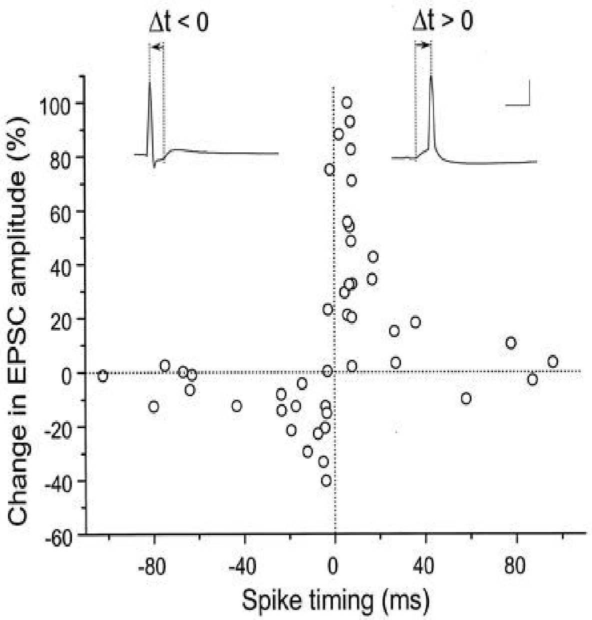
Learning and Memory in an Exactly Solvable Stochastic Spiking Network

Surya Ganguli

Sloan-Swartz Center for Theoretical Neurobiology

UCSF

Problem: Understand the Cooperative Dynamics of Many Synapses in a Network



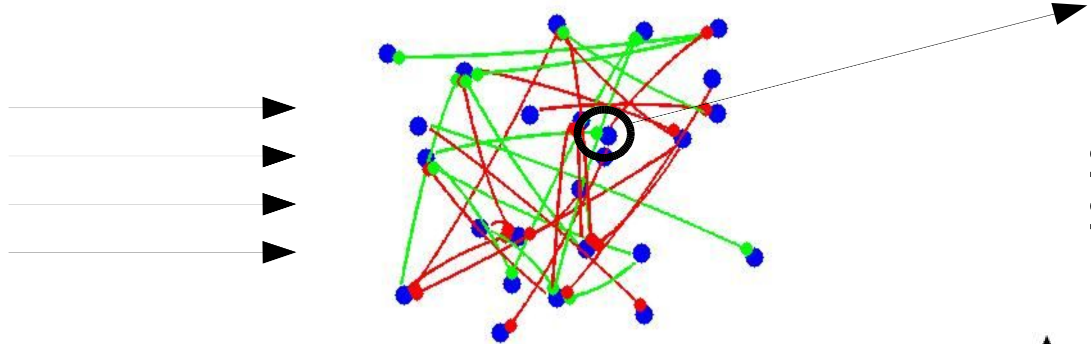
Bi & Poo 98

Memory = Stable patterns of many synaptic efficacies in a spiking network.

Learning = Collective changes in these patterns in response to new input correlations.

One plasticity rule for a single synapse.

The Complexity of the Problem



To LTP or to LTD??

Correlated Inputs: C_{ab}

Recurrent Dynamics: w_{ij}

Synaptic Dynamics: Slow
Spiking Dynamics: Fast

$$\Delta w_{ij} \sim \langle \rho_i(t)\rho_j(t+\tau)\rangle$$

$$\langle \rho_i(t)\rho_j(t+\tau)\rangle = F_{ij}[\{C_{ab}, w_{kl}\}]$$

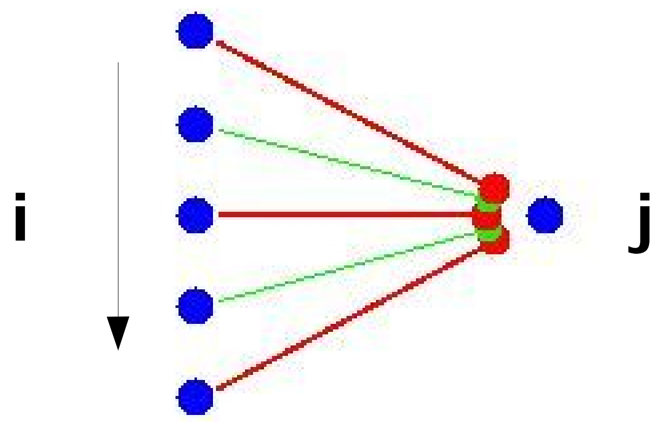
???

