Strain analysis for the plateau of Iran

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Lecture for the Seminar course

Contents

Foreword

- Deformation of the earth is directly related to geodetic observations.
- Increase in the number and accuracy of geodetic observations, has resulted in various displacement and velocity observations.
- This observations can describe various geodynamic processes.
- The strain tensor of the earth's crust is one of the direct results of geodetic observations.

Displacement Gradient Tensor & Deformation Gradient Tensor

- **Displacement vector:** $$\vec{u} = \left[u_x, u_y, u_z \right]^T \quad (1)$$

$$\nabla \vec{u} = \begin{bmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{bmatrix} \quad (2)$$

$$F = I + \nabla \vec{u} \quad (3)$$

Strain Concepts & Strain Tensors

• **Strain**: Relative deformation, compared to a reference position configuration.

I. Cauchy-Green strain tensor:

$$C = F^T F \quad (4)$$

II. Lagrangian finite strain tensor:

$$\varepsilon^* = \frac{\nabla\vec{u} + \nabla\vec{u}^T + \nabla\vec{u}^T \nabla\vec{u}}{2} \quad (5)$$

Strain Concepts & Strain Tensors

III. Eulerian finite strain tensor:

$$\varepsilon = \frac{\nabla\vec{u} + \nabla\vec{u}^T}{2} \quad (6)$$

$$\varepsilon = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} e_{xx} & \frac{1}{2}\left(e_{yx} + e_{xy}\right) & \frac{1}{2}\left(e_{zx} + e_{xz}\right) \\ \frac{1}{2}\left(e_{xy} + e_{yx}\right) & e_{yy} & \frac{1}{2}\left(e_{zy} + e_{yz}\right) \\ \frac{1}{2}\left(e_{xz} + e_{zx}\right) & \frac{1}{2}\left(e_{yz} + e_{zy}\right) & e_{zz} \end{bmatrix} \quad (7)$$

Strain Concepts & Strain Tensors

IV. Principal strain tensor: strain tensor in principal planes, where the shear strains are zero.

$$\theta = \frac{1}{2}\arctan\left(\frac{2\varepsilon_{xy}}{\varepsilon_{xx} - \varepsilon_{yy}}\right) \quad (8)$$

$$\varepsilon_1, \varepsilon_2 = \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2} \pm \sqrt{\left(\frac{\varepsilon_{xx} - \varepsilon_{yy}}{2}\right)^2 + \varepsilon_{xy}{}^2} \quad (9)$$

Strain Concepts & Strain Tensors

- **Strain invariants:**

$$\left|\varepsilon-\lambda I\right|=\begin{vmatrix}\varepsilon_{xx}-\lambda & \varepsilon_{xy} & \varepsilon_{xz}\\ \varepsilon_{xy} & \varepsilon_{yy}-\lambda & \varepsilon_{yz}\\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz}-\lambda\end{vmatrix}=0 \qquad (10)$$

$$\begin{cases} I_1 = trace\left(\varepsilon\right)\\ I_2 = \begin{vmatrix}\varepsilon_{xx} & \varepsilon_{xy}\\ \varepsilon_{xy} & \varepsilon_{yy}\end{vmatrix}+\begin{vmatrix}\varepsilon_{xx} & \varepsilon_{xz}\\ \varepsilon_{xz} & \varepsilon_{zz}\end{vmatrix}+\begin{vmatrix}\varepsilon_{yy} & \varepsilon_{yz}\\ \varepsilon_{yz} & \varepsilon_{zz}\end{vmatrix}\\ I_3 = \left|\varepsilon\right| \end{cases} \qquad (11)$$

