Determining the People Capacity of a Structure

Samuel W. Malone
W. Garrett Mitchener
John Alexander Thacker
Duke University
Durham, NC

Advisor: David P. Kraines

Summary

Many public facilities are assigned a "maximum legal occupancy" for how many people may be in the facility at one time. For typical facilities, we consider personal space, evacuation time, and ventilation to determine this number. We present several models of evacuation and flow of people to determine how quickly a given number of people can leave a room or complex of rooms in case of an emergency. We estimate the time for a room to become dangerous when toxins are leaking into the atmosphere, including the carbon dioxide produced by human respiration and by fire. In addition to an emergency situation, we investigate how the ventilation through a room might limit its maximum occupancy.

We expect each person to need 0.5 to 1 m^2 of personal space. For an elevator or a concert, in which close contact is not considered uncomfortable, smaller values may be used. For a swimming pool, where people need more room to maneuver, we recommend more.

We use three models of flow of people out of a room with a door. One assumes that the flow rate is constant, the second bounds it by a linear function of people-density (people per unit area) in the room, and the third bounds it by a concave-down quadratic function of people-density. In each case, the rate at which people exit is roughly proportional to the combined flow rates of all the doors. A room with a lot of small furniture is similar to an empty room, since people are not heavily restricted in direction of travel; but a room with large furniture that restricts motion is better considered as a complex of connected rooms. The space taken up by furniture must be subtracted from the whole when calculating capacity.

The UMAP Journal 20 (3) (1999) 273–286.

A series of rooms can be represented as a graph with nodes for rooms and edges (marked with a flow rate) for doors. For constant flow rate, the Ford-Fulkerson algorithm gives the maximum flow through the room and hence an estimate of the time for any given number of people to evacuate.

For the constant and quadratic bound models, a computer simulation gives consistent results for a complicated cafeteria on campus. Unless there is a bottleneck somewhere inside, the limiting factor on the evacuation rate seems to be the flow rates of the doors.

Once we know how long it takes to evacuate N people is known, we can back-solve to determine the maximum number of people who can evacuate in time T. The problem is determining how much time to allow for evacuation. Based on the combustion of wood, we estimate that the sample cafeteria would take 2.5 min to evacuate but 2.5 h to fill with carbon dioxide.

Our evacuation models are flexible, in good agreement with each other for the sample buildings we used, and give reasonable times for evacuation. The ventilation model is likewise reasonable and flexible. Although we had to guess many of the parameter values used in the models, we designed experiments to determine some of these parameters. In particular, the estimate of time until fatality for a fire was extremely rough and should be refined.

We recommend that personal space be used as a first estimate of capacity. The evacuation models should then be applied to be sure that there are no bottlenecks. The ventilation system should be examined to ensure that enough fresh air comes in and that the room dissipates heat quickly enough.

Introduction

Two important factors affect capacity:

- **The Emergency Problem:** What should be the maximum capacity in terms of minimizing the time for every occupant to exit without sustaining injury?
- **The Comfort Problem:** How many people can fit in a room, for a given interval, before the room becomes overheated or the carbon dioxide level rises significantly above normal?

We present two models for the emergency problem, both of which give a method for determining the minimum time for a specified number people to exit a specified structure. Conversely, we use these methods to determine the maximum number of people who can exit a structure in a given period of time.

For the comfort problem, we estimate the maximum number N of people who can comfortably occupy a given space for a period of time T.

To avoid ambiguity, we use the following definitions:

- A *structure* is an assortment of interconnected spaces, each of which leads to at least one other space or an exit.

- An *emergency* is a situation that poses sufficient potential or actual harm to the well-being of the group within a structure to require its complete evacuation.
- The assumption of *orderly movement* states that no personal injuries or other accidents occur that affect the minimum time to evacuate the structure.
- A *panic* is a situation in which orderly movement does not hold.
- A room is *comfortable* if the quality of its air is acceptable and its temperature falls within a specified range.