$$\{C_{ab}, w_{kl}\} \qquad \langle \rho_i(t)\rho_j(t+\tau)\rangle$$

$$\Delta w_{ij}$$

The Synapse-Spike Waltz

The Model



If neuron i sends a spike to neuron j at time t,
then neuron j will fire at time $t+\epsilon$ with probability p_{ij}

If simultaneous spikes arrive at neuron j on different synapses, the probability neuron j will fire sums linearly.

Neurons are independent, conditioned on past inputs.

Linear Poisson neuron (Kistler and van Hemmen 99
Gutig, et. al. 03)

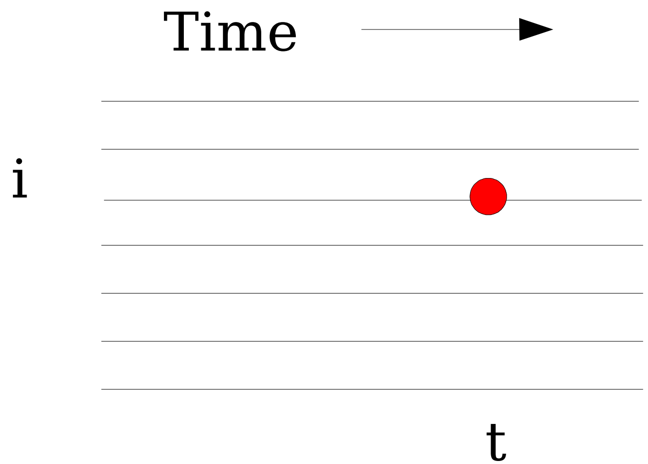
The First Order Statistics: Average Firing Probability

Imagine N neurons driven by uncorrelated poisson inputs at rate λ_i. Calculate the firing rate r_i of neuron i.

$r_i \Delta t \equiv \langle \rho_i(t) \rangle \Delta t$ = Prob neuron i spikes in window t..t+Δt.

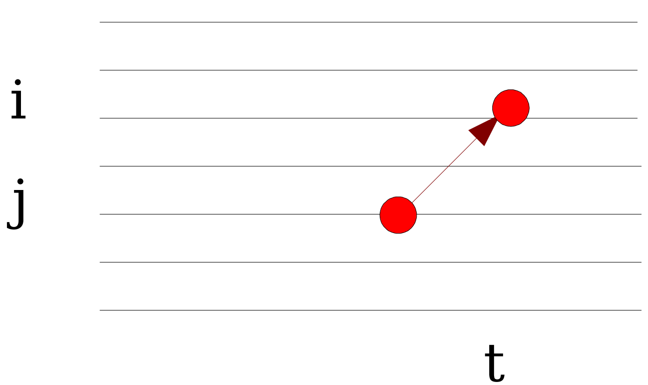
Enumerate all the ways neuron i could spike around time t, calculate each of these probabilties, and sum them up.

What are all the ways neuron i could fire at t?



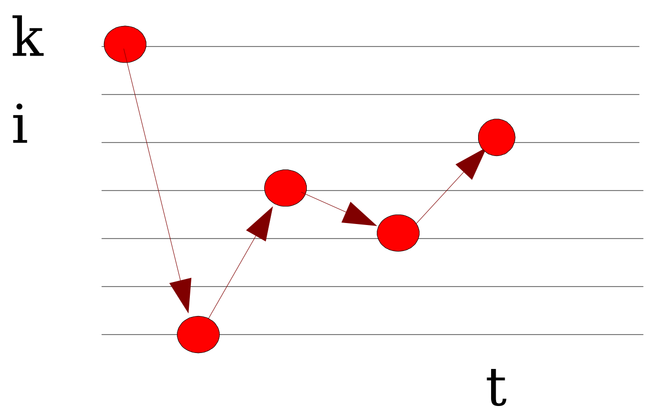
Neuron i receives an input around t-ε and then fires at t.

