Fundamental Mathematics (Engineering Mathematics)

Shinichi Nishizawa

Course schedule

- Guidance + Differential equations (#1,2)
- Differential equations and physics (#3)
- Array and vector (#4, 5)
- Vector analysis (#6, 7)
- Complex function theory (#8, 9)
- Fourier transform (#10, 11)
- Laplace transform (#12, 13)
- Final examination and explanation(#14)

- Score: Exam (70%) + Report (20%) + Attendance (10%)

Fundamental Mathematics

- Laplace transform 1-

Laplace transform (ラプラス変換)

- Assume function $f(x)$ within infinite range $(0, \infty)$. If its integral $F(s)$ is finite determinate for complex value s, this is called Laplace transform

- $F(s) = \int_0^\infty f(x)e^{-st}dt = \lim_{\substack{T\to\infty \\ \epsilon\to+0}} \int_\epsilon^T f(x)e^{-st}dt = \mathcal{L}f(t)$
  - $f(t)$: time domain function
  - $F(s)$: complex domain (s-domain) function
  - $\mathcal{L}$: Laplace-operator(ラプラス演算子), or Laplacian
  - Useful to calculate differential equation

Laplace transform definition

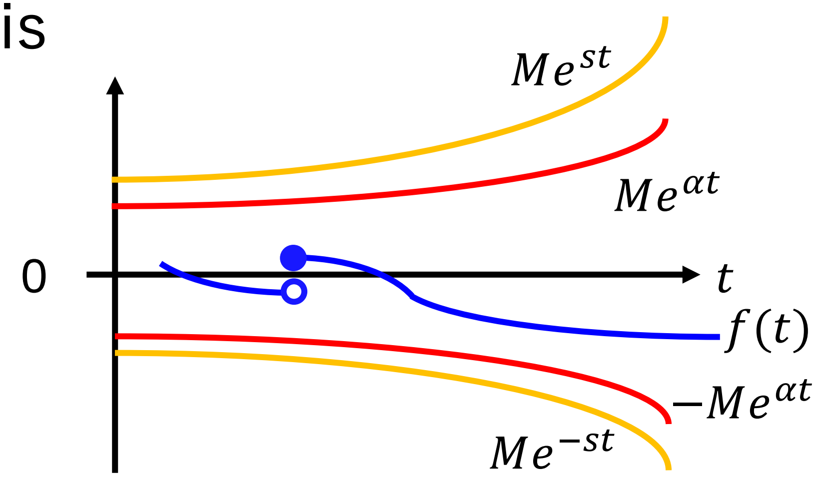
- Preliminaries:
  - If $\mathrm{Re}(s) > 0$
    - $\lim_{t\to\infty} t^n e^{-st} = 0 \ (n \in \mathbb{N})$
  - Proof:
    - Assume $s = a + bi$ and $t > 0$, since $\mathrm{Re}(s) = a > 0$
    - $|t^n e^{-st}| = t^n e^{-at}$
    - $\lim_{t\to\infty} |t^n e^{-st}| = \lim_{t\to\infty} \frac{t^n}{e^{at}} = \lim_{t\to\infty} \frac{nt^{n-1}}{ae^{at}} = \cdots = \lim_{t\to\infty} \frac{n!}{a^n e^{at}} = 0$

Examples (condition: $\mathrm{Re}[s] > 0$)

- $\mathcal{L}1 = \frac{1}{s}$
  - $\mathcal{L}1 = \int_0^\infty 1 \cdot e^{-st} dt = \left[-\frac{1}{s} e^{-st}\right]_0^\infty = -\frac{1}{s} \lim_{t\to\infty} e^{-st} + \frac{1}{s} = \frac{1}{s}$
- $\mathcal{L}t = \frac{1}{s^2}$
  - $\mathcal{L}t = \int_0^\infty t \cdot e^{-st} dt = \left[-\frac{t}{s} e^{-st}\right]_0^\infty + \frac{1}{s} \int_0^\infty e^{-st} dt$
  - $= -\frac{1}{s} \lim_{t\to\infty} t e^{-st} + \frac{1}{s} \mathcal{L}1 = \frac{1}{s^2}$
- $\mathcal{L}t^n = \frac{n!}{s^{n+1}}$: use inductive method, $n = 1$ satisfy, assume $n = k$ satisfy $\mathcal{L}t^k = \frac{k!}{s^{k+1}}$
  - $\mathcal{L}t^{k+1} = \int_0^\infty t^{k+1} \cdot e^{-st} dt = \left[-\frac{t^{k+1}}{s} e^{-st}\right]_0^\infty + \frac{k+1}{s} \int_0^\infty t^k e^{-st} dt$
  - $= -\frac{1}{s} \lim_{t\to\infty} t^{k+1} e^{-st} + \frac{k+1}{s} \mathcal{L}t^k = \frac{k+1}{s} \cdot \frac{k!}{s^{k+1}} = \frac{(k+1)!}{s^{k+2}}$