Further Considerations

One difficulty in developing a model for the emergency problem is deciding how different types of emergencies affect the rate at which people can exit a given structure. A bomb threat and a fire are both pressing reasons to evacuate a building. Imminent danger of smoke inhalation is more serious than the knowledge that five hours later a bomb may or may not explode; but a bomb threat called in five minutes before detonation could cause a panic that might leave many people injured in the rush to exit, whether or not the threat is real. The dynamics of the exiting processes for each of these situations present distinctly different modeling situations.

In addressing the emergency problem, we first consider orderly movement and then extend our analysis to what might happen in a panic.

Assumptions and Hypotheses

- The people in our models are adults weighing between 100 and 300 lbs.
- There are no "security guards" or individuals responsible for regulating evacuation. That is, every individual desires to exit the structure as quickly as possible and employs the same process for deciding on the best route.
- The ceilings are of normal height, and the uppermost floor is not extremely distant from ground level (i.e., the rooms are not crawl spaces nor are they penthouses of skyscrapers).
- The time for a person to move from one room to another is negligible compared to the time to evacuate all people from a room.
- The room is in a modern building in a town or city. We do not expect our results to apply to submarines, space stations, or other unusual structures.

Personal Space Constraints

The simplest constraint on the capacity of any room is space. Each person requires about 1 m^2 (9 ft^2) to stand and move around comfortably. So if a room is designed for standing or sitting in an upright chair, an upper bound on the room's capacity is given by its area (less any area occupied by furniture) in square meters.

In special cases, such as a rock concert or an elevator, in which people are willing to stand closer together, the maximum capacity may allow for only 0.75 or 0.5 m^2 per person.

Evacuation Models

- How long would it take all the people in a full room to exit?
- What is the risk that someone would be injured during the evacuation? (by being trampled, left in the building, etc.)
- In an emergency, how long do people have to get out of the room?

To answer these questions, we develop several models of evacuation based on assumptions about kinds of emergencies and how people move through doors.

The Constant Rate Model

The constant rate model is based on the following assumptions:

- A door lets people through at a constant flow rate.
- The time for a person to get in line at a door is negligible compared to the time to evacuate the room.
- Doors do not become blocked during the evacuation.
- People are crowded around each door. Until the room is almost empty, there are enough people standing close to the door to use it to full capacity. When someone exits, the crowd pushes forward to fill the gap.
- People tend to go either to a nearest door or to a door that will allow them to exit the fastest.

First we analyze a room containing only people; later we add furniture. Similarly, we initially ignore the possibility of a panic.

Single Room with One Door

For a single room with one door, we assume that there are always enough people to use the door to capacity. If the door allows people through at rate r and there are n people in the room, it takes $t = n/r$ time for the room to empty.

Single Room with Multiple Doors

If the room has multiple doors, each person initially goes toward the nearest door. If it becomes clear that one crowd is moving faster than the others, people at the end of slow lines move to the end of the fast line. In this way, all doors are crowded until the room is empty. Suppose that there are k doors with flow rates $r_1, \ldots, r_k$ and that $n_1, \ldots, n_k$ people exit through the doors, respectively. All lines finish at the same time, yielding

$$t = \frac{n_1}{r_1} = \frac{n_2}{r_2} = \cdots = \frac{n_k}{r_k}.$$

If we let n be the sum of the n_i, we have

$$n = tr_1 + tr_2 + \cdots + tr_k.$$

Defining r to be the total number of people divided by the total time of evacuation and substituting yields

$$r = \frac{n}{t} = r_1 + r_2 + \cdots + r_k. \qquad \textbf{(1)}$$

That is, a room with many doors is equivalent to a room with a single larger door whose flow rate is the sum of the rates of all the smaller doors.

Subroom and Corridor Decomposition

Now we consider furniture and other obstacles. First, imagine a dining room with a large number of tables and chairs (see **Figure 1**).

