# Part 1, Background

The term 'radar' is an acronym for **RA**dio **D**etection **A**nd **R**anging. Radar is a type of wireless sensor that operates by emitting electromagnetic signals and receiving the scattered echoes (散射回波) from targets and environment. From those echoes, radar extracts information about the targets and the surrounding environment.

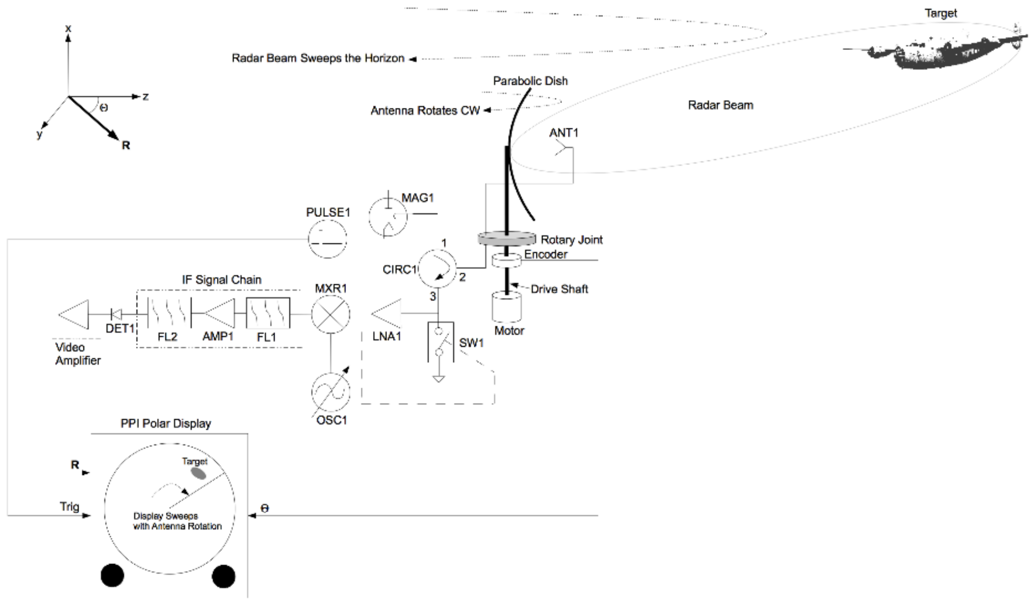



Figure 1：ART Midrange High-Resolution Ground Surveillance Radar

The transmitted signal of a radar is typically represented as a periodic pulse (周期脉冲), with a single pulse expressed as:

$$x(t) = A\cos\left(2\pi f_c t + \theta(t)\right)[u(t) - u(t-\tau)]$$

In the above equation:

- **A** is the amplitude, which is usually a constant
- $f_c$ is operating frequency of the radar, i.e. carrier frequency (载波频率).
- $\theta(t)$ represents the frequency modulation (频率调制) or phase coding (相位编码) part.
- $\tau$ is the pulse width.

This project will focus on FMCW（Frequency Modulated Continuous Wave，调频连续波）radar. FMCW radar measures the distance and velocity of targets by varying the frequency of transmitted signal. As shown in Figure 2, the radar system first transmit a signal, which frequency increases from a relatively low value to a relatively high value in a cyclic manner. When this signal encounters a target and is reflected back, the received signal goes through mixing (混频), to produce an intermediate frequency (IF, 中频) signal. By processing and analyzing the IF signal, the system can measure the distance and velocity of the target.

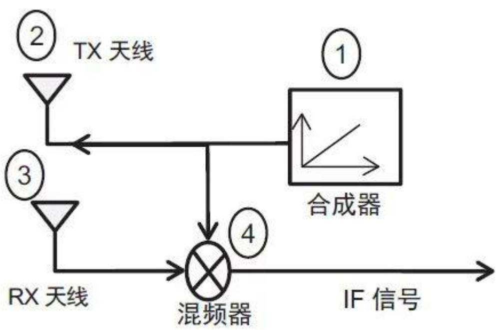



Figure 2：Simplified block diagram of FMCW radar

## 1）Transmitted Signal

Within one period, the frequency of the transmitted signal changes linearly (as shown in following equation), producing a sawtooth wave (i.e. chirp) in the frequency domain (as shown in Figure 3).

