Problem 1

(1). $\alpha_1 = 2[1-\Phi(k)] = 0.0027$

$\alpha_2 = 0.001$

(2) $ARL_1 = \frac{1}{\alpha} = 370$

$ARL_2 = \frac{1}{\alpha} = 1000.$

(3). mean change $= 104$ mm 1σ mean shift.

$\beta_1 = Pr\{LCL \le \bar{X} \le UCL \mid \bar{X} \sim N(\mu_1, \sigma_0^2/n)\}.$

$= Pr\{LCL \le \bar{X} \le UCL \mid X \sim N(\mu_0, \sigma_0^2)\} = Pr\{\mu_0 - k\sigma_0/\sqrt{n} \le \bar{X} \le \mu_0 + k\sigma_0/\sqrt{n} \mid \bar{X} \sim N(\mu_0, \sigma_0^2/n)\}$

$= Pr\{\frac{(\mu_0 - k\sigma/\sqrt{n}) - \mu_1}{\sigma_0/\sqrt{n}} \le \frac{\bar{X} - \mu_1}{\sigma_0/\sqrt{n}} \le \frac{(\mu_0 + k\sigma_0/\sqrt{n}) - \mu_1}{\sigma_0/\sqrt{n}}\}.$

$= Pr\{-k - \frac{\delta}{\sigma_0/\sqrt{n}} \le Z \le k - \frac{\delta}{\sigma_0/\sqrt{n}}\}$

$= \Phi(k - \frac{\delta}{\sigma_0/\sqrt{n}}) - \Phi(-k - \frac{\delta}{\sigma_0/\sqrt{n}}) = \Phi(3 - \frac{\sigma_0}{\sigma_0/2}) - \Phi(-3 - \frac{\sigma_0}{\sigma_0/2})$

$= \Phi(-1) - \Phi(-7) = 0.1586$

$\beta_2 = \Phi(k - \frac{\delta}{\sigma_0/\sqrt{n}}) - \Phi(-k - \frac{\delta}{\sigma_0/\sqrt{n}})$

$2[1-\Phi(k)] = 0.001$ $\Phi(k) = 0.9995$ $k \approx 3.3$

$\therefore \beta_2 = \Phi(k-5) - \Phi(-k-5) = 0.04368$

(4) $ARL_{out1} = \frac{1}{1-\beta_1} = 1.18$

$ARL_{out2} = \frac{1}{1-\beta_2} = 1.0456$

(5) option 1 :

Pros: with smaller sample sizes, the chart can quickly signal an out-of-control condition, allowing for rapid response.

Cons: 3-sigma limit are quite wide, leading to a higher risk of Type I errors. when the process is actually in control.

Option 2 :

Pros: The probability limits allow for a more controlled Type I error rate, reducing the risk of false alarms compared to the 3-sigma limits.

Cons: The larger sample size may result in a slower response to change in the process.

Problem 2.

(1) Since $n=1$. So we can only use $X(I)$-MR chart.

(2)

Problem 3.

(1). $k=3$, $n=6$. $\sum_{i=1}^{30} \bar{X}_i = 6000$ & $\sum_{i=1}^{30} R_i = 150$ $\Rightarrow$ $\bar{\bar{X}} = 200$; $\bar{R} = 5$.

For $\bar{X}$
$$\begin{cases} UCL = \bar{\bar{X}} + A_2 \bar{R} = 200 + 0.483 \times 5 = 202.415 \\ CL = \bar{\bar{X}} = 200 \\ LCL = \bar{\bar{X}} - A_2 \bar{R} = 200 - 0.483 \times 5 = 197.585 \end{cases}$$

For R:
$$\begin{cases} UCL = D_4 \bar{R} = 2.004 \times 5 = 10.02. \\ CL = \bar{R} = 5 \\ LCL = D_3 \bar{R} = 0. \end{cases}$$

(2) $\mu = 200$ $\sigma = \frac{\bar{R}}{d_2} = \frac{5}{2.534} = 1.97$

(3). shift $\delta = \frac{200-199}{1.97} \approx 0.5068.$

$\beta = \Phi(k - \delta\sqrt{n}) - \Phi(-k - \delta\sqrt{n}) \approx 0.9606$

Problem 4

(1) $\mu = 10$. $\sigma = 2.5$ $n = 6$.

For S chart:
$$\begin{cases} UCL = B_6 \sigma_0 = 2.088 * 2.5 = 5.22 \\ CL = c_4 \sigma_0 = 0.9213 * 2.5 = 2.30325 \\ LCL = B_5 \sigma_0 = 0. \end{cases}$$

(2) For S chart:
$$\begin{cases} UCL = c_4 \sigma_0 + 2 \cdot \sigma_0 \sqrt{1-c_4^2} = 0.9213 * 2.5 + 3 * 2.5 * \sqrt{1-(0.9213)^2} = 4.2475 \\ CL = c_4 \sigma_0 = 0.9213 * 2.5 = 2.303 \\ LCL = c_4 \sigma_0 - 2 \cdot \sigma_0 \sqrt{1-c_4^2} = 0.9213 * 2.5 + 3 * 2.5 * \sqrt{(1-0.9213)^2} = 0.3590 \end{cases}$$

Problem 5:

(1) $\sigma \approx \frac{\bar{S}}{C_4} = \frac{0.3252}{0.9400} = 0.3460$

(2). For $\bar{X}$ chart: $\begin{cases} UCL = 1.9697 \\ CL = 1.5056 \\ LCL = 1.0415 \end{cases}$

(3) For $\bar{S}$ chart: $\begin{cases} UCL = 0.6794 \\ CL = 0.3252 \\ LCL = 0.0 \end{cases}$

(4) There is one point out of control

(5) There is no point out of control.

(6) XR chares have a narrow limit while X-S chares have a wider limit. So XR chares is more sensitive. to changes in process variability. Besides, mean values are consistent in both chares, indicate that process central tendency is stable. So R chart is more accurate in reflecting changes.