Utilize the Limited Frequency Resources Efficiently

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Introduction

For Requirements A, B, and C, we find an optimal assignment. We find efficient strategies for assignment and the spans for $k = 2$ (9), $k = 3$ (12), and $k \geq 4$ ($2k + 7$) for the case of two levels of interference.

For Requirement D, to check all possible assignments is impractical. Instead, we devise a heuristic algorithm to produce a near-optimal assignment and span.

We also consider other important factors, such as cell-splitting and duopoly.

Analysis of the Problem

We first define

- **successful assignment**: An assignment of channels that satisfies all constraints.
- **span**: The minimum, over all successful assignments, of the largest channel used.

Our goal is to find the span and a successful assignment under given constraints:

- **Constraint 1:** The channels of transmitters within distance $2s$ differ by at least k.
- **Constraint 2:** No transmitters within $4s$ of each other can use the same channel.

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Assumptions

- The region is partitioned into a grid of regular hexagons (called *cells*), and each transmitter is located at the center of a hexagon.
- The frequency spectrum is divided into regularly spaced channels represented by positive integers.
- A channel can be used by more than one transmitter, provided interference is avoided.
- Each transmitter is assigned a single channel.
- Each transmitter covers its entire cell, and the effect of landform can be ignored.

Table 1.

Notation used.

s	length of a side of a hexagon
$A, B, \ldots$	cells or transmitters
$a, b, \ldots$	channels
$\omega_i s$	distance constraint of the ith level of interference
k_i	frequency constraint of the ith level of interference
k	frequency constraint of first level of interference in Requirement C
p, q	shift parameters
$d(A, B)$	distance between cells A and B

Model Design and Results

Requirement A

We find an optimal result via a recursive backtracking algorithm. Our goal is to see if we can use the integers up through n to satisfy the constraints. We loop from $n = 1$ to $n =$ number of cells (giving every cell a distinct channel must work). We order the cells and try each in turn to see if it can be numbered with an integer between 1 and n: if so, we proceed to the next cell; if not, we renumber the previous cell. If all cells can be numbered, then we have a successful assignment with n channels; otherwise, n channels are not enough.

Proposition 1. *For requirement A, the span is 9.*

Proof: Every cell is adjacent to six others, and these cells are all within $4s$ distance of each other; so according to **Constraint 2**, these seven cells must be assigned different channels. Furthermore, if one cell has channel m, the six adjacent cells cannot have $m + 1$ or $m - 1$, according to **Constraint 1**. Hence, the span must be at least 9. Our algorithm finds a successful assignment with 9 channels (**Figure 1**), so 9 is the span. □

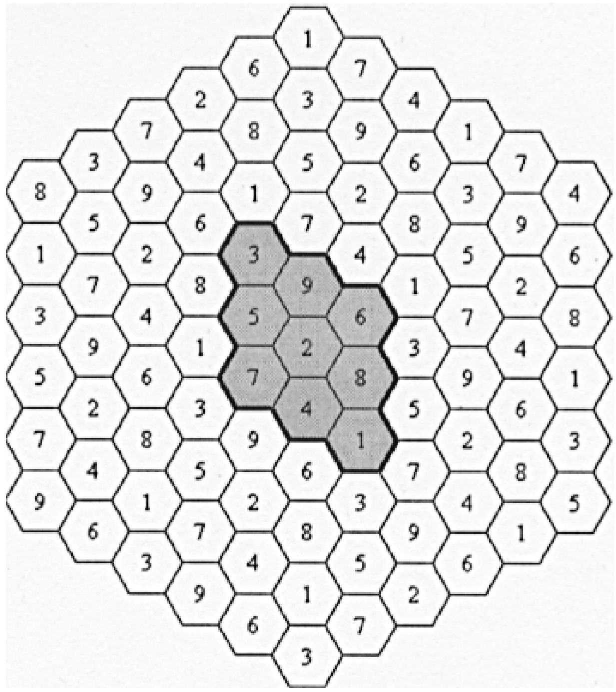

Figure 1. For $k = 2$, the span is 9.

Requirement B

The shaded part in **Figure 1** can be expanded arbitrarily far in all directions, so for Requirement B the span is still 9.

Requirement C

For Requirement C, there are still two levels of interference, but the k of **Constraint 1** is allowed to vary.

