Pursuit–Evasion Games in the Late Cretaceous

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Summary

Using techniques from differential game theory, we model the velociraptor hunting problem by means of a semi-discrete computer algorithm.

By defining predator and prey behaviors in terms of simple, intuitive principles, we identify a set of strategies designed to counter one another, such that no one pure predator strategy or prey strategy defines an optimal behavior pattern. Instead, the ideal strategy switches between two or more pure strategies, in an essentially unpredictable, or protean, manner. The resulting optimum behaviors show a mixture of feints, bluffs, and true turns for the thescelosaurs, and a mixture of predictive interception and simple pursuit for the velociraptors.

Finally, using these strategies, we demonstrate a conclusive advantage for velociraptors hunting in pairs over velociraptors hunting in isolation.

Introduction

We describe hunting strategies for a velociraptor and flight strategies for its prey using a computational semi-discrete representation of a differential game of pursuit and evasion.

First, we review the formalism of traditional non-differential game theory and the extension of its principles into the analysis of differential systems, taking careful note of the unique aspects of the velociraptor problem.

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Second, we propose a minimal set of assumptions required to reduce the analytical game to a numerical time-iterated computer algorithm, and submit a small set of intuitively simple strategies for each participant to execute.

Third, we examine the implications of both the assumptions of perfect and imperfect information for the optimization of strategies for a single pursuer and single evader and then extend these conclusions to more pursuers.

Finally, we comment on the inherent limitations of this model and offer our evaluation. We conclude that for the game of imperfect information there exists no pure strategy for the prey, but that the best alternative is to behave in an unpredictable fashion with respect to the alternation of turns and feints. The predator, however, has a clearly dominant strategy of compromising between a predictive algorithm and simple pursuit.

Mathematical Formalization in Game-Theoretic Terms

Contests of interception and avoidance between a predator and its prey fall into the broad and diverse category of linear differential games. In a differential game of pursuit and evasion, two or more opposing players attempt either to maximize or to minimize their separation, subject to certain limitations on their motion.

Unlike the games of classical game theory, differential games involve the application of methodology from differential equations and make use of continuous fluctuations that define the states and objectives of the players.

In either a traditional or differential game, players seek to maximize some value, known as the *payoff function*, by selecting among a set of alternatives, with the payoff to each player determined by some function of all the players' choices. In a zero-sum game, all players are in direct competition with one another, such that the objectives expressed by their payoff functions are exactly opposite. Games of pursuit and evasion fall within this classification; the pursuer seeks to minimize the distance between itself and the evader, and the evader seeks to maximize this distance.

In the case of the velociraptor–thescelosaur pursuit–evasion game, the payoff function is a simple binary function: The only relevant outcomes are capture or escape. Such games are referred to as *games of kind*, as opposed to *games of degree*, in which the payoff can take on a larger set of possible values. In some cases, it may be helpful to consider a game of kind as being embedded in a game of degree, with the time to capture (or the separation at evasion) as the payoff function. Although this in not necessary in purely deterministic trials, the average time to capture for a particular strategy pair may provide a useful statistical measure of the efficacy of a strategy.

There are two classes of variables in a differential game: state variables, which specify the complete configuration of the entire system at any given

point in time, and control variables, which participants in the game use to alter the state function in ways favorable to their attempts to maximize their own payoff function. Common state variables in traditional pursuit–evasion games are the spatial coordinates of the participants; control variables may include an angle of maximum turn, or acceleration vectors.

Games in which all players have access to a complete set of state variables at any given point in time are known as games of *perfect information*; games in which this is not true are referred to as games of *imperfect information*. Typically, games of imperfect information lack exact analytical methods of solution. Thus, one of the best methods of evaluating such games is to use a discrete model, which can be implemented in terms of a simple computational algorithm. Unfortunately, purely discrete models often run the risk of obscuring essential details of the system that may depend on continuity of values without quantization. A reasonable compromise that still permits a computational method of solution is a semi-discrete method, in which time is iterated discretely, but spatial values are allowed to extend over the entire domain of real numbers [Isaacs 1967, 42]. We chose to implement the raptor hunting game as a semi-discrete computational algorithm.

The velociraptor problem embodies one of the most interesting situations in a two-player pursuit–evasion game. If the maximum velocity of the evader is greater than the maximum velocity of the pursuer, then the evader has an optimal strategy of moving directly away from the pursuer at maximal velocity and will always successfully evade capture. Similarly, if the pursuer is superior in both speed and maneuverability, then the pursuer has an optimal strategy of moving directly toward the evader and is guaranteed of a successful capture. The case in which the pursuer is swifter while the evader is more agile, however, has no trivial solution and may require complex or nondeterministic strategies for optimal play.

Assumptions and Development of the Model

Initial Configuration

Prey animals generally ignore predators until they have moved within a well-defined flight radius; when the distance between themselves and a predator falls below this value, a flight response is triggered. We are given that the flight distance of a thescelosaurus is less than 50 m and greater than 15 m, such that the moment a velociraptor is detected within this distance, the thescelosaur will immediately flee. Similarly, we assume that the raptor has been deliberately stalking the thescelosaur with the intent of capturing it; thus, the predator will be in a set crouch position and will pursue at the first sign of flight. Although the thescelosaur may be startled or surprised, it initiates the flight and chase sequence, and thus both it and the raptor begin moving (nearly) simultaneously.

Our model allows us to specify the initial state either through selection of locations and facings for each predator and prey, or by implementing a simple probabilistic stalking model to determine the initial separation. In most cases, we set the starting separation of the predator and prey close to the minimum possible value, given to be 15 m. This is because, for most large separations, the optimal strategy of the prey is to move directly away from the predator at maximum speed until the predator is very close. Thus, differences in strategic behavior do not typically become manifest until the prey is forced to deviate from a linear path due to the proximity of the predator.

In the case of multiple predators, we assume either that they have stalked out nearly opposite sides of a circle of radius 15 m or that they start from approximately the same position.

Sensory Acuity

For the simplest level of approximation, it is sufficient to assume that all participants in a pursuit–evasion game have complete and instantaneous access to the state function for all times, such that they are involved in a game of perfect information. However, this is a relatively inaccurate assumption for most real physical systems. The ability to estimate distances and directions is always subject to random error, and vision is limited to a field of sight subtending an angle of less than 180°. Being forced to rely on other senses, usually hearing, to track a moving object is less than ideal, and can lead to sizable error in the assessment of distances in particular. These limitations are particularly problematic for predators, who have a narrowly focused forward field of vision and depend heavily on being able to estimate not only the present but the future locations of their prey.

Our model functions under assumptions of perfect information but with the introduction of both random and systematic error in sensory perception. The former is enforced by multiplying the magnitude and angle of the displacement vector between predator and prey each by a different random number between 0.95 and 1.05 before passing it to the routine governing the selection of control variables. Additionally, uncertainty in these equations due to the limited field of sight (systematic errors) are estimated using a linearly increasing statistical spread proportional to the angular displacement between the direction of sight and the observed object. The distance and angle are each multiplied by a different random number in the range $1.00 \pm s\theta/\pi$, where $s = 0.05$ for angular displacement and 0.25 for linear displacement. Finally, to emphasize the importance of visual contact, these factors are further increased for the predator by 50% to $s = 0.075$ and 0.375, respectively.

A related issue is reaction time. In the perfect information case, knowledge of the state vector is imparted instantly. A more realistic assumption is that these data become available for use only after they have been psychologically processed by the brain. Ordinarily, the process of updating awareness of the environment could be considered to be immediate; but in a contest measured in

hundredths of a second, to deny the prey the ability to have an advantage over the predator in its knowledge of its own actions would obscure an essential element of the model. To prevent either dinosaur from being able to react instantaneously to changes in its environment, we delay information about the state variables of other dinosaurs by 0.05 s for the raptor and 0.037 s for the thescelosaur.

Physical Constraints on Motion

The center of mass of each of the dinosaurs is assumed to be a point particle moving in a two-dimensional plane in accordance with Newtonian mechanics. Two available options in altering the motion of a point (subject to Newtonian kinematic equations) are to impose either a linear or an angular acceleration.

The data provided by biomechanical analysis indicate that the maximum turn radius of the raptor was 1.5 m and that of the thescelosaur was only 0.5 m. (We understand the problem statement to mean that these values apply at top speed, even though this leads to the thescelosaur being capable of a 32-g turn.) This suggests, in each case, that a maximum possible angular displacement in any given time interval may be defined as $d\theta = a(dt)/v$, where v is the current velocity, dt is the length of the time interval, and a is the centripetal acceleration, where $a = v^2/r$, and v and r here are the speed and maximum radius of curvature at maximum velocity.

