Logic Final Assessment

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1 Q1

$$\overline{Fa \vdash Fa}\ A \qquad \overline{Fb \vdash Fb}\ A$$

$$\vdash Fa, Fa, Fb \qquad \overline{Fab, A \vdash A, Rba}\ A \qquad \overline{Fab, A \vdash A\cancel{\phi}, Fb}\ A$$

$$Fa \to Rab \vdash Fa, Fb, Rba \ \vdash\vee \qquad Fab, Fb \vdash \qquad \overline{Rba \vdash Rba}\ A$$

$$\vee\vdash$$

$$Fa \to Rab \vdash Fa, Fb \vee Rba \qquad Fb \vee Rba, Fab \vdash \quad \to\vdash$$

$$Fa \to Rab, Fb \vee Rba, Fa \to Fab \vdash Fb \vee \neg Rba \quad \forall\vdash$$

$$\forall y(Fa \to Ray), \forall y(Fy \vee Rya), Fa \to Fab \vdash Fb \vee \neg Rba \quad \forall\vdash$$

$$\forall x \forall y(Fx \to Rxy), \forall x \forall y(Fy \vee Ryx), Fa \to Fab \vdash Fb \vee \neg Rba \quad \vdash\to$$

$$\forall x \forall y(Fx \to Rxy), \forall x \forall y(Fy \vee Ryx) \vdash (\cancel{Fb}\, Fa \to Rab) \to (Fb \vee \neg Rba) \quad \vdash\forall$$

$$\forall x \forall y(Fx \to Rxy), \forall x \forall y(Fy \vee Ryx) \vdash \forall y((Fa \to Ray) \to (Fy \vee \neg Rya)) \quad \vdash\forall$$

$$\forall x \forall y(Fx \to Rxy), \forall x \forall y(Fy \vee Ryx) \vdash \forall x \forall y((Fx \to Rxy) \to (Fy \vee \neg Ryx))$$

2 Q2

Intuitionistic logic restricts sequents to have only a single conclusion. This reflects that it is more natural than multiple-conclusion sequent calculus from a logical point of view, since arguments in natural language usually have only one conclusion, and ensure that every conclusion can be constructed.

3 Q3

3.1

$p \to (p \to q) \vdash p \to q$
invalid in fuzzy logic: suppose p is i and q is false, this assignment gives to$p \to (p \to q)$ is true, but gives to $(p \to q)$ is false.
valid in relevant logic: in any assignment of values, the value of $(p \to q)$ is at least good as the value of $p \to (p \to q)$.

3.1.1

$\neg\neg p \vdash p$
invalid in intuitionistic logic: 1. Assign p the value of i. 2. $\neg p$ now has a value of F. 3. Therefore $\neg\neg p$ has value T, but p has a value of i.
valid in fuzzy logic: this sequent is valid iff any assignment of values that gives T to $\neg\neg p$ also gives T to p, and any assignment of values that gives F to p gives F to $\neg\neg p$.

3.1.2

$(p \wedge q) \to r \vdash p \to (q \to r)$
invalid in relevant logic: If we assign T to p and i to both q and r, it is not valid in the three-valued semantics for relevant logic because the premise of the sequent is i, while the conclusion is F. **valid in intuitionistic logic:** every interpretation that makes $(p \wedge q) \to r$ true also makes $p \to (q \to r)$ true.

4 Q4

Relevant logic is a logical system focused on the relevance among assumptions and conclusions, which is crucial for logical soundness and completeness. Here are detailed arguments on why relevant logic might be considered superior to classical logic, fuzzy logic, and intuitionistic logic.

4.1 Superiority Over Classical Logic

Imagine that we have these statements,

p: It is snowy
¬p: It is not snowy
r: Humans are carbon-based organisms

4.1.1 The Principle of Explosion

Classical logic allows that conclusions are irrelevant to promises, which leads to logical paradoxes and meaningless arguments, since it suffers from the principle of explosion. From p and $\neg q$ we can derive contradiction($\perp$), and classical logic would allow us to deduce any conclusion through natural deduction. So r is legal in classical logic, even

the content inside is not relevant to p.
Relevant logic addresses the principle of explosion in classical logic because it is paraconsistent, which lead to avoid the paradoxes of implication. Vacuous discharge would not appear. In relevant logic, we consider that assumptions are combined with each other. This ensures a more robust system and realistic logical application. In this way, contradictions would not bring arbitrary conclusions anymore.

