SSN | Name | Salary |
|---|---|---|
| 1234545 | John | 200000 |
| 5423341 | John | 600000 |
| 4352342 | John | 200000 |

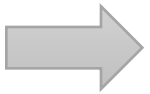
$\Pi_{Name,Salary}$ (Employee)

| Name | Salary |
|---|---|
| John | 200000 |
| John | 600000 |

# Note that RA Operators are Compositional!

```
Students(sid,sname,gpa)
```

```
SELECT DISTINCT
   sname,
   gpa
FROM Students
WHERE gpa > 3.5;
```



$$\Pi_{sname,gpa}(\sigma_{gpa>3.5}(Students))$$

$$\sigma_{gpa>3.5}(\Pi_{sname,gpa}(Students))$$

How do we represent this query in RA?

Are these logically equivalent?

# 3. Cross-Product (×)

- Each tuple in R1 with each tuple in R2
- Notation: R1 × R2
- Example:
  - Employee × Dependents
- Rare in practice; mainly used to express joins

```
Students(sid,sname,gpa)
People(ssn,pname,address)
```

*SQL:*

```
SELECT *
FROM Students, People;
```

*RA:*

$$Students \times People$$

Another example:

People

| ssn | pname | address |
|---|---|---|
| 1234545 | John | 216 Rosse |
| 5423341 | Bob | 217 Rosse |

$\times$

Students

| sid | sname | gpa |
|---|---|---|
| 001 | John | 3.4 |
| 002 | Bob | 1.3 |

$$Students \times People$$

| ssn | pname | address | sid | sname | gpa |
|---|---|---|---|---|---|
| 1234545 | John | 216 Rosse | 001 | John | 3.4 |
| 5423341 | Bob | 217 Rosse | 001 | John | 3.4 |
| 1234545 | John | 216 Rosse | 002 | Bob | 1.3 |
| 5423341 | Bob | 216 Rosse | 002 | Bob | 1.3 |

# Natural Join (⋈)

- Notation: $R_1 \bowtie R_2$
- Our first example of a *derived* RA operator:
  - Meaning: $R_1 \bowtie R_2 = \Pi_A(\sigma_C(R_1 \times R_2))$
- Where:
  - The selection $\sigma_C$ checks equality of all common attributes
  - The projection eliminates the duplicate common attributes

```
Students(sid,name,gpa)
People(ssn,name,address)
```

*SQL:*

```
SELECT DISTINCT
  ssid, S.name, gpa,
  ssn, address
FROM
  Students S,
  People P
WHERE S.name = P.name;
```

*RA:*

$$Students \bowtie People$$

Another example:

Students S

| sid | S.name | gpa |
|---|---|---|
| 001 | John | 3.4 |
| 002 | Bob | 1.3 |

$\bowtie$

People P

| ssn | P.name | address |
|---|---|---|
| 1234545 | John | 216 Rosse |
| 5423341 | Bob | 217 Rosse |

$$Students \bowtie People$$

| sid | S.name | gpa | ssn | address |
|---|---|---|---|---|
| 001 | John | 3.4 | 1234545 | 216 Rosse |
| 002 | Bob | 1.3 | 5423341 | 216 Rosse |

# Natural Join

- Given schemas R(A, B, C, D), S(A, C, E), what is the schema of R ⋈ S ?
- Given R(A, B, C), S(D, E), what is R ⋈ S ?
- Given R(A, B), S(A, B), what is R ⋈ S ?

# Renaming ($\rho$)

- Changes the schema, not the instance
- A 'special' operator- neither basic nor derived
- Notation: $\rho_{B1,\ldots,Bn}(R)$

- **Note: this is shorthand for the proper form (since names, not order matters!):**
  - $\rho_{A1 \rightarrow B1,\ldots,An \rightarrow Bn}(R)$

```
Students(sid,sname,gpa)
```

*SQL:*

```
SELECT
  sid AS studId,
  sname AS name,
  gpa AS gradePtAvg
FROM Students;
```

*RA:*

$$\rho_{studId,name,gradePtAvg}(Students)$$

We care about this operator *because* we are working in a *named perspective*

Another example:

Students

| sid | sname | gpa |
|---|---|---|
| 001 | John | 3.4 |
| 002 | Bob | 1.3 |

$$\rho_{studId,name,gradePtAvg}(Students)$$

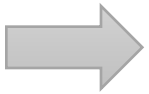
Students

| studId | name | gradePtAvg |
|---|---|---|
| 001 | John | 3.4 |
| 002 | Bob | 1.3 |

# Example: Converting SFW Query -> RA

```
Students(sid,sname,gpa)
People(ssn,sname,address)
```

```
SELECT DISTINCT
   gpa,
   address
FROM Students S,
      People P
WHERE gpa > 3.5 AND
    sname = pname;
```



$$\Pi_{gpa,address}(\sigma_{gpa>3.5}(S \bowtie P))$$

How do we represent this query in RA?

