

Lectures 5 & 7: Design Theory

Lecture 5: Design Theory I

Today's Lecture

1. Normal forms & functional dependencies
 - ACTIVITY: Finding FDs
2. Finding functional dependencies
 - ACTIVITY: Compute the closures
3. Closures, superkeys & keys
 - ACTIVITY: The key or a key?

1. Normal forms & functional dependencies

What you will learn about in this section

1. Overview of design theory & normal forms
2. Data anomalies & constraints
3. Functional dependencies
4. ACTIVITY: Finding FDs

Design Theory

- Design theory is about how to represent your data to avoid ***anomalies***.
- It is a mostly mechanical process
 - Tools can carry out routine portions
- *We have a notebook implementing all algorithms!*
 - *We'll play with it in the activities!*

Normal Forms

- 1st Normal Form (1NF) = All tables are flat
- 2nd Normal Form = *disused*

- Boyce-Codd Normal Form (BCNF)
- 3rd Normal Form (3NF)

DB designs based on
functional dependencies,
intended to prevent
data *anomalies*

- 4th and 5th Normal Forms = see text books

*Our focus in
this lecture
+ next one*

1st Normal Form (1NF)

Student	Courses
Mary	{CS145,CS229}
Joe	{CS145,CS106}
...	...

Violates 1NF.

Student	Courses
Mary	CS145
Mary	CS229
Joe	CS145
Joe	CS106

In 1st NF

1NF Constraint: Types must be atomic!

Data Anomalies & Constraints

Constraints Prevent (some) Anomalies in the Data

A poorly designed database causes *anomalies*:

Student	Course	Room
Mary	CS145	B01
Joe	CS145	B01
Sam	CS145	B01
..

If every course is in only one room, contains redundant information!

Constraints Prevent (some) Anomalies in the Data

A poorly designed database causes *anomalies*:

Student	Course	Room
Mary	CS145	B01
Joe	CS145	C12
Sam	CS145	B01
..

If we update the room number for one tuple, we get inconsistent data = an *update anomaly*

Constraints Prevent (some) Anomalies in the Data

A poorly designed database causes *anomalies*:

Student	Course	Room
..

If everyone drops the class, we lose what room the class is in! = a **delete anomaly**

Constraints Prevent (some) Anomalies in the Data

A poorly designed database causes *anomalies*:

...	CS229	C12
-----	-------	-----

Student	Course	Room
Mary	CS145	B01
Joe	CS145	B01
Sam	CS145	B01
..

Similarly, we can't reserve a room without students
= an insert anomaly

Constraints Prevent (some) Anomalies in the Data

Student	Course
Mary	CS145
Joe	CS145
Sam	CS145
..	..

Course	Room
CS145	B01
CS229	C12

Is this form better?

- Redundancy?
- Update anomaly?
- Delete anomaly?
- Insert anomaly?

Today: develop theory to understand why this design may be better **and** how to find this *decomposition*...

Functional Dependencies

Functional Dependency

Def: Let A,B be sets of attributes

We write $A \rightarrow B$ or say A *functionally determines* B if, for any tuples t_1 and t_2 :

$$t_1[A] = t_2[A] \text{ implies } t_1[B] = t_2[B]$$

and we call $A \rightarrow B$ a functional dependency

$A \rightarrow B$ means that

“whenever two tuples agree on A then they agree on B.”

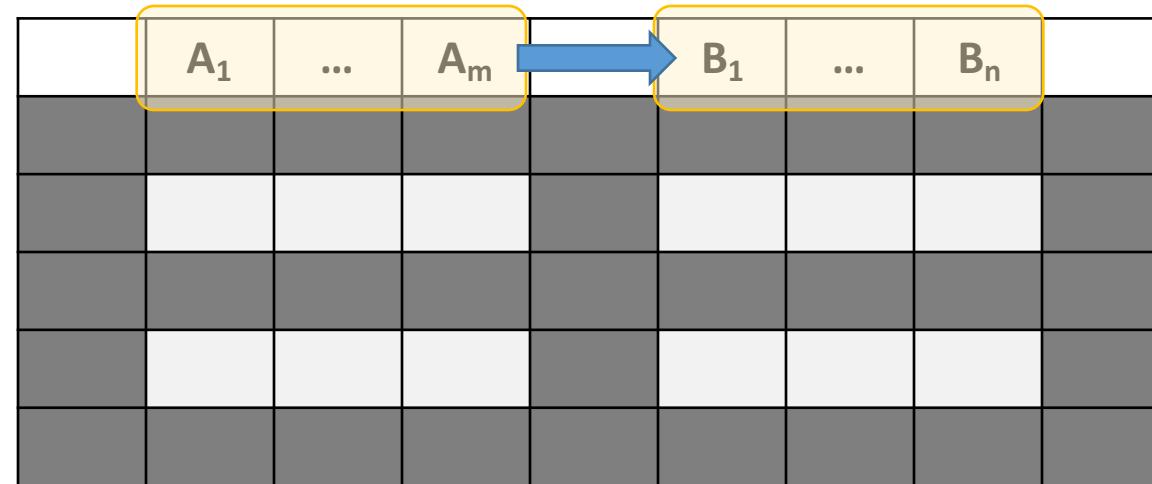
A Picture Of FDs

	A_1	...	A_m		B_1	...	B_n	

Defn (again):

Given attribute sets $A = \{A_1, \dots, A_m\}$ and $B = \{B_1, \dots, B_n\}$ in R ,

A Picture Of FDs



Defn (again):

Given attribute sets $A = \{A_1, \dots, A_m\}$ and $B = \{B_1, \dots, B_n\}$ in R ,

The *functional dependency* $A \rightarrow B$ on R holds if for *any* t_i, t_j in R :

A Picture Of FDs

	A_1	...	A_m		B_1	...	B_n	
t_i								
t_j								

If t_1, t_2 agree here..

Defn (again):

Given attribute sets $A = \{A_1, \dots, A_m\}$ and $B = \{B_1, \dots, B_n\}$ in R ,

The *functional dependency* $A \rightarrow B$ on R holds if for *any* t_i, t_j in R :

$$t_i[A_1] = t_j[A_1] \text{ AND } t_i[A_2] = t_j[A_2] \text{ AND } \dots \text{ AND } t_i[A_m] = t_j[A_m]$$

A Picture Of FDs

	A_1	...	A_m		B_1	...	B_n	
t_i								
t_j								

If t_1, t_2 agree here.. ...they also agree here!

Defn (again):

Given attribute sets $A = \{A_1, \dots, A_m\}$ and $B = \{B_1, \dots, B_n\}$ in R ,

The *functional dependency* $A \rightarrow B$ on R holds if for *any* t_i, t_j in R :

if $t_i[A_1] = t_j[A_1]$ AND $t_i[A_2] = t_j[A_2]$ AND ... AND $t_i[A_m] = t_j[A_m]$

then $t_i[B_1] = t_j[B_1]$ AND $t_i[B_2] = t_j[B_2]$ AND ... AND $t_i[B_n] = t_j[B_n]$

FDs for Relational Schema Design

- High-level idea: **why do we care about FDs?**
 1. Start with some relational *schema*
 2. Find out its *functional dependencies (FDs)*
 3. Use these to *design a better schema*
 1. One which minimizes the possibility of anomalies

Functional Dependencies as Constraints

A **functional dependency** is a form of **constraint**

- *Holds* on some instances not others.
- Part of the schema, helps define a valid *instance*.

Recall: an instance of a schema is a multiset of tuples conforming to that schema, i.e. a table

Student	Course	Room
Mary	CS145	B01
Joe	CS145	B01
Sam	CS145	B01
..

Note: The FD {Course} \rightarrow {Room} *holds on this instance*

Functional Dependencies as Constraints

Note that:

- You can check if an FD is **violated** by examining a single instance;
- However, you **cannot prove** that an FD is part of the schema by examining a single instance.
 - *This would require checking every valid instance*

Student	Course	Room
Mary	CS145	B01
Joe	CS145	B01
Sam	CS145	B01
..