Probability: $\lambda_i \Delta t$.



Neuron j receives an input around t-2ε, fires at time t-ε, and this spike causes neuron i to fire at time t.

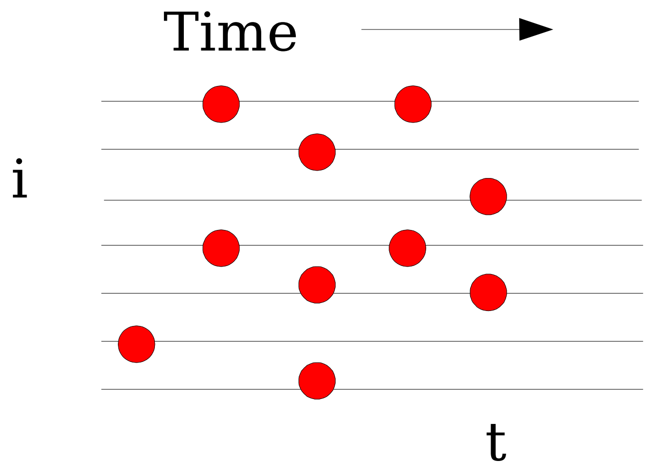
Probability: $\lambda_j \, p_{ji} \Delta t$.



Neuron k fires in the distant past at time t-nε, due to input at k, and it causally influences i to spike at time t through a network path from k to i.

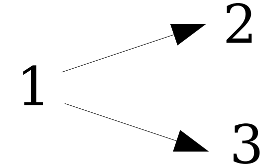
Probability: λ_k * (path from k to i) * Δt .

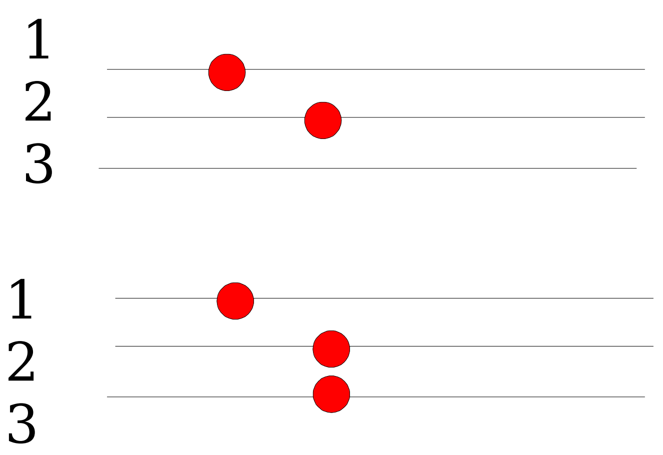
Contributions from simultaneous spikes



In principle, we should sum over an exponential number of patterns at each step. When we do, they all cancel, leaving only the single spike paths seen previously!

Simple Three Neuron Example:



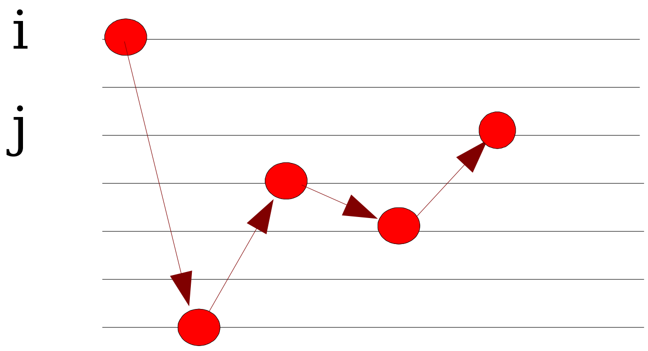


$\lambda_1 p_{12}(1-p_{13})\Delta t$

$\lambda_1 p_{12}(p_{13})\Delta t$

$\lambda_1 p_{12}\Delta t$

The Upshot: Average Firing Prob = Sum over Paths



$[P^n]_{ij}$ = Weighted sum of all paths of length n through the network starting at i and ending at j.