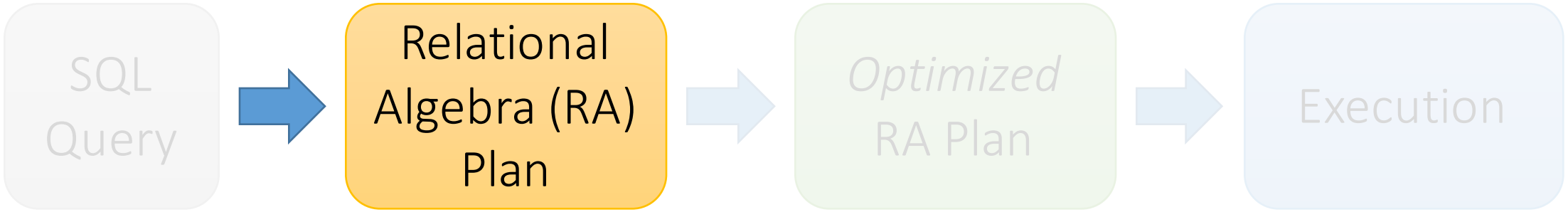
# Logical Equivalece of RA Plans

- Given relations R(A,B) and S(B,C):
  - Here, projection & selection commute:
    - $\sigma_{A=5}(\Pi_A(R)) = \Pi_A(\sigma_{A=5}(R))$
  - What about here?
    - $\sigma_{A=5}(\Pi_B(R)) \,?= \Pi_B(\sigma_{A=5}(R))$

We'll look at this in more depth later in the lecture…

# RDBMS Architecture

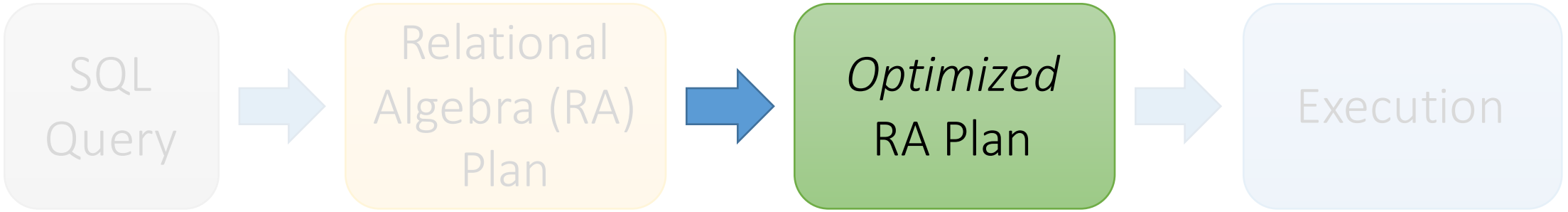
## How does a SQL engine work ?




We saw how we can transform declarative SQL queries into precise, compositional RA plans

# RDBMS Architecture

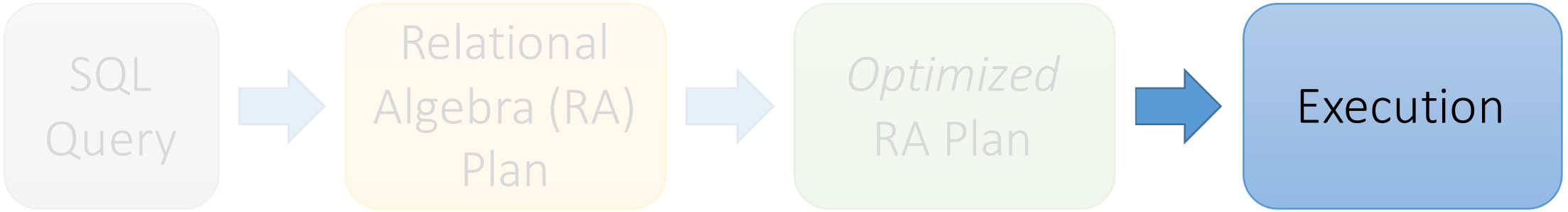
## How does a SQL engine work ?




We'll look at how to then optimize these plans later in this lecture

# RDBMS Architecture

How is the RA "plan" executed?




We already know how to execute all the basic operators!

# RA Plan Execution

- Natural Join / Join:
  - We saw how to use **memory & IO cost considerations to pick the correct algorithm to execute a join with (BNLJ, SMJ, HJ…)!**
- Selection:
  - We saw how to use **indexes to aid selection**
  - Can always fall back on scan / binary search as well
- Projection:
  - The main operation here is finding *distinct* values of the project tuples; we briefly discussed how to do this with e.g. **hashing** or **sorting**

We already know how to execute all the basic operators!