However, cannot *prove* that the FD {Course} -> {Room} is *part of the schema*

More Examples

An FD is a constraint which holds, or does not hold on an instance:

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E3542	Mike	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234	Lawyer

More Examples

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E3542	Mike	9876 ←	Salesrep
E1111	Smith	9876 ←	Salesrep
E9999	Mary	1234	Lawyer

$$\{\text{Position}\} \rightarrow \{\text{Phone}\}$$

More Examples

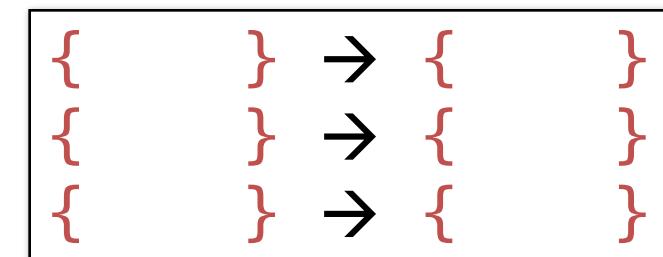
EmpID	Name	Phone	Position
E0045	Smith	1234 →	Clerk
E3542	Mike	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234 →	Lawyer

but *not* $\{\text{Phone}\} \rightarrow \{\text{Position}\}$

ACTIVITY

A	B	C	D	E
1	2	4	3	6
3	2	5	1	8
1	4	4	5	7
1	2	4	3	6
3	2	5	1	8

Find at least *three* FDs which hold on this instance:



2. Finding functional dependencies

What you will learn about in this section

1. “Good” vs. “Bad” FDs: Intuition
2. Finding FDs
3. Closures
4. ACTIVITY: Compute the closures

“Good” vs. “Bad” FDs

We can start to develop a notion of **good** vs. **bad** FDs:

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E3542	Mike	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234	Lawyer

Intuitively:

$\text{EmpID} \rightarrow \text{Name, Phone, Position}$ is “*good FD*”

- *Minimal redundancy, less possibility of anomalies*

“Good” vs. “Bad” FDs

We can start to develop a notion of **good** vs. **bad** FDs:

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E3542	Mike	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234	Lawyer

Intuitively:

$\text{EmpID} \rightarrow \text{Name, Phone, Position}$ is “*good FD*”

But $\text{Position} \rightarrow \text{Phone}$ is a “*bad FD*”

- *Redundancy!*
Possibility of data anomalies

“Good” vs. “Bad” FDs

Student	Course	Room
Mary	CS145	B01
Joe	CS145	B01
Sam	CS145	B01
..

Returning to our original example... can you see how the “bad FD” $\{\text{Course}\} \rightarrow \{\text{Room}\}$ could lead to an:

- Update Anomaly
- Insert Anomaly
- Delete Anomaly
- ...

Given a set of FDs (from user) our goal is to:

1. Find all FDs, and
2. Eliminate the “Bad Ones”.

FDs for Relational Schema Design

- High-level idea: **why do we care about FDs?**

1. Start with some relational *schema*
2. Find out its *functional dependencies (FDs)*
3. Use these to *design a better schema*
 1. One which minimizes possibility of anomalies

This part can be tricky!

Finding Functional Dependencies

- There can be a very **large number** of FDs...
 - *How to find them all efficiently?*
- We can't necessarily show that any FD will hold **on all instances**...
 - *How to do this?*

We will start with this problem:

Given a set of FDs, F , what other FDs ***must*** hold?

Finding Functional Dependencies

Equivalent to asking: Given a set of FDs, $F = \{f_1, \dots, f_n\}$, does an FD g hold?

Inference problem: How do we decide?

Finding Functional Dependencies

Example:

Products

Name	Color	Category	Dep	Price
Gizmo	Green	Gadget	Toys	49
Widget	Black	Gadget	Toys	59
Gizmo	Green	Whatsit	Garden	99

Provided FDs:

1. $\{\text{Name}\} \rightarrow \{\text{Color}\}$
2. $\{\text{Category}\} \rightarrow \{\text{Department}\}$
3. $\{\text{Color}, \text{Category}\} \rightarrow \{\text{Price}\}$

Given the provided FDs, we can see that $\{\text{Name}, \text{Category}\} \rightarrow \{\text{Price}\}$ must also hold on **any instance...**

Which / how many other FDs do?!?

Finding Functional Dependencies

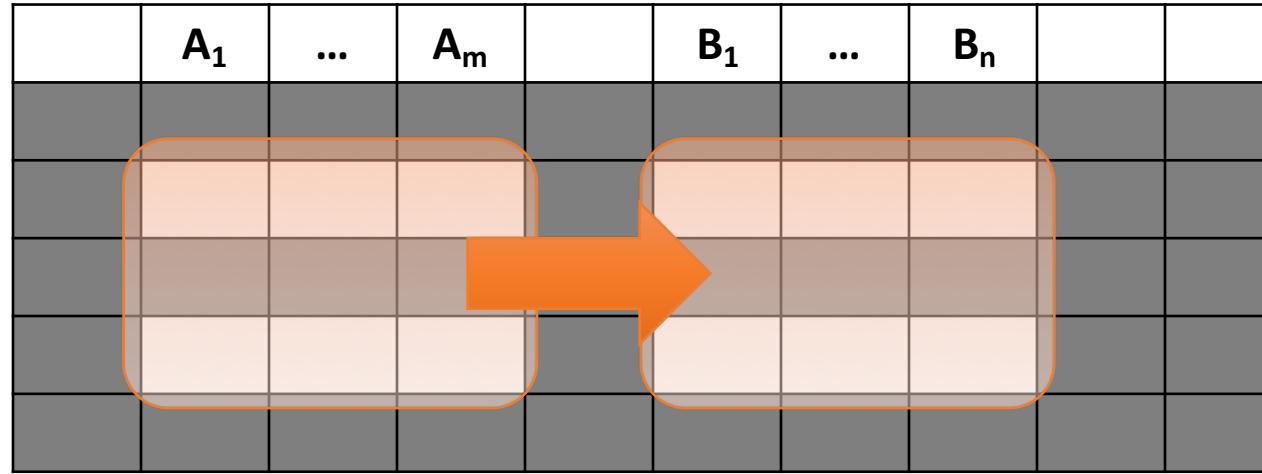
Equivalent to asking: Given a set of FDs, $F = \{f_1, \dots, f_n\}$, does an FD g hold?

Inference problem: How do we decide?

Answer: Three simple rules called **Armstrong's Rules**.

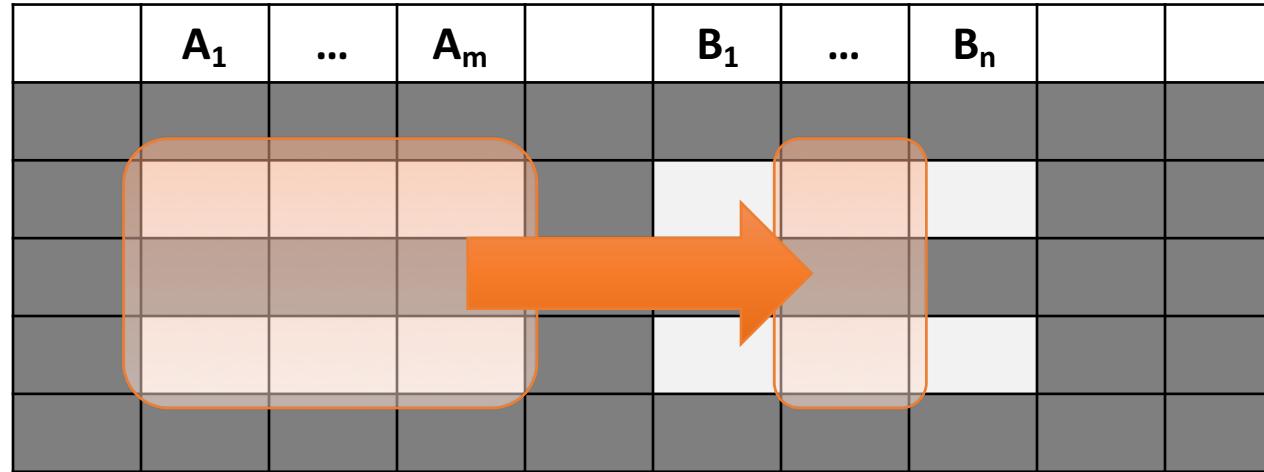
1. Split/Combine,
2. Reduction, and
3. Transitivity... *ideas by picture*