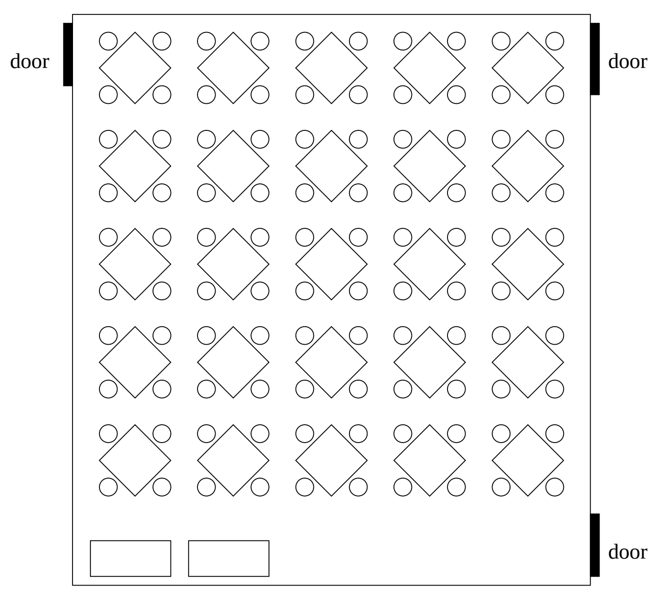

Figure 1. A dining room, view from above.

The furniture restricts people to certain paths, but the assumptions of the open-room model still hold. People can generally move in whatever direction they want, there is always a crowd at each door, and each door flows at maximum capacity. It is the combined flow rate of all the doors that determines the evacuation time, as in **(1)**.

Alternatively, obstacles can divide a room into smaller rooms and corridors, a situation that requires a significantly different model. For example, consider a small lecture hall with rows of seats, a table, and several doors (see **Figure 2**). People would likely walk between the chairs rather than leap over them. So, the single room is broken up by the furniture into smaller "subrooms" and "corridors," as shown in **Figure 3**. This situation is different from the dining hall because the furniture of a lecture hall more severely restricts the directions people can move in. A person in the hall must first exit a row of seats, then go down one of the outside aisles. If one end of an aisle is blocked, it takes longer for the last person on that aisle to exit the room. In the dining hall, a blocked passageway is less critical because there are so many other passages.

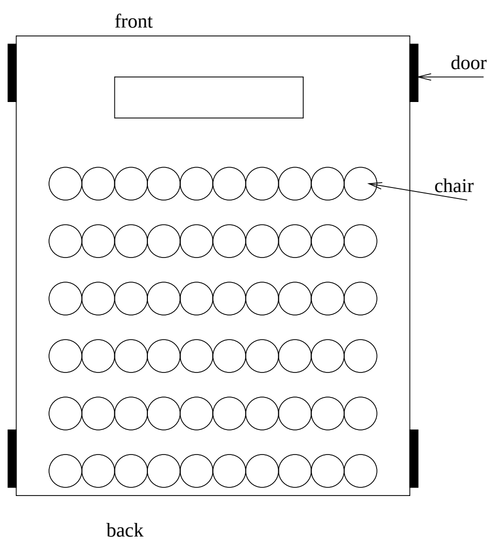

Figure 2. View of a lecture hall from above.

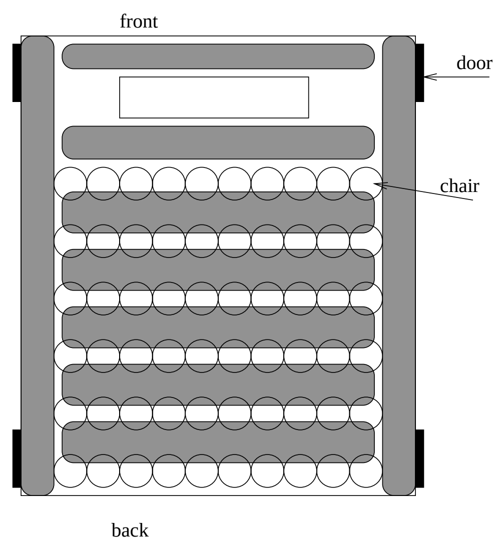

Figure 3. Corridors of movement (in gray) in the lecture hall.

