Lecture 11 – RANSAC & Robust Estimator

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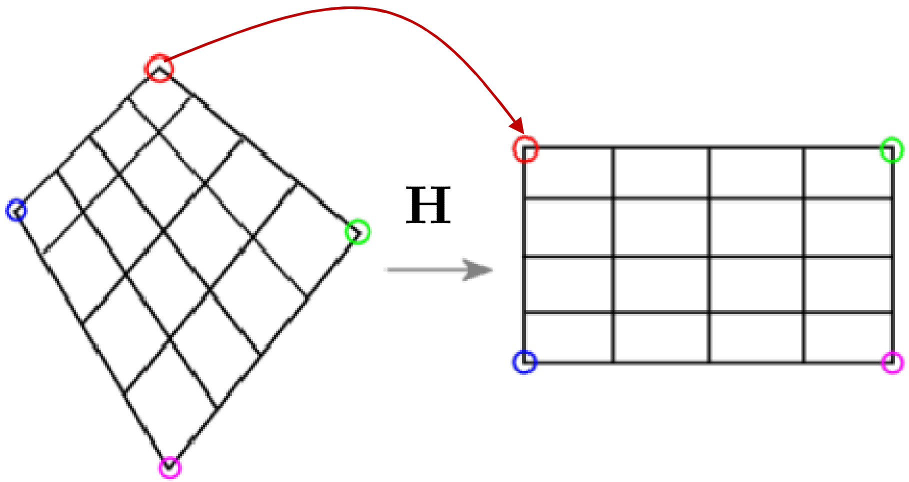
Outline

- Outliers
  - Homography estimation
  - Pose estimation
  - Fundamental/Essential matrix estimation
- RANSAC
  - Example - Fundamental/Essential matrix estimation

