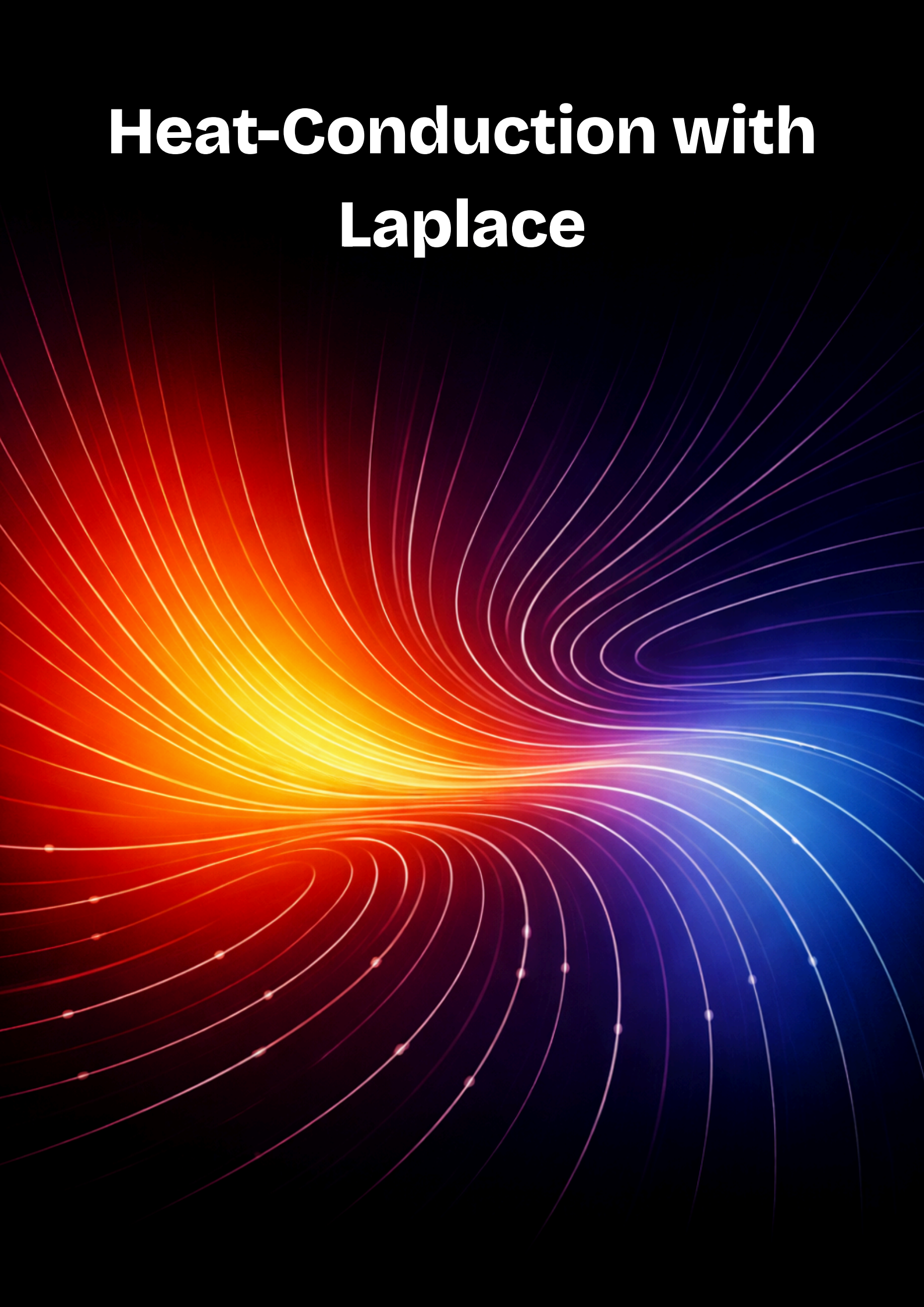
Heat-Conduction with Laplace



LAPLACE EQUATION

This equation represents steady-state heat conduction in a two-dimensional domain with no internal heat generation.

- The temperature field does not change with time.
- Heat diffusion in the x and y directions is perfectly balanced.
- The system has reached thermal equilibrium.

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

T : Temperature (°C or K)

x, y : Spatial coordinates (m)

FINITE DIFFERENCE DISCRETIZATION

This equation is the finite difference approximation of the Laplace equation at an interior grid point.

- The temperature at a grid point is influenced by its four nearest neighbors.
- Temperature curvature in both spatial directions must balance to zero.