1. Split/Combine



$$A_1, \dots, A_m \rightarrow B_1, \dots, B_n$$

1. Split/Combine

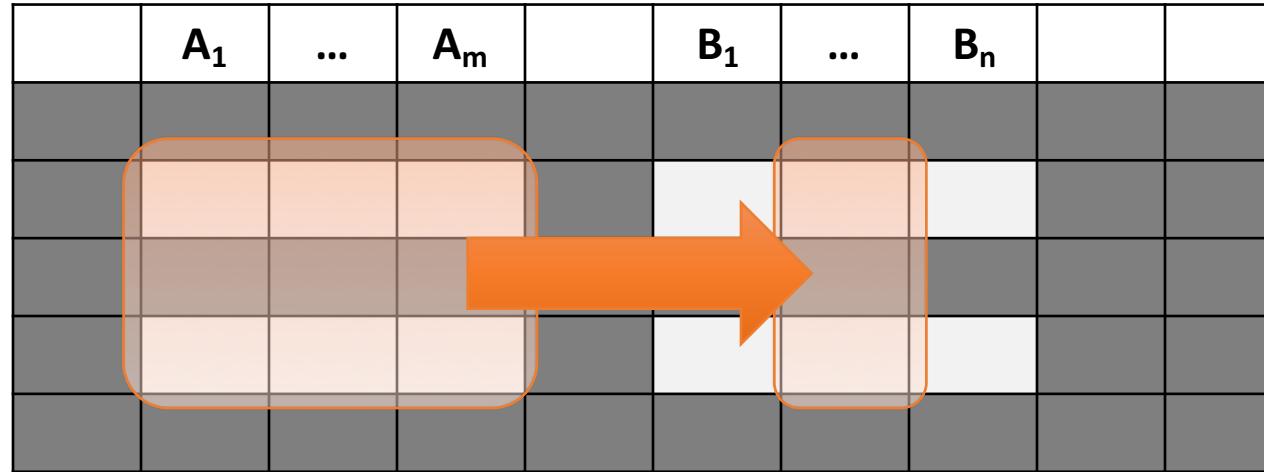


$$A_1, \dots, A_m \rightarrow B_1, \dots, B_n$$

... is equivalent to the following n FDs...

$$A_1, \dots, A_m \rightarrow B_i \text{ for } i=1, \dots, n$$

1. Split/Combine

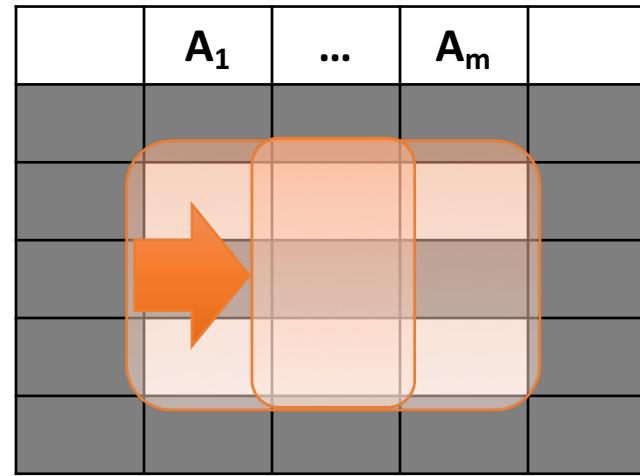


And vice-versa, $A_1, \dots, A_m \rightarrow B_i$ for $i=1, \dots, n$

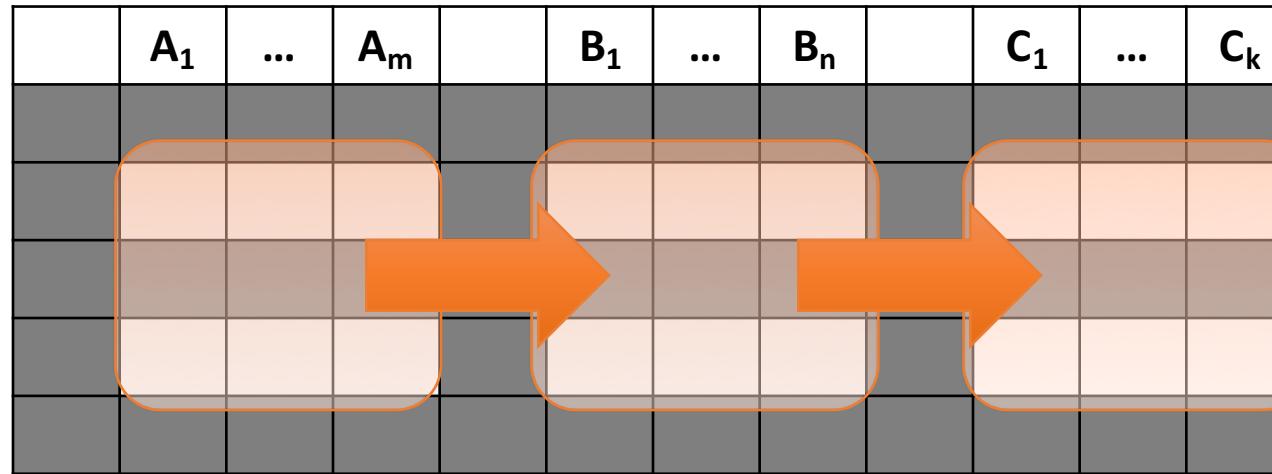
... is equivalent to ...

$$A_1, \dots, A_m \rightarrow B_1, \dots, B_n$$

Reduction/Trivial

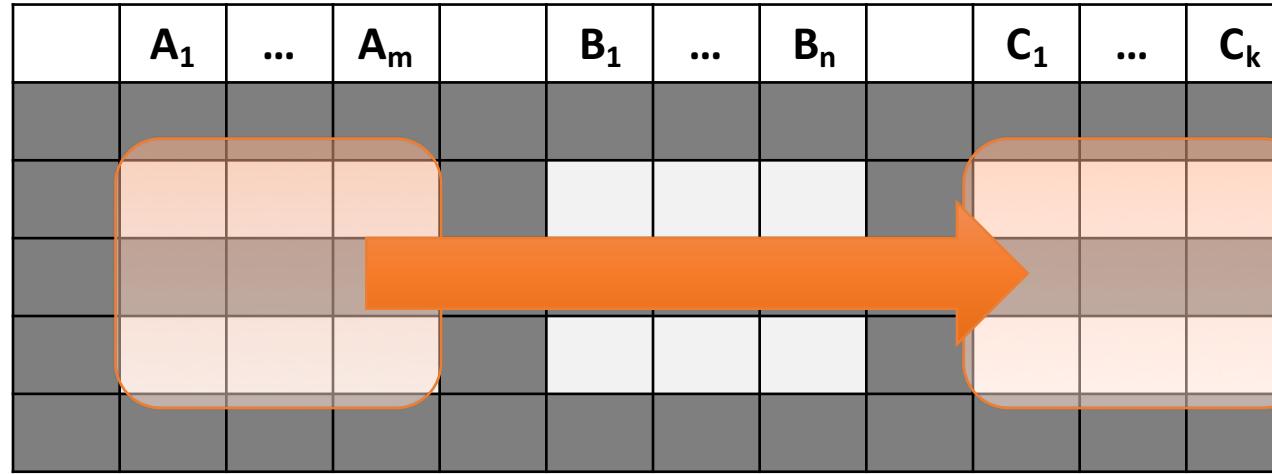

$$A_1, \dots, A_m \rightarrow A_j \text{ for any } j=1, \dots, m$$

3. Transitive Closure



$A_1, \dots, A_m \rightarrow B_1, \dots, B_n$ and
 $B_1, \dots, B_n \rightarrow C_1, \dots, C_k$

3. Transitive Closure



$A_1, \dots, A_m \rightarrow B_1, \dots, B_n$ and
 $B_1, \dots, B_n \rightarrow C_1, \dots, C_k$

implies

$A_1, \dots, A_m \rightarrow C_1, \dots, C_k$

Finding Functional Dependencies

Example:

Products

Name	Color	Category	Dep	Price
Gizmo	Green	Gadget	Toys	49
Widget	Black	Gadget	Toys	59
Gizmo	Green	Whatsit	Garden	99

Provided FDs:

1. $\{\text{Name}\} \rightarrow \{\text{Color}\}$
2. $\{\text{Category}\} \rightarrow \{\text{Department}\}$
3. $\{\text{Color}, \text{Category}\} \rightarrow \{\text{Price}\}$

Which / how many other FDs hold?

Finding Functional Dependencies

Example:

Inferred FDs:

Inferred FD	Rule used
4. $\{Name, Category\} \rightarrow \{Name\}$?
5. $\{Name, Category\} \rightarrow \{Color\}$?
6. $\{Name, Category\} \rightarrow \{Category\}$?
7. $\{Name, Category\} \rightarrow \{Color, Category\}$?
8. $\{Name, Category\} \rightarrow \{Price\}$?

Provided FDs:

1. $\{Name\} \rightarrow \{Color\}$
2. $\{Category\} \rightarrow \{Dept.\}$
3. $\{Color, Category\} \rightarrow \{Price\}$

Which / how many other FDs hold?