Model estimation

- Homography estimation

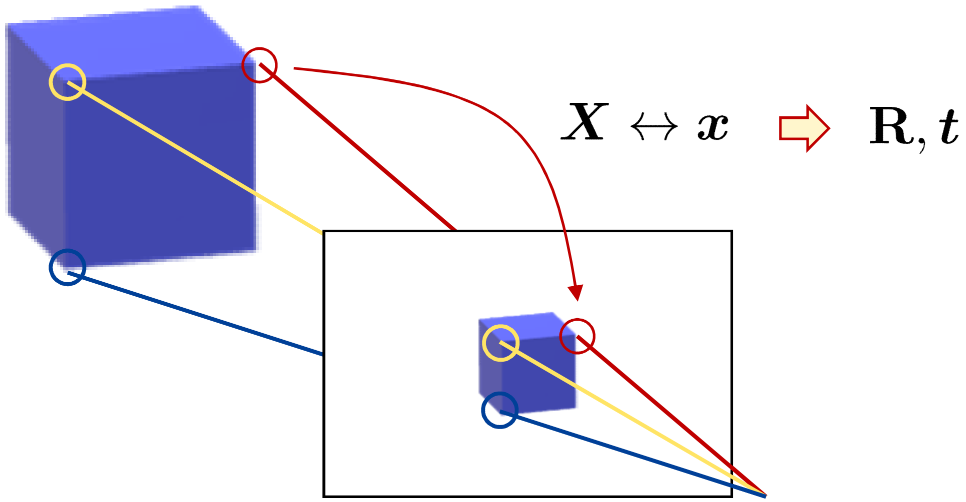


$$\boldsymbol{x} \leftrightarrow \boldsymbol{x}' \in \mathbb{R}^{3\times 1}$$

$$\Rightarrow \quad \boldsymbol{x}' \sim \mathbf{H}\boldsymbol{x}$$

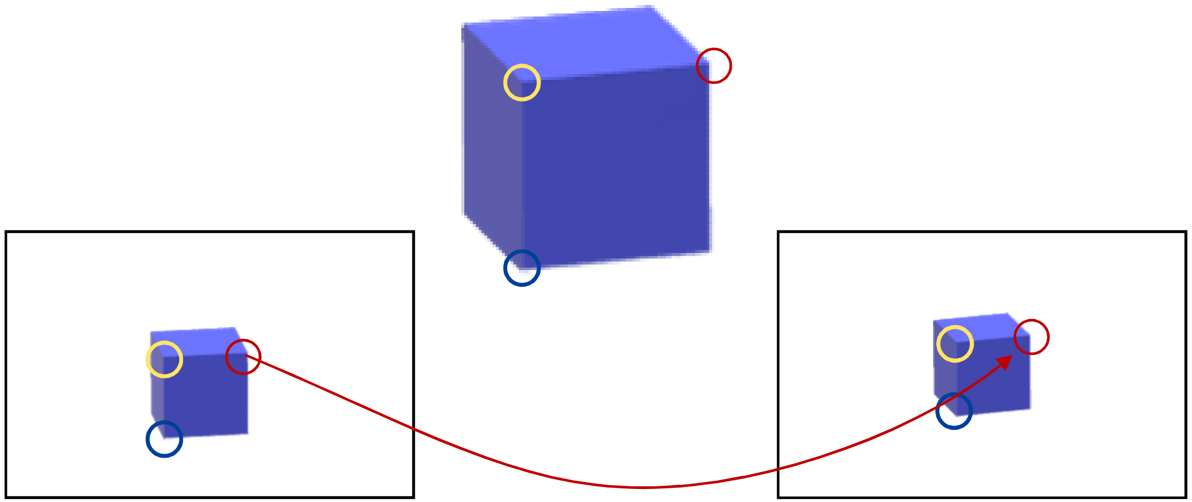
Model estimation

- Pose estimation



Model estimation

- Fundamental/Essential matrix estimation



$$x \leftrightarrow x'$$

$$\Rightarrow \mathbf{R}, t$$

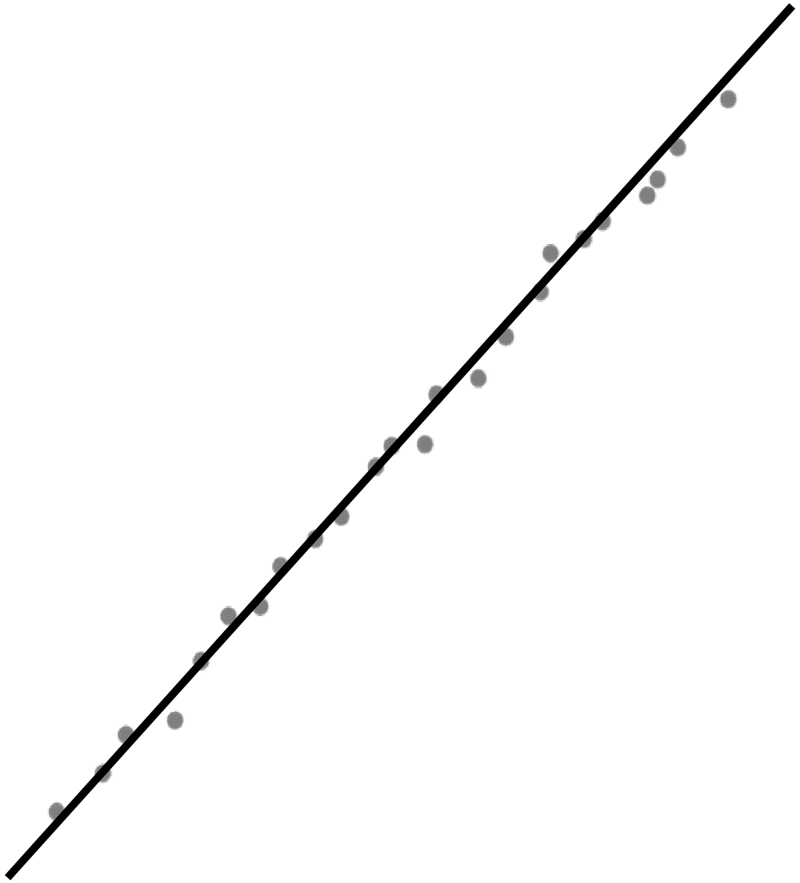
Model estimation

- Linear regression

$$x \leftrightarrow y \in \mathbb{R}$$

$$\Rightarrow \quad a, b$$

$$\boldsymbol{y} = a\boldsymbol{x} + b$$

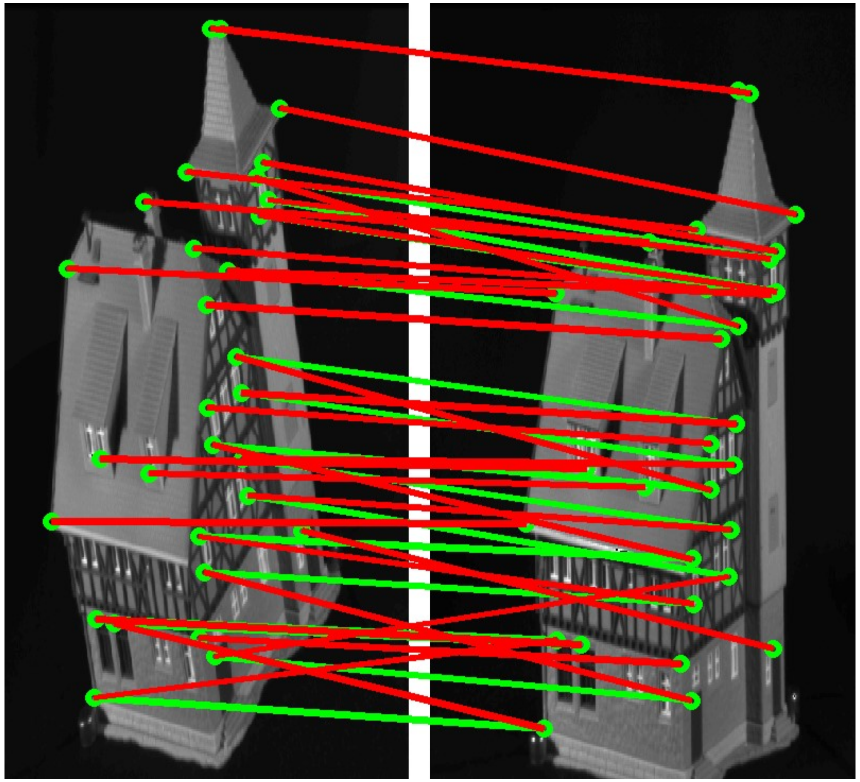


Model estimation

Problem	Correspondences	Model	DoG
Homography estimation	$\boldsymbol{x}_i \in \mathbb{R}^{2\times 1} \leftrightarrow \boldsymbol{x}'_i \in \mathbb{R}^{2\times 1}$	$\mathbf{H} \in \mathbb{R}^{3\times 3}$	8
Pose estimation	$\boldsymbol{X}_i \in \mathbb{R}^{3\times 1} \leftrightarrow \boldsymbol{x}_i \in \mathbb{R}^{2\times 1}$	$\mathbf{R} \in SO(3), \boldsymbol{t} \in \mathbb{R}^{3\times 1}$	6
Fundamental/Essential matrix estimation	$\boldsymbol{x}_i \in \mathbb{R}^{2\times 1} \leftrightarrow \boldsymbol{x}'_i \in \mathbb{R}^{2\times 1}$	$\mathbf{F}, \mathbf{E} \in \mathbf{R}^{3\times 3}$	$7/5$
Linear regression	$x \in \mathbb{R} \leftrightarrow y \in \mathbb{R}$	$(a, b) \in \mathbb{R}^{2\times 1}$	2

Outliers

- What if there exists outliers in the correspondences ?
  - Feature matching could not be always correct.