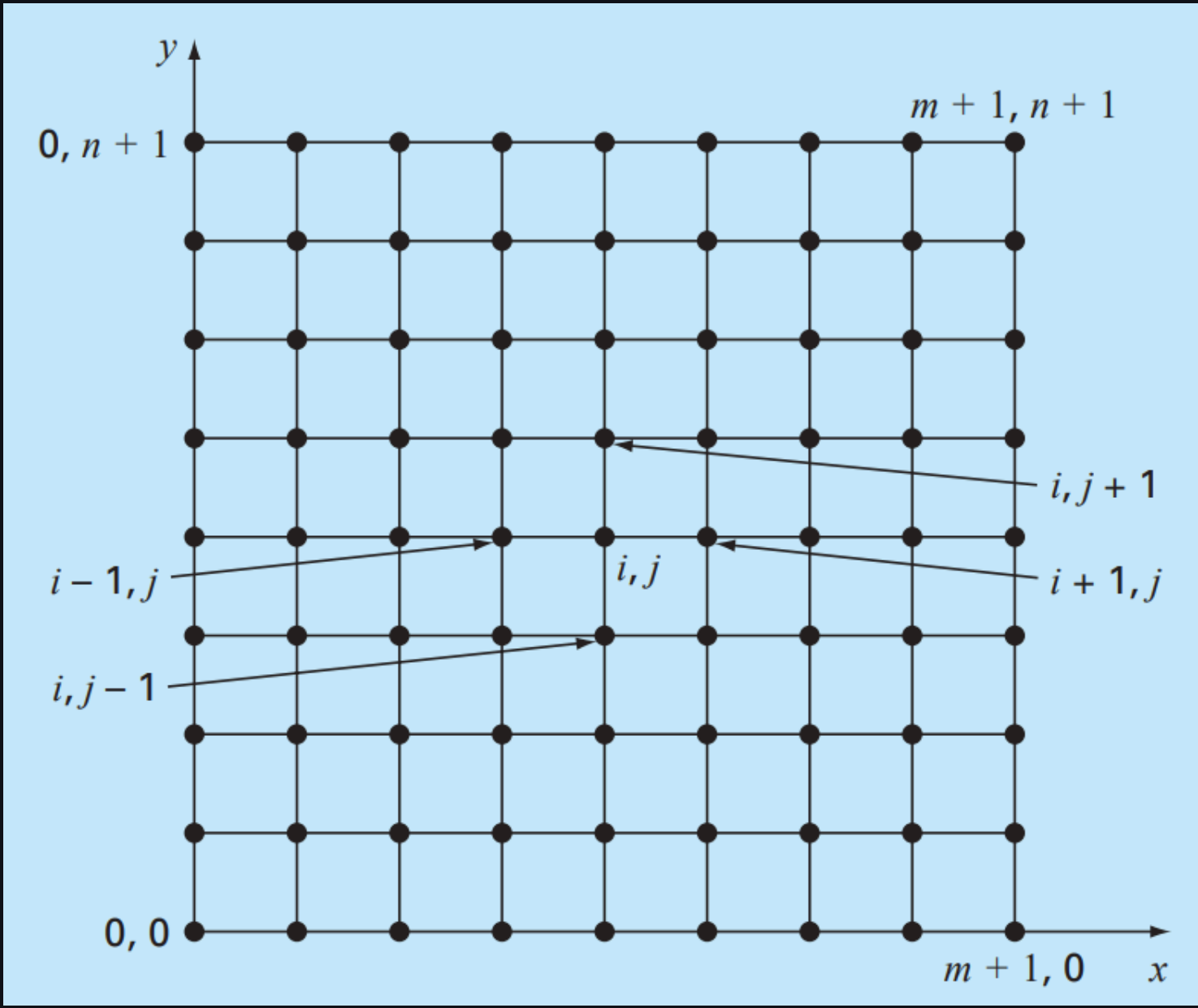
$$\frac{T_{i+1,j} - 2T_{i,j} + T_{i-1,j}}{\Delta x^2} + \frac{T_{i,j+1} - 2T_{i,j} + T_{i,j-1}}{\Delta y^2} = 0$$

$T_{i,j}$: Temperature at grid point (i, j)

Δx : Grid spacing in the x-direction (m)

Δy : Grid spacing in the y-direction (m)

i, j : Grid indices



A grid used for the finite-difference solution of elliptic PDEs in two independent variables such as the Laplace equation.

Refrence:
Chapra, S. C. ., & Canale, R. P. . (2010). Numerical methods for engineers. McGraw-Hill Higher Education.

GAUSS-SEIDEL

This equation updates the temperature at each grid point using the Gauss–Seidel iterative method.