$$f(t) = f_c + Kt$$

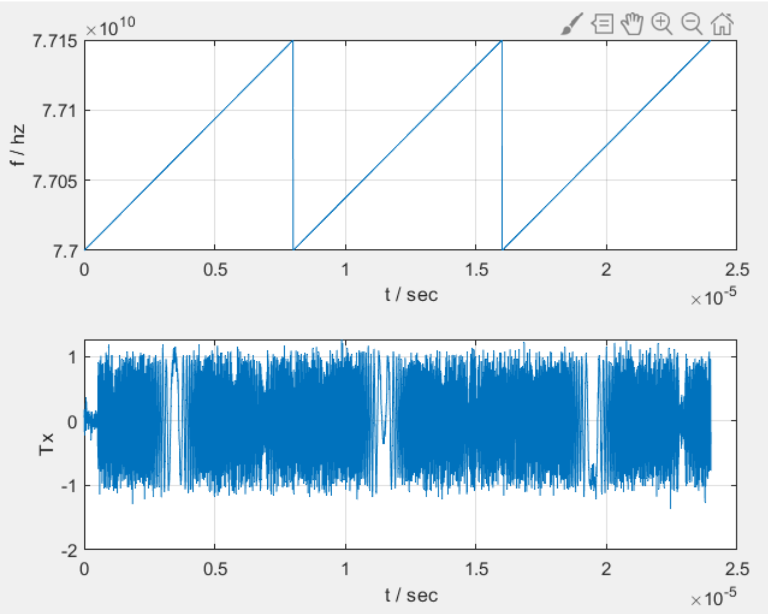



Figure 3: Frequency and Amplitude of the Transmitted Signal

For example, as shown in Figure 3:

- The carrier frequency fc = 77GHz
- The chirp period Tchirp = 8us
- The bandwidth B=150MHz

Thus, the frequency change rate K = B / Tchirp = 18.75 MHz/us

Assuming the linearly frequency-modulated signal (线性调频信号) is in the form of a cosine, the transmitted signal within one period (where $\varphi_t(t)$ is the phase) can be expressed as:

$$s(t) = \cos(\varphi_t(t)) = \cos\left( 2\pi \int_0^t f(t)\, dt \right)$$

$$s(t) = \cos\left(2\pi(f_c t + \frac{K}{2} t^2)\right)$$

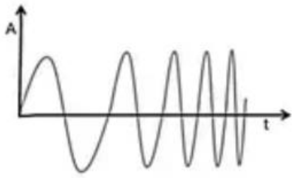
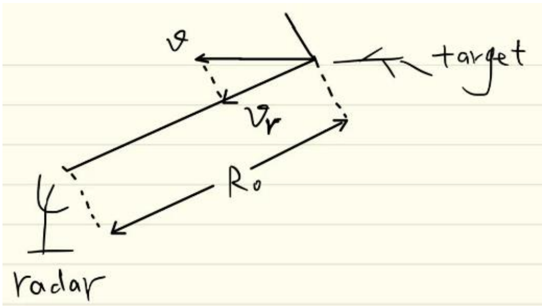


Figure 4：Simplified Diagram of the Transmitted Signal

## 2）Received Signal




As shown in above figure, assume the target is at the distance $R_0$ from the radar and has a radial velocity $V_r$ (经向速度, the sign indicates direction, positive for moving away from the radar). The received signal experiences a delay $\tau$ relative to the transmitted signal, where $c$ is the speed of light (electromagnetic wave speed).

$$\tau = \frac{2(R_0 + V_r t)}{c}$$

The received signal within one period can be expressed as:

$$r(t) = \cos\big(\varphi_r(t)\big) = \cos\big(\varphi_t(t-\tau)\big)$$
$$= \cos\left(2\pi(f_c(t-\tau) + \frac{K}{2}(t-\tau)^2)\right)$$

## 3) IF (Intermediate Frequency) Signal

The received signal is mixed with the transmitted signal (i.e., the two signals are multiplied) to obtain the mixed signal (混频信号):

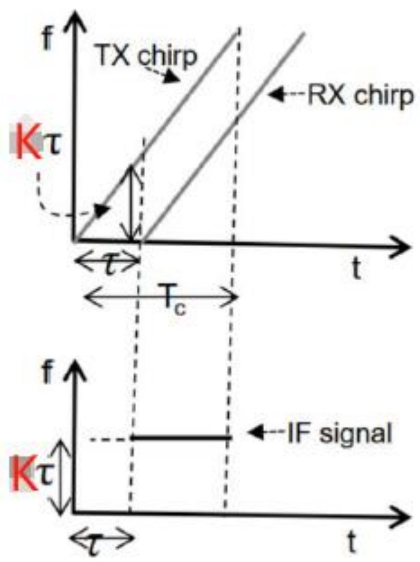
$$x(t) = \cos(\varphi_t(t))\cos(\varphi_r(t))$$
$$= \frac{1}{2}\cos(\varphi_t(t) + \varphi_r(t)) + \frac{1}{2}\cos(\varphi_t(t) - \varphi_r(t))$$