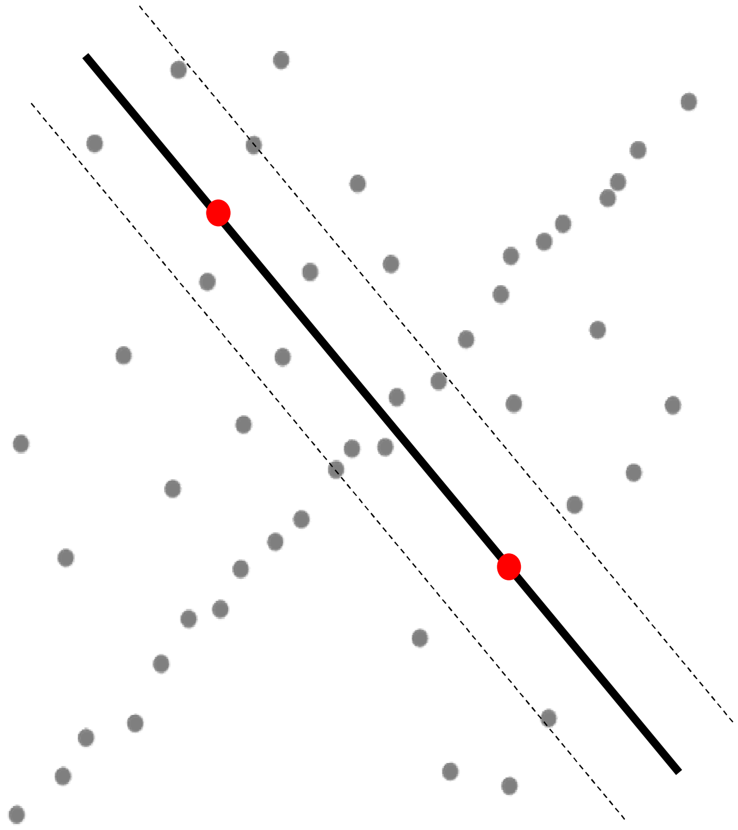


RANdom SAmple Consensus(RANSAC)

- Repeat the following steps for N times
  - **Hypothesis**
    - Randomly select the minimum number of data required to determine the model parameter
    - Solve the model parameters
  - **Verification**
    - Determine the inliers (consensus set) that fit with the solved model.
    - The consensus set with the largest number of elements is recorded.
- The model is finally refined using the elements within the consensus set.

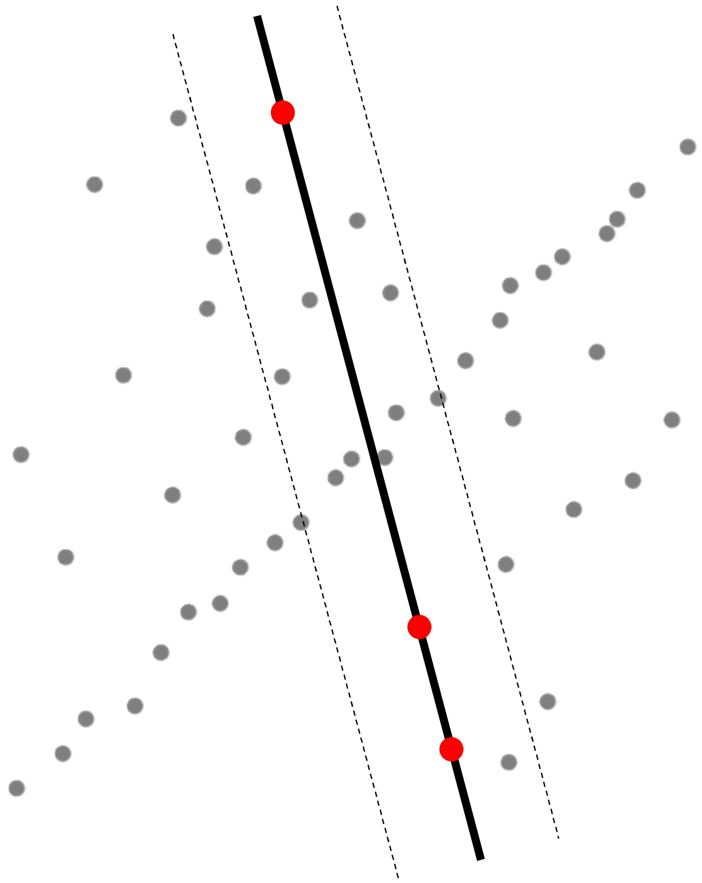
RANSAC

- Select **m** samples randomly
- Estimate the model from the sampled points
- Find the consensus set (inliers)



