# On the global economic potentials and marginal costs of non-renewable resources and the price of energy commodities, supplementary material

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This documents presents supplementary information to the paper entitled 'On the global economic potentials and marginal costs of non-renewable resources and the price of energy commodities', *Energy Policy*. While this additional material would reduce the clarity of the main paper with unnecessary detail, it is essential for anyone wishing to explore further the assumptions of the model, the data justifying its theoretical structure and the mathematical properties of the equations. Thus, this document presents, in order, an exploration of BP reserve data in their 2009 and 2013 versions, the differences observed and how these are used to determine the parameter $\nu_0$ of the model (section S.1), a sensitivity analysis of the model over the value of $\nu_0$ for oil and gas consumption and price paths (section S.2) and an exploration of the mathematical properties of the model, in particular hysteresis and path dependence (section S.3).

## S.1. Discussion of Reserve to Production ratios and the determination of $\nu_0$

### *S.1.1. Reserve data and its reliability*

Reserve data is subject to continuous revisions and changes due not only to energy commodity price changes in time but also from political motives. Such data is therefore in general difficult to use and interpret. For the model presented in this paper, it is however necessary to use reserve data since a statement is made on the dynamics of reserves and their relation to commodity prices. The data used, taken from the BP Statistical Review of World Energy Workbook, was explored using two different versions of the workbook, those of 2009 and 2013. In the case of oil, differences were observed between the two versions of the BP data, over historical trends.[1] Therefore, it is apparent that BP changed its historical oil reserve data *retrospectively*. Our interpretation of historical reserve data concerns what we consider that the oil industry thought the reserves were each year in the past. However, from this observation we know that BP data are subject to later revisions and thus this interpretation of the data is only partially reliable. No such significant changes were observed in the case of gas.

The details of the changes however, present in every region in almost every year, can be analysed and the most significant changes can be traced to two regions and particular cases: unconventional oil in NAM and SAM. These correspond to special categories in the BP workbook for Canadian tar sands and Venezuelan heavy oil, of which the reserves have been revised by very large amounts retrospectively (e.g. the 2013 workbook reports much larger tar sands reserves for 1999-2012 than the 2009 workbook). When removing these unconventional reserves from the data, however, the changes are much less significant. This is shown in figure 1. The top panels show total reserves in the 2009 workbook (left) and the 2013 workbook (right), while the bottom panels show the same data minus unconventional reserves and highlights the addition of unconventional oil . It is noted that without unconventional reserves, no significant data changes are apparent, and that large fluctuations appear in recent years for unconventional oil. The retrospective data change justifies to characterise these increases as fluctuation in this context.

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[1]Thanks to comments from the anonymous referee, this might have otherwise escaped our attention.