Once a room has been broken up into subrooms and corridors, it is useful to think of them each as being separate rooms with doors connecting them, and the evacuation problem becomes one of evacuating a whole complex of rooms (see **Figure 4**). The diagram can be simplified somewhat by combining doors that lead to the same place as in **(1)**. In this case, the exit doors can operate at maximum capacity the whole time, so the time for evacuation is determined entirely by their combined flow rate.

For a more complex example, consider the cafeteria floor plan shown in **Figure 5** (this is based on an actual building on campus). Most rooms are connected by open arches that function as doors with large flow rates. The

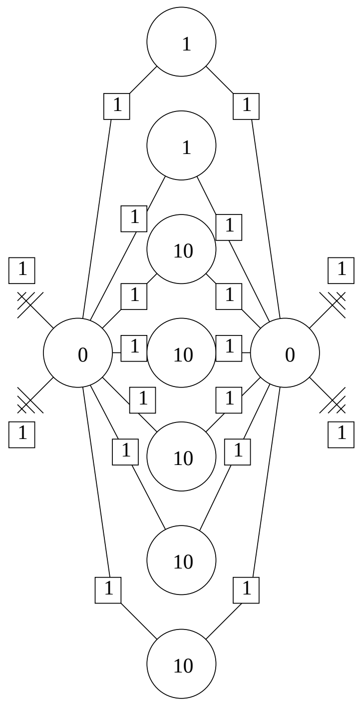

Figure 4. Schematic diagram of subrooms of the lecture hall. Circles represent subrooms, lines represent passage from one subroom to the next, and ground symbols represent doors leading to the outside. Each subroom is marked with how many people are in it and each connection is marked with how many people per second can flow through it.

cafeteria reduces to the schematic diagram shown in **Figure 6**. Here it is not so clear that the flow rate of the four exit doors determines the evacuation time, although our simulations and a method that we will describe show that this is in fact the case. If we had a large room connected to a lobby by a single small door, and a large door connecting the lobby to the outside, the evacuation time would be more dependent on the flow of people into the lobby. In other words, sometimes a small interior door is a bottleneck, but sometimes it is not. For a complicated network like the cafeteria, whether or not there is an interior bottleneck is not immediately apparent.

Maximum Flow Model

Curiously, the evacuation problem for a complex of rooms can be solved by ignoring the numbers of people in the rooms. Suppose that people constantly flow out of the complex and other people emerge inside at the same rate (think of people falling out of the ceiling as fast as other people exit). The rooms will have constant numbers of people, since people are replaced as fast as they leave. The problem is to find the flow rate of people through a complex.

The Ford-Fulkerson algorithm finds the maximum flow through a graph. Suppose that in a directed graph each connection has a known maximum capacity (e.g., people per second who can pass through a crowded door). One of the

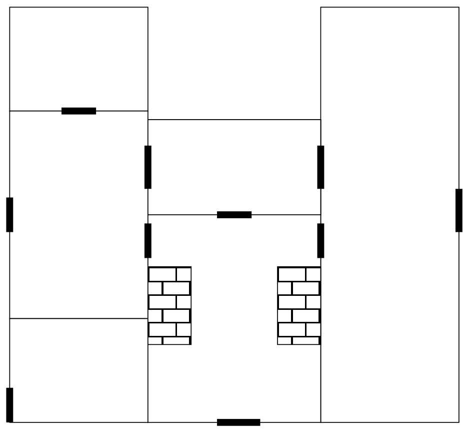

Figure 5. A large cafeteria viewed from above.