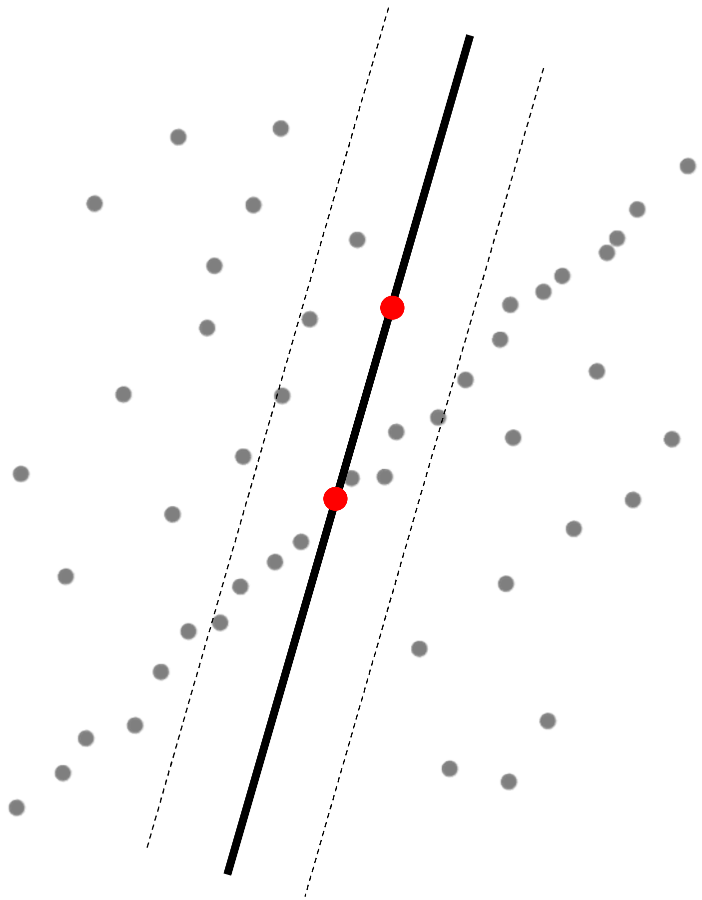
RANSAC

- Select m samples randomly
- Estimate the model from the sampled points
- Find the consensus set (inliers)
- **Repeat sampling**



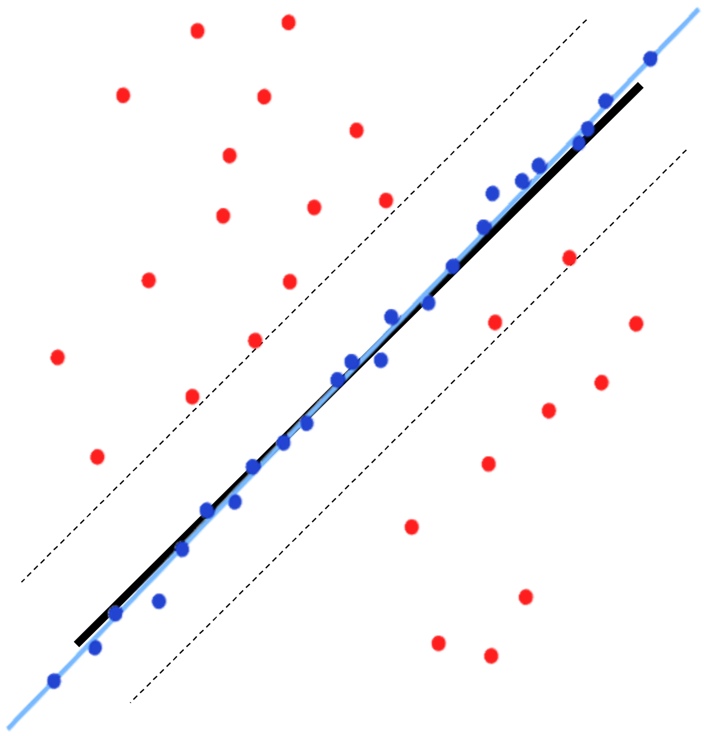
RANSAC

- Select m samples randomly
- Estimate the model from the sampled points
- Find the consensus set (inliers)
- **Repeat sampling**



RANSAC

- Select m samples randomly
- Estimate the model from the sampled points
- Find the consensus set (inliers)
- Repeat sampling
- **A best model is the one with maximum number inliers**



RANSAC

- How many times do we need to do sampling to get a noisy-free subset ?

