PROJECT REPORT

MINIMIZING WAREHOUSE SETUP COST USING CVRP

OR6205: DETERMINISTIC OPERATIONS RESEARCH SEC 08

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ABSTRACT

With an ever-increasing demand for vehicle routing in logistics, delivery, and transportation systems, it has been challenging to optimize the delivery routes and costs considering the limited capacity and while multiple companies expecting a specified demand. Regular hindrances could be traversing unwanted routes and increasing the fixed costs and as well the inconsistency of the fixed costs.

Our project aims at finding the optimal location for a warehouse such that all the designated demand from each point is satisfied in a specified amount of time considering the capacity of the vehicle. The model is focused on minimizing the total costs which include reducing the distance traversed by vehicles and costs of setting up the warehouse.
All the required data was collected and computed on Python to find an optimized solution to set up a warehouse.

Table of Contents

I. Introduction **6**

i. Project Statement 6

ii. Background 6

II. Literature Review **7**

i. Logistics in the United States 7

ii. Cost Parameters (Setup, Operation, Transport) 7

iii. Cost Minimization……………………………………………..………………8

III. Preliminary Mathematical Model………………………………………………………**9**

IV. Final Mathematical Model **14**

V. Sensitivity Analysis **22**

VI. Future Scope and Improvement **27**

VII. References **28**

List of Figures

Figure 3.1: Warehouse 1 optimal route.. 12

Figure 3.2: Warehouse 2 optimal route.. 13

Figure 3.3: Warehouse 3 optimal route.. 13

Figure 4.1: Warehouse 1 optimal route and requirements.. 19

Figure 4.2: Warehouse 2 optimal route and requirements.. 19

Figure 4.3: Warehouse 3 optimal route and requirements.. 20

Figure 4.4: LP Formulation for Warehouse Decision……………………………………………..20

Figure 4.5: Optimal Solution…………………………………………………………….………..21

Figure 5.1: New Optimal Solution (Warehouse 1) ………..…………………………….………..23

Figure 5.2: New Optimal Solution (Warehouse 2) ………..…………………………….………..24

Figure 5.3: New Optimal Solution (Warehouse 3) ………..…………………………….………..25

Figure 5.4: New IP Model………..…………………………………………...…….….………..26

Figure 5.5: New Optimal Solution after parameter change………..…………….…...…….….………..26

I. INTRODUCTION

i. Project Statement

The project aims to find an optimal location to set up a warehouse out of the three potential locations in Boston considering fixed locations of demand points i.e., minimizing the setup cost of a warehouse for a delivery company. The demand locations are obtained from a dataset with their coordinates and the required demand at a particular location. The location of the warehouse is determined from one of the three potential locations in a way that the distance traversed by the vehicles is minimal and the setup cost of the warehouse and number of vehicles minimized. Three locations of the warehouses to be determined are chosen by the user by assigning their coordinates. The demands at each location are met by choosing the routes actively such that each route is traversed only once by any one of the vehicles. The distances are computed for decisions by implementing an algorithm using mathematical expressions and computed using Python.

The motivation for this project involves the understanding of the present situation of logistics and delivery systems and the challenges faced primarily traversing a single route by multiple vehicles and failure to meet the demand from a particular point. Often, we see delivery delays from major logistics companies which damages the reputation and customer satisfaction. The research then led to developing a model for an optimized network having multiple demand locations and transporting without exceeding the vehicle capacity.

We then computed the formulated algebraic Integer Linear Programming model and sensitivity analysis is performed to determine the changes in total route distances for each potential warehouse location.

ii. Background

Vehicle Routing problem as stated by Graham. K is a 'set of customers with known location and demand are to be supplied from a depot by delivery vehicles of known capacity subject to all customer demand being met, vehicle capacity not being exceeded and total trip length not exceeding some specified level. The routes begin and end at the depot'. Although various computational algorithms have been employed in the past few decades, a reliable network is difficult to determine, or the problem complexity is too expensive to solve. The recent technological advancements have made it less challenging for real-world applications with different sets of data to be considered with considerable changes in mathematical programming. The method of capacitated vehicle routing has been implemented in the last few years, but we look forward to making advancements by minimizing the setup costs and trying to operate on multiple routes.

II. LITERATURE REVIEW

i. Logistics in the United States

According to the research firm Armstrong & Associates Inc, the US logistics market was closing to $2 trillion which attributed to about 10% of the total GDP in the year 2019 and is estimated to raise over $12 trillion by 2025. Boston is one of the most expensive and densely populated cities in the US, the decision to set up warehouses and determining their optimal location is an important financial decision. Achieving this while being able to satisfy all demand points is a complicated problem that we aim to solve.

With the increasing demand, it has become important to add supply warehouses to the logistics supply chain while maintaining an optimal network. Our focus of this project is to find a suitable location to construct a warehouse considering all the transport and setup costs and constraints. A total of 3 potential warehouse locations were chosen and a set of 10 demand points was fixed for this project.

ii. Cost Parameters

The characteristics and structure of a vehicle delivery problem must be learned before studying the design. The process of a vehicle routing problem involves multiple stages like setting up a warehouse in an economical location, calculating the distance traveled by each delivery vehicle and cost for each route, and as well the fixed costs to procure the required number of vehicles.

