计算物理第八次作业

近代物理系 张浩然 SA21004048

Question ▷ Caldeira-Leggett Model

Solve

$$H = \frac{P^2}{2M} + V(X) + \sum_{j=1}^{N}\left[\frac{p_j^2}{2m_j} + \frac{1}{2}k_j(q_j - X)^2\right],$$

for $N = 100$ and random m_j, k_j's.

Solution

设

$$V(X) = \frac{1}{2}KX^2, \qquad F(X) = -V'(X) = -KX \tag{1}$$

运动方程为

$$\begin{cases} \dot{X} = \dfrac{\partial H}{\partial P} = \dfrac{P}{M}, \\ \dot{P} = -\dfrac{\partial H}{\partial X} = F(X) - \sum_j k_j(X - q_j), \\ \dot{q}_i = \dfrac{\partial H}{\partial p_i} = \dfrac{p_i}{m_i} \\ \dot{p}_i = -\dfrac{\partial H}{\partial q_i} = -k_i(q_i - X) \end{cases} \tag{2}$$

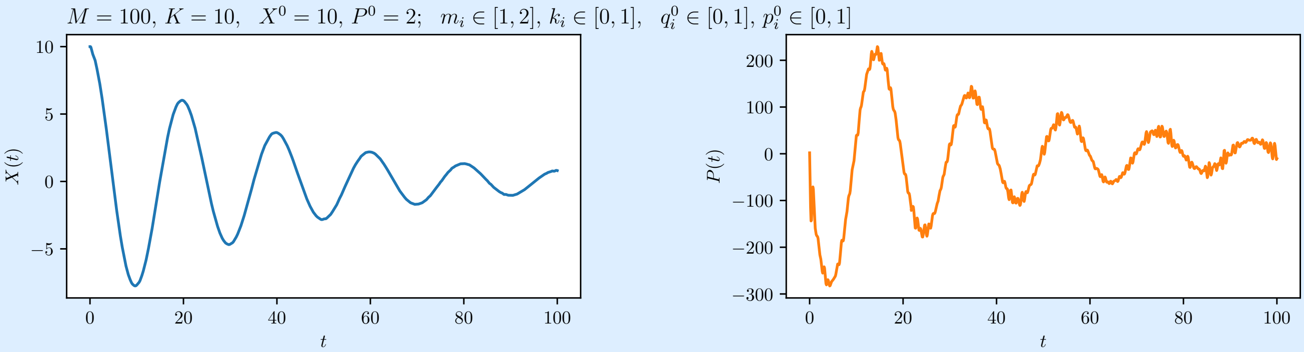
离散化后为(上指标代表时间, 下指标为bath中的谐振子编号)

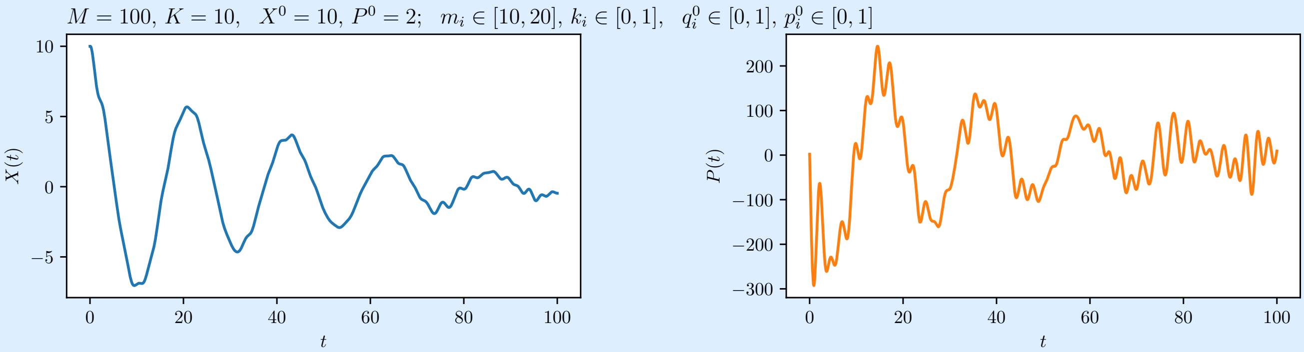
$$\begin{cases} X^{n+1} = X^n + \dfrac{P^n}{M}\mathrm{d}t, \\ P^{n+1} = P^n + \left[F(X^n) - \sum_j k_j(X^n - q_j^n)\right]\mathrm{d}t, \\ q_i^{n+1} = q_i^n + \dfrac{p_i^n}{m_i}\mathrm{d}t, \\ p_i^{n+1} = p_i^n - k_i(q_i^n - X^n)\mathrm{d}t. \end{cases} \tag{3}$$