RANSAC

- Let p be the probability of getting a noisy-free subset. What we want maybe

$$p > 99\%$$

- Let the inlier ratio of the data be :

$$u = \frac{\#\text{noisy-free data points}}{\#\text{total points}}$$

RANSAC

- Repeat sampling until noisy-free model has been sampled…
  - #1 => contain noisy points $(1-u^M)$
  - #2 => contain noisy points $(1-u^M)$
  - … … ……
  - #N => contain noisy points $(1-u^M)$
  - #N+1=> only noisy free points
- The probability of getting N times noisy model is

$$(1-u^M)^N$$

- The probability of getting a noisy-free model after N times sampling is :

$$p = 1-(1-u^M)^N$$

RANSAC

- Therefore, the number of sampling required is

$$N = \frac{\log(1-p)}{\log(1-u^M)}$$

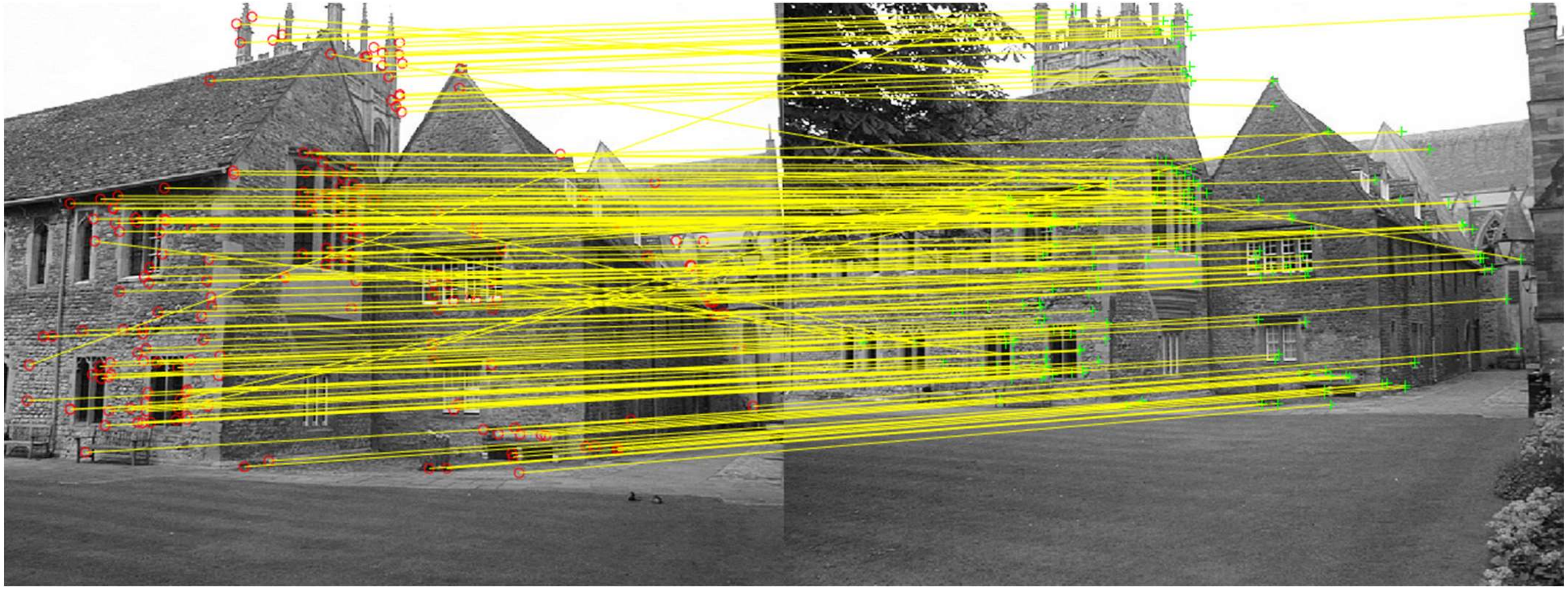
- Example:
  - $p = 0.99, m = 3, u = 0.5 \rightarrow N \approx 35$
  - $p = 0.95, m = 3, u = 0.5 \rightarrow N \approx 22$
  - $p = 0.99, m = 3, u = 0.8 \rightarrow N \approx 6$
  - $p = 0.99, m = 5, u = 0.8 \rightarrow N \approx 12$