- The new temperature is computed as a weighted average of neighboring temperatures.
- Newly updated values are used immediately, accelerating convergence.

$$T_{i,j}^{(k+1)} = \frac{\left(T_{i+1,j}^{(k)} + T_{i-1,j}^{(k+1)}\right)\Delta y^2 + \left(T_{i,j+1}^{(k)} + T_{i,j-1}^{(k+1)}\right)\Delta x^2}{2\left(\Delta x^2 + \Delta y^2\right)}$$

k : Iteration index

$T^{(k+1)}$: Updated temperature value

$T^{(k)}$: Temperature from the previous iteration

DIRICHLET BOUNDARY CONDITION

This boundary condition specifies a fixed temperature at the boundary.

- The boundary is maintained at a constant temperature.
- Commonly represents contact with a heat reservoir or controlled surface.

$$T = T_{\text{bc}}$$

T_{bc} **: Prescribed boundary temperature**

NEUMANN BOUNDARY CONDITION

This boundary condition specifies zero heat flux across the boundary.

- No heat flows in or out of the boundary.
- Represents an adiabatic or insulated surface.

$$\frac{\partial T}{\partial n} = 0 \quad \Rightarrow \quad T_{\text{boundary}} = T_{\text{adjacent}}$$

$\frac{\partial T}{\partial n}$ **: Temperature gradient normal to the boundary**

T_{boundary} **: Temperature at the boundary node**

T_{adjacent} **: Temperature at the adjacent interior node**

CONVERGENCE CRITERIA

This criterion determines when the iterative solution has converged.

- The solution is considered stable when temperature changes between iterations become negligible.
- Ensures numerical accuracy and stability.

$$\max \left| T^{(k+1)} - T^{(k)} \right| < \varepsilon$$

$$\varepsilon = 10^{-6}$$

ε	**: Convergence tolerance**
$\max \ \lvert\cdot\rvert$	**: Maximum absolute difference over all grid points**

TUTORIAL

1

First, choose whether each boundary (top, bottom, left, and right) uses a Dirichlet or Neumann boundary condition (°C for Dirichlet, W/m^2 for Neumann

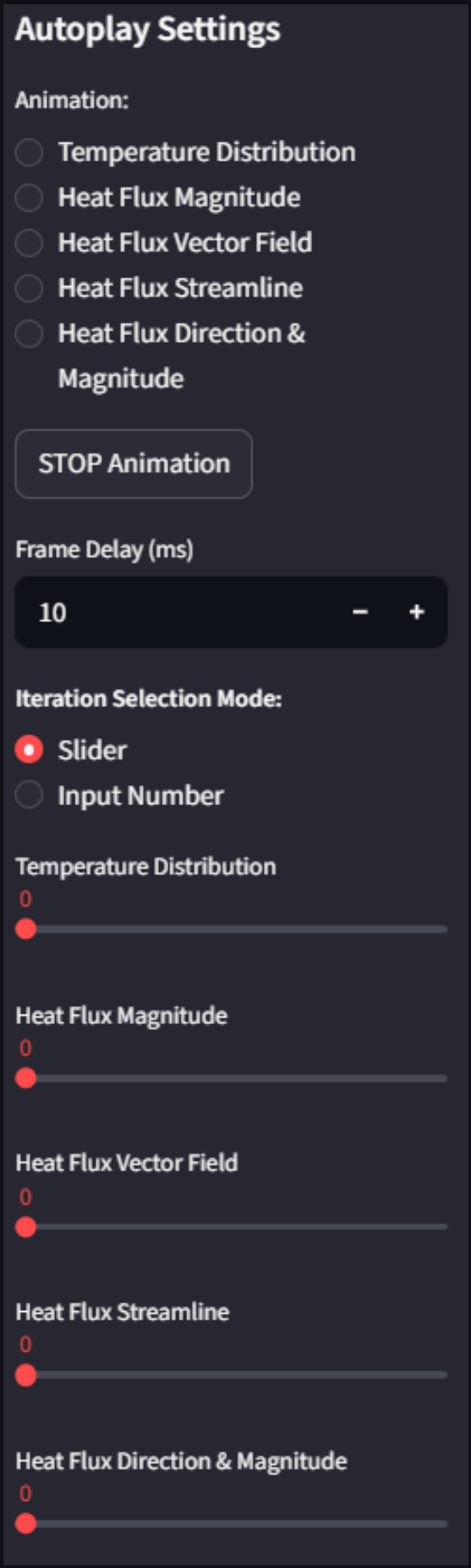
2

Next, enter the material properties and the simulation iteration parameters. The list of required property values is as follows:

- K = Thermal conductivity (W/m·K)
- Lx = Length in the x-direction (m)
- Ly = Length in the y-direction (m)
- Nx = Number of grid points in the x-direction
- Ny = Number of grid points in the y-direction

Once all values have been entered, press the “Run Simulation” button to execute the simulation.