# Activity-16-1.ipynb

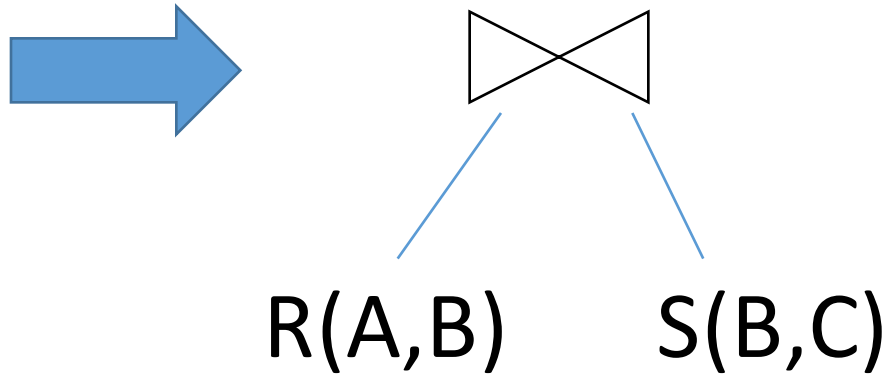
# 2. Optimization of RA Plans

# What you will learn about in this section

1. Optimization of RA Plans
2. ACTIVITY: RA Plan Optimization

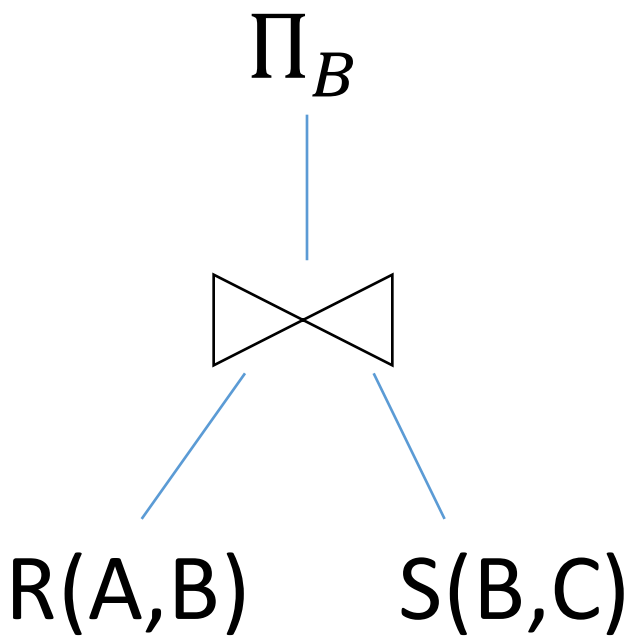
# Note: We can visualize the plan as a tree

$$\Pi_B(R(A,B) \bowtie S(B,C))$$




"bottom-up evaluation"

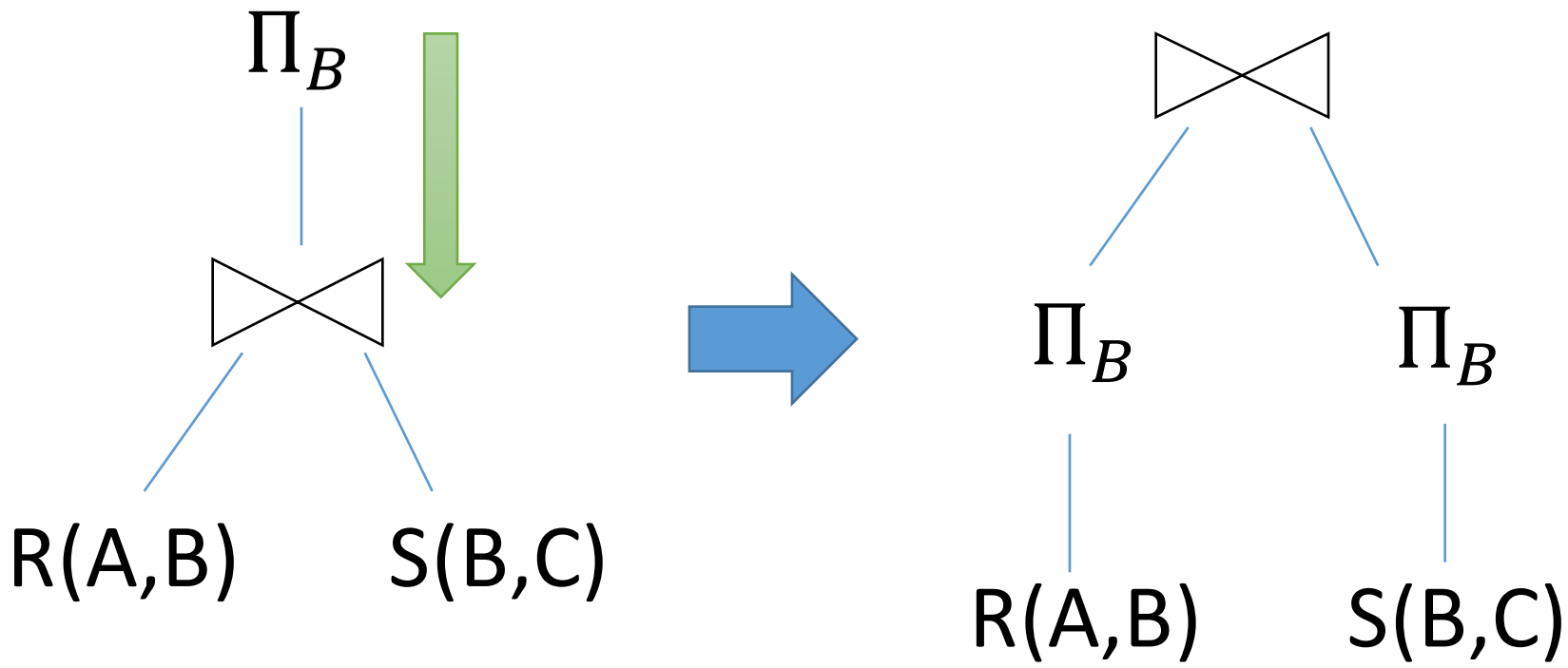
# A simple plan




What SQL query does this correspond to?

