

# Engineering Thermodynamics

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## 1 Basic Definition

### 1.1 能量转换装置

- 热能动力装置:
  - 蒸汽动力装置(Vapor Power Systems): 工质: Steam/Water
  - 燃气动力装置(Gas Power Systems): 工质: 燃气
    - 内燃机
    - 燃气轮机
    - 喷气发动机
- 制冷和热泵装置 (Refridgeration and Heat Pump Systems): 工质: 制冷剂

### 1.2 热力系

物质交换:

- 闭口系 (closed system): 控制质量 (C.M.) ——与外界无物质交换
- 开口系 (open system): 控制体积 (C.V.) ——与外界有物质交换

能量交换:

- 简单热力系: 与外界只交换热量和一种形式的准静功
- (简单压缩系 (simple compressive system)): 准静功为可逆体积变化功
- 绝热系 (adiabatic system): 与外界无热量交换
- 孤立系 (isolated system): 与外界无能量交换也无物质交换

### 1.3 热力状态与基本状态参数

#### • Thermodynamic State

- steady state: 状态参数不随时间变化
- equilibrium state: 所有参数达到平衡, 即不再变化

#### • State Properties

- extensive property:  $M, V, U, E, H, S$
- intensive property:  $T, p, \text{广延量的比参数 } (v, u, h, s)$
- Basic Properties:  $T, p, v$

#### • Process

irreversibility:

- reversible:(quasi-steady & non-dissipative) (准稳态及无耗散)
- irreversible

thermodynamic:

- quasi-steady 混静态
- isothermal 等温
- adiabatic 绝热

- isentropic 等熵

- Process para

- $Q, W$
- 

## 1.4 Zeroth law of thermodynamics

“It’s a matter of experience that when two objects are in thermal equilibrium with a third object, they are in thermal equilibrium with one another.”

## 1.5 Temperature Scales

- Kelvin scale

- Rankine scale

$$T(^{\circ}R) = 1.8T(K)$$

- Celsius scale

$$T(^{\circ}C) = T(K) - 273.15$$

- Fahrenheit scale

$$T(^{\circ}F) = T(^{\circ}R) - 459.67 = 1.8T(^{\circ}C) + 32$$

开尔文温标	摄氏度	华氏度
0K	-273.15°C	-459.67°F
273.15K	0°C	32°F
373.15K	100°C	212°F

## 2 Energy and the First law of Thermodynamics

### 2.1 Energy

- Internal energy:  $U$
- Kinetic energy:  $E_k$
- Potential energy:  $E_p$

Total energy:

$$E = U + E_k + E_p$$

$$\Delta E = \Delta U + \Delta E_k + \Delta E_p$$

### 2.2 Work

- 输出功 Output work (对外输出总功量) :  $W'$
- 净输出功:  $W_{net}$ , 轴功 (叶轮机械净输出功):  $W_s$
- 膨胀功/压缩功/体积功 Expansion or Compression work:

$$W = p\Delta V$$

$$\delta W = pdV$$

$$\delta w = pdv$$

- **推动功/推挤功:**

流体进出系统系统所对流体做出的推动功量

$$W_p = pV$$

- **流动功 Flow work/energy(State property):**

$$W_f = \sum_i W_{p,i} = \Delta(pV)$$

$$\delta W_f = Vdp + pdV$$

$$\delta w_f = vdp + pdv$$

- **技术功 Technical work:**

$$W_t = \Delta E_k + \Delta E_p + W_{net}$$

$$\delta W_t = \delta W - \delta W_f = -Vdp$$

$$\delta w_t = -vdp$$

**Relation:**

$$W' = W_{net} + W_f$$

$$W_t = \Delta E_k + \Delta E_p + W_{net}$$

$$W = W_t + W_f$$

## 2.3 Heat

Conduction:

$$\dot{Q} = -\kappa A \frac{dT}{dx}$$

Radiation:

$$\dot{Q} = \varepsilon\sigma AT^4$$

Convection:

$$\dot{Q} = hA\Delta T$$

## 2.4 The first law of Thermodynamics

“During an interaction, energy can change from one form to another but the total amount of energy remains constant.”

general form:

$$\Delta E = Q - W$$

$$\Delta E_k + \Delta E_p + \Delta U = Q - W$$

differential form:

$$dE = \delta Q - \delta W'$$

time rate form:

$$\frac{dE}{dt} = \dot{Q} - \dot{W}'$$

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### CLOSED SYSTEMS(C.M.)

1. differential form:

闭口系热一定律基本表达式:

$$dU = \delta Q - \delta W \quad (1)$$

$$du = dq - pdv \quad (2)$$

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### OPEN SYSTEMS(C.V.)

1. differential form:

热一定律:

$$dE = \delta Q - \delta W'$$

输出总功量转化为净功量和流动功:

$$\delta W' = \delta W_{net} + d(pV)$$

技术功、焓的定义:

$$\begin{aligned}\delta W_t &= \delta W_{net} + dE_k + dE_p \\ dH &= dU + d(pV)\end{aligned}$$

开口系-热力学第一定律基本表达式:

$$dH = \delta Q - \delta W_t \quad (1.1)$$

$$dU = \delta Q - \delta W \quad (1.2)$$

$$\begin{aligned}dh &= du + vdp + pdv \\ dh &= dq + vdp\end{aligned} \quad (2.1)$$

$$du = dq - pdv \quad (2.2)$$

## 2. time rate & integral form:

热一定律:

$$\begin{aligned} \frac{dE}{dt} &= \dot{Q} - \dot{W}' \\ e &= u + \frac{V^2}{2} + gz \\ E &= \int_{CV} edm = \int_{CV} e\rho d\gamma \end{aligned} \quad (1)$$

differential form:

$$dE_{CV} = \delta Q - \delta W + d(ms)$$

Steady Flow:

$$0 = \delta Q - \delta W + mds$$

积分形式(Reynolds Transport Theorem)

$$\frac{d}{dt}(E_{sys}) = \dot{Q} - \dot{W}' = \frac{d}{dt}\left(\int_{CV} e\rho d\gamma\right) + \int_{CS} e\rho V_n dA$$

输出总功量转化为净功量和流动功:

$$\dot{W}' = \dot{W}_{net} + \int_{CS} pV_n dA$$

焓的定义:

$$\begin{aligned} h &= u + pv \\ \int_{CS} h\rho V_n dA &= \int_{CS} (u + \frac{p}{\rho})\rho V_n dA \end{aligned}$$

连续介质力学形式:

$$\begin{aligned} \dot{Q} - \dot{W}_{net} &= \frac{d}{dt}\left(\int_{CV} e\rho d\gamma\right) + \int_{CS} \left(h + \frac{V^2}{2} + gz\right)\rho V_n dA \\ \text{or} \\ \dot{Q} - \dot{W}_{net} &= \frac{\partial}{\partial t} \int_{CV} e\rho d\gamma + \int_{CS} \left(h + \frac{V^2}{2} + gz\right)\rho V_n dA \\ \text{or} \\ \dot{Q} - \dot{W}_{net} &= \frac{\partial}{\partial t} \int_{CV} e\rho d\gamma + \sum (h + \frac{V^2}{2} + gz)_e \dot{m}_e - \sum (h + \frac{V^2}{2} + gz)_i \dot{m}_i \end{aligned}$$

## 2.5 application

- 气轮机叶轮 Turbine Wheel

$$\dot{W}_{net} = \dot{m} \left( \frac{V_1^2 - V_2^2}{2} \right)$$

- 热力发动机 Gas And Steam Turbine

$$0 = \dot{Q}_{CV} - \dot{W}_{net} + \dot{m}(h_1 - h_2)$$

$$\dot{W}_{net} = \dot{m}(h_1 - h_2)$$

- 压气机/泵 Compressor and Pump

$$0 = \dot{Q}_{CV} - \dot{W}_{net} + \dot{m}(h_1 - h_2)$$

$$\dot{W}_{net} = \dot{m}(h_1 - h_2)$$

- 喷管/扩散器 Nozzle/Diffuser

$$0 = Q_{CV} + \dot{m}(h_1 + \frac{V_1^2}{2}) - \dot{m}(h_2 + \frac{V_2^2}{2})$$

$$0 = (h_1 - h_2) + (\frac{V_1^2 - V_2^2}{2})$$

- 换热器 Heat Exchanger

$$\dot{Q}_{CV} = \dot{m}(h_2 - h_1)$$

- 节流 Throttling Process

$$h_1 = h_2$$


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### 3 Entropy and the Second Law of Thermodynamics

#### 3.1 The Second Law of Thermodynamics

**Clausius statement:**

Heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time.

**Kelvin Statement:**

It is impossible, by means of inanimate material agency, to derive mechanical effect from any portion of matter by cooling it below the temperature of the coldest of the surrounding objects.

**Planck's Proposition:**

It is impossible to construct an engine which will work in a complete cycle, and produce no effect except the raising of a weight and cooling of a heat reservoir.

- 克劳修斯说法:

- 热传导的不可逆性
- 无法制造理想制冷机

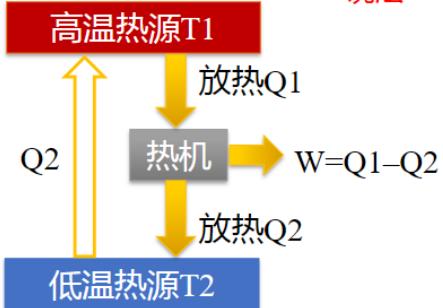
- 开尔文-普朗克说法:

- 热工转换的不可逆性
- 无法制造第二类永动机
- 单一热源下不可能做正输出功

$$\oint W_{cycle} = \oint Q_{cycle} \leq 0$$

开尔文说法成立  
 克劳修斯说法不成立

高温热源吸热  
 $Q_1 - Q_2 = W$   
 有悖于开尔文  
 说法!



- 熵增原理（能量贬值原理）：孤立系统总是朝着熵增方向进行
- 永动机：
  - 第一类永动机（违背热一定律）凭空产生能量
  - 第二类永动机（违背热二定律）热量100%转化为功
- 卡诺定理：
  - 卡诺第一定理：不可能制造出在两个温度不同的热源间工作的热机，而使其效率超过在同样热源间工作的可逆热机
  - 卡诺第二定理：在两个热源间工作的一切可逆热机具有相同的效率
  - 卡诺定理推论：在相同两个恒温热源间工作的一切不可逆热机的效率必小于可逆热机的效率
- 能量品位：

能量转换方向性的  
 实质是能质有差异

$\left\{ \begin{array}{l} \text{无限可转换能—机械能, 电能} \\ \text{部分可转换能—热能} \\ \text{不可转换能—环境介质的热力学能} \end{array} \right.$
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### 3.2 Reversible Process

Reversible Process  $\iff$  No internal irreversibilities

$$\oint W_{cycle} \leq 0 \begin{cases} < 0 : & \text{Internal irreversibilities present} \\ = 0 : & \text{No internal irreversibilities} \end{cases} \quad (\text{single reservoir})$$

### 3.3 Propositions of The Second Law of Thermodynamics

#### I. Kelvin Scales

In Carnot Heat Engines:

$$\left(\frac{Q_C}{Q_H}\right)_{cycle}^{rev} = \phi(T_C, T_H) \stackrel{\text{def}}{=} \frac{T_C}{T_H}$$

#### II. Clausius Inequality

$$\oint \left(\frac{Q}{T}\right)_b = -\sigma_{cycle}, \sigma_{cycle} \begin{cases} > 0 : & \text{Internal irreversibilities present} \\ = 0 : & \text{No internal irreversibilities} \end{cases} \quad (\text{system boundary heat exchange})$$

#### III. Entropy

Definition:

$$dS = \left(\frac{\delta Q}{T}\right)_{rev}$$

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T}\right)_{rev}$$

$TdS$  equation in reversible system:

$$TdS = dU + pdV$$

$$Tds = dH - Vdp$$

$$Tds = du + pdv$$

$$Tds = dh - vdp$$

#### Closed System

basic relation:

$$\oint \left(\frac{Q}{T}\right)_b = \int_1^2 \left(\frac{Q}{T}\right)_b + \int_2^1 \left(\frac{Q}{T}\right)_{rev}$$

$$\int_1^2 \left(\frac{Q}{T}\right)_b + S_1 - S_2 = -\sigma$$

differential form:

$$dS = \left( \frac{\delta Q}{T} \right)_b + \delta\sigma$$

or

$$dS = dS_f + \delta S_g$$

#### Condition Relation:

Common:

$$dS = \left( \frac{\delta Q}{T} \right)_b + \delta\sigma$$

Adiabatic:

$$dS = \delta\sigma$$

Reversible:

$$dS = \left( \frac{\delta Q}{T} \right)_b$$

Isentropic: ( $\iff$  Adiabatic+Reversible)

$$dS = 0$$


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#### IV Increase of Entropy Principle

In isolated system:

$$\left( \frac{\delta Q}{T} \right)_b = 0$$

$$dS_{iso} = \delta S_g \geq 0$$

#### Open System

differential form:

$$dS_{CV} = \frac{\delta Q}{T} + \delta S_{g,CV} + d(ms)$$

Steady Flow:

$$0 = \frac{\delta Q}{T} + \delta S_{g,CV} + mds$$

time rate & intergral form:

$$(\frac{dS}{dt})_{sys} = \sum \frac{\dot{Q}}{T} + \dot{\sigma}$$

$$\sum \frac{\dot{Q}}{T} + \dot{\sigma} = \frac{\partial}{\partial t} \int_{CV} \rho s dV + \int_{CS} s \rho V_n dA$$

or

$$\sum \frac{\dot{Q}}{T} + \dot{\sigma} = \frac{\partial}{\partial t} \int_{CV} \rho s dV + \sum \dot{m}_e s_e - \sum \dot{m}_i s_i$$

