

1 Data structures

1.1 Table

A Table stores prefixes with the values associated to them. It is represented as follows:

$$Table : T(E, m),$$

where m , the maximum number of entries in the table, is a positive integer and

$$E = \{(P_1, v_1), (P_2, v_2), \dots\}, P_i = p_{i0}p_{i1}\dots$$

is the collection of entries, where the K_i s are keys and the v_i s are the integer values associated with those keys.

2 Functions

2.1 initialize

The initialize function creates a new empty table with a given maximum number of entries. It is specified as follows:

$$\text{initialize}(m) = T(E, m),$$

where $m \in \mathbb{N} \setminus \{0\}$ and $|E| = 0$.

2.2 update

The update function inserts a new key-value pair in a table or updates the value associated with a key, if the key given as argument is already part of the entries in the table. The table given as argument must have space for at least one more entry. The update function must not modify table entries that do not correspond to the key given as argument. It is specified as follows:

$$\text{update}(T(E, m), (K, v)) = T(E', m),$$

where $|E| < m$, $(K, v) \in E'$, $E' \setminus \{(K, v)\} = E \setminus \{(K, -)\}$ and

$$|E'| = \begin{cases} |E| + 1, & \text{if } (K, -) \notin E \\ |E|, & \text{otherwise} \end{cases}$$

2.3 delete

The delete function removes the entry corresponding to a specified key from the table. The key given as argument must belong to the entries of the table given as argument. The delete function must not modify entries in the table that do not correspond to the key given as argument. It is specified as follows:

$$\text{delete}(T(E, m), K) = T(E', m),$$

where $(K, -) \in E$, $E' = E \setminus \{(K, -)\}$ and $|E'| = |E| - 1$.

2.4 lookup

lookup is the main function used to perform the actual longest prefix match. It takes as arguments a non-empty table and a key and returns the value associated with the table entry that best matches the key given as argument. For the purpose of specifying the lookup function, we introduce the matchLength function, defined as follows:

2.4.1 matchLength

The matchLength function takes two keys as parameters and returns the number of symbols in the first key matching the prefix in the second key.

$$\text{matchLength}(K(P, l), K'(P', l')) = i,$$

where i is the largest integer such that $p_1 p_2 \dots p_i = p'_1 p'_2 \dots p'_i$, and $i \leq \min(l, l')$.

With the matchLength function, we can specify the lookup function as follows:

$$\text{lookup}(T(E, m), K) = v,$$

where $|E| > 0$, $(K', v) \in E$, and

$$\forall (K'', -) \in E \text{ such that } K'' \neq K' :$$

$$\text{matchLength}(K, K'') \leq \text{matchLength}(K, K')$$