Our algorithm finds the successful assignments of **Figure 2** for $k = 3, 4, 5$, for which the given spans can be proved easily.

The shaded parts of **Figure 2** show patterns that allow the same radio frequencies to be reused throughout. We systematize the reuse pattern. We begin with a pair of integers $p \geq q$, which we call *shift parameters*. In a hexagonal tiling there are six "chains" of hexagons emanating in different directions from each hexagon. Starting at any cell, we proceed as follows: Move p cells along any chain, turn CCW 60°, then move q cells along the chain in that direction. The original cell and the qth cells so located in each direction are *co-channel* cells (**Figure 3**).

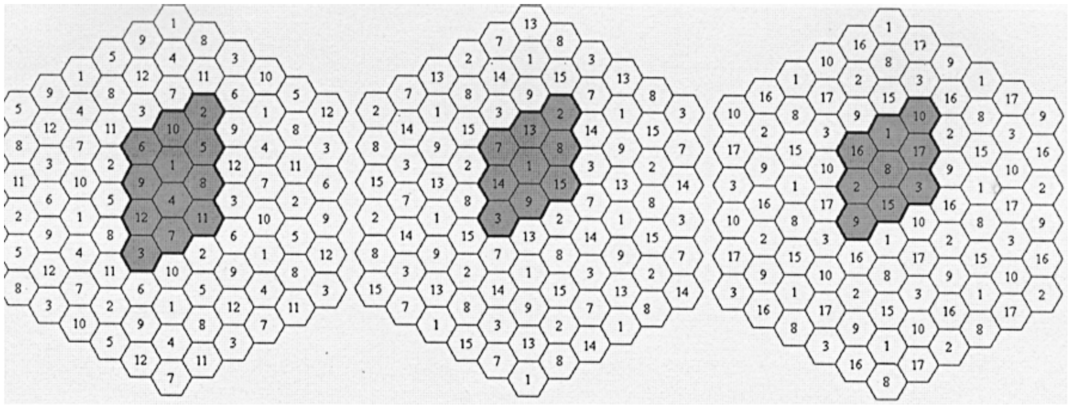

Figure 2. **a.** $k = 3$: the span is 12. **b.** $k = 4$: the span is 15. **Figure c.** $k = 5$: the span is 17.

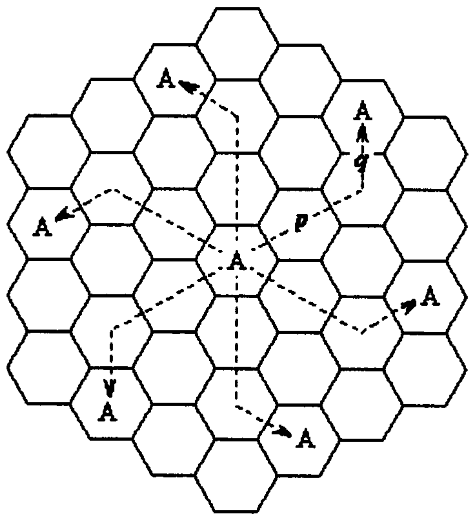

Figure 3. A cell and co-channel cells.

We repeat the procedure for a different starting cell until all cells in the region are assigned. The region can then be divided into a *cluster* of cells, such that transmitters in the same cluster are assigned different channels. The form of the cluster is determined by the shift parameters p and q, and it can be proved that the number of cells in a cluster is $p^2 + pq + q^2$.

In particular, we want to find a suitable cluster of cells for all $k \geq 4$. To minimize the width of the interval of the frequency spectrum, we adopt the cluster of 9 cells in **Figure 2c**; it can be proved that $2k + 7$ is the best result for the span. This reuse pattern is shown in **Figure 4**.