The mixed signal x(t) is then sampled by an ADC. At the relatively low sampling rate (i.e., $\ll \boldsymbol{f_c}$), the high-frequency component $\cos(\varphi_t(t) + \varphi_r(t))$ is filtered out due to under-sampling (欠采样), or in practical systems it is removed by using a bandpass filter, retaining only the $\cos(\varphi_t(t) - \varphi_r(t))$ component. This result in the IF signal (note that the constant A does not affect the measurement result):

$$IF(t) = A\cos(\varphi_t(t) - \varphi_r(t))$$

Also note that, the phase difference between transmitted signal and the echo signal from a single target is:

$$\varphi_t(t) - \varphi_r(t) = 2\pi\left(f_c t + \frac{K}{2}t^2\right) - 2\pi\left(f_c(t-\tau) + \frac{K}{2}(t-\tau)^2\right)$$
$$= 2\pi f_c\tau + 2\pi K\tau\, t + \pi K\tau^2$$




If the target is stationary (静止), $\tau$ is a constant value, and the frequency of IF(t) is:

$$f_m = K\tau$$

Given K, measuring $f_m$ allow us to determine: $\tau = \frac{f_m}{K}$ .

And subsequently, the distance $R_0 = \frac{c\tau}{2} = \frac{cf_m}{2K}$ .

Additionally, the analysis for moving targets and multiple targets will be further explored in the project.

# Part 2, Project Requirements

## 1) Basic Tasks

Given the radar parameters:

- Carrier frequency fc = 77GHz
- Bandwidth B =150MHz
- Chirp period Tchirp = 8us
- Chirp slope K = B / Tchirp = 18.75 MHz/us

Using MATLAB to simulate and sample the mixed signal with following sampling parameters:

- Number of samples per chirp period: Nr = 1024
- Number of chirps per sample: Nd = 128

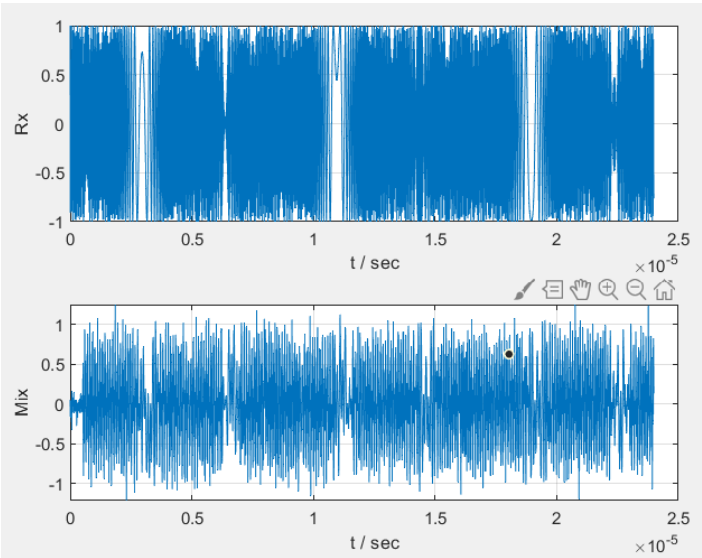
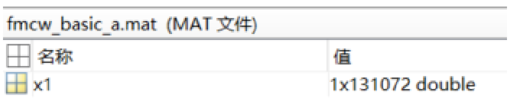
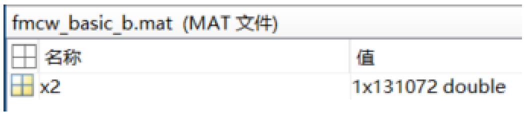



Figure 5: Sampled Received and Mixed Signals

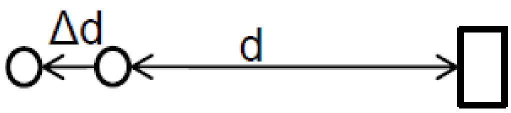
**<u>Problem a)</u>**: For a single stationary target (see Blackboard, vector x1), solve for the target distance from radar.