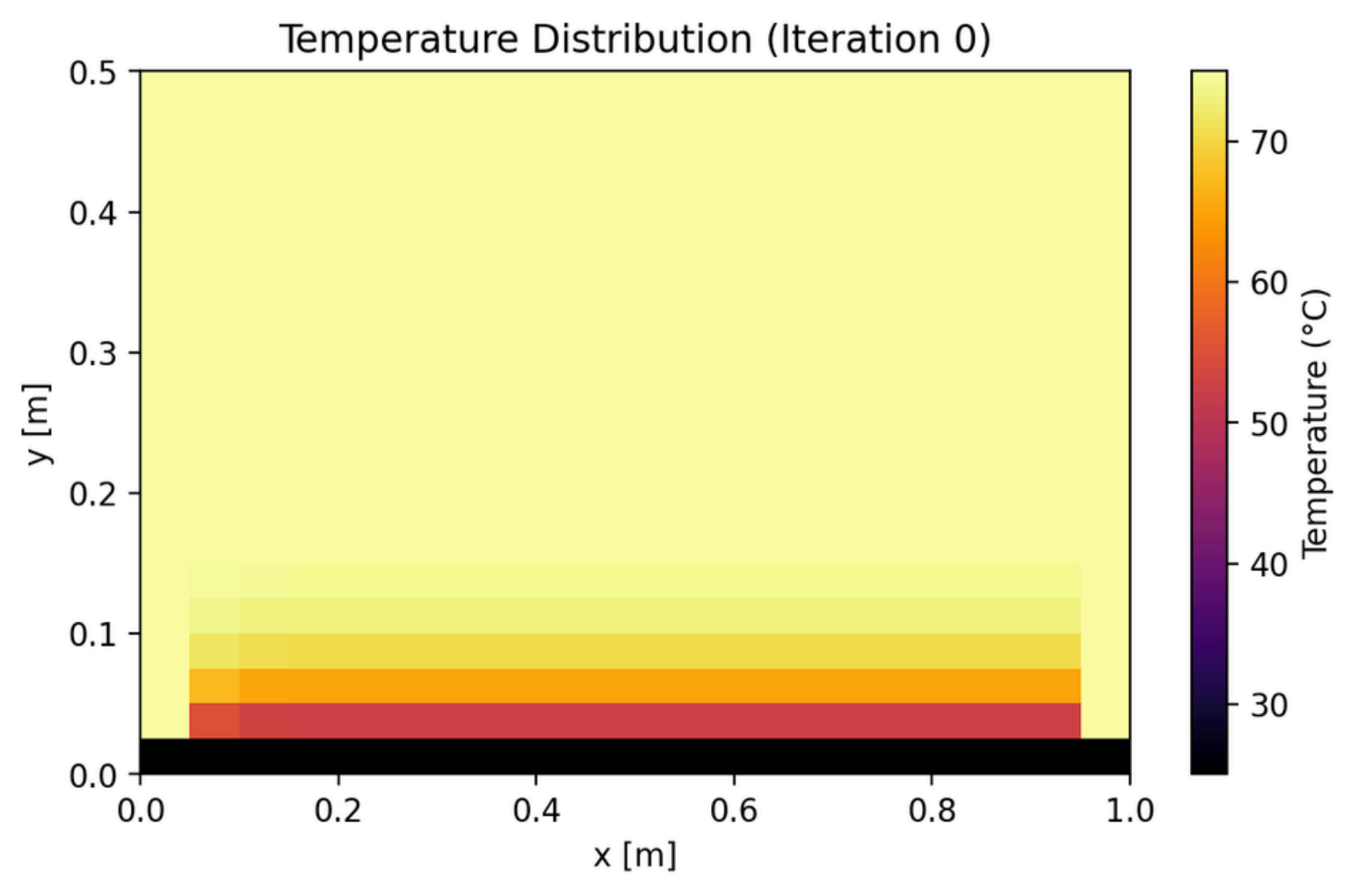
RESULT



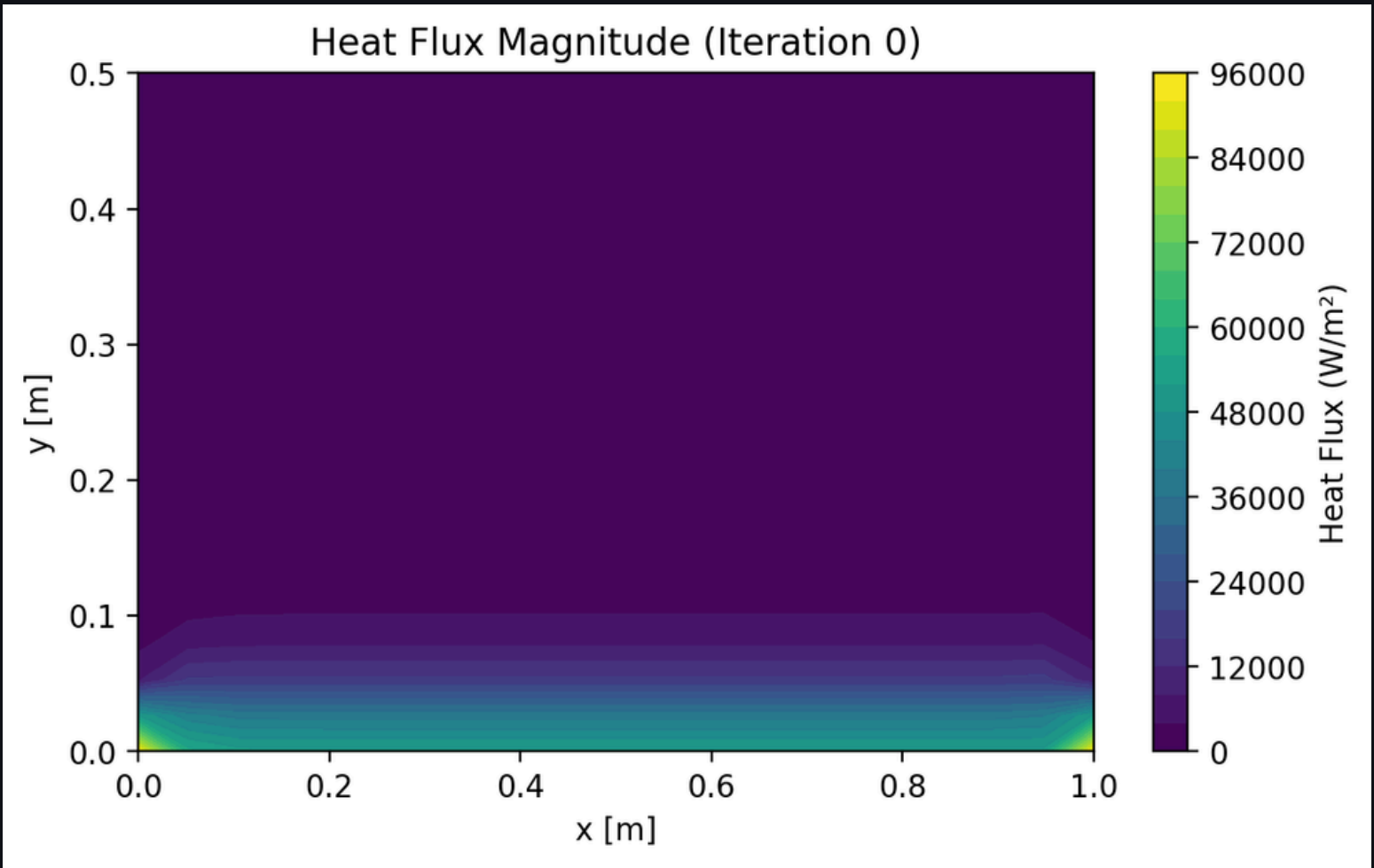
After the simulation converges, the results are displayed in the Results section. Users can visualize the temperature distribution and heat flux behavior using several animation options, including temperature distribution, heat flux magnitude, vector field, streamlines, and combined direction–magnitude plots.

The animation speed can be adjusted using the Frame Delay setting, and playback can be stopped at any time using the STOP Animation button. Specific iterations can be selected using either the Slider or Input Number mode for detailed analysis.