Finding Functional Dependencies

Example:

Inferred FDs:

Inferred FD	Rule used
4. $\{Name, Category\} \rightarrow \{Name\}$	Trivial
5. $\{Name, Category\} \rightarrow \{Color\}$	Transitive ($4 \rightarrow 1$)
6. $\{Name, Category\} \rightarrow \{Category\}$	Trivial
7. $\{Name, Category\} \rightarrow \{Color, Category\}$	Split/combine (5 + 6)
8. $\{Name, Category\} \rightarrow \{Price\}$	Transitive ($7 \rightarrow 3$)

Provided FDs:

1. $\{Name\} \rightarrow \{Color\}$
2. $\{Category\} \rightarrow \{Dept.\}$
3. $\{Color, Category\} \rightarrow \{Price\}$

Can we find an algorithmic way to do this?

Closures

Closure of a set of Attributes

Given a set of attributes A_1, \dots, A_n and a set of FDs F :

Then the closure, $\{A_1, \dots, A_n\}^+$ is the set of attributes B s.t. $\{A_1, \dots, A_n\} \rightarrow B$

Example: $F =$

$$\begin{aligned}\{name\} &\rightarrow \{color\} \\ \{category\} &\rightarrow \{department\} \\ \{color, category\} &\rightarrow \{price\}\end{aligned}$$

*Example
Closures:*

$$\begin{aligned}\{name\}^+ &= \{name, color\} \\ \{name, category\}^+ &= \\ \{name, category, color, dept, price\} & \\ \{color\}^+ &= \{color\}\end{aligned}$$

Closure Algorithm

Start with $X = \{A_1, \dots, A_n\}$ and set of FDs F .

Repeat until X doesn't change; do:

if $\{B_1, \dots, B_n\} \rightarrow C$ is entailed by F

and $\{B_1, \dots, B_n\} \subseteq X$

then add C to X .

Return X as X^+

Closure Algorithm

Start with $X = \{A_1, \dots, A_n\}$, FDs F .

Repeat until X doesn't change; **do**:

if $\{B_1, \dots, B_n\} \rightarrow C$ is in F **and** $\{B_1, \dots, B_n\} \subseteq X$:

then add C to X .

Return X as X^+

$\{\text{name}, \text{ category}\}^+ =$
 $\{\text{name}, \text{ category}\}$

$F =$

$\{\text{name}\} \rightarrow \{\text{color}\}$

$\{\text{category}\} \rightarrow \{\text{dept}\}$

$\{\text{color}, \text{ category}\} \rightarrow$
 $\{\text{price}\}$

Closure Algorithm

Start with $X = \{A_1, \dots, A_n\}$, FDs F .

Repeat until X doesn't change; **do**:

if $\{B_1, \dots, B_n\} \rightarrow C$ is in F **and** $\{B_1, \dots, B_n\} \subseteq X$:
then add C to X .

Return X as X^+

$F =$

$\{name\} \rightarrow \{color\}$

$\{category\} \rightarrow \{dept\}$

$\{color, category\} \rightarrow \{price\}$

$\{name, category\}^+ =$
 $\{name, category\}$

$\{name, category\}^+ =$
 $\{name, category, color\}$

Closure Algorithm

Start with $X = \{A_1, \dots, A_n\}$, FDs F .

Repeat until X doesn't change; **do**:

if $\{B_1, \dots, B_n\} \rightarrow C$ is in F **and** $\{B_1, \dots, B_n\} \subseteq X$:
then add C to X .

Return X as X^+

$F =$

$\{\text{name}\} \rightarrow \{\text{color}\}$

$\{\text{category}\} \rightarrow \{\text{dept}\}$

$\{\text{color}, \text{category}\} \rightarrow \{\text{price}\}$

$\{\text{name}, \text{category}\}^+ =$
 $\{\text{name}, \text{category}\}$

$\{\text{name}, \text{category}\}^+ =$
 $\{\text{name}, \text{category}, \text{color}\}$

$\{\text{name}, \text{category}\}^+ =$
 $\{\text{name}, \text{category}, \text{color}, \text{dept}\}$

Closure Algorithm

Start with $X = \{A_1, \dots, A_n\}$, FDs F .

Repeat until X doesn't change; **do**:

if $\{B_1, \dots, B_n\} \rightarrow C$ is in F **and** $\{B_1, \dots, B_n\} \subseteq X$:
then add C to X .

Return X as X^+

$F =$

$\{\text{name}\} \rightarrow \{\text{color}\}$

$\{\text{category}\} \rightarrow \{\text{dept}\}$

$\{\text{color}, \text{category}\} \rightarrow \{\text{price}\}$

$\{\text{name}, \text{category}\}^+ =$
 $\{\text{name}, \text{category}\}$

$\{\text{name}, \text{category}\}^+ =$
 $\{\text{name}, \text{category}, \text{color}\}$

$\{\text{name}, \text{category}\}^+ =$
 $\{\text{name}, \text{category}, \text{color}, \text{dept}\}$

$\{\text{name}, \text{category}\}^+ =$
 $\{\text{name}, \text{category}, \text{color}, \text{dept}, \text{price}\}$

Example

R(A,B,C,D,E,F)

{A,B} → {C}
{A,D} → {E}
{B} → {D}
{A,F} → {B}

Compute $\{A,B\}^+ = \{A, B, \}$

Compute $\{A, F\}^+ = \{A, F, \}$

Example

R(A, B, C, D, E, F)

{A, B} → {C}
{A, D} → {E}
{B} → {D}
{A, F} → {B}

Compute $\{A, B\}^+ = \{A, B, C, D\}$

Compute $\{A, F\}^+ = \{A, F, B\}$

Example

R(A, B, C, D, E, F)

$\{A, B\} \rightarrow \{C\}$
 $\{A, D\} \rightarrow \{E\}$
 $\{B\} \rightarrow \{D\}$
 $\{A, F\} \rightarrow \{B\}$

Compute $\{A, B\}^+ = \{A, B, C, D, E\}$

Compute $\{A, F\}^+ = \{A, B, C, D, E, F\}$

[Activity-5-2.ipynb](#)

3. Closures, Superkeys & Keys

What you will learn about in this section

1. Closures Pt. II
2. Superkeys & Keys
3. ACTIVITY: The key or a key?

Why Do We Need the Closure?

- With closure we can find all FD's easily

- To check if $X \rightarrow A$

- Compute X^+

- Check if $A \in X^+$

Note here that X is a *set* of attributes, but A is a *single* attribute. Why does considering FDs of this form suffice?

Recall the Split/combine rule:
 $X \rightarrow A_1, \dots, X \rightarrow A_n$
implies
 $X \rightarrow \{A_1, \dots, A_n\}$

Using Closure to Infer ALL FDs

Example:

Given $F =$

$\{A, B\}$	→	C
$\{A, D\}$	→	B
$\{B\}$	→	D

Step 1: Compute X^+ , for every set of attributes X :

$$\{A\}^+ = \{A\}$$

$$\{B\}^+ = \{B, D\}$$

$$\{C\}^+ = \{C\}$$

$$\{D\}^+ = \{D\}$$

$$\{A, B\}^+ = \{A, B, C, D\}$$

$$\{A, C\}^+ = \{A, C\}$$

$$\{A, D\}^+ = \{A, B, C, D\}$$

$$\{A, B, C\}^+ = \{A, B, D\}^+ = \{A, C, D\}^+ = \{A, B, C, D\}$$

$$\{B, C, D\}^+ = \{B, C, D\}$$

$$\{A, B, C, D\}^+ = \{A, B, C, D\}$$

No need to
compute these-
why?

Using Closure to Infer ALL FDs

Example:

Given $F =$

$\{A, B\}$	\rightarrow	C
$\{A, D\}$	\rightarrow	B
$\{B\}$	\rightarrow	D

Step 1: Compute X^+ , for every set of attributes X :

$$\begin{aligned}\{A\}^+ &= \{A\}, \quad \{B\}^+ = \{B, D\}, \quad \{C\}^+ = \{C\}, \quad \{D\}^+ = \\ &\{D\}, \quad \{A, B\}^+ = \{A, B, C, D\}, \quad \{A, C\}^+ = \{A, C\}, \\ &\{A, D\}^+ = \{A, B, C, D\}, \quad \{A, B, C\}^+ = \{A, B, D\}^+ = \\ &\{A, C, D\}^+ = \{A, B, C, D\}, \quad \{B, C, D\}^+ = \{B, C, D\}, \\ &\{A, B, C, D\}^+ = \{A, B, C, D\}\end{aligned}$$

Step 2: Enumerate all FDs $X \rightarrow Y$, s.t. $Y \subseteq X^+$ and $X \cap Y = \emptyset$:

$$\begin{aligned}\{A, B\} &\rightarrow \{C, D\}, \quad \{A, D\} \rightarrow \{B, C\}, \\ \{A, B, C\} &\rightarrow \{D\}, \quad \{A, B, D\} \rightarrow \{C\}, \\ \{A, C, D\} &\rightarrow \{B\}\end{aligned}$$

Using Closure to Infer ALL FDs

Example:

Given $F =$

$\{A, B\} \rightarrow C$
$\{A, D\} \rightarrow B$
$\{B\} \rightarrow D$

Step 1: Compute X^+ , for every set of attributes X :

$$\begin{aligned}\{A\}^+ &= \{A\}, \quad \{B\}^+ = \{B, D\}, \quad \{C\}^+ = \{C\}, \quad \{D\}^+ = \\ &\{D\}, \quad \{A, B\}^+ = \{A, B, C, D\}, \quad \{A, C\}^+ = \{A, C\}, \\ &\{A, D\}^+ = \{A, B, C, D\}, \quad \{A, B, C\}^+ = \{A, B, D\}^+ = \\ &\{A, C, D\}^+ = \{A, B, C, D\}, \quad \{B, C, D\}^+ = \{B, C, D\}, \\ &\{A, B, C, D\}^+ = \{A, B, C, D\}\end{aligned}$$

Step 2: Enumerate all FDs $X \rightarrow Y$, s.t. $Y \subseteq X^+$ and $X \cap Y = \emptyset$:

"Y is in the closure of X"

$$\begin{aligned}\{A, B\} \rightarrow \{C, D\}, \quad \{A, D\} \rightarrow \{B, C\}, \\ \{A, B, C\} \rightarrow \{D\}, \quad \{A, B, D\} \rightarrow \{C\}, \\ \{A, C, D\} \rightarrow \{B\}\end{aligned}$$

Using Closure to Infer ALL FDs

Example:

Given $F =$

$\{A, B\} \rightarrow C$
$\{A, D\} \rightarrow B$
$\{B\} \rightarrow D$

Step 1: Compute X^+ , for every set of attributes X :

$$\begin{aligned}\{A\}^+ &= \{A\}, \quad \{B\}^+ = \{B, D\}, \quad \{C\}^+ = \{C\}, \quad \{D\}^+ = \\ &\{D\}, \quad \{A, B\}^+ = \{A, B, C, D\}, \quad \{A, C\}^+ = \{A, C\}, \\ &\{A, D\}^+ = \{A, B, C, D\}, \quad \{A, B, C\}^+ = \{A, B, D\}^+ = \\ &\{A, C, D\}^+ = \{A, B, C, D\}, \quad \{B, C, D\}^+ = \{B, C, D\}, \\ &\{A, B, C, D\}^+ = \{A, B, C, D\}\end{aligned}$$

Step 2: Enumerate all FDs $X \rightarrow Y$, s.t. $Y \subseteq X^+$ and $X \cap Y = \emptyset$:

$$\begin{aligned}\{A, B\} \rightarrow \{C, D\}, \quad \{A, D\} \rightarrow \{B, C\}, \\ \{A, B, C\} \rightarrow \{D\}, \quad \{A, B, D\} \rightarrow \{C\}, \\ \{A, C, D\} \rightarrow \{B\}\end{aligned}$$

The FD $X \rightarrow Y$
is non-trivial

Superkeys and Keys

Keys and Superkeys

A superkey is a set of attributes A_1, \dots, A_n s.t.
for *any other* attribute B in R ,
we have $\{A_1, \dots, A_n\} \rightarrow B$

i.e. all attributes are
functionally determined
by a superkey

A key is a *minimal* superkey

Meaning that no subset of
a key is also a superkey

Finding Keys and Superkeys

- For each set of attributes X
 1. Compute X^+
 2. If $X^+ = \text{set of all attributes}$ then X is a **superkey**
 3. If X is minimal, then it is a **key**

Do we need to check all sets of attributes? Which sets?

Example of Finding Keys

Product(name, price, category, color)

{name, category} → price
{category} → color

What is a key?

Example of Keys

Product(name, price, category, color)

{name, category} → price
{category} → color

$\{name, category\}^+ = \{name, price, category, color\}$
= the set of all attributes
⇒ this is a **superkey**
⇒ this is a **key**, since neither **name** nor **category**
alone is a superkey

[Activity-5-3.ipynb](#)

Lecture 7: Design Theory II

Today's Lecture

1. Boyce-Codd Normal Form
 - ACTIVITY

2. Decompositions & 3NF
 - ACTIVITY

3. MVDs
 - ACTIVITY

1. Boyce-Codd Normal Form

What you will learn about in this section

1. Conceptual Design
2. Boyce-Codd Normal Form
3. The BCNF Decomposition Algorithm
4. ACTIVITY

Conceptual Design

Back to Conceptual Design

Now that we know how to find FDs, it's a straight-forward process:

1. Search for “bad” FDs
2. If there are any, then *keep decomposing the table into sub-tables* until no more bad FDs
3. When done, the database schema is *normalized*

Recall: there are several normal forms...

Boyce-Codd Normal Form (BCNF)

- Main idea is that we define “good” and “bad” FDs as follows:
 - $X \rightarrow A$ is a “*good FD*” if X is a (*super*)key
 - In other words, if A is the set of all attributes
 - $X \rightarrow A$ is a “*bad FD*” otherwise
- We will try to eliminate the “bad” FDs!

Boyce-Codd Normal Form (BCNF)

- Why does this definition of “good” and “bad” FDs make sense?
- If X is *not* a (super)key, it functionally determines *some* of the attributes
 - Recall: this means there is redundancy
 - And redundancy like this can lead to data anomalies!

EmpID	Name	Phone	Position
E0045	Smith	1234	Clerk
E3542	Mike	9876	Salesrep
E1111	Smith	9876	Salesrep
E9999	Mary	1234	Lawyer

Boyce-Codd Normal Form

BCNF is a simple condition for removing anomalies from relations:

A relation R is in BCNF if:

if $\{A_1, \dots, A_n\} \rightarrow B$ is a *non-trivial* FD in R

then $\{A_1, \dots, A_n\}$ is a superkey for R

Equivalently: \forall sets of attributes X, either $(X^+ = X)$ or $(X^+ = \text{all attributes})$

In other words: there are no “bad” FDs

Example

Name	SSN	PhoneNumber	City
Fred	123-45-6789	206-555-1234	Seattle
Fred	123-45-6789	206-555-6543	Seattle
Joe	987-65-4321	908-555-2121	Westfield
Joe	987-65-4321	908-555-1234	Westfield

$\{SSN\} \rightarrow \{Name, City\}$

This FD is *bad*
because it is not a
superkey

\Rightarrow Not in BCNF

What is the key?
 $\{SSN, PhoneNumber\}$

Example

Name	<u>SSN</u>	City
Fred	123-45-6789	Seattle
Joe	987-65-4321	Madison

$$\{\text{SSN}\} \rightarrow \{\text{Name, City}\}$$

This FD is now
good because it is
the key

<u>SSN</u>	<u>PhoneNumber</u>
123-45-6789	206-555-1234
123-45-6789	206-555-6543
987-65-4321	908-555-2121
987-65-4321	908-555-1234

Let's check anomalies:

- Redundancy ?
- Update ?
- Delete ?

Now in BCNF!

BCNF Decomposition Algorithm

BCNFD**e**comp(R):

BCNF Decomposition Algorithm

BCNFD**e**comp(R):

Find a set of attributes X s.t.: $X^+ \neq X$ and $X^+ \neq [$ all attributes $]$

Find a set of attributes X which has non-trivial “bad” FDs, i.e. is not a superkey, using closures

BCNF Decomposition Algorithm

BCNFD**e**comp(R):

Find a *set of attributes* X s.t.: $X^+ \neq X$ and $X^+ \neq [$ all attributes $]$

if (not found) **then Return** R

If no “bad” FDs found, in BCNF!