Are there any logically equivalent RA expressions?

# "Pushing down" projection




Why might we prefer this plan?

# Takeaways

- This process is called **logical optimization**
- Many equivalent plans used to search for “good plans”
- Relational algebra is an important abstraction.

# RA commutators

- The basic commutators:
  - Push **projection** through **(1) selection**, **(2) join**
  - Push **selection** through **(3) selection, (4) projection, (5) join**
  - *Also:* Joins an be re-ordered!

- Note that this is not an exhaustive set of operations
  - This covers *local re-writes; global re-writes possible but much harder*

This simple set of tools allows us to greatly improve the execution time of queries by optimizing RA plans!

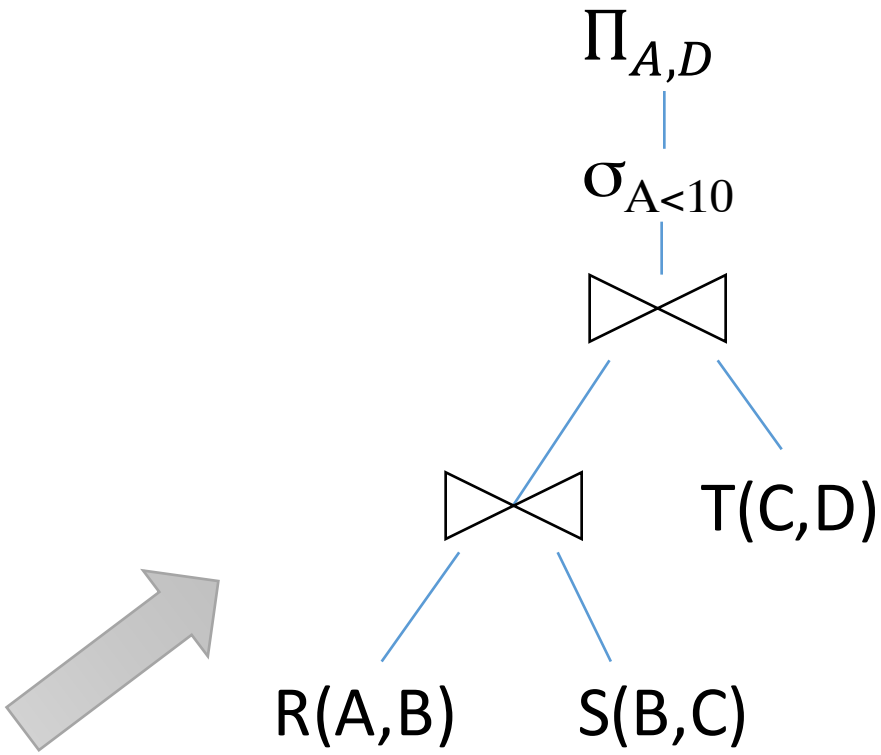
# Optimizing the SFW RA Plan

# Translating to RA

```
R(A,B)  S(B,C)  T(C,D)
```

```
SELECT DISTINCT R.A,S.D
FROM R,S,T
WHERE R.B = S.B
  AND S.C = T.C
  AND R.A < 10;
```

$$\Pi_{A,D}(\sigma_{A<10}(T \bowtie (R \bowtie S)))$$



# Logical Optimization

- Heuristically, we want selections and projections to occur as early as possible in the plan
  - Terminology: “push down **selections**” and “pushing down **projections.”**

- **Intuition:** We will have fewer tuples in a plan.
  - Could fail if the selection condition is very expensive (say runs some image processing algorithm).
  - Projection could be a waste of effort, but more rarely.

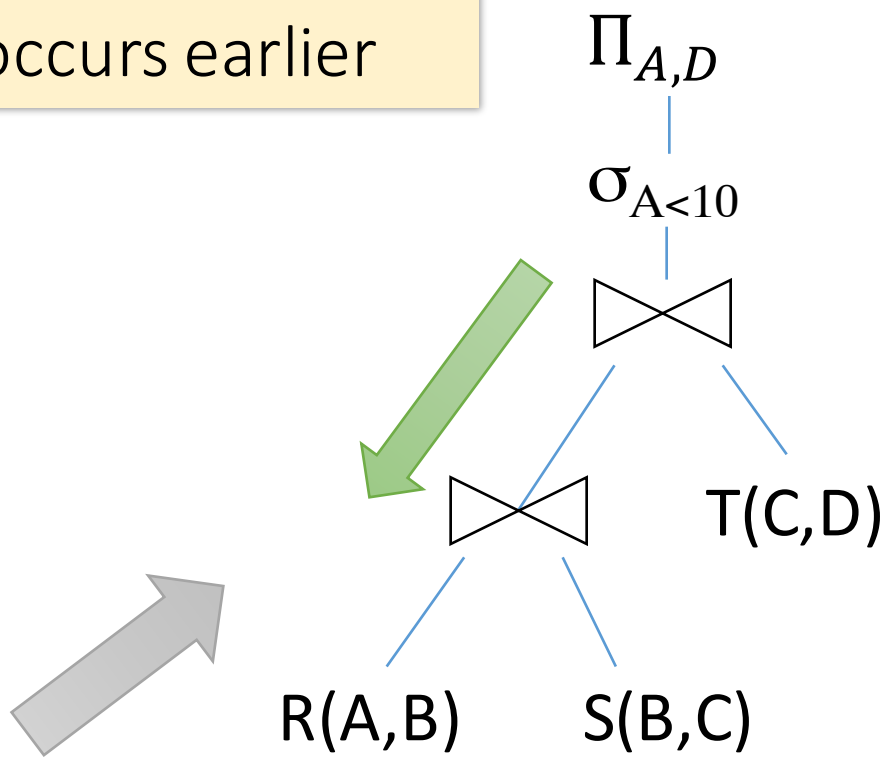
# Optimizing RA Plan

```
R(A,B)  S(B,C)  T(C,D)
```

```
SELECT DISTINCT R.A,S.D
FROM R,S,T
WHERE R.B = S.B
  AND S.C = T.C
  AND R.A < 10;
```