Setup Costs

For the project, to choose an economical location to set up a warehouse, we have three potential locations for warehouses based on the geographical locations of the demand points. The warehouse locations are considered are as follows (co-ordinates).

WAREHOUSE	LATITUDE	LONGITUDE
1	42.340335	-71.131830
2	42.710020	-71.740000
3	41.814841	-71.508300

Table 2.1: Warehouse Locations

A set of assumptions have been made for the initial setup costs of the warehouses and they are $150,000, $180,000, $120,000 for warehouse 1, warehouse 2, and warehouse 3 respectively. A distance algorithm has been deployed using Python to compute routes from between warehouse and demand locations and the transport costs are computed accordingly.

Transportation and Operational Costs

The model has 11 demand locations obtained from a dataset along with their demand as follows.

	latitude	longitude	demand
0	42.368197	-71.019922	13
1	41.927897	-70.141155	14
2	42.306619	-71.473414	10
3	41.982628	-71.790283	16
4	42.792484	-71.404969	15
5	42.722427	-71.368172	17
6	42.752336	-71.705351	15
7	41.677468	-71.308487	15
8	41.463087	-70.631341	18
9	42.840534	-70.956215	11

Table 2.2: Demand Points Location

iii. Cost Minimization

All the data was used to form an Integer Linear Programming (ILP) model to determine the minimum total cost from the network considering all the assumptions. The warehouse location has been selected based on the cost per distance traversed algorithm. The distances traversed by the vehicles to satisfy all the demands are obtained as 688.77 km, 755.93 km, and 777.61 km for warehouses 1,2,3 respectively.

III. PRELIMINARY MATHEMATICAL MODEL

The aim of this project is to determine the optimal set of routes from several warehouses to a set of demand points for satisfying the delivery demands while minimizing the total cost.

The most common routing problem to find the shortest route of one vehicle is the Traveling Salesman Problem (TSP). Since we have more than one vehicle and demand points, the computation for the optimal path using the Traveling Salesman Problem becomes difficult. So, we decided to use a generalized version of the TSP that is Vehicle Routing Problem (VRP) for the preliminary model.

In-Vehicle Routing Problem, we are given a set of warehouses and demand points and we assume a fixed cost of traversing each warehouse and demand point. The objective is to find the path that reaches all the locations at minimum cost. VRP is called an NP-hard problem in mathematical terms because the solution time increases exceptionally with size. The number of solutions for the VRP is n! where n is the number of nodes or demand points. Some common objectives of VRP are:

- Minimize the overall transportation cost based on the total distance traveled and the fixed costs allied with the used vehicles and drivers.
- Minimize the total number of vehicles required to serve all the demand points.

i. Assumptions and Justifications

Some Assumptions of Vehicle Routing Problem are as follows:

- Vehicles start at a particular warehouse and finish at the same warehouse.
- The number of vehicles that can be activated to service at a certain warehouse is limited.
- Every demand point can be visited once by a specific vehicle.
- Any vehicle from any warehouse can satisfy the demand of each demand point.
- The Vehicle fleet is homogenous.

ii. Vehicle Flow Formulation

The following notations and parameters were used to develop the model.

INDEX SETS

W	Set of Warehouses	{W = 0,1,2}
P	Set of Demand Points	{P=1, 2,…20}
V	Set of total nodes	{V= 0,1,2,3…..22}, indexed by i and j

PARAMETERS AND DECISION VARIABLES

c_{ij}	Represents the cost of going from node i to node j
K	Represents the number of available vehicles
r(S)	Corresponds to the minimum number of vehicles needed to serve set N

BINARY VARIABLES

x_{ij} =1 if the arc going from i to j is considered as part of the solution and

=0 otherwise

OBJECTIVE FUNCTION

The objective function was to minimize the cost.

$$\min \sum_{i \in V} \sum_{j \in V} c_{ij} x_{ij}$$

FUNCTIONAL CONSTRAINTS

This objective function is subject to some constraints. These constraints are discussed below.

Constraint 1: This constraint indicates that exactly one arc enters, and one leaves each node associated with a demand point.

$$\sum_{i \in V} x_{ij} = 1 \quad \forall j \in V \setminus \{0\} \tag{1}$$

Constraint 2: This constraint also indicates that exactly one arc enters and exactly one leaves each vertex associated with a demand point.

$$\sum_{j \in V} x_{ij} = 1 \quad \forall i \in V \setminus \{0\} \tag{2}$$

Constraint 3: This constraint indicates that the number of vehicles leaving the warehouse is the same as the number incoming.

$$\sum_{i \in V} x_{i0} = K \tag{3}$$

Constraint 4: This constraint is same as constraint 3.

$$\sum_{j \in V} x_{0j} = K \tag{4}$$

Constraint 5: It indicates routes must be connected and the demand on each route must not exceed the vehicle capacity.

$$\sum_{i \notin S} \sum_{j \in S} x_{ij} \geq r(S), \ \ \forall S \subseteq V \setminus \{0\}, S \neq \emptyset \tag{5}$$

Constraint 6: Non-negativity constraint for binary variables.

$$x_{ij} \in \{0, 1\} \quad \forall i, j \in V \tag{6}$$

iii. Model Input Data and Functions

Python is a programming language that can be used to solve highly complex optimization problems. The Delivery Route Optimization Project was developed using Python. The input data used in VRP is made up and not obtained from anywhere. The following are the inputs for the preliminary model.