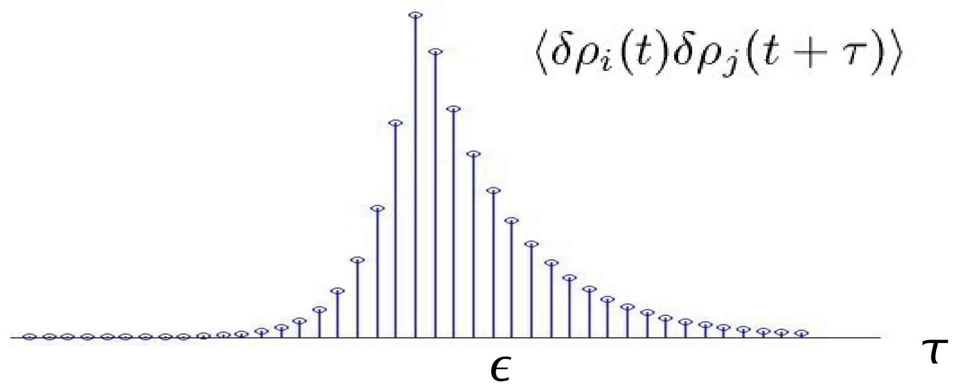
$$\vec{r} = \vec{\rho}(t) = \vec{\lambda}(1 + P + P^2 + P^3 + \cdots)$$

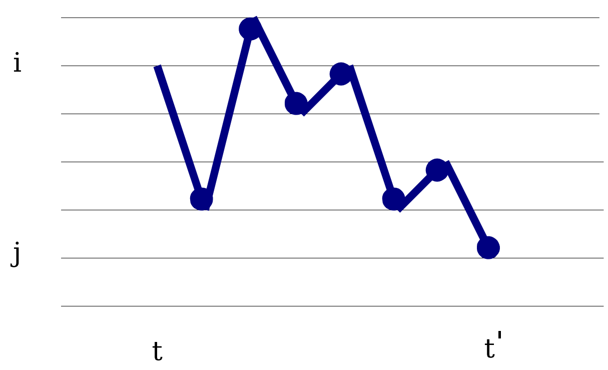
$$= \vec{\lambda}\frac{1}{1-P}$$

This is independent of the latency and is the same answer you you get from a rate model!

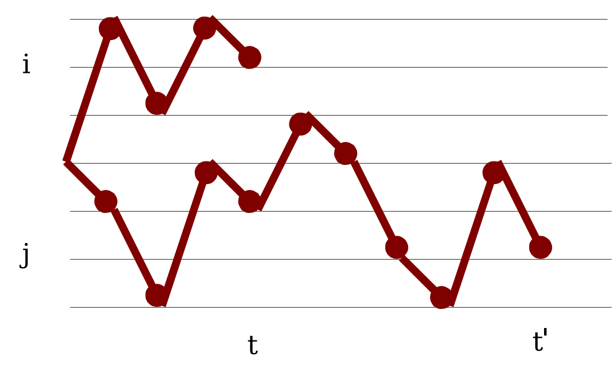
Second Order: The Spike Train Cross Correlation Function

$$\langle \rho_i(t)\rho_j(t+\tau)\rangle = r_i r_j + \langle \delta\rho_i(t)\delta\rho_j(t+\tau)\rangle$$





Excess correlation due to direct interaction through the network.



Excess correlation due to common drive.

One Application: The Formation and Stability of Memories with Binary Synapses

Symmetry between learning and forgetting of everyday memories:

Learn quickly => Plastic Synapses => Forget Quickly
Forget Slowly => Rigid Synapses => Learn Slowly

Forgetting = Diffusion on the space of synaptic patterns due to on going spiking activity: synapses fluctuate until the initial memory trace is lost....

Fusi, Drew, Abbott 05: Multiple time scales of synaptic plasticity.

Complementary mechanism: Fluctuations in synapses are correlated with each other because they talk to each other using spikes.

Such correlated fluctuations can yield long time scales: synapses cooperate to keep the memory trace alive.

