

# A Theory of Neural Dimensionality and Measurement

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## Summary

In many experiments, neuroscientists tightly control behavior, record many trials, and obtain trial-averaged firing rates from hundreds of neurons in circuits containing millions of behaviorally relevant neurons. Dimensionality reduction has often shown that such datasets are strikingly simple; they can be described using a much smaller number of dimensions (principal components (PCs)) than the number of recorded neurons, and the resulting projections onto these components yield a remarkably insightful dynamical portrait of circuit computation.

This ubiquitous simplicity raises several profound and timely conceptual questions. What is the origin of this simplicity and its implications for the complexity of brain dynamics? Would neuronal datasets become more complex if we recorded more neurons? How and when can we trust dynamical portraits obtained from only hundreds of neurons in circuits containing millions of neurons? We present a theory that answers these questions, and test it using data from reaching monkeys.

We derive a theoretical upper bound on the dimensionality of data. Our bound has a natural interpretation as a quantitative measure of task complexity. Interestingly, the dimensionality of motor cortical data is close to this bound, indicating neural activity is as complex as possible, given task constraints. Our theory provides a general analytic framework to ascertain whether neural dimensionality is constrained by task complexity or intrinsic brain dynamics, furthering our ability to interpret large-scale datasets.

We also describe sufficient conditions on PCs underlying neural activity so that low dimensional dynamical portraits remain unchanged as we record more neurons, and show that they are satisfied by motor cortical data. This theory yields a picture of the neural measurement process as a random projection of neural dynamics, conceptual insights into how we can reliably recover dynamical portraits in such under-sampled measurement regimes, and quantitative guidelines for the design of future experiments.