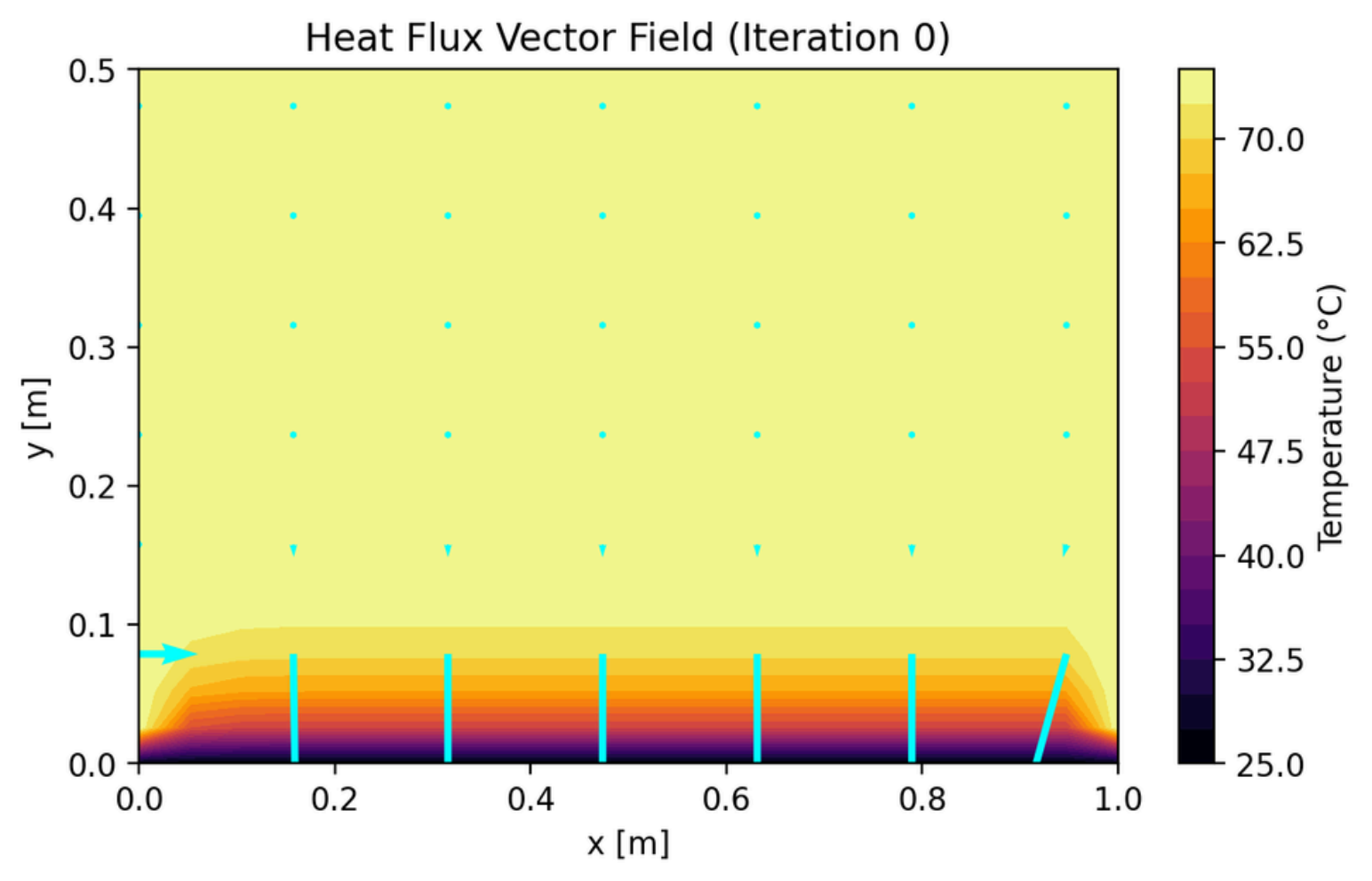
TEMPERATURE DISTRIBUTION



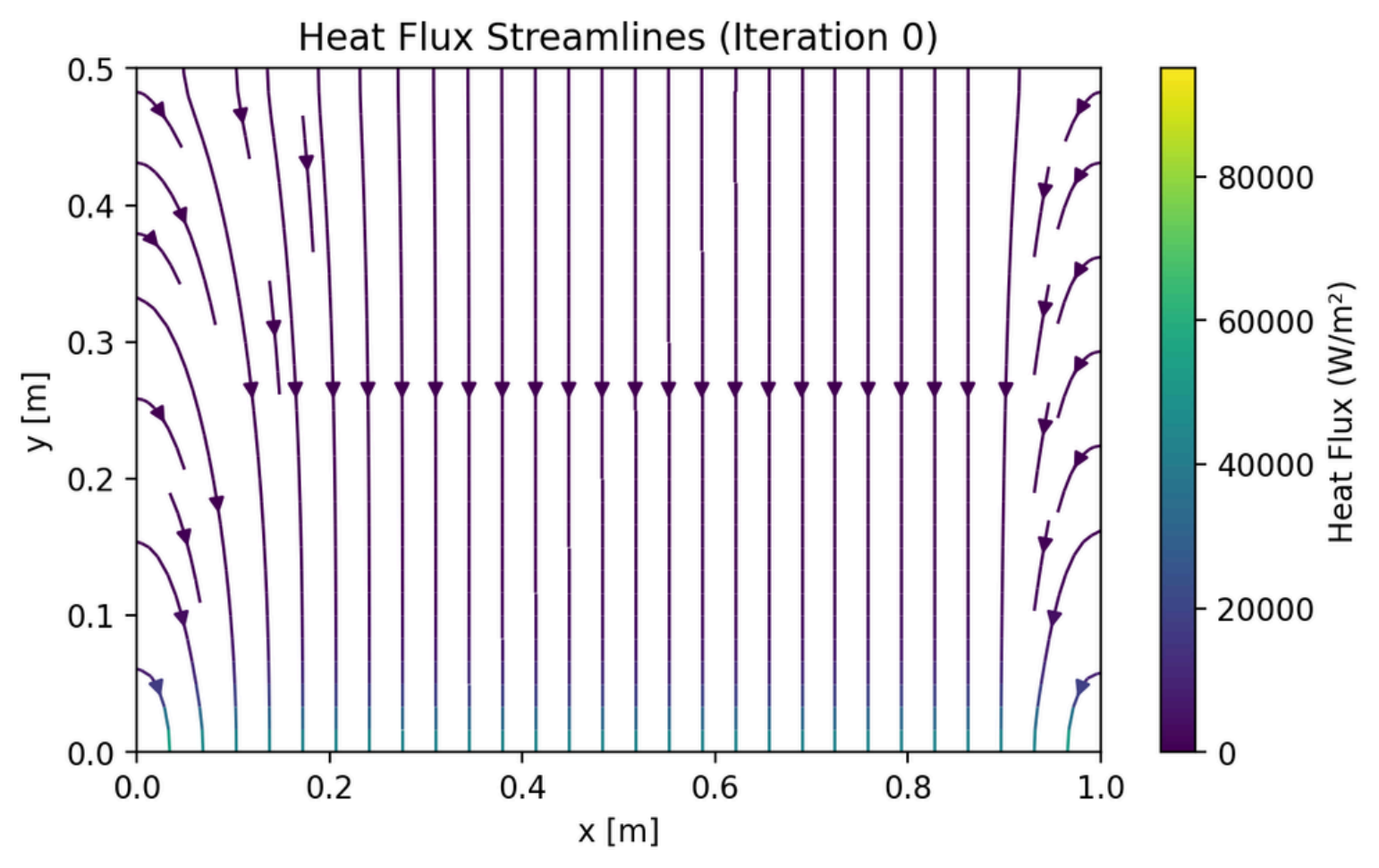
HEAT FLUX MAGNITUDE



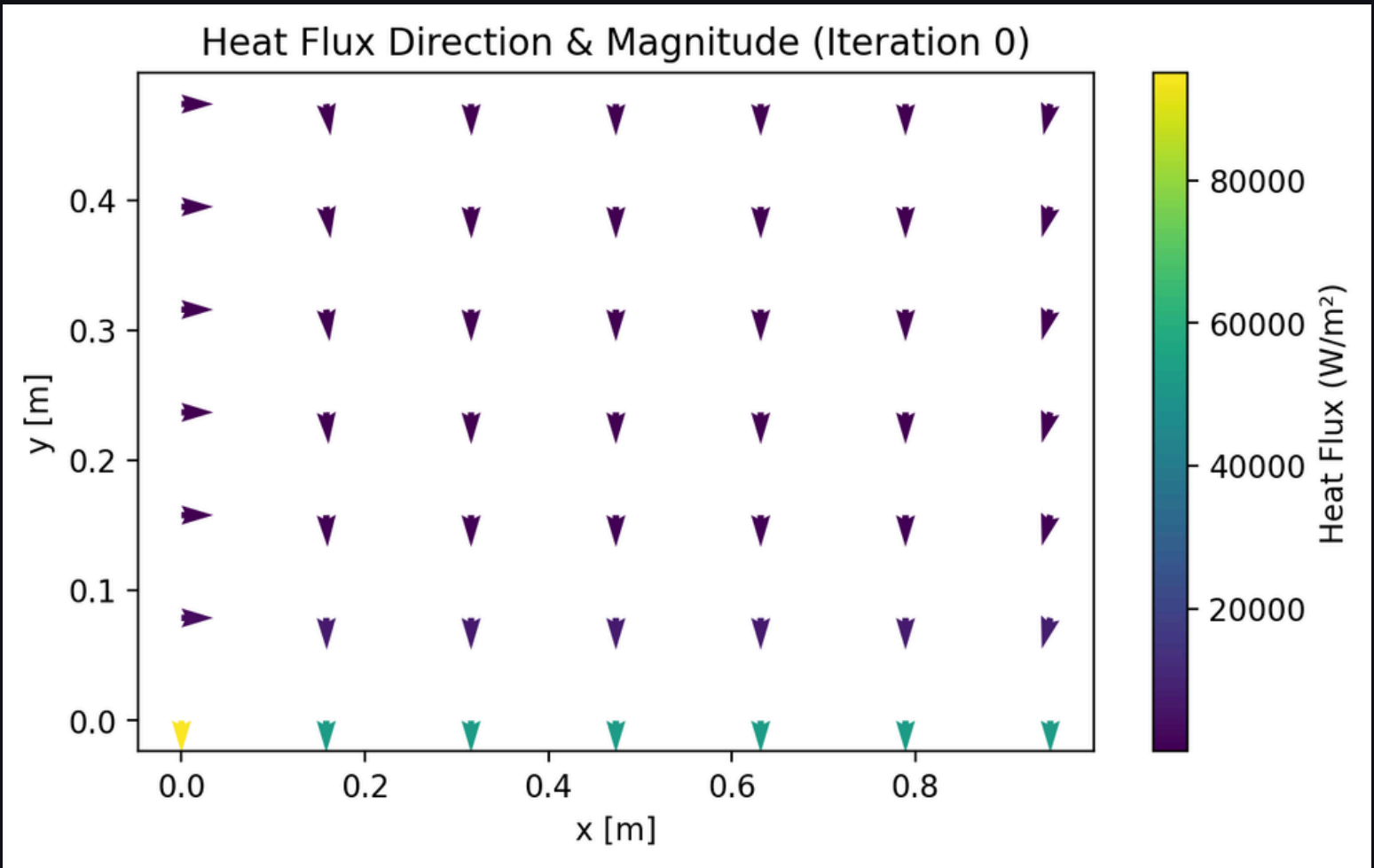
HEAT FLUX VECTOR FIELD



HEAT FLUX STREAMLINES



HEAT FLUX DIRECTION & MAGNITUDE



TEMPERATURE DISTRIBUTION FOR EACH SECTION AT CONVERGENCE

	x=0.00m	x=0.05m	x=0.11m	x=0.16m	x=0.21m	x=0.26m	x=0.32m	x=0.37m	x=0.42m	x=0.47
y=0.00m	25	25	25	25	25	25	25	25	25	
y=0.03m	25	25	25	25	25	25	25	25	25	
y=0.05m	25	25	25	25	25	25	25	25	25	
y=0.08m	25	25	25	25	25	25	25	25	25	
y=0.11m	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.00
y=0.13m	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.00
y=0.16m	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.00
y=0.18m	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.00
y=0.21m	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.00
y=0.24m	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.0001	25.00

