Logic Mid-term Assessment

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1 Q1

1.1 sequent

$$\forall x(Fx \land \neg Gx) \vdash \neg\exists x(Fx \to Gx) \land \exists x Fx$$

1.2 semantic tableaux

$\forall x(Fx \land \neg Gx) \vdash \neg\exists x(Fx \to Gx) \land \exists x Fx$

(1) T: $\forall x(Fx \land \neg Gx)$ ✓

(2) F: $\neg\exists x(Fx \to Gx) \land \exists x Fx$ ✓

(3) F: $\neg\exists x(Fx \to Gx)$ ✓ from (2)	(4) F: $\exists x Fx$ from (2)
(5) T: $\exists x(Fx \to Gx)$ ✓ from (3)	(13) F: Fa from (4)
(6) T: $Fa \to Ga$ ✓ from (5)	(14) T: $Fa \land \neg Ga$ ✓ from (1)
(7) T: $Fa \land \neg Ga$ ✓ from (1)	(15) T: Fa ↯ from (14)
(8) T: Fa from (7)	(16) T: $\neg Ga$ from (14)
(9) T: $\neg Ga$ ✓ from (7)	(17) F: Ga from (16)
(10) F: Ga from (9)	

(11) F: Fa ↯ from (6)	(12) T: Ga ↯ from (6)

1.3 natural deduction

$\forall x(Fx \land \neg Gx) \vdash \neg \exists x(Fx \to Gx) \land \exists x Fx$

α_1	(1)	$\forall x(Fx \land \neg Gx)$	A
α_2	(2)	$\exists x(Fx \to Gx)$	A
α_3	(3)	$Fa \to Ga$	A
α_1	(4)	$Fa \land \neg Ga$	1 $\forall E$
α_1	(5)	Fa	4 $\land E$
α_1	(6)	$\neg Ga$	4 $\land E$
α_1, α_3	(7)	Ga	3,5 $\to E$
α_1, α_3	(8)	$\bot$	6,7 $\neg E$
α_1, α_2	(9)	$\bot$	2,8 $[\alpha_3]$ $\exists E$
α_1	(10)	$\neg \exists x(Fx \to Gx)$	9 $[\alpha_2]$ $\neg I$
α_1	(11)	$\exists x Fx$	5 $\exists I$
α_1	(12)	$\neg \exists x(Fx \to Gx) \land \exists x Fx$	10,11 $\land I$

2 Q2

Natural deduction is not always intuitive and not close to human's reasoning. It is pure syntax manipulation and acts as proof system with a formal notion of proof as a mathematical entity. However, semantic tableaux can be easier to understand, particularly for logic beginners.
Natural deduction is less expressive than semantic tableaux because semantic tableaux could show all branches. Facing complex sequents, if we don't know whether it's valid or invalid, semantic tableaux is more prioritised. Natural deduction is difficult to decide validity. So Natural deduction is not suited for proving complex sequents.
Natural deduction proofs can be shorter and flexible rather than longer and cumbersome. We can discharge a vacuous discharge - assumptions that are not there. As semantic tableaux, if some interpretations cause branches, different possibilities should be considered, and the progresses are not very quick and easy.

3 Q3

3.1 Introduction

The sorites-type paradoxes were created in the ancient puzzles. They seem to be generated by vagueness, with ambiguous edges of application[1].
An example using some arguments that can introduced the sorites-type paradoxes. Basically, we agree that only one book(L_1) does not make a library. So we can get this pattern:

$$\begin{array}{l} \neg L_1, \\ \neg L_1 \rightarrow \neg L_2, \\ \neg L_2 \rightarrow \neg L_3, \\ \neg L_3 \rightarrow \neg L_4, \\ \vdots \\ \neg L_{n-1} \rightarrow \neg L_n \\ \hline \neg L_n \text{(where } n \text{ can be arbitrarily large)} \end{array}$$

According to the obviously true premises, we can get the conclusion: n books(L_n) do not make a library. However, it is common sense that a huge collection of books can make a library, so L_n should be true. Intuitively, L_{n-1} should also be true.
From facts that are undoubted, we draw unacceptable conclusions. This cause contradiction. In this essay, I will propose a method for solving these paradoxes, consider counter-arguments against my approach, then conclude analyse the response.