Without associated data on the linear acceleration of the dinosaurs, it is necessary to invoke an argument by analogy. The African cheetah is a modern predator filling an ecological role similar to that of the raptor. Cheetahs also share many of the same strategic attributes with velociraptors (high speed, limited endurance, and a turning radius inferior to that of their primary prey). One might reasonably assume that the linear acceleration capabilities of the raptor would have been similar. A cheetah can accelerate to peak velocity (over 90 km / hr) in about 2 s. However, the velociraptor has a lower maximum speed, and is also lighter than the cheetah. This suggests that the acceleration for a raptor may be somewhat greater. Operating under the assumption that the velociraptor possesses the same relative acceleration ability compared to its body size, and recognizing that the force exerted by muscle tissue is proportional to the square of the linear dimensions of the body, while body mass is equal to the cube, a factor of $(2/3)/(1.25)^{2/3} \approx .57$ is not an unreasonable correction to the acceleration of the lighter raptor (where 1.25 is an approximate ratio of the mass of a cheetah to the mass of a raptor).

Strategy

An animal might be expected to behave in accordance with straightforward heuristic principles, and each of the strategies that we tested reflects such a principle. For the sake of the simulation, we assume each raptor and each

thescelosaur must adopt only one strategy at the start of a pursuit–evasion game, although we later consider the advantages that could be gained by switching strategies during the course of a chase.

Predator Strategies

- Strategy Predator-0 (default): Move directly toward the present location of the prey at the maximum possible speed without deceleration (i.e., speed cannot be decreased, even if facing away from the prey).
- Strategy Predator-1 (predictive): Estimate the future position of the prey by assuming that it will continue moving in its current direction at the present velocity, and plot an intercept course.
- Strategy Predator-2 (half-predictive): Take the average of the angles indicated by strategy Predator-0 and strategy Predator-1.

Prey Strategies

- Strategy Prey-0 (default): Move directly away from the present location of the predator at the maximum possible speed without deceleration (i.e., speed cannot be decreased, even if facing toward the predator).
- Strategy Prey-1 (constant turn): Similar to Prey-0, but every time the predator comes within 1.5 m, make a sharp 90° turn, and then go straight again.
- Strategy Prey-2 (variable turn): Similar to Prey-1, but instead of turning at 90°, turn constantly at a rate proportional to the distance to the predator.
- Strategy Prey-3 (feint and bluff): Similar to Prey-1, but instead of going on straight after the 90° turn, turn back at maximum rate by 270°.

Capture and Non-Capturability

Conventionally, in analytical studies of differential games of pursuit and evasion, the condition for capture is just the decrease of the distance between the pursuer and evader below some set value, the radius of capture. In our model, we permit capture more realistically by one of two mechanisms.

- If the raptor is within one meter of the thescelosaur, it may attempt to "lunge" and score a critical strike, thrusting the parts of its body (jaws and talons) employed to injure or subdue prey forward with a sudden burst of acceleration. This is represented by a probabilistic capture condition consisting of two factors, one dependent on distance varying linearly from 0.5 to 1.0 m, and one dependent on angle varying linearly from 0 radians to π radians. If a random number generated between 0 and 1 is less than the product of these two factors, then capture occurs.

- Alternatively, if the raptor and the thescelosaur move within 0.5 m of one another, they physically collide, with invariably catastrophic consequences for the prey. We choose not to model the possibility that the raptor may be physically injured in this case.

If capture has not been attained within 15 s, the simulation ends, and the thescelosaur is assumed to escape, as the raptor is forced to stop and rest. Although the raptor may not spend all of the 15 s at top speed, the process of accelerating and decelerating is at least as energetically demanding as running at a constant speed.

The Simulator

The simulator consists of two main portions: the outer loop in charge of updating the game status at each iteration, and the movement generators. These last, one each for the predator(s) and prey, implement the strategies. This is performed in five steps:

1. Data Acquisition Phase: The bearing and distance to each opponent is determined and recorded.
2. Data Manipulation Phase: Each of the above values is randomly permuted to simulate inaccuracies in sensory acuity.
3. Strategizing Phase: Based on the data and the chosen strategy, a "best-case" move is chosen.
4. Limitation Phase: Physical limitations are imposed on the chosen move in accordance with the associated capabilities of the animal.
5. Movement Phase: This final value is passed back to the outer loop for implementation.

If at any point the outer loop determines that capture has occurred, or that the 15-s time limit has expired, the simulation is halted and the final status is reported.

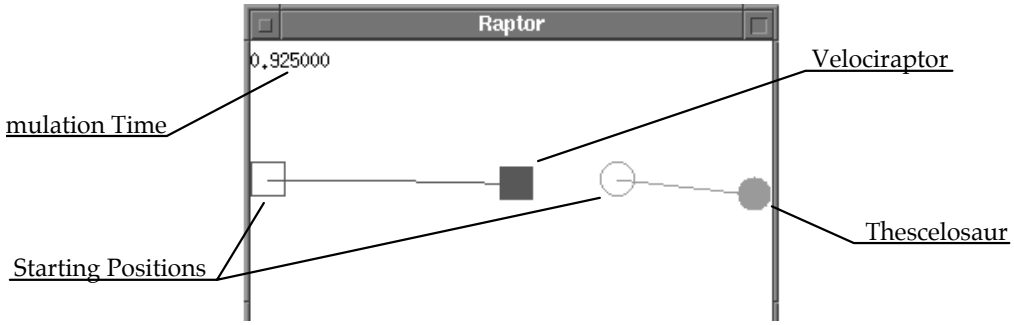

Figure 1. The simulator window.

The simulator was written using ANSI C++ (with the Hewlett-Packard standard template library). The program uses an X Windows (X11R5) display interface to graphically track the positions of the predator(s) and prey. The code was compiled and linked using the freely available C++ compiler from the Gnu Software Foundation and was executed on a Sun Microsystems SparcStation 5 workstation. Source code is available via e-mail to `smenni23@calvin.edu`.

Results

One Raptor

We studied the case of a solitary hunter in detail for both the assumptions of perfect information and for the restrictions on sensory acuity defined above. Under the first assumption, the outcome of every initial position is deterministic and repeatable. Thus every strategy by the predator is either successful, in that it results in the capture of the prey, or unsuccessful, because it does not lead to a capture within the allotted time. This leads to a natural formulation in terms of a binary matrix (**Table 1**) where 1 represents capture and 0 denotes an escape.

Table 1.
Capture in a game of perfect information.

	Prey-0	Prey-1	Prey-2	Prey-3
Predator-0	1	0	0	1
Predator-1	1	1	1	0
Predator-2	1	1	1	1

The intransitivity of the relative ranking of Predator-0 and Predator-1 (or Prey-1, Prey-2, and Prey-3) merits further comment. This corresponds to a situation in which no optimum strategy exists for the predator (or the prey) with respect to the reduced game containing only those choices. Due to the tighter turning radius of the thescelosaur, the only way the raptor can catch it as it moves through its sharp turn is by anticipating its future position. However, this leads to the potential that the thescelosaur may switch to Prey-3, a devious strategy whereby the predator is forced to overcommit as a result of a false turn by the prey, something to which the less sophisticated strategy Predator-0 was immune. The same analysis applies in reverse to the thescelosaurus; for any one strategy she commits to consistently, the raptor can switch to a new strategy capable of beating her every time. Thus, if either player insists on playing with a single deterministic strategy, the other can take advantage of it.

The most obvious option is to switch to a random selection of bluffs and real turns. This would at least guarantee that the other dinosaur could not anticipate the tactic consistently and preemptively account for it. Fortunately, in a game of perfect information, the raptor has another option: Predator-2, a deterministic strategy that combines the other two in a nonrandom way. This

same alternative would not be available if the raptor suffered from a reaction time delay, as is the case with the imperfect-information variant described next.

With the introduction of effects resulting in imperfect information, the deterministic outcomes are replaced by probabilistic outcome distributions. Some strategies will still virtually always succeed in defeating others, but in many cases the outcome will be sufficiently random that it can be treated only by statistical methods. To get some idea of the relative probabilities in the game of imperfect information, we have run 10 tests at each pair of strategies, with random variation in the starting configuration. The matrix values now reported are the probability of a kill as determined by the above testing (**Table 2**). These values should be taken as rough approximations only.

Table 2.
Capture in a game of imperfect information.

	Prey-0	Prey-1	Prey-2	Prey-3
Predator-0	100%	30%	100%	70%
Predator-1	10%	70%	90%	10%
Predator-2	100%	90%	90%	30%

Many of the essential features of the game of perfect information remain: Predator-2 is still an excellent strategy, and the intransitivity between Predator-0 and Predator-1 with respect to Prey-0 and Prey-1 is also evident. However, Predator-2 is no longer a strictly dominant strategy, and Prey-2 loses much of its merit. In purely statistical terms, the optimal strategy is not to select any one pattern of behavior, but to mix them randomly. This game in intensive form has no saddle-point in pure strategies. By defining the probabilistic outcome in terms of a payoff function, however, an optimum randomized behavior defining a saddle-point in mixed strategies could probably be found.

Two Raptors

An identical analysis can be carried out for the two-raptor case (**Table 3**).

Table 3.
Capture with two predators.