- Distance Matrix: The distance matrices created for the 3 warehouses represent the array of distances between the depot and 20 demand points.
- Num_vehicles: This variable represents the number of vehicles in the fleet.
- Depot: It represents the index of the depot, the location where all vehicles start and end their routes.

The following functions are used in the preliminary model:

- A distance callback function is used to returns the distances between the nodes and pass them to the solver. It also sets the edge costs, which define the cost of travel, to be the distances of the edges.
- SetGlobalSpanCostCoefficient function sets a large coefficient (100) for the global span of the routes, which in this is the maximum of the distances of the routes. This makes the global span the major factor in the objective function, so the program minimizes the length of the longest route.

iv. Preliminary Result Display and Analysis

The figures below show the output of the Vehicle Routing Problem. It shows the optimal route of all the 4 vehicles from the warehouse to demand points.

```
In [79]: get_optimal_routes(1)

Route for vehicle 0:
 0 ->  5 ->  1 ->  8 ->  16 ->  10 -> 0
Distance of the route: 8m

Route for vehicle 1:
 0 ->  19 ->  7 ->  15 ->  6 ->  13 ->  12 -> 0
Distance of the route: 10m

Route for vehicle 2:
 0 ->  9 ->  11 ->  14 -> 0
Distance of the route: 8m

Route for vehicle 3:
 0 ->  2 ->  20 ->  18 ->  3 ->  4 ->  17 -> 0
Distance of the route: 10m

Maximum of the route distances: 10m
Total distance: 36m
```

Figure 3.1: Warehouse 1 optimal route

```
In [80]: get_optimal_routes(2)

Route for vehicle 0:
 0 ->  9 ->  11 ->  23 ->  2 ->  6 ->  14 ->  7 ->  15 ->  5 -> 0
Distance of the route: 13m

Route for vehicle 1:
 0 -> 0
Distance of the route: 0m

Route for vehicle 2:
 0 ->  19 ->  8 ->  1 ->  22 ->  17 ->  21 -> 0
Distance of the route: 13m

Route for vehicle 3:
 0 ->  16 ->  10 ->  24 ->  4 ->  20 ->  18 ->  3 ->  13 ->  12 -> 0
Distance of the route: 13m

Maximum of the route distances: 13m
Total distance: 39m
```

Figure 3.2: Warehouse 2 optimal route

```
In [81]: get_optimal_routes(3)

Route for vehicle 0:
 0 ->  5 ->  1 ->  8 ->  16 ->  10 -> 0
Distance of the route: 8m

Route for vehicle 1:
 0 ->  19 ->  7 ->  15 ->  6 ->  13 ->  12 -> 0
Distance of the route: 10m

Route for vehicle 2:
 0 ->  9 ->  11 ->  14 -> 0
Distance of the route: 8m

Route for vehicle 3:
 0 ->  2 ->  20 ->  18 ->  3 ->  4 ->  17 -> 0
Distance of the route: 10m

Maximum of the route distances: 10m
Total distance: 36m
```

Figure 3.3: Warehouse 3 optimal route

v. Limitations of the Preliminary Model

- In the preliminary model, different constraints of VRP like vehicle capacity or time window are not taken into consideration.
- Vehicle Routing problems can sometimes take more time to solve because the length of time grows exponentially with the size of the problem. As a result, the output for VRP is sometimes not optimal.

IV. FINAL MATHEMATICAL MODEL

The capacitated vehicle routing problem (CVRP) is a combination of combinatorial optimization and integer programming that aims to find a set of routes at a minimal cost (beginning the route at the warehouse) so that the known demand of all nodes is fulfilled. Each node is visited only once, by only one vehicle, and each vehicle has a limited capacity. In a way, we can say that It generalizes the well-known traveling salesman problem (TSP).

i. Assumptions and Justifications

Here, the scope of our problem statement has increased from just finding the shortest path and minimizing the number of vehicles, to pick up or deliver the items with the least cost, while never exceeding the capacity of the vehicles. For that, we set a demand point number, vehicle number, demand of each point, and capacity of vehicles and made the following assumptions:

- Number of demand points: 10 (plus one depot)
- Number of vehicles: 4
- Demand of each point: random values between 10 to 20
- Vehicle capacity: 50 (assumed that all vehicles have the same capacity)

ii. Variable Definition, Objective Function and Constraints

INDEX SETS

G	Graph showing location and route of vehicles	G={V,E}
V	A collection of all nodes, where:	
	V=0 represents the Warehouse and	V={0, 1,…, n}
	V=1…n represents the Demand Points	
E	Set of arcs connecting each node	E_{ij}= {i,j}
K	Set of vehicles	K={1, 2…, |K|}

PARAMETERS

d_i	Demand of point i
Q	Maximum vehicle capacity
c_{ij}	Travel cost (Distance) between demand point i and j

DECISION VARIABLES

x^k_{ij} A decision variable that indicates weather the vehicle k has taken route e_{ij}.