Convergence of Laplace transform

- Discuss the requirement of $f(x)$ to achieve $\int_0^\infty f(x)e^{-st}dt = F(s)$ ($\cancel{F(s)}$ ~~is const~~., not diverge)
- Assume function $f(x)$ is piecewise smooth within infinite range $(0,\infty)$. If $f(x)$ satisfy following conditions, $f(x)$ is called "exponential w/ index number α"
  - $|f(t)| < Me^{\alpha t}$ $(M > 0) \leftrightarrow -Me^{\alpha t} < f(t) < Me^{\alpha t}$
  - Speed of diverge of $|f(x)|$ is slower than $Me^{\alpha t}$
- $\int_0^\infty f(x)e^{-st}dt$ $(\mathrm{Re}(s) > \mathrm{Re}(\alpha))$ is the condition to converge



Convergence theorem 1

- Theorem 1: Assume function $f(x)$ is piecewise smooth within infinite range $(0, \infty)$, and "exponential w/ index number a". Laplace transform $F(s)$ is available for any complex number s which satisfy $\mathrm{Re}(s) > \gamma$
- Proof: Assume $0 < T < T'$, $\mathrm{Re}(s) = \alpha$.
  - $\int_0^{T'} f(x)e^{-st}dt = \int_T^{T'} f(x)e^{-st}dt + \int_0^T f(x)e^{-st}dt$
    - $\int_0^T f(x)e^{-st}dt < Me^{\alpha t}$
    - $\left|\int_T^{T'} f(x)e^{-st}dt\right| \leq M \int_T^{T'} e^{-(a-\gamma)t}dt = \frac{M(e^{-(a-\gamma)T} - e^{-(a-\gamma)T'})}{a-\gamma}$
      - $T' \to \infty$, $\mathrm{Re}(s) = \alpha > \gamma$ means $\alpha - \gamma > 0$, then
    - $\left|\int_T^{T'} f(x)e^{-st}dt\right| \leq \frac{Me^{-(a-\gamma)T}}{a-\gamma}$
- Both term are less than exponential (Bounded)

Convergence theorem 2

- Theorem 2: Assume function $f(x)$ is piecewise smooth within infinite range $(0,\infty)$, and "exponential w/ index number γ". If Laplace transform $F(s_0)$ is available, Laplace transform $F(s)$ is also available for $\mathrm{Re}[s] > \mathrm{Re}[s_0]$.
- Proof: Assume $g(t) = \int_0^t f(u)e^{-s_0 u}du$, this is Laplace transform and bounded. $\mathcal{L}g(s)$ is available for any complex s
  - $\int_0^T f(x)e^{-st}dt = \int_0^T e^{-(s-s_0)t}e^{-s_0 t}f(t)dt$
    - $= e^{-(s-s_0)T}g(T) + (s-s_0)\int_0^T e^{-(s-s_0)t}g(x)\,dt$
  - Assume $T \to \infty$, $e^{-(s-s_0)T}g(T) \to 0$, $(s-s_0)\int_0^T e^{-(s-s_0)t}g(x)\,dt \to (s-s_0)\mathcal{L}g(s)$
- Thus, $\int_0^{\infty} f(x)e^{-st}dt$ has some limit value $F(s)$

Convergence coordinate, range

- Theorem 1,2, if $f(x)$ satisfy condition in theorem 1, and its Laplace transform $F(s_0)$ is available, Laplace transform $F(s)$ is available for $\mathrm{Re}[s] > \mathrm{Re}[s_0]$.
  - Guess boundary α which (1) satisfy $F(s) = \mathcal{L}f(t)$ for $\mathrm{Re}[s] > a$ and (2) α is the lowest boundary of a (a is real value)
    - Convergence range: $\mathrm{Re}[s] > \alpha$
    - Convergence coordinate: α (Outside of convergence range or $\mathcal{L}f(t)$ is not available)

Characteristics of Laplace transform

- Laplace transform $F(s) = \int_0^\infty f(x) e^{-st} dt = \mathcal{L}f(t)$ has following characteristics
  1. $\mathcal{L}(af(t) + bg(t)) = a\mathcal{L}f(t) + b\mathcal{L}g(t)$
  2. $\mathcal{L}f(at) = \frac{1}{a} F\left(\frac{s}{a}\right)$
  3. $\mathcal{L}\{e^{at} f(t)\} = F(s-a)$

  where $a > 0$

Characteristics of Laplace transform (cont)

- Characteristics of integrals, differentials
  - If $f(t)$ is continuous,
    1. $\mathcal{L}f'(t) = sF(s) - f(+0)$
  - If $f(t)$, $f'(t)$, $\cdots f^{(n-1)}(t)$ are continuous,
    2. $\mathcal{L}f^{(n)}(t) = s^n F(s) - f(+0)s^{n-1} - f'(+0)s^{n-2} \ldots - f^{(n)}(+0)$