	Prey-0	Prey-1	Prey-2	Prey-3
Predators 0,0	90%	50%	100%	40%
Predators 1,1	90%	100%	100%	100%
Predators 2,2	100%	100%	100%	70%
Predators 0,1	100%	100%	100%	70%

In all but two cases, the two-raptor strategy is superior to the one-raptor strategy, assuming both raptors behave the same way. If one functions in

a predictive fashion (Predator-1), while the other accepts the default strategy (Predator-0), the outcome is remarkably similar to two "half-predictive" (Predator-2) raptors, suggesting that specialization of hunting roles might offer considerable advantages. (Examine, for example, the difference between the outcome of a 0,0 strategy vs. the 0,1 strategy.) These results suggest that velociraptors would have strong incentives to hunt in groups, particularly if less experienced raptors (using Predator-0) could cooperate with more experienced raptors (using Predator-1). The total number of kills by mixing two "0" raptors with two "1" raptors is not only greater than for all four hunting in isolation, it is also greater than the total kills with non-experience-mixed pairings.

Weaknesses

- Because of the lack of direct evidence for the biomechanical properties of dinosaurs, the numerical values for certain parameters, particularly the maximum linear acceleration and the reaction time delay, are matters of speculation, and subject to legitimate dispute. Arguments by analogy to modern predators are of dubious value, and if the intent is to compare the results of the simulation to the experimentally observed behavior of modern mammalian predators, then using such references as empirical parameters could be a source of positive bias.
- Because of the intrinsic limitations of a semi-discrete methodology, we cannot be assured that we have really found an optimal solution, only that a given solution is optimal with respect to the others considered.
- The strategies for the two-raptor case are unrealistic, in that they assume that the thescelosaur completely ignores the more distant of the two raptors at any time.
- The introduction of a reaction time delay substantially complicates the task of the raptor(s), and leaves then vulnerable to deception due to a fairly simple feinting turn. The model lacks the ability to "learn" even repeated behaviors without having a new strategy explicitly designed to counter them incorporated into the text of the program.
- By omitting factors such as terrain, visibility, and obstacles, the model gives considerably more advantage to predators than is the case in nature. This accounts for the unreasonably high capture percentages.

Conclusion

Our model provides conclusive support for the hypothesis that the raptors would be advantaged by hunting in pairs. Moreover, it demonstrates that there

is no one optimum pursuit or evasion strategy, and that the flexibility to switch between techniques would serve to favor both the predator and the prey.

Appendix: Typical Simulator Output Illustrating Prey Strategies in Progress

While the predator strategies are fairly obvious, the prey strategies may be more difficult to visualize. The following figures illustrate each of the non-default prey strategies in the best possible situation, that is, against the correspondingly weakest predator strategy. Additionally, **Figure 4** shows the complications arising from imperfect information. Note, especially as compared to **Figure 2**, the thescelosaur's tendency to turn too far and the raptor's delayed reaction to the turn.

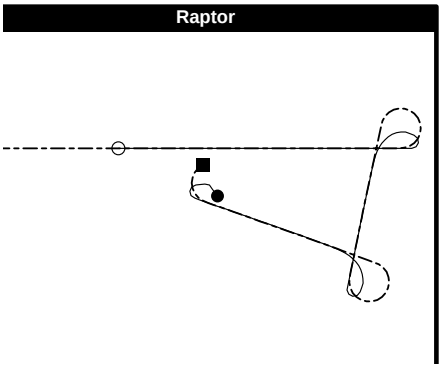

Figure 2. Prey-1 with perfect information.

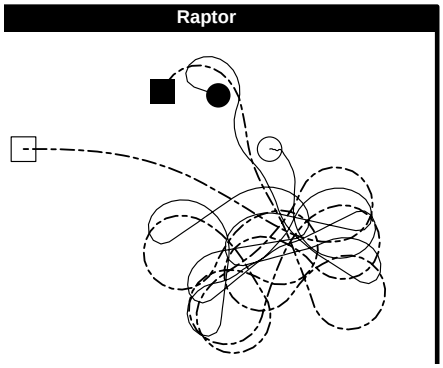

Figure 3. Prey-2 with perfect information.

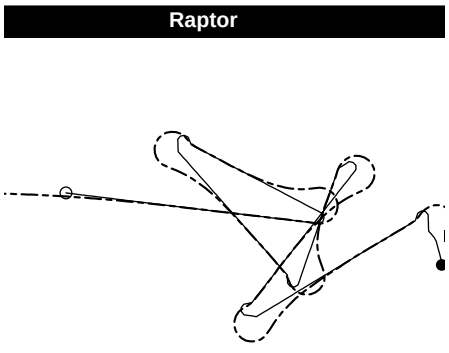

Figure 4. Prey-1 with imperfect information.

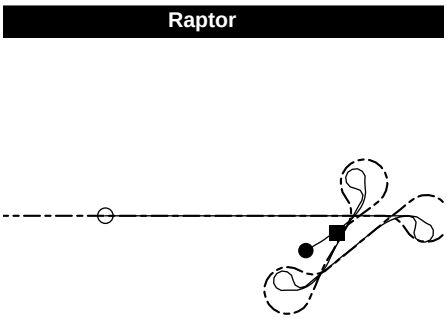

Figure 5. Prey-3 with perfect information.

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原论文及程序

Introduction

We have been asked by a group of paleontologists to model the hunting and evasion strategies of the predator *Velociraptor mongoliensis* as, alone or with a friend, he pursues the prey species *Thescelosaurus neglectus*; both extinct dinosaurs. The predator is 6/5ths faster than the prey, but has a 3-times larger turning radius at top speed. The chase continues for only 15 seconds, whereupon the predator must stop because of a lactic acid buildup in his muscles.

We devised two different models for this pursuit— a purely mathematical model, and a computer model. Both models use the following assumptions:

1.) All animals are represented as points at their respective centers of mass.
2.) The simulations and chase both begin when the predator, slowly sneaking up on the prey, is spotted by the prey. The distance between the animals at this instant is [hi].
3.) The chase lasts a maximum of 15 seconds, or until the prey is killed.
4.) No animal may travel at more than his maximum speed
5.) Each time the prey's distance to the predator is at a local minimum, the prey takes a chance of being killed, p(x).

Introduction to the Mathematical Model

The mathematical model makes the additional assumption that it takes a negligible amount of time to start and stop moving, but that the turning radius consideration limits the maximum angular velocity the animals are able to sustain. This assumption has the advantage of making the analysis possible without a computer, but neglects acceleration. This model is also only capable of analyzing one predator, one prey situations, but for those situations in which it works, it works quite quickly and well.

Using the mathematical model, we analyzed two predator strategies and one prey strategy: the prey strategy is to run away until the predator gets closer than some distance [hb], then make a sharp turn to the left or right. The prey can just squeak around the predator, whose larger turn radius makes him lose a few feet. The predator strategies considered were a hungry predator, who head straight for the current position of the prey, and the maximum turn predator, who uses his knowledge about how the prey reacts to make his turn sharper than the hungry predator.

Introduction to the Computer Model

In the computer model, we start with the assumption that the animals have an ability to accelerate in the direction of their choice. We then stick with a physics-based model of velocity and position, and use an iterative algorithm to figure out where predators and prey go, and which survive. By running several hundred thousand 15-second scenarios, we were able to determine how survival rates change depending on the predator strategy, the prey strategy, the initial separation [hi], the elevation and terrain, animal reaction times, etc.

The computer model supports N predators chasing M prey across any given terrain, and takes into account acceleration/deacceleration, reaction times, two predator strategies, and three prey strategies. Written in C++, and running on a PowerPC 604-based 132 mHz workstation, the program can calculate six hundred 15-second long scenarios using a 1/1000 of a second timestep in one minute. This enabled us to quickly and easily experiment with different strategies, and get statistically meaningful results.

The Velociraptor Problem *or* Lunch on the Run

We have been asked by a group of paleontologists to model the hunting and evasion strategies of the predator *Velociraptor mongoliensis* as, alone or with a friend, he pursues the prey species *Thescelosaurus neglectus*; both extinct dinosaurs. The predator is faster than the prey, but has a larger turning radius at top speed. The chase can continue for only 15 seconds.

Using both mathematical and computer models, we determined how the following factors affect survival rates:

1.) Predator Strategy
2.) Prey Strategy
3.) Reaction Time
4.) Elevation/Terrain
5.) Distance between the animals when the chase starts, h_i.

Since our models are able to produce probability of survival rates, we were able to graphically find which of the above factors most influences survival. We discovered that, despite quite different assumptions in our mathematical and computational models, both models agreed that the single most important factor in determining survival rates is the initial separation, h_i.

This is because our most successful prey strategy was to flee the predator directly, making a sharp turn at the critical distance h_b (1.619 m) from the predator. By utilizing its smaller turn radius in this fashion, the prey can get slightly ahead of the predator. The predator rapidly closes this distance, but then the prey repeats the same trick. Each time he does so, however, he takes another chance that the predator will kill him. Hence the number of breakaway turns he has to make is the most important factor in determining his chances for survival.