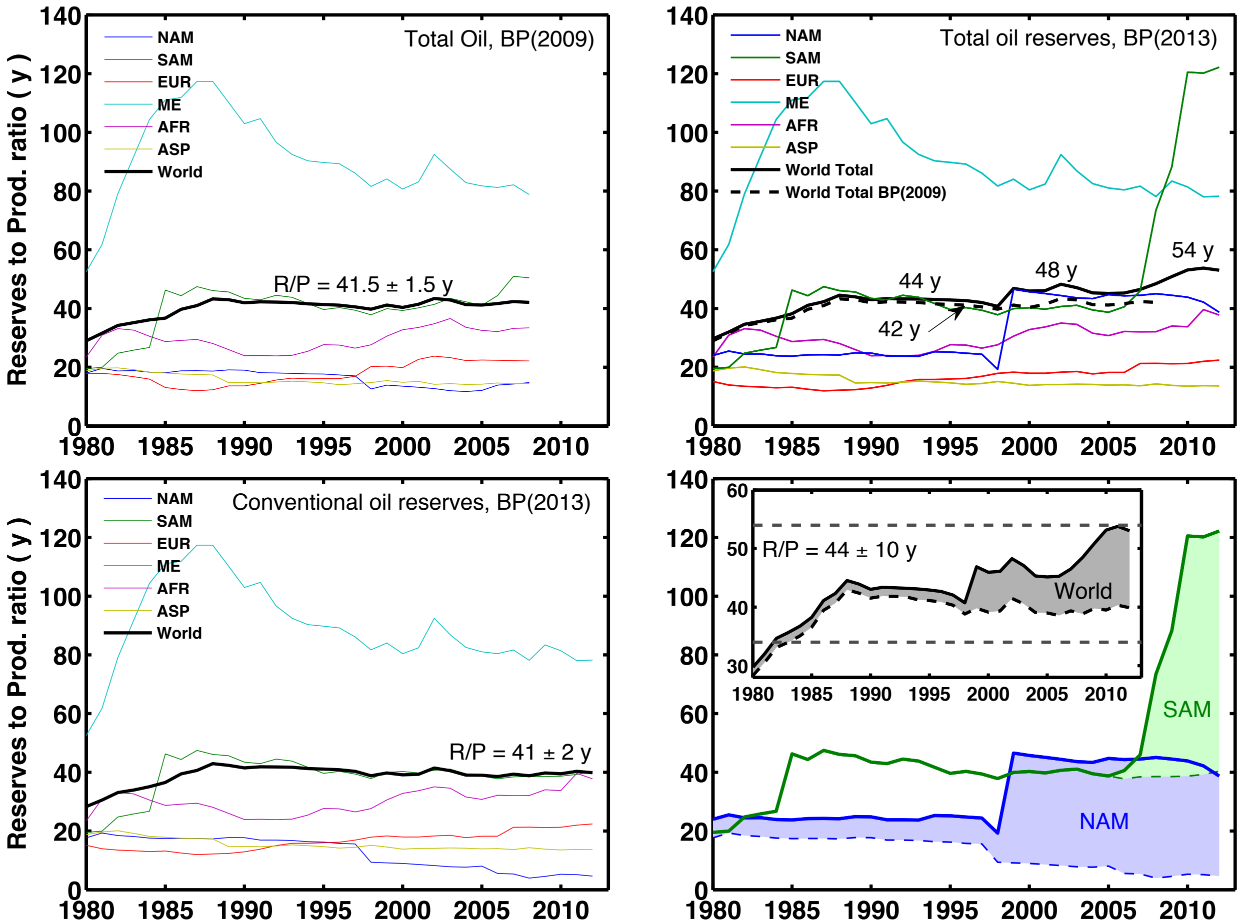


Figure 1: Reserve to production ratios for stock resources for Oil calculated from the 2009 version of the BP Statistical Review of World Energy Workbook (BP, 2009) (*top left*) and its 2013 (BP, 2013) version (*top right*). Changes are apparent. Reserves excluding unconventional oil in the 2013 version are given in the *bottom left*. Important changes in the data for NAM and SAM, and their impact on the World value, are highlighted in the panel on the *bottom right*. The legend abbreviations correspond to North America (NAM), South America (SAM), Europe (EUR), Middle East (ME), Africa (AFR) and Asia-Pacific (ASP). Note how changes have been made to the data retrospectively over past years for both NAM and SAM. The dashed line on the right panel is a copy of the world total of the left panel for comparison.

Unconventional reserves are known to be on the edge of the profitability threshold, and may actually well set the marginal cost. In these high cost ranges, large amounts of unconventional resources exist, in other words, the cost-distributed density sees a very sharp rise at these cost values. Therefore, small variations in the estimated *economic* extraction cost (or changes in the value of existing fossil fuel subsidies) may signify bringing in large amounts of resources into reserves, the actual real amount of unconventional oil that is economical to exploit being highly speculative. In addition to this, important (direct or indirect) subsidies exist in unconventional oil producing countries with the specific goal to help bring their exploitation cost within the economic threshold. All of this contributes to generate large fluctuations in the current amounts of oil reserves, and since production does not fluctuate in this way, the R/P ratio fluctuates along with reserves.

### *S.1.2. The determination of $v_0$*

These fluctuations indicate uncertainty in the determination of the $v_0$ parameter. We nevertheless interpret the data as indicating that $v_0$ *is a constant of time as long as there is no change of regime for the global energy market*, but that fluctuations make its determination difficult. If no monopolistic behaviour existed in the oil market, $v_0$ would represent the rate at which energy reserves can be extracted from the ground. If that rate could be very fast (large

$\nu_0$) in comparison to the demand, energy resources would be consumed in perfect order of cost, within an uncertainty cost range. However, even without monopolistic behaviour, since any individual oil and gas wells or coal and uranium mines can only produce at a certain finite rate (small $\nu_0$) which is lower than global demand, a number of wells and mines will remain under exploitation covering a range of costs of extraction, where expensive projects are undertaken *before* resources run out in low cost projects. Adding monopolistic behaviour means increasing a local value of the R/P ratio (e.g. in ME), leading to an increase of the global value as well, indicating that some resource owners delay production while some others fill in that gap. This forces a change of regime in the world market and has to be interpreted with a change in the value of $\nu_0$. In principle, the value of $\nu_0$ need not be a constant of time; however there is no credible way by which to predict how its value could change in the future and the best approach is to keep it constant at its current value within an uncertainty range (see next section for a sensitivity analysis on changes in the value of $\nu_0$). This correspond to taking the assumption that *no change of regime in the energy market* occurs in the future. The impact of the value of $\nu_0$ onto the properties of the model are described below in section S.3. The values for $\nu_0$ for oil are indicated in figure 1 with uncertainty ranges. Note that the impact of OPEC formation is excluded from the analysis.