3. $\mathcal{L}\left[\int_0^t f(t)dt\right] = \frac{F(s)}{s}$
4. $\mathcal{L}[tf(t)] = -\frac{dF(s)}{ds}$
5. $\mathcal{L}\left[\frac{f(t)}{t}\right] = \int_0^\infty F(s)ds$

Composite product (合成積)

- Assume $f(x)$ and $g(x)$ are defined within infinite range $(0, \infty)$, if $h(t)$ is available, $h(t) = f(t) * g(t)$ is composite product
  - $h(t) = \int_0^t f(\tau)g(t-\tau)d\tau$
- Composite product support commutation relations (交換則)
  - $f(t) * g(t) = g(t) * f(t)$
- Can calculate Laplace transform by multiply of individual Laplace transform
  - $\mathcal{L}h(t) = \mathcal{L}(f(t) * g(t)) = \mathcal{L}\big(f(t)\big)\mathcal{L}\big(g(t)\big)$

Inverse Laplace transform

- Theorem 3: Assume function $f(x)$, $g(x)$ are piecewise smooth within infinite range $(0, \infty)$, and exponential. If $\mathcal{L}f(t) = \mathcal{L}g(t)$, $f(t) = g(t)$
  - If Laplace transform of $\mathcal{L}f(t)$ is known, $f(t)$ can be calculated
  - Inverse Laplace transform

Table: cheat sheet of Laplace transform

$F(s) = \mathcal{L}f(t)$	$f(t) = \mathcal{L}^{-1}F(\boldsymbol{s})$
$1/s$	1
$1/s^n$	$t^{n-1}/(n-1)!$
$1/(s-a)$	e^{at}
$\omega/(s^2+\omega^2)$	$\sin \omega t$
$s/(s^2+\omega^2)$	$\cos \omega t$
$\omega/((s-a)^2+\omega^2)$	$e^{at} \sin \omega t$
$(s-a)/((s-a)^2+\omega^2)$	$e^{at} \cos \omega t$

Exercise

- Proof following relationships
  - $\mathcal{L}e^{at} = 1/(s-a)$ (where $\mathrm{Re}[s] > a$)
  - $\mathcal{L}\sin\omega t = \omega/(s^2+\omega^2)$ (where $\mathrm{Re}[s] > 0$)
  - $\mathcal{L}\cos\omega t = s/(s^2+\omega^2)$ (where $\mathrm{Re}[s] > 0$)
- Calculate following Laplace transform
  - $*$: composite product
  - $f(t) = (t^2) * (te^{-t})$
  - $f(t) = (e^{at}\sin\omega t) * (e^{at}\cos\omega t)$

Solutions

Proof followings

(1) $\mathcal{L} e^{at} = \frac{1}{s-a}$ $(Re[s] > a)$

$$\mathcal{L} e^{at} = \int_0^\infty e^{at} e^{-st} dt = \int_0^\infty e^{-(s-a)t} dt = \left[-\frac{e^{-(s-a)t}}{s-a} \right]_0^\infty$$

$$= \underbrace{-\frac{1}{s-a} \lim_{t\to\infty} e^{-(s-a)t}}_{\to 0} + \frac{1}{s-a} = \frac{1}{s-a}$$

(2) $\mathcal{L} \sin\omega t = \frac{\omega}{s^2+\omega^2}$, $\mathcal{L} \cos\omega t = \frac{s}{s^2+\omega^2}$ $(Re[s] > 0)$

$$\mathcal{L} \sin\omega t = \int_0^\infty \sin\omega t \, e^{-st} dt = \left[-\frac{e^{-st}}{s} \sin\omega t \right]_0^\infty + \frac{\omega}{s} \int_0^\infty \cos\omega t \, e^{-st} dt$$

$$= \underbrace{-\frac{1}{s} \lim_{t\to\infty} e^{-st} \sin\omega t}_{\to 0} + \frac{\omega}{s} \mathcal{L} \cos\omega t$$

$$\therefore \mathcal{L} \sin\omega t = \frac{\omega}{s} \mathcal{L} \cos\omega t$$

$$\mathcal{L} \cos\omega t = \left[-\frac{e^{-st}}{s} \cos\omega t \right]_0^\infty - \frac{\omega}{s} \int_0^\infty \sin\omega t \cdot e^{-st} dt$$

$$= \underbrace{-\frac{1}{s} \lim_{t\to\infty} e^{-st} \cos\omega t}_{\to 0} + \frac{1}{s} - \frac{\omega}{s} \mathcal{L} \sin\omega t$$

$$\therefore \mathcal{L} \cos\omega t = \frac{1}{s} - \frac{\omega}{s} \mathcal{L} \sin\omega t$$

$$\frac{s}{\omega} \mathcal{L} \sin\omega t = \frac{1}{s} - \frac{\omega}{s} \mathcal{L} \sin\omega t$$

$$\left(\frac{s}{\omega} + \frac{\omega}{s} \right) \mathcal{L} \sin\omega t = \frac{1}{s} \quad \mathcal{L} \sin\omega t = \frac{\omega}{s^2+\omega^2}, \ \mathcal{L} \cos\omega t = \frac{s}{s^2+\omega^2}$$