BCNF Decomposition Algorithm

BCNFDekomp(R):

Find a *set of attributes* X s.t.: $X^+ \neq X$ and $X^+ \neq$
[all attributes]

if (not found) then Return R

let $Y = X^+ - X$, $Z = (X^+)^C$

Let Y be the attributes that
 X *functionally determines*
(+ that are not in X)

And let Z be the other
attributes that it *doesn't*

BCNF Decomposition Algorithm

BCNFDekomp(R):

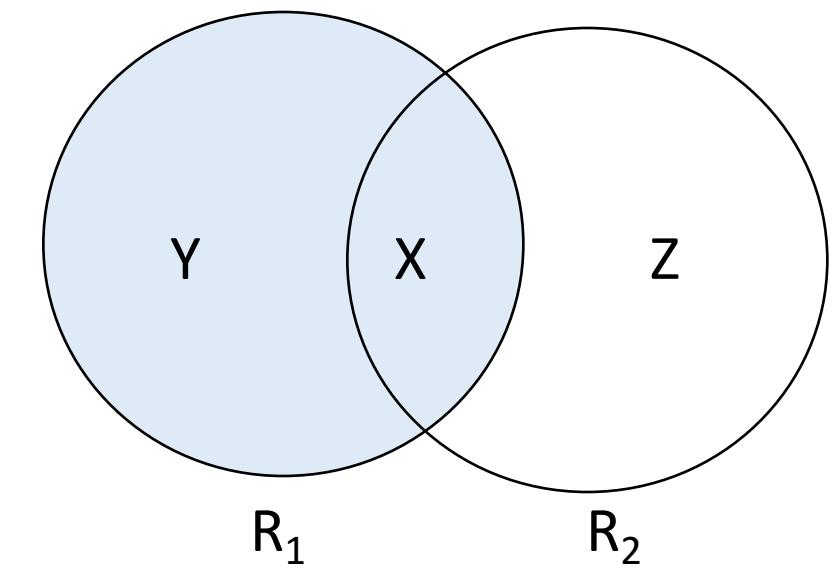
Find a *set of attributes* X s.t.: $X^+ \neq X$ and $X^+ \neq$
[all attributes]

if (not found) then Return R

let $Y = X^+ - X$, $Z = (X^+)^C$

decompose R into $R_1(X \cup Y)$ and $R_2(X \cup Z)$

Split into one relation (table)
with X plus the attributes
that X determines (Y)...



BCNF Decomposition Algorithm

BCNFDekomp(R):

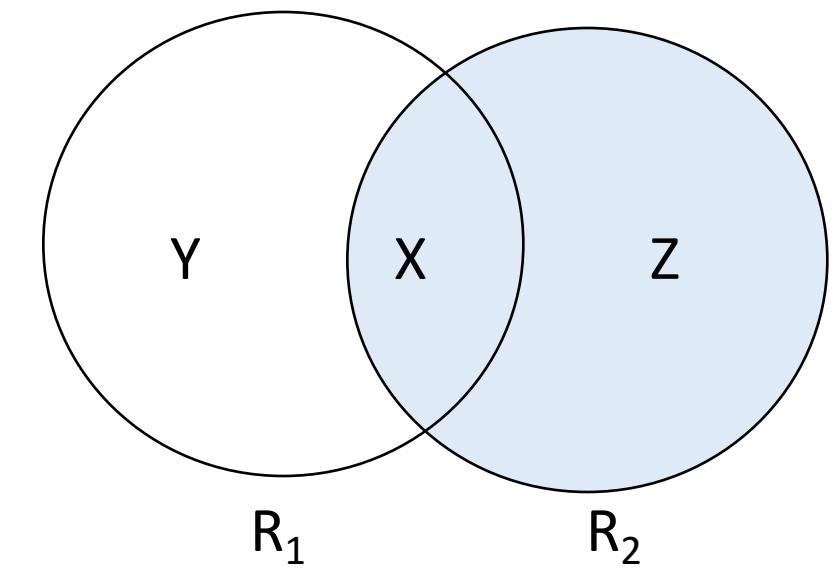
Find a set of attributes X s.t.: $X^+ \neq X$ and $X^+ \neq$
[all attributes]

if (not found) then Return R

let $Y = X^+ - X$, $Z = (X^+)^C$

decompose R into $R_1(X \cup Y)$ and $R_2(X \cup Z)$

And one relation with X plus
the attributes it *does not*
determine (Z)



BCNF Decomposition Algorithm

BCNFDekomp(R):

Find a set of attributes X s.t.: $X^+ \neq X$ and $X^+ \neq [all\ attributes]$

if (not found) then Return R

let $Y = X^+ - X$, $Z = (X^+)^C$

decompose R into $R_1(X \cup Y)$ and $R_2(X \cup Z)$

Return BCNFDekomp(R_1), BCNFDekomp(R_2)

Proceed recursively until no more “bad” FDs!

Example

BCNFDekomp(R):

Find a set of attributes X s.t.: $X^+ \neq X$ and $X^+ \neq$
[all attributes]

if (not found) then Return R

let $Y = X^+ - X$, $Z = (X^+)^C$

decompose R into $R_1(X \cup Y)$ and $R_2(X \cup Z)$

Return BCNFDekomp(R_1), BCNFDekomp(R_2)

$R(A, B, C, D, E)$

$\{A\} \rightarrow \{B, C\}$
 $\{C\} \rightarrow \{D\}$

Example

$R(A, B, C, D, E)$

$\{A\} \rightarrow \{B, C\}$
 $\{C\} \rightarrow \{D\}$

$R(A, B, C, D, E)$

$\{A\}^+ = \{A, B, C, D\} \neq \{A, B, C, D, E\}$

$R_1(A, B, C, D)$

$\{C\}^+ = \{C, D\} \neq \{A, B, C, D\}$

$R_{11}(C, D)$

$R_{12}(A, B, C)$

$R_2(A, E)$

[Activity-7-1.ipynb](#)

2. Decompositions

What you will learn about in this section

1. Lossy & Lossless Decompositions

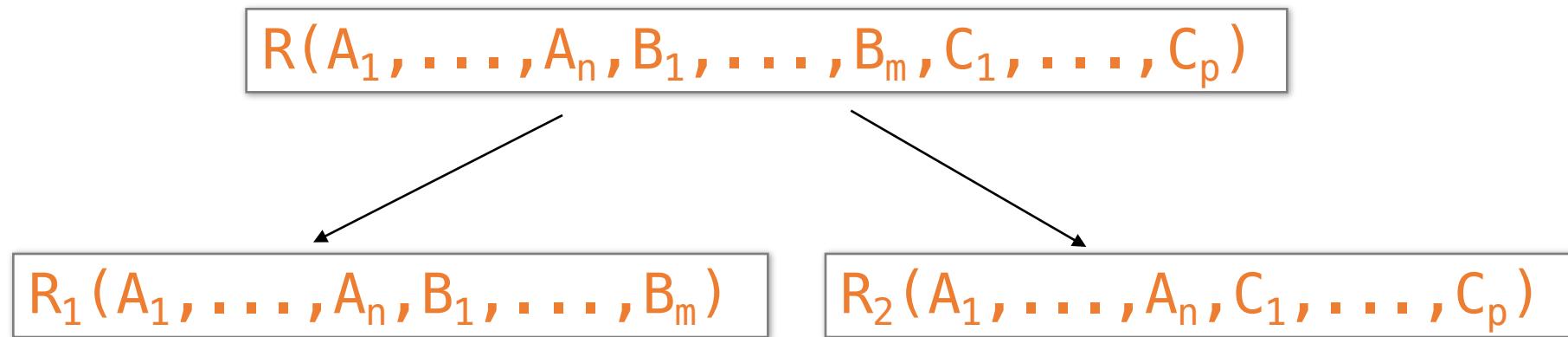
2. ACTIVITY

Recap: Decompose to remove redundancies

1. We saw that **redundancies** in the data (“bad FDs”) can lead to data anomalies
2. We developed mechanisms to **detect and remove redundancies by decomposing tables into BCNF**
 1. BCNF decomposition is *standard practice*- very powerful & widely used!
3. However, sometimes decompositions can lead to **more subtle unwanted effects...**

When does this happen?

Decompositions in General



R_1 = the *projection* of R on $A_1, \dots, A_n, B_1, \dots, B_m$

R_2 = the *projection* of R on $A_1, \dots, A_n, C_1, \dots, C_p$

Theory of Decomposition

Name	Price	Category
Gizmo	19.99	Gadget
OneClick	24.99	Camera
Gizmo	19.99	Camera

Sometimes a decomposition is “correct”

i.e. it is a Lossless decomposition



Name	Price
Gizmo	19.99
OneClick	24.99
Gizmo	19.99

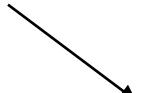
Name	Category
Gizmo	Gadget
OneClick	Camera
Gizmo	Camera

Lossy Decomposition

Name	Price	Category
Gizmo	19.99	Gadget
OneClick	24.99	Camera
Gizmo	19.99	Camera

However
sometimes it isn't

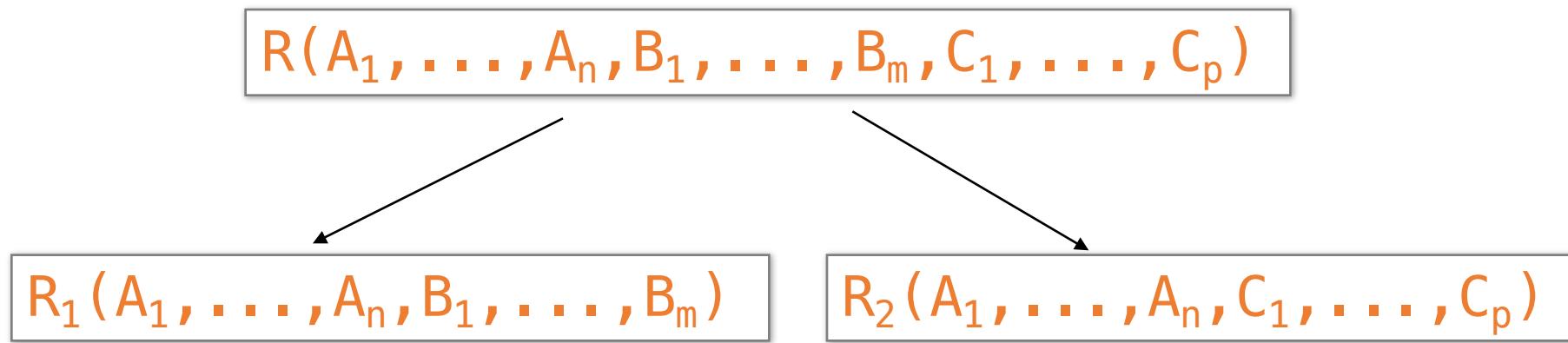
What's wrong
here?