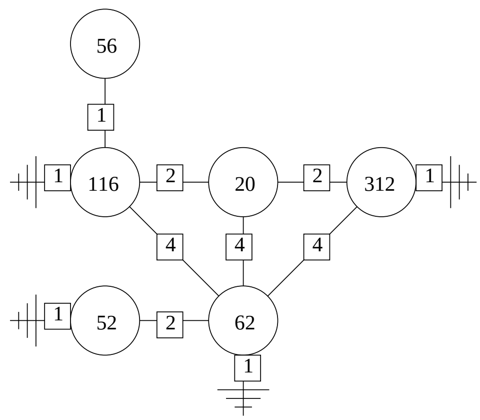

Figure 6. Schematic diagram of the cafeteria.

nodes is designated the "source" (people falling from the ceiling) and another is designated the "sink" (the outside). We assign to each connection the actual flow through it. Such an assignment can be improved if there is a path from source to sink in which the flow through every connection can be increased. An assignment is maximal if there is no such path. The Ford-Fulkerson algorithm looks at all possible paths until no improvements can be made.

The time for n people to leave the building can be estimated by dividing n by the maximum flow. To use the Ford-Fulkerson algorithm on a room graph, we must add two nodes: a source is connected to all rooms with lines of infinite capacity; and a sink node representing the outside is connected to all exits from the complex, with connection capacities equal to those of the exit doors.

For a continuation of the cafeteria example, see **Figure 7**. This graph is marked with a maximum flow. The flow cannot be improved because all the connections leading to the sink are at their maximum. The figure confirms that the rate of evacuation is determined by the flow rate of the exit doors; in other words, there are no internal bottlenecks. The same technique can be applied to any room graph.

Quadratic Rate Model

Motivation for the Negative Quadratic Model

A linear rate model proposes that the rate at which people can exit a room, $f(t)$, is bounded by a linear function of the number of people in the room. The evacuation problem can be stated as:

$$\text{maximize} \quad \int_0^T f(s)ds,$$

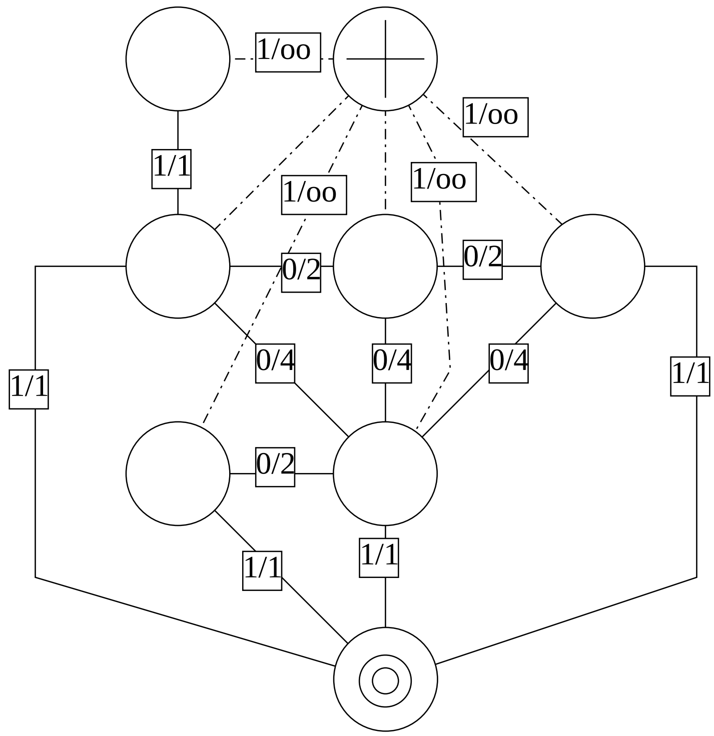

Figure 7. The graph for the Ford-Fulkerson algorithm for the dining hall. The + node represents the source and the bull's eye represents the sink.

that is, maximize the number of people who can evacuate in time T,

$$\text{subject to} \quad 0 < f(t) < a \int_t^T f(s)\,ds + b, \text{ for } 0 < t < T.$$

The integral gives the total number of people evacuated after time T minus the number of people who evacuated up to time t; in other words, it is the number of people in the room at time t.