OBJECTIVE FUNCTION

The objective function was to minimize the total cost where:

Total Cost (Z)= Transportation Cost (from source i destination j)

$$\min \sum_{k\in K} \sum_{(i,j)\in E} c_{ij} x_{ij}^k$$

That is, minimize the travel cost (sum of traveling distance) of all vehicles.

CONSTRAINTS

This objective function was subject to many constraints. These constraints are discussed below:

Constraint 1: Ensures that we only make one visit per vehicle per demand point.

$$\sum_{k\in K} \sum_{i\in V, i\neq j} x_{ij}^k = 1 \qquad \forall j \in V \setminus \{0\}$$

……..(1)

Constraint 2: This constraint indicates that all vehicles depart from warehouse only.

$$\sum_{j\in V\setminus\{0\}} x_{0j}^k = 1 \qquad \forall k \in K$$

……..(2)

Constraint 3: This constraint is for making sure that the number of vehicles coming in and out of each demand point is the same.

$$\sum_{i\in V, i\neq j} x_{ij}^k - \sum_{i\in V} x_{ji}^k = 0 \qquad \forall j \in V, \forall k \in K$$

……..(3)

Constraint 4: The delivery capacity of each vehicle should not exceed the maximum capacity.

$$\sum_{i\in V}\sum_{j\in V\setminus\{0\},i\neq j} q_j x_{ij}^k \leq Q \qquad \forall k \in K$$

……..(4)

Constraint 5: Removal of subtours. This means that in the final output, each route is traversed exactly once by only one vehicle.

$$\sum_{k\in K}\sum_{(i,j)\in S,i\neq j} x_{ij}^k \leq |S|-1 \qquad S \subseteq V \setminus \{0\}$$

……..(5)

Constraint 6: Non negativity constraint.

$$x_{ij}^k \in \{0,1\} \qquad \forall k \in K, \forall(i,j) \in E$$

……..(6)

$di \geq 0$ and integer ……..(7)

$Q \geq 0$ and integer ……..(8)

$Cij \geq 0$ and integer ……..(9)

The second step is deciding on the most economical warehouse location based on the distances computed using CVRP model.

Assumptions & Justifications:

- We consider the warehouse fixed cost as 150000, 180000 and 120000 respectively for each warehouse
- Cost of operating delivery vehicle as $2 per KM which Includes fuel and toll costs.
- Cost of procuring vehicle Is kept same for all vehicles as $2000.

Variable Definition, Objective Function and Constraints

INDEX SETS

W	Warehouse	W = {1,2,3}

PARAMETERS

v_i	Vehicle cost for warehouse i	$i \in W$

c_i Fixed cost of setting up warehouse i $i \in W$

DECISION VARIABLES

x_i A decision variable that indicates whether the warehouse is built.

y_i A decision variable that indicates whether vehicles for warehouse i are to be procured.

OBJECTIVE FUNCTION

The objective function was to minimize the total cost i.e. fixed costs and variable costs (Calculated from the distance and number of vehicles from part 1):

$$\min \quad z = \sum_{i=1}^{3} c_i x_i + \sum_{i=1}^{3} v_i \cdot y_i$$

That is, minimize the travel cost (sum of traveling distance) of all vehicles.

CONSTRAINTS

This objective function was subject to many constraints. These constraints are discussed below:

Constraint 1: If warehouse Is built then the vehicles need to be procured.

$$x_i - y_i = 0$$

Constraint 2: Ensuring only one warehouse Is made.

$$\sum_{i=1}^{3} x_i = 1$$

Constraint 3: Setting the operation budget which should be 10% of the capital cost.

$$\sum_{i=1}^{3} v_i y_i \leq (0.1) c_i$$

Constraint 4: Binary Variable

$xi \in \{0,1\}$

$yi \in \{0,1\}$

iii. Model Input Data and Functions

Our data file contains the data which was collected through a through quantitative research. Initially, the model was tested on a sample data consisting of 9 demand points and one warehouse. Various scenarios were considered for various demand points and how to satisfy them. The preliminary model file was then analysed on each scenario and checked for its robustness. After completing this phase of analysis, the model was tested on the actual assumed set of data. Finally, after a bunch of trial and errors, our final data file now consists of 10 demand points. Our model comes up with optimal solution that gives us the shortest distance and tells us which warehouse to build from 3 potential locations.

iv. Result Display and Analysis

Shortest route for warehouse locations and the vehicle requirements

```
%%time
W1_vehicles,W1_route,W1_plot = get_warehouse_optimal_dist(df1,vehicle_count=4,vehicle_capacity=50)
```

```
Vehicle Requirement: 3
Total route distance kms: 688.7710718382834
Wall time: 55 s
```