Push down selection on A so it occurs earlier

$$\Pi_{A,D}(\sigma_{A<10}(T \bowtie (R \bowtie S)))$$



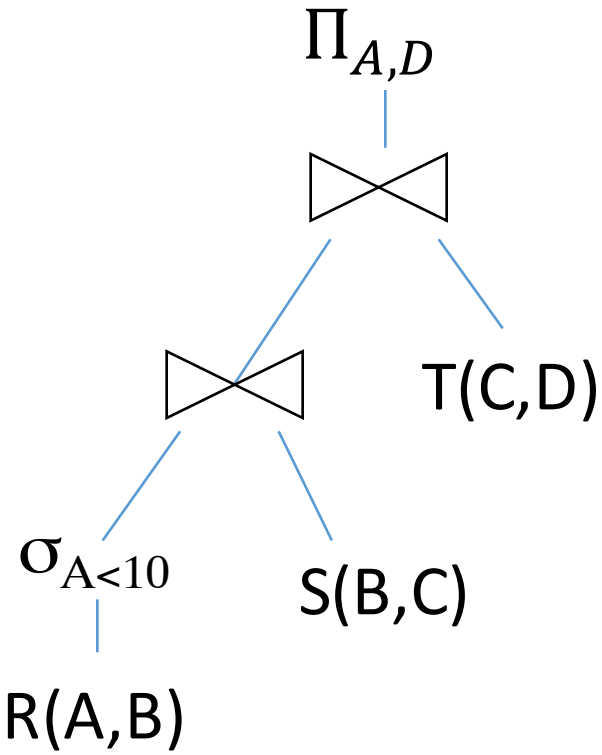
# Optimizing RA Plan

```
R(A,B)  S(B,C)  T(C,D)
```

```
SELECT DISTINCT R.A,S.D
FROM R,S,T
WHERE R.B = S.B
  AND S.C = T.C
  AND R.A < 10;
```

Push down selection on A so it occurs earlier

$$\Pi_{A,D}\left(T \bowtie \left(\sigma_{A<10}(R) \bowtie S\right)\right)$$



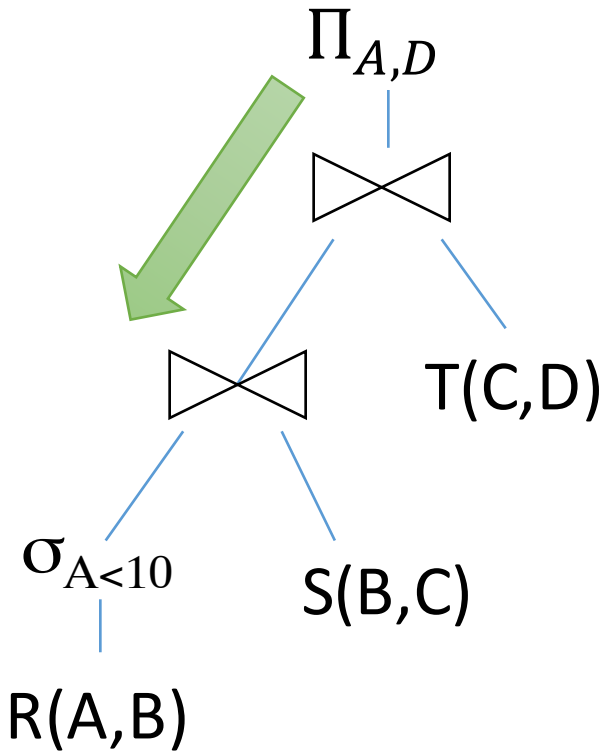
# Optimizing RA Plan

```
R(A,B)  S(B,C)  T(C,D)
```

```
SELECT DISTINCT R.A,S.D
FROM R,S,T
WHERE R.B = S.B
  AND S.C = T.C
  AND R.A < 10;
```

