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CS 416 HW2.
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Problem 1:

1.1 $g(z) = \frac{1}{1+e^{-z}} = (1+e^{-z})^{-1}$

$g(z)(1-g(z)) = \frac{1}{1+e^{-z}} * \frac{e^{-z}}{1+e^{-z}} = \frac{e^{-z}}{(1+e^{-z})^2}$

$\frac{d}{dz} g(z) = e^{-z}(1+e^{-z})^{-2} = \frac{e^{-z}}{(1+e^{-z})^2}$

1.2. $1 - g(z) = 1 - \frac{1}{1+e^{-z}} = \frac{e^{-z}}{1+e^{-z}}$

$g(-z) = \frac{1}{1+e^{z}} = \frac{1}{1+e^{z}} \cdot \frac{e^{-z}}{e^{-z}} = \frac{e^{-z}}{e^{-z}+e^{z-z}} = \frac{e^{-z}}{e^{-z}+1}$

1.3 $\frac{\partial}{\partial \theta_j} J(\theta) = -y \cdot \frac{1}{h_\theta(x)} \cdot \left(\frac{\partial h_\theta(x)}{\partial \theta_j}\right) - (1-y) \cdot \frac{1}{1-h_\theta(x)} \cdot \frac{\partial (1-h_\theta(x))}{\partial \theta_j}$

$= \left[-y \cdot \frac{1}{h_\theta(x)} \cdot g(z)(1-g(z)) \cdot \frac{\partial(\theta^T x)}{\partial \theta_j}\right] - \left[(1-y) \cdot \frac{1}{1-h_\theta(x)} \cdot (g(z)(1-g(z)) \frac{\partial(\theta^T x)}{\partial \theta_j}\right]$

$= \left[-y \cdot \frac{1}{h_\theta(x)} \cdot h_\theta(x)(1-h_\theta(x)) \cdot x_j\right] - (1-y) \frac{1}{1-h_\theta(x)} (-h_\theta(x)(1-h_\theta(x)) \cdot x_j)$

$= -y \cdot (1-h_\theta(x)) \cdot x + (1-y) h_\theta(x) \cdot x_j$

$= \left[-y + y h_\theta(x) + h_\theta(x) - y h_\theta(x)\right] \cdot x_j$

$= (-y + h_\theta(x)) \cdot x_j = (h_\theta(x) - y) \cdot x_j$