ME 453: Data Science in Manufacturing Quality Control
Homework 3

Assigned: October 27, 2023 Due: November 10, 2023

Homework guidelines:

1. The total number of points is 100. There are also 10 bonus points available from the bonus problems. The assigned points are given beside questions. To get full credit you must SHOW ALL OF YOUR WORK.
2. Bonus points will be used to compensate for lost points in homework assignments.
3. 💻 Indicates the problem which you need to use Python.
4. A complete submission features the following items: (a) a brief report including all figures and results, and explanations of necessary steps taken to obtain them; and (b) and the source code (Python is recommended).
5. Item (a) can be scanned copies of handwritten or typeset reports. Both items shall be submitted through Canvas.

Review the following topics discussed in lecture:

- Control Charts for Variables (lecture videos 11, 12).

Problem 1(20 points + 3 points)

A factory plans to use a Shewhart control chart to monitor one quality characteristic of a product. It is known that the quality characteristic follows a normal distribution N(100, 16). The quality control team proposes two designs for the control chart as follows:

Option 1: 3-sigma limits, sample size = 16
Option 2: probability limits with p = 0.001, sample size = 25

(1) Calculate Type I error rate for both control charts. (4 points)

(2) Calculate the in-control average run length (ARL) for both control charts. (4 points)

Assume that the process mean changes to 104 mm, i.e., one-sigma mean shift. Complete the following questions.

(3) Calculate the type II error rate for both control charts. (4 points)

(4) Calculate the out-of-control ARL for both control charts. (4 points)

(5) Assume that one-sigma mean shift is the most commonly encountered out-of-control scenario, discuss the pros and cons of both control charts. (4 points)

(6) [Bonus] 💻 Use software to generate an operating characteristic (OC) curve for each control chart. To facilitate better comparison, plot two OCs in a single graph and provide a legend. (3 points)

Problem 2 (20 points)

Suppose you would like to track and monitor the amount of exercise you take per day using the step counter in your smartphone. If you don't have access to such data, you can use the data provided on Canvas `StepCounter.csv`.

(1) Which control charts should be used? Justify your answer. (5 points)

(2) 💻 Establish Phase I trial limits for the control charts you selected in (1). Use step counts from 20 workdays: Aug. 22 - 26, Aug. 29 - Sept. 2, Sept. 5 - 9, and Sept. 12 - 16. If any out-of-control points are present, assume that you can identify the root causes and eliminate these points. (10 points)

(3) 💻 Use the control limits obtained in (2) to monitor your steps on Sept. 19 - 23 and Sept. 26 - 30 (Phase II). Report your observations. (5 points)

Problem 3 (15 points + 3 points)

Control charts for $\bar{X}$ and R are to be established to control the tensile strength of a metal part. Assume that tensile strength is normally distributed. Thirty samples of size n = 6 parts are collected over a period of time with the following results:

$$\sum_{i=1}^{30} \bar{X}_i = 6000 \text{ and } \sum_{i=1}^{30} R_i = 150$$

(1) Use 3-sigma to calculate control limits for $\bar{X}$ and R. (6 points)

(2) Estimate the process mean and standard deviation when the process in control. (4 points)

(3) For the above $\bar{X}$ chart, find the β-risk when the true process mean is 199 and the standard deviation remains unchanged. (5 points)

(4) [Bonus] 💻 Perform a simulation study on β-risk obtained from (3) following the steps listed below. (3 points)

   (a) Generate 1000 samples with sample size n=6 using $X \sim N(199, \hat{\sigma}^2)$, where $\hat{\sigma}$ is obtained from (2);

   (b) For each sample, compute $\bar{X}$ and check if it falls within the control limits;

   (c) Count how many samples fall within the control limits and calculate the β error rate;

   (d) Compare the β error rate obtained in (c) with the expected β-risk obtained from (3).

Problem 4 (15 points)

A process is to be monitored with distribution parameters $\mu = 10$ and $\sigma = 2.5$. The sample size is $n = 4$.

(1) Find the center line and the 3-sigma control limits for the S chart. (9 points)

(2) Find the 2-sigma warning limits for the S chart. (6 points)

Problem 5 (30 points + 4 points)

Revisit the lecture example at P.33 in Topic 5:
Construct 3-sigma $\bar{X}$ and S control charts using the data in attached `sample.csv` file.
💻 Use Python to calculate the information you need.

(1) What is the estimated standard deviation of the process? (5 points)

(2) What are the control limits in $\bar{X}$ chart? (5 points)

(3) What are the control limits in S chart? (5 points)

(4) Are there any out-of-control points in either chart? What are the final Phase I control limits? (5 points)

(5) Use the control charts obtained in (4) for Phase II monitoring. Use data provided in `additional sample.csv`. Are there any out-of-control samples? If so, specify the sample number and which control charts detects the out-of-control. (5 points)

(6) Compare your results with that of $\bar{X} - R$ charts obtained in the class example. (5 points)

(7) [Bonus] 💻 Use software to visualize $\bar{X}$ and S control charts for both Phase I and Phase II and mark out-of-control samples, if any. See slide P.38 in Topic 5 lecture notes for an example. (4 points)