Push down projection so it occurs earlier

$$\Pi_{A,D}\left(T \bowtie \left(\sigma_{A<10}(R) \bowtie S\right)\right)$$



# Optimizing RA Plan

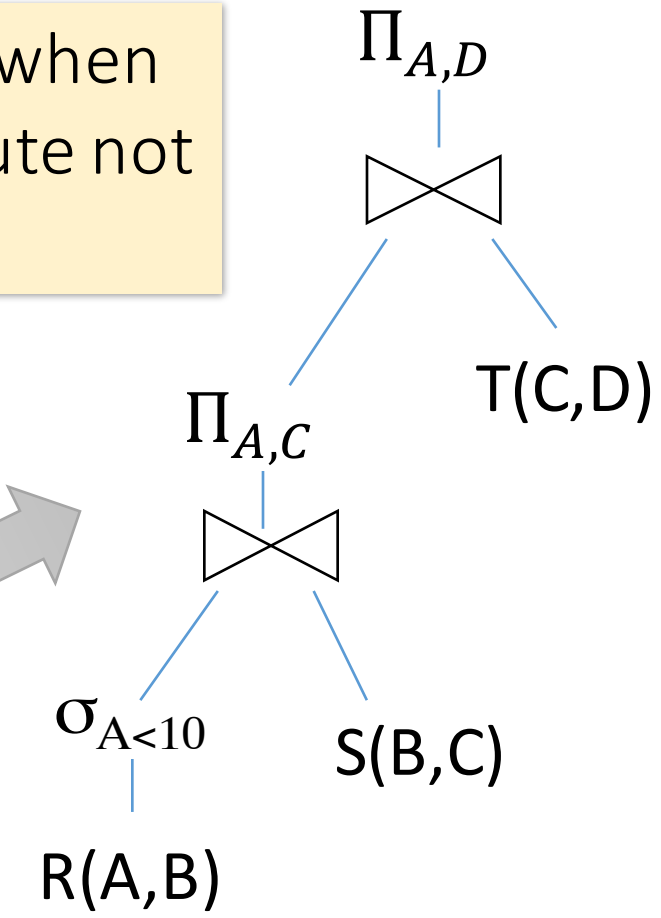
```
R(A,B)  S(B,C)  T(C,D)
```

```
SELECT DISTINCT R.A,S.D
FROM R,S,T
WHERE R.B = S.B
  AND S.C = T.C
  AND R.A < 10;
```

We eliminate B earlier!

In general, when is an attribute not needed…?

$$\Pi_{A,D}\left(T \bowtie \Pi_{A,C}(\sigma_{A<10}(R) \bowtie S)\right)$$



# Activity-16-2.ipynb

# 3. Adv. Relational Algebra

# What you will learn about in this section

1. Set Operations in RA
2. Fancier RA
3. Extensions & Limitations

# Relational Algebra (RA)

- Five **basic** operators:
  1. Selection: $\sigma$
  2. Projection: $\Pi$
  3. Cartesian Product: $\times$
  4. Union: $\cup$
  5. Difference: -

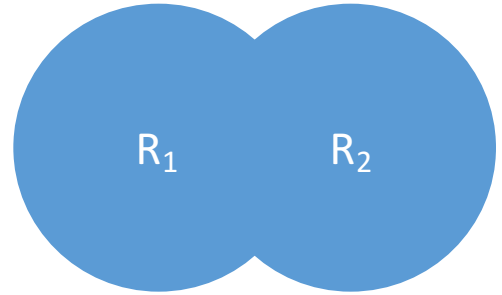
We'll look at these

- Derived or auxiliary operators:
  - Intersection, complement
  - Joins (natural,equi-join, theta join, semi-join)
  - Renaming: $\rho$
  - Division

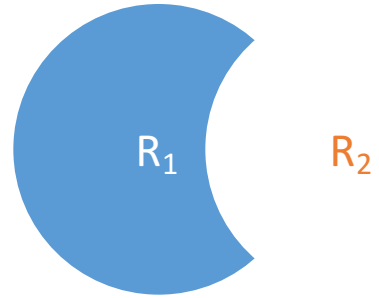
*And also at some of these derived operators*

# 1. Union (∪) and 2. Difference (–)

- R1 ∪ R2
- Example:
  - ActiveEmployees ∪ RetiredEmployees

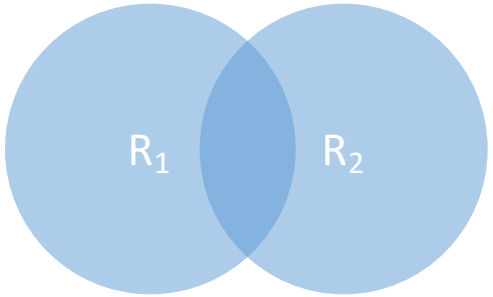



- R1 – R2
- Example:
  - AllEmployees -- RetiredEmployees



# What about Intersection (∩) ?

- It is a derived operator
- R1 ∩ R2 = R1 – (R1 – R2)
- Also expressed as a join!
- Example
  - UnionizedEmployees ∩ RetiredEmployees



# Fancier RA

# Theta Join ($\bowtie_\theta$)

- A join that involves a predicate
- R1 $\bowtie_\theta$ R2 = $\sigma_\theta$ (R1 × R2)
- Here $\theta$ can be any condition

Note that natural join is a theta join + a projection.

```
Students(sid,sname,gpa)
People(ssn,pname,address)
```

*SQL:*

```
SELECT *
FROM
  Students,People
WHERE θ;
```

*RA:*

$$Students \bowtie_\theta People$$

# Equi-join ($\bowtie_{A=B}$)

- A theta join where $\theta$ is an equality
- $R1 \bowtie_{A=B} R2 = \sigma_{A=B}(R1 \times R2)$
- Example:
  - Employee $\bowtie_{SSN=SSN}$ Dependents