Calculate following Laplace transform

(1) $f(t) = (t^2) * (te^{-t})$

$$\mathcal{L} t^2 = \frac{2}{s^3}, \ \mathcal{L}(te^{-t}) = -\left[\frac{1}{s+1} \right]' = \frac{1}{(s+1)^2}$$

$$\mathcal{L} f(t) = \frac{2}{s^3(s+1)}$$

(2) $f(t) = (e^{at} \sin\omega t) * (e^{at} \cos\omega t)$

$$\mathcal{L}(e^{at} \sin\omega t) = \frac{\omega}{(s-a)^2 + \omega^2}$$

$$\mathcal{L}(e^{at} \cos\omega t) = \frac{s-a}{(s-a)^2 + \omega^2}$$

from cheat sheet

$$\mathcal{L} f(t) = \frac{\omega(s-a)}{\{(s-a)^2 + \omega^2\}}$$

Fundamental Mathematics

- Laplace transform 2-

Laplace transform (recall)

- Assume function $f(x)$ within infinite range $(0, \infty)$. If its integral $F(s)$ is finite determinate, this is called Laplace transform
  - $F(s) = \int_0^\infty f(x)e^{-st}dt = \mathcal{L}f(t)$
    - $f(t)$: time domain function
    - $F(s)$: complex domain (s-domain) function
    - $\mathcal{L}$: Laplace-operator
- Useful to calculate differential equation

Inverse Laplace transform

- Assume $\mathcal{L}f(t) = \mathcal{L}g(t)$, $f(t) = g(t)$
  - If Laplace transform of $\mathcal{L}f(t)$ is known, $f(t)$ can be calculated
  - Inverse Laplace transform

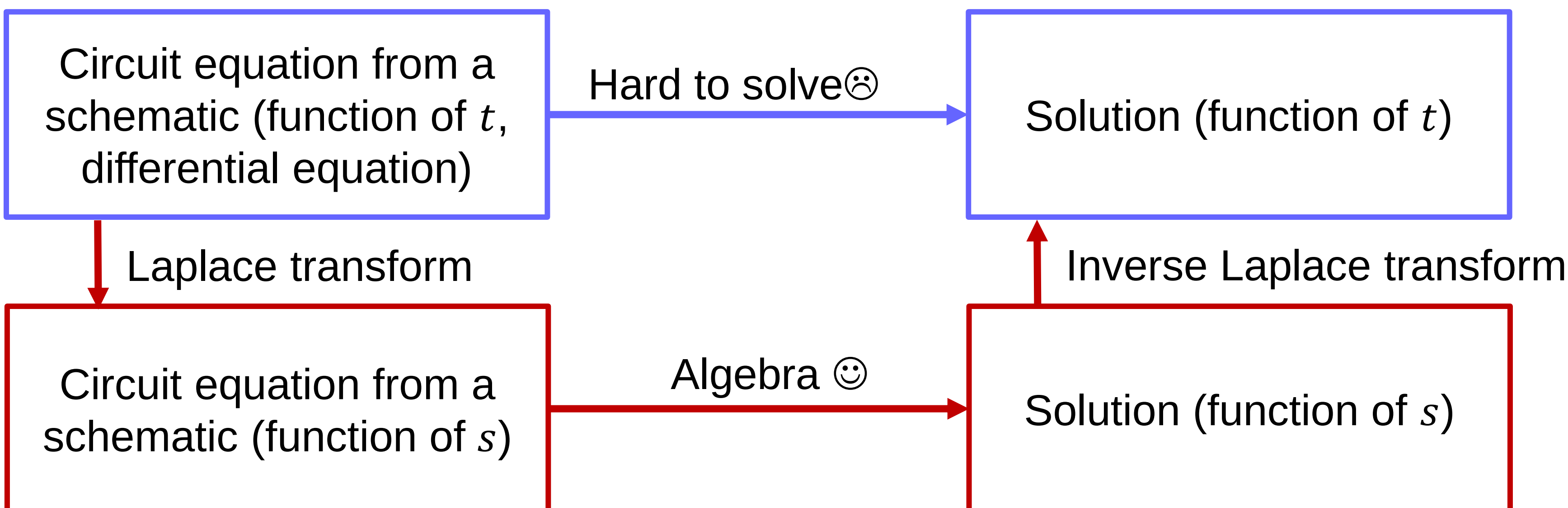
Table: cheat sheet of Laplace transform

s function	t function
$1/s$	1
$1/s^n$	$t^{n-1}/(n-1)!$
$1/(s-a)$	e^{at}
$\omega/(s^2+\omega^2)$	$\sin\omega t$
$s/(s^2+\omega^2)$	$\cos\omega t$
$\omega/(s^2-\omega^2)$	$\sinh\omega t$
$s/(s^2-\omega^2)$	$\cosh\omega t$
$\omega/((s^2-a^2)+\omega^2)$	$e^{at}\sin\omega t$
$(s-a)/((s^2-a^2)+\omega^2)$	$e^{at}\cos\omega t$