We define the three integer sets N_1, N_2, N_3:

$$N_1 = \{1, 2, \ldots, k\},\ N_2 = \{k{+}1, k{+}2, \ldots, 2k\},\ N_3 = \{2k{+}1, 2k{+}2, \ldots, 2k{+}6\}.$$

Proposition 2. *In a successful assignment with largest channel no more than $2k + 6$ ($k \geq 6$), channels for two transmitters at a distance of $3s$ must belong to the same set N_i.*

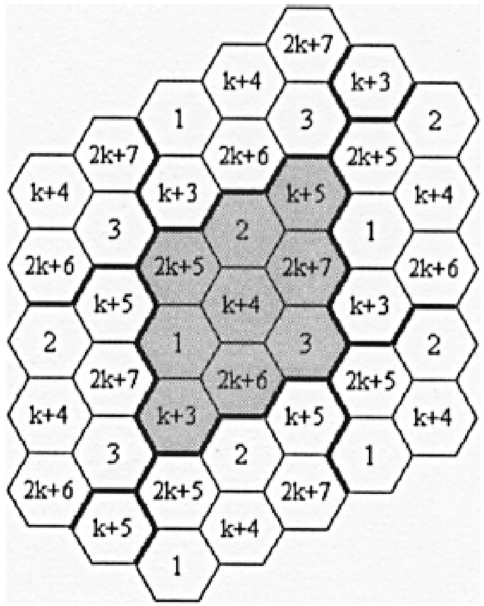

Figure 4. Cell reuse pattern using 9 channels.

Proof: Let A, B be the transmitters, and let C, D be transmitters adjacent to both (**Figure 5**); a, b, c, d are the corresponding channels in a successful assignment. Considering **Constraint 1**, b, c, d must belong to three different sets N_1, N_2, N_3; similarly, a, c, d must belong to three different sets. Hence a, b must belong to the same set. □

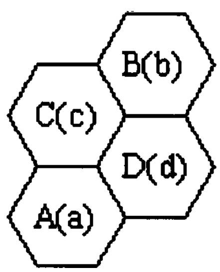

Figure 5. Situation of **Proposition 2**.

Proposition 3. *The span is* $2k + 6$ *for* $k \geq 6$.

Proof: Suppose not. Select any cell A not at the edge of the given region. If its channel a is in N_2, then the six channels of the cells adjacent to A must belong to N_1 or N_3. By **Proposition 2**, three of the six channels belong to N_1, and they are different from each other; hence, we deduce that $a \geq k + 3$ by **Constraint 1**.

From the shaded part of **Figure 6**, select any cell B whose channel b is in N_3. Let the channels for the cells adjacent to B be c, d, e, f, g, h. From **Proposition 2** and **Constraint 1**, we may assume that $d, f, h \in N_1$ and $c, e, g \in N_2$; thus, we have

$$\min\{c, e, g\} \geq k + 3 \longrightarrow \max\{c, e, g\} \geq k + 5 \longrightarrow b \geq \max\{c, e, g\} + k \geq 2k + 5.$$

So, in N_3 there are only two integers ($2k+5$, $2k+6$) that can be assigned to the shaded part of **Figure 6**, which is impossible.

Hence, the supposition that the span is less than or equal to $2k+6$ is not true. We already have a successful assignment with largest channel $2k+7$, so the span is $2k+7$. $\square$

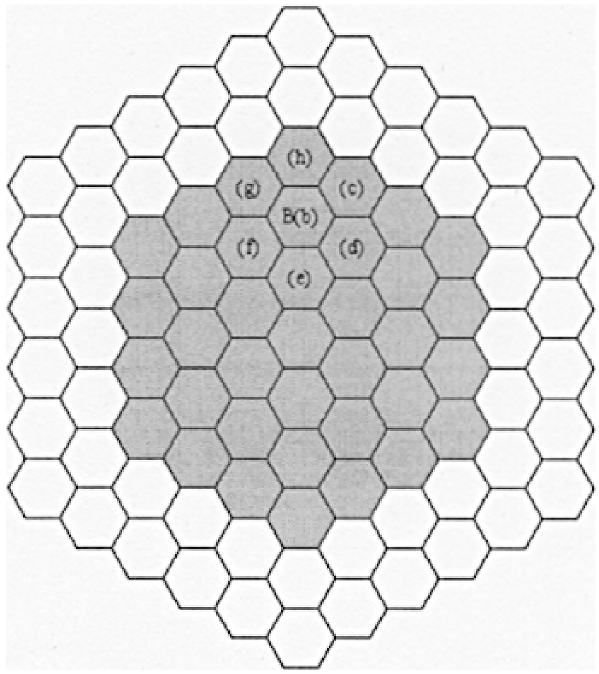

Figure 6. Situation of **Proposition 3**.

Requirement D

Several Levels of Interference

Our backtracking algorithm's workload increases rapidly with the number of interference levels, and we don't know what kind of clustered region could attain the span. So we turn to an approximate algorithm.