## S.2. Sensitivity analysis for the value of $\nu_0$ on future oil/gas prices and flows

According to data from BP (2013) with the analysis given above, the global ratio of conventional oil reserves to production has not changed by more than 5% during the last two decades (see Figure 1, bottom left chart). If unconventional oil is included, then the variations in the ratio increase up to 23%, mostly due to the reclassification of Canadian tar sands and Venezuelan heavy oil from resources to reserves (see Figure 1, upper and lower right charts). In order to understand how corresponding variations in $\nu_0$ affect the results of the model, we carried out a sensitivity analysis over possible values for $\nu_0$ for oil and gas within uncertainty ranges that reflect the variations in BP data.

### *S.2.1. Sensitivity for oil and its $\nu_0$ value*

In the model, the parameter $\nu_0^{-1}$ is considered constant and equal to 44 years for the case of oil. For the sensitivity analysis, we varied $\nu_0^{-1}$ within its uncertainty interval of 44 ± 10 years. Following the approach of sections 3.2 and 3.3 of the paper, figure 2 presents the evolution in the flow of oil for the same exogenous price (left chart), and the evolution in the marginal cost of oil for the same exogenous flow (right chart). The resulting values can be directly compared to the originals of the main paper in order to estimate the level of uncertainty in comparison to other factors such as the uncertainty generated by the actual amounts of resources available. In both cases, three curves are presented, each of them corresponding to the minimum, the central and the maximum value of $\nu_0^{-1}$.

As the left chart of figure 2 shows, a rise in $\nu_0^{-1}$ delays the production of oil in the model, moving the peak from 2059 ± 3 years for minimum and maximum values of $\nu_0^{-1}$. The average flow differences are approximately of 10%, with a maximum of 18% in 2026. Comparing the left chart of figure 2 with the upper left chart of figure 4 in the paper, it is clear that the impact in the oil flow associated to changes in $\nu_0^{-1}$ are much less important than those associated to uncertainty in the amount of oil resources. For the exogenous flow case, we can see the impact of changes in $\nu_0^{-1}$ in the marginal cost of oil in the right chart of figure 2. A rise in $\nu_0^{-1}$ increases the marginal cost of oil in 8% average. Again, these changes are much less important than those associated to uncertainty in the amount of resources, presented in the upper left chart of figure 5 in the paper.

### *S.2.2. Sensitivity for gas and its $\nu_0$ value*

Using the same methodology, we extended the sensitivity analysis for the natural gas industry. The parameter $\nu_0^{-1}$ presented in equation (1) of the paper is considered constant and equal to 56 years. For the sensitivity analysis, we varied $\nu_0^{-1}$ within the quoted uncertainty interval of 56 ± 6 years, in accordance with the data presented in figure 2 of the paper. Following the approach of sections 3.2 and 3.3 of the paper, figure 2, bottom panels, presents the flow of gas for the same exogenous price (left chart), and the evolution in the marginal cost of gas for the same exogenous flow (right chart).

In the case of an exogenous gas price presented in figure 2 (left chart), the rise in $\nu_0^{-1}$ causes a very small delay in production, estimated at ± 1 year for minimum and maximum values of $\nu_0^{-1}$. Regarding the flow differences, these are smaller than 6%. For the exogenous flow case presented in figure 2 (right chart), a rise in $\nu_0^{-1}$ increases the marginal