s function	t function
$aF(s)+bF(s)$	$af(t)+bf(t)$
$\frac{1}{a}F\left(\frac{s}{a}\right)$	$f(at)$
$e^{-as}F(s)$	$f(t-a)$
$F(s+a)$	$e^{-at}f(t)$
$sF(s)-f(+0)$	$f'(t)$
$\frac{F(s)+\int_0^t f(0)dt}{s}$	$\int_0^t f(t)dt$

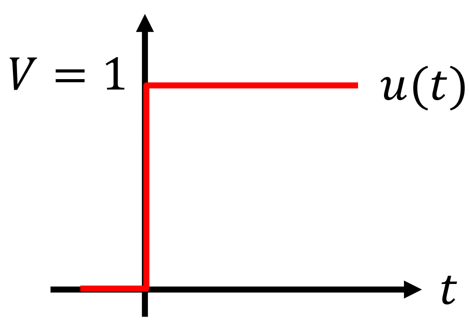
Laplace transform for practical analysis

- Laplace transform and inverse Laplace transform help to solve differential equation
  - Convert differential equation in time domain to s domain
  - Get solution (no need to solve differential equation)
  - Re-convert equation from s domain to time domain
    - This is the solution

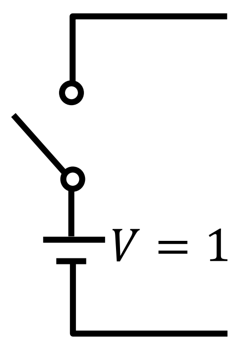


Power source 1

- Example 1: calc. Laplace transform of unit function $u(t)$
  - $u(t) = \begin{cases} 0 \ (t < 0) \\ 1 \ (0 \leq t) \end{cases}$
- Solution 1:
  - $U(s) = \int_0^\infty u(t)e^{-st}dt = \int_0^\infty e^{-st}dt = \left[-\frac{1}{s}e^{-st}\right]_0^\infty = 0 + \frac{1}{s} = \frac{1}{s}$
- Unit function: ideal switch w/ DC supply



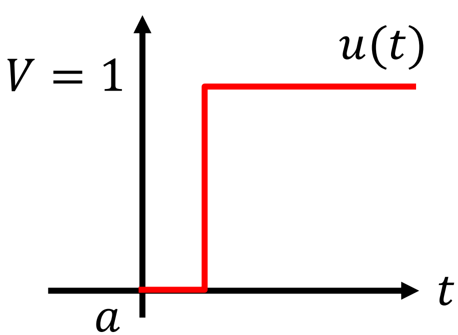
Unit function

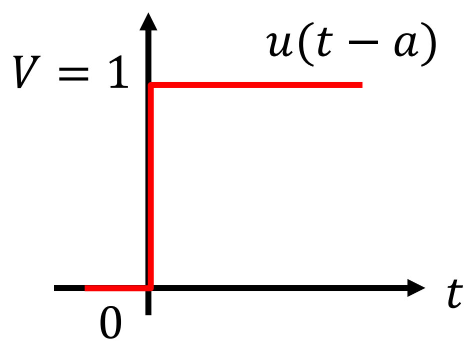


Ideal switch w/ DC power

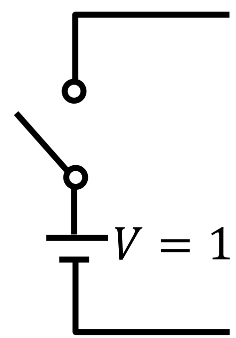
Power source 2

- Example 2: calc. Laplace transform of unit function $u(t)$
  - $u(t) = \begin{cases} 0 \ (t < a) \\ 1 \ (a \leq t) \end{cases}$
- Solution 2:
  - $U(s) = \int_0^\infty u(t)e^{-st}dt = \int_0^\infty u(t-a)e^{-s(t-a)}dt =$
    $\int_0^\infty e^{-s(t-a)}dt = \left[-\frac{1}{s-a}e^{-st}\right]_0^\infty = 0 + \frac{1}{s-a} = \frac{1}{s-a}$
- Unit function: ideal switch w/ DC supply