Strain Concepts & Strain Tensors

- **Dilation (2D):**

$$\sigma = \frac{1}{2}\left(e_{xx} + e_{yy}\right) \qquad (12)$$

- **Pure Shear (2D):**

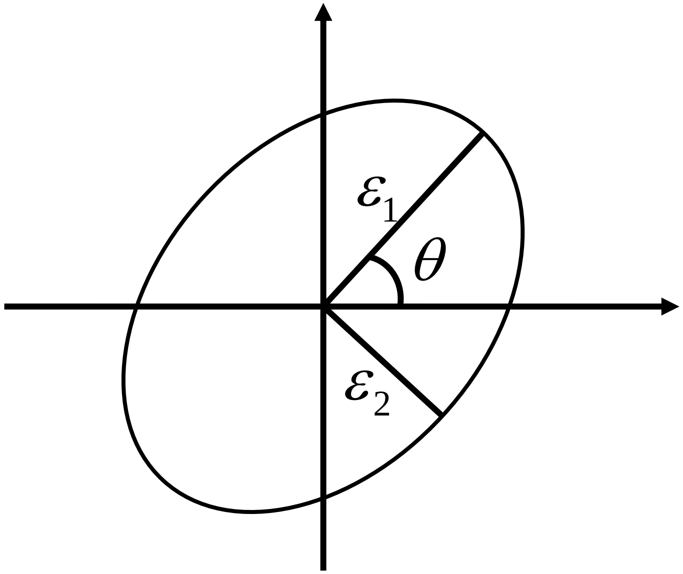
$$\tau = \frac{1}{2}\left(e_{xx} - e_{yy}\right) \qquad (13)$$

- **Simple Shear (2D):**

$$\upsilon = \frac{1}{2}\left(e_{xy} + e_{yx}\right) \qquad (14)$$

Strain Concepts & Strain Tensors

- **Strain ellipse** (in case of 2D strain tensor):



Strain Tensor From Displacement Observations

I. Finite element method (2D):

$$u_x = ax + by + c = e_{xx}x + e_{xy}y + d_x \qquad (15)$$

$$u_y = e_{yx}x + e_{yy}y + d_y \qquad (16)$$

$$\begin{bmatrix} u_{1,x} \\ u_{1,y} \\ u_{2,x} \\ u_{2,y} \\ u_{3,x} \\ u_{3,y} \end{bmatrix} = \begin{bmatrix} x_1 & 0 & y_1 & 0 & 1 & 0 \\ 0 & y_1 & 0 & x_1 & 0 & 1 \\ x_2 & 0 & y_2 & 0 & 1 & 0 \\ 0 & y_2 & 0 & x_2 & 0 & 1 \\ x_2 & 0 & y_3 & 0 & 1 & 0 \\ 0 & y_3 & 0 & x_3 & 0 & 1 \end{bmatrix} \begin{bmatrix} e_{xx} \\ e_{yy} \\ e_{xy} \\ e_{yx} \\ d_x \\ d_y \end{bmatrix} \qquad (17)$$

Strain Tensor From Displacement Observations

I. Finite element method (2D):

$$\begin{cases} u_{i,x} = \varepsilon_{xx} x_i + \varepsilon_{xy} y_i + \omega y_i + d_x \\ u_{i,y} = \varepsilon_{xy} x_i + \varepsilon_{yy} y_i - \omega x_i + d_y \end{cases} \quad (18)$$