Fundamental matrix estimation

- Step 1 – Feature matching
  - SIFT,SURF,BRISK,ORB

Fundamental matrix estimation

- Step 1 – Feature matching



Fundamental matrix estimation

- Set the maximum number of sampling steps

$$N = \frac{\log(1-p)}{\log(1-u^M)}$$

$$(p = 0.99, u = 0.8, M = 8)$$

$$\Rightarrow \quad N \approx 25$$

Fundamental matrix estimation

- Step 2 – Sample 8 corresponding points randomly

Fundamental matrix estimation

- Step 3 - Compute the fundamental matrix from selected corresponding points

$$\boldsymbol{x}_i'^{\mathrm{T}}\mathbf{F}\boldsymbol{x}_i = \mathbf{0}$$

$$\mathbf{F} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

$$\begin{cases} x_1'x_1f_{11} + x_1'y_1f_{12} + x_1'f_{13} + y_1'x_1f_{22} + y_1'y_1f_{23} + x_1f_{31} + y_1f_{32} + f_{33} = 0 \\ \cdots\cdots \\ x_i'x_if_{11} + x_i'y_if_{12} + x_i'f_{13} + y_i'x_if_{22} + y_i'y_if_{23} + x_if_{31} + y_if_{32} + f_{33} = 0 \\ \cdots\cdots \end{cases}$$

$$\mathbf{A}_{8\times 9}\boldsymbol{f} = \mathbf{0}_{8\times 1}$$

Fundamental matrix estimation

- Use SVD to get the solution

$$\mathbf{A}_{8\times 9} = [\boldsymbol{u}_1, \boldsymbol{u}_2, \ldots \boldsymbol{u}_8] \begin{bmatrix} \sigma_1 & 0 & \ldots & 0 \\ 0 & \sigma_2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \sigma_8 & 0 \end{bmatrix} [\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots \boxed{\boldsymbol{v}_9}]^{\mathrm{T}}$$

$$\boldsymbol{f} = \boldsymbol{v}_9$$

$$\Rightarrow \quad \mathbf{F}^*$$

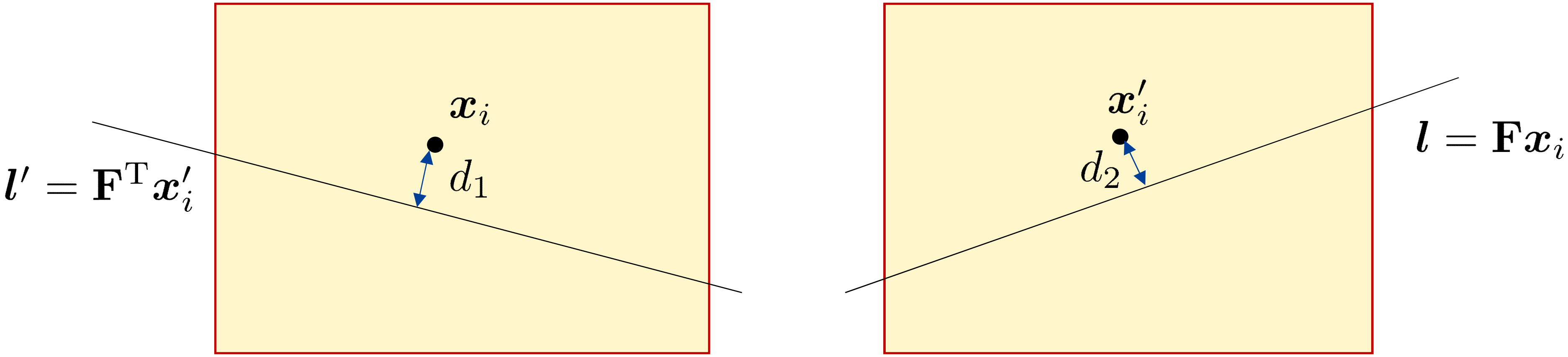
$$\mathbf{F}^* = \mathbf{U} \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} \mathbf{V}^{\mathrm{T}} \quad \Rightarrow \quad \mathbf{F} = \mathbf{U} \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{V}^{\mathrm{T}}$$