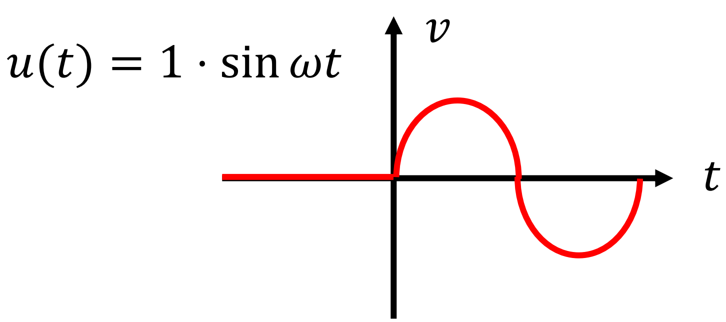
Unit function



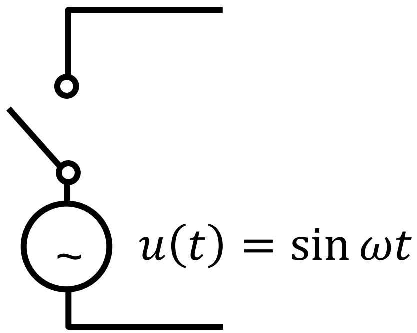
Ideal switch w/ DC power

Power source 3

- Example 3: calc. Laplace transform of sine function $u(t)$
  - $u(t) = \sin \omega t$
- Solution 3: (refer the last exercise)
  - $U(s) = \int_0^{\infty} \sin \omega t \, e^{-st} dt = \frac{\omega}{s^2+\omega^2}$
- sine function: AC supply
  - Note: Laplace transform has not only steady-state solution
    - It also has transient solution (cause it has switch)



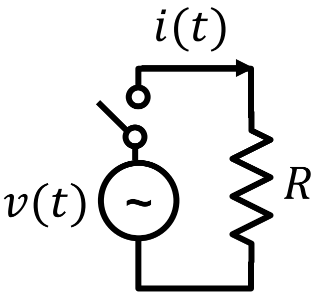
sine function



Ideal switch w/ AC power

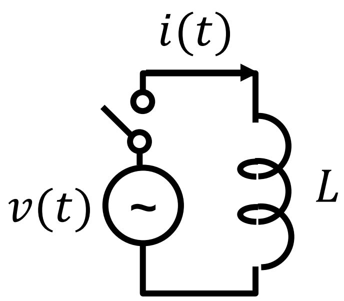
Impedance 1

- Example 4: calc. circuit equation of resistance R after the Laplace transform
- Solution 4:
  - R, current $i(t)$, electromotive force $v_R(t)$ satisfy
    - $v_R(t) = Ri_R(t)$
  - Laplace transform of $i(t)$ and $v(t)$ are:
    - $\mathcal{L}i(t) = I(s)$, $\mathcal{L}v(t) = V(s)$
    - $V_R(s) = RI_R(s)$
    - $I_R(s) = V_R(s)/R$



Impedance 2

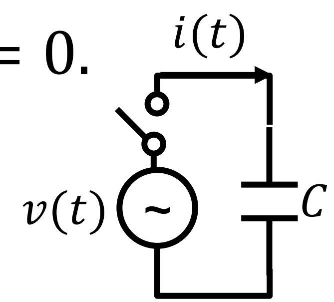
- Example 5: derive current $i(t)$ by circuit equation of resistance L by the Laplace transform. Assume $i_L(0) = 0$.
- Solution 5:
  - L, current $i(t)$, electromotive force $v_L(t)$ satisfy
    - $v_L(t) = L\frac{d}{dt}i_L(t)$
  - Laplace transform $v_L(t)$ is:
    - $V_L(s) = L\{sI_L(s) - i_L(0)\} = sLI_L(s)$
    - $I_L(s) = V_L(s)/sL$



s function	t function
$sF(s) - f(+0)$	$f'(t)$

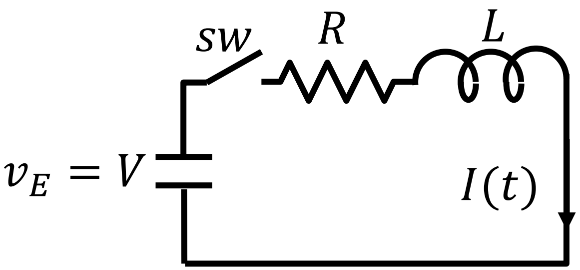
Impedance 3

- Example 6: calc. circuit equation of capacitance C after the Laplace transform. Assume $v_C(0) = 0$, $i_C(0) = 0$.
- Solution 6:
  - Current $i_C(t)$, charge $Q_C(t)$, electromotive force $v_C(t)$ satisfy
    - $v_C(t) = \frac{1}{C} Q_C(t) = \frac{1}{C} \int_{-\infty}^{t} i_C(t) dt$
    - Divide charge (integ. of current) to $(-\infty, 0)$ and $[0, t]$
      - $v_C(t) = \frac{1}{C} \int_{-\infty}^{0} i_C(t) dt + \frac{1}{C} \int_{0}^{t} i_C(t) dt = v_C(0) + \frac{1}{C} \int_{0}^{t} i_C(t) dt$
  - Laplace transform: $V_C(s) = \frac{v_C(0)}{s} + \frac{1}{C}\left\{\frac{I_C(s)}{s} + \frac{\int i_C(0) dt}{s}\right\}$
    - Use initial conditions $v_C(0) = 0$, $i_C(0) = 0$.
      - $V_C(s) = \frac{1}{C} \frac{I_C(s)}{s}$