The linear model represents the situation where the number of people in the room has a "forcing" effect on the flow rate through the exits; the greater the constant a, the greater the forcing effect. The constant b represents the normal rate of flow when the forcing effect is negligible. Assuming that people exit the room in an efficient and orderly manner, the linear model hypothesizes that the maximum flow rate out of the room increases linearly with the number of people in the room.

However, this model does not take into account that for sufficiently large flows, the capacity function representing the upper bound of the flow rate should decrease to zero. The evacuation dynamics of an emergency require an upper bound model that takes large flow values into consideration: When, other than an emergency or a panic, would such large flow values occur and evacuation time be more crucial?

Developing the Negative Quadratic Model

We pose a model that assumes that the upper bound of the flow rate is a negative quadratic function of the number of people in the room at time t. The evacuation problem becomes:

$$\text{maximize} \quad \int_0^T f(s)ds$$

$$\text{subject to} \quad 0 < f(t) < q - r\left(\int_t^T f(s)ds - p\right)^2, \text{ for } 0 < t < T. \qquad \textbf{(2)}$$

The maximum flow rate q occurs when the room is occupied by an optimal capacity p of people. The motivation for the negative quadratic rests on two assumptions:

- The upper bound decreases when the number of people in the room is substantially less than p, because the time for people to walk to and through the exit becomes nonnegligible compared to the total time required to evacuate all people from the room.
- Conversely, when the number of people in the room noticeably exceeds p, the jostling, discomfort, and limitation of movement that occurs reduces the flow rate through the exits.

The value of p for a room depends on its floor space A and a critical density d (the number of people per area beyond which impediment to motion increases and flow efficiency decreases), with $p = Ad$. We assume that $d = 0.75$ people/ft^2.

To solve the evacuation problem using the quadratic model, we assume that maximum flow occurs. The constraint **(2)** becomes

$$f(t) = q - r\left(\int_0^T f(s)ds - \int_0^t f(s)ds - p\right)^2, \qquad 0 < t < T.$$

Differentiating both sides twice with respect to t leads to

$$f''(t)f(t) - f'(t)^2 + 2rf(t)^3 = 0.$$

Using the initial values $f(T) = q - rp^2$ and $f'(T) = 0$ and the package Maple, we get the following solution for the flow rate out of the room at time t:

$$f(t) = \left(\frac{q - rp^2}{\cos\big((t-T)\sqrt{-qr + r^2p^2}\big)}\right).$$

From this result, we compute the maximum number of people N who can exit the room in a time interval T:

$$N(T) = \int_0^T f(t)dt = \left(\frac{-\tan\left(T\sqrt{r(-q+rp^2)}\right)(-q+rp^2)}{\sqrt{r(-q+rp^2)}}\right).$$

Solving for T, we have

$$T(N) = \frac{\left(\frac{\arctan\left(-N\sqrt{r(-q+rp^2)}\right)}{-q+rp^2}\right)}{\sqrt{r(-q+rp^2)}}.$$

The Relevance of the Negative Quadratic Model

In a panic, some people may sustain injury, fall down, or disrupt the flow of the crowd. Our justification for the quadratic model assumes something similar: People packed together at a density greater than the critical density slow each other down in their attempt to evacuate a room. The difference between the impediments to flow caused by crowding and the impediments caused by panic is one of degree.

To illustrate the predictions of the negative quadratic model, consider a room of size $A = 1{,}000$ square feet and suppose that the optimum flow rate is $q = 90$ people/min, that optimum flow occurs with $p = Ad = (1000)(0.75) = 750$ people, and that we have $T = 6$ min to evacuate. We take the value for r to be $a/p^2 = .01/(750^2) = 1.8 \times 10^{-8}$. Doing so yields $N(6) = 540$ for the quadratic model and $N(6) = 557$ for the linear model. It makes sense that these numbers are not too far apart, since we are not dealing with an extreme case where the number of people evacuated greatly exceeds or undercuts the critical value p. When p does not deviate significantly from Ad, this will usually be the case. However, if we set p, for example, to $10{,}000$ and calculate as above with all else held constant, we get $N(6) = 501$ for the quadratic model; if we set $p = 100{,}000$, we get $N(6) = 195$. The negative quadratic model suggests that efforts of a packed crowed to evacuate may actually decrease the number of people evacuated, by causing injuries and inefficient flow.

