The design of the buffer cardboard boxes

Summary

In this paper, in order to protect the stunt person, we design a NLP (Non Linear programming) model based on impulse theory and the critical compressive stress theory etc.

Firstly, according to the landing process and required to use relatively few boxes, we receive a series of equations and inequalities to get the connection between variables such as the jumping height, the combined weights, the size and the amounts of the boxes. Then based on the cushion theory, we design a best stack shape and correspondingly modify the shape of the box. Through comparing the rectangle stacked shape and triangle stacked shape, we conclude that when the jumping height higher than 9.6 meters, it had better use triangle boxes and the stacked shape as **Picture 5.**

Importantly, we use some practical data to test our model and analysis its stability. We generalize it to different combined weights and different jumping heights. Moreover we get some useful and practical conclusions. At the same time, we simulate this model and receive a well effect.

At last, we discuss the strength and the weakness of the model.

Restatement of the Problem

A stunt person on a motorcycle will jump over an elephant and land in a pile of cardboard boxes to cushion the fall. We need to protect the person, and also use relatively few cardboard boxes (lower cost, not seen by camera, etc) .Our job is to determine the size of the boxes, the quantity of the boxes, and how the boxes to be stacked, if any modification to the boxes would help, and to generalize to different combined weights (stunt person & motorcycle) and different jump heights.

Basic Assumptions

- We are not considering the frictional force, when the motorcycle contacts the boxes.
- The masses of boxes are much smaller than that of the motorcycle and the stunt person, so we omit them.
- The highest moving height of the motorcycle must be higher than elephant, so it couldn’t collide with the elephant. But in the course of the computing, we suppose the highest moving height is the height of the elephant.
- Because the landing time is too small and the horizontal velocity is approximately to be zero, we do not consider the length the motorcycle moving on the boxes and assume that the length of the total boxes is a little bigger than the length of the motorcycle, the constant is 1.5. At the same, the total width of a layer boxes is 1.5 times bigger than the width of the motorcycle.
- We do not consider the resistance of the atmosphere, which is much smaller than the gravity of the motorcycle and person.
- When the motorcycle contacts those boxes, the static friction is bigger than moving friction, so the boxes aren’t gliding.

- We can't consider the buffer effect of the motorcycle and only analyse the buffer function of the boxes;
- We don't consider that there will be different box in a putting and suppose all the boxes have the same size.
- The cardboard box is a buffer object, but when it is on the action with the motorcycle, it can't rebound.

Symbols

l length of a box;
w width of a box;
h height of a box;
N quantity of the boxes;
H total height of the boxes;
V velocity when the motorcycle lands on the first box;
S proportion of the motorcycle contacting the box;
m total mass of the motorcycle and the person;
g the gravity acceleration;
la layer of the boxes.

Analysis of the Problem

When the motorcycle is jumping over an elephant, in order to achieve the best effect, its highest point should be above the elephant, where the vertical velocity of the motorcycle is zero and only have the horizontal velocity. In the course of the motorcycle descending on the boxes, in the horizontal direction, because we omit the resistance of the atmosphere, it moving under the same velocity, at the same time, the motorcycle starts to brake. When the motorcycle is contacting boxes, the level velocity is very small. So we can suppose the action of motorcycle is vertical descending on the cardboard boxes in the last time when the motorcycle is going to contact the boxes. We simulate this process (see **Figure 1**).

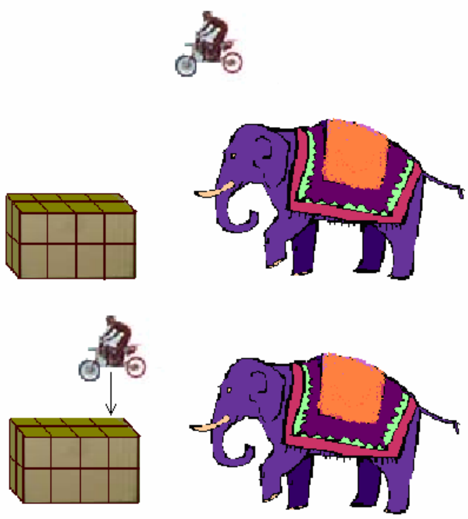


Figure1 the simulating process

On the other hand in the vertical direction, because the effect of the gravity force and omit the resistance of the atmosphere, the acceleration of motorcycle is the gravity acceleration and its beginning speed is zero. When the motorcycle is contacting the boxes, it suffers a buffer function from the boxes relative to the length, width, height and amount of the boxes. When the layer of the boxes increases, the total buffer function also increases. At the same time, we must consider the cost, which requires less, and be sure the boxes are not be seen by the camera.