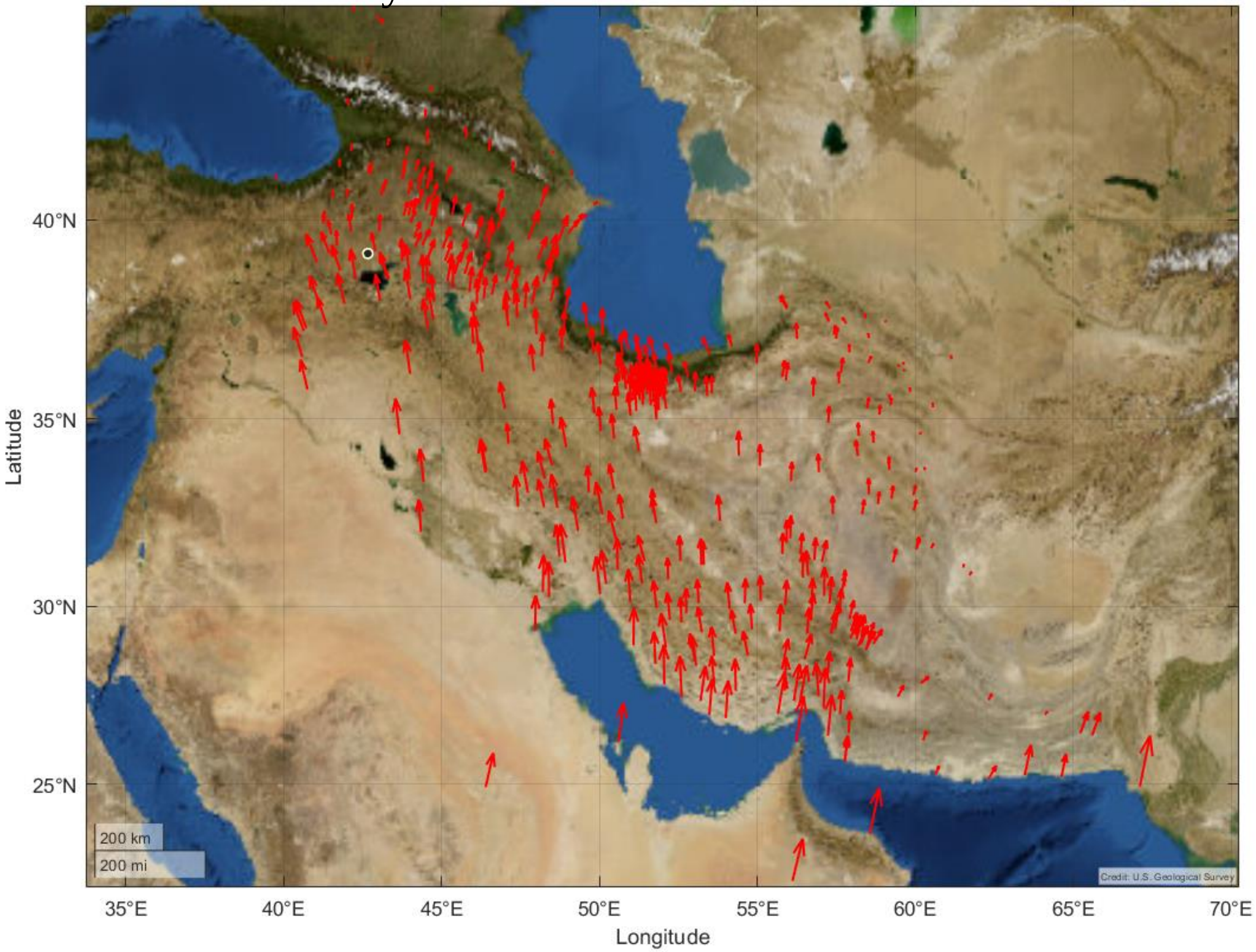
II. Finite difference method (2D):

$$\begin{cases} u_{i,x} = \varepsilon_{xx} \Delta x_i + \varepsilon_{xy} \Delta y_i + \omega \Delta y_i + d_x \\ u_{i,y} = \varepsilon_{xy} \Delta x_i + \varepsilon_{yy} \Delta y_i - \omega \Delta x_i + d_y \end{cases} \quad (19)$$

$$\begin{cases} \Delta x_i = x_i - x_A \\ \Delta y_i = y_i - y_A \end{cases} \quad (20)$$