### 3.4 Heat Engine and Heat Pump/Refrigerator

#### Heat Engine

Thermal efficiency:

$$\eta = \frac{W_{cycle}}{Q_H} = 1 - \frac{Q_C}{Q_H}$$

$$\eta_{max} = 1 - \frac{T_C}{T_H}$$

#### Refrigerator

Thermal efficiency:

$$\varepsilon = \frac{Q_C}{W_{cycle}} = \frac{Q_C}{Q_H - Q_C}$$

$$\varepsilon_{max} = \frac{T_C}{T_H - T_C}$$

#### Heat Pump

Thermal efficiency:

$$\varepsilon' = \frac{Q_H}{W_{cycle}} = \frac{Q_H}{Q_H - Q_C}$$

$$\varepsilon'_{max} = \frac{T_H}{T_H - T_C}$$

### 3.5 application-efficiency

- isentropic turbine efficiency

$$\frac{\dot{W}_{net}}{\dot{m}} = (h_1 - h_2)$$

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

- isentropic compressor and pump efficiency

$$\frac{-\dot{W}_{net}}{\dot{m}} = (h_2 - h_1)$$

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1}$$

- isentropic nozzle efficiency

$$\frac{V_2^2}{2} = h_1 - h_2 + \frac{V_1^2}{2}$$

$$\eta_{nozzle} = \frac{V_2^2/2}{(V_2^2/2)_s}$$

### 3.6 Exergy 焰 Anergy 燐

- Exergy and Anergy in Heat

Environment at  $T_0$ , System at  $T$

Exergy:

$$E_{x,Q} = W_{max} = \left(1 - \frac{T_0}{T}\right)Q$$

Anergy:

$$E_{n,Q} = Q - E_{x,Q} = \frac{T_0}{T}Q = Q_{emit}$$

- Exergy and Anergy in System

**Definition:** Maximum theoretical work from system and environment to reach an equilibrium

**Closed System**

Overall:

$$0 = dU_s + dU_e + dW_x$$

$$\delta S_g = dS_{iso} = dS_s + dS_e$$

System:

$$\delta Q_{rev} = dU_s + \delta W_{max}$$

$$\delta W_{max} = \delta W_x + p_0 dV_s$$

Environment, at  $T_0, p_0$ :

$$-\delta Q_{rev} = dU_e + p_0 dV_e$$

$$-\delta Q_{rev} = T_0 dS_e$$

Maximum Work:

$$\delta W_x = -dU_s - p_0 dV_s - T_0(dS_g - dS_s)$$

$$W_x = (U - U_0) + p_0(V - V_0) - T_0(S - S_0) + (E_k + E_p) - T_0 S_g$$

Exergy(a kind of Potential):

$$E_x = (U - U_0) + p_0(V - V_0) - T_0(S - S_0) + (E_k + E_p)$$

or

$$E_x = (U - U_0) + p_0(V - V_0) - T_0(S - S_0)$$

$$e_x = (u + p_0v - T_0s) - (u_0 + p_0v_0 - T_0s_0)$$

Exergy Change:

$$\begin{aligned} dE_x &= dU + p_0dV - T_0dS \\ dE_x &= \delta Q - (\delta W - p_0dV) - T_0\left(\frac{\delta Q}{T_b} + \delta S_g\right) \end{aligned}$$

$$\begin{aligned} dE_x &= \left(1 - \frac{T_0}{T_b}\right)\delta Q - (\delta W - p_0dV) - T_0\delta S_g \\ E_2 - E_1 &= \int_1^2 \left(1 - \frac{T_0}{T_b}\right)\delta Q - [W - p_0(V_2 - V_1)] - T_0\Delta S_g \end{aligned}$$

- Exergy transfer of Heat transfer

$$\int_1^2 \left(1 - \frac{T_0}{T_b}\right)\delta Q$$

- Exergy transfer of Work

$$[W - p_0(V_2 - V_1)]$$

- Exergy destruction

$$T_0\Delta S_g$$

Simplification

$$E_2 - E_1 = E_q - E_w - E_d$$

Isolated and no work process:

$$\Delta E = -E_d$$

time rate form:

$$\dot{E}_x = \sum_j \left(1 - \frac{T_0}{T}\right)\dot{Q}_j - [\dot{W} - p_0\left(\frac{dV}{dt}\right)] - T_0\dot{\sigma}$$

steady state:

$$0 = \sum_j \left(1 - \frac{T_0}{T}\right)\dot{Q}_j - \dot{W} - T_0\dot{\sigma}$$

### Open System

Exergy:

$$\begin{aligned} E_x &= (H - H_0) - T_0(S - S_0) + (E_k + E_p) \\ &\quad \text{or} \\ E_x &= (H - H_0) - T_0(S - S_0) \end{aligned}$$

$$e_x = (h - T_0 s) - (h_0 - T_0 s_0)$$

Exergy Change  
time rate form:

$$\sum_j \left(1 - \frac{T_0}{T_j}\right) \dot{Q}_j - (\dot{W}_{CV} - p_0 \frac{dV_{CV}}{dt}) - T_0 \dot{\sigma} = \frac{d}{dt} \int_{CV} e_x \rho d\gamma + \int_{CA} e_x \rho dV_n$$

steady state:

$$0 = \sum_j \left(1 - \frac{T_0}{T_j}\right) \dot{Q}_j - \dot{W}_{CV} + \sum_i \dot{m}_i e_{fi} - \sum_e \dot{m}_e e_{fe} - T_0 \dot{\sigma}$$

specific flow exergy:

$$e_f = (h - h_0) - T_0(s - s_0) + \frac{V^2}{2} + gz$$

$$e_f = e_x + v(p - p_0)$$

### Other System

存在非体积功输出:

$$TdS = dU + \delta W + \delta W_u$$

$$\delta W_{u,max} = TdS - dU - pdV$$

- 定温-定容

$$F = U - TS$$

$$f = u - Ts$$

$$E_x = W_{u,v,max} = (U - TS) - (U_0 - T_0 S_0) = F - F_0$$

$F$ 为亥姆霍兹自由能[强度量]

- 定温-定压

$$G = H - TS$$

$$g = h - Ts = u + pv - Ts$$

$$E_x = W_{u,p,max} = (H - TS) - (H_0 - T_0 S_0) = G - G_0$$

$G$ 为吉布斯自由能（自由焓） [强度量]

- Exergetic Efficiency

Turbines

$$e_{f1} - e_{f2} = \frac{\dot{W}}{\dot{m}} + \frac{\dot{E}_d}{\dot{m}}$$

$$\varepsilon = \frac{\dot{W}/\dot{m}}{e_{f1} - e_{f2}}$$

Compressors and Pumps

$$-\frac{\dot{W}}{\dot{m}} = e_{f2} - e_{f1} + \frac{\dot{E}_d}{\dot{m}}$$

$$\varepsilon = \frac{e_{f2} - e_{f1}}{-\dot{W}/\dot{m}}$$

Heat Exchanger Without Mixing

$$\dot{m}_h(e_{f1} - e_{f2}) = \dot{m}_c(e_{f4} - e_{f3}) + \dot{E}_d$$

$$\varepsilon = \frac{\dot{m}_c(e_{f4} - e_{f3})}{\dot{m}_h(e_{f1} - e_{f2})}$$

Heat Exchanger With Mixing

$$\dot{m}_h(e_{f1} - e_{f3}) = \dot{m}_c(e_{f3} - e_{f2}) + \dot{E}_d$$

$$\varepsilon = \frac{\dot{m}_c(e_{f3} - e_{f2})}{\dot{m}_h(e_{f1} - e_{f3})}$$

- Cost Rate

---

## 4 Thermodynamcis Relation

### 4.1 Legendre Transformation

$$f^*(x_1^*, x_2^*, \dots, x_n^*) = \sum x_i^* x_i - f(x_1, x_2, \dots, x_n)$$

$$du = Tds - pdv$$

$$dh = Tds + vdp \Leftrightarrow H = U + PV$$

$$df = -sdT - pdv \Leftrightarrow F = U - TS$$

$$dg = -sdT + vdp \Leftrightarrow G = H - TS$$

### 4.2 Drivation

$$du = Tds - pdv$$

$$\left\{ \begin{array}{l} T = \frac{\partial u}{\partial s} \\ p = -\frac{\partial u}{\partial v} \end{array} \right. _v$$

$$dh = Tds + vdp$$

$$\left\{ \begin{array}{l} T = \frac{\partial h}{\partial s} \\ v = \frac{\partial h}{\partial p} \end{array} \right. _p$$

$$df = -sdT - pdv$$

$$\left\{ \begin{array}{l} s = -\frac{\partial f}{\partial T} \\ p = -\frac{\partial f}{\partial v} \end{array} \right. _T$$

$$dg = -sdT + vdp$$

$$\left\{ \begin{array}{l} s = -\frac{\partial g}{\partial T} \\ v = \frac{\partial g}{\partial p} \end{array} \right. \begin{array}{l} p \\ T \end{array}$$

### 4.3 Maxwell Relation

$$\left( \frac{\partial T}{\partial v} \right)_s = -\left( \frac{\partial p}{\partial s} \right)_v$$

$$\left( \frac{\partial T}{\partial p} \right)_s = \left( \frac{\partial v}{\partial s} \right)_p$$

$$\left( \frac{\partial s}{\partial v} \right)_T = \left( \frac{\partial p}{\partial T} \right)_v$$

$$\left( \frac{\partial s}{\partial p} \right)_T = -\left( \frac{\partial v}{\partial T} \right)_p$$

### 4.4 Basic Parameters

$$dv = \left( \frac{\partial v}{\partial T} \right)_p dT + \left( \frac{\partial v}{\partial p} \right)_T dp$$

**Volume Expansivity**

$$\alpha_V = \frac{1}{v} \left( \frac{\partial v}{\partial T} \right)_p$$

**Isothermal Compressibility**

$$\kappa_T = -\frac{1}{v} \left( \frac{\partial v}{\partial p} \right)_T$$

**Isentropic Compressibility**

$$\kappa_s = -\frac{1}{v} \left( \frac{\partial v}{\partial p} \right)_s$$

**Temperature Coefficient of Pressure**

$$\beta = \frac{1}{p} \left( \frac{\partial p}{\partial T} \right)_v$$

$$\frac{\alpha_V}{\kappa_T} = p\beta$$

$$du = c_V dT + [T \left( \frac{\partial p}{\partial T} \right)_v - p] dv$$

$$dh = c_p dT - [T \left( \frac{\partial v}{\partial T} \right)_p - v] dp$$

$$ds = \frac{c_V}{T} dT + \left( \frac{\partial p}{\partial T} \right)_v dv$$

$$ds = \frac{c_p}{T} dT - \left( \frac{\partial v}{\partial T} \right)_p dp$$

#### 4.5 Specific Heat

$$\begin{aligned} c_v &= \left( \frac{\partial u}{\partial T} \right)_v = T \left( \frac{\partial s}{\partial T} \right)_v \\ c_p &= \left( \frac{\partial h}{\partial T} \right)_p = T \left( \frac{\partial s}{\partial T} \right)_v + T \left( \frac{\partial v}{\partial T} \right)_p \left( \frac{\partial p}{\partial T} \right)_v \\ \left( \frac{\partial c_v}{\partial v} \right)_T &= T \left( \frac{\partial^2 p}{\partial T^2} \right)_v \\ \left( \frac{\partial c_p}{\partial p} \right)_T &= -T \left( \frac{\partial^2 v}{\partial T^2} \right)_p \end{aligned}$$

$$c_p - c_v = T \left( \frac{\partial v}{\partial T} \right)_p \left( \frac{\partial p}{\partial T} \right)_v = T p v \alpha_V \beta = T v \frac{\alpha^2}{\kappa_T}$$

$$\gamma = \frac{c_p}{c_v} = \left( \frac{\partial v}{\partial p} \right)_T \left( \frac{\partial p}{\partial v} \right)_s = \frac{\kappa_T}{\kappa_s}$$

#### Velocity of Sound

$$c = \sqrt{-v^2 \left( \frac{\partial p}{\partial v} \right)_s} = \sqrt{v/\kappa_s} = \sqrt{\gamma v / \kappa_T}$$

#### Joule-Thomson Coefficient

$$\mu_J = \left( \frac{\partial T}{\partial p} \right)_h = \frac{1}{c_p} \left[ T \left( \frac{\partial v}{\partial T} \right)_p - v \right] = \frac{v}{c_p} (T \alpha_v - 1)$$

## 5 Evaluating Properties

### 5.1 Ideal Gas & Real Gas

#### Equation of State

- Ideal Gas

$$\begin{aligned}pv &= R_g T \\pV &= NkT = nRT = mR_g T \\pV_m &= RT\end{aligned}$$

- Van der Waals

$$\begin{aligned}(p + \frac{a}{V_m^2})(V_m - b) &= RT \\p &= \frac{RT}{V_m - b} - \frac{a}{V_m^2} \\(p + \frac{a}{M^2 v^2})(v - \frac{b}{M}) &= R_g T\end{aligned}$$

- virial EoS

$$pV_m = RT\left(1 + \frac{B}{V_m} + \frac{C}{V_m^2} + \frac{D}{V_m^3} + \dots\right)$$

#### Properties of Ideal Gas

$$\alpha_V = \frac{1}{T}$$

$$\kappa_T = \frac{1}{p}$$

$$\kappa_s = \frac{1}{\gamma p}$$

$$\beta = \frac{1}{T}$$

$$\mu_J = 0$$

$$c = \sqrt{\gamma R_g T}$$

$$c_p - c_v = R_g$$

$$du = c_v dT$$

$$dh = c_p dT$$

$$ds = \frac{c_v}{T} dT + R_g \frac{dv}{v}$$

$$ds = \frac{c_p}{T} dT - R_g \frac{dp}{p}$$

### The Law of Corresponding State (Real Gas)

$$\begin{cases} p_r = \frac{p}{p_C} \\ T_r = \frac{T}{T_C} \\ v_r = \frac{v}{v_C} = \frac{V_m}{V_{m,C}} \end{cases}$$

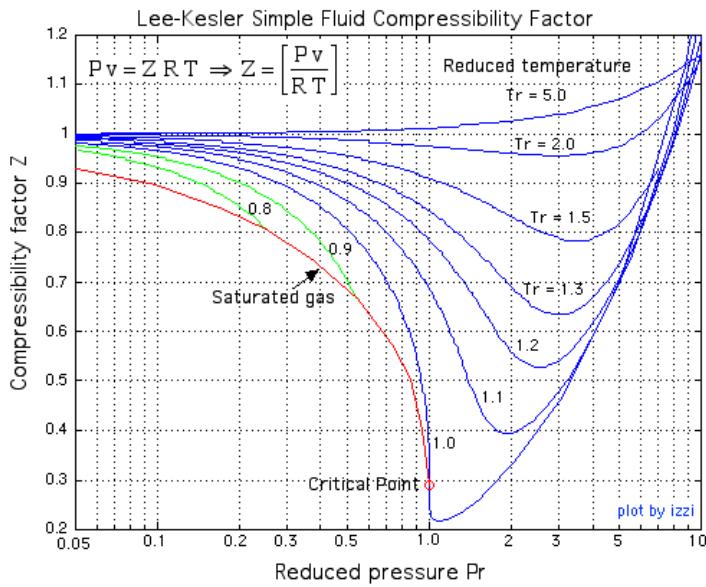
- Compressibility factor

$$Z = \frac{pV_m}{RT}$$

$$Z = f(p_r, T_r)$$

- pseudoreduced specific volume 拟对比态体积

$$v_r = \frac{p_C V_m}{R T_C}$$



## Properties of Real Gas

$$\alpha_V = \frac{1}{T} + \frac{1}{z} \left( \frac{\partial z}{\partial T} \right)_p = \frac{1}{T} \left[ 1 + T \left( \frac{\partial \ln z}{\partial T} \right)_p \right]$$

$$\kappa_T = \frac{1}{p} - \frac{1}{z} \left( \frac{\partial z}{\partial p} \right)_T = \frac{1}{p} \left[ 1 - p \left( \frac{\partial \ln z}{\partial p} \right)_T \right]$$

$$\kappa_s = \frac{\kappa_T}{\gamma}$$

$$\beta = \frac{1}{T} + \frac{1}{z} \left( \frac{\partial z}{\partial T} \right)_v = \frac{1}{T} \left[ 1 + T \left( \frac{\partial \ln z}{\partial T} \right)_v \right]$$

$$\mu_J = \frac{vT}{c_p} \left( \frac{\partial \ln z}{\partial T} \right)_p$$

$$c = \sqrt{\gamma \frac{z R_g T}{1 - p \left( \frac{\partial \ln z}{\partial p} \right)_T}}$$