Various hunting strategies are also considered, as well as the two-predator case (whereupon sharply veering is less effective because a second predator, following the first, can make the kill). Nonetheless; because h_i is so important, our advice is this:

Predator, be stealthy. Prey, be vigilant.

The Computer Simulation Model:

Our simulator is physics-based, modeling each animal as a point at its center of mass, which is capable of choosing a direction in which to accelerate. This model outputs smooth curves representing the path of the center of motion of each animal. It does so by iteratively solving these vector differential equations using Euler's method (outlined below):

1.) $$\frac{d\vec{P}}{dt} = \vec{V} \qquad \frac{d\vec{V}}{dt} = \vec{A}$$

(where P,V, and A are the position, velocity, and acceleration as functions of time)

The simulation cycle begins by determining the optimal direction for the animal to accelerate based on the animal's strategy. The animal then accelerates at his maximum acceleration in that direction.

Values for this upper bound on acceleration are parameters to the simulation, derived from the minimum turn radius at top speed given for each animal in the problem description. We can work this out with the central acceleration formula, $a=v^2/r$.

The acceleration vector is then added to the local elevation gradient multiplied by the acceleration of gravity, making it easier to go down a hill than up. The elevation gradient is determined from a bi-linearly interpolated elevation grid, which is read in from an elevation file.

Once the animal has decided on a direction to accelerate in, the simulation applies one step of Euler's method to update his position and velocity vectors. This method of solving differential equations works by noting that the first two terms of the Taylor series expansion of a function of time depend only on the present condition of the system.

2.) $$f(t+h)=f(t)+h*f'(t)$$

Hence, if we know an animal's position, velocity, and acceleration at some time t, we can figure out a first-order approximation for where the animal will be at time $t+\Delta t$ by substituting equation 1.) into 2.):

$$\vec{P}(t+\Delta t)=\vec{P}(t)+\Delta t*\vec{V}(t)$$

$$\vec{V}(t+\Delta t)=\vec{V}(t)+\Delta t*\vec{A}(t)$$

In this way, using a Δt of 1/1000 of a second, our simulation works out, given the initial conditions and an acceleration vector, the velocity and position of each animal at any instant.

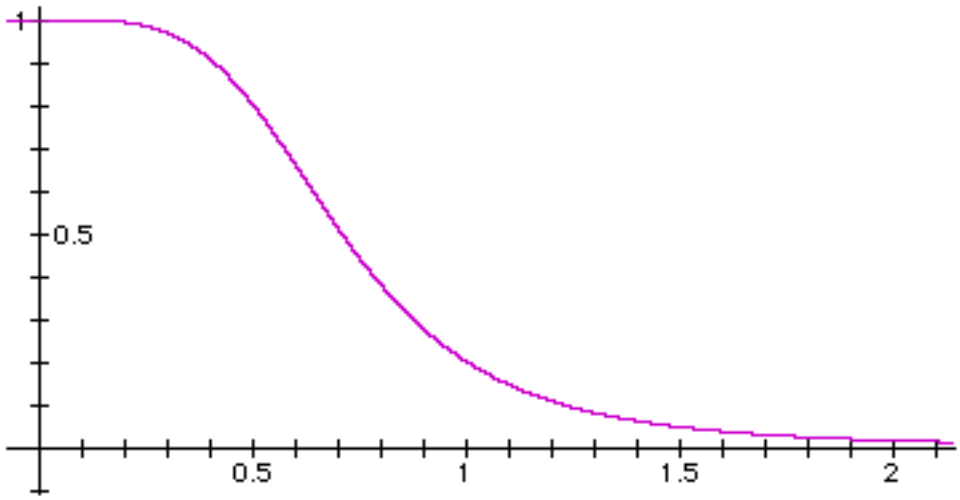
This simulation method is quite general, currently modeling several hunting and evasion strategies, elevation, reaction times, numeric or graphical output, and N predators vs. M prey. It is written in C++, and the entire program (10 pages) appears in appendix [C].

Life and Death in the Computer:

In the computer model, each time the prey passes near the predator, a probability of death function is evaluated. The probability of death given closest distance formula we chose is:

$$P(x)=\frac{1}{1+4*x^4}$$

This probability function looks like:



This function was chosen because the raptor's head and upper body is small and lightweight, while the main killing machinery are attached to the raptor's feet in the form of a large, curved hook. Hence it was assumed that the raptor would have to get his center of mass one leg-length (0.5 m) away from his prey's center of mass in order to substantially ensure a kill, but there remains a reasonable probability of a kill farther away if the predator uses his head or arms, or if the prey's tail is caught.

Since there is always some chance of a kill, at any distance (due to events such as the prey tripping, or breaking his leg, etc. which are not otherwise addressed), the probability function is asymptotic with respect to the x axis.

Simulated Hunting and Evasion Strategies:

In the simulation, each animal has perfect awareness of its own position and velocity, but only knows other animal's position and velocity one reaction time ago (e.g. 20 milliseconds).

Visually, this assumption is less justifiable for the prey than for the predator, since the prey would have to look backwards to see the predator, while the predator keeps his gaze fixed on the prey. But if we consider auditory perception as well, this "delayed telepathic knowledge" is fairly reasonable— the predator both sees and hears the prey, and the prey hears the predator's footsteps behind him.

The interesting question is: if you have some idea of where your prey or predator is, then how do you respond? Various modeled strategies for both predator and prey are discussed next.

Modeled Hunting Strategies:

Every orientation of two animals can be simulated by starting the predator at the origin and the prey a distance hi along the x-axis, where hi varies in this simulation from 15m to 50m.

The Hungry Hunter:

A hungry hunter heads straight for the current position of the closest prey. This hunter is subject only to the constraints of his knowledge about the prey's position and his own ability to get there. This is the only strategy modeled in the mathematical model, and has been rigorously analyzed there.

The Smart Hunter:

A smart hunter determines the point where he can intercept the closest prey, and heads straight for that point. A detailed derivation of the quadratic equation the smart hunter solves to determine the intercept point is given in Appendix [A].

Modeled Evasion Strategies (with 1-predator graphical results):

The Frightened Prey:

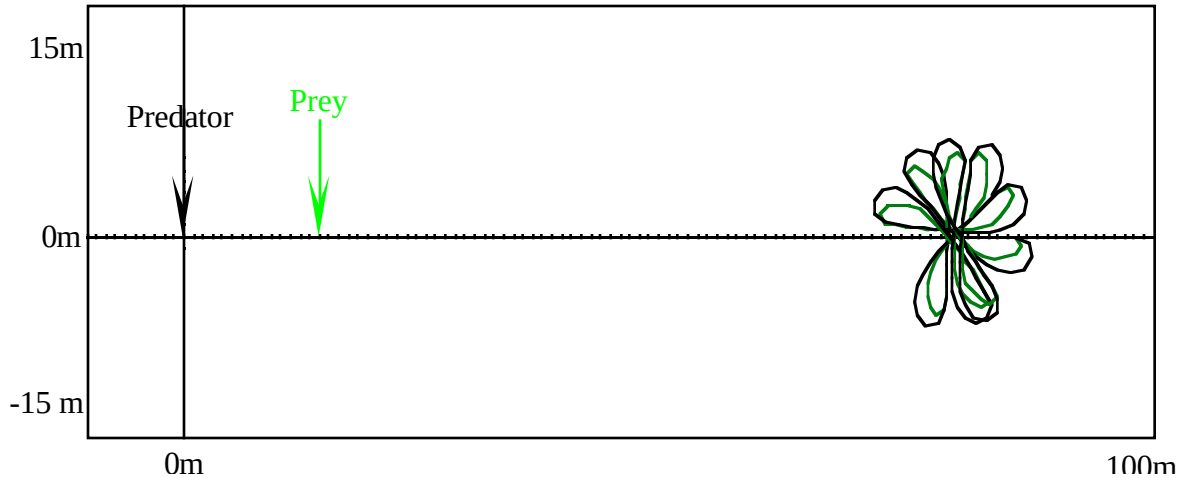
Frightened prey flee straight away from the nearest predator. These prey will always die if the predator can close the distance between them before his 15 seconds are up. At a closing rate of 2.7777 m/s, the prey will always be overtaken and die if hi (the initial separation of the predator and prey) is less than 41.6666 meters.