**<u>Problem b)</u>**: For a single moving target, given the mixed signal (see Blackboard, vector x2), solve for the target distance and velocity.




Note: Starting from problem b, a deeper investigation into FMCW radar is required to find the solution. Hint for velocity calculation: due to the Doppler effect, the phase of the IF signal from adjacent chirps will show significant changes, but the frequency change will be less noticeable.




## 2) Advanced Tasks

Given the radar parameters:

- Carrier frequency fc = 2.253 GHz
- Bandwidth B =178 MHz (i.e. working frequency range 2.253 ~ 2.431 GHz)
- Chirp period Tchirp = 1/65 s
- Sampling frequency fs = 50 KHz

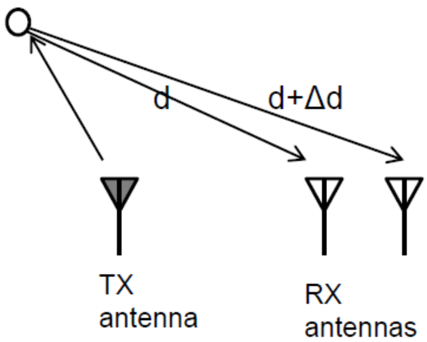
Use MATLAB to simulate the signal transmission, reception, mixing, and sampling of an FMCW radar system. Analyze the mixed signal and combine it with theoretical analysis to solve the following problems:

**Problem c)**: what are the **measurement ranges and accuracies** for the distance and velocity of a single target? Compare to the radar in 1) basic tasks, which radar in better in those metrics respectively? why?

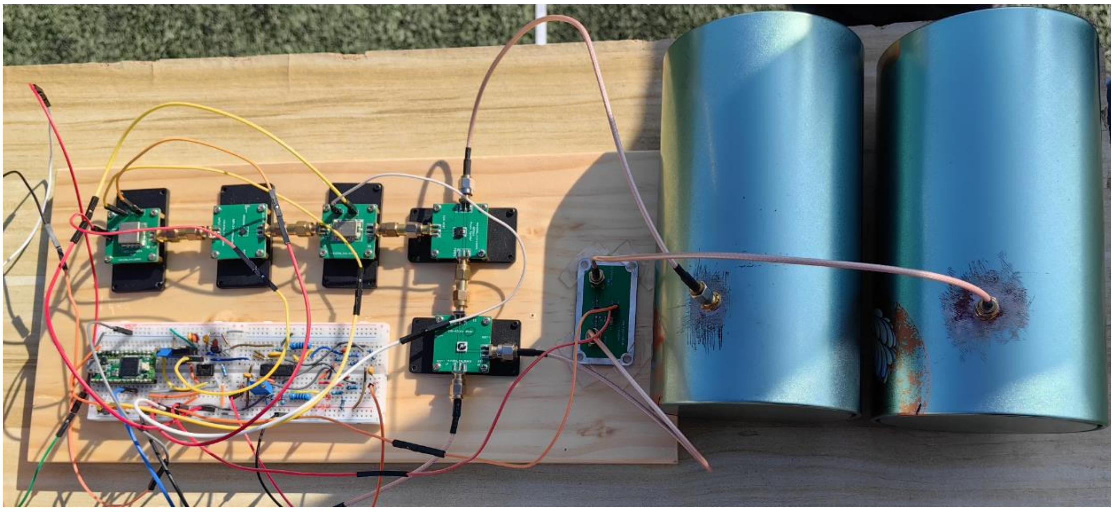
**Problem d)**: Select a set of radar parameters, and design a method to measure the distance and velocity of M (where M >=2) moving objects, illustrate the details with a MATLAB simulation case.

