ME 453: Data Science in Manufacturing Quality Control
Homework 2

Assigned: October 13, 2023 $\hfill$ Due: October 27, 2023

Homework guidelines:

1. The total number of points is 100. There are also 10 bonus points available from the bonus problems. The assigned points are given beside questions. To get full credit you must SHOW ALL OF YOUR WORK.

2. Bonus points will be used to compensate for lost points in homework assignments.

3. ⌨ Indicates the problem which you need to use Python.

4. A complete submission features the following items: (a) a brief report including all figures and results, and explanations of necessary steps taken to obtain them; and (b) and the source code (Python is recommended).

5. Item (a) can be scanned copies of handwritten or typeset reports. Both items shall be submitted through Canvas.

Review the following topics discussed in lecture:

1. Method and Philosophy of SPC (lecture videos 8, 10–11).

Problem 1 (15 points)

⌨ Perform a simulation study on confidence interval. Take $X \sim N(10, 0.01)$.

(1) Generate 100 random samples using software, with each sample of size $n = 10$. Take $\alpha = 0.05$, compute CI for all samples. (5 points)

(2) Visualize CIs with an error bar plot (Hint: check `matplotlib.pyplot.errorbar()`), with sample ID on horizontal axis, and CI on vertical axis. Indicate the location of true mean μ. (5 points)

(3) How many CIs failed to capture μ? Distinguish them in the error bar plot. (5 points)

Problem 2 (15 points + 5 points)

Suppose we wish to test the hypothesis:

$$H_0 : \mu = 20 \qquad H_1 : \mu \neq 20$$

It is given that $\sigma^2 = 4.0$, true mean $\mu_1 = 16$, and $\alpha = 0.05$.

(1) What sample size n must be used to ensure that the probability of type II error is no greater than 0.1? (5 points)

(2) Under the sample size calculated in (1), what is the resulting type II error? (5 points)

(3) 🖳 Use software to plot type II error vs. sample size n. Do you observe a monotonic trend? (5 points)

(4) [Bonus] What is the probability of type II error when the sample size approaches infinity, i.e., $n \to \infty$? What does this imply? (5 points)

Problem 3 (10 points + 5 points)

(1) What is the type I error for X-bar control charts with probability limit of 0.001 and sample size of 4? (5 points)

(2) Assuming the mean shift is 1.5σ, what is the type II error for detecting this mean shift by using this control chart? (5 points)

(3) [Bonus] 🖳 Use software to generate a plot for type II error vs. mean shift. (5 points)

Problem 4 (30 points)

An X-bar control chart with 1% probability limit is used to monitor process mean. Inspection decision is made based on four successive samples with sample size n = 9 using the following rules:

Rule 1: If the mean of any sample exceeds the upper or lower control limit.
Rule 2: If the sample means of four samples in a row fall on the same side of the center line.

(1) What is type I error rate using Rule 1 alone? (5 points)

(2) What is type I error rate using Rule 2 alone? (5 points)

(3) If the process has a mean shift of 1σ (one process standard deviation), what is type II error rate using Rule 1? (5 points)

(4) If the process has a mean shift of 1σ, what is type II error rate using Rule 2? (5 points)

(5) 🖳 Use software to generate plots of type II error rate vs. mean shift vs. for both Rules 1 and 2. Please provide a figure with both plots and provide a legend. (5 points)

(6) Based on the above results, discuss the pros and cons of Rules 1 and 2. Note that this is an open-ended question. Any answer that is well justified will receive full credit. (5 points)

Problem 5 (30 points)

For a 3-sigma $\bar{X}$ chart with $n = 9$, it is given that UCL = 104, LCL = 96. If a 2σ shift occurs in the process mean,

(1) What is the detection power of this control chart? (10 points)

(2) What is the probability of detecting the shift by the 3rd sample after the shift? (10 points)

(3) What is expected length to detect this change? (10 points)