The Smart Prey:

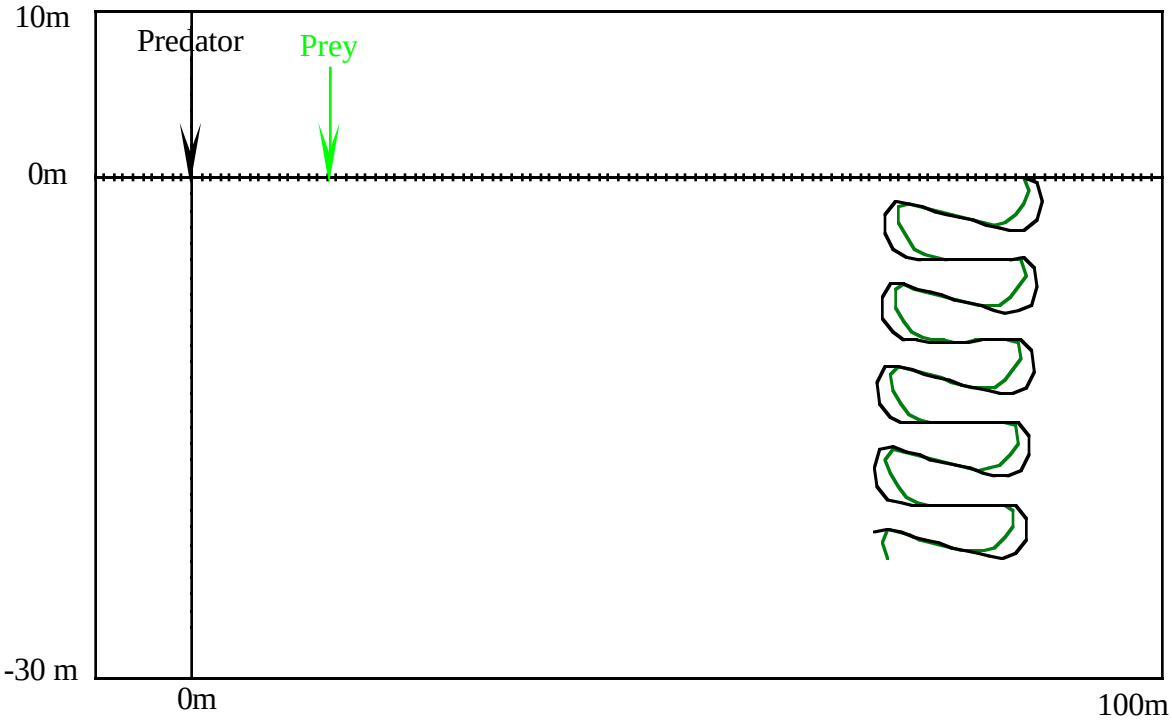
When the nearest predator is far away, smart prey act like frightened prey and flee straight away. But when the predator closes to a critical distance hb (1.619 m— see Appendix [B] for derivation), the prey will dart either to the left or to the right. By using his much smaller turn radius, the prey buys some distance, which is again closed by the predator, whereupon the prey can dart again.

For the two-animal case, there are two possibilities for what this looks like: smart prey versus hungry predator, and smart prey versus smart predator.

Smart Prey (grey) versus Hungry Predator (black), starting at hi=15 meters.



Smart Prey vs. Smart Predator, starting at hi=15 meters:
(once again, the prey makes it).



The Gradient Prey:

Gradient prey are the same as smart prey when the nearest predator is very close (less than hb) or when there is only one predator. When there are two or more predators, the gradient prey runs in the direction of least danger— i.e. along the gradient of the danger function. If the danger from each predator decreases with the inverse square of its distance, and the danger from each predator is added to produce the danger function, then the danger gradient can be computed by adding the gradient of the danger from each predator. I.E.

$$\nabla \sum \frac{1}{\mathbf{d}_i^2(x,y)} = \sum \nabla \frac{1}{\mathbf{d}_i^2(x,y)}$$

This can be quickly and easily done in the simulation. The gradient prey is only different from the smart prey when there are two predators, so their graphs and analysis come in the next section.

The 2-Predator Situation:

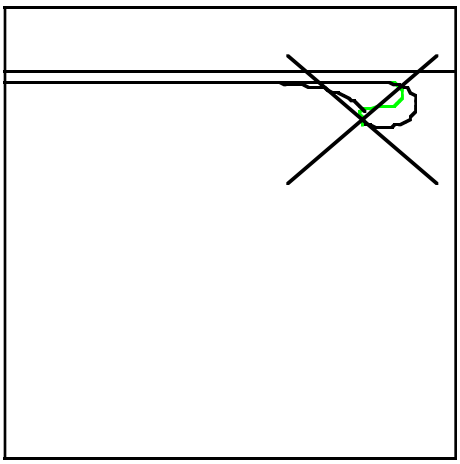
After some experimentation, we've determined that the best strategy for the predators is for the second predator to follow the first one (by about 8 meters), since our prey's strategies hinge around getting behind the first predator.

Frightened Prey:

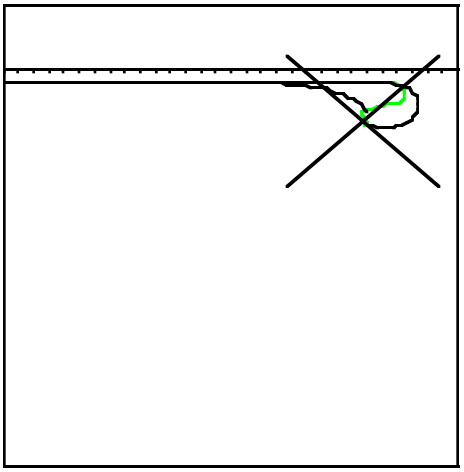
Because frightened prey always die unless they detect the predator(s) more than 41.6666 meters away, there is no advantage to be had in chasing them with 2 predators. (plots from 60 to 90 m X, 5 to -30 m Y, predator 1 at -8 m, predator 2 at 0m, prey at 15 m)

Smart Prey:

vs. 2 Smart Predators,

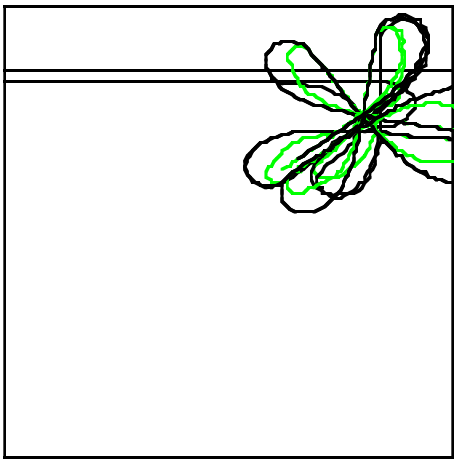


vs. 2 Hungry Predators

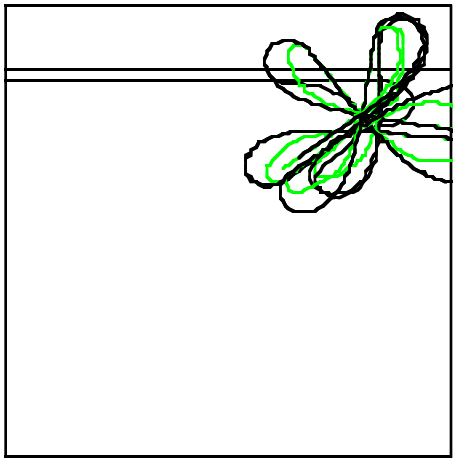


Gradient Prey:

vs. 2 Smart Predators,



vs. 2 Hungry Predators



Computational Results:

If the animals started running at a separation with the beta distribution, the overall survival rates (out of 2,000 runs each) and their standard errors were:

1-on-1:

vs.	Hungry Predator	Smart Predator
Frightened Prey	3.20 ±0.39%	3.85 ±0.43%
Smart Prey	35.7 ±1.1%	35.5 ±1.1%
Gradient Prey	35.7 ±1.1%	35.5 ±1.1%

2-on-1:

vs.	2 Hungry Predators	2 Smart Predators
Frightened Prey	3.20 ±0.39%	3.85 ±0.43%
Smart Prey	3.70 ±0.42%	4.00 ±0.44%
Gradient Prey	3.45 ±0.41%	9.40 ±0.65%

We can see from these tables how well each strategy worked:

Frightened Prey: Always fleeing the predator unconditionally leads to the lowest survival rates. This is a bad choice.

Smart Prey: The smart prey, which darts to the side when the predator closes to some critical distance behind him, did quite well in the single-predator runs. By the numbers, this is the best overall evasion strategy for the Thescelosaurus.

Gradient Prey: As discussed above, the gradient prey is the same as the smart prey for the single-predator runs. On the two-predator runs, the gradient prey did quite well against the smart predators, but more poorly than the smart prey against the hungry ones!

Hungry Predator: This predator killed the most prey in both the single-predator and hungry-predator scenarios. Because he's never mislead by the prey's movements, he relentlessly brings them down better. Our data indicates that this is the best overall hunting strategy for the Velociraptor— always head straight for where the prey currently is.

Smart Predator: This predator is too easily mislead by the prey's movements, and, except alone against smart prey, is always worse than the hungry predator. He is especially poor in the two-predator scenario against the gradient prey.

Computer Model Overview and Assumptions:

In the computer model, we decided to use physics-based motion, where accelerations were finite, instead of the more-theoretical motion of the mathematical model (in which acceleration is not considered). While there is no explicit limit on the turn radius (it isn't even calculated in the simulation), when we restrict our creatures' acceleration to the assumed value, then at top speed the animals can turn at no less than their minimum turning radius.

