CS 416 HW1

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Problem 1.

1. $\frac{\partial}{\partial u} f(u,v) = 16uv^4 + 6$.

2. $\frac{\partial}{\partial v} f(u,v) = 32u^2v^3 + 12v^2$.

3. $\frac{\partial}{\partial u} g(u,v,w) = \frac{x}{u \ln 2} + yvw^3$

4. $\frac{\partial}{\partial v} g(u,v,w) = yuw^3$

5. $\frac{\partial}{\partial w} g(u,v,w) = 3yuvw^2$

6. $\frac{\partial}{\partial u} h(u,v) = \frac{1}{2} \sum_{i=1}^{m} (2x^{(i)^2}u + 2x^{(i)}y^{(i)}v) = \sum_{i=1}^{m} (x^{(i)^2}u + x^{(i)}y^{(i)}v)$

7. $\frac{\partial}{\partial v} h(u,v) = \frac{1}{2} \sum_{i=1}^{m} (2y^{(i)^2}v + 2x^{(i)}uy^{(i)}) = \sum_{i=1}^{m} (y^{(i)^2}v + x^{(i)}y^{(i)}u)$

Problem 2.

1. $\frac{\partial}{\partial u} f(-2,-2)$.

   At point $(-2,-2)$, fix v, as u increases, we can see that the value of f decreases from around 30 to 15, therefore, $\frac{\partial}{\partial u} f(-2,-2) < 0$.

2. $\frac{\partial}{\partial v} f(-2,-2)$

   At point $(-2,-2)$. fix u, as v increases, the value of f decreases from around 30 to 15, therefore, $\frac{\partial}{\partial v} f(-2,-2) < 0$.

3. $\frac{\partial}{\partial u} f(3,-3)$.

   At point $(3,-3)$, fix v, as u increases, we can see that the value of f increases from around 30 to 40, therefore, $\frac{\partial}{\partial u} f(3,-3) > 0$.

4. $\frac{\partial}{\partial u} f(3,-3)$.

At point $(3,-3)$, fix u, as V increases, we can see that the value of f decreases from around 50 to 30, therefore, $\frac{\partial}{\partial v} f(3,-3) < 0$.

5. $\min f(u,v) = f(1,2)$.

From the contour plot, we can find that the f values decrease when the points tend to be at the center.

Problem 3.

1. $\begin{bmatrix} 3 & -1 \\ 2 & 5 \\ -2 & 2 \end{bmatrix} \cdot \begin{bmatrix} u & a \\ v & b \end{bmatrix} = \begin{bmatrix} 3u-v & 3a-b \\ 2u+5v & 2a+5b \\ -2u+2v & -2a+2b \end{bmatrix}$

2. Yes. 2×4.

3. No.

4. $y^T \in R^{1\times3}$ then $y^TA \in R^{1\times2}$, which is a row vector

5. $Ax \in R^{3\times1}$, which is a column vector.

6. $(Bx+y)^T A^T = 0$

$\Rightarrow [(Bx+y)A]^T = 0$

$(Bx+y)A = 0$

$(Bx+y)A \cdot A^{-1} = 0$

$Bx+y = 0$

$Bx = -y$

$B^{-1} \cdot Bx = B^{-1}(-y)$

$x = -B^{-1}y$

Problem 5:

1. $h_\theta(2) = \theta_0 + 2\theta_1$

$h_\theta(-1) = \theta_0 - \theta_1$

$h_\theta(1) = \theta_0 + \theta_1$.

$J(\theta_0,\theta_1) = \frac{1}{2}\left[(\theta_0+2\theta_1-5)^2 + (\theta_0-\theta_1+1)^2 + (\theta_0+\theta_1-3)^2\right]$.

$= \frac{1}{2}[\theta_0^2 - 10\theta_0 + 4\theta_1\theta_0 + 4\theta_1^2 - 20\theta_1 + 25 + \theta_0^2 - 2\theta_0\theta_1 + 2\theta_0 + \theta_1^2 - 2\theta_1$
$+ 1 + \theta_0^2 + 2\theta_0\theta_1 - 6\theta_0 + \theta_1^2 - 6\theta_1 + 9]$.

$= \frac{1}{2}[3\theta_0^2 + 4\theta_1\theta_0 - 14\theta_0 + 6\theta_1^2 - 28\theta_1 + 35]$

$\frac{\partial}{\partial\theta_0} J(\theta_0,\theta_1) = \frac{1}{2}(6\theta_0 + 4\theta_1 - 14) = 3\theta_0 + 2\theta_1 - 7$

$\frac{\partial}{\partial\theta_1} J(\theta_0,\theta_1) = \frac{1}{2}(12\theta_1 + 4\theta_0 - 28) = 6\theta_1 + 2\theta_0 - 14$

Let $\frac{\partial}{\partial\theta_0} J(\theta_0,\theta_1) = 0$. $\Rightarrow 3\theta_0 + 2\theta_1 - 7 = 0$

$\Rightarrow 2\theta_1 = 7 - 3\theta_0$

Let $\frac{\partial}{\partial\theta_1} J(\theta_0,\theta_1) = 0 \Rightarrow 6\theta_1 + 2\theta_0 - 14 = 0$

Solve for this two equation.