According to our analysis, we found that the putting of triangle puts of the boxes can produce large buffer function than other mode of putting, so we design a stacked shape of the boxes in order to achieve the best buffer effect.

Because different combined weights (stunt person & motorcycle) and different jumping heights influence the impulse of the motorcycle when it contacts the boxes, so we generalize their influence to the results based on impulse theory and the critical compressive stress theory etc.

The Design of the Model

The obstruction of the boxes

When the motorcycle lands on the box, it suffers a buffer function and the velocity suddenly changed. This phenomenon is called impulsion. As the effect of inertia, the motorcycle gets large force that is defined in the physics as strain force. The mathematics expression is:

$$P_d = (1+\sqrt{1+\frac{2(Hh-H)}{Y}} \times \frac{6m\times g\times w}{\Delta^2} \qquad (1)$$

P_d : is the biggest impulsion load of the cardboard box;

Δ : is the thickness of the box, we suppose it is 5mm;

H : is the total height of the box;

Hh : is the height of the elephant;

Y : is the biggest displacement of the cardboard box, which is defined that

$$Y = \frac{m\times g\times w^3}{3\times E_m \times I_1} \qquad (2)$$

$$I_1 = \frac{l\times \Delta^3}{12} \qquad (3)$$

I_1 : is the moment of inertia of the box;

E_m : is the elasticity modulus, by checking some data, E_m equals 50000;

A box has four flanks. They bear the vertical force, so we must be sure that the impulsion force is smaller than the total sustained forces from the four flanks. The mathematical expression is:

$$P_d \times S \le 4P \tag{4}$$

$$P = \frac{\pi \times E_m \times I_2}{h^2} \tag{5}$$

$$I_2 = \frac{h \times \Delta^3}{12} \tag{6}$$

P : is the critical compressive stress of the box;
I : is the moment of inertia of the box;

E_m: is the elastic modulus;

When the motorcycle lands on the box, it produces a vertical stress. According to the law of action and reaction, those boxes must sustain these forces. At the same time, because the velocity changes in sudden, the acceleration will be big. But for the physical limits of a people, the acceleration can't bigger than the high-point acceleration that a people can hold. According to some data (Wang Yulan, Cheng Zilong, Han Yanfang etc, the Analysis to the Physical Effect of Landing Impulsion and the Characteristic of the Dynamic Reaction, 1990.3 , Space Medicine & Medical Engineering), we know that the high-point acceleration is 14g. If the acceleration is more than 14g, the person is easy to hurt himself. It can't protect the person well.

So we can get that:

$$P_d \times S - m \times g \le m \times a \tag{7}$$

$$a \le 14g \tag{8}$$

a : is the high-point acceleration which the people can hold.

On the other hand, the impulse of the motorcycle suddenly changes under the buffer force from boxes. In order to protect the stunt person, the velocity must be decreased to zero when the motorcycle lands on the ground. According to the impulse theory, we can gain that:

$$m \times v \le F \times \Delta t \tag{9}$$

$$F = P_d \times S - m \times g \tag{10}$$

F : is the force of boxes giving to motorcycle;

V : is the velocity when the motorcycle lands on the first box, according to the second motive law of Newton and the assumption, we can get that:

$$\left.\begin{aligned} Hh &= \frac{1}{2} \times g \times t^2 \\ v &= g \times t \end{aligned}\right\} \Rightarrow Hh = \frac{v^2}{2g} \tag{11}$$