3.2 Addressing Sorites-type Paradoxes

Due to one of the most debatable points on sorites-type paradoxes is infinite, I will set the boundaries, also can be called thresholds, to define the precise amount of books. In other words, I assign a specific value to n. The amount of books less than n could be considered as a library, i.e. libraries can be built if the amount of books is greater than or equal to n.
For instance, a small-scale collection at 1,000 can be a library books in a village. However, if a national library has a collection of less than 100,000 books, it is hard for people to agree that it deserves the title.
The terminology for the approach I have listed can be called "contextual logic" in borderline cases[2]. In this way, fuzzy predicates are turned into precise predicates. Contextual logic is applied in a wide areas and easy to find out. For example, scientists have defined the range of different wavelengths of visible light based on different colors[3]. The ISO technical committee suggests a fine border between visible—beginning at $380nm$[4].

3.3 Advantages of Contextual Logic

This rule can be flexible under different background. In formal logic, we can introduce another function B_x into the process for our determination on n. B is the elemental background that we need to be considered. Then our proposition becomes L_{nx}. Even more thinkable is the owner's willing. If someone in a village owns over 1,000 books but just for personal collection, the space where he place these books could not be considered as a library. If that person likes to share his books and establishes some lending policy, he becomes a librarian as well. In this situation, a boolean function has been introduced.

In addition, this method allows us for a practical comprehension to reality. Semantic and syntax part in languages must be connected with the context. By using contextual logic, we can better adapt the ambiguity and blur of daily language application. It is worth mentioning that these thresholds are not set at random, but are generally determined through consensuses, scales or functions.

3.4 Counter-Arguments

It is not a bad way for us to address some uncertain problems in decision making. However, definition itself is too hard to establish accuracy. How to determine the border? How can these edges make sense? Why is $L_{1,000}$ the standard for a library for a village? We can discover that defining the border is a complex task. If $L_{1,000}$ is true, most people probably don not even notice the difference between L_{999} and $L_{1,001}$. Finally, we also struggle in convincing ourselves why L_{999} is invalid.

Furthermore, that is not only numerical or physical parameters, but multi-dimension consideration. Assuming that there are two villages which are next to each other. They have very different residential populations. There is no doubt that the thresholds of two library would show the discrepancy. It is beyond the relative standard background $B_{village}$. Does that should introduce a new function or variable? Change is a constant, dynamic process so that a simple division is not persuasive enough. It is an expediency for practical convenience. Are there non-practical reasons driving us to do so?

The individual cognitive precision is limited. It is not a coincidence that we ask such question: why we can not see the difference between A and B, and some properties A has but not B. We know there exist the light with a wavelength differentiate infinitely small from visible light. These lights are still visible to human eyes. But they are defined as invisible lights. Although we suppose the wavelength of visible light is specific, we can actually see some lights whose wavelengths are not in visible light range. This makes it unavoidable that individual judgements include luck.

3.5 Responses

The concern about two neighbouring village libraries' standards points to the necessity of contextual combination. This reflects the dynamism in human affairs. The standard for library is related to its function in operation. To serve the residential better, the least specific number is required, which leads to high-quality common resource.

I acknowledge that human cognition and language are limited. Actually, luck plays the important part in judgement. Luck acknowledge the uncertainties and limitations of human perception. The decision-making process, as well as contextual division, is in order to decease this uncertainty. Our borders are based on general experience, not on special circumstances. We define concepts not only on the basis of scientific precision, but also on the basis of the application in real life.

Contextual logic serves us by providing useful ways to organise knowledge. Combining with the situation and background also maintains a logical consistency. Contextual logic can address the sorites-paradoxes and can stand up in philosophy.

3.6 Conclusion

In any case, the Sorites-paradoxes reminds us of deepening our understanding of fuzzy statements and their boundaries. Contextual logic offers us an applicable solution. It not only comprises the practical utility of division, but also respects inherent flexibility in real life. This approach based on practical and formal logical considerations. It provides us with an effective idea for navigating the paradoxes without being afraid of their confusions. In addition, we can still comprehend of the intrinsic properties. By carefully considering and applying formal logic, we can continue to refine our understanding of ambiguity and precision in language and thought. Last but not least, we can keep dialectical perspective to regard dynamism into relative stasis.

References

[1] Sergi Oms and Elia Zardini. "An introduction to the sorites paradox". In: *The sorites paradox* (2019), pp. 3–18.

[2] Haim Gaifman. "Vagueness, tolerance and contextual logic". In: *Synthese* 174 (2010), pp. 5–46.

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[4] International Standards Organization. *ISO 20473:2007, Optics and Photonics — Spectral Bands*. International Standard. ISO: Geneva, Switzerland. Geneva, Switzerland: International Organization for Standardization, 2007.