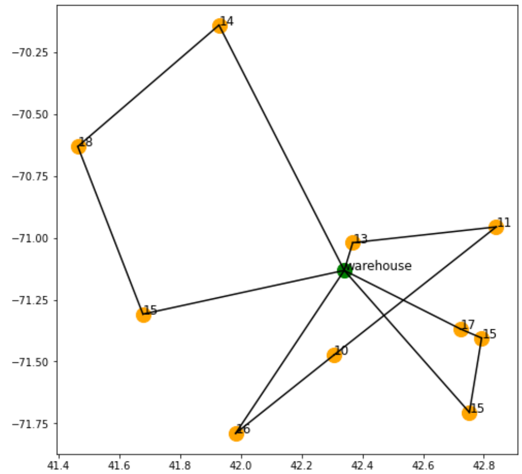

Figure 4.1: Warehouse 1 optimal route and requirements

```
%%time
W2_vehicles,W2_route,W2_plot = get_warehouse_optimal_dist(df2,vehicle_count=4,vehicle_capacity=50)
```

```
Vehicle Requirement: 3
Total route distance kms: 755.9301714050924
Wall time: 49.5 s
```

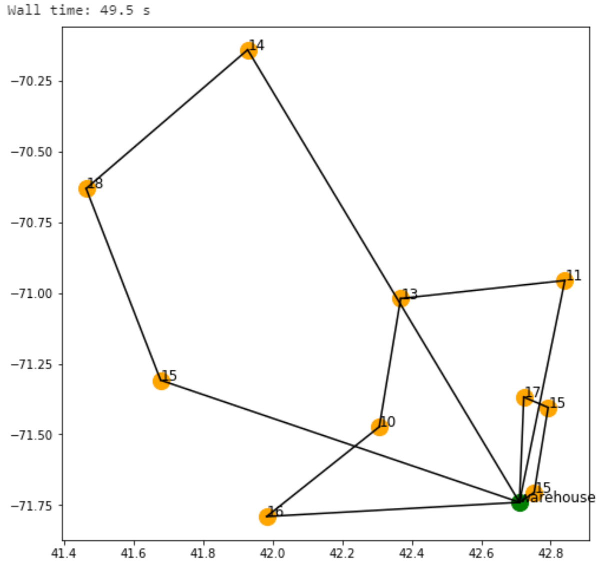

Figure 4.2: Warehouse 2 optimal route and requirements

```
%%time
W3_vehicles,W3_route,W3_plot = get_warehouse_optimal_dist(df3,vehicle_count=4,vehicle_capacity=50)
```

```
Vehicle Requirement: 3
Total route distance kms: 777.6152120033222
Wall time: 59.6 s
```

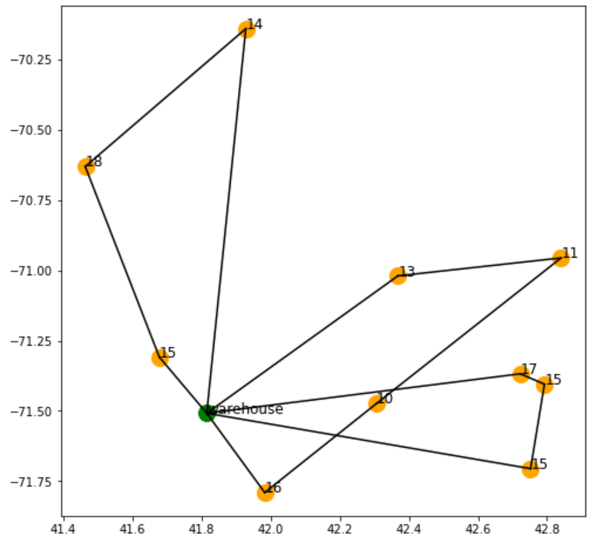

Figure 4.3: Warehouse 3 optimal route and requirements

```
model
```

```
Warehouse_Decision_and_Cost:
MINIMIZE
150000*X_1 + 180000*X_2 + 120000*X_3 + 7377.54*Y_1 + 7511.86*Y_2 + 7555.24*Y_3 + 0.0
SUBJECT TO
Delivery_Vehicles_Purchased_for_Warehouse0: X_1 - Y_1 = 0

Delivery_Vehicles_Purchased_for_Warehouse1: X_2 - Y_2 = 0

Delivery_Vehicles_Purchased_for_Warehouse2: X_3 - Y_3 = 0

Only_one_warehouse_is_made: X_1 + X_2 + X_3 = 1

Operations_budget_wrt_capital1: 7377.54 Y_1 <= 15000

Operations_budget_wrt_capital2: 7511.86 Y_2 <= 18000

Operations_budget_wrt_capital3: 7555.24 Y_3 <= 12000

VARIABLES
0 <= X_1 <= 1 Integer
0 <= X_2 <= 1 Integer
0 <= X_3 <= 1 Integer
0 <= Y_1 <= 1 Integer
0 <= Y_2 <= 1 Integer
0 <= Y_3 <= 1 Integer
```