The limits of the boxes

1. the lower cost of the boxes

If the stuff of the box fixedness, the cost of box per proportion is also constant, so if the total proportion of the boxes is the fewest, the cost of these boxes is also the fewest. The total of the proportion is:

$$C = 2\times N\times(l\times w + l\times h + w\times h) \tag{12}$$

According to the assumption, the total length of a layer box is 1.5 times more than that of the motorcycle and the total width is 1.5 times more than that of the motorcycle, the mathematical expression is

$$la = \frac{H}{h} \tag{13}$$

$$N = la\times\frac{1.5\times L_m}{l}\times\frac{1.5\times W_m}{w} \tag{14}$$

C: is the total proportion;

L_m : is the length of the motorcycle;

W_m : is the width of the motorcycle;

2. the boxes must not be seen by camera

A film composes of a series of distant photographs. When the interval time between two distant photographs less than 0.1 seconds, the eyes can’t identify those distant photographs and consider them to be a continuous photograph. So we must be sure that the time from the motorcycle contacting the first box to landing on ground should be less than 0.1 seconds. Only like this, these boxes can’t be seen by camera. The mathematical expression is:

$$\Delta t \le 0.1 \tag{15}$$

On the other hand, the total height of the box should not be too big in order to satisfy the descending time. So the total height of the boxes must be fulfilled that:

$$H \le \frac{1}{2}\times v\times\Delta t \tag{16}$$

Conclusion

According to the upper analysis, we design a Non Linear Programming (NLP) below:

Objective function:

$$\textbf{Min}\ \ C = 2\times N\times(l\times w + l\times h + w\times h)$$

s.t.

$$P_d = (1+\sqrt{1+\frac{2(Hh-H)}{Y}}\times\frac{6m\times g\times w}{\Delta^2}$$

$$Y = \frac{m \times g \times w^3}{3 \times E_m \times I_1} \qquad I_1 = \frac{l \times \Delta^3}{12}$$

$$P_d \times S \le 4P \qquad P = \frac{\pi \times E_m \times I_2}{h^2}$$

$$I_2 = \frac{h \times \Delta^3}{12} \qquad P_d \times S - m \times g \le m \times a$$

$$a \le 14g \qquad m \times v \le F \times \Delta t$$

$$Hh = \frac{v^2}{2g} \qquad F = P_d \times S - m \times g$$

$$I_2 = \frac{h \times \Delta^3}{12} \qquad \Delta t \le 0.1 \qquad l > 0 \quad h > 0 \quad w > 0 \quad H > h \quad N > 0$$

The test of the model

The results of the model

We use four group data to test our model (see **Table 1**). The mass of the motorcycle uses the mass of the **YAMAHA** motorcycle and the **ZongShen** motorcycle. The mass of a people is about 70kg. In this table the front 2 heights are gained by the zoo, the latter 2 are discretional. We use the LINGO to solve that NLP model and receive the results (**Table 1**).

We can know, from the results, that the model our design is reasonable. In practice, when **Ke Shouliang** jumps over the **Yellow River,** it uses hundreds of boxes and extends 6 layers. These data is approximately accord with our results. So we can think our model is correct and practical.

Table 1 the results

m (kg)	H (m)	l (cm)	w (cm)	h (cm)	La	N
215	5.0	45.3	37.3	37.1	2	124
240	5.0	40.2	31.2	32.2	3	171
225	4.5	58.2	42.6	50.1	1	36
225	7.0	50.1	38.1	29.5	2	90

The Simulation of the Model

In order to test our model farther, we choose a group results (m=240kg, H=5.0m, l=40.2cm, w=31.2cm, h=32.2cm, N=171cm) to simulate the model. The program flow chart is the following chart 1.

