A Level Mathematics - Mechanics

Xingzhi Lu (2129570@concordcollege.org.uk)

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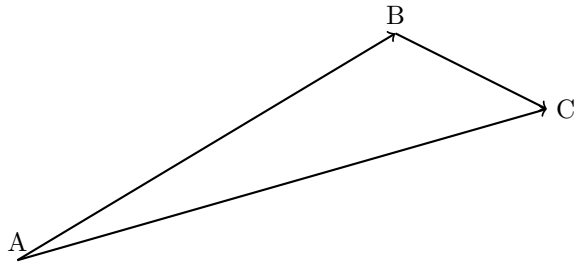
1 Vectors

1.1 Calculations

- $\vec{a} = \vec{a_x} + \vec{a_y}$
- $|\vec{a_x}| = |\vec{a}| \cos\theta$
- $|\vec{a_y}| = |\vec{a}| \sin\theta$
- $\tan\theta = \dfrac{|\vec{a_y}|}{|\vec{a_x}|}$
- $|\vec{a}|^2 = |\vec{a_x}|^2 + |\vec{a_y}|^2$
- $\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta = x_1x_2 + y_1y_2$

  If $a \perp b$: $\vec{a} \cdot \vec{b} = 0$
- $\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$
- Unit vector (magnitude = 1) $= \dfrac{\vec{a}}{|\vec{a}|}$

1.2 Find the resultant of two vectors



$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$
$|\overrightarrow{AC}|$ can be found by sine or cosine rule

2 Forces and motion

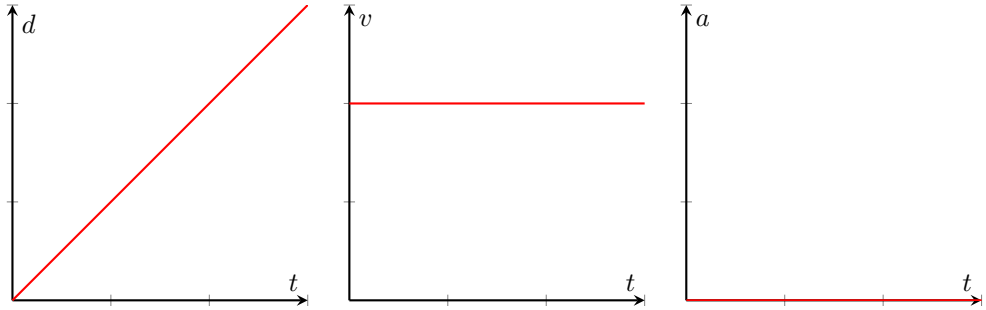
2.1 Types of motion

2.1.1 Constant speed motion

Calculations:

- v is constant, $a = 0$
- $d = vt$

Motion graphs:



2.1.2 Uniform acceleration motion

Calculations:

- $d = v_i t + \frac{1}{2}at^2$
- $v_f = v_i + at$
- $v_f^2 = v_i^2 = 2as$
- $d = \overline{v}t$
- $\overline{v} = \frac{v_i + v_f}{2}$

2.1.3 Free fall

Air resistance is ignored, so $a = g$
Calculations:

- $v_i = 0$
- $v_f = gt$
- $h = \frac{1}{2}gt^2$

2.1.4 Vertically upward

Calculations:

- $v = u - gt$

Rising and falling at the same height: speed same, opposite direction

2.1.5 Projectile

Calculations:

- $y = \tan\theta x - \frac{g}{2u^2}(1 + \tan^2\theta)x^2$
- range $= \frac{u^2 \sin 2\theta}{g}$
- greatest height: $\frac{u^2 \sin^2\theta}{2g}$

- Time to flight (back to x-axis) = $\dfrac{2u\sin\theta}{g}$
- Time to greatest height: $\dfrac{u\sin\theta}{g}$

2.2 Types of forces

Weight: $W = mg$

Normal contact force: symbol = R or N

Static friction: Depends on driving force, $F \leq \mu R$

Dynamic friction: $f = \mu R$ (μ=coefficient of kinetic friction)

Tension: T = elastic coefficient $\times$ extension = $k \times \Delta x$

2.3 Common scenarios

2.3.1 Lift

Rising: $R - W = ma$

Moving down: $W - R = ma$

On rest: $R = W$

2.3.2 Slope

- Coordinate: centre = object, x-axis = slope surface, y-axis = perpendicular to slope surface
- Calculate resultant force in x and y direction
- Use SUVAT equations to find distance / speed / time

2.3.3 One whole system

e.g. on a train / car

- Acceleration is the same across the whole system
- Internal force can be ignored
- Tension at the same rope has the same magnitude

2.3.4 Fixed pulley

- Same tension
- Same magnitude for acceleration (different direction)
- Use simultaneous equations to find tension

3 Momentum

3.1 Definitions

- $p = mv$
- $\Delta p = m\Delta v = mat = Ft$
- Impulse $= Ft = \delta p = m(v_f - v_i)$

3.2 Collision

Elastic: KE conserved

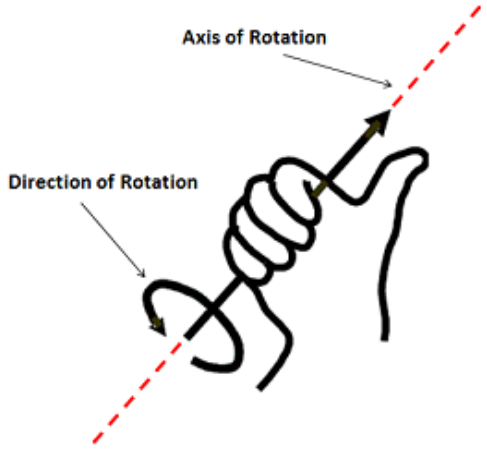
Inelastic: KE not conserved

4 Moments

4.1 Definition

Turning effect of the force on a rigid body.
Clockwise moment of F about P: $|F| \times d = \vec{F} \times \vec{d} = |F||d| \sin\theta$

4.2 Right hand rule



$\vec{a} \times \vec{b}$= from $\vec{a}$ to $\vec{b}$
$\vec{b} \times \vec{a}$= from $\vec{b}$ to $\vec{a}$

4.3 Tilting about a pivot

Support / tension force at any point = 0

5 Common questions

5.1 Projectile

5.1.1 Asking for improvements

- Ball modelled as particle / no volume
- Air resistance ignored
- Ball doesn't spin / spin ignored

5.2 Moment

5.2.1 Plank modelling

Modelled as rod: Plank remains straight

Mass modelled as particles: Weights act at the ends of plank