Figure 4.4: LP Formulation for Warehouse Decision

Value of decision variable helps us determine that warehouse 3 was the most economical to build.

```python
status = pulp.LpStatus[model.status]
status
```

```
'Optimal'
```

```python
model.objective.value()
```

```
127555.24
```

```python
# Objective Func and DV values
print("Total Cost:", model.objective.value())
# Decision Variables

for v in model.variables():
    try:
        print(v.name,"=", v.value())
    except:
        print("error couldnt find value")
```

```
Total Cost: 127555.24
X_1 = 0.0
X_2 = 0.0
X_3 = 1.0
Y_1 = 0.0
Y_2 = 0.0
Y_3 = 1.0
```

Figure 4.5: Optimal Solution

v. Limitations of the Preliminary Model

- The fixed costs of building each warehouse are decided beforehand.
- The vehicle costs consist of 2 factors. Cost to procure and cost to operate.
- The cost per km is assumed which is a rough estimation of the fuel and toll costs only.
- Vehicle range for CVRP is not considered due to computation restrictions.
- The model is robust enough to handle only 10 demand points and 1 warehouse at a time as the computation time increases exponentially.
- The distance computed in CVRP is using haversine which is the straight-line distance and not actual path that the vehicle will take.

V. SENSITIVITY ANALYSIS

i. Selection of sensitive parameters

Sensitivity analysis is useful because it improves the prediction of the model, by studying qualitatively and quantitatively the model response to change in input variables, or by understanding the phenomenon studied by the analysis of interactions between variables.

Various algorithms and heuristics were applied for solving our capacitated vehicle routing problem. Once an optimal solution for the problem was obtained, we made changes in the parameter values for our final model and observed the outputs. Changes in parameter values resulted in a new optimal solution which was different from the initial optimal solution. We also observed the quantitative impact of parameter changes on the value of the objective function.

In our problem, each route has a start instant at the warehouse and this instant is subject to capacity constraints for handling the route. Hence, we had to choose one or more of the following parameters for sensitivity analysis:

- Increasing/Decreasing the number of warehouses or demand points.
- Increasing/Decreasing the number of vehicles
- Increasing/ Decreasing the capacity of each vehicle.

But during heuristic evaluations, we found out that changing the number of warehouses or increasing the number of demand points over 9 or 10, increased the complexity and the calculation time exponentially. In-order to avoid that, we chose to perform sensitivity analysis on the other two parameters. Therefore, our sensitivity analysis is based on a limited number of loading vehicles and a limited size of loading capacity.

After we receive our optimal solution, we can get account of the total number of vehicle requirements and the total route distance in Kms. Hence, we start our sensitivity analysis by:

1. Changing the number of vehicles: Here, we impose a restriction on the number of vehicles to be used for transportation of goods. Hence, the total number of loading activities should not exceed the number of vehicles available to us. In this model, we lose the transportation capacity of our model.

2. Changing the capacity of vehicles : Here, we impose a restriction on the transportation capacity of each vehicle. Hence, the total amount of amount loaded on each vehicle should not exceed the total transportation capacity of the vehicle. In this model, increasing or decreasing the transportation capacity of our vehicles creates a significant impact on the number of vehicle required and the distance each vehicle has to travel. For example, if we increase the capacity of each vehicle, we need less vehicles to satisfy our demand points, but if we decrease the capacity of each vehicle, we need more vehicles to satisfy our

demand points. Based on the total number of vehicles we are using, we might also have to change the routes and this impacts the distance travelled in Kms.

ii. Visualization of sensitivity analysis

As discussed earlier, our model output starts by telling us where our warehouse should be located in the first place. Once we fix our warehouse and calculate the shortest route distance from a warehouse to fulfil the requirements of all the demand points, we can decide the number of vehicles required and also the capacity of each vehicle. Therefore, we performed the following sensitivity analysis trials:

1. Setting vehicle count to 4 and vehicle capacity to 50 units each for warehouse 1.

   i. Input code: Here, we try the optimal distance from our warehouse 1 (df1) to all our demand points, set the vehicle count to 4 and vehicle capacity to 50.

```
%%time
W1_vehicles,W1_route,W1_plot = get_warehouse_optimal_dist(df1,vehicle_count=4,vehicle_capacity=50)
```

   ii. Output received:

```
Vehicle Requirement: 3
Total route distance kms: 688.7710718382834
Wall time: 1min
```

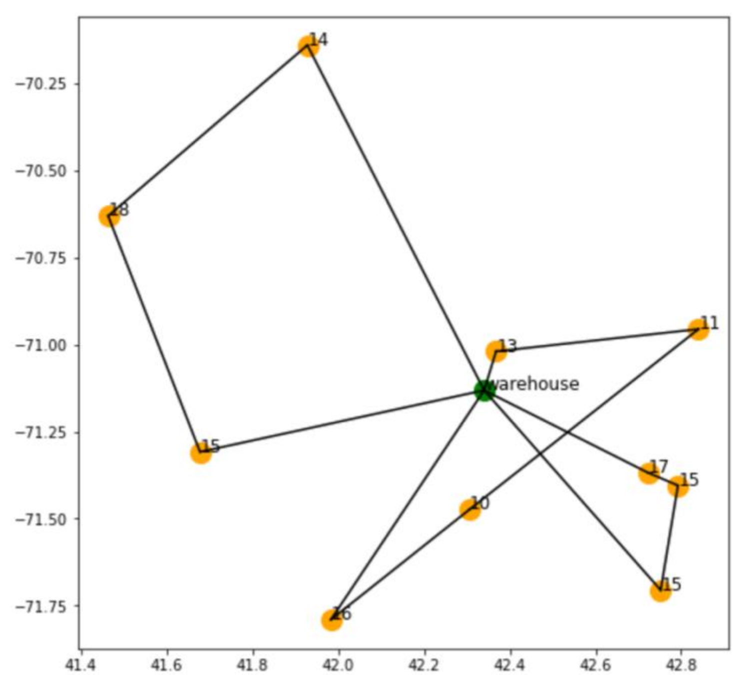

Figure 5.1: New Optimal Solution (Warehouse 1)