Example: RL circuit

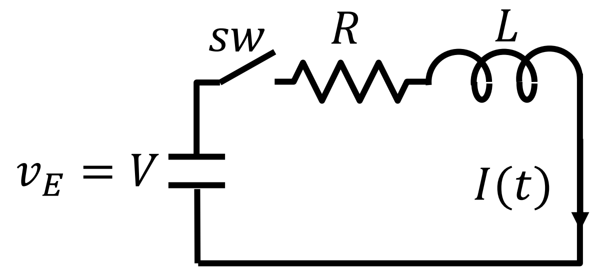
- Derive current $i(t)$ of RL circuit
  - Switch on at $t = 0$
  - Initial condition: $i(0) = 0$,
- Voltage of R (v_R) L (v_L) are:
  - $v_R = Ri(t)$, $v_L = L\dfrac{di(t)}{dt}$,
  - $v_R + v_L = v_E = V$
- Laplace transformation
  - $\dfrac{V}{s} = RI(s) + L\{sI(s) + i(0)\}$



s function	t function
$sF(s) - f(+0)$	$f'(t)$

Example: RL circuit (2)

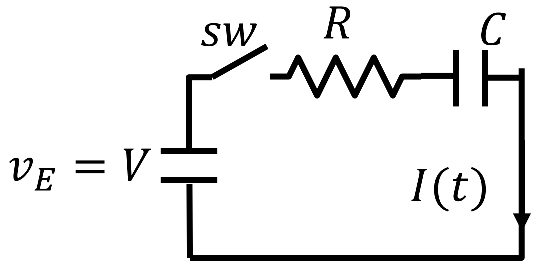
- Change the position of current $I(s)$
  - $I(s) = \frac{V}{Ls^2+Rs} = \frac{V}{L}\frac{1}{s\left(s+\frac{R}{L}\right)} = \frac{V}{L}\left(\frac{1}{s} - \frac{1}{s+\frac{R}{L}}\right)$



- Inverse Laplace transform
  - $i(t) = \frac{V}{L}\left\{1 - e^{-\frac{R}{L}t}\right\}$

s function	t function
$1/s$	1
$1/(s-a)$	e^{at}

Example: RC circuit

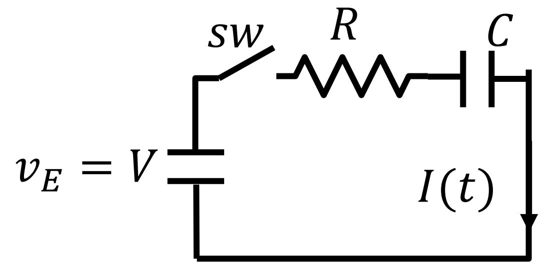


- Derive current $i(t)$ of RC circuit
  - Switch on at $t = 0$
  - Initial condition: $i(0) = 0$, $v_C(0) = 0$
- Voltage of R (v_R) C (v_C) are:
  - $v_R = Ri(t)$, $v_C = \frac{q(t)}{C} = \frac{\int i(t)dt}{C}$
  - $v_R + v_C = v_E = V$
- Laplace transformation
  - $\frac{V}{s} = RI(s) + \frac{I(s)+\int_0^t i(0)dt}{s} = RI(s) + \frac{I(s)}{s}$

s function	t function
$\frac{F(s) + \int_0^t f(0)dt}{s}$	$\int_0^t f(t)dt$

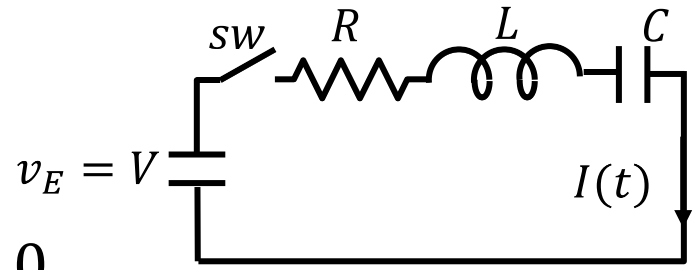
Example: RC circuit (2)