Hence, although the mathematical and computation models' methods differ, our results (minimum turn radius at top speed) are the same. This seemed to recur throughout the course of our numerical and mathematical experimentation— the mathematical and computational models would attack the same problem from different directions, and end up with the same answer.

Computer Model Assumptions:

1.) The predator (or predators) will slowly sneak up on the prey as long as the prey doesn't start to run away. At some distance hi (where 15m<hi<50m), the prey will notice that it is being approached and start running. This is where the simulation starts.

2.) Each animal has a maximum speed.

3.) Each animal has a maximum acceleration, derived from the equation from physics:

$$a=v^2/r$$

where v is the animal's maximum speed (see assumption 2) and r is their minimum turn radius at top speed.

4.) At each instant, each animal figures out where they would like to apply their acceleration, based on their present position, and the positions of every other animal. However, they only know the position of the other animals 20 milliseconds (their reaction time) ago.

5.) Whenever a predator stops getting closer to the prey (i.e., the predator has made a close pass by the prey) a probability-of-death function of the smallest distance between the predator and prey is evaluated (the prey then has a p(x) chance of death). That function is

$$p(x)=1/(1+4*x^4)$$

where x is the predator's closest approach to the prey, and p(x) is the prey's chance of death.

6.) Simulation continues until the prey dies, or 15 seconds elapse.

Conclusion:

Since our models are both able to produce probability of survival rates, we were able to graphically find which of the above factors most influences survival. We discovered that, despite rather large differences between our mathematical and computational models, in both models the single most important factor in determining survival rates is the distance between the prey and predator at the moment the prey detects him, h_i.

This is because the most successful prey strategy was to flee the predator directly until it closed to other critical distance h_b (1.619 m), then make a sharp turn. By utilizing its smaller turn radius in this fashion, the prey can get slightly ahead of the predator. The predator rapidly closes this distance, whereupon the prey can pull the same trick again. Each time he does so, however, he takes another chance that the predator will kill him. Hence the number of breakaway turns he has to make is the single most important factor in determining his chances for surviving one predator. His chances of survival are very low (<4%) in the two-predator case, when one predator trails the first by 8 m.

On the following pages, you will find graphs of the probability of survival versus [hi]. Figure 11 is predicted probability of survial graph produced by the mathematical model, for the case of a smart prey being pursued by a hungry hunter. Figure 12 is the graph of the actual probability, produced with 50,000 runs of the computer model for various [hi]s. The graphs, aside from the static produced by the random variation of our small sample size, are nearly identical.

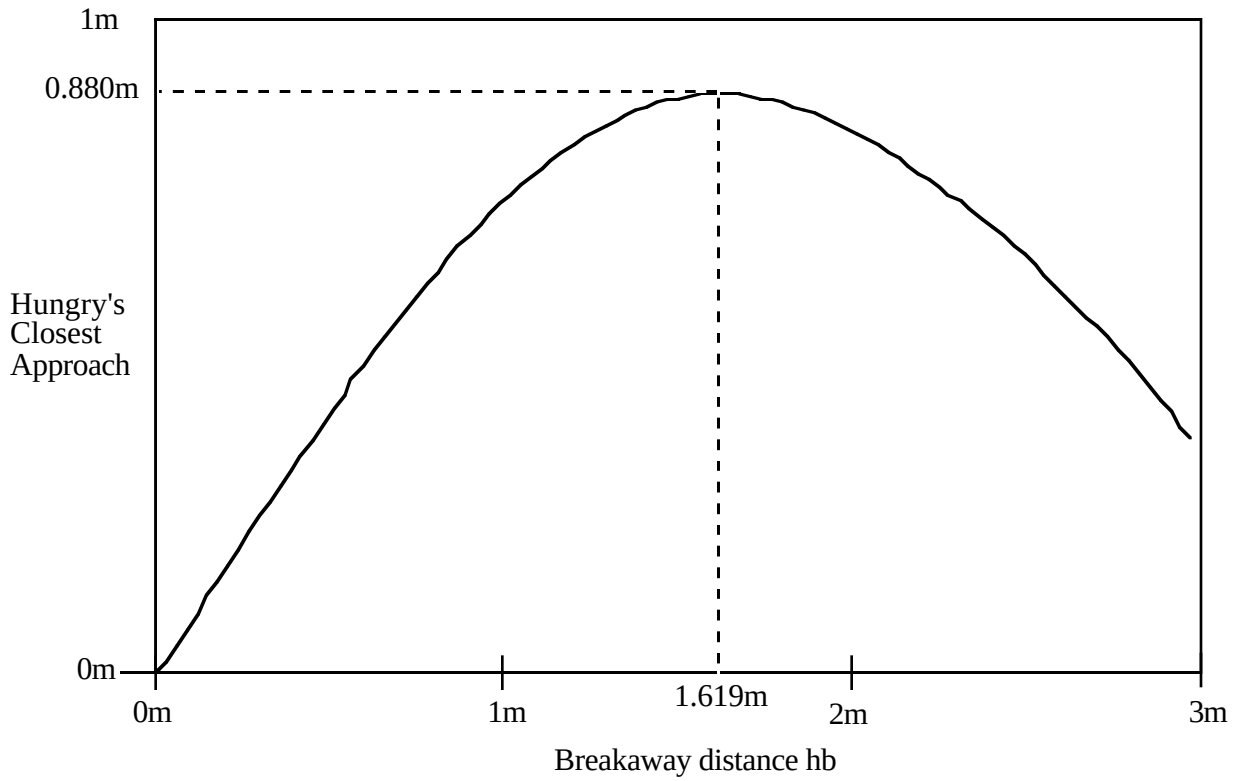
Using the mathematical model, we found that while changing the turn radii affected the sharpness of curvature of the probability graphs, it did not diminish the importance of [hi]. Using the computational model, we tried many different hunting and evasion strategies, and still [hi] was the most important factor.

Hence, it seems that in the final analysis, the prey's most important task is to be as paranoid as possible, and detect the predator as soon as possible. The predator's most important task is to blend in as well as possible, and not be detected.

Finding the breakaway distance hb:

In both the one- and two-predator models, the prey is often chased down. Since the predator(s) are faster, if the prey doesn't veer sharply from a straight line, it will die (this is just what the frightened prey does). To avoid this fate, both the smart and gradient prey veer sharply to the left or right if the nearest predator is less than some optimal distance hb away.

We found the optimal value for hb by plotting it versus the closest approach a hungry predator makes.



Hence, we can see that versus a hungry predator, if the prey makes his turn to the left or right when the predator is exactly 1.619m away, the prey will have the best probability of sucess.

A similar curve was plotted for the smart predator, and though the value for hb was nearly the same, the predator got closer.

Simulator Code in C++:

Salient Features:

- Models several hunting and evasive strategies.
- Models NxM predator/prey situations.
- Models elevation.
- Models reaction time.

Program code:

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include "Plot.h"//Proprietary graphical output library

//Program Control:
const char *animalString="sH";//lowercase--prey.  Uppercase--predator.
//s,S=smart    f=frightened prey    H=hungry predator   t,T=turning

typedef enum {drawActivity,drawGraph}drawWhatType;
const drawWhatType drawWhat=drawActivity;

typedef enum {doActivity,doHistogram}doWhatType;
const doWhatType doWhat=doHistogram;
const int numHistoBuckets=100;
real histo[numHistoBuckets];

//Constants:
/*Critical animal parameters: speeds are in m/s, accellerations in m/(s*s),
reaction times in secs.*/
const real maxPredKMPH=60,predTurnRadius=1.5,predReactTime=0.02;
const real maxPreyKMPH=50,preyTurnRadius=0.5,preyReactTime=0.02;
const real maxTime=15;//run is for 15 seconds.
const real gravAccelleration=9.80;//in m/(s*s)
const long steps_per_sec=1000;//simulation steps per second

const real timeStep=1.0/steps_per_sec;
const real maxReactionTime=0.03;
const long maxReactionSteps=maxReactionTime*steps_per_sec;
const real maxPredSpeed=maxPredKMPH/3.6;
const real maxPredAccell=maxPredSpeed*maxPredSpeed/predTurnRadius;//a=v^2/R
const real maxPreySpeed=maxPreyKMPH/3.6;
const real maxPreyAccell=maxPreySpeed*maxPreySpeed/preyTurnRadius;//a=v^2/R
const long predReactSteps=predReactTime*steps_per_sec;
const long preyReactSteps=preyReactTime*steps_per_sec;

//Global:
int drawThisRun=0;

//Type definitions:
typedef struct point { real x,y;} point;
#define vector point

//Elevation Utilities:
real *elevation=NULL;

void readInElevationFile(char *name,real scaleBy,short drawAfterwards);
real getHeight(real inx,real iny);
vector getGravAccelleration(point p);
```