iii. Inference: From this, we observe that increasing vehicle count and reducing the capacity will help us reduce the total route distance in Kms.

2. Decreasing vehicle count and increasing vehicle capacity for warehouse 2.

   i. Input code: Here, we try the optimal distance from our potential warehouse 2 (df2) to all our demand points, set the vehicle count to 2 and vehicle capacity to 120.

```
%%time
W2_vehicles,W2_route,W2_plot = get_warehouse_optimal_dist(df2,vehicle_count=2,vehicle_capacity=120)
```

   ii. Output received:

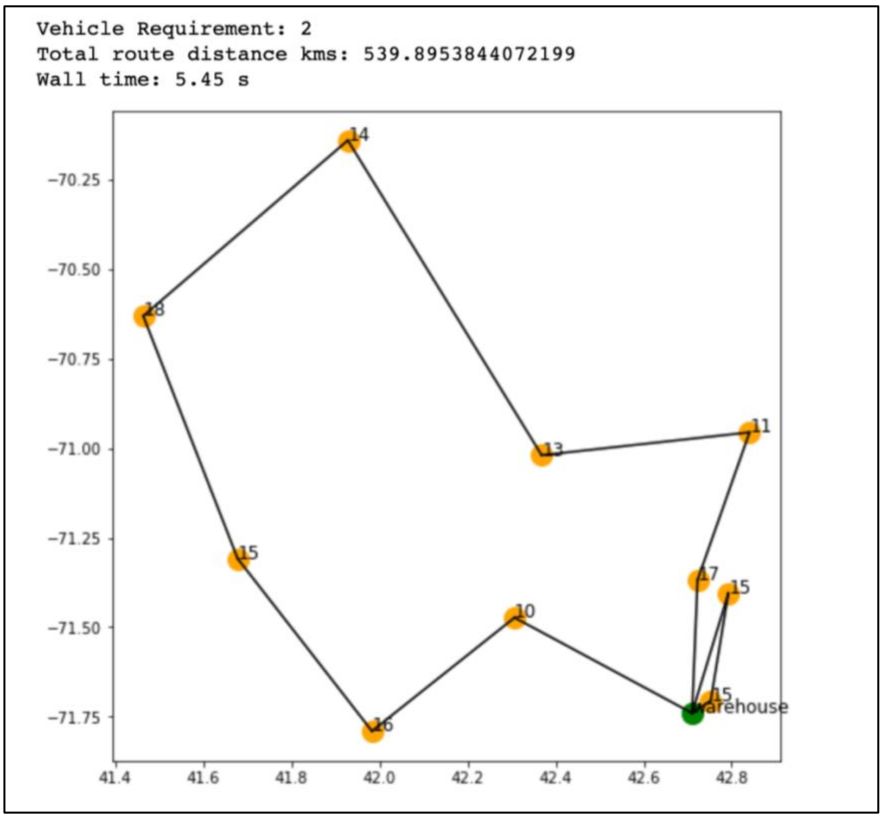

Figure 5.2: New Optimal Solution (Warehouse 2)

   iii. Inference: From this, we infer that reducing vehicle count and increasing the capacity will further reduce the total route distances in Kms.

3. Increasing vehicle count and decreasing vehicle capacity for warehouse 3 (df3).

    1. Input code: Here, we try the optimal distance from our warehouse(df3) to all our demand points, set the vehicle count to 6 and vehicle capacity to 30.

```
%%time
W3_vehicles,W3_route,W3_plot = get_warehouse_optimal_dist(df3,vehicle_count=6,vehicle_capacity=30)
```