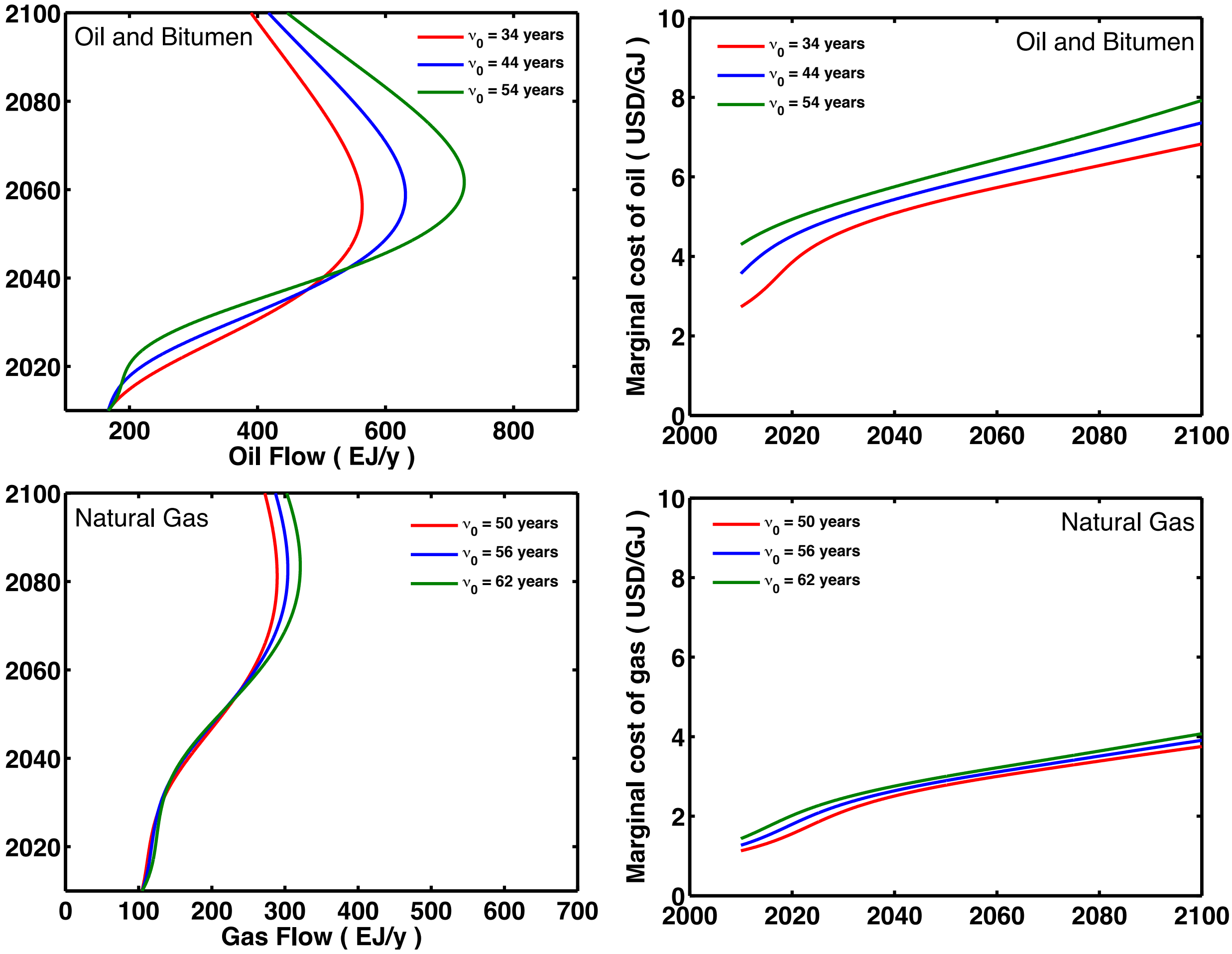


Figure 2: (*Top panels*) Sensitivity analysis for the parameter $\nu_0^{-1}$ for the oil case. In the left chart, the flow of oil is estimated using an exogenous price of oil, following the assumptions of section 3.2 of the main paper. In the right chart, the marginal cost of oil is estimated using an exogenous flow, following the assumptions of section 3.3 of the main paper. In both cases, the red, blue and green curves represent the minimum, the central and maximum values for $\nu_0^{-1}$. (*Bottom panels*) Sensitivity analysis for the parameter $\nu_0^{-1}$ for natural gas.

cost of gas in less than 6%. Comparing the charts in figure 2 with figures 4 and 5 in the paper, it is clear that changes in $\nu_0^{-1}$ have much smaller impacts on the flow and price of gas than those associated to uncertainty in the amount of resources.

## S.3. Mathematical properties of the resource flow equation

### *S.3.1. Introduction*

We present in this section of the supplementary material a mathematical digression that explores the mathematical properties of equations 1 and 2 of the main text, which establish a relationship between the price $P(t)$ of an energy carrier derived from a particular type of non-renewable energy resource (e.g. coal, oil, gas, U) and its consumption, or flow, $F(t)$. As stated in the main text, the relationship is not functional, i.e. $F(t)$ cannot be written as a single valued function of $P(t)$ or the reverse. The relationship is *path dependent*, and therefore depends on the history of the system and on its starting point. It thus features hysteresis. The model is slightly inspired from a physical model of energy activated processes in chemistry, as given by Mercure et al. (2005). This material also presents the different limiting behaviours of this system for various values of its parameter $\nu_0$.