$$c_p - c_v = R_g \left[ 1 + T \left( \frac{\partial \ln z}{\partial T} \right)_p \right] \left[ 1 + T \left( \frac{\partial \ln z}{\partial T} \right)_v \right]$$

$$du = c_v dT + p T \left( \frac{\partial \ln z}{\partial T} \right)_v dv$$

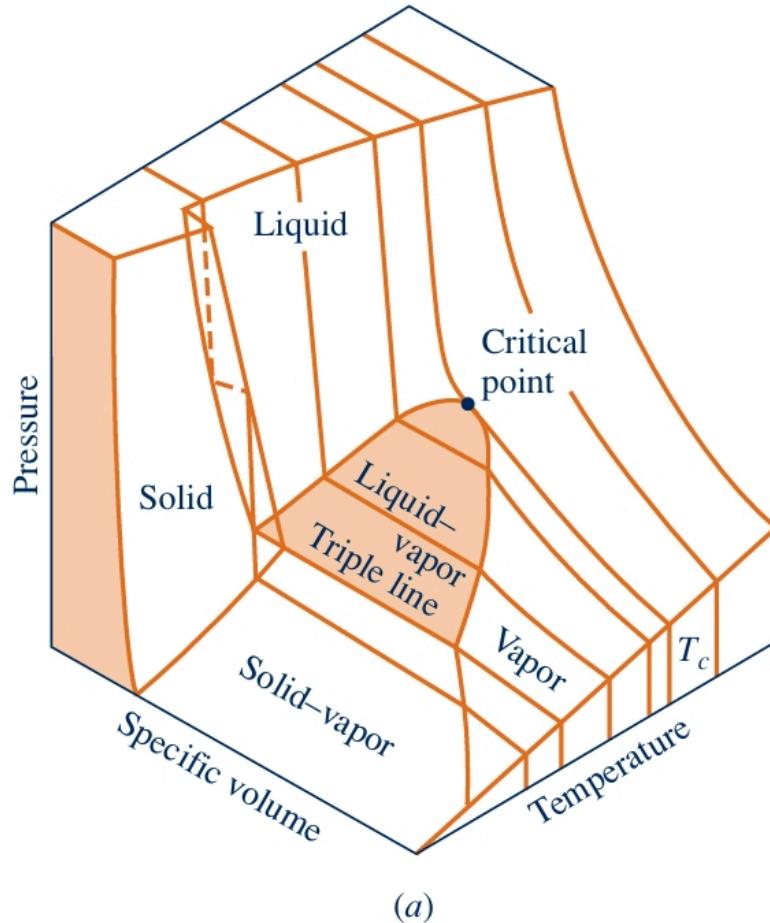
$$dh = c_p dT - v T \left( \frac{\partial \ln z}{\partial T} \right)_p dp$$

$$ds = \frac{c_v}{T} dT + z R_g \left[ 1 + T \left( \frac{\partial \ln z}{\partial T} \right)_v \right] \frac{dv}{v}$$

$$ds = \frac{c_p}{T} dT - z R_g \left[ 1 + T \left( \frac{\partial \ln z}{\partial T} \right)_p \right] \frac{dp}{p}$$

## 6 Vapour

### 6.1 Phase Graph



### 6.2 Equilibrium Conditions

#### Thermodynamics Criterion

- Entropy Criterion

$$dS \Big|_{iso} \geq 0$$

$$dS \Big|_{iso} = 0$$

- Other Criterion

- Exergy Criterion

$$dE = dU + p_0 dV - T_0 dS \leq 0$$

$$dE = dU + p_0 dV - T_0 dS = 0$$

- o Isothermal & Volume-Const

$$dF \Big)_{T,V} = dU - d(TS) \Big)_{T,V} \leq 0$$

$$dF \Big)_{T,V} = dU - d(TS) \Big)_{T,V} = 0$$

- o Isothermal & Pressure-Const

$$dG \Big)_{T,p} = dH - d(TS) \Big)_{T,p} \leq 0$$

$$dG \Big)_{T,p} = dH - d(TS) \Big)_{T,p} = 0$$

### Chemical Potential:

$$dU = TdS - pdV + \mu dm$$

$$dS = \frac{1}{T}dU + \frac{p}{T}dV - \frac{\mu}{T}dm$$

$$\mu = \frac{\partial G}{m} \Big)_{T,p} = g$$

### Phase Equilibrium Condition-Single Unit Multi-Phases

- Thermodynamics Equilibrium

$$T^\alpha = T^\beta$$

- Mechanics Equilibrium

$$p^\alpha = p^\beta$$

- Phase Equilibrium

$$\mu^\alpha = \mu^\beta$$

### \*Phase Equilibrium Condition-Multi-Unit Multi-Phases

- Thermodynamics Equilibrium

$$T_i^\alpha = T_i^\beta$$

- Mechanics Equilibrium

$$p_i^\alpha = p_i^\beta$$

- Phase Equilibrium

$$\mu_i^\alpha = \mu_i^\beta$$

- Chemical Equilibrium

$$\sum_{i,\alpha} \nu_{i,\alpha} \mu_i^\alpha = 0$$

### 6.3 Gibbs Phase Law

$$F = C - P + 2$$

### 6.4 Clausius-Clapeyron Equation

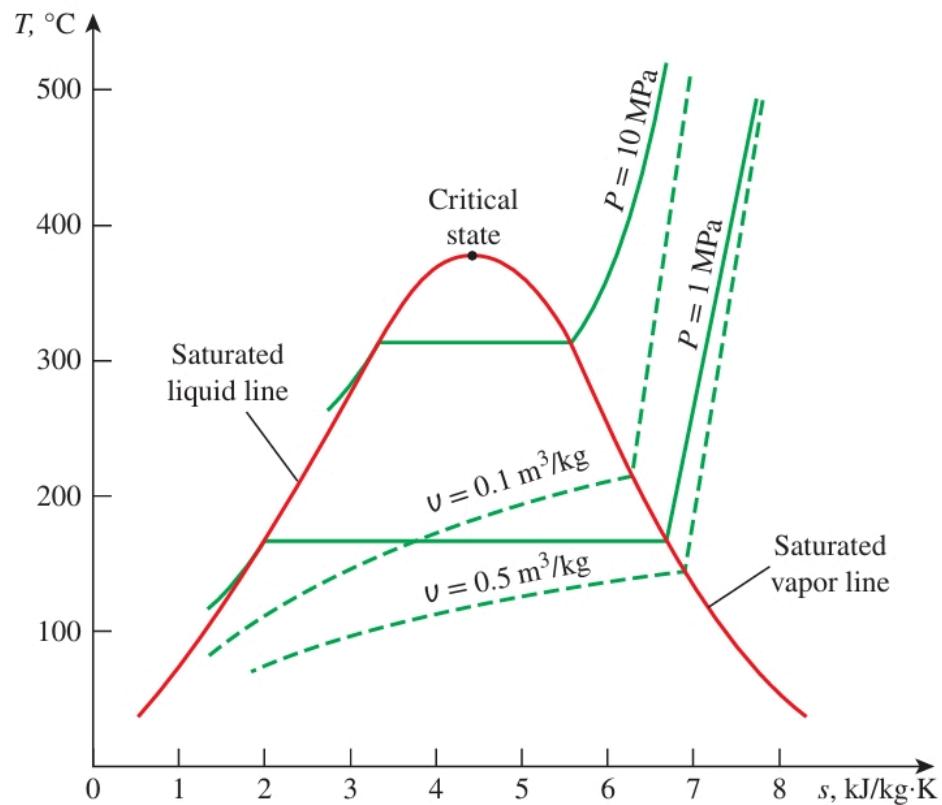
$$\begin{aligned} \mu^\alpha &= \mu^\beta \\ &\downarrow \\ d\mu^\alpha &= d\mu^\beta \\ &\downarrow \\ v^\alpha dp_s - s^\alpha dT_s &= v^\beta dp_s - s^\beta dT_s \\ &\downarrow \\ \frac{dp_s}{dT_S} &= \frac{s^\beta - s^\alpha}{v^\beta - v^\alpha} = \frac{l}{(v^\beta - v^\alpha)T_s} \end{aligned}$$

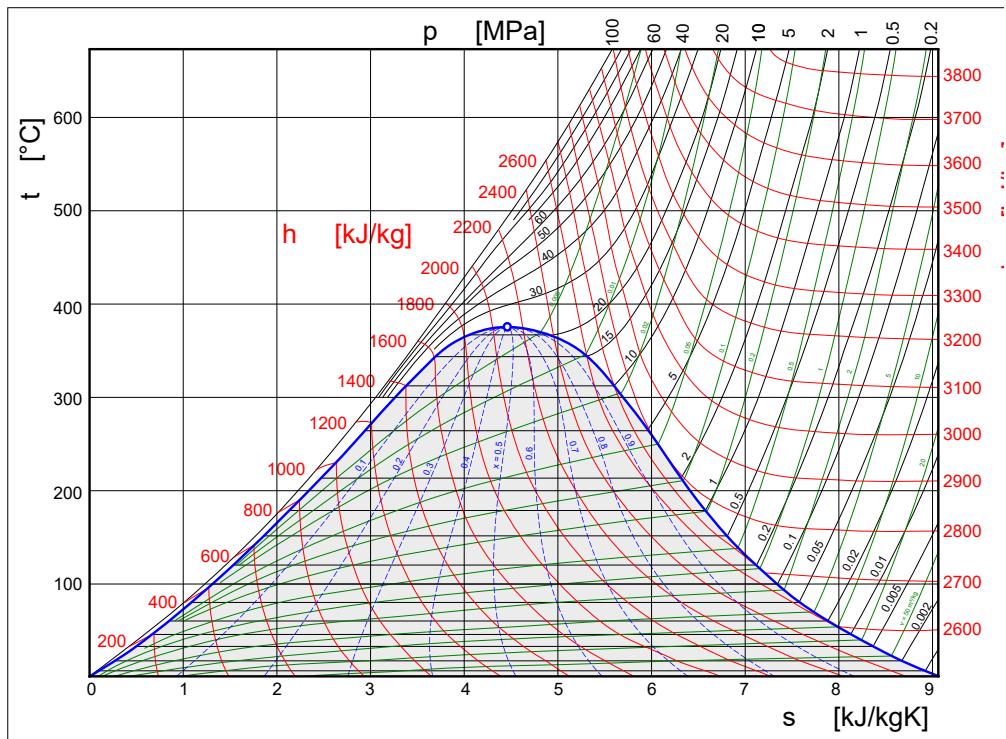
#### Low-Pressure Approximation

$$\ln\left(\frac{p_{s,2}}{p_{s,1}}\right) = -\frac{l}{R_g}\left(\frac{1}{T_{s,2}} - \frac{1}{T_{s,1}}\right)$$

## 6.5 Process of Evaporation (Constant Pressure)

- 6.6 T-s图——Cryogenic Engineering





- Heating of Liquid

Liquid:Subcooled  $\longrightarrow$  Saturated

$$q_l = h_{l,s} - h_0$$

Slope:定压液线下凸

$$\left( \frac{\partial T}{\partial s} \right)_p = \frac{T}{c_{p,l}}$$

- Evaporation

Saturated Liquid  $\longrightarrow$  Saturated Vapor

$$l = h_{v,s} - h_{l,s} = T_s(s_{v,s} - s_{l,s})$$

干度Dryness

$$x = \frac{m_v}{m_l + m_v}$$

- Heating of Vapour

Vapour:Saturated  $\longrightarrow$  Overheated

$$q_v = h - h_{v,s}$$

Slope: 定压气线下凸，通常斜率大于同温度的定压液线

$$\left( \frac{\partial T}{\partial s} \right)_p = \frac{T}{c_{p,v}}$$

过热度 Degree of Overheat

$$\Delta t = t - t_s$$

- Varied Pressure

**Liquid:**

液体热膨胀性弱——熵随压力变化弱，T-s图中低于临界压力的不同定压液线近似和饱和液线重合

**Wet Vapour:**

T-s图中随压力增大向上平行平移

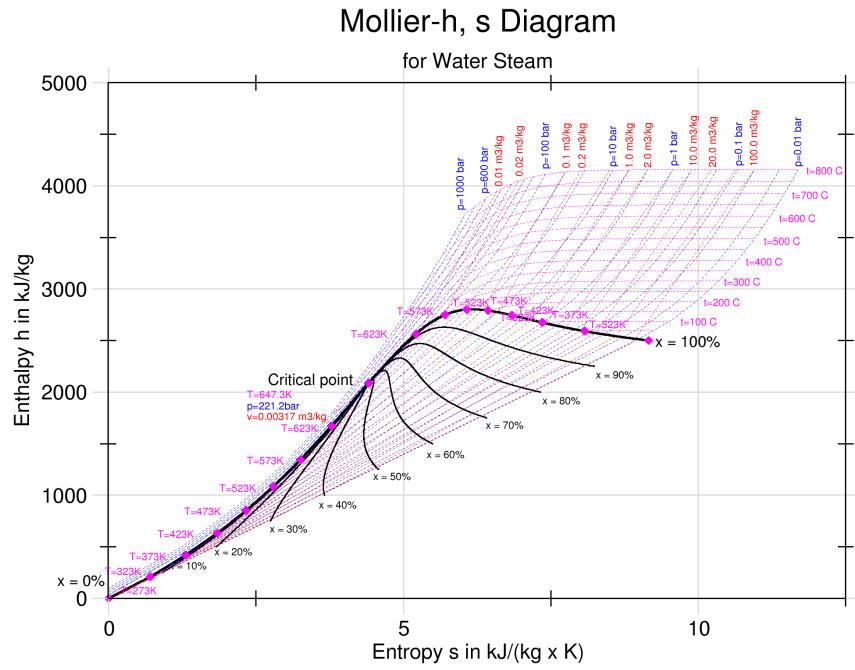
**Dry Vapour:**

气体定压热容随压力变化小，T-s图中随压力增大沿着饱和气线向上近似平行平移

**Supercritical Fluid:**

T-s图中不经过液-气平衡两相区

• 6.7 h-s图——Thermal Engineering



### 饱和液线

下凸单调上升，临界点斜率为临界温度

### 饱和液线

先增大后减小，有极大值

### 定压线

$$\left( \frac{\partial h}{\partial s} \right)_p = T$$

液相区与饱和液线近乎重合

湿蒸气区为斜直线

气相区下凸单调递增

### 定温线

$$\left( \frac{\partial h}{\partial s} \right)_T = T - \frac{1}{\alpha_V}$$

液相区斜率较小，温度较低时可能为负

湿蒸气区为斜直线，与定压线重合

气相区为缓增线

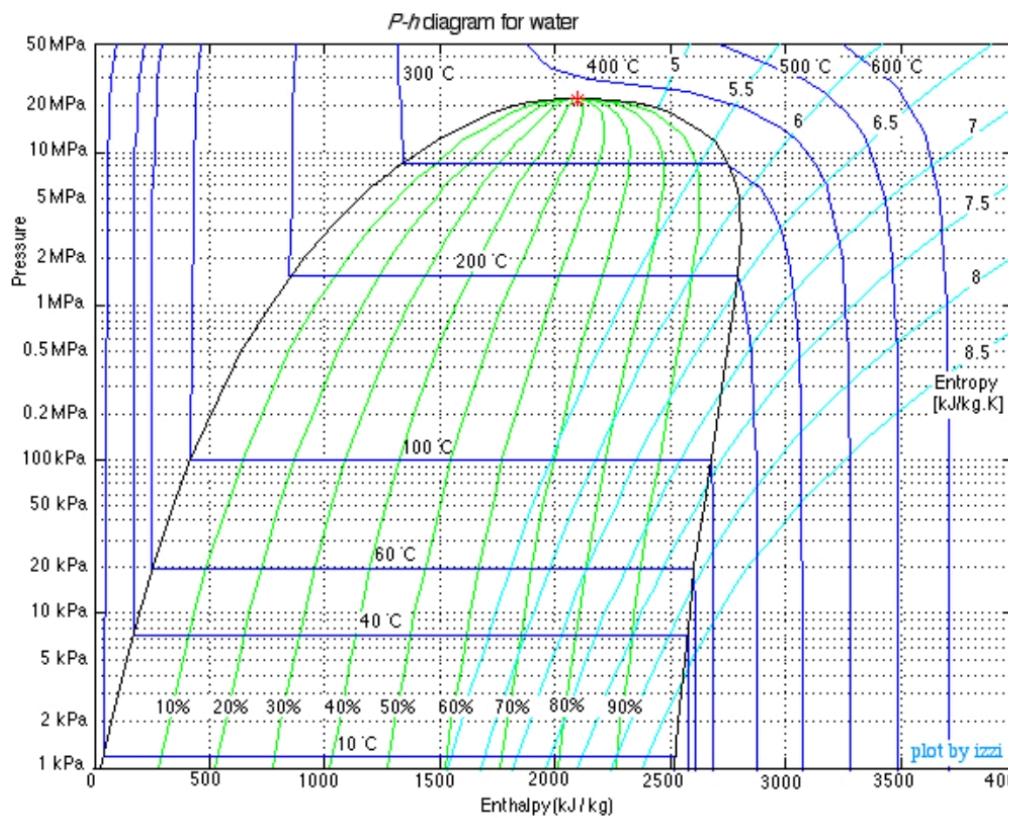
### 定容线

$$\left(\frac{\partial h}{\partial s}\right)_v = T \left[ 1 + \frac{v}{c_v} \left( \frac{\partial p}{\partial T} \right)_v \right]$$