We notice from our results for Requirement C that

- There are several groups of channels; channels in the same group are close and evenly distributed, while channels in different groups differ greatly.
- In a single cluster, the channels are all different.

Considering these observations, we guess that the distance between any two adjacent channel-reuse cells should be constant. We use this as the basis for the

Heuristic Skip Algorithm (HSA)

Let there be n levels of interference, and let the channels for transmitters within $\omega_i s$ of each other have to differ by at least k_i. Let Ω be a "cell set" and let $\alpha \in \{1, \ldots, n\}$ be a control parameter.

Step 1. Choose the center cell A of the region as the initial cell:

$$j := \alpha, \qquad l := 1.$$

Step 2. If $j = 0$, stop;
else, pick out all cells A_i that have not been numbered but satisfy $\omega_{j-1}s \le d\,(A_i, A) \le \omega_j s$.

Step 3. If there are not such A_i, then set $j := j - 1$ and go to Step 2;
else, choose one cell nearest to A from A_i and assign the minimal feasible channel to it. If $l = 1$, denote the selected cell by B and denote the shift parameters from A to B by p, q.

Step 4. Add the selected cell to Ω.

Step 5. Start with any cell in Ω as a reference, move p cells along any chain of hexagons, turn CCW 60°, and move q cells along the chain in the new direction. Assign the minimal feasible channel to this new cell and add it to Ω.

Step 6. If there is no starting cell in Step 5, set $\Omega := \phi$ and $l := l + 1$ and return to Step 2;
else, repeat Step 5.

When the algorithm ends, every cell is numbered. The control parameter α determines the distance between any two adjacent channel-reuse cells, and we can execute the algorithm repeatedly with different values of α to get the best result.

Results

- 2 levels of interference (e.g., $\omega_1 = 2, \omega_2 = 4, k_1 = 5, k_2 = 1$): The largest channel assigned by HSA is 17, which agrees with the optimal result earlier.
- 3 levels of interference (e.g., $\omega_1 = 2, \omega_2 = 4, \omega_3 = 6, k_1 = 5, k_2 = 3, k_3 = 1$): The largest channel assigned by HSA is 33; the cluster pattern is shown in **Figure 7a**.
- 4 levels of interference (e.g., $\omega_1 = 2, \omega_2 = 3, \omega_3 = 4, \omega_4 = 6, k_1 = 4, k_2 = 3, k_3 = 2, k_4 = 1$): The largest channel assigned by HSA is 29; the cluster pattern is shown in **Figured 7b**.

Irregular Transmitter Placement

Let r be the largest distance between a transmitter and the center of its hexagon. For $r \le 0.134s$ and two levels of interference, the results apply as before, and a similar analysis can be made for other cases and numbers of levels of interference.

For larger r, we still use HSA. But first, since the position of transmitters is irregular, some might be missed when we use HSA to assign channels. We