HEAT FLUX DATA FRAME (X-DIRECTION) AT CONVERGENCE

	x=0.00m	x=0.05m	x=0.11m	x=0.16m	x=0.21m	x=0.26m	x=0.32m	x=0.37m	x=0.42m	x=0.47
y=0.00m	0	0	0	0	0	0	0	0	0	
y=0.03m	0.00008	0.00008	0.00008	0.00007	0.00007	0.00006	0.00006	0.00005	0.00005	0.000
y=0.05m	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.00009	0.000
y=0.08m	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0001	0.00
y=0.11m	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002	0.0002	0.0002	0.0002	0.00
y=0.13m	0.0004	0.0004	0.0004	0.0003	0.0003	0.0003	0.0003	0.0002	0.0002	0.00
y=0.16m	0.0005	0.0005	0.0004	0.0004	0.0004	0.0003	0.0003	0.0003	0.0003	0.00
y=0.18m	0.0005	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0003	0.0003	0.00
y=0.21m	0.0006	0.0006	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0003	0.00
y=0.24m	0.0007	0.0006	0.0006	0.0006	0.0005	0.0005	0.0004	0.0004	0.0004	0.00

HEAT FLUX DATA FRAME (Y-DIRECTION) AT CONVERGENCE

	x=0.00m	x=0.05m	x=0.11m	x=0.16m	x=0.21m	x=0.26m	x=0.32m	x=0.37m	x=0.42m	x=0.47
y=0.00m	-0.0303	-0.0301	-0.0299	-0.0298	-0.0297	-0.0295	-0.0294	-0.0293	-0.0292	-0.02
y=0.03m	-0.0301	-0.0299	-0.0298	-0.0296	-0.0295	-0.0294	-0.0292	-0.0291	-0.029	-0.0
y=0.05m	-0.0296	-0.0294	-0.0293	-0.0292	-0.029	-0.0289	-0.0288	-0.0287	-0.0286	-0.02
y=0.08m	-0.0289	-0.0288	-0.0286	-0.0285	-0.0284	-0.0282	-0.0281	-0.028	-0.0279	-0.02
y=0.11m	-0.0281	-0.0279	-0.0278	-0.0276	-0.0275	-0.0274	-0.0273	-0.0272	-0.0271	-0.0
y=0.13m	-0.027	-0.0268	-0.0267	-0.0266	-0.0265	-0.0263	-0.0262	-0.0261	-0.0261	-0.0
y=0.16m	-0.0258	-0.0256	-0.0255	-0.0254	-0.0252	-0.0251	-0.025	-0.0249	-0.0249	-0.02
y=0.18m	-0.0243	-0.0242	-0.0241	-0.024	-0.0239	-0.0238	-0.0237	-0.0236	-0.0235	-0.02
y=0.21m	-0.0228	-0.0226	-0.0225	-0.0224	-0.0223	-0.0222	-0.0221	-0.022	-0.022	-0.02
y=0.24m	-0.021	-0.0209	-0.0208	-0.0207	-0.0206	-0.0205	-0.0205	-0.0204	-0.0203	-0.02

THANK YOU