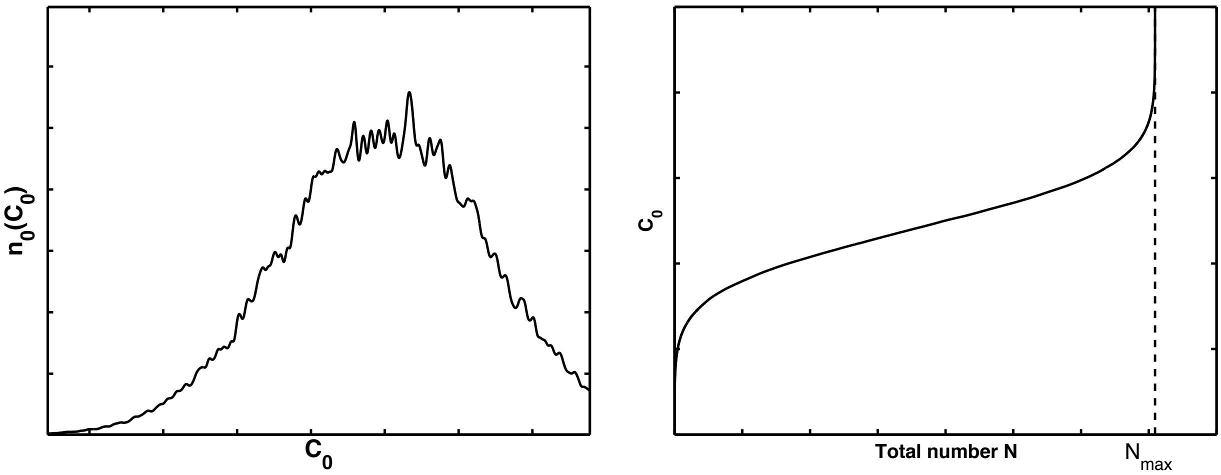


Figure 3: *Left* Example of cost distribution for a specific natural resource $n_0(C)$ as a function of exploitation cost $C$. *Right* Cost-quantity curve for that resource type.

### *S.3.2. Cost-supply curves*

As was introduced in Mercure (2012) and used in Mercure and Salas (2012), we define an initial (present day) distribution of non-renewable (stock) resource $n_0(C)$ function of cost $C$. This is a histogram of the number of resource units between cost values of $C$ and $C + dC$. This is a density function; the amount of resources available at costs between the values of $C_1$ and $C_2$ is

$$N_{1,2} = \int_{C_1}^{C_2} n_0(C) dC. \tag{1}$$

An example of such a starting resource distribution is given in the left panel of figure 3. At the present day, given the cost of extraction of every resource unit, the total amount $N$ of resources available below a cost value of $C$ is

$$N(C) = \int_0^C n_0(C') dC'. \tag{2}$$

Associated to this is the inverse relationship, the marginal cost of the resource $C(N)$ given that $N$ units have already been exploited; this is the cost-quantity curve. This is shown in the right panel of figure 3.

The amount $N$ of stock resources available below cost $C$ cannot be exploited instantaneously however, and furthermore, their owner might not be willing to extract and sell them at the current price of the associated commodity. Therefore, this amount $N$ will most likely no be sold for the price associated with the marginal cost of production $C$. If less resources are extracted at prices below $C$ than the total amount available in this cost range, resources situated higher up along the distribution must be used. Thus the cost-supply curve framework is not appropriate to use for stock resources with an international market. A similar statement could be invoked for renewable resources, however it has much less impact and the cost-supply curve framework is much more appropriate there. This is discussed in section S.3.8.

### *S.3.3. The probability of extraction*

Each resource unit has at every instant a probability of being extracted, and this probability depends on its probable individual cost of extraction $C$ and the market price of the commodity $P(t)$, $f(C, P(t))$. Its cost of extraction is uncertain; therefore a probability distribution exists for the value of its cost of extraction $h(C)$. Additionally, the market price is stochastic and has a certain volatility, with a certain standard deviation and probability distribution $g(P(t))$. The probability $f(C, P(t))$ is related to $h(C)$ and $g(P)$ through the integral of their convolution, providing the ‘rounded step-like’ probability function depicted in figure 2 of the main text. If $h$ and $g$ are normal distributions, then $f$ is an error function with width equal to the root of the sum of the squares of the widths of $h$ and $g$. These assumptions enable to avoid the use of a sharp step function, which would introduce unwanted kinks into calculations.

The probability of a resource unit of being extracted can moreover be expressed in terms of the difference between its most probable marginal cost of production and the mean value of the price, $f(P(t) - C)$.

### *S.3.4. A differential equation for resource flows*

While the initial (present day) distribution of resources is denoted with $n_0(C)$, the distribution of future amounts of resources left as they are gradually consumed is denoted as the time dependent function $n(C,t)$. This function depends on time in two ways, by itself and through the value of the price $P(t)$, and thus strictly speaking, should be written as $n(C, t, P(t))$. This property is the one that leads to path dependence, since the time derivative involves two terms, shown below.