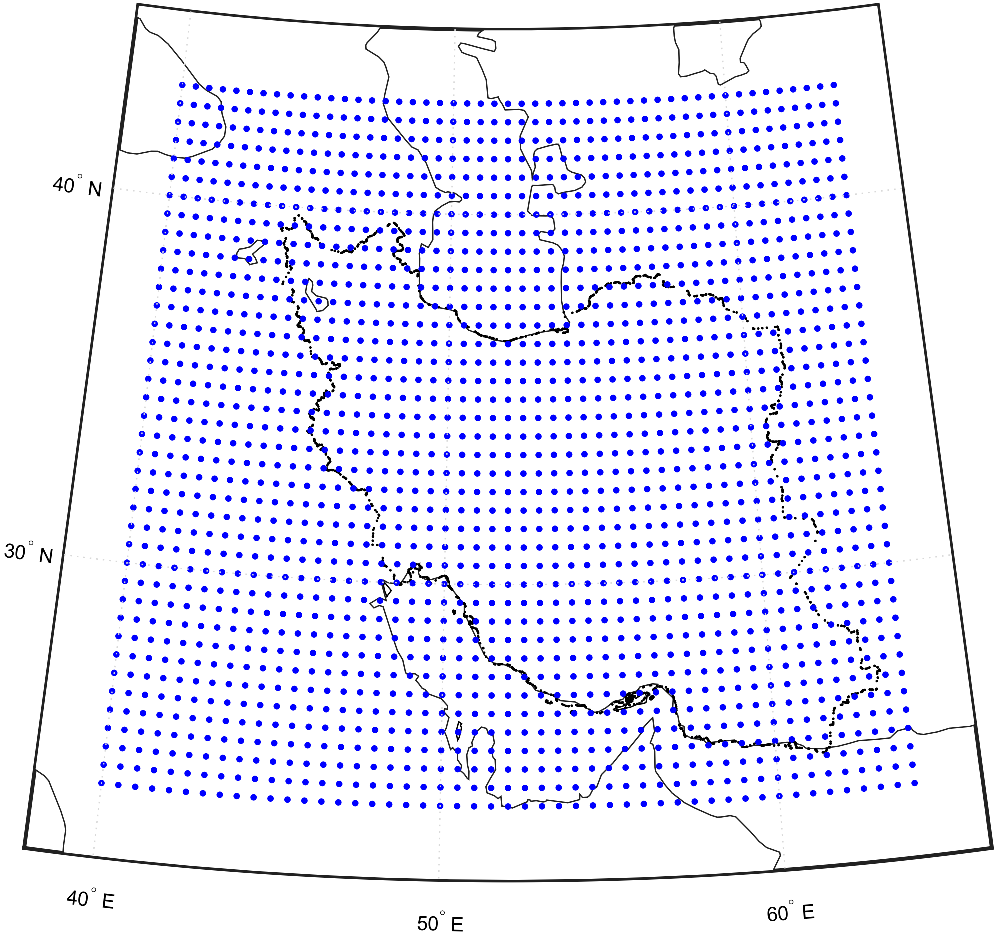
Strain Analysis in Action

GPS velocity field relative to the Eurasian fixed frame



Strain Analysis in Action

Grid Latitude: 24° to 43° Grid Longitude: 40° to 64° Step: 0.25°



Strain Analysis in Action

- **Using finite difference method:**

$$\begin{cases} \dot{u}_{i,x} = \Delta x_i \dot{\varepsilon}_{xx} + \Delta y_i \dot{\varepsilon}_{xy} + \Delta y_i \dot{\omega} + \dot{d}_x \\ \dot{u}_{i,y} = \Delta x_i \dot{\varepsilon}_{xy} + \Delta y_i \dot{\varepsilon}_{yy} - \Delta x_i \dot{\omega} + \dot{d}_y \end{cases} \quad ; \quad i = 1, 2, \cdots, 399 \qquad (21)$$

$$\begin{bmatrix} u_{1,x} \\ u_{1,y} \\ \vdots \\ u_{m,x} \\ u_{m,y} \end{bmatrix} = \begin{bmatrix} \Delta x_1 & 0 & \Delta y_1 & \Delta y_1 & 1 & 0 \\ 0 & \Delta y_1 & \Delta x_1 & -\Delta x_1 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \Delta x_m & 0 & \Delta y_m & \Delta y_m & 1 & 0 \\ 0 & \Delta y_m & \Delta x_m & -\Delta x_m & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\varepsilon}_{xx} \\ \dot{\varepsilon}_{yy} \\ \dot{\varepsilon}_{xy} \\ \dot{\omega} \\ \dot{d}_x \\ \dot{d}_y \end{bmatrix} \qquad (22)$$