较定压线更陡

### 定干度线

- 6.8 p-h图——Refrigeration Engineering



## 7 Ideal Gas Mixture and Moist Air

- Mass Percent

$$w_i = \frac{m_i}{\sum_i m_i}$$

$$R_g = \frac{R}{M} = \frac{\sum_i n_i M_i R_{g,i}}{m} = \sum_i w_i R_{g,i}$$

$$w_i = \frac{R_{g,i}}{R_g}$$

- Mole Percent

$$x_i = \frac{n_i}{\sum_i n_i}$$

$$M = \frac{m}{n} = \frac{\sum_i n_i M_i}{n} = \sum_i x_i M_i$$

$$x_i = \frac{M_i}{M}$$

- Dalton's Law (Law of Partial Pressures)

恒容恒温

$$x_i = \frac{p_i}{p} = \frac{n_i}{n}$$

- Law of Partial Volumes

恒压恒温

$$x_i = \frac{V_i}{V} = \frac{n_i}{n} \triangleq \varphi_i$$

- Thermal Properties of Ideal Mixtures

$$u = \sum_i w_i u_i \text{ or } U_m = \sum_i x_i U_{m,i}$$

$$h = \sum_i w_i h_i \text{ or } H_m = \sum_i x_i H_{m,i}$$

$$c_v = \sum_i w_i c_{v,i} \text{ or } C_{V,m} = \sum_i x_i C_{V,m,i}$$

$$c_p = \sum_i w_i c_{p,i} \text{ or } C_{p,m} = \sum_i x_i C_{p,m,i}$$

$$s = \sum_i w_i s_i \text{ or } S_m = \sum_i x_i S_{m,i}$$

同温同压理想气体混合熵变：

$$\Delta S_{m,mix} = -R \sum_i x_i \ln x_i$$

## 7.1 Moist Air

- Absolute Humidity

$$\rho_v = \frac{1}{v_v} = \frac{p_v}{R_{g,v}T}$$

- Maximum Absolute Humidity(Saturated Air)

等温过程中水蒸气分压不大于其饱和压力(Saturated Vapour Pressure)

$$\rho_s = \frac{p_s}{R_{g,v}T}$$

at  $T_s$ ,  $p_v \leq p_s$

- Relative Humidity

$$\varphi = \frac{\rho_v}{\rho_s} = \frac{p_v}{p_s}$$

- Dew Point and Dew Point Temperature

等压过程仅降温至饱和状态(等p等d), 该状态称为湿空气露点d

等压过程中水蒸气温度不小于其露点温度(Dew Point Temperature)

at  $p_v$ ,  $T \geq T_d$

绝热过程至饱和状态, 该状态成为绝热饱和状态w

绝热过程下水蒸气温度不小于其绝热饱和温度(Adiabatic Saturated Temperature)

at  $s_v$ ,  $T \geq T_w$

- Moisture(含湿量)

$$d = \frac{m_v}{m_a} = 0.622 \frac{p_v}{p_a} = 0.622 \frac{p_v}{p - p_v} = 0.622 \varphi \frac{p_s}{p - \varphi p_s}$$

- Enthalpy of Wet Air

$$H = m_v h_v + m_a h_a$$

$$h \triangleq \frac{H}{m_a} = h_a + d h_v$$

## 7.2 Bulb Temperature

Dry-bulb Temperature

等温过程

$t \rightarrow$  空气温度, 对应饱和温度  $T_s$

Wet-bulb Temperature

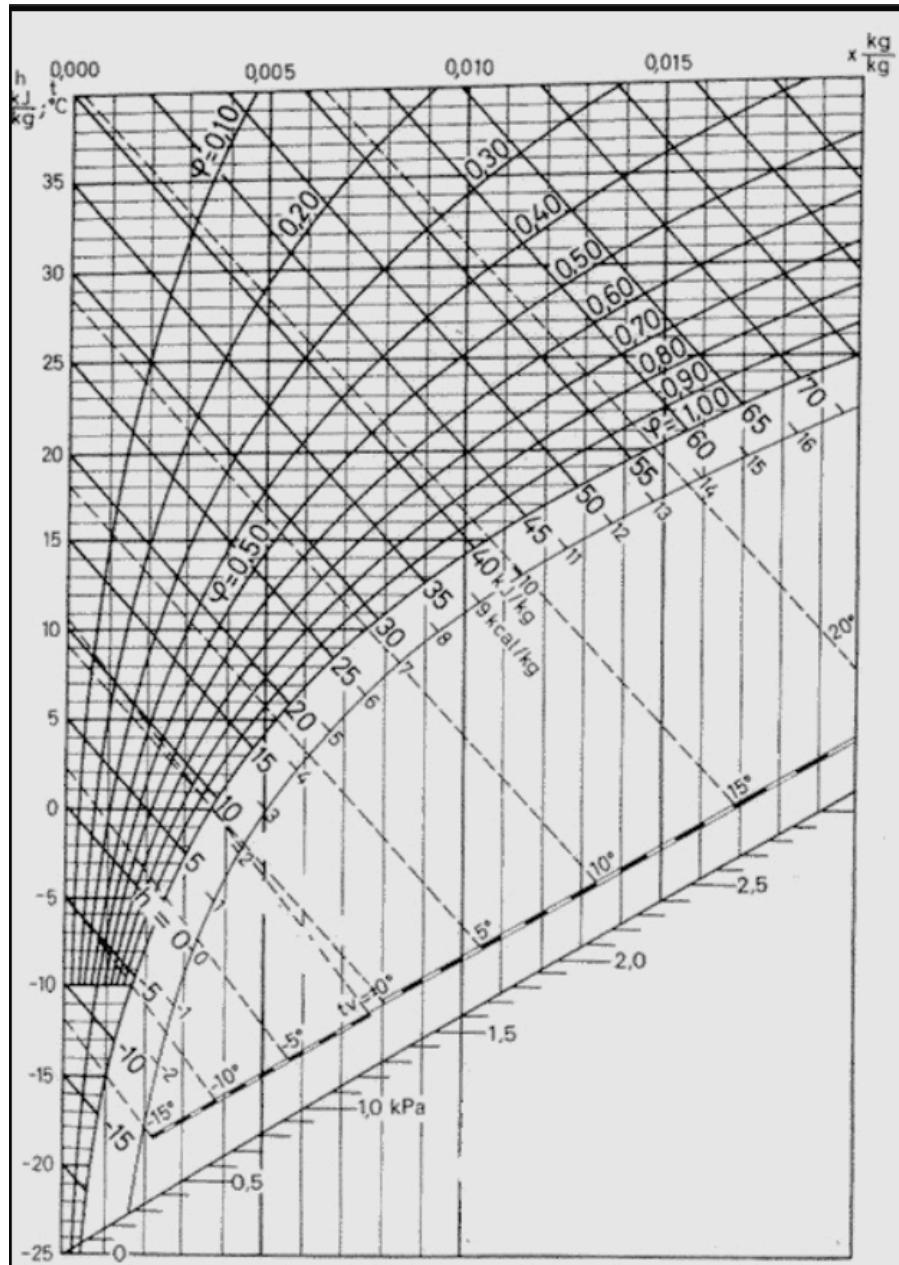
近似等焓过程

$t_w \rightarrow$  吸液芯水温, 近似为绝热饱和温度  $T_w$

$$T_d \leq T_w \leq T_s$$

当且仅当空气饱和时, 取得等号

### 7.3 焓-湿图



- 定含湿量线

$$\text{定}d \Rightarrow \text{定}p_v \Rightarrow \text{定}T_d$$

- 定焓线

$$h_a + dh_v + (d_w - d)h' = h_{a,w} + d_w h_{v,w}$$

↓

$$h + (d_w + d)h' = h_w \approx h$$

$$\text{定}h \Rightarrow \text{定}T_w$$

- 定温线

$$h = c_{p,a}t + d(h_0 + c_{p,v}t)$$

$$\text{定}t \Rightarrow \text{定斜率}h_0 + c_{p,v}t$$

- 定相对湿度线

$$\varphi = \frac{dp}{(d + 0.622)p_s}$$

## 7.4 湿空气过程

- 热湿变化比

$$\varepsilon = \frac{\Delta h}{\Delta d}$$

- 加热/冷却过程

$$q = h_2 - h_1$$

$$\varepsilon = \pm\infty(\text{定}d)$$

- 绝热加湿(蒸发冷却)

$$h_1 + (d_2 - d_1)h_w = h_2 \Rightarrow h_1 \approx h_2$$

$$\varepsilon = 0$$

- 加热加湿/冷却加湿

$$\varepsilon > 0 / \varepsilon < 0$$

- 冷却去湿

达到露点温度后继续受冷冷凝，空气总处于饱和状态，沿着相对湿度100%线，含湿量减小、温度降低方向移动

$$q + (d_1 - d_2)h_w = (h_1 - h_2)$$

### • 绝热混合

$$q_{m,a1} + q_{m,a2} = q_{m,a3}$$

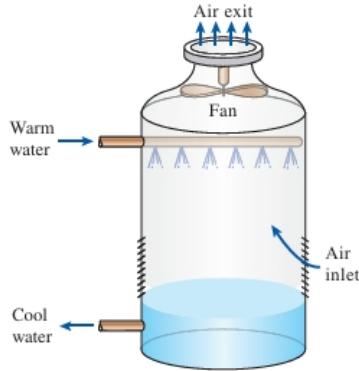
$$q_{m,a1}d_1 + q_{m,a2}d_2 = q_{m,a3}d_3$$

$$q_{m,a1}h_1 + q_{m,a2}h_2 = q_{m,a3}h_3$$

$$\frac{q_{m,a1}}{q_{m,a2}} = \frac{d_3 - d_2}{d_1 - d_3} = \frac{h_3 - h_2}{h_1 - h_3}$$

等比切分

### 冷却塔



利用蒸发冷却原理，已知大气状态、热水状态、冷水参数、空气出口参数条件下，可计算出所需要的空气进口参数

理论上可将水冷却至湿球温度，实际生产上选用比湿球温度高8°C左右的冷水温度。

$$q_{m,a}(h_4 - h_3) = q_{m,w1}h_{w1} - q_{m,w2}h_{w2}$$

$$q_{m,a}(d_4 - d_3) = q_{m,w1} - q_{m,w2}$$

$$q_{m,a} = \frac{q_{m,w1}(h_{w1} - h_{w2})}{(h_4 - h_3) - (d_4 - d_3)h_{w2}}$$

## 8 Ideal Gas Thermodynamic Process

$$\Delta u = c_v(T_2 - T_1)$$

$$\Delta h = c_p(T_2 - T_1)$$

$$\begin{aligned}\Delta s &= c_v \ln \frac{T_2}{T_1} + R_g \ln \frac{v_2}{v_1} \\ &= c_p \ln \frac{T_2}{T_1} - R_g \ln \frac{p_2}{p_1} \\ &= c_p \ln \frac{v_2}{v_1} + c_v \ln \frac{p_2}{p_1}\end{aligned}$$

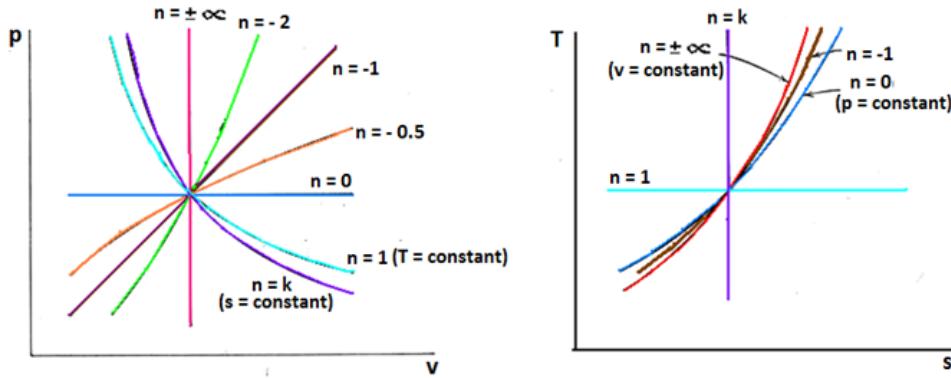
$$q = \int_1^2 cdT = \int_1^2 Tds$$

Assuming Reverse

$$w = \int_1^2 pdv$$

$$w_t = \int_1^2 -vdp$$

## 8.1 Polytropic Process



$$pv^n = \text{const}$$

$$q = c(T_2 - T_1)$$

$$c = \frac{n - \gamma}{n - 1} c_v \quad (\text{理想气体多变过程比热容})$$

$$\frac{\partial p}{\partial v} = n \frac{p}{v}$$

$$\frac{\partial T}{\partial s} = \frac{T}{c}$$

$$\frac{p_2}{p_1} = \left( \frac{v_1}{v_2} \right)^n$$

$$\frac{T_2}{T_1} = \left( \frac{v_1}{v_2} \right)^{n-1}$$

$$\frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}}$$

$$\begin{aligned}
w &= \frac{1}{n-1}(p_1v_1 - p_2v_2) \\
&= \frac{1}{n-1}R_g(T_1 - T_2) \\
&= \frac{1}{n-1}RgT_1\left(1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right) \\
&= \frac{1}{n-1}p_1v_1\left(1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right)
\end{aligned}$$

$$\begin{aligned}
\frac{dp}{p} + n\frac{dv}{v} &= 0 \\
\downarrow \\
w_t &= nw
\end{aligned}$$

$$\begin{aligned}
w_t &= \frac{n}{n-1}(p_1v_1 - p_2v_2) \\
&= \frac{n}{n-1}R_g(T_1 - T_2) \\
&= \frac{n}{n-1}RgT_1\left(1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right) \\
&= \frac{n}{n-1}p_1v_1\left(1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right)
\end{aligned}$$