Short time scale fluctuations can also occur, but only to synaptic patterns or memories that are correlated with the original memory.

Mathematical Realization

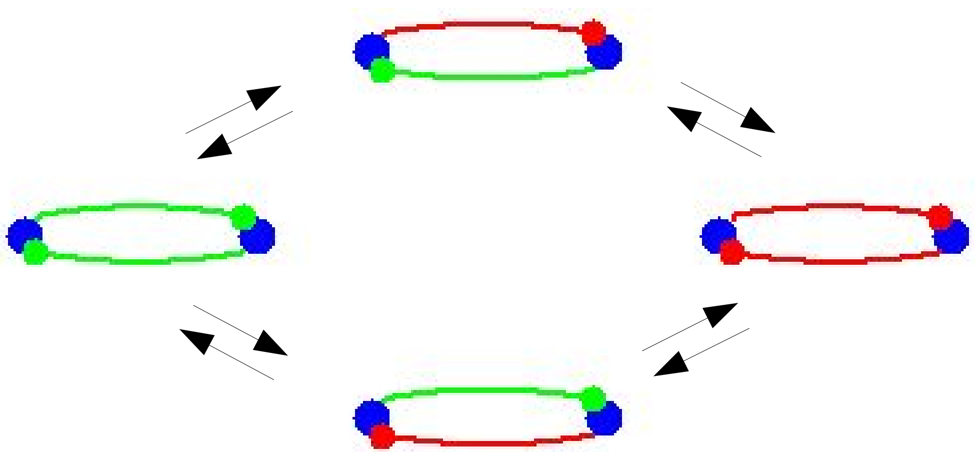
N neurons

S binary synapses $(0 < S < N^2 - N)$ (no autapses)

2^S synaptic patterns (memories)

Each memory can stochastically transition to one of S neighboring memories.

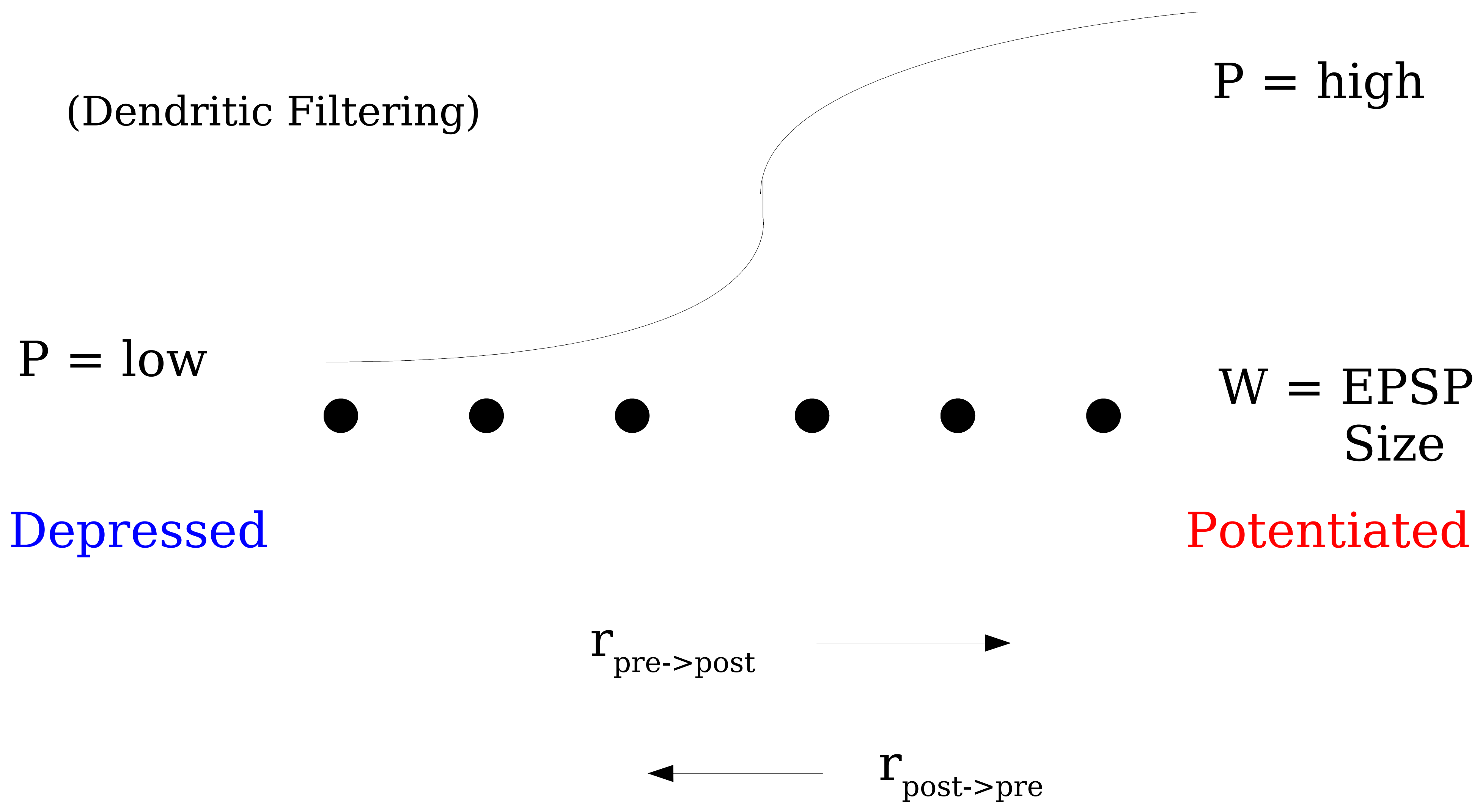
Example: 2 neuron network, 2 synapses, 4 memories:



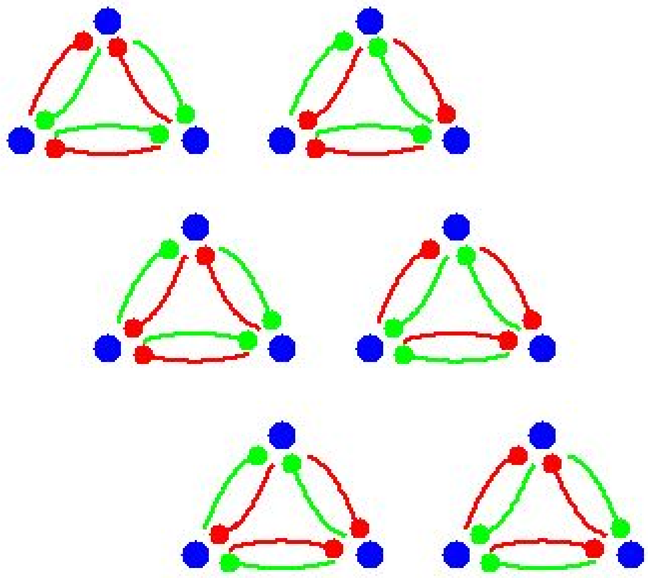
What is the stationary distribution?

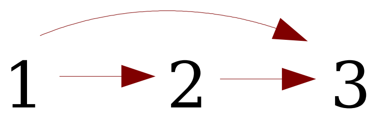
What are the eigenvalues? (Decay rates -> long time scales?)

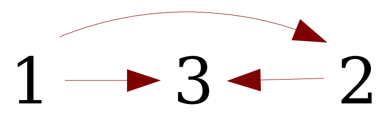
Coupling Binary Synapses to Pre/Postsynaptic Spikes