$3(7-3\theta_0) + 2\theta_0 - 14 = 0 \Rightarrow 21 - 9\theta_0 + 2\theta_0 - 14 = 0$.

$\Rightarrow 7 = 7\theta_0 \Rightarrow \theta_0 = 1$

$\theta_1 = \frac{1}{2}(7 - 3\theta_0) = \frac{1}{2}(7-3) = \frac{1}{2} \times 4 = 2$. $\therefore \begin{cases} \theta_0 = 1 \\ \theta_1 = 2 \end{cases}$

2. $J(\theta_0,\theta_1) = \frac{1}{2}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2$

$= \frac{1}{2}\cdot\sum_{i=1}^{m}(\theta_0 + x^{(i)}\theta_1 - y^{(i)})^2$

$\frac{\partial}{\partial\theta_0} = \theta_0 + \theta_1 x^{(1)} - y^{(1)} + \theta_0 + \theta_0 x^{(2)} - y^{(2)} + \theta_0 + \theta_0 x^{(3)} - y^{(3)}$

$= 3\theta_0 + \theta_1(\Sigma x) - \Sigma y = 0$. ①

$$\frac{\partial}{\partial\theta_1} = x^{(1)}(\theta_0+\theta_1 x^{(1)}-y^{(1)}) + x^{(2)}(\theta_0+\theta_0 x^{(2)}-y^{(2)}) + x^{(3)}(\theta_0+\theta_0 x^{(3)}-y^{(3)})$$

$$= 0. \quad ②$$

from ① we have $\theta_1 = \dfrac{\Sigma y - 3\theta_0}{\Sigma x}$

plug it into ②, we have.

$$\theta_0 (x^{(1)}+x^{(2)}+x^{(3)}) + \theta_1((x^{(1)})^2+(x^{(2)})^2+(x^{(3)})^2) - x^{(1)}y^{(1)} - x^{(2)}y^{(2)} - x^{(3)}y^{(3)} = 0$$

$$\theta_0 \Sigma x + \theta_1 \Sigma x^2 + \Sigma xy = 0.$$

$$\Rightarrow \theta_0 \Sigma x + \frac{\Sigma y - 3\theta_0}{\Sigma x}\cdot \Sigma x^2 - \Sigma xy = 0$$

$$\Rightarrow \frac{\theta_0 (\Sigma x)^2 + (\Sigma y - 3\theta_0)\cdot \Sigma x^2}{\Sigma x} = \Sigma xy$$

$$\Rightarrow \frac{\theta_0 (\Sigma x)^2 - 3\theta_0 \cdot \Sigma x^2}{\Sigma x} = \Sigma xy - \frac{\Sigma y \cdot \Sigma x^2}{\Sigma x}$$

$$\Rightarrow \theta_0 (\Sigma x)^2 - 3\theta_0 \cdot \Sigma x^2 = \Sigma x \Sigma xy - \Sigma y \cdot \Sigma x^2$$

$$\Rightarrow \theta_0 [(\Sigma x)^2 - 3\Sigma x^2] = \Sigma x \Sigma xy - \Sigma y \cdot \Sigma x^2.$$

$$\Rightarrow \theta_0 = \frac{\Sigma x \Sigma xy - \Sigma y \cdot \Sigma x^2}{(\Sigma x)^2 - 3\Sigma x^2}$$

$$\theta_1 = \frac{\Sigma y - 3\theta_0}{\Sigma x} = \frac{\Sigma y}{\Sigma x} - \frac{3}{\Sigma x}\cdot \frac{\Sigma x \Sigma xy - \Sigma y \cdot \Sigma x^2}{(\Sigma x)^2 - 3\Sigma x^2}$$

$$= \frac{\Sigma y [(\Sigma x)^2 - 3\Sigma x^2] - 3(\Sigma x \Sigma xy - \Sigma y \cdot \Sigma x^2)}{\Sigma x [(\Sigma x)^2 - 3\Sigma x^2]}$$

plug in the data, $\theta_0 = \dfrac{2\cdot(14) - 7\times 6}{2\times 2 - 18} = \dfrac{-14}{-14} = 1$

$$\theta_1 = \frac{\Sigma y - 3\theta_0}{\Sigma x} = \frac{7-3(1)}{2} = \frac{7-3}{2} = \frac{4}{2} = 2.$$