$y \qquad\qquad A \qquad\qquad x$

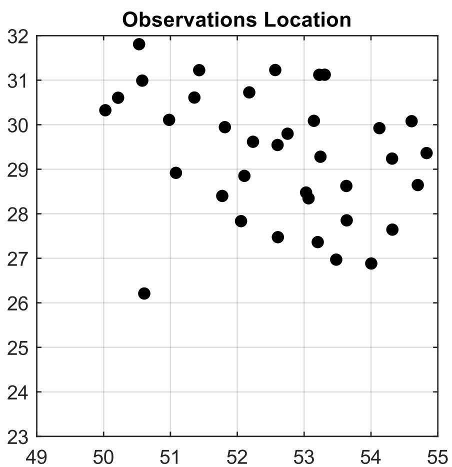
Strain Analysis in Action

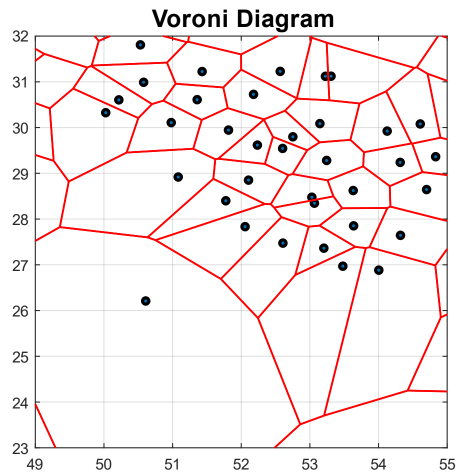
- **Weight matrix:**

  I. Based on distance:

$$L_i = \exp\left\{\frac{-\Delta R_i^2}{D^2}\right\} \qquad (23)$$

  II. Based on clustering:





$$Z_i = \frac{nS_i}{\Sigma S_k} \qquad (24)$$

Strain Analysis in Action

- **Weight matrix:**

  III. Based on STD of observations: C_i^{-1}

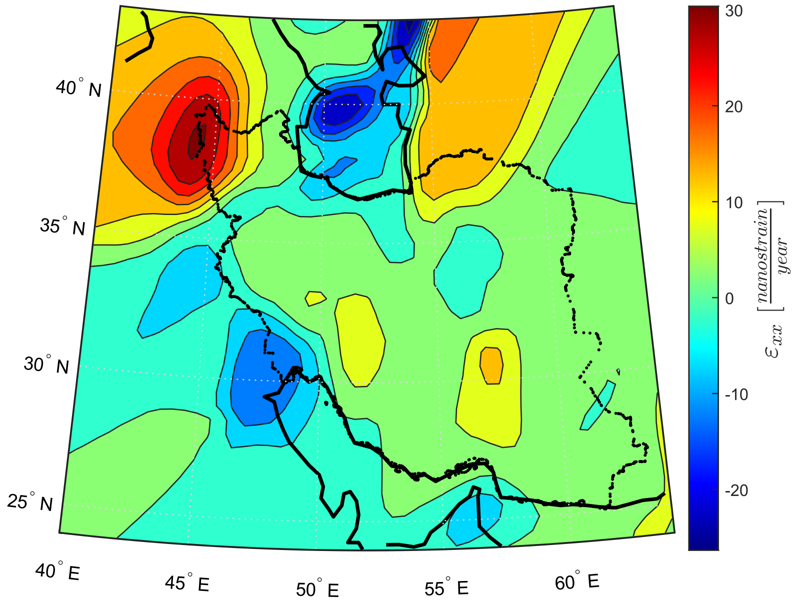
$$W_i = C_i^{-1} \times L_i \times Z_i \qquad (25)$$