- Change the position of current $I(s)$
  - $I(s) = \frac{V}{Rs+\frac{1}{C}} = \frac{V}{R}\frac{1}{(s+1/RC)}$
- Inverse Laplace transform
  - $i(t) = \frac{V}{R}e^{-\frac{1}{RC}t}$



s function	t function
$1/s$	1
$1/(s-a)$	e^{at}

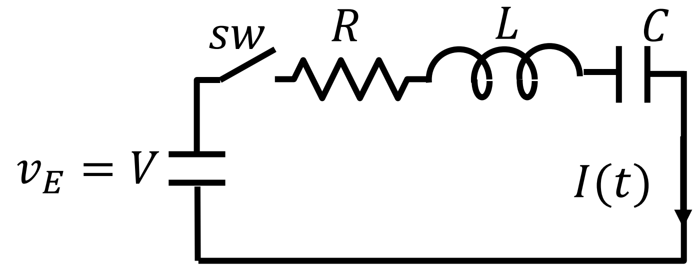
Example: RLC circuit



- Derive current $i(t)$ of RLC circuit
  - Switch on at $t = 0$
  - Initial condition: $i(0) = 0$, $v_C(0) = 0$
- Voltage of R (v_R) L (v_L) C (v_C) are:
  - $v_R = Ri(t)$, $v_L = L\dfrac{di(t)}{dt}$, $v_C = \dfrac{q(t)}{C} = \dfrac{\int i(t)dt}{C}$
  - $v_R + v_L + v_C = v_E = V$
- Laplace transformation
  - $\dfrac{V}{s} = RI(s) + L\{sI(s) + i(0)\} + \dfrac{1}{C}\dfrac{I(s)}{s}$

s function	t function
$sF(s) - f(+0)$	$f'(t)$
$\dfrac{F(s) + \int_0^t f(0)dt}{s}$	$\int_0^t f(t)dt$

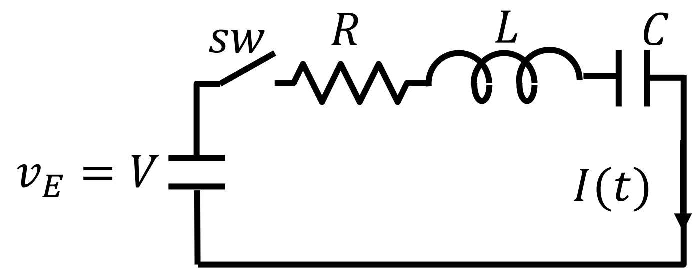
Example: RLC circuit (2)



- Change the position of current $I(s)$
  - $I(s) = \frac{V}{Ls^2+Rs+\frac{1}{C}} = \frac{V}{L}\frac{1}{s^2+\frac{R}{L}s+\frac{1}{LC}}$
- Here
  - $s^2 + \frac{R}{L}s + \frac{1}{LC} = \left(s + \frac{R}{2L}\right)^2 - \left\{\left(\frac{R}{2L}\right)^2 - \frac{1}{CL}\right\}$
  - Assume $\left\{\left(\frac{R}{2L}\right)^2 - \frac{1}{CL}\right\} = D$ and branch
- Case: $D > 0$
  - $I(s) = \frac{V}{L}\frac{1}{\left(s+\frac{R}{2L}\right)^2 - D} = \frac{V}{L\sqrt{D}}\frac{\sqrt{D}}{\left(s+\frac{R}{2L}\right)^2 - \left(\sqrt{D}\right)^2}$
  - $i(t) = \frac{V}{L\sqrt{D}}e^{-\frac{R}{2L}t}\sinh\sqrt{D}\,t$

s function	t function
$1/(s-a)$	e^{at}
$\omega/(s^2-\omega^2)$	$\sinh \omega t$

Example: RLC circuit (3)



- Case: $D = 0$
  - $I(s) = \frac{V}{L}\frac{1}{\left(s+\frac{R}{2L}\right)^2}$
  - $i(t) = \frac{V}{L\sqrt{D}} t e^{-\frac{R}{2L}t}$
- Case: $D < 0$
  - $I(s) = \frac{V}{L}\frac{1}{\left(s+\frac{R}{2L}\right)^2 - D} = \frac{V}{L\sqrt{-D}}\frac{\sqrt{-D}}{\left(s+\frac{R}{2L}\right)^2 + \left(\sqrt{-D}\right)^2}$ (Note $-D > 0$)
  - $i(t) = \frac{V}{L\sqrt{-D}} e^{-\frac{R}{2L}t} \sin\sqrt{-D}\, t$

s function	t function
$1/(s-a)$	e^{at}
$\omega/(s^2+\omega^2)$	$\sin\omega t$

Conclusion

- Use Laplace transform to solve practical differential equation
- Next: exam (60min) + summary (40min)
  - You can use your calculator, note/papers/prints
  - Cannot use phone, iPads
  - Same or similar difficulty to the exercise
  - nishizawa@aoni.waseda.jp