While the cost distributed amounts of resources *left* is $n(C, t, P(t))$, not all resources are exploited but only those which have a probability of being exploited, given by

$$n(C, t, P(t)) f(P(t) - C). \tag{3}$$

This corresponds to cost distributed reserves, and the size of the reserves depend on the price, expanding when the price increases. If a constant fraction $\nu_0$ of reserves are consumed with the time interval $dt$, then the flow of resources during that interval is[2]

$$dn(C, t, P(t)) = -\nu_0 n(C, t) f(P(t) - C) dt, \tag{4}$$

which is the main equation of the model. This equation has no complete analytical solution but can be evaluated numerically, which is done for instance in FTT:Power using a discrete time step. Note that the negative sign stems from the fact that $n$ corresponds to cost distributed resources that are *left* at time $t$, and that the change in resources left is negative. The flow of resources $F(t)$ is the time derivative of the total amount of resources left at all cost values $N(C,t)$, which itself is the cost integral of the resource distribution $n(C,t)$.

$$F(P(t), t) = -\frac{dN}{dt} = -\int_0^\infty \frac{dn(C,t)}{dt} dC = \int_0^\infty \nu_0 n(C, t) f(P(t) - C) dC. \tag{5}$$

It is important to emphasise that the notation $F(P(t), t)$ signifies that $F$ is not a single function of the price $P(t)$, or conversely, since an infinite number of distributions, or integrands of eq. 5, can produce the same value for $F$.

### *S.3.5. Mathematical properties when the price is constant*

A few important properties of equation 4 may be derived from the simple situation where the current price of oil $P$, is independent of time. Eq. 4 is solved simply:

$$\frac{dn}{n} = -\nu_0 f(P - C) dt, \tag{6}$$

$$\Rightarrow n(C, P, t) = n_0(C) e^{-t\nu_0 f(P-C)}, \tag{7}$$

where $n_0$ is the initial distribution of the resource previously defined, which we take, for simplicity for now, as a constant extending to very large values. We observe from this result that for a constant price, the amount of resource left at each cost value decreases exponentially in time, more and more slowly at higher and higher values of $C$.

The behaviour of $n$ as a function of $C$ is not quite as simple as this, but may be calculated numerically using a simple functional form for $f$,[3] and is shown in the left panel of fig. 4 for different times after exploitation began. The times noted as $t_1$ to $t_6$ increase exponentially by a factor of 2 between each. Starting from the initial distribution $n_0$, a certain range of low cost units are first extracted, until they are depleted at time $t_3$. From then on, units at higher costs start to be exploited and the mid-point of $n$ starts to move towards higher values of $C_0$ up to $t_4$. $t_6$ shows $n$ after a very

[2] The fraction of cost distributed reserves consumed is actually $\nu_0 f(P(t) - C)$, a slight subtlety, which could become important is $f(P(t) - C)$ is very 'rounded'.

[3] A logistic function was used in this particular case; however an error function may be as appropriate.

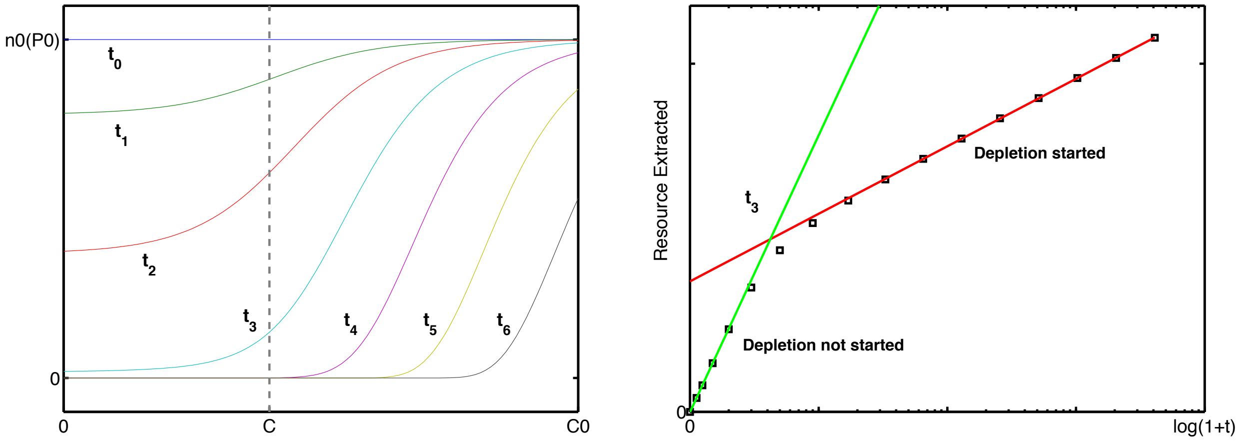


Figure 4: *Left* Distribution of oil resources $n$ as a function of extraction cost $C$ at different times. *Right* Total oil extracted as a function of time.

long time. We observe that for $t \to \infty$, all units are eventually used up and $n = 0$ for all values of $C_0$. This is due to a non-vanishing value of the probability distribution function $f(P - C)$.[4]

Furthermore, by calculating the area underneath $n_0 - n(C_0, t)$, we find the time dependence of the total number of units extracted. Fig. 4, right panel, shows this value against $\ln(1 + t)$. We observe that before the beginning of depletion, which occurs at around time $t_3$, the extraction is fast and linear against $\ln(1+t)$. After time $t_3$, it slows down but is again perfectly linear against $\ln(1 + t)$. We conclude that even though for $t \to \infty$, the system uses all existing resources, the progression of the extraction becomes exponentially slower and slower. At any practical time, the shape of $n(C_0, C, t)$ actually corresponds qualitatively to that at time $t_3$ in fig. 4, where low cost units have been consumed, and progresses very slowly.