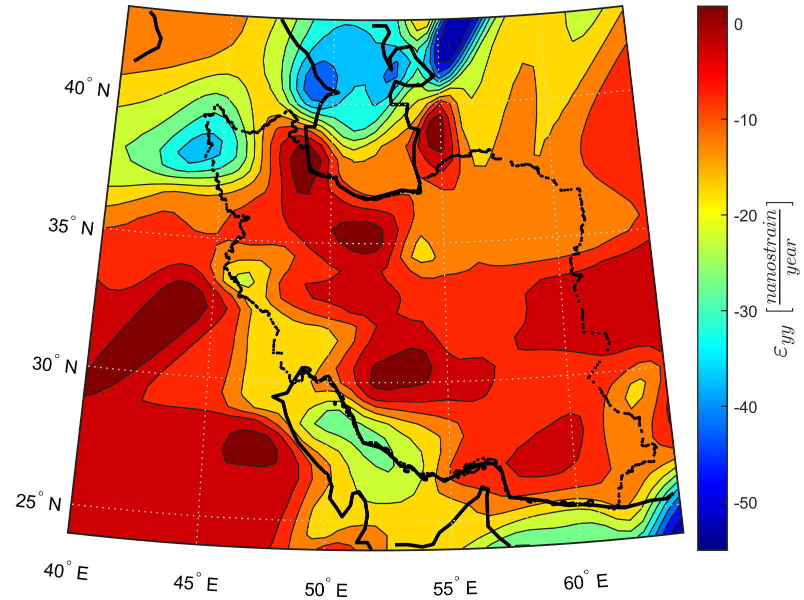
- **Weighted Least Squares:**

$$\hat{x} = \left(A^T W A\right)^{-1} A^T W y \qquad (26)$$

Strain Analysis in Action

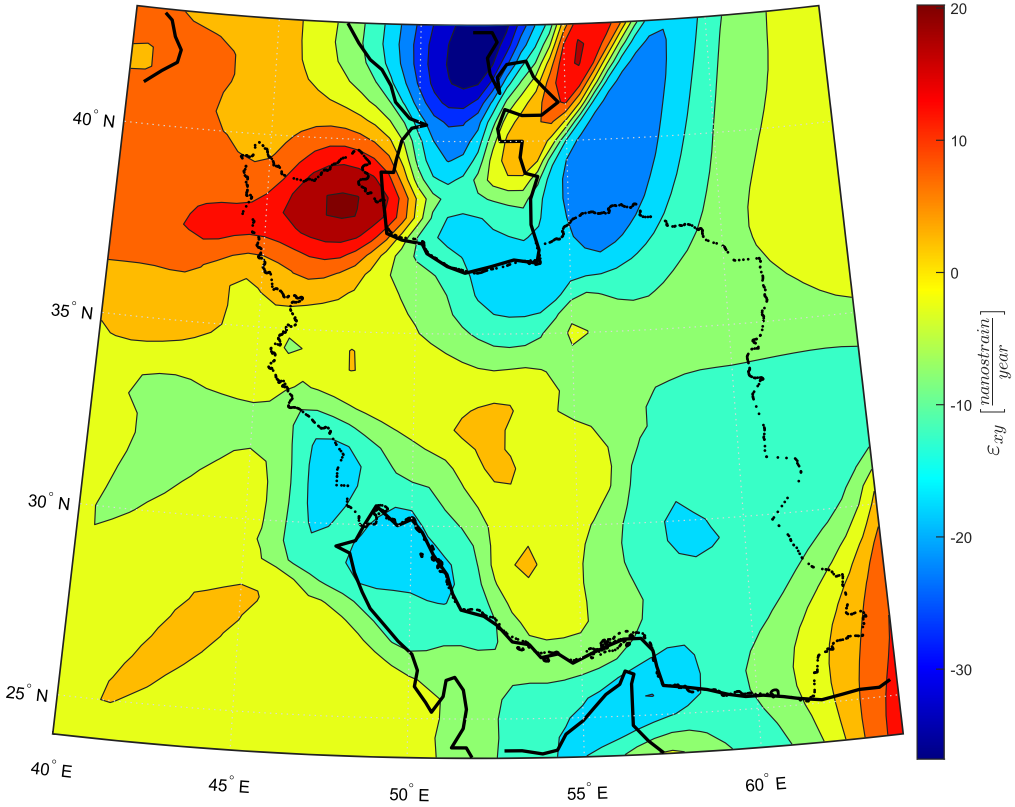
Normal strain maps





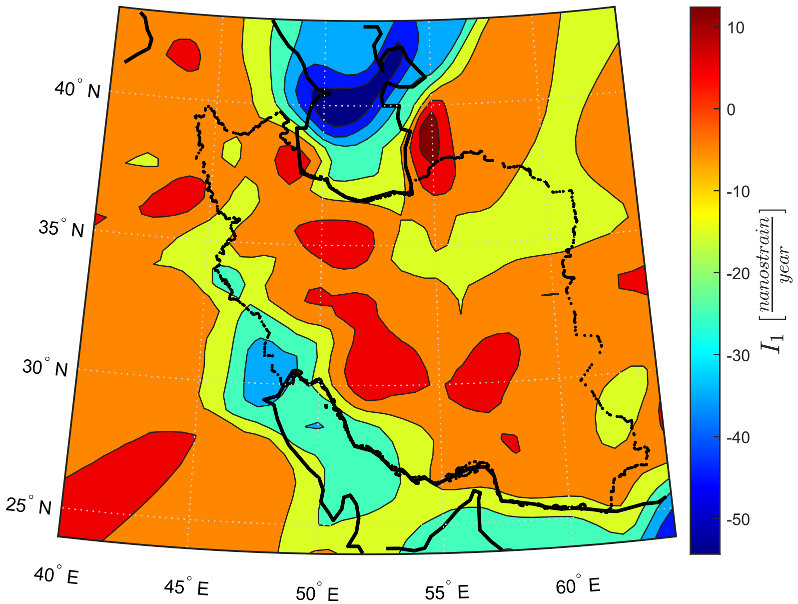
Strain Analysis in Action