```
void readInElevationFile(char *name,real scaleBy,short drawAfterwards)
{
	if (!name)
		return;
	FILE *f=fopen(name,"r+");
	if (!f)
	{
		printf("Elevation file %s not found.  Continuing anyway.\n",name);
		return;
	}
	elevation=(real *)malloc(sizeof(real)*10*10);
	real maxElev=-10000000;
	for (int y=0;y<10;y++)
		for (int x=0;x<10;x++)
		{
			float inValue;
			fscanf(f,"%f",&inValue);
			inValue*=scaleBy;
			elevation[y*10+x]=inValue;
			if (inValue>maxElev)
				maxElev=inValue;
		}
	fclose(f);
	real maxInverse=1/maxElev;
}
real getHeight(real inx,real iny)
{//return a bilinearly interpolated elevation.
	real x=(inx+250)/500*9;
	real y=(iny+250)/500*9;
	if (x<0) x=0;
	else if (x>=9) x=8.9999999;
	if (y<0) y=0;
	else if (y>=9) y=8.9999999;
	int ix=x,iy=y;
	real fx=x-ix,fy=y-iy;
	register real e00=elevation[iy*10+ix+00],
		e10=elevation[iy*10+ix+01],
		e01=elevation[iy*10+ix+10],
		e11=elevation[iy*10+ix+11];
	return (e11*fx+e01*(1-fx))*fy+(e10*(fx)+e00*(1-fx))*(1-fy);
}
vector getGravAccelleration(point p)
{
	vector ret;
	if (elevation)
	{
//Force vector due to surface reaction=Gradient at surface*Gravitational accelleration*Mass
//(But because mass cancels out of the final motion equations, we're sticking with
//  accelleration);
		ret.x=(getHeight(p.x+0.5,p.y)-getHeight(p.x-0.5,p.y))
			*gravAccelleration;
		ret.y=(getHeight(p.x,p.y+0.5)-getHeight(p.x,p.y-0.5))
			*gravAccelleration;
	} else {ret.x=ret.y=0;}
	return ret;
}
```

```
//Vector/point utilities:
#define sqrt(x) (double)(sqrt((float)(x)))
real ptDist(const point &a,const point &b);
real ptDist(const point &a,const point &b)
{
	real xDel=a.x-b.x,yDel=a.y-b.y;
	return sqrt(xDel*xDel+yDel*yDel);
}
real vecLen(const vector &v);
real vecLen(const vector &v)
{
	return sqrt(v.x*v.x+v.y*v.y);
}
void limitVector(vector *v,const real maxLen);
void limitVector(vector *v,const real maxLen)
{
	real vecLength=vecLen(*v);
	if (vecLength>maxLen)
	{//scale the vector back so its length is maxLen.
		real scaleFactor=maxLen/vecLength;
		v->x*=scaleFactor;
		v->y*=scaleFactor;
	}
}
#undef sqrt
//Probability:
real getNormalRand(void);
inline real getNormalRand(void){return (real(rand())/real(RAND_MAX));}
real getRandProd(int numProducts);
real getRandProd(int numProducts)
{
	real ret=1;
	for (int i=0;i<numProducts;i++)
		ret*=getNormalRand();
	return ret;
}
real getRand(void);
real getRand(void) {return getNormalRand()*2-1;}
int did_I_die(real closestApproachToPredator);
int did_I_die(real x)
{
	real probabilityOfDeath=1.0/(1.0+4.0*x*x*x*x);
	if (getNormalRand()<=probabilityOfDeath)
		return 1;
	else return 0;
}
```

```
//The Animal Class:
class animal
{
	public:
		int isAlive,isPrey;
		real totalDistanceTravelled;
		point oldDrawnPos,pos;
		vector vel;
		real topSpeed,maxAccell;
		color animalColor;

		point oldPositions[maxReactionSteps];
		long oldPosIndex;
		long reactionSteps;
#define getReactPos(anim) anim->oldPositions[(anim->oldPosIndex \
			+reactionSteps)%maxReactionSteps]
#define getReactPos2(anim) anim->oldPositions[(anim->oldPosIndex \
			+reactionSteps+1)%maxReactionSteps]

		animal(real x,real y,real xv,real yv)
		{
			isAlive=1;
			pos.x=x;
			pos.y=y;
			oldDrawnPos=pos;
			vel.x=xv;
			vel.y=yv;
			totalDistanceTravelled=0;
			oldPosIndex=maxReactionSteps-1;
			for (int i=0;i<maxReactionSteps;i++) oldPositions[i]=pos;
		}
		void advance(const vector &force)//go in the direction of force.
	{
			vector accell=force;
//scale accelleration so that we're always running at full speed
			accell.x*=10000;
			accell.y*=10000;
//limit accelleration to the animal's maximum
			limitVector(&accell,maxAccell);
			vector gravAccell=getGravAccelleration(pos);
//use Euler's method to find new position
			pos.x+=vel.x*timeStep;
			pos.y+=vel.y*timeStep;
			oldPosIndex--;//update reaction time index
			if (oldPosIndex<0) oldPosIndex=maxReactionSteps-1;
			oldPositions[oldPosIndex]=pos;
			totalDistanceTravelled+=vecLen(vel)*timeStep;
//use Euler's method on the velocity
			vel.x+=(gravAccell.x+accell.x)*timeStep;
			vel.y+=(gravAccell.x+accell.y)*timeStep;
//Clip the velocity to the maximum speed
			limitVector(&vel,topSpeed);
		}
		virtual void decide(animal **animals,int numAnimals) {}
		void draw(void)
		{
			PlotColor(animalColor);
```

```
                  PlotStart(oldDrawnPos.x,oldDrawnPos.y);
                  PlotPoint(pos.x,pos.y);
                  oldDrawnPos=pos;//Save old Position
            }
            virtual void die(void) //kills the simulated animal
            {
                  isAlive=0;
                  if (drawThisRun)//Mark a big black X where we died.
                  {
                        PlotColor(black);
                        PlotStart(pos.x+10,pos.y+10);
                        PlotPoint(pos.x-10,pos.y-10);
                        PlotStart(pos.x-10,pos.y+10);
                        PlotPoint(pos.x+10,pos.y-10);
                  }
            }
            virtual void destroy(void) //destroys the C++ object
            {
                  delete this;
            }
};
//The Prey class: animals that can be eaten.
class prey:public animal{public:

typedef enum {predGettingCloser,predGettingFarther} predSlopeType;
      real lastPredDist;
      predSlopeType slope;
      prey(real x,real y,real xv,real yv):animal(x,y,xv,yv){
            animalColor=green;topSpeed=maxPreySpeed;maxAccell=maxPreyAccell;
            isPrey=1;reactionSteps=preyReactSteps;
            lastPredDist=100000000;slope=predGettingCloser;
      }
      virtual void flee(animal *predator,real distance)=0;
      virtual void decide(animal **animals,int numAnimals)
      {
      //Find the distance to the closest predator
            real closestPredatorDist=100000000;
            real trueClosestDist=10000000;
            animal *closestPredator=NULL;
            for (int i=0;i<numAnimals;i++)
                  if (!animals[i]->isPrey)//(only worry about predators).
                  {
                        real
distToPredator=ptDist(getReactPos(animals[i]),pos);
                        if (distToPredator<closestPredatorDist)
                        {
                              closestPredatorDist=distToPredator;
                              closestPredator=animals[i];
                        }
                        real trueDist=ptDist(animals[i]->pos,pos);
                        if (trueDist<trueClosestDist)
                              trueClosestDist=trueDist;
                  }
      //Check to see if we've been killed.
            if (trueClosestDist<(lastPredDist))//a predator is gaining on us
                  slope=predGettingCloser;
            else if (trueClosestDist>(lastPredDist))//we're getting farther
```

```
away from the predator
            {
                  if (slope==predGettingCloser)
                  {//we've had a minima in the last step, so we now
                        //take the chance of getting killed.
                        if (drawThisRun)
                        {
                              PlotColor(blue);
                              #define boxSize 0.5
                              PlotBox(pos.x-boxSize,pos.x+boxSize,
                                    pos.y-boxSize,pos.y+boxSize);
                        }
                        if (did_I_die(lastPredDist))
                        {
                              die();//if we lost the gamble, we're dead.
                              return;
                        }
                  }
                  slope=predGettingFarther;
            }
            lastPredDist=trueClosestDist;
      //Otherwise, we have to flee the closest predator.
            if (closestPredator)
                  flee(closestPredator,closestPredatorDist);
      }
};
//scared prey flee straight away from predator
class scaredPrey:public prey {public:
      scaredPrey(real x,real y,real xv,real yv):prey(x,y,xv,yv) {}
      virtual void flee(animal *predator,real distance)
      {
            vector away;
            away.x=pos.x-getReactPos(predator).x;
            away.y=pos.y-getReactPos(predator).y;
            advance(away);
      }
};
//Smart prey flee straight away when predator
//is far away, but veer off to the side when he's close.
class smartPrey:public prey {public:
      int headedLeft;
      smartPrey(real x,real y,real xv,real yv):prey(x,y,xv,yv) {headedLeft=0;}
      virtual void flee(animal *predator,real distance)
      {
            vector accell,away,perp;
            away.x=pos.x-getReactPos(predator).x;
            away.y=pos.y-getReactPos(predator).y;
            perp.x=away.y;
            perp.y=-away.x;
            if (headedLeft)
            {
                  perp.x*=-1.0;
                  perp.y*=-1.0;
            }
            if (distance<1.619)
            {//Imminent death! Better veer now.
                  accell.x=perp.x;//-away.x*0.5;
```