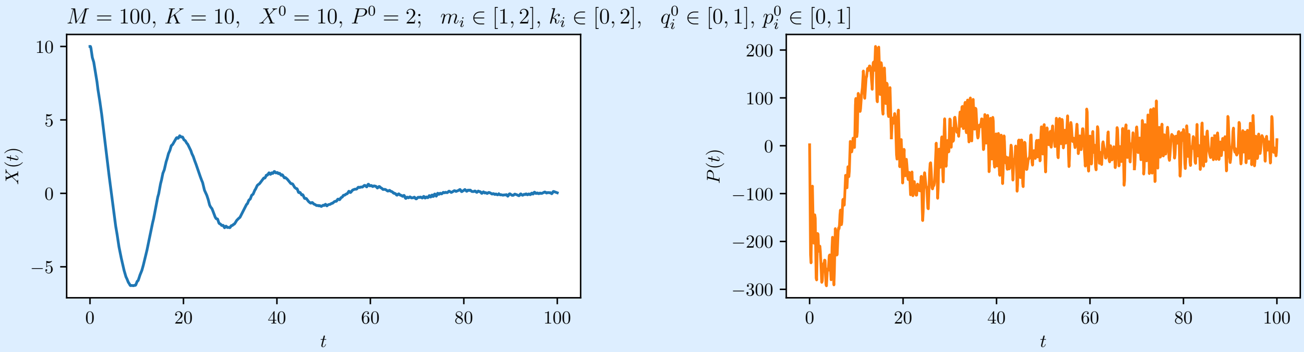
取 $M=100, K=10$, m_i, k_i 及bath中各谐振子的初始位置和动量均为随机数. 使用python进行计算, 代码如下:

```
import numpy as np
from numpy import random
import matplotlib.pyplot as plt

def F(X):
    return - K * X

def evolve(M, K, m, k, X0, P0, q0, p0):
    X = X0; P = P0;
    q = q0; p = p0;
    Xs = [X]; Ps = [P];
    for n in range(t_steps):
        X += P / M * dt
        P += ( F(X) - np.dot(k,X-q) ) * dt
        q += p/m
        p += - k * (q - X)
        Xs.append(X)
        Ps.append(P)
    return Xs, Ps

t = 100; t_steps = 1000;
dt = t / t_steps;
ts = [i*dt for i in range(t_steps+1)]

N = 100
M = 100; K = 10
m = 1+random.rand(N); k = random.rand(N)
X0 = 10; P0 = 2
q0 = random.rand(N); p0 = random.rand(N)

Xs, Ps = evolve(M, K, m, k, X0, P0, q0, p0)
```

取不同的 m_i, k_i 范围, 结果如下图所示.







可以看到, 保持 $k_i \in [0, 1]$ 不变, 而将 $m_i \in [1, 2]$ 变为 $m_i \in [10, 20]$ 时, 所研究系统的振幅衰减速度不变, 但坐标和动量的高频小幅震荡的频率变慢、幅度变大.

而保持 $m_i \in [1, 2]$ 不变, 将 $k_i \in [0, 1]$ 变为 $k_i \in [0, 2]$ 时, 所研究系统的振幅衰减变快.