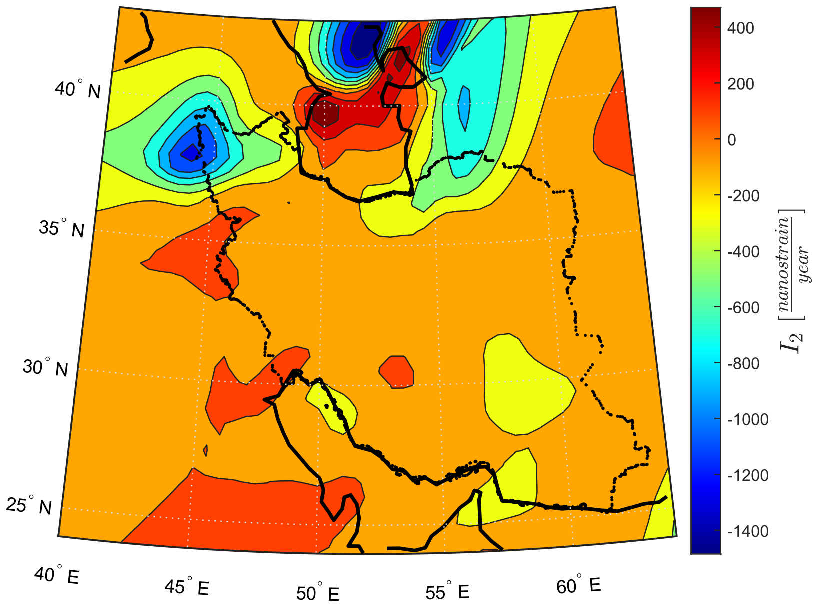
Shear strain map



Strain Analysis in Action

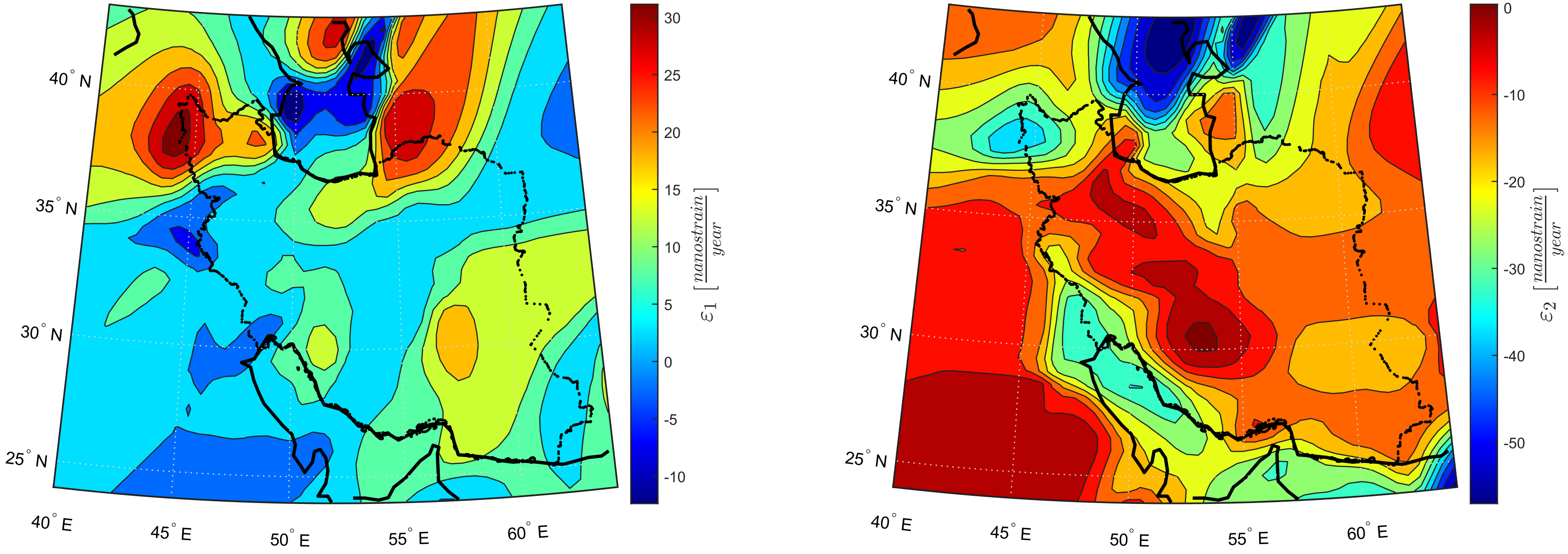
Strain invariant maps





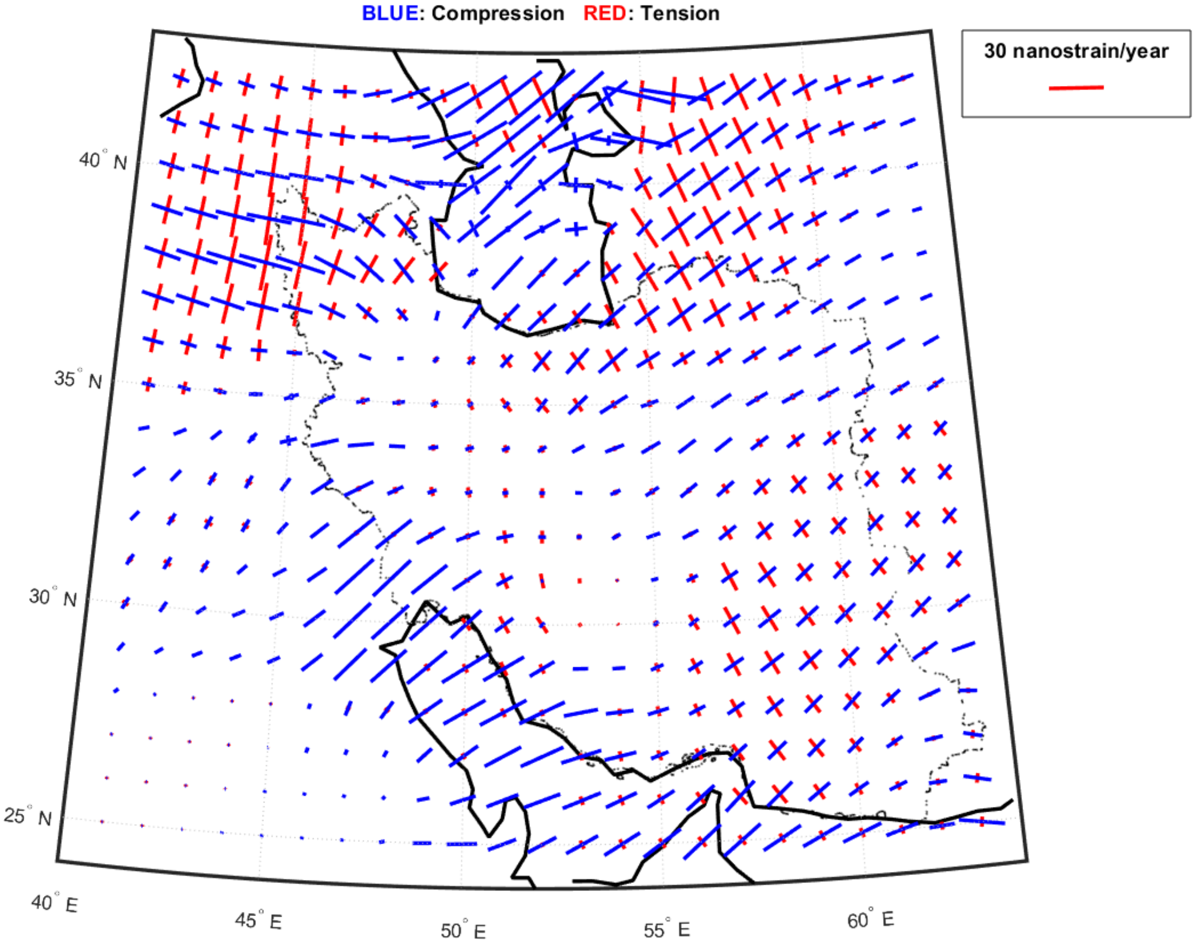
Strain Analysis in Action

Principal strain maps



Strain Analysis in Action

Strain ellipse map



References

- **Donald L. Turcotte and Gerald Schubert, 1982:** Geodynamics third edition.

- **Kurt Stiiwe, 2007:** Geodynamics Of The Lithosphere second edition.

- **Fatemeh Khorrami et al. 2019:** An up-to-date crustal deformation map of Iran using integrated campaign-mode and permanent GPS velocities.