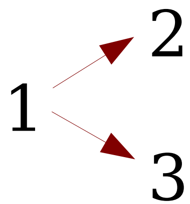


The Topological Origins of Stable Memories

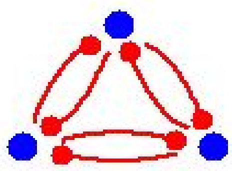






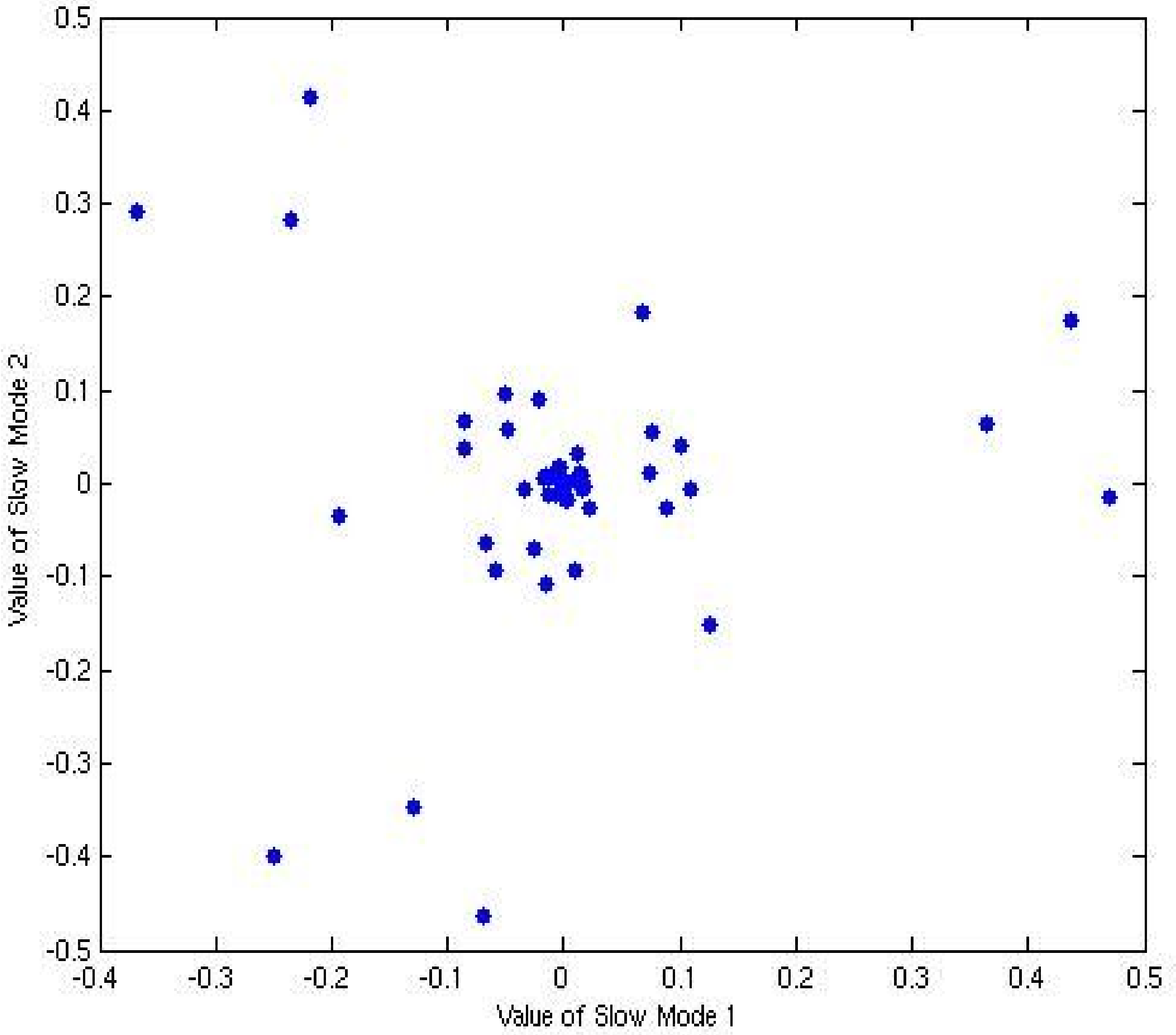


N=3
S=6
64 memories



The most unstable memory

Hierarchial Structure on the Space of Memories induced by STDP



Acknowledgements

UCSF

Loren Frank
Philip Sabes

Sen Cheng
Yuri Dabhagian
Leslie Osborne
Brian Wright

Columbia

Larry Abbott
Stefano Fusi
Ken Miller
Michael Eisele