Most common join in practice!

```
Students(sid,sname,gpa)
People(ssn,pname,address)
```

*SQL:*

```
SELECT *
FROM
  Students S,
  People P
WHERE sname = pname;
```

*RA:*

$$S \bowtie_{sname=pname} P$$

# Semijoin ($\ltimes$)

- $R \ltimes S = \Pi_{A1,\ldots,An}(R \bowtie S)$
- Where $A_1, \ldots, A_n$ are the attributes in R
- Example:
  - Employee $\ltimes$ Dependents

```
Students(sid,sname,gpa)
People(ssn,pname,address)
```

*SQL:*

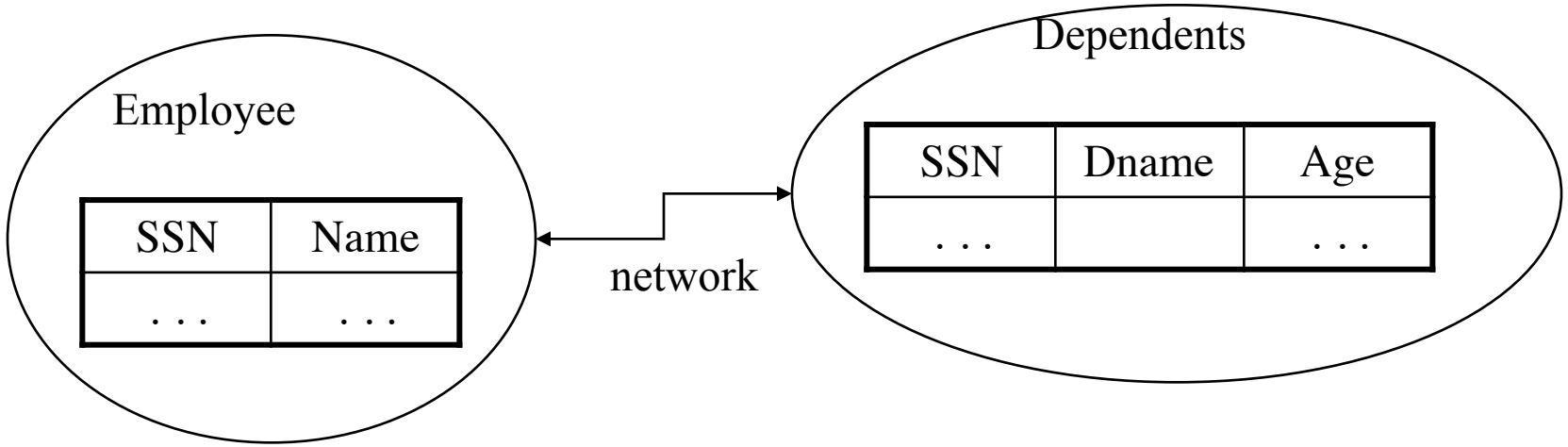
```
SELECT DISTINCT
  sid,sname,gpa
FROM
  Students,People
WHERE
  sname = pname;
```

*RA:*

$$Students \ltimes People$$

# Semijoins in Distributed Databases

- Semijoins are often used to compute natural joins in distributed databases




Send less data to reduce network bandwidth!