Limitations of the Negative Quadratic Model

The negative quadratic model is designed to model the evacuation of a space, not of an entire structure. Applying it to a cafeteria on our campus gave results that agree with the constant rate model. An extension of our project would be to simulate a variety of panic situations using the negative quadratic model, the linear model, and the constant rate model and compare the results.

Our simulation works by computing the probability that a person leaves a room at a given time step. The quadratic model breaks down by giving zero or negative probability when the number of people inside is small, so the program switches to a linear model when there are 10 or fewer people in a room.

We estimated p and d. Since the results from the model depend heavily on the values of these parameters, it is important to estimate them accurately.

Ventilation Models

Comfort level is another consideration for maximum capacity:

- The temperature should be between 65° and 90° F. In particular, the ventilation system should be able to dissipate the heat produced by the bodies of the people inside.
- Toxins in the air should be kept to harmless levels. The only one likely to apply to all situations is carbon dioxide (CO_2), produced naturally by human respiration. Jones [1973] recommends that the CO_2 level should be below 0.1%; at 8%, it can be fatal.
- If smoking is allowed, additional circulation must be allowed for.

Human bodies produce heat at a rate from 60 W (asleep) to 600 W (strenuous activity), with 100 W for moderate activity [Jones 1973]. Heat dissipation from a room depends upon its insulation, windows, and any air conditioning. Rooms that are used for several hours at a time should be able to dissipate 100 W/person so that the temperature remains roughly constant.

Jones [1973] recommends at least 0.2 l/s per person of fresh air, to dilute the CO_2 concentration and unpleasant odors, and 25 l/s if smoking is allowed.

The fraction of oxygen in the air can decrease to 13% before it becomes dangerous, so the presence of toxins is the limiting factor [Jones 1973]. In a tightly enclosed space, the CO_2 produced naturally by human respiration becomes important. A normal human breath is about 500 cc, 4.1% of which is CO_2, and the breath takes 4 s [Hughes 1963]. Thus, humans produce CO_2 at a rate of 5×10^{-3} mol/s.

Given a room of volume V, the amount N of air molecules is given by the gas law $PV = NRT$, where P is pressure, T is the room temperature in Kelvins, and R is the gas constant. Denote by r the constant rate (in moles per second) of creation of a toxin, by q the fraction of the air that is toxic, and by t elapsed. Then we have

$$qN = rt \qquad \text{or} \qquad t = \left(\frac{qV}{r}\right)\left(\frac{P}{RT}\right). \tag{3}$$

At room temperature and pressure of 1 atmosphere, $P/RT = 41.4$ mol/m^3. Substituting for q the lethal concentration of the toxin yields as t the time for the toxin to reach it.

Consider, for example, an elevator 3 m by 3 m by 3 m carrying 12 people that becomes stuck and is somehow completely air-tight. The people take up about half its volume. Using **(3)**, we find that in 2.5 h the the CO_2 level reaches 8%. Hence, we might limit the capacity of the elevator by the time that it takes to get a rescue crew in to open it up. However, elevators are usually well vented, so CO_2 buildup will normally not be a significant constraint.

Swimming Pools

For an outdoor swimming pool, evacuation is not much of a consideration. For an indoor swimming pool, evacuation is basically the same as for an open room. People can exit the pool itself on all sides, except for weaker swimmers who may have to use a ladder, and then flow through the exit doors.

Personal space is the important safety issue. In the water, people must move their arms and legs over a greater range of motion to maneuver than in walking on land. Many swimming strokes limit a swimmer's vision and make collisions more likely. Some swimmers wear floats, which take up additional space.