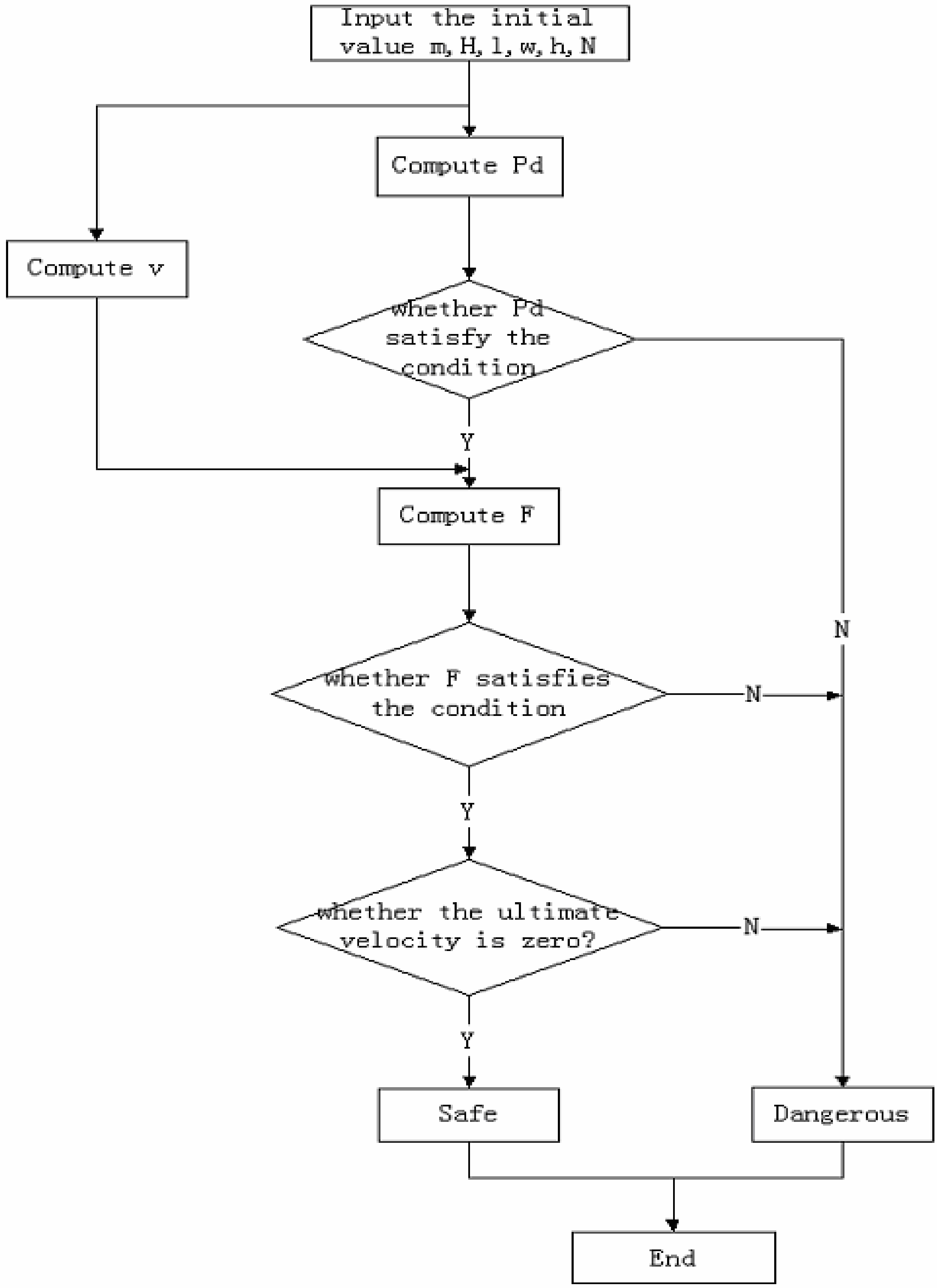


Chart 1: the program flow chart

Through operation, we find that the stunt person can safely land on the ground. So our model is proved to be correct and useful. In practice, because the effect of the friction and the buffer function of the motorcycle, the stunt person will be more safe. So our model not only can protect the person, but also have some bounds to cope with the emergencies

The shape of the boxes stacked

According to the commonly instance, the boxes are pileup (see **Picture 1**). Although this mode of stacked is convenience, its buffer affect is not the best. According to the buffer theory, the buffer efficiency is

$$\rho = \frac{E}{D_d \times P_C} \tag{17}$$

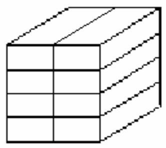
ρ : is the buffer efficiency;