Fundamental matrix estimation

- Step 4 – find all the inliers (consensus set) using the current fundamental matrix

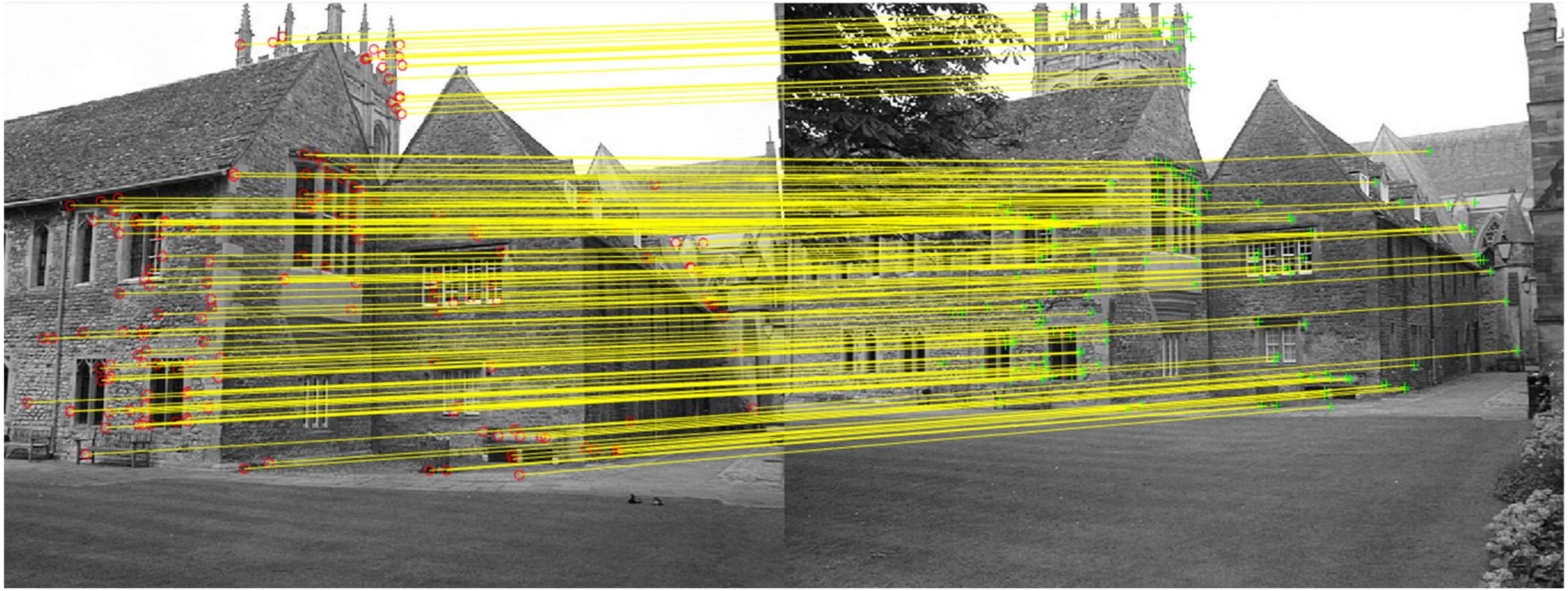
$$d = d_1 + d_2 < \theta \Rightarrow \boldsymbol{x}_i \leftrightarrow \boldsymbol{x}'_i$$

$$d_1 = d(\boldsymbol{x}_i, \mathbf{F}^{\mathrm{T}}\boldsymbol{x}'_i) \qquad d_2 = d(\boldsymbol{x}'_i, \mathbf{F}\boldsymbol{x}_i)$$



Fundamental estimation

- Step 5 – repeat above steps until reaching the maximum number of steps
- Record the consensus set with the maximum number of inliers



Summary

- **RAN**dom **SA**mple **C**onsensus(RANSAC)
- For model estimation among noisy data
- Can handle data with a lot of noises
- Typically used in Fundamental matrix estimation