Name	Category
Gizmo	Gadget
OneClick	Camera
Gizmo	Camera

Price	Category
19.99	Gadget
24.99	Camera
19.99	Camera

Lossless Decompositions



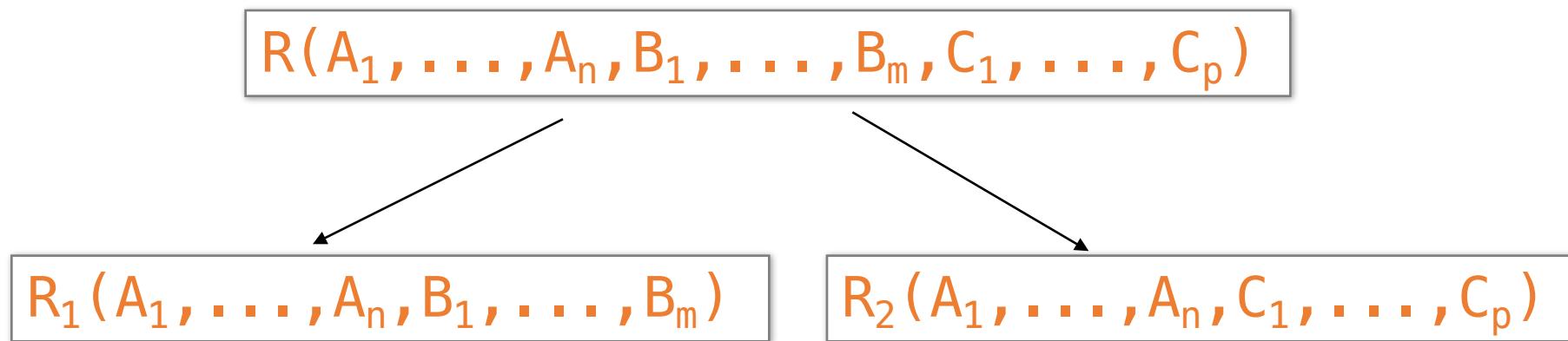
What (set) relationship holds between R1
Join R2 and R if lossless?

Hint: Which tuples of R will be present?



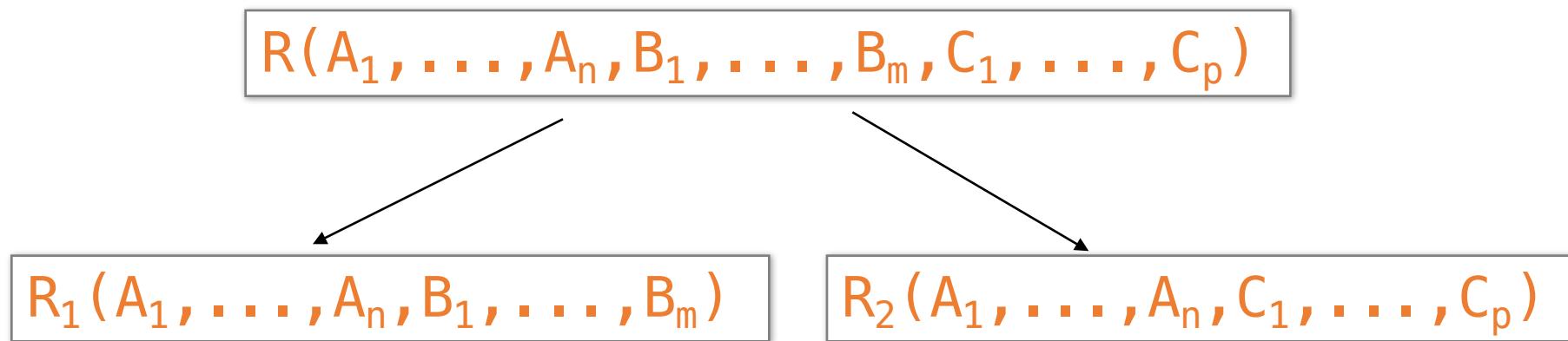
It's lossless
if we have
equality!

Lossless Decompositions



A decomposition R to (R_1, R_2) is lossless if $R = R_1 \text{ Join } R_2$

Lossless Decompositions



If $\{A_1, \dots, A_n\} \rightarrow \{B_1, \dots, B_m\}$
Then the decomposition is lossless

Note: don't need
 $\{A_1, \dots, A_n\} \rightarrow \{C_1, \dots, C_p\}$

BCNF decomposition is always lossless. Why?

A problem with BCNF

Problem: To enforce a FD, must reconstruct original relation—*on each insert!*

Note: This is historically inaccurate, but it makes it easier to explain

A Problem with BCNF

Unit	Company	Product
...

$\{Unit\} \rightarrow \{Company\}$
 $\{Company, Product\} \rightarrow \{Unit\}$

Unit	Company
...	...

Unit	Product
...	...

$\{Unit\} \rightarrow \{Company\}$

We do a BCNF decomposition
on a “bad” FD:
 $\{Unit\}^+ = \{Unit, Company\}$

We lose the FD $\{Company, Product\} \rightarrow \{Unit\}!!$

So Why is that a Problem?

Unit	Company
Galaga99	UW
Bingo	UW

Unit	Product
Galaga99	Databases
Bingo	Databases

No problem so far.
All *local* FD's are satisfied.

$\{Unit\} \rightarrow \{Company\}$

Unit	Company	Product
Galaga99	UW	Databases
Bingo	UW	Databases

Let's put all the data back into a single table again:

Violates the FD $\{Company, Product\} \rightarrow \{Unit\}$!!

The Problem

- We started with a table R and FDs F
- We decomposed R into BCNF tables R_1, R_2, \dots with their own FDs F_1, F_2, \dots
- We insert some tuples into each of the relations—which satisfy their local FDs but when reconstruct it violates some FD **across** tables!

Practical Problem: To enforce FD, must reconstruct
R—on each insert!

Possible Solutions

- Various ways to handle so that decompositions are all lossless / no FDs lost
 - For example 3NF- stop short of full BCNF decompositions. See Bonus Activity!
- Usually a tradeoff between redundancy / data anomalies and FD preservation...

BCNF still most common- with additional steps to keep track of lost FDs...

[Activity-7-2.ipynb](#)

3. MVDS

What you will learn about in this section

1. MVDs
2. ACTIVITY

Multi-Value Dependencies (MVDs)

- A multi-value dependency (MVD) is another type of dependency that could hold in our data, ***which is not captured by FDs***
- Formal definition:
 - Given a relation R having attribute set A , and two sets of attributes $X, Y \subseteq A$
 - The ***multi-value dependency (MVD)*** $X \Rightarrow\!\!> Y$ holds on R if
 - ***for any tuples*** $t_1, t_2 \in R$ s.t. $t_1[X] = t_2[X]$, there exists a tuple t_3 s.t.:
 - $t_1[X] = t_2[X] = t_3[X]$
 - $t_1[Y] = t_3[Y]$
 - $t_2[A \setminus Y] = t_3[A \setminus Y]$
 - *Where $A \setminus B$ means “elements of set A not in set B ”*

Multi-Value Dependencies (MVDs)

- One less formal, literal way to phrase the definition of an MVD:
- ***The MVD $X \twoheadrightarrow Y$ holds on R if for any pair of tuples with the same X values, the “swapped” pair of tuples with the same X values, but the other permutations of Y and A\Y values, is also in R***

Ex: $X = \{x\}$, $Y = \{y\}$:

x	y	z
1	0	1
1	1	0

For $X \twoheadrightarrow Y$ to hold must have...

x	y	z
1	0	1
1	1	0
1	0	0
1	1	1

Note the connection to a local *cross-product*...