Thus this demonstrates that consumption occurs even when the price does not increase. The supply, however, most likely does not meet the demand, since it monotonically decreases.

### *S.3.6. Consumption for a changing price*

In the general case where the market price of units varies in time, many types of behaviour can be observed. The important mechanism at work, however, is that as low cost units are depleted, in order to keep a supply of units which does not plummet, the price of the resource must increase in time in order to make the extraction of more expensive units economical. There is thus a direct relation between supply and price, but it cannot be expressed by a simple curve. We derive here the general solution to eq. 4.

The general solution to eq. 4 is as follows:

$$\int_{n_0}^{n} \frac{dn'}{n'} = -\nu_0 \int_0^t f(P(t') - C)dt', \tag{8}$$

which yields

$$n(C, P(t), t) = n_0(C)e^{-\nu_0 \int_0^t f(P(t')-C)dt'}. \tag{9}$$

Two time dependences are specifically denoted, that inherent to the price $P(t)$, and that associated to the exponentially decreasing exploitation that occurs at constant $C$. It is thus clear that the number of units $n(C, P(t), t)$ at any time strongly depends on the path taken by $P(t)$. The magnitude of the flow of energy extracted depends directly on the rate of change of $P(t)$.

[4]The non-zero value of $f$ concerns the non-zero but very small probability that firms extract resources at a loss, due to a lack of information or miscalculations of exploitation costs.

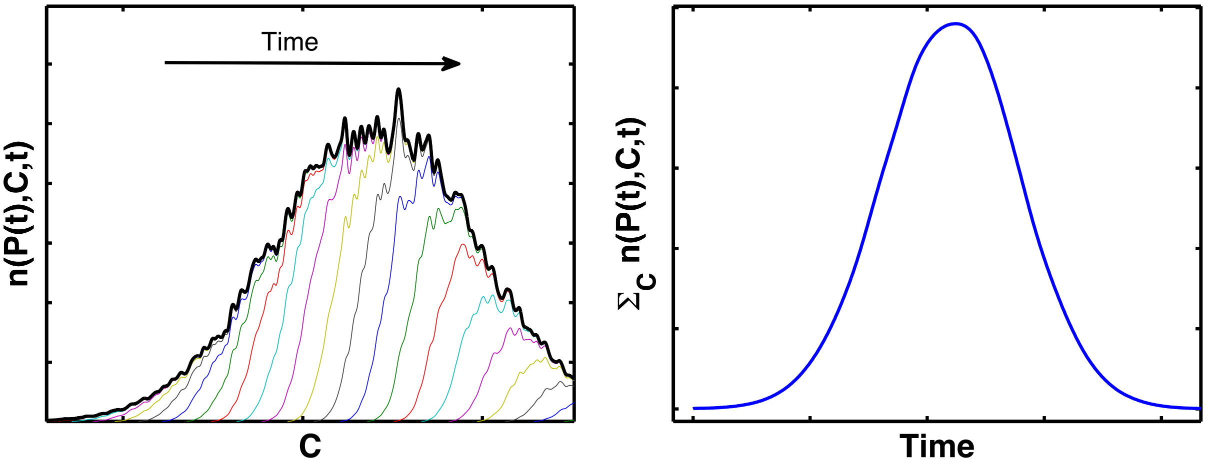


Figure 5: *Left* Progression of the extraction of units of a resource for a linearly increasing price $P(t)$, where vertical slices of the distribution are gradually removed. *Right* Resulting supply of units as a function of time.

As an example, figure 6, left panel, depicts the progression of exploitation in time using a linearly increasing cost $P(t)$. The black curve corresponds to the initial distribution, $n_0$. Each colour curve represents the number of units as extraction progresses after a certain time. These are equally spaced in time, which results from using a linear cost progression. The right panel of the figure shows the resulting supply of units as a function of time, calculated by taking the integral of the remaining number of units $n(C, P(t), t)$ at each time value.

However, when requiring a certain supply of units, the change in $P(t)$ depends on the magnitude of $n_0(C)$ at $C$ values near that of $P(t)$. For a large numbers of units situated narrowly in cost values $C$, the value of the market cost of extraction $P(t)$ may be almost stationary. For low amounts of cheap units, the rate of change of $P(t)$ will be very large. In this theory, therefore, the value of $P(t)$ never endogenously decreases unless demand decreases. This is due to the fact that we have made abstraction of the effect of hoarding, and we thus assume that only the amount of energy intended to be consumed immediately is generated by producer firms.