    2. Output received:

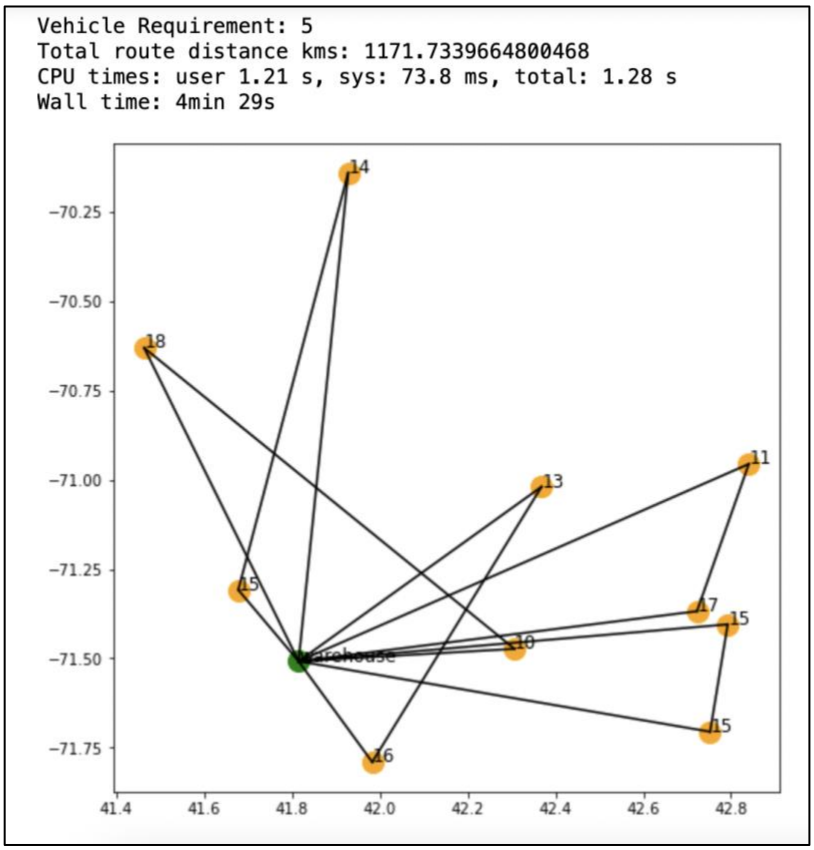

Figure 5.3: New Optimal Solution (Warehouse 3)

    3. Inference: From the result we can observe that increasing the number of vehicles and reducing the vehicle capacity increased the total route distance, but it also increases the calculation complexity.

iii. Results of Sensitivity Analysis

The impact of changing the parameter values modified our final model to:

```
Warehouse_Decision_and_Cost:
MINIMIZE
150000*X_1 + 180000*X_2 + 120000*X_3 + 7377.54*Y_1 + 5079.8*Y_2 + 12343.46*Y_3 + 0.0
SUBJECT TO
Delivery_Vehicles_Purchased_for_Warehouse0: X_1 - Y_1 = 0

Delivery_Vehicles_Purchased_for_Warehouse1: X_2 - Y_2 = 0

Delivery_Vehicles_Purchased_for_Warehouse2: X_3 - Y_3 = 0

Only_one_warehouse_is_made: X_1 + X_2 + X_3 = 1

Operations_budget_wrt_capital1: 7377.54 Y_1 <= 15000

Operations_budget_wrt_capital2: 5079.8 Y_2 <= 18000

Operations_budget_wrt_capital3: 12343.46 Y_3 <= 12000

VARIABLES
0 <= X_1 <= 1 Integer
0 <= X_2 <= 1 Integer
0 <= X_3 <= 1 Integer
0 <= Y_1 <= 1 Integer
0 <= Y_2 <= 1 Integer
0 <= Y_3 <= 1 Integer
```

Figure 5.4: New IP Model

The value of the objective function was also changed to 157377.54.The decision variable change resulted in building warehouse 1 instead of 3 for the new parameters.

```
Total Cost: 157377.54
X_1 = 1.0
X_2 = 0.0
X_3 = 0.0
Y_1 = 1.0
Y_2 = 0.0
Y_3 = 0.0
```

Figure 5.5: New Optimal Solution after parameter change

VI. FUTURE SCOPE & IMPROVEMENT

For this project we were successfully able to select the best location for warehouse with the least setup cost and operation costs. However, there were a few assumptions that were made which can be eliminated and the model can estimate the costs more accurately.

We are considering cost per km which includes the cost of fuel and tolls, there are other added costs that have not been considered such as the cost of hiring the driver which can be make the model more practical.

The first part of the project calculates the optimal route all vehicles are supposed to take for a potential warehouse location. The distance between the warehouse and the demand points were calculated using the haversine formula. This distance is the straight-line distance, however in the future iterations of this model can include an integration with Google Maps API which can calculate the actual distance based on the roads the trucks can take.

There are a lot of factors that were overlooked when considering the fixed and variable costs in order to limit the complexity of the project. Factors such as hiring and salary of employees, cost of electricity, rental and utility deposits, security, insurance costs, taxes etc. This project acts as a good foundation to build a comprehensive model in the future which considers all the costs and aids the decision making to find the best location for setting up a warehouse.

VII. REFERENCES

1. Fisher, M., Vehicle routing. In Ball M., Magnanti T., Monma C., and Nemhauser G. (Eds.), Network Routing, Handbooks in Operations Research and Management Science, 8, 1–33, Amsterdam, Elsevier Science, 1995.
2. Petch R. J. and Salhi S., “A multi-phase constructive heuristic for the vehicle routing problemwith multiple trips,” Discrete Applied Mathematics, 133, 69-92, 2003
3. Schwind M, Kunkel M. Collaborative Optimization of Last Mile Networks for Courier, Express and Parcel Delivery Services. 2010,1961-1973.
4. Google OR Tools - https://developers.google.com/optimization
5. PuLP Library for Python - https://coin-or.github.io/pulp/