- **8.2 Constant Volume**

$$n = \pm\infty, \quad pv^{\pm\infty} = const, \quad v = const$$

$$\begin{aligned}
\frac{\partial p}{\partial v} &= \pm\infty \\
\frac{\partial T}{\partial s} &= \frac{T}{c_v}
\end{aligned}$$

$$\begin{aligned}
c &= c_v \\
\Delta u &= c_v(T_2 - T_1) \\
w &= 0, \quad w_t = -v(p_2 - p_1) \\
q &= c_v(T_2 - T_1)
\end{aligned}$$

- **8.3 Constant Pressure**

$$n = 0, \quad pv^0 = const, \quad p = const$$

$$\begin{aligned}
\frac{\partial p}{\partial v} &= 0 \\
\frac{\partial T}{\partial s} &= \frac{T}{c_p}
\end{aligned}$$

$$c = c_p$$

$$\begin{aligned}\Delta h &= c_p(T_2 - T_1) \\ w &= p(v_2 - v_1), \quad w_t = 0 \\ q &= c_p(T_2 - T_1)\end{aligned}$$

- 8.4 Isothermal

$$n = 1, \quad pv = R_g T = \text{const}, \quad T = \text{const}$$

$$\begin{aligned}\frac{\partial p}{\partial v} &= -\frac{p}{v} \\ \frac{\partial T}{\partial s} &= 0\end{aligned}$$

$$\begin{aligned}c &= \pm\infty \\ \Delta u &= 0, \quad \Delta h = 0 \\ w &= R_g T \ln\left(\frac{v_2}{v_1}\right) = w_t = R_g T \ln\left(\frac{p_1}{p_2}\right) \\ q &= T(s_2 - s_1)\end{aligned}$$

- 8.5 Adiabatic (and Reverse)

$$n = \gamma, \quad pv^\gamma = \text{const}, \quad s = \text{const}$$

$$\begin{aligned}\frac{\partial p}{\partial v} &= -\gamma \frac{p}{v} \\ \frac{\partial T}{\partial s} &= \pm\infty\end{aligned}$$

$$\begin{aligned}c &= \frac{n-\gamma}{n-1} \\ \Delta u &= c_v(T_2 - T_1), \quad \Delta h = c_p(T_2 - T_1) \\ w &= \frac{1}{\gamma-1}(p_1 v_1 - p_2 v_2), \quad w_t = \frac{\gamma}{\gamma-1}(p_1 v_1 - p_2 v_2) \\ q &= 0\end{aligned}$$

$$(\text{绝热系数})\gamma = \frac{c_p}{c_v} = 1 + \frac{R_g}{c_v}$$

$$\begin{aligned}c_v &= \frac{1}{\gamma-1} \\ c_p &= \frac{\gamma}{\gamma-1}\end{aligned}$$

## 9 Compressible Flow

### 9.1 Steady Flow Basic Equations

Continuity Equation

$$\begin{aligned}\nabla(\rho \mathbf{V}) &= 0 \\ \downarrow \\ d(\rho c_f A) &= 0 \\ \downarrow \\ \frac{dA}{A} + \frac{dc_f}{c_f} - \frac{dv}{v} &= 0\end{aligned}$$

Energy Conservation Equation

$$\delta q = dh + \delta w_t = dh + \delta w_{net} + \frac{1}{2}dc_f^2 + gdz$$

Entropy Conservation Equation

$$ds = ds_f + \delta s_g \geq \frac{\delta q}{T}$$

Equation of State

$$f(p, v, T) = 0$$

### 9.2 Isentropic Flow

Characteristic Equation of Isentropic Flow

$$(马赫数) Ma \triangleq \frac{c_f}{c}$$

$$\frac{dA}{A} = (Ma^2 - 1) \frac{dc_f}{c_f}$$

- Categories

流动分类	马赫数大小	流速与声速大小
亚声速流动	$Ma < 1$	$c_f < c$
超声速流动	$Ma > 1$	$c_f > c$
临界流动	$Ma = 1$	$c_f = c$

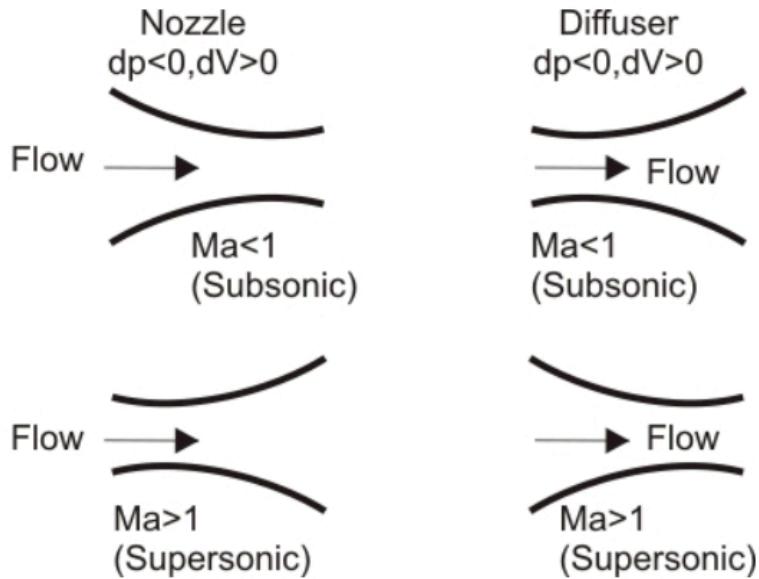
- Heat Capacity Ratio-Ideal Gas

$$\gamma = -\frac{v}{p} \left( \frac{\partial p}{\partial v} \right)_s$$

- Speed of Sound-Ideal Gas

$$c = \sqrt{\gamma p v} = \sqrt{\gamma R_g T} (\text{Local Speed of Sound})$$

### 9.3 Nozzles and Diffusers



#### Basic Equation

$$\frac{1}{2}(c_{f2}^2 - c_{f1}^2) = \int_2^1 -v dp = h_2 - h_1$$

分类	速度	马赫数	压力	体积	流道截面
渐缩喷管	$dc_f > 0$	$Ma < 1$ (亚声速流动)	$dp < 0$	$dv > 0$	$dA < 0$
渐扩喷管	$dc_f > 0$	$Ma > 1$ (超声速流动)	$dp < 0$	$dv > 0$	$dA > 0$
渐缩扩压管	$dc_f < 0$	$Ma > 1$ (超声速流动)	$dp > 0$	$dv < 0$	$dA < 0$
渐扩扩压管	$dc_f < 0$	$Ma < 1$ (亚声速流动)	$dp > 0$	$dv < 0$	$dA > 0$

#### 缩放喷管（拉伐尔喷管）

- 滞止参数

$$h^* = h + \frac{c_f^2}{2}$$

$$T^* = T + \frac{c_f^2}{2c_p}$$

$$p^* = p \left( \frac{T^*}{T} \right)^{\frac{\gamma}{\gamma-1}}$$

$$v^* = v \left( \frac{T}{T^*} \right)^{\frac{1}{\gamma-1}}$$

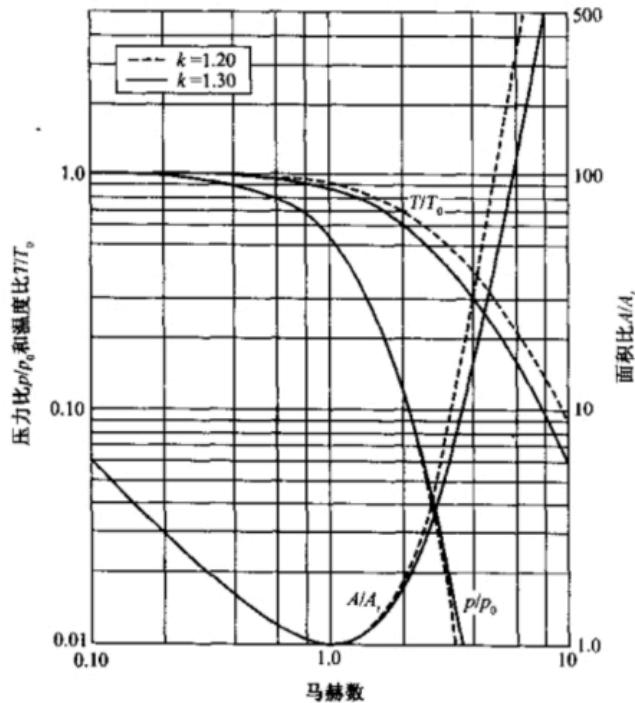
### • 气体流速

$$c_f = \sqrt{2(h^* - h)} = \sqrt{2\frac{\gamma}{\gamma-1} R_g T^* \left[ 1 - \left( \frac{p}{p^*} \right) \right]}$$

理论最大值（条件：截面压力为0，无法达到）

$$c_{f,max} = \sqrt{2\frac{\gamma}{\gamma-1} R_g T^*}$$

等熵流动特性曲线



### • 临界压力比

临界截面上临界流速等于当地声速

$$\nu_{cr} = \frac{p_{cr}}{p^*} = \left( \frac{2}{\gamma+1} \right)^{\frac{\gamma}{\gamma-1}}$$

临界声速：

$$c_{f,cr} = c = \sqrt{\gamma R_g T_{cr}} = \sqrt{\frac{2\gamma}{\gamma+1} R_g T^*}$$

设计管型	压比与临界压比大小
渐缩喷管	$\nu = \frac{p_B}{p^*} \geq \nu_{cr}$
缩放喷管	$\nu = \frac{p_B}{p^*} < \nu_{cr}$

- 气体流量及喷管尺寸

- 喷管喉部截面积

$$A_{min} = \frac{q_m v_{cr}}{c_{f,cr}}$$

- 出口截面面积

$$A_2 = \frac{q_m v_2}{c_{f2}}$$

- 经验长度

$$l = \frac{d_2 - d_{min}}{2 \tan \frac{\varphi}{2}} \quad (\varphi \text{为顶锥角})$$

- 气体流量

$$q_m = A \sqrt{2 \frac{\gamma}{\gamma - 1} \frac{p^*}{v^*} \left[ \left( \frac{p_2}{p^*} \right)^{\frac{2}{\gamma}} - \left( \frac{p_2}{p^*} \right)^{\frac{\gamma+1}{\gamma}} \right]}$$

流量曲线

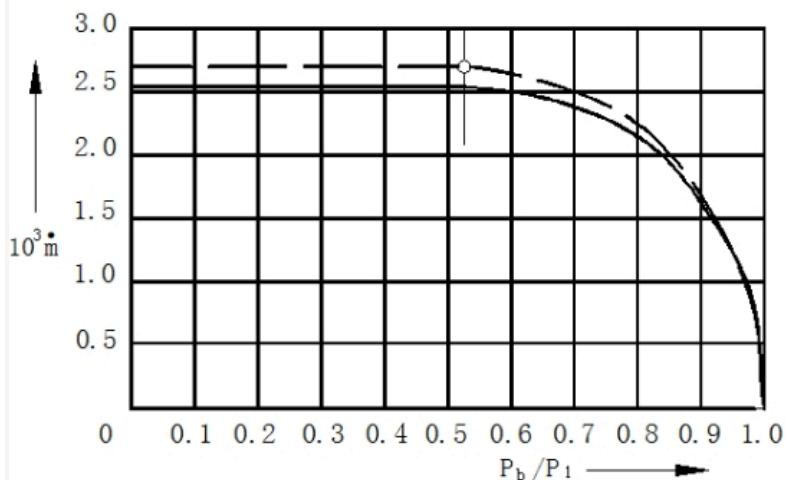


图8 渐缩喷管流量曲线(当 $P_1=1\text{bar}$ ,  $t_a=20^\circ\text{C}$ )

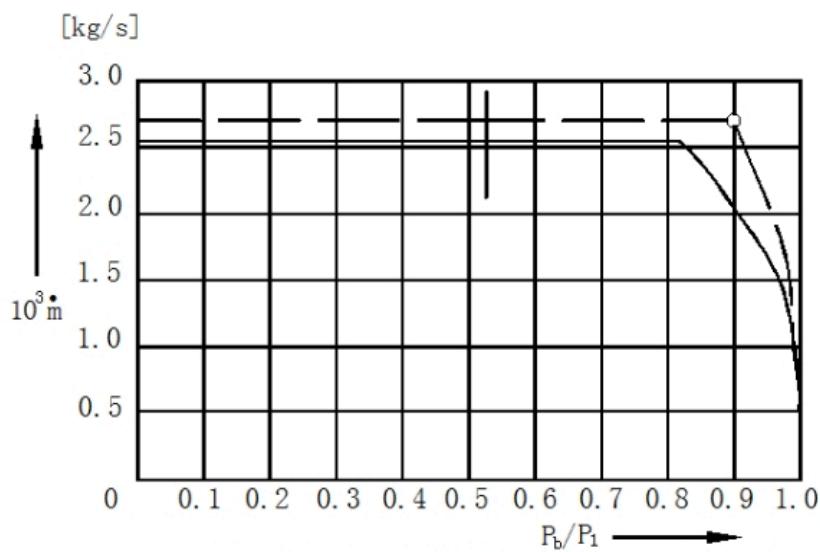


图9 缩放喷管流量曲线(当 $P_1=1\text{bar}$ ,  $t_1=20^\circ\text{C}$ )

在临界压比处有流量最大值

$$q_{m,max} = A_{min} \sqrt{2 \frac{\gamma}{\gamma+1} \left( \frac{2}{\gamma+1} \right)^{\frac{2}{\gamma-1}} \frac{p^*}{v^*}}$$

- 渐缩喷管截面压力无法降低至临界压力以下，故出口压力即为临界压力
- 缩放喷管截面压力可以降低至临界压力以下，但流量保持最大流量不变
- 渐缩喷管校核

背压	膨胀程度	流量
$\frac{p_B}{p^*} > \nu_{cr}$	完全膨胀	流量逐渐增大，出口流量未达理论最大值
$\frac{p_B}{p^*} = \nu_{cr}$	完全膨胀	出口流量达到理论最大值
$\frac{p_B}{p^*} < \nu_{cr}$	不完全膨胀，出口处发生自由膨胀	出口流量达到理论最大值

- 缩放喷管校核

设计压力 $p_d$ : 出口截面设计压力

临界压力 $p_{cr}$ : 喉部截面设计压力

背压	膨胀程度	流量
$p_B >> p_d$ 且喉部面积设计大于理论值	全程亚音速流动	/
$p_B > p_d$	过度膨胀, 可能产生激波	出口流量达到理论最大值
$p_B = p_d$	完全膨胀	流量达到最大值
$p_B < p_d$	不完全膨胀, 出口处发生自由膨胀	流量达到最大值