D_d : is the limit distortion of box;

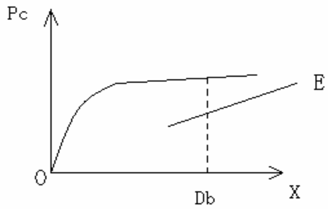
P_c : is the compress load;

E : is the press energy of the box;

From the formula, we know that in order to improve ρ, we must increase E, but E is relative to the distortion of the box.(see **Picture 2)**. In picture 2, x is the distortion of box. We find that if the structure of box stacked is designed in reason, the buffer function will be better.

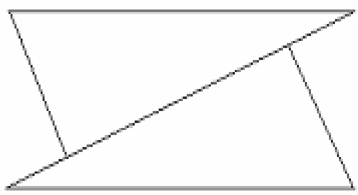


Picture 1
Commonly pile up

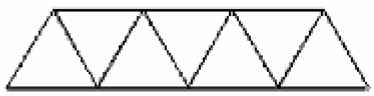


Picture 2
The relatives between compress load and distorion

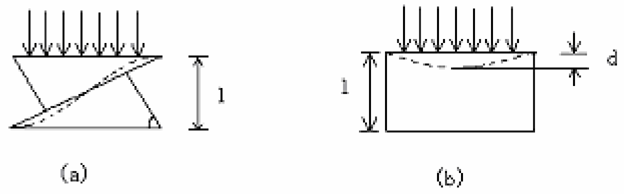
According to buffer theory, dual tangent is the best buffer structure (see **Picture 3,4**), so we designed **Picture 5** to simulate it. The buffer function producing by this stacked shape of the boxes is better than other stacked shapes.



Picture 3
The best buffer structure



Picture 5
The simulation structure



Picture 4
The buffer strueture

The modification of the boxes

According to the upper analysis and **Picture 5**, we can consider changing the shape of the box. The rectangle boxes are changed to triangle boxes.

On the other hand, we find that if keep the same layers of boxes, the total surface proportion bigger than that of rectangle putting. So we think that if the height is too high, we should adopt the triangle boxes, by the contrary, if the height is not high, we should use the rectangle boxes.

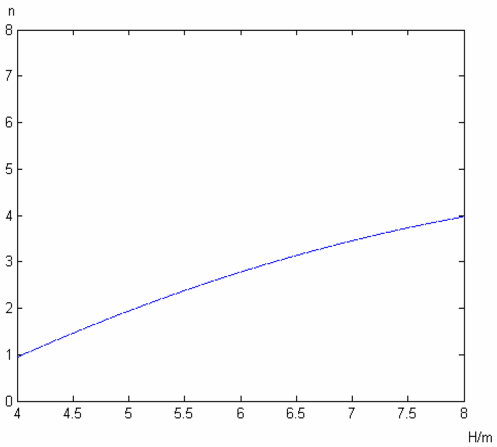
If the cost and suffer function have the same importance to the coordinator, we can know that if the jumping height is higher than 9.6 meters, we had better use the triangle boxes.

The influence of the weights and the jumping heights

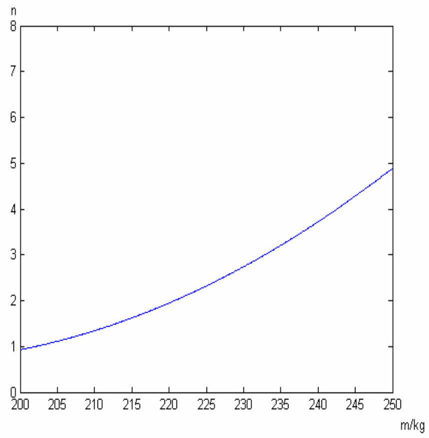
If the combined weights (motorcycle and person) are different, the impulse is different and the influence to box is also different.