Multi-Value Dependencies (MVDs)

- Another way to understand MVDs, in terms of *conditional independence*:
- **The MVD $X \twoheadrightarrow Y$** holds on R if given X, Y is conditionally independent of $A \setminus Y$ and vice versa...

Here, given $x = 1$, we know for ex. that:
 $y = 0 \rightarrow z = 1$

I.e. z is conditionally *dependent* on y given x

x	y	z
1	0	1
1	1	0

Here, this is not the case!

I.e. z is conditionally *independent* of y given x

x	y	z
1	0	1
1	1	0
1	0	0
1	1	1

MVDs: Movie Theatre Example

Movie_theater	film_name	snack
Rains 216	Star Trek: The Wrath of Kahn	Kale Chips
Rains 216	Star Trek: The Wrath of Kahn	Burrito
Rains 216	Lord of the Rings: Concatenated & Extended Edition	Kale Chips
Rains 216	Lord of the Rings: Concatenated & Extended Edition	Burrito
Rains 218	Star Wars: The Boba Fett Prequel	Ramen
Rains 218	Star Wars: The Boba Fett Prequel	Plain Pasta

Are there any functional dependencies that might hold here?

No...

And yet it seems like there is some pattern / dependency...

MVDs: Movie Theatre Example

Movie_theater	film_name	snack
Rains 216	Star Trek: The Wrath of Kahn	Kale Chips
Rains 216	Star Trek: The Wrath of Kahn	Burrito
Rains 216	Lord of the Rings: Concatenated & Extended Edition	Kale Chips
Rains 216	Lord of the Rings: Concatenated & Extended Edition	Burrito
Rains 218	Star Wars: The Boba Fett Prequel	Ramen
Rains 218	Star Wars: The Boba Fett Prequel	Plain Pasta

For a given movie theatre...

MVDs: Movie Theatre Example

Movie_theater	film_name	snack
Rains 216	Star Trek: The Wrath of Kahn	Kale Chips
Rains 216	Star Trek: The Wrath of Kahn	Burrito
Rains 216	Lord of the Rings: Concatenated & Extended Edition	Kale Chips
Rains 216	Lord of the Rings: Concatenated & Extended Edition	Burrito
Rains 218	Star Wars: The Boba Fett Prequel	Ramen
Rains 218	Star Wars: The Boba Fett Prequel	Plain Pasta

For a given movie theatre...

Given a set of movies and snacks...

MVDs: Movie Theatre Example

Movie_theater	film_name	snack
Rains 216	Star Trek: The Wrath of Kahn	Kale Chips
Rains 216	Star Trek: The Wrath of Kahn	Burrito
Rains 216	Lord of the Rings: Concatenated & Extended Edition	Kale Chips
Rains 216	Lord of the Rings: Concatenated & Extended Edition	Burrito
Rains 218	Star Wars: The Boba Fett Prequel	Ramen
Rains 218	Star Wars: The Boba Fett Prequel	Plain Pasta

For a given movie theatre...

Given a set of movies and snacks...

Any movie / snack combination is possible!

MVDs: Movie Theatre Example

	Movie_theater (A)	film_name (B)	Snack (C)
t_1	Rains 216	Star Trek: The Wrath of Kahn	Kale Chips
	Rains 216	Star Trek: The Wrath of Kahn	Burrito
	Rains 216	Lord of the Rings: Concatenated & Extended Edition	Kale Chips
t_2	Rains 216	Lord of the Rings: Concatenated & Extended Edition	Burrito
	Rains 218	Star Wars: The Boba Fett Prequel	Ramen
	Rains 218	Star Wars: The Boba Fett Prequel	Plain Pasta

More formally, we write $\{A\} \twoheadrightarrow \{B\}$ if for any tuples t_1, t_2 s.t. $t_1[A] = t_2[A]$

MVDs: Movie Theatre Example

	Movie_theater (A)	film_name (B)	Snack (C)
t_1	Rains 216	Star Trek: The Wrath of Kahn	Kale Chips
t_3	Rains 216	Star Trek: The Wrath of Kahn	Burrito
	Rains 216	Lord of the Rings: Concatenated & Extended Edition	Kale Chips
t_2	Rains 216	Lord of the Rings: Concatenated & Extended Edition	Burrito
	Rains 218	Star Wars: The Boba Fett Prequel	Ramen
	Rains 218	Star Wars: The Boba Fett Prequel	Plain Pasta

More formally, we write $\{A\} \twoheadrightarrow \{B\}$ if for any tuples t_1, t_2 s.t. $t_1[A] = t_2[A]$ there is a tuple t_3 s.t.

- $t_3[A] = t_1[A]$

MVDs: Movie Theatre Example

	Movie_theater (A)	film_name (B)	Snack (C)
t_1	Rains 216	Star Trek: The Wrath of Kahn	Kale Chips
t_3	Rains 216	Star Trek: The Wrath of Kahn	Burrito
	Rains 216	Lord of the Rings: Concatenated & Extended Edition	Kale Chips
t_2	Rains 216	Lord of the Rings: Concatenated & Extended Edition	Burrito
	Rains 218	Star Wars: The Boba Fett Prequel	Ramen
	Rains 218	Star Wars: The Boba Fett Prequel	Plain Pasta

More formally, we write $\{A\} \twoheadrightarrow \{B\}$ if for any tuples t_1, t_2 s.t. $t_1[A] = t_2[A]$ there is a tuple t_3 s.t.

- $t_3[A] = t_1[A]$
- $t_3[B] = t_1[B]$

MVDs: Movie Theatre Example

	Movie_theater (A)	film_name (B)	Snack (C)
t_1	Rains 216	Star Trek: The Wrath of Kahn	Kale Chips
t_3	Rains 216	Star Trek: The Wrath of Kahn	Burrito
	Rains 216	Lord of the Rings: Concatenated & Extended Edition	Kale Chips
t_2	Rains 216	Lord of the Rings: Concatenated & Extended Edition	Burrito
	Rains 218	Star Wars: The Boba Fett Prequel	Ramen
	Rains 218	Star Wars: The Boba Fett Prequel	Plain Pasta

More formally, we write $\{A\} \twoheadrightarrow \{B\}$ if for any tuples t_1, t_2 s.t. $t_1[A] = t_2[A]$ there is a tuple t_3 s.t.

- $t_3[A] = t_1[A]$
- $t_3[B] = t_1[B]$
- and $t_3[R \setminus B] = t_2[R \setminus B]$

Where $R \setminus B$ is “R minus B” i.e. the attributes of R not in B

MVDs: Movie Theatre Example

	Movie_theater (A)	film_name (B)	Snack (C)
t_2	Rains 216	Star Trek: The Wrath of Kahn	Kale Chips
	Rains 216	Star Trek: The Wrath of Kahn	Burrito
t_3	Rains 216	Lord of the Rings: Concatenated & Extended Edition	Kale Chips
t_1	Rains 216	Lord of the Rings: Concatenated & Extended Edition	Burrito
	Rains 218	Star Wars: The Boba Fett Prequel	Ramen
	Rains 218	Star Wars: The Boba Fett Prequel	Plain Pasta

Note this also works!

Remember, an MVD holds over a *relation or an instance*, so defn. must hold for every applicable pair...

MVDs: Movie Theatre Example

	Movie_theater (A)	film_name (B)	Snack (C)
t ₂	Rains 216	Star Trek: The Wrath of Kahn	Kale Chips
	Rains 216	Star Trek: The Wrath of Kahn	Burrito
t ₃	Rains 216	Lord of the Rings: Concatenated & Extended Edition	Kale Chips
t ₁	Rains 216	Lord of the Rings: Concatenated & Extended Edition	Burrito
	Rains 218	Star Wars: The Boba Fett Prequel	Ramen
	Rains 218	Star Wars: The Boba Fett Prequel	Plain Pasta

This expresses a sort of dependency (= data redundancy) that we *can't* express with FDs

*Actually, it expresses conditional independence (between film and snack given movie theatre)!

Connection to FDs

If $A \rightarrow B$ does $A \gg B$?

Hint: sort of like multiplying by one...

Comments on MVDs

- MVDs have “rules” too!
 - **Experts:** Axiomatizable
- 4th Normal Form is “non-trivial MVD”
- *For AI nerds:* MVD is conditional independence in graphical models!

See the MVDs IPython notebook
for more examples!

[Activity-7-3.ipynb](#)

Summary

- Constraints allow one to reason about **redundancy** in the data
- Normal forms describe how to **remove** this redundancy by **decomposing** relations
 - Elegant—by representing data appropriately certain errors are essentially impossible
 - For FDs, BCNF is the normal form.
- A tradeoff for insert performance: 3NF