Problem 1

(3) From the error bar plot, I find there is no CI failed to capture μ.

Problem 2.

(1). $H_0: \mu = 20 \quad H_1: \mu \neq 20. \quad \sigma^2 = 4.0 \quad \mu_1 = 16 \quad \alpha = 0.05$

$\beta = \Phi(Z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}) - \Phi(-Z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma})$

$= \Phi(Z_{0.025} - \frac{(-4)\sqrt{n}}{2}) - \Phi(-Z_{0.025} - \frac{(-4)\sqrt{n}}{2})$.

$= \Phi(1.96 + 2\sqrt{n}) - \Phi(2\sqrt{n} - 1.96) \leq 0.1$

We can get : $n = 3$.

$\beta = \Phi(1.96 + 2\sqrt{3}) - \Phi(-1.96 + 2\sqrt{3}) = 0.06627\cdots$

(2) From the previous problem:

$\beta = \Phi(1.96 + 2\sqrt{3}) - \Phi(-1.96 + 2\sqrt{3}) = 0.06627\cdots$

(4). When $n \to \infty$, type II error $\to 0$.

Because when the $n \uparrow$, the CI will approach to 0. In that case, the probility of failing to reject a false null hypothesis will reduce. So the type II error approaches to 0.

Problem 3.

(1). The type I error will be 0.001.

(2) $\beta = Pr(LCL \leq \bar{X} \leq UCL \mid \bar{X} \sim N(\mu_0, \frac{\sigma_0^2}{n}))$

$= Pr(\frac{LCL - \mu}{\sigma_0/\sqrt{n}} \leq \frac{\bar{X} - \mu}{\sigma_0/\sqrt{n}} \leq \frac{UCL - \mu}{\sigma_0/\sqrt{n}})$

$= Pr(\frac{\mu_0 - k\sigma_0/\sqrt{n} - \mu}{\sigma_0/\sqrt{n}} \leq Z \leq \frac{\mu_0 + k\sigma_0/\sqrt{n} - \mu}{\sigma_0/\sqrt{n}})$

$= Pr(-k - \frac{\delta}{\sigma_0/\sqrt{n}} \leq Z \leq k - \frac{\delta}{\sigma_0/\sqrt{n}})$

$= \Phi(k - \frac{\delta}{\sigma_0/\sqrt{n}}) - \Phi(-k - \frac{\delta}{\sigma_0/\sqrt{n}})$

$k = 3.29 \quad \delta = 1.5\sigma \quad n = 4$.

$\therefore \beta = 0.614$

Problem 4.

(1) type I error is 0.01.

(2) type I error rate is $P = 2 * (\frac{1}{2})^4 = \frac{1}{8}$

(3) $\beta = Pr\{ LCL \leq \bar{X} \leq UCL \mid \bar{X} \sim N(\mu_1, \sigma_0^2/n) \}$.

$= Pr\{ \frac{LCL - \mu_1}{\sigma_0/\sqrt{n}} \leq \frac{\bar{X} - \mu_1}{\sigma_0/\sqrt{n}} \leq \frac{UCL - \mu_1}{\sigma_0/\sqrt{n}} \}$.

$= Pr\{ \frac{(\mu_0 - k\sigma_0/\sqrt{n}) - \mu_1}{\sigma_0/\sqrt{n}} \leq \frac{\bar{X} - \mu_1}{\sigma_0/\sqrt{n}} \leq \frac{UCL - \mu_1}{\sigma_0/\sqrt{n}} \}$.

$= Pr\{ -k - \frac{\delta}{\sigma_0/\sqrt{n}} \leq Z \leq k - \frac{\delta}{\sigma_0/\sqrt{n}} \} = \Phi(k - \frac{\delta}{\sigma_0/\sqrt{n}}) - \Phi(-k - \frac{\delta}{\sigma_0/\sqrt{n}})$

Now shift is 1σ. $k = Z_{\alpha/2} = 1.64$ $n = 9$

$\beta = \Phi(1.64 - 3) - \Phi(-1.64 - 3) = 0.335 \cdots$

(4) $Z_{\alpha/2} = 1.534 \cdots$ $k = 1.53$

$\therefore \beta = \Phi(1.53 - 3) - \Phi(-1.53 - 3) = 0.0713 \cdots$

(6). ① We will detect larger error rate when mean shif is small. when using rule 1

② But we will find large difference of error rate when mean shif is change when using rule 2.

Problem 5

(1) $UCL = \mu_w + k\sigma_w = 104$

$CL = \mu_w$ $\Rightarrow CL = \mu_w = 100$

$LCL = \mu_w - k\sigma_w = 96$

$Z = \frac{\delta}{\sigma_0/\sqrt{n}} = 6$ $\Rightarrow$ detection power ≈ 1.

(2) $\beta = \Phi(k - \frac{\delta}{\sigma_0/\sqrt{n}}) - \Phi(-k - \frac{\delta}{\sigma_0/\sqrt{n}})$ $k = 3$

$\therefore \beta = \Phi(-3) - \Phi(-9) = 0.000134 \cdots$

$\therefore$ probability of detecting the shift by 3rd sample $= 1 - \beta^3 \approx 1$.

(3). $ARL_1 = \frac{1}{1-\beta} \approx 1.00135 \cdots$