Fundamental Mathematics (Engineering Mathematics)

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Guidance

- 集積システムを設計する上で前提となる物理現象を扱う数学について学ぶ．微分方程式から始まり，ベクトル解析，複素関数，フーリエ変換・ラプラス変換について学ぶことで，これらの知識を利用するアナログ回路設計および高周波回路設計を可能にする．

- This course introduces the mathematics for physics for integrated system design. It includes differential equations, vector analysis, complex function theory, Fourier transform and Laplace transform for analog circuit design and RF circuit design.

Course schedule

- Guidance + Differential equations (#1,2)
- Differential equations and physics (#3)
- Array and vector (#4,5)
- Vector analysis (#6,7)
- Complex function theory (#8,9)
- Fourier transform (#10,11)
- Laplace transform (#12,13)
- Final examination and explanation(#14)

- Score: Exam (70%) + Report (20%) + Attendance (10%)

Guidance

- Target:
  - Student who have **NO** background for engineering math.
  - Potential circuit designer; s.t. RF-analog circuit design, image processor design, Neural-Network design
- Non-Target:
  - Student who have background for engineering math.
    - Contents are very wide (2~3 class)
    - Introduce brief overview of this field
    - If you already took math. class in your bachelor, you should take another class
  - IPS has a lot of interesting class for students

Fundamental Mathematics
- Differential equations 1 -

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Differential equation

- Differential equation for engineering
  - Express natural behaviors (physics, electrics) as equation
    - Handle simple models
    - Easy to understand
    - Show very primitive “solution”

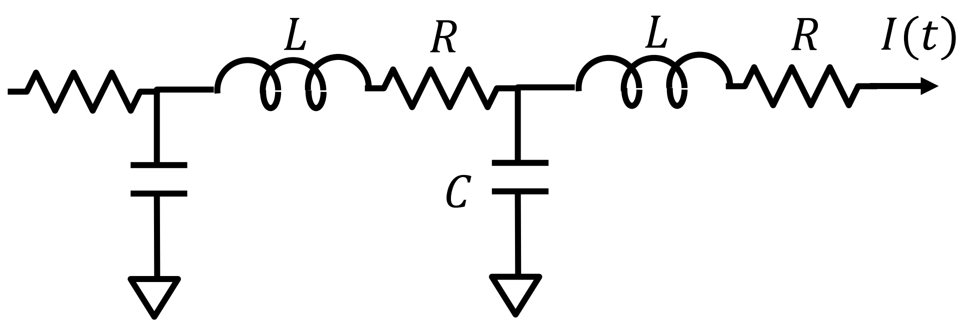
Model v.s. simulation

- We have simulator to simulate physical behavior
  - ANSIS (field solver) : electromagnetics
  - SPICE: circuit include FETs
  - These simulator handle a lot of variables, complex conditions, it should be perfect -> NO
  - Simulator show some values, how to validate them?

- Model equation is used to validate the tendency of simulation result

Application

- In semiconductor field…
  - Analog circuit design
  - Transmission line design
  - Transistor modeling
  - Modulation (Wireless transmitter)
  - Process and device simulation
  - …



Linear differential equation

- Differential equation defined by linear polynomial in the unknown function and its derivatives
  - $y' + ay = 0$ (a is constant) (eq.1.1)
    - If $y(x)$ satisfy above, $y(x)$ is called as solution
    - Solve above equation to obtain y(x)
    - Before that, y is called as unknown function

  - How to solve the equation?
    - Algebraically
      - Formula
      - Use assumption (Method of undetermined multiplier)
    - Program solver (Mathematica, Maxima)

Linear differential equation (cont.)

- Mission: solve function $y' + ay = 0$ (a is constant) (eq.1.1)
  - Use nature of exponential function
    - $(e^{ax})' = ae^{ax}$
  - Multiple e^{ax} to (eq.1.1)
    - $e^{ax}y' + ae^{ax}y = 0$
  - Recall the differential for products
    - $\big(g(x)h(x)\big)' = g(x)h(x)' + g(x)'h(x)$
  - (e.q.1.1) should be
    - $(e^{ax}y)' = 0.$ -> $e^{ax}y = c$ (c is <u>arbitrary constant</u>)
    - Solution w/o constant: <u>a general solution</u>
    - If c has some specific value -> <u>a particular solution</u>

Initial value problem

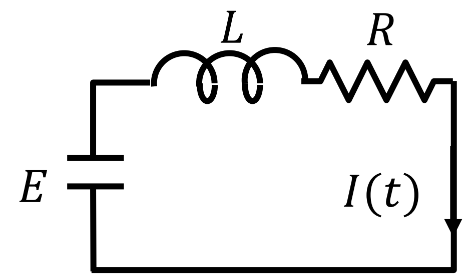
- Shape of function depends arbitrary constant
  - We may don't know the arbitrary constant itself
  - We may know the value(y_0) on specific point(x_0)
    - y_0: Initial value or initial condition
  - e.x. $y' + ay = 0,\ y_0 = y(x_0) = ce^{-ax_0}$
    - $c = y_0 e^{ax_0}$

  - General form of (eq.1.1) should be
    - $y = y_0 e^{-a(x-x_0)}$

Homogeneous differential eq.

- Think about following equation
  - $y' + ay = r(x)$ (eq.1.7)
- A differential equation is homogeneous when
  - $f(x,y)dy = -g(x,y)dx$ -> $f(x,y) + g(x,y)\frac{dx}{dy} = 0$
  - If $r(x) = 0$, eq.1.7 is homogeneous
- If not, a differential equation is inhomogeneous
  - If $r(x) \neq 0$, eq.1.7 is inhomogeneous
- A general solution of inhomogeneous function eq.1.7 is
  - $y = \left(\int r(x)e^{ax}dx + c\right)e^{-ax}$ (c is constant)
  - No need to understand (I'll introduce more easy way)

Example: RL circuit



- Derive current $I(t)$ of RL circuit
  - Initial condition: $I(0) = 0$
- Voltage of R (V_R) and L (V_L) are:
  - $V_R = RI(t)$, $V_L = L\frac{dI(t)}{dt}$, $E = V_R + V_L$, thus $\frac{dI(t)}{dt} + \frac{R}{L}I(t) = \frac{E}{L}$
  - Equation is same as eq.1.7 ->
  - From general solution: $I(t) = \left(\int \frac{E(t)}{L} e^{\left(\frac{R}{L}\right)t} dt + c\right) e^{-\left(\frac{R}{L}\right)t}$,
  - E is constant: $I(t) = \frac{E}{R} + ce^{-\left(\frac{R}{L}\right)t}$
  - Apply initial condition, and final result should be
    - $I(t) = \frac{E}{R}\left(1 - e^{-\left(\frac{R}{L}\right)t}\right)$

Conclusion

- Introduce (1st-order) differential equation
- Differential equation is widely used for engineering
  - Express natural behaviors (physics, electrics) as equation
  - Build equation, solve equation, and obtain its solution
    - Solution w/o constant: a general solution
    - Solution for specific value: a particular solution
    - A particular solution is obtained using initial condition
- Introduce RL-circuit case
- Nest: 2nd –order differential equation
- Contact: nishizawa@aoni.waseda.jp

Quiz

- Solve a general form of following equations
  1. $y' - y = 0$
  2. $y' + y = 0$
- Solve a initial value problem of following equations
  1. $y' + y = 0, y(0) = 2$
  2. $y' - 2y = 0, y(1) = 2$