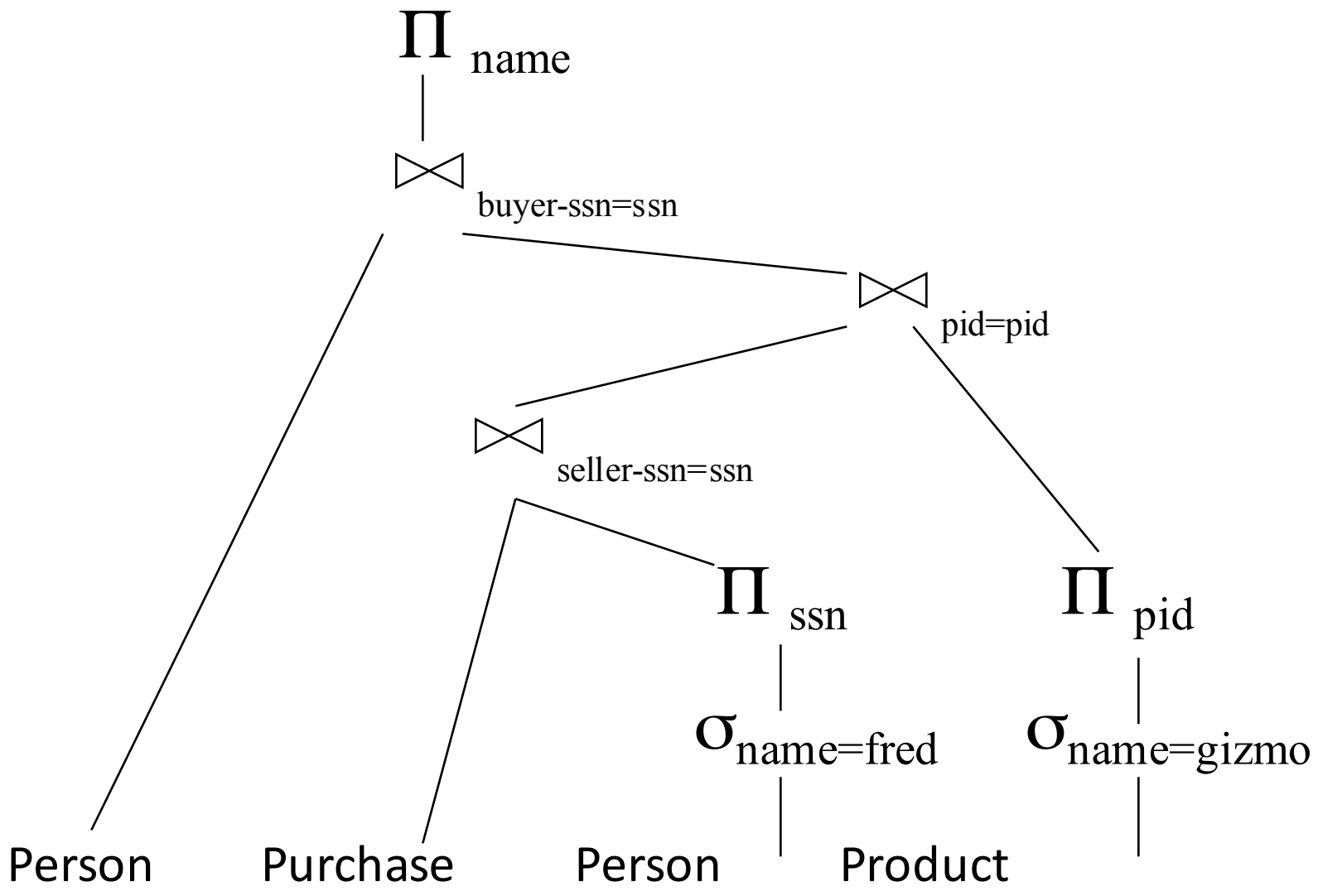
$$\text{Employee} \bowtie_{ssn=ssn} (\sigma_{age>71} (\text{Dependents}))$$

$$T = \Pi_{SSN} \sigma_{age>71} (\text{Dependents})$$

$$R = \text{Employee} \ltimes T$$

$$\text{Answer} = R \bowtie \text{Dependents}$$

# RA Expressions Can Get Complex!



# Multisets

# Recall that SQL uses Multisets

$\boldsymbol{\lambda(X)}$= *"Count of tuple in X" (Items not listed have implicit count 0)*

Multiset X

| Tuple |
|---|
| (1, a) |
| (1, a) |
| (1, b) |
| (2, c) |
| (2, c) |
| (2, c) |
| (1, d) |
| (1, d) |

Equivalent Representations of a **Multiset**

Multiset X

| Tuple | $\boldsymbol{\lambda(X)}$ |
|---|---|
| (1, a) | 2 |
| (1, b) | 1 |
| (2, c) | 3 |
| (1, d) | 2 |

*Note: In a set all counts are {0,1}.*

# Generalizing Set Operations to Multiset Operations

Multiset X

| Tuple | $\lambda(X)$ |
|---|---|
| (1, a) | 2 |
| (1, b) | 0 |
| (2, c) | 3 |
| (1, d) | 0 |

$\cap$

Multiset Y

| Tuple | $\lambda(Y)$ |
|---|---|
| (1, a) | 5 |
| (1, b) | 1 |
| (2, c) | 2 |
| (1, d) | 2 |

$=$

Multiset Z

| Tuple | $\lambda(Z)$ |
|---|---|
| (1, a) | 2 |
| (1, b) | 0 |
| (2, c) | 2 |
| (1, d) | 0 |

$$\lambda(Z) = min(\lambda(X), \lambda(Y))$$

For sets, this is **intersection**

# Generalizing Set Operations to Multiset Operations

Multiset X

| Tuple | $\lambda(X)$ |
|---|---|
| (1, a) | 2 |
| (1, b) | 0 |
| (2, c) | 3 |
| (1, d) | 0 |

$\cup$

Multiset Y

| Tuple | $\lambda(Y)$ |
|---|---|
| (1, a) | 5 |
| (1, b) | 1 |
| (2, c) | 2 |
| (1, d) | 2 |

$=$

Multiset Z

| Tuple | $\lambda(Z)$ |
|---|---|
| (1, a) | 7 |
| (1, b) | 1 |
| (2, c) | 5 |
| (1, d) | 2 |

$$\lambda(Z) = \lambda(X) + \lambda(Y)$$

For sets,
this is **union**

# Operations on Multisets

All RA operations need to be defined carefully on bags

- $\sigma_C(R)$: preserve the number of occurrences
- $\Pi_A(R)$: no duplicate elimination
- Cross-product, join: no duplicate elimination

This is important- relational engines work on multisets, not sets!

# RA has Limitations !

- Cannot compute “transitive closure”

| Name1 | Name2 | Relationship |
|---|---|---|
| Fred | Mary | Father |
| Mary | Joe | Cousin |
| Mary | Bill | Spouse |
| Nancy | Lou | Sister |

- Find all direct and indirect relatives of Fred
- Cannot express in RA !!!
  - Need to write C program, use a graph engine, or modern SQL…