We recommend 3 m^2, giving each swimmer 1 m in all directions to move. A large space should left open around diving boards and slides, perhaps a circle of 4 m.

Capacities for Elevators

Elevators usually have very wide doors and hold only a few people. Thus, evacuation time is negligible in case of an emergency. (The real time constraint will be getting the people down the stairs and out of the building, a problem that is similar to the room problem.)

We already considered the limitations imposed by possible lack of fresh air. More important factors would seem to be weight and space. Elevators have a weight limit supplied by the manufacturer, and a simple elbow-room constraint of 0.5 m^2 per person should provide sufficient personal space.

Strengths and Weaknesses

Our models are fairly robust, with the negative quadratic model being a more realistic tool than the linear model, since the former more accurately simulates panic. However, the negative quadratic model yields questionable results for large values of room occupancy.

Recommendations

For the negative quadratic model, we could extend an analysis of how to determine the value of p to a more comprehensive understanding of how the "forcing effect" operates to slow the evacuation of a panicked crowd. Also, we could develop techniques for measuring the value of the critical density d, such as observing how many people can evacuate a building in different time intervals T, and using those data to estimate the critical value at which maximum flow occurs.

For improving our analysis of the comfort problem, we could develop ways to estimate better how long it takes a room takes to become overheated or stuffy.

Appendix: Computer Simulation

To test the evacuation times of complexes of rooms, we wrote a simulation engine in Python. Object-oriented programming techniques allow us to use different kinds of doors (always open, sometimes blocked, variable flow rate, etc.) and different strategies of selecting a path out of the building with the same structural models. Each door has a queue of people waiting to get through. At each time step, all the doors "warp" some number of people into the next room. Then everyone in line is given the opportunity to move to a different queue, based on their perception of the room. A special room object is designated the "outside" and throws an exception to halt the simulation when a specified number of people have arrived outside. A class diagram for the simulation is given in **Figure A1**.

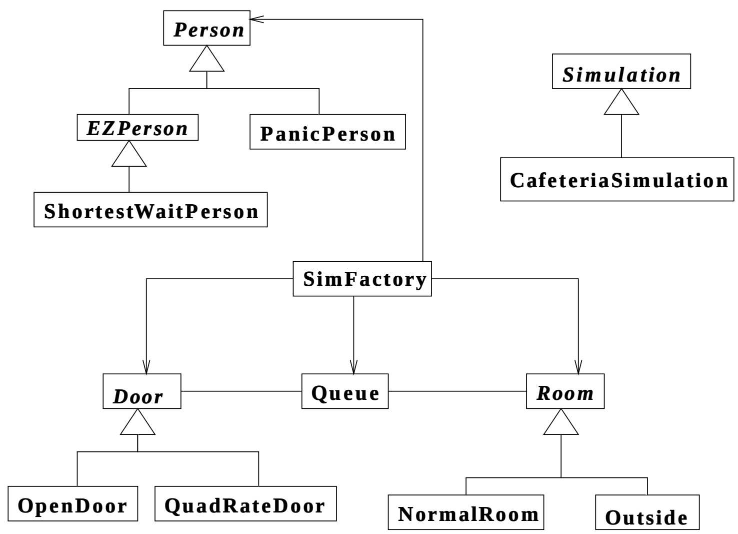

Figure A1. Class diagram of the simulation in abbreviated UML. Triangles indicate inheritance, hairline arrows indicate "creates."

References

Francis, R.L. 1984. A negative exponential solution to an evacuation problem. Research Report No. 84–36. Gaithersburg, MD: U.S. Dept. of Commerce National Bureau of Standards Center for Fire Research.

Hughes, G. M. 1963. *Comparative Physiology of Vertebrate Respiration.* Cambridge, MA: Harvard University Press.

Jones, W.P. 1973. *Air Conditioning Engineering.* London: Edward Arnold.