At the same, if the jumping height changes, the velocity when the motorcycle lands on the first box is also different, so the impulse is different and the influence to box is also different including the width, height, length, quantity, and the layer of the boxes.

In order to find their relatives, we mainly analyses upper NLP model. Through operating by **Matlab**, we can gain the **Picture 6 and 7.**



Picture 6 the connection between jumping height and amount of boxes



Picture 7 the connection between weight and the amount of boxes.

Picture 6 expressions the relatives between the jumping height and the amount of the boxes. **Picture 7** expressions the relatives between the weights and the amount of box. From the two pictures, we can get some conclusions as that:

1. The relation between the jumping height and the amount box is approximately direct ratio. When jumping height increase, the amount of the box is also increase, but it increases relativity slowly. If the height increases one meter, the layers corresponding increases about one layer and the amount of the boxes increases differently which is relatively to the size of the box. In general, the higher of the height, the faster increases of the amount of boxes.
2. The curve of the weight and the amount of the boxes is approximately a quadratic function. The increasing velocity of the amount boxes is smaller than that of the weight. So if the weight increases a little, the influence will be much small.

On the other hand, based on the model, we generalize 15 group results (see **Table 2**) and analyse them.

Table 2 the data of the results

Number	**H (m)**	**m (kg)**	**l (cm)**	**w (cm)**	**h (cm)**	**N**
1	**4.3**	**232**	**30**	**25.1**	**26.7**	**350**
2	**4.5**	**223**	**56**	**40.3**	**50**	**39**
3	**5**	**208**	**52.5**	**45.5**	**48.5**	**36**
4	**5.3**	**230**	**40.9**	**32**	**29.9**	**200**
5	**5.5**	**225**	**42.8**	**35.5**	**31.6**	**110**
6	**5.7**	**218**	**46.2**	**37.4**	**36.1**	**112**
7	**5.8**	**228**	**37.4**	**30.3**	**31.8**	**132**
8	**6.2**	**220**	**50.2**	**38.5**	**32**	**90**
9	**6.3**	**216**	**45.9**	**36.9**	**32.6**	**98**
10	**6.5**	**226**	**27.7**	**20.9**	**27.4**	**54**
11	**6.7**	**215**	**46.4**	**30.6**	**28.9**	**218**
12	**6.9**	**205**	**65.8**	**48.5**	**31.5**	**27**
13	**7**	**210**	**37.6**	**25.4**	**29**	**70**
14	**7.5**	**238**	**25.4**	**19.8**	**22.7**	**596**
15	**7.8**	**240**	**20.1**	**28**	**20.5**	**764**

Based on these data, we can conclude that:

1. In generally, the change of the combined weight (motorcycle and person) is not to be big, So, to the size of box, the influence of it is very small.
2. According to the formula of biggest impulsion load P_d and the critical compressive stress P, we can know that the size of a box (length, width, height) decreases when the jumping height increases. But when the jumping height achieve to a high-point, the size of the box can not to decrease, for it limits by the following two factors:

(1) The limit acceleration of the person can afford. Because some physical factors, the biggest acceleration can’t be bigger than 14g, or else the person will be likely to be hurt.

(2) The boxes must be not seen by the camera, so the height of the box can't be too high, or else, the land time from contacting the first box to landing on the ground will be more than 0.1 seconds.

Strength and Weakness

Strength

1.Our model is applicable to any instances including different combined weights (motorcycle and person) and different jumping height.

2. According to our model, we generalize the influence of the combined weights and the jumping height.

3. According to the buffer theory we provide another shape of the box stacked. At the same time, we compare it to the usually stacked shape and get a useful result.

4. Our model builds based on the correct formula, through our test, the model is proved to be correct and practical. At the same time, through simulation, we receive a better result and a good effect. So in another aspect, our model is proved to be correct and practical.

Weakness

At the course of designing model, we omit some factors such as the friction force between the box and the motorcycle and the obstruction of the atmosphere. So the results will be a little different from the practical instance. But these factors are difficult to estimate and have little influence to the results, the difference between the results and the practice is so small that they can be omit in commonly occasion except for requiring current results.

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