压力曲线

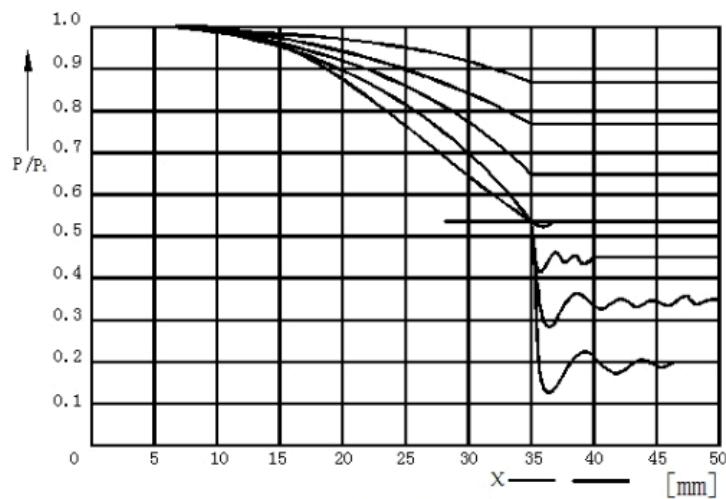


图5 渐缩喷管压力曲线

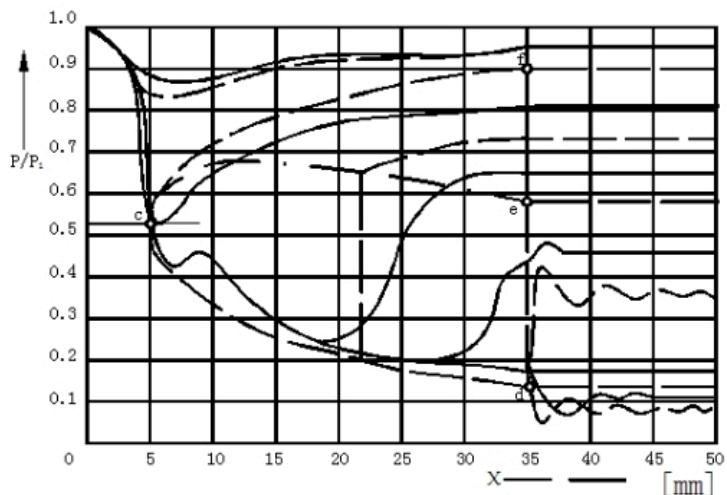


图6 缩放喷管压力曲线

## 缩放扩压管

### Irreversible Adiabatic Flow

- 速度系数

$$\varphi = \frac{c_{f2'}}{c_{f2}}$$

- 喷管效率

$$\eta_N = \frac{c_{f2}^2/2}{c_{f2'}^2/2} = \frac{h^* - h_{2'}}{h^* - h_2}$$

$$\eta_N = \varphi^2$$

### 9.4 Adiabatic Throttle(绝热节流)

$$h_2 \approx h_1$$

$$s_2 > s_1$$

对于降压很小的节流过程：

- 节流热效应

$$\mu_J < 0$$

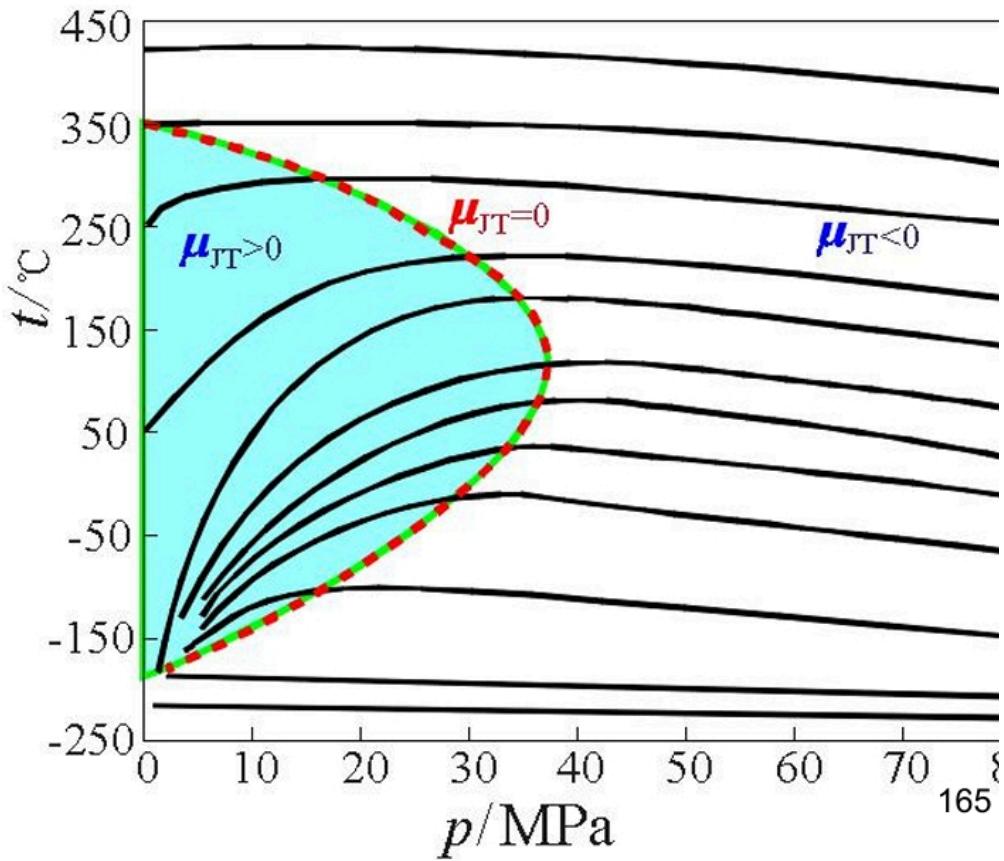
- 节流冷效应

$$\mu_J > 0$$

- 节流零效应

$$\mu_J = 0$$

T-p图绝热节流曲线：



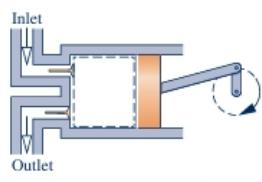
每一条定焓线上温度最大值点为转变点，温度称为回转温度 ( $\mu_J = 0$ )

- 转变曲线
- 热效应区
- 冷效应区
- 最大转变压力 $p_N$
- 上转变温度、下转变温度
- 最大转变温度 $T_N$ ，节流致冷温度需设定在最大转变温度之下

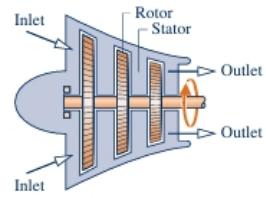
## 10 Compression and Compressor

### 10.1 工作原理

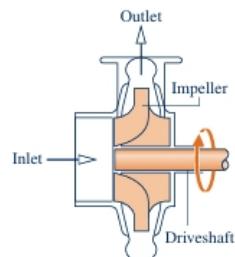
- 单级活塞式
- 叶轮式



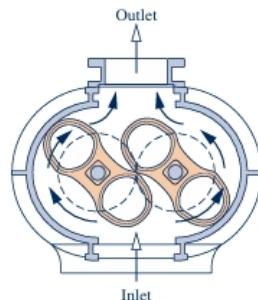
(a) Reciprocating



(b) Axial flow

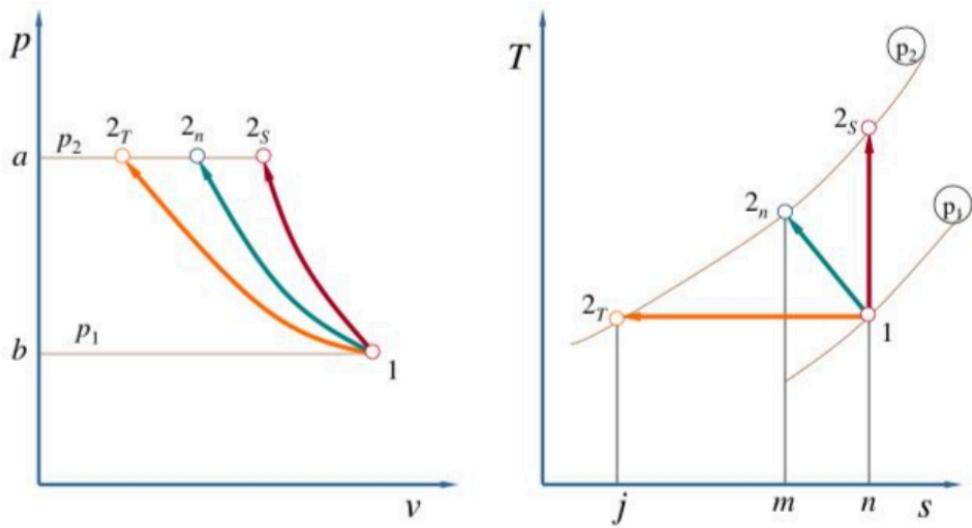


(c) Centrifugal



(d) Roots type

## 10.2 压缩过程热力学分析



- **增压比**

$$\pi = \frac{p_2}{p_1}$$

- **能量转化**

$$q = (h_2 - h_1) + w_t$$

- **可能的压缩过程**

- **绝热压缩**

压缩过程可近似看作无热量交换过程

$$w_t = \frac{\gamma}{\gamma-1} R_g T_1 \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

$$q = 0$$

- **多变压缩**

采用一定冷却措施，但气体温度仍然升高

$$w_t = \frac{n}{n-1} R_g T_1 \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right]$$

$$q = \frac{n-\gamma}{n-1} c_v (T_2 - T_1)$$

- **定温压缩**

理想冷却条件

$$w_t = R_g T_1 \ln \frac{p_2}{p_1}$$

$$q = R_g T_1 \ln \frac{p_2}{p_1}$$

$$|w_{t,T}| < |w_{t,n}| < |w_{t,s}|$$

$$T_T < T_n < T_s$$

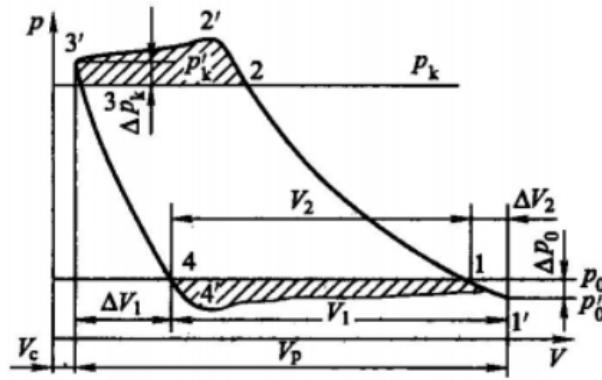
### • 不可逆压缩过程

绝热效率:

$$\eta_{C,s} = \frac{w_t}{w'_t} = \frac{h_2 - h_1}{h'_2 - h_1}$$

轴流式压气机一般  $\eta_{C,s}$  为  $0.80 \sim 0.90$

## 10.3 压缩循环



### • 余隙容积

容积效率

$$\eta_V = \frac{V(\text{有效进气容积})}{V_h(\text{排气容积})}$$

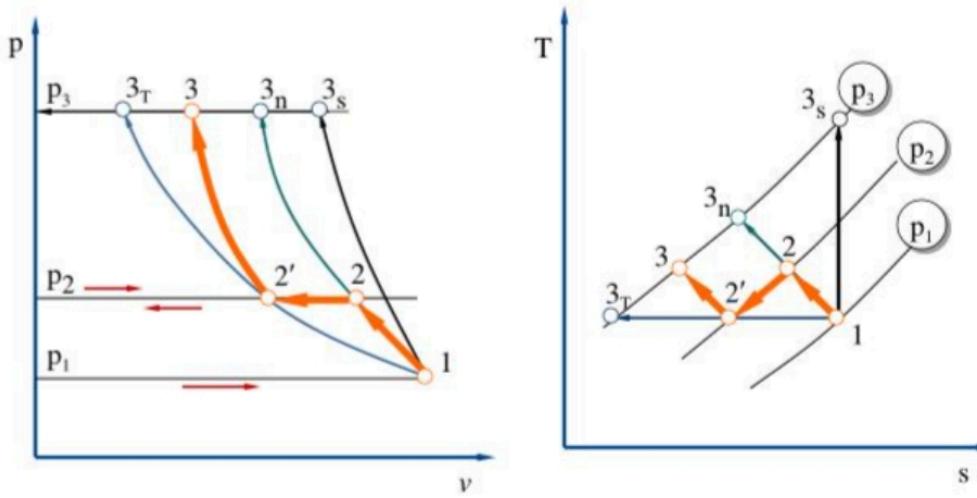
$$w_{t,n} = \frac{n}{n-1} p_1 V \left[ 1 - \left( \frac{p_2}{p_1} \right)^{\frac{n-1}{n}} \right]$$

余隙容积一定时，增大增压比，有效进气容积减小，直至为0

通常余隙容积的百分比

$$c = \frac{V_c}{V_h} = 0.03 \sim 0.08$$

- 多级压缩中间冷却



- 多级压缩过程总耗功量

$$w_t = \sum_i w_{t,i}$$

- 中间冷却

通常冷却至上一级进气温度

- 最有利分级增压比

$$\pi = \sqrt[3]{\pi_{total}}$$

## 11 Vapour Power Cycle

- 循环效率

$$\eta_t = \frac{w}{q_1} = 1 - \frac{q_2}{q_1} = 1 - \frac{\bar{T}_2}{\bar{T}_1}$$

- 卡诺循环效率

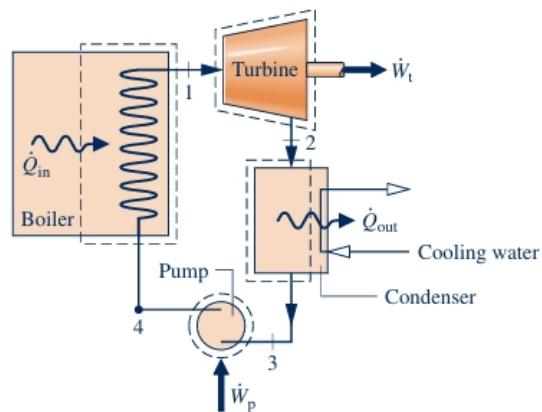
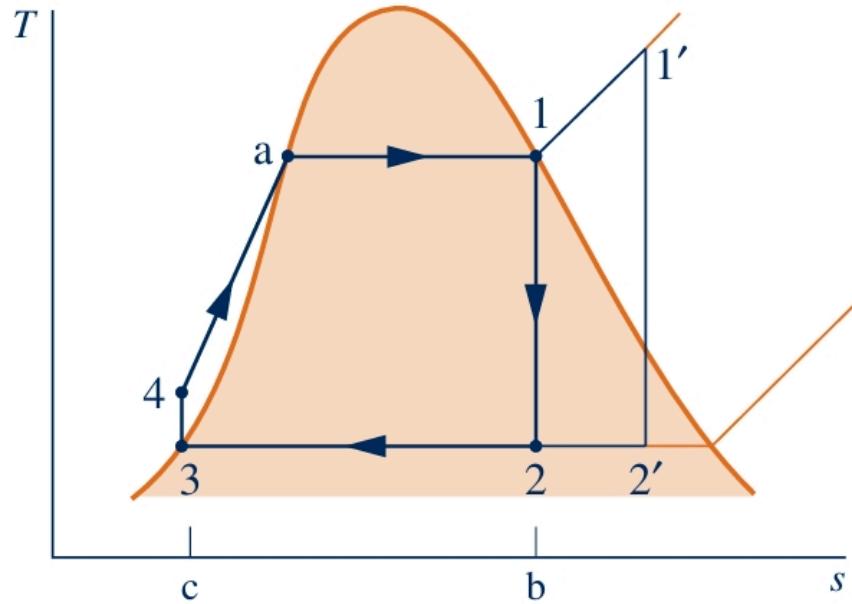
$$\eta_t = 1 - \frac{T_2}{T_1}$$