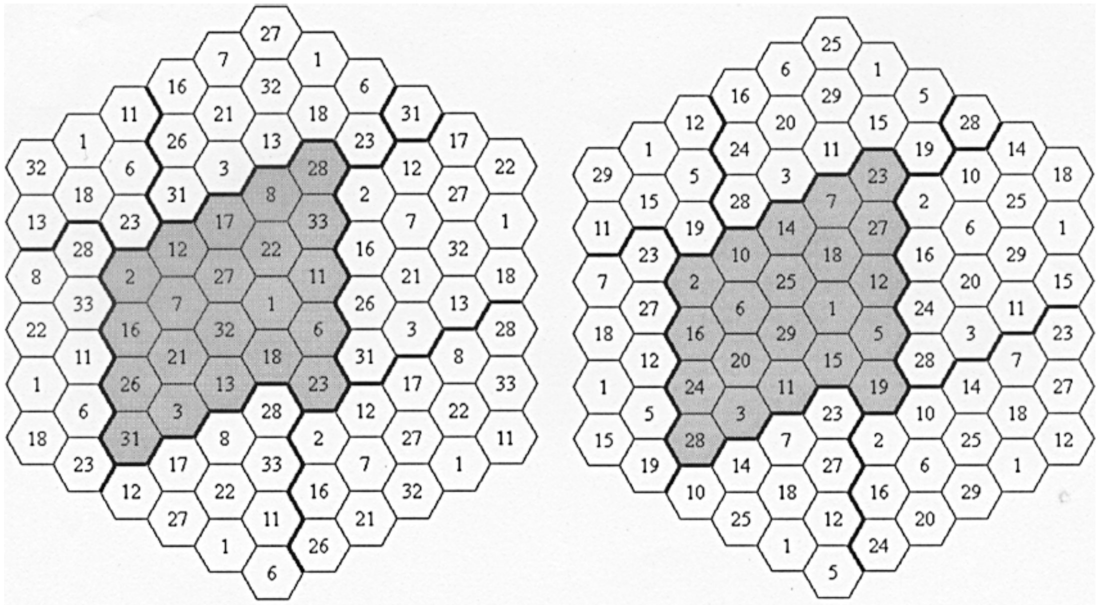

Figure 7. a. Assignment for 3 levels of interference. **b.** Assignment for 4 levels of interference.

overcome this problem by improving HSA so that when it determines which transmitter is to be assigned a channel, it ignores shift parameters, but it does consider the position of the transmitter when assigning a channel.

Result

- 2 levels of interference (e.g., $\omega_1 = 2, \omega_2 = 4, k_1 = 4, k_2 = 1$): To simulate reality, we randomly choose 80% of the transmitters move by $0.134s$ and the others to move by $0.3s$. The largest channel is 16, compared with 15 for regular placement.

Analysis of Results

We use HSA to solve the problems under various conditions. Attenuation of radio signals follows a log normal distribution. Since the radius of a cell is several kilometers, we solve only the problem of $6s$ interference, which should suffice in reality.

Two or More Levels

In Requirements A, B, and C, only two levels of interference are taken into account, with $\omega_1 = 2$ and $\omega_2 = 4$. For $k = 2$, HSA gives 11 channels, while the span is 9; for $k = 3$, HSA gives 13, while the span is 12; for $k = 4, \ldots, 10$, HSA gives the span.

For Requirement D, with three levels of interference ($\omega_1 = 2, \omega_2 = 3, \omega_3 = 4$), HSA gives a largest channel of $3k_1 + 6$, which we feel is very close to the span. **Figure 8** shows the frequency reuse pattern of a cluster of 12 cells.

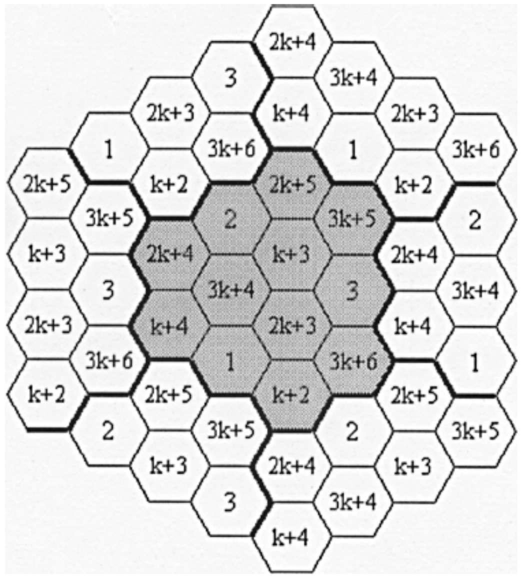

Figure 8. Cellular reuse pattern using a cluster of 12 channels.

Table 1 gives results for various combinations of the parameters for the case of 3 levels of interference; **Table 2** gives results for 4 levels.

For three levels and k_2 small compared to k_1, and for four levels with $k_3 = 2$, the HSA results are smaller than for other cases, because the algorithm can adopt a more rational cluster structure and assign channels more economically. This fact indicates that the result of HSA is determined not by one or two parameters but by all parameters and that HSA makes full use of the information of the constraints; hence, HSA may give a comparatively good result.

Irregular Transmitter Placement

Tables 3–4 give the results, which varies only slightly when the proportion of transmitters moved more than $0.3s$ is less than 10%.

Table 1.

