Problem 1

(1). With known $p_0$, the control chart:

$$\begin{cases} UCL_p = p_0 + L\sqrt{\frac{p_0(1-p_0)}{n}} = 0.0962 \\ CL_p = p_0 = 0.05 \\ LCL_p = p_0 - L\sqrt{\frac{p_0(1-p_0)}{n}} = 0.0038 \end{cases}$$

In that case: $\sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.05(1-0.05)}{400}}$

$\therefore L = \frac{UCL_p - CL_p}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.0462}{\sqrt{\frac{0.05(1-0.05)}{400}}} \approx 4.24.$

(2). type I error for p chart:

$$\begin{aligned} \alpha &= Pr\{\hat{P} \le LCL_p | p_0\} + Pr\{\hat{P} \ge UCL_p | p_0\}. \\ &= Pr\{D \le nLCL_p | p_0\} + 1 - Pr\{D < nUCL_p | p_0\}. \\ &= Pr\{D < 400 * 0.0038 | 0.05\} + 1 - Pr\{D < 400 * 0.0962 | 0.05\}. \\ &= Pr\{D < 1.52 | 0.05\} + 1 - Pr\{D < 38.48 | 0.05\}. \\ &= Pr\{D \le 1 | Binomial(400, 0.05)\} + 1 - Pr\{D \le 38 | Binomial(400, 0.05)\}. \\ &= 6.74 \times 10^{-5} \end{aligned}$$

(3) type II error: $p_1 = 0.15$

$$\begin{aligned} \beta &= Pr(LCL_p < \hat{P} < UCL_p | p_1\}. \\ &= Pr\{D \le nUCL_p | p_1\} - Pr\{D \le nLCL_p | p_1\}. \\ &\approx 0.00076 \end{aligned}$$

$\therefore$ detect shfie $= 1 - \beta = 0.99924$

Problem 2.

(1) $(1-p)^n \le 1-0.9 \Rightarrow n_{min} = 230.$

(2) np chart:

$$\begin{cases} UCL = np_0 + L\sqrt{np_0(1-p_0)} = 230 * 0.01 + 3 * \sqrt{230 * 0.01 * 0.99} = 6.82 \\ CL = np_0 = 2.3 \\ LCL = np_0 - L\sqrt{np_0(1-p_0)} = 0. \end{cases}$$

(3) type I error:

$$\begin{aligned} \alpha &= Pr\{D < n * LCL | p_0\} + 1 - Pr\{D < n * UCL | p_0\}. \\ &\approx 0.0991 \end{aligned}$$

Problem 3.

(1) 3-sigma chart:

$$\begin{cases} UCL = \bar{c} + 3\sqrt{\bar{c}} = 4 \\ CL = \bar{c} = 1 \\ LCL = \bar{c} - 3\sqrt{\bar{c}} = 0 \end{cases}$$

(2) $\alpha = Pr\{X \le LCL \mid c_0\} + Pr\{X \ge UCL \mid c_0\}$

$= 1 - Pr\{X < 4 \mid c_0 = 1\}$

$= 1 - Pr\{X \le 3 \mid c_0 = 1\} = 0.0189$

(3) sample size = 2.

(4) chart: $\begin{cases} UCL_u = \frac{c_0}{n} + 3\sqrt{\frac{c_0}{n^2}} = 2.0 \\ CL_u = \frac{c_0}{n} = 0.5 \\ LCL_u = \frac{c_0}{n} - 3\sqrt{\frac{c_0}{n^2}} = 0. \end{cases}$

Problem 4:

(1) $\begin{cases} UCL = \bar{c} + 3\sqrt{\bar{c}} \\ CL = \bar{c} \\ LCL = \bar{c} - 3\sqrt{\bar{c}} \end{cases}$ c monitoring chart

(2) $LCL = \bar{c} - 3\sqrt{\bar{c}} > 0$ ; $\bar{c} = \frac{40}{1000/n} = 9$.

$\Rightarrow n = 225$

(3) For monitoring the number of defects, c-chart is better. Since this chart is better when dealing with count data and sample size is constant.