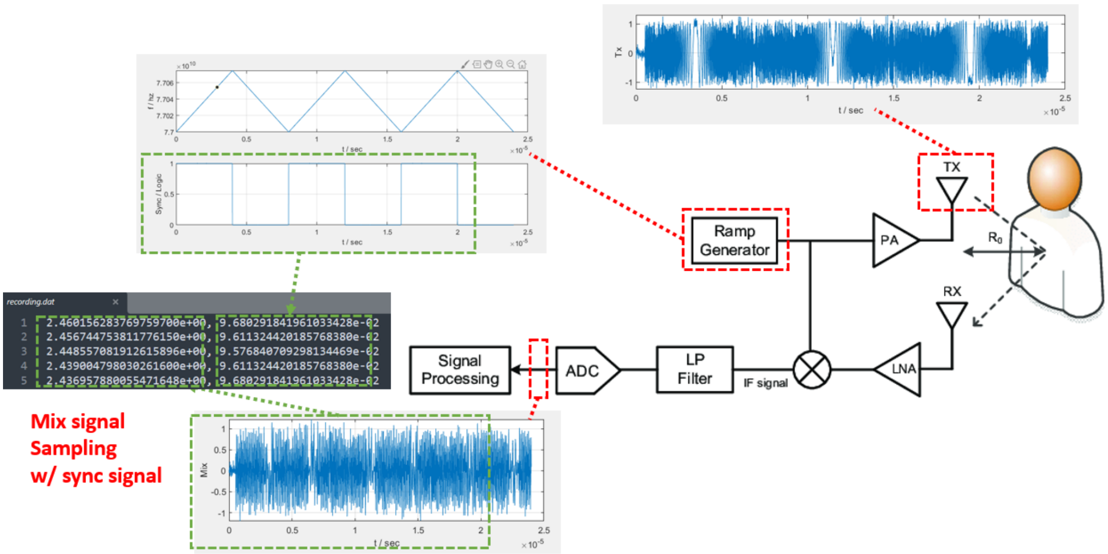
## 3) Bonus Tasks

**Problem e)**: For a multi-antenna（SIMO, Single-Input-Multiple-Output）FMCW radar system, design a method to locate M（M>=2）moving objects in 2D plane (i.e. distance and angle) and measure their velocities. Illustrate the details with a MATLAB simulation case. You can select reasonable radar parameters.




**Problem f)**: Consider a real single-antenna FMCW radar system using triangular wave modulation. Note that the principle is similar to the sawtooth wave modulation described above. The hardware setup and system diagram are given below.






System parameters：

- Carrier frequency fc = 2.253 GHz
- Bandwidth B = 178 MHz (i.e. working frequency range 2.253 ~ 2.431 GHz)
- Triangle wave period Tt = 1/65 s
- Triangle wave slope K = 2B / Tt = 0.2314 MHz/us
- Sampling frequency fs = 50 KHz

As shown in the figure, the system is used to conduct real measurements. The tester moves back and forth on a sports field. The radar system simultaneously samples the mixed the synchronization signals, collecting real data over a period of 30 seconds (see blackboard). **Analysis the distance changes and velocity range of the tester throughout the whole process**. Note: the real data may contain significant noise and environment influences compared to simulation data.


# Part 3，Evaluation criteria

## 1) Presentation，40 points

- Completion status：solution approaches and results for each problems.

  【30 points, 5, 5, 10, 10 points for problem a, b, c, d respectively】

  Note: ***MATLAB simulation results (including plotting graph etc. ) and theoretical analysis both are required*** in each problems.

- Project summary【5 points】
  - Division of labor among team members
  - What problems were encountered during the process ?
  - What was learned ?
  - Further reflection and thoughts, etc.
- On-site Q & A【5 points】

## 2) Project report, 60 points

Organize the presentation materials into a document and add the following content:

- Background explanation for FMCW radar after your investigation

- Contribution ratio of team members
- MATLAB codes

- There is no fixed structure for the document, but it must be clear and logically organized to explain the solution approaches, methods, and related results.
- Points distribution in the report：Problems a) to d) each account for 20%, and others (such as background explanation) account for 20%.
- Bonus tasks can earn up to an additional 20 points (including presentation and report parts), with a maximum total score of 100 points.

## Others

- Team work with 2 members.

  (只需要提交一份报告，并说明小组成员（姓名和学号），以及两人的贡献度（如：张三 50%，李四 50%）。两人的得分，将基于贡献度分配，如下

$$\text{个人得分} = \text{小组}50\%\text{得分} + \text{小组}50\%\text{得分} \times \min\left(\frac{\text{个人贡献}}{\text{队友贡献}}, 1\right)$$

- Presentation schedule：Lab class in 15th -16th week.

  (汇报先后顺序请填写腾讯表格, 详见 QQ 群公布, slot 先到先得)

- Report Due：2025/1/12

## Reference

[1] Introduction to mmwave Sensing: FMCW Radars (PDF shared in blackboard)

[2] Frequency-Modulated Continuous-Wave Radar (FMCW Radar): a brief introduction of the principle of FMCW