Results for 3 levels ($\omega_1 = 2, \omega_2 = 4, \omega_3 = 6$).

k_1	k_2	k_3	HSA
3	2	1	27
4	2	1	27
	3	1	33
5	2	1	33
	3	1	33
	4	1	39
6	2	1	38
	3	1	39
	4	1	39
	5	1	45
7	2	1	38
	3	1	45
	4	1	45
	5	1	45
	6	1	51
8	2	1	40
	3	1	51
	4	1	51
	5	1	51
	6	1	51
	7	1	57
9	2	1	42
	3	1	53
	4	1	57
	5	1	57
	6	1	57
	7	1	57
	8	1	63

Table 2.
Results for 4 levels ($\omega_1 = 2, \omega_2 = 3, \omega_3 = 4, \omega_4 = 6$).

k_1	k_2	k_3	k_4	HSA
4	3	2	1	29
5	3	2	1	32
	4	2	1	34
		3	1	35
6	3	2	1	37
	4	2	1	37
		3	1	39
	5	2	1	37
		3	1	41
		4	1	41
7	3	2	1	38
	4	2	1	38
		3	1	45
	5	2	1	38
		3	1	45
		4	1	45
	6	2	1	40
		3	1	47
		4	1	47
		5	1	47

Table 3.
Irregular transmitter placement, 2 levels of interference ($\omega_1 = 2, \omega_2 = 4$).

% moved > $0.3s$	$k_1 = 4, k_2 = 1$	$k_1 = 6, k_2 = 1$
0	15	19
5	15	19
10	15	19
20	16	27
25	20	29
30	21	33
35	21	31
40	21	31
45	15	29
50	21	26

Table 4.
Irregular transmitter placement, 3 levels of interference ($\omega_1 = 2, \omega_2 = 4, \omega_3 = 6$).

% moved > $0.3s$	$k_1 = 4, k_2 = 3, k + 3 = 1$	$k_1 = 6, k_2 = 4, k_3 = 1$
0	33	50
5	33	41
10	33	42
20	38	44
25	38	46
30	38	46
35	34	43
40	34	54
45	36	51
50	39	57

Further Discussion

Cell Splitting

One advantage of cellular service is its ability to keep up with changing customer demands. If the customer base approaches full capacity, key cells can be divided into a number of smaller cells, each broadcasting at lower power, and channels can be reassigned to increase the volume of customers (**Figure 9**).

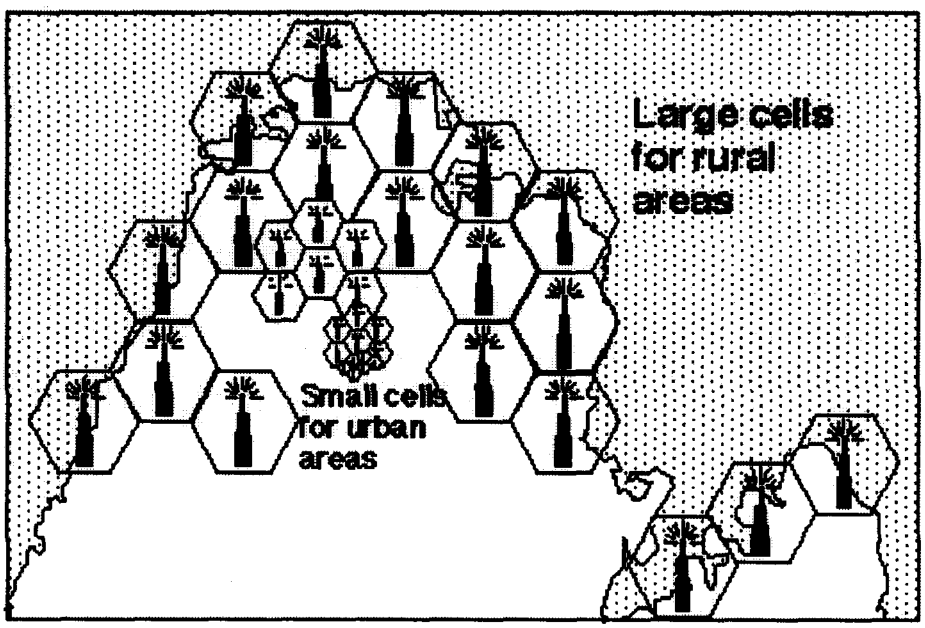

Figure 9. Cell-splitting.

Assumptions

- The radius of a new cell is half that of an old one, and the power of the new transmitter is half that of the old one; as a result, the distance constraints between new cells become half that of the old.
- The new cells plus unsplit old ones cover the entire region.
- The number of levels of interference between any two old cells are unchanged.
- The number of levels of interference between an old cell and a new cell are the same as between the old cells.