### 11.1 Carnot Cycle of Vapor

难以采用：

- 压缩低干度湿蒸汽耗功量大，且对压缩机工作不利
- 水临界温度较低，平均吸热温度较低，效率远低于卡诺循环效率

## 11.2 Rankine Cycle



- 热效率

$$\eta_t = \frac{w}{q_1} = \frac{w_T - w_P}{q_1} \approx \frac{w_T}{q_1} \text{(忽略泵功条件下)}$$

- 功比

$$r_w = \frac{w}{w_T} = \frac{w_T - w_P}{w_T}$$

- 泵功

$$w_P \approx v\Delta p$$

- 汽耗率

$$d = \frac{3600}{w}$$

- 相对内效率

$$\eta_T = \frac{w'_T}{w_T} \text{(表示膨胀的理想程度)}$$

### 蒸汽参数分析

- 蒸汽初压力

- 提升初压力，锅炉蒸发吸热温度提高，平均吸热温度提高，循环热效率增大
- 低压提升效率较高压更为显著
- 仅提高初压力会导致乏汽干度下降，危害汽轮机安全（应不低于88%）

- 蒸汽初温度

- 提高初温度，平均吸热温度提高，循环热效率增大
- 提高初温度，乏汽干度亦提高，有利于汽轮机工作
- 过高初温度受到成本及材料的限制

- 蒸汽终参数

- 降低终压力，平均放热温度降低，循环热效率增大
- 终压对应饱和温度应高于冷凝器冷却水温度

### 11.3 Improving Performance Cycle

#### Reheat

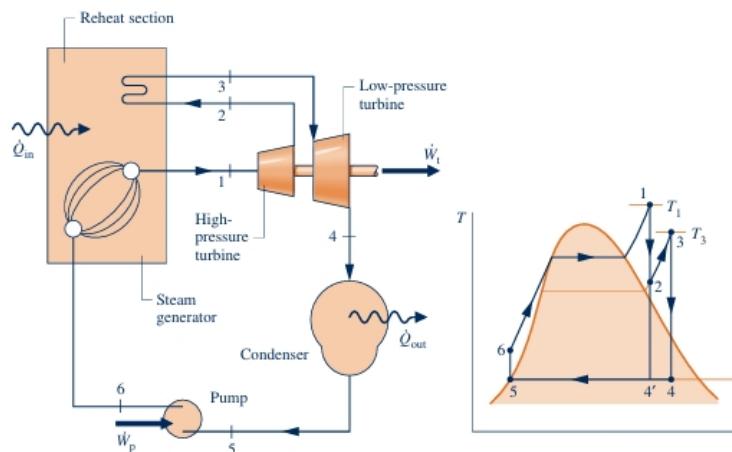


Fig. 8.7 Ideal reheat cycle.

- 增大乏汽干度
- 提高平均吸热温度，增大循环热效率

### Regeneration

- 极限回热循环

回热水套，乏汽全部用于加热水至给水回热温度

- 分级抽汽回热循环

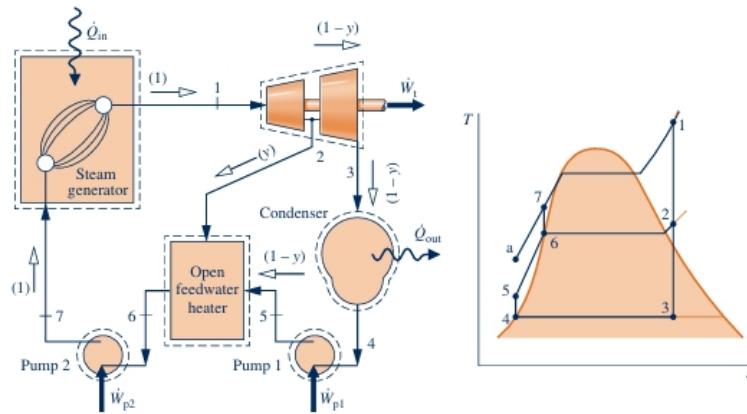


Fig. 8.9 Regenerative vapor power cycle with one open feedwater heater.

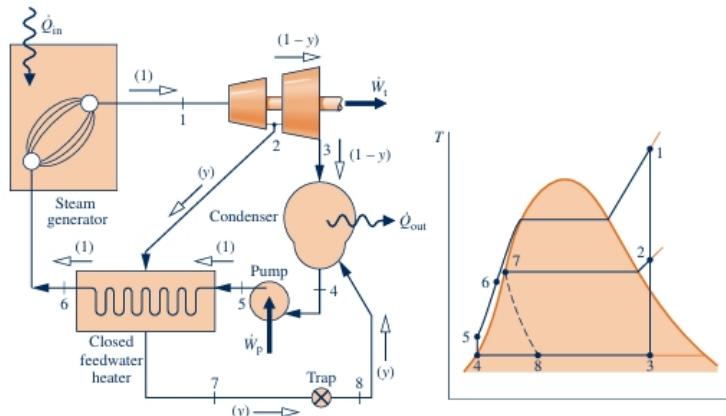


Fig. 8.11 Regenerative vapor power cycle with one closed feedwater heater.

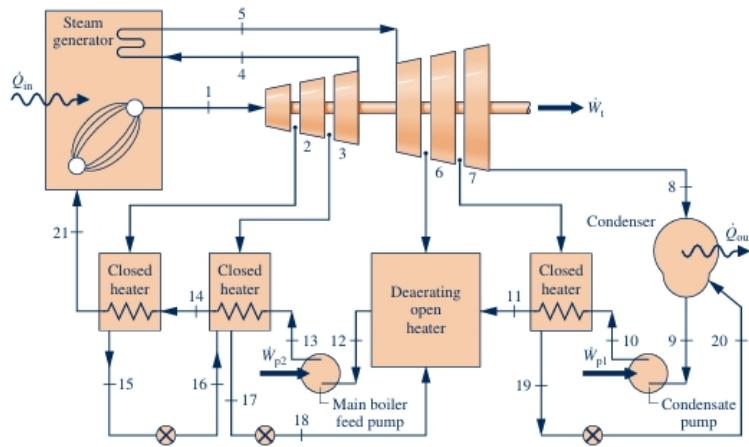
- 抽汽流量系数

$$\alpha_i = \frac{q_{m,i}}{q_m}$$

- 凝汽系数

$$\alpha_C = 1 - \sum_i \alpha_i$$

- 涉及计算：湿空气混合过程

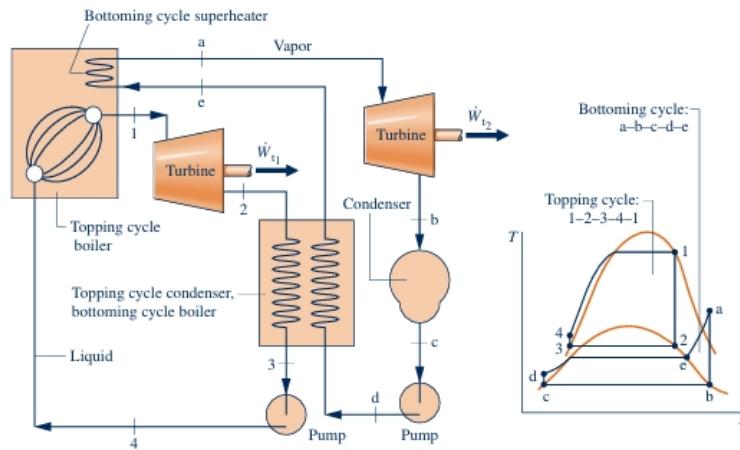


- 热电循环

- 引射器

- 有机朗肯循环

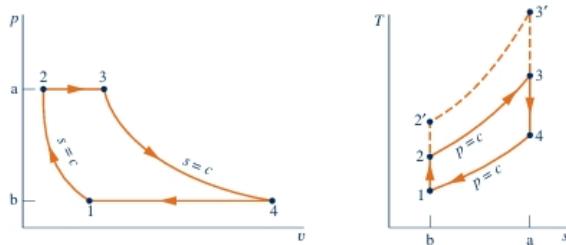
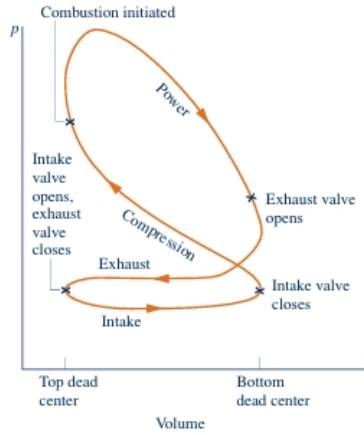
- 两气循环



- 过热循环、超临界循环

## 12 Gas Power Cycle

### 12.1 Brayton Cycle



- 热效率

$$\eta_t = 1 - \frac{1}{\pi^{\frac{2-1}{\gamma}}}$$

热效率随增压比增大而提高

- 循环净功量

$$w = w_T - w_C$$

当循环净功量达到最大值时，增压比为最佳增压比：

$$\pi_{opt} = \left( \frac{T_H}{T_C} \right)^{\frac{\gamma}{2(\gamma-1)}} = \tau^{\frac{\gamma}{2(\gamma-1)}} \quad (\tau \text{ 为循环增温比})$$

- 相对内效率

$$\eta_T = \frac{w'_T}{w_T} (\text{燃气轮机摩擦损失})$$

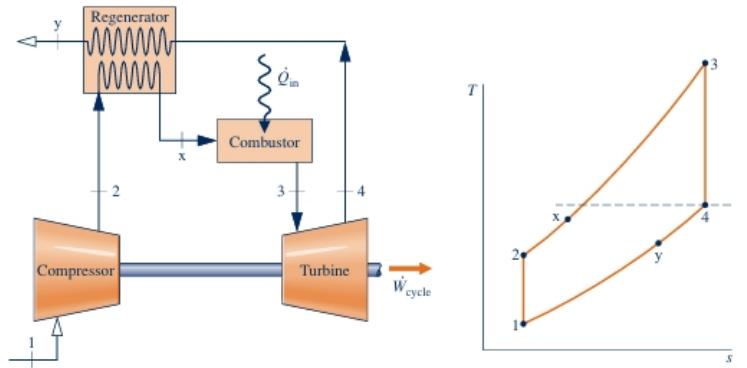
$$\eta_C = \frac{w'_C}{w_C} (\text{压气机摩擦损失})$$

- 燃气轮机摩擦损失使循环净功量减少，循环热效率降低
- 压气机摩擦损失使循环净功量和吸热量减少相同数值，循环热效率降低，次于燃气轮机摩擦损失降低效果
- 烟效率

$$\eta_{e_x} = \frac{w}{e_{x,Q}} (\text{循环烟效率})$$

$$\eta_t = \eta_{e_x} \eta_{Carnot}$$

## 12.2 Regeneration

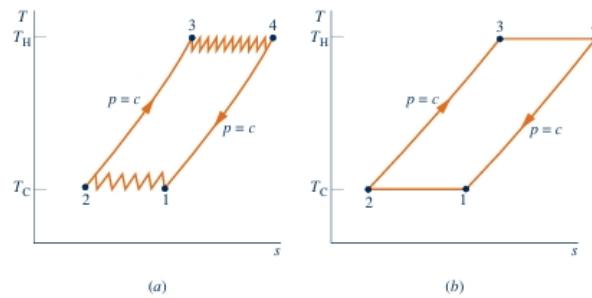
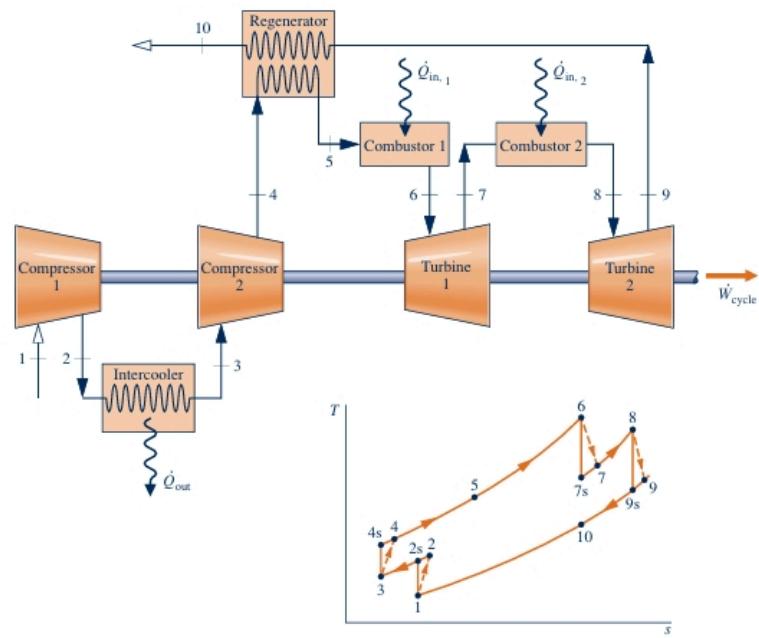


- 回热度

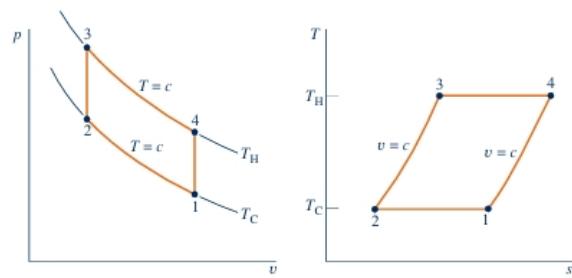
$$\sigma = \frac{h_x - h_2}{h_4 - h_2}$$