```
                  accell.y=perp.y;//-away.y*0.5;
                  advance(accell);
            } else {//A big separation-- run away!
                  headedLeft=!headedLeft;//Randomize next bolt direction.
                  advance(away);
            }
      }
};
//Turning prey run in little clockwise loops.
class turningPrey:public prey {public:
      turningPrey(real x,real y,real xv,real yv):prey(x,y,xv,yv) {}
      virtual void flee(animal *predator,real distance)
      {
            vector accell;
            accell.x=vel.y;
            accell.y=-vel.x;
            advance(accell);
      }
};

//The Predator class
class predator:public animal {public:
      predator(real x,real y,real xv,real yv):animal(x,y,xv,yv)
      {     animalColor=red;topSpeed=maxPredSpeed;maxAccell=maxPredAccell;
            isPrey=0;reactionSteps=predReactSteps;}
      virtual void chase(animal *prey,const vector &toPrey)=0;
      virtual void decide(animal **animals,int numAnimals)
      {
      //Find the closest prey...
            real closestPreyDist=100000000;
            animal *closestPrey=NULL;
            vector toClosestPrey;
            for (int i=0;i<numAnimals;i++)
                  if (animals[i]->isPrey)
                  {
                        vector toPrey;
                        toPrey.x=getReactPos(animals[i]).x-pos.x;
                        toPrey.y=getReactPos(animals[i]).y-pos.y;
                        real distToPrey=vecLen(toPrey);
                        if (distToPrey<closestPreyDist)
                        {
                              closestPreyDist=distToPrey;
                              toClosestPrey=toPrey;
                              closestPrey=animals[i];
                        }
                  }
      //... then chase it (however we happen to chase it).
            if (closestPrey)
                  chase(closestPrey,toClosestPrey);
      }
};
```

```
//Hungry predators run straight for prey.
class hungryPredator:public predator {public:
      hungryPredator(real x,real y,real xv,real yv):predator(x,y,xv,yv) {}
      virtual void chase(animal *prey,const vector &toPrey)
      {
            advance(toPrey);
      }
};
//Smart predators predict the future position
//of their prey.

class smartPredator:public predator {public:
      smartPredator(real x,real y,real xv,real yv):predator(x,y,xv,yv) {}
      virtual void chase(animal *prey,const vector &toPrey)
      {
            point nowPos=getReactPos(prey);
            point lastPos=getReactPos2(prey);
            vector velPrey,predictedPrey;
            velPrey.x=nowPos.x-lastPos.x;
            velPrey.y=nowPos.y-lastPos.y;
      //Compute quadratic coefficients for prey predictor:
            const real
rSqOverTSq=maxPredSpeed*maxPredSpeed/(maxPreySpeed*maxPreySpeed);
            real a=(velPrey.x*velPrey.x+velPrey.y*velPrey.y)*(1-rSqOverTSq);
            real b=2*velPrey.x*toPrey.x+2*velPrey.y*toPrey.y;
            real c=toPrey.x*toPrey.x+toPrey.y*toPrey.y;
            real d=b*b-4*a*c;
            if ((d<0.0)||(a==0.0))
                  advance(toPrey);
            else
            {
                  real t=(-b+sqrt(d))/(2*a);//quadratic time of intercept
                  predictedPrey.x=velPrey.x*t+toPrey.x;
                  predictedPrey.y=velPrey.y*t+toPrey.y;
                  advance(predictedPrey);
            }
      }
};

//Turning predators run in larger clockwise loops.
class turningPredator:public predator {public:
      turningPredator(real x,real y,real xv,real yv):predator(x,y,xv,yv) {}
      virtual void chase(animal *prey,const vector &toPrey)
      {
            vector accell;
            accell.x=vel.y;
            accell.y=-vel.x;
            advance(accell);
      }
};
```

```
//The main simulator loop: run all the animals around until either all
//the prey are dead, or our 15 seconds are up.
void runFor15s(animal **animals,int numAnimals);
void runFor15s(animal **animals,int numAnimals)
{
      for (long time=0;time<maxTime*steps_per_sec;time++)
      {
            int livingPreyExist=0;
            for (int animalNo=0;animalNo<numAnimals;animalNo++)
            {
                  animal *thisAnimal=animals[animalNo];
                  if (!thisAnimal->isAlive)
                        continue;
                  thisAnimal->decide(animals,numAnimals);
                  if (thisAnimal->isPrey)
                        livingPreyExist=1;
            }
            if (!livingPreyExist)//out of prey-- it's over.
                  return;
            if (drawThisRun)
                  if ((time&0x0f)==0)
                        for (int drawNo=0;drawNo<numAnimals;drawNo++)
                              if (animals[drawNo]->isAlive)
                                    animals[drawNo]->draw();
      }
}

animal *animals[10];
int numAnimals;
int killedPrey=0,totalPrey=0;

void createAnimals(real minSeparationBtwPredatorAndPrey);
void createAnimals(real separation)
{
      numAnimals=strlen(animalString);
      for (int i=0;animalString[i];i++)
      {
            real px=0,py=0;
            const real predSep=5;//Distance between each predator
            if (!doWhat==doHistogram)
                  {px=getRand()*20;py=getRand()*20;}
            switch(animalString[i])
            {
      //s,S=smart    f=frightened prey    H=hungry predator   t,T=turning

case 's':animals[i]=new smartPrey(separation,py,0,0);totalPrey++;break;
case 'f':animals[i]=new scaredPrey(separation,py,0,0);totalPrey++;break;
case 't':animals[i]=new turningPrey(separation,py,0,0);totalPrey++;break;
case 'S':animals[i]=new smartPredator(-(i-1)*predSep,py,0,0);break;
case 'H':animals[i]=new hungryPredator(-(i-1)*predSep,py,0,0);break;
case 'T':animals[i]=new turningPredator(-(i-1)*predSep,py,0,0);break;
                  default:
                        printf("Unrecognized animal type: %c.\n");
                        exit(0);
            }
      }
}
```

```
void destroyAnimals(void);
void destroyAnimals(void)
{
      for (int i=0;i<numAnimals;i++)
      {
            if (animals[i]->isPrey&&!animals[i]->isAlive)
                  killedPrey++;
            animals[i]->destroy();
      }
}
//getBetaH and getGammaH return an inital separation,
//    appropriately distributed.
real getBetaH(void);
real getBetaH(void)
{
      const int v=3,w=4;
      real m=-log(getRandProd(v)),n=-log(getRandProd(w));
      return 15.0+35.0*m/(m+n);
}
real getGammaH(void);
real getGammaH(void)
{
      const int a=6,b=5;
      return -b*log(getRandProd(a));
}
//Main runs and plots some number of simulation runs, possibly plotting them.
main()
{
      srand(2);
      readInElevationFile(NULL,0,0);//"Mountain.elev",2000,0);
      const real maxCoord=150;
      PlotAxes(-maxCoord,maxCoord,-maxCoord,maxCoord,300,300);
      for (long whichRun=0;whichRun<500;whichRun++)
      {
            createAnimals(getGammaH());
            if (drawWhat==drawActivity)
                  drawThisRun=whichRun%20==0;
            runFor15s(animals,numAnimals);
            destroyAnimals();
      }
      if (drawWhat==drawActivity)
            printf("Survival Rate: %.3f%%\n",(1-
real(killedPrey)/real(totalPrey))*100.0);
      return 0;
}
```

Appendix D:
Rather uninteresting results:

Our computer model was capable of handling any reaction time, and any elevation map. However, preliminary experimentation with these factors did not lead to any interesting or useful results.

Making the reaction time much longer (greater than 0.1 s) tended to hurt the predator's chances of sucess (especially the smart predator, which is then predicting based on older data), but this can probably be attributed to mere increased difficulty in locating anything when you're moving at 60 kmph and you can only see where things were 0.1 s (or 1.6 meters) ago. So this factor didn't seem very crucial or interesting.

Elevation, while having many interesting possiblities for research, is usually too dependant on specific terrain features combined with pre-hunt predator collusion (e.g. to run the prey into a narrowing, steep-walled valley) to produce any generally useful results. The only general observations we can offer is that if there is a steep peak between the predator and prey, it makes the predator's job harder (and decreases his chances of a kill); and if the predator gets to start down the slope of a bowl, and the prey has to climb up the slope, that makes the prey's job harder (and increases his chances of a kill).

Elevation would be a very interesting aspect of a site-specifice modelling problem, but tended not to lead to any general results.