This also demonstrates the irreversibility in the process; if the price remains low, only low cost resources are used, while if the price increases, for example faster than consumption can occur, then significant amounts of low cost resources may remain in place while more expensive resources are consumed, and these two price behaviour assumptions could be chosen such that they generate exactly the same flow $F$. This corresponds to hysteresis and path dependence, akin to the change in entropy that occurs in irreversible processes in physical systems.

*S.3.7. The role of the parameter $v_0$*

The parameter $v_0$ controls equation 4 by determining the fraction of reserves per unit time the system is allowed to consume during a unit of time (e.g. a year), for a multitude of internal reasons that need not be known in detail. All that is required to be known is that $v_0$ is indeed a constant of time, a fact fairly well demonstrated for oil and gas in the main paper. But what is the effect of changing the value of $v_0$? This can be illustrated by the use of limiting values.

For a value of $v_0 = 1$ (corresponding to a R/P ratio of 1), the complete amount of reserves can be consumed during a unit of time without requiring the price to increase. This results in the resources being exploited in perfect order of cost, with perfectly vertical slices of the distribution (as in figure 6) being consumed in order of cost.

For a low value of $v_0$, the opposite behaviour occurs, where a very small fraction of reserves at each value of $C$ below $P$, producing a very low supply. This thus requires the price $P$ to increase to values high enough that the supply meets the demand. At the extreme situation where the flow does not meet the demand, the price diverges to infinity and all resources of the distribution are exploited at equal rates. The slices of figure 6 thus become more or less horizontal.

Thus it can be inferred intuitively that the role of the parameter $v_0$ controls the size of the reserves and how high the price is required to remain in order to supply the demand. It represents to some extent the rate at which resources

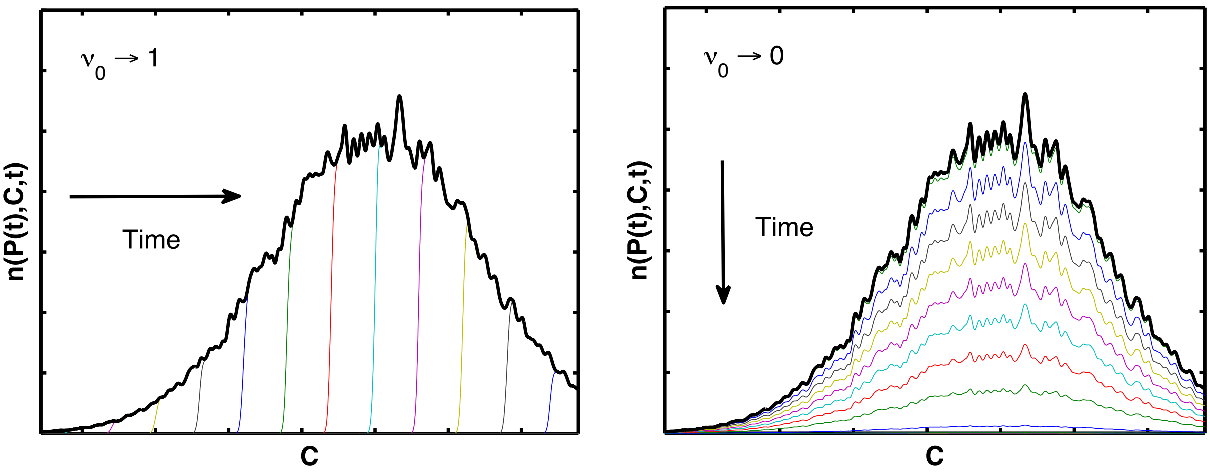


Figure 6: (*Left*) Limit for $\nu_0 \to 1$. (*Right*) Limit for $\nu_0 \to \infty$.

can physically be taken out of the ground, but also to a certain degree how willing resource rich land owners are to exploit their resources instead of keeping them for a future where they expect a higher price for them.

### *S.3.8. Renewable resources*

This theoretical framework could also be used for renewable resources. In this case, $n(C,t)$ would correspond to a distribution of resource producing units, and the price $P(t)$ would be the price of electricity. Such a model would have quite different properties, stemming from the fundamental difference in the definition of $n(C,t)$: it concerns units of flows of resources rather than resource units. Therefore, for a constant supply of resource, i.e. a constant flow, the level of resource use does not need to change, and thus the price $P$ does not need to increase but simply to converge towards a constant value. This therefore shows that this model used for renewables would be very close to equivalent to a cost-supply curve framework, where a supply corresponds closely to a single cost value. There would be little gain in attempting to define such a model for renewables, and thus this is not done in the FTT model.

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