表示实际回热程度，通常0.5~0.7

### 12.3 Ericsson Cycle (回热+多级压缩+中间冷却)

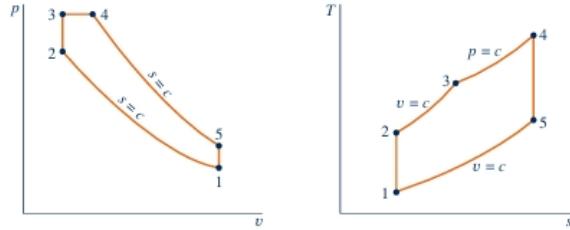


### 12.4 Stirling Cycle



## 12.5 内燃机循环

### 混合加热循环



- 压缩比(区分压气机, 此处为体积压缩)

$$\varepsilon = \frac{v_1}{v_2}$$

- 定容升压比

$$\lambda = \frac{p_3}{p_2}$$

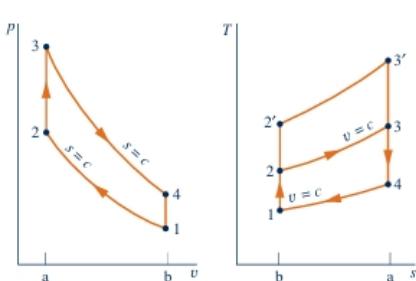
- 定压预胀比

$$\rho = \frac{v_4}{v_3}$$

- 热效率

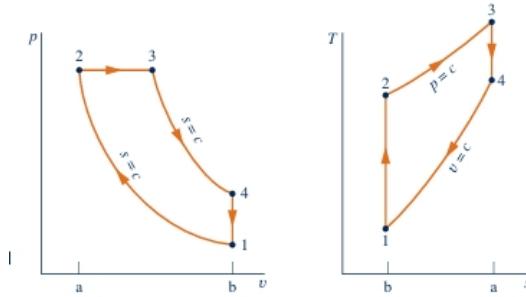
$$\eta_t = 1 - \frac{\lambda \rho^\gamma - 1}{\varepsilon^{\gamma-1} [(\lambda - 1) + \gamma \lambda (\rho - 1)]}$$

Otto Cycle (定容加热循环) (点燃式内燃机(煤气机、汽油机))



$$\eta_{t,V} = 1 - \frac{1}{\varepsilon^{\gamma-1}}$$

## Diesel Cycle (定压加热循环) (压燃式内燃机(柴油机))



$$\eta_{t,V} = 1 - \frac{1}{\varepsilon^{\gamma-1}} \frac{\rho^\gamma - 1}{\gamma(\rho - 1)}$$

### 条件分析

- 相同压缩比及吸热量

$$q_{2,V} < q_2 < q_{2,p}$$

$$\bar{T}_{1,V} > \bar{T}_1 > \bar{T}_{1,p}, \quad \bar{T}_{2,V} < \bar{T}_2 < \bar{T}_{2,p}$$

$$\eta_{t,2,V} > \eta_{t,2} > \eta_{t,2,p}$$

点燃式内燃机压缩比通常5~9  
压燃式内燃机压缩比通常13~18

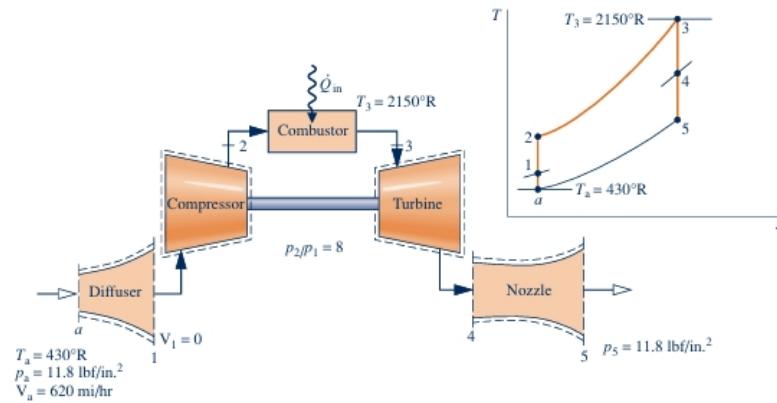
- 相同最高压力及最高温度

$$q_{1,V} < q_1 < q_{1,p}$$

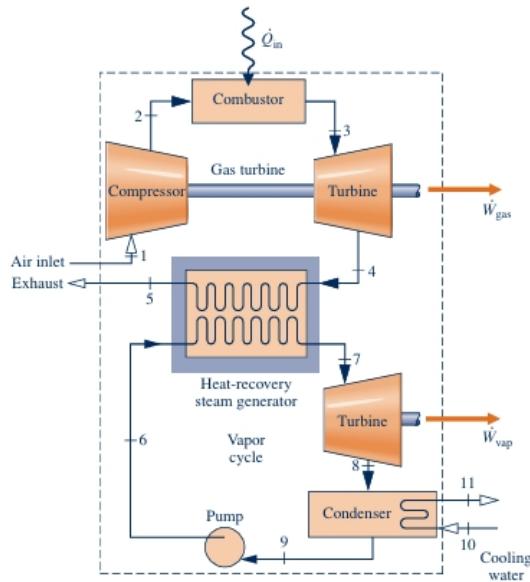
$$\bar{T}_{1,V} < \bar{T}_1 < \bar{T}_{1,p}, \quad \bar{T}_{2,V} = \bar{T}_2 = \bar{T}_{2,p}$$

$$\eta_{t,2,V} < \eta_{t,2} < \eta_{t,2,p}$$

## 12.6 喷气式发动机



## 12.7 燃-蒸联合循环



## 13 Refrigerator and Heat Pump Cycle

- 制冷系数

$$\varepsilon = \frac{q_2}{w}$$

- 供暖系数

$$\varepsilon' = \frac{q_1}{w}$$

- 性能系数

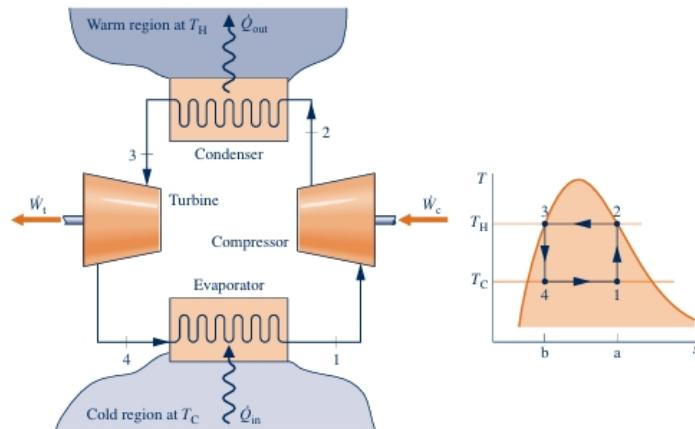
$$COP = \frac{Profit}{Cost}$$

$$COP_{HP} = COP_R + 1$$

- 制冷量

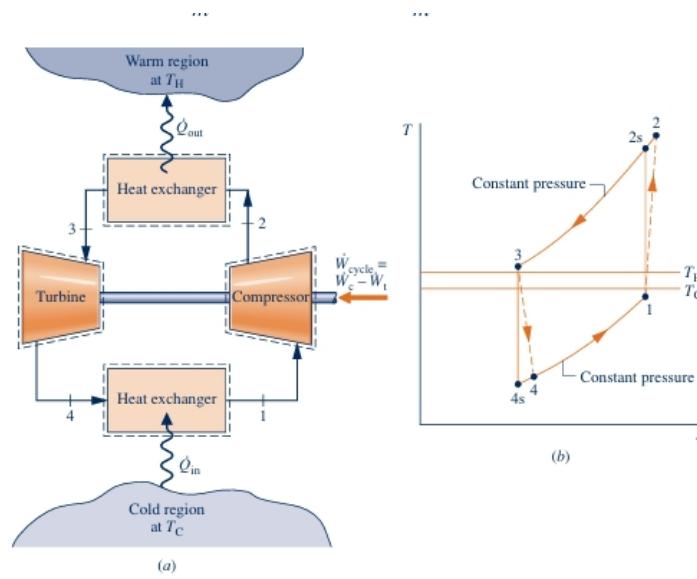
制冷装置每小时从冷源吸收的热量

### 13.1 Reversed Carnot Cycle

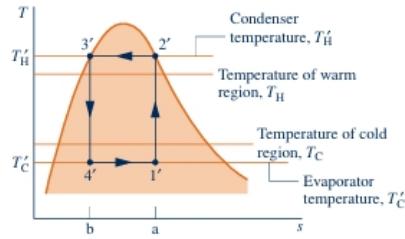


$$\varepsilon_C = \frac{T_2}{T_1 - T_2}$$

### 13.2 空气制冷循环(Brayton Cycle)



空气在冷藏室出口温度理论上等于冷藏室温度，实际上应低于冷藏室温度  
 空气在冷却器出口温度理论上等于冷却水温度（环境室温），实际上应高于冷却水温度



**Fig. 10.2** Comparison of the condenser and evaporator temperatures with those of the warm and cold regions.

(蒸汽制冷Carnot Cycle演示)

#### • 制冷系数

逆卡诺循环温差（高温热源为室温，低温热源为冷藏室温）  
 实际循环温差（高温热源为室温，低温热源为冷藏室进口温度）

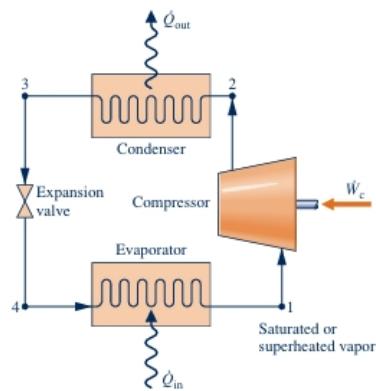
$$\varepsilon = \frac{1}{\frac{\gamma-1}{\pi^{\frac{1}{\gamma}}}-1}$$

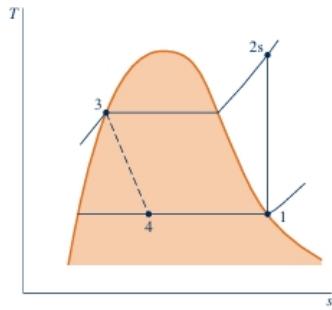
增压比越小，制冷系数越大，但制冷能力变小

#### Regeneration

- 回热前后制冷系数相同
- 相同制冷量及制冷系数，回热后增压比小很多
- 低增压比可减小压缩及膨胀过程中不可逆性的影响，提高制冷装置实际工作有效性

### 13.3 蒸气压缩制冷循环()





- 湿压缩

压缩机吸入蒸气干度较低，易损坏压缩机

- 干压缩

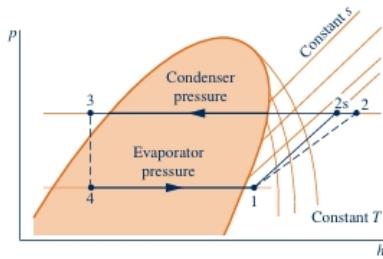
压缩机吸入干饱和蒸气进行压缩

制冷机膨胀过程在节流阀中进行，膨胀功无回收，故耗功量仅为压缩机耗功量

$$\varepsilon = \frac{q_2}{w_T}$$

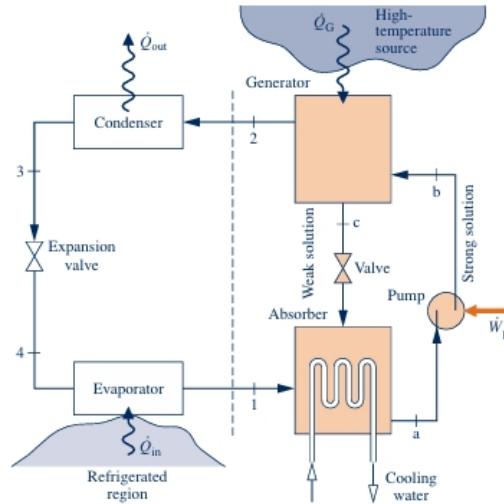
### 13.4 Refrigerant

- 压-焓图

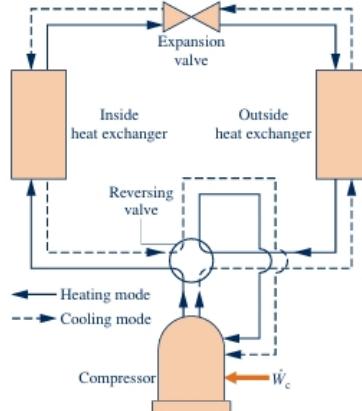


### 13.5 蒸汽喷射制冷循环

### 13.6 吸收式制冷循环



### 13.7 热泵供热循环



## 14 Chemical Reaction

### 14.1 化学平衡条件

(前文已叙述)

- **生成热:**

由单质化合生成1mol化合物的热效应称为该化合物的生成热

- **燃烧热:**

由1mol物质全部氧化燃烧的热效应

- **定容热效应及定压热效应**

$$Q_p - Q_V = RT\Delta n \approx 0$$

## 14.2 Hess's Law

- **生成热计算热效应 (生成物生成热减去反应物生成热)**

$$Q_p = \Delta H = \sum_j (n_j \Delta H_{f,j})_{pr} - \sum_i (n_i \Delta H_{f,i})_{re}$$

- **燃烧热计算热效应 (反应物燃烧热减去生成物燃烧热)**

$$Q_p = \Delta H = \sum_i (n_i \Delta H_{c,i})_{re} - \sum_j (n_j \Delta H_{c,j})_{pr}$$

## 14.3 Kirchhoff's Law

热效应与温度的关系:

$$\frac{dQ_p}{dT} = \sum_j n_j C_{m,p,j} - \sum_i n_i C_{m,p,i}$$

$$\frac{dQ_V}{dT} = \sum_j n_j C_{m,V,j} - \sum_i n_i C_{m,V,i}$$

$$\frac{dQ_x}{dT} = \sum_j n_j C_{m,x,j} - \sum_i n_i C_{m,x,i}$$

## 14.4 理论燃烧温度

$$-\Delta U_b^0 = \Delta U_{pr} = \int_{T_0}^{T_t} \sum_{j,pr} n_j C_{m,V,j} dT$$

$$-\Delta H_b^0 = \Delta H_{pr} = \int_{T_0}^{T_t} \sum_{j,pr} n_j C_{m,p,j} dT$$

## 14.5 反应最大功

定温-定容过程

$$\Delta F = \Delta U + T \left( \frac{\partial \Delta F}{\partial T} \right)_V$$

$$W_{u,V,max} = -\Delta F = -Q_v - T(S_1 - S_2) = -Q_v + T \left( \frac{\partial W_{u,V,max}}{\partial T} \right)_V$$

定温-定压过程

$$\Delta G = \Delta H + T \left( \frac{\partial \Delta G}{\partial T} \right)_p$$

$$W_{u,p,max} = -\Delta G = -Q_p - T(S_1 - S_2) = -Q_p + T \left( \frac{\partial W_{u,p,max}}{\partial T} \right)_p$$

一般过程

$$W_{u,x,max} = -Q_x - T(S_1 - S_2) = -Q_x + T \left( \frac{\partial W_{u,x,max}}{\partial T} \right)_x$$

## 14.6 Chemical Equilibrium

### 化学平衡常数

$$\Delta G = \Delta G^0 + RT \ln \frac{p_C^c p_E^e}{p_A^a p_B^b}$$

$$\Delta G^0 = -RT \ln K_p$$

$$K_p = K_c (RT)^{\Delta n}$$

### 平衡常数与最大有用功

$$W_{u,p,max} = RT (\ln K_p - \ln \frac{p_C^c p_E^e}{p_A^a p_B^b})$$

Le Chatelier, H. Principle

Nernst's Law(The Third law of Thermodynamics)

$$\lim_{T \rightarrow 0} (Q + W_{u,max}) = 0$$

$$\left( \frac{\partial W_{u,max}}{\partial T} \right)_{T \rightarrow 0} = \left( \frac{\partial Q}{\partial T} \right)_{T \rightarrow 0} = 0$$

凝聚态在可逆定温过程中的熵变随绝对温度趋于零而趋于零

- 绝对零度下凝聚物体熵为零
- 绝对零度下凝聚物体热容为零
- 绝对零度无法达到

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