Strategies

- Strategy 1: Assign the new cells first, then the old ones.
- Strategy 2: Assign the old cells first, then the new ones.

Results

We allot channels by HSA in the area with split shells that is shown in **Figure 10**. For two levels of interference ($\omega_1 = 2, \omega_2 = 4, k_1 = 4, k_2 = 1$), Strategy 1 uses 28 channels and Strategy 2 uses 27.

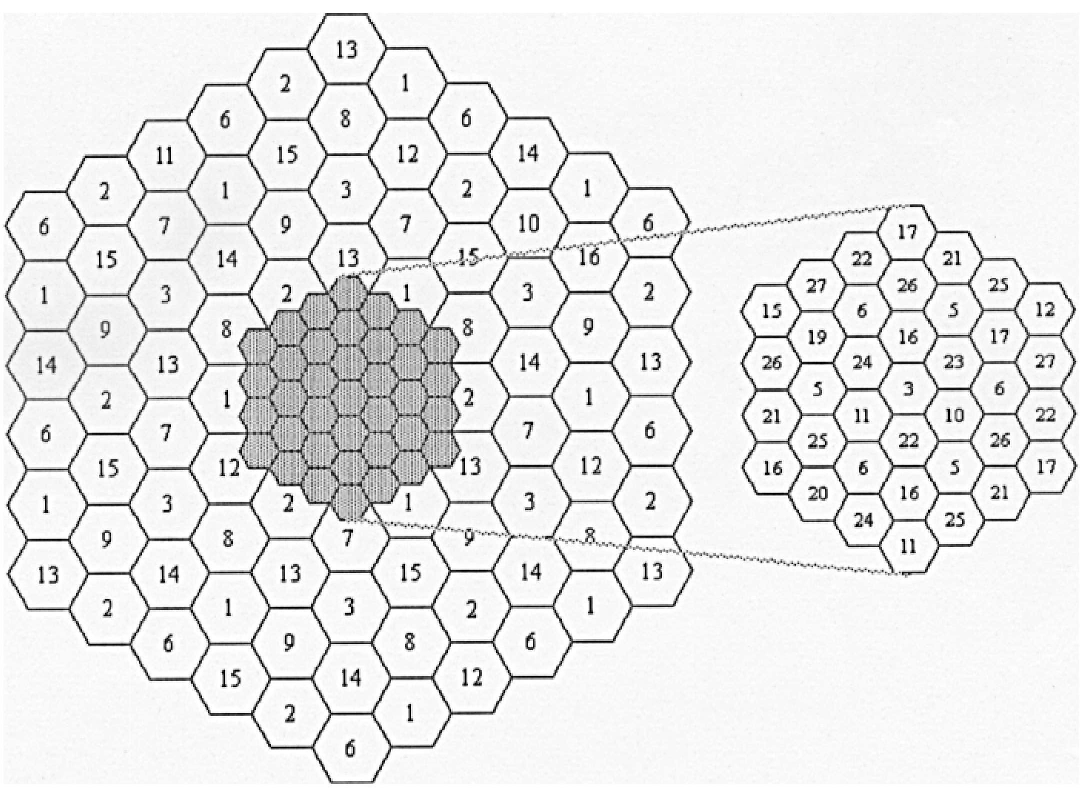

Figure 10. Test region for splitting strategies; result for Strategy 2 is shown (largest channel is 27).

Duopoly

If there are two providers in a region, we must assign their channels at the same time to avoid cross interference between the two systems.

Strengths

For two levels of interference, we determine the span as a function of k, prove optimality of the result, and give an efficient strategy for assigning channels.

The HSA algorithm is adaptable to large areas, irregular transmitter placement, and splitting cells. It seems to give results close to the span, together with a cluster of cells that allows simple assignment of channels.

The HSA algorithm is polynomial-bounded (in class $\mathcal{P}$); for the test cases examined, it gave results (on a PC) within 5 sec.

Weaknesses

The result of the HSA algorithm may not be optimal.

Although we conjecture that the span is $3k_1 + 6$ for the situation with parameters $\omega_1 = 2, \omega_2 = 3, \omega_3 = 4$, we cannot prove it.

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