4.1.2 Anti-Intuitive

Consider the implication $A \to B$, which is the statement *"if A, then B"*. This implication is valid if A is really relevant to B based on our intuitions. However, classical logic often leads to anti-intuitive conclusions from assumptions. In classical, a false premise is always true in the conclusion. This can be anti-intuitive, because we can always get irrelevant conclusion as long as the premise is true. for instance, suppose A is the statement *"it is snowy"* and B is the statement *"Humans are carbon-based organisms"*. If it is sunny (not snowy) in reality, then humans are carbon-based organisms. The compound statement above is true in classical logic because A is a false premise. It is natural for us to find this is very anti-intuitive. It does not sound like natural human language.
Relevant logic addressed this awkward situation by requiring the relevance between premises and conclusions. In same implication $A \to B$, suppose A is the statement *"it is snowy"* and B is the statement *"I should wear more clothes"*. This makes the logical implications more intuitive in actual inferential relationships, as well as easier to be understand because it is closer to natural language.

4.2 Superiority Over Fuzzy Logic

Except from classical logic, there are three values in non-classical logic. Each of them is different from others. Fuzzy logic gives a logical account of vague predicates.

4.2.1 Truth Value of Fuzzy Logic And Relevant Logic

Consider about the implication in fuzzy logic:

$\to$	T	i	F
T	T	i	F
i	T	T	i
F	T	T	T

Figure 1: Implication for fuzzy logic[Maruyama, 2024b].

We may think of the intermediate value i as "half-true" (same as "half-false"). This approach is useful for dealing with vagueness. In this case, if the consequent is at least as good as the antecedent, the implication is true. The consequent is a half-step down from the antecedent leads to half-true conclusion. Moreover, i(half-true) conclusions appear more in fuzzy logic than in relevant logic. The comparison will be shown in next sub-subsection. Consider about the implication in relevant logic:

→	T	i	F
T	T	F	F
i	T	i	F
F	T	T	T

Figure 2: Implication for relevant logic[Maruyama, 2024a].

We consider i to be a "confused" value between true and false. T is better than i, and i is better than F. And we can recognise that if there is an i appear in the semantics, and there exist T or F, we consider i as opposite (i.e., i is true when there exist$F$ and vice versa). For instance, in the truth table we can see that $i \to F$ is false. More deeper, it is easily to find that the truth values in relevant logic follows the rules in classical logic under this regulation.

4.2.2 Argument for Relevant Logic over Fuzzy Logic

Fuzzy logic gives a method to deal with vagueness, especially in Sorites-type paradoxes. The truth value can be a intermediate value between completely true and completely false. However, this can make it more complicated and ambiguous to think about things because the probability to half-true(half-false) conclusion is higher than in relevant logic. On the other hand, unlike fuzzy logic, although the truth value of i is "confused", it still means binary truth values. After inference, we can find that the truth value of fuzzy logic conforms to classical logic to some extent. This is not only simpler and more certain, but easier for learners to understand and mastered based on classical logic.

4.3 Superiority Over Intuitionistic Logic

Intuitionistic logic emphasize on the "construction", which means if something exists, we must know what that "something" really is. And then we can justify the existence.

4.3.1 the Law of the Excluded Middle

"The law in classical logic stating that one of the two statements *"A"* or *"not A"* is true. The law of the excluded middle is expressed in mathematical logic by the formula $A \vee \neg A$[Brouwer and van Dalen, 1981]." So the law of excluded middle can be expressed as: Every statement is either true or false.
Suppose A to be *"it is snowy"*, then $A \vee \neg A$ is the statement *"it is snowy or it is not snowy"*. Intuitionistic logic does not accept the law of excluded middle as its general

principle, so $A \vee \neg A$ is invalid. $A \vee \neg A$ is only valid if there is a constructive proof for A or a constructive proof for $\neg A$. However, relevant logic does not reject the law of the excluded middle so $A \vee \neg A$ is valid. This aligns the expressive power of classical logic while adding a layer of relevance. Moreover, it is more intuitive in human language.

4.3.2 Approach to Contradiction

Relevant logic is paraconsistent logic. It ensures that a contradiction $A \vee \neg A$ does not lead to explosion, i.e., it does not imply any arbitrary statement B. Intuitionistic logic does not handle contradictions in a paraconsistent manner. The existence of both A and $\neg A$ is problematic, and the system requires handle such contradictions. We don't really need the negation rules in intuitionistic logic, since we have a equivalence between $\neg A$ and $A \to \perp$. This equivalence does not inherently support paraconsistency. Paraconsistency provides a more flexible logical system for dealing with contradictions.

References

[Brouwer and van Dalen, 1981] Brouwer, L. and van Dalen, D. (1981). *Brouwer's Cambridge Lectures on Intuitionism.* Cambridge University Press.

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