# 8 Unlawful Assembly[①]

## 8.1 竞赛题

Many facilities for public gatherings have signs that state that it is "unlawful" for their rooms to be occupied by more than a specified number of people. Presumably, this number is based on the speed with which people in the room could be evacuated from the room's exits in case of an emergency. Similarly, elevators and other facilities often have "maximum capacities" posted.

Develop a mathematical model for deciding what number to post on such a sign as being the "lawful capacity." As part of your solution, discuss criteria—other than public safety in the case of a fire or other emergency—that might govern the number of people considered "unlawful" to occupy the room (or space). Also, for the model that you construct, consider the differences between a room with movable furniture such as a cafeteria (with tables and chairs), a gymnasium, a public swimming pool, and a lecture hall with a pattern of rows and aisles. You may wish to compare and contrast what might be done for a variety of different environments: elevator, lecture hall, swimming pool, cafeteria, or gymnasium. Gatherings such as rock concerts and soccer tournaments may present special conditions.

Apply your model to one or more public facilities at your institution (or neighboring town). Compare your results with the stated

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capacity, if one is posted. If used, your model is likely to be challenged by parties with interests in increasing the capacity. Write an article for the local newspaper defending your analysis.

## 赛题简析

此题要求解决如何确定一个公共建筑物的合法容纳量。

一个建筑物的合法容纳量可以受很多因素影响,其中最重要的,正如题目中提到的那样,是安全因素(即,要控制建筑物中的人数,以保证发生紧急状况时,所有的人都可以安全地撤离);舒适程度也是衡量公共建筑物的合法容纳量的重要指标。对于安全因素应解决两个问题,给出一定人员撤离建筑物所需的时间,反过来确定在一定的时间内撤离的最多人数。对于舒适因素,给出在一定时间内可以舒适地占用给定空间的最多人数。

题目中所要建立的数学模型还应体现其他影响因素,如房间内的人员流动,由房间形状及家具等形成的瓶颈造成的拥挤,以及人群的初始分布等。建模过程还应参考已有的规范,对聚集的人群进行观察,收集数据,另外建立的模型是否易于应用也是不可忽视的。

# How to Calculate the "Lawful Capacity" in the Constrained Condition

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**Summary**

We formulate two models to solve the "lawful capacity" problem. In the first model we give the detailed procedure to determine the "lawful capacity" of a general public building. In order to simplify the problem and to consider the buildings with regular structure and more factors, we construct Model II to make the calculation simple and fast.

During the course of model formulation we chiefly considered "public safety" on which "lawful capacity" mainly depends. This idea is successfully realized by introducing the concept of "node" and its "grading rules".

In addition, we still consider other factors, such as convenience, psychology, and financial income, which influence "lawful capacity" And we look the problem as a constrained multi-object optimization.

In this paper, we give the difference between the buildings with movable furniture and unmovable furniture. By applying our models, we solve the calculating problem on "lawful capacity" in various buildings and environment mentioned in the problem. Espe-

cially, we give an example of a cafe, the result we obtained is close to the reality.

## Ⅰ Comprehension to the problem

The problem given requires us to determine the lawful capacity of a public building. The quantity depends mainly on the speed at which the people evacuate the building during an emergency.

Besides the safety of the crowd, which means the people can escape safely during the emergency, what affects "the lawful capacity" includes also the mood of the people (for example, as there are too many people in the cafe or it is too noisy inside so that the customers will not enter it), safety factor of the surrounding environment, (even without an emergency, it is likely to have such events as fighting because there are too many people), financial factor (for instance, the boss wants to make more money), effect factor (for instance, in the music concert or speech hall, only with a considerable audience, can the stirring effect be aroused), convenience factor (for example, in order to bring convenience to the customer, it is likely to increase a fixed space, thus resulting in the decrease of the number of the people), etc. Consequently, when making a model, it is necessary to bring into consideration not only the criterion of the safety of the crowd, but also the factors mentioned above.

When making the model, the difference between the movable and immovable facilities in the building should be considered, and the diverse buildings in different environments should be considered.

When applying an established model to a certain public building, the model can be testified by comparing the lawful capacity obtained with the stated one.

## Ⅱ Assumption of the problem

From the requirements of the problem, the following assumptions can be made:

1. Before the jam takes place at the "bottle neck", a person can pass through that place at the original speed. He can get out as soon as he arrives.

2. Before the evacuation of the people, the density of the people is uniformly distributed in the buildings.

3. When there is an emergency, the escaping ability of different individuals is the same.

4. The factors caused by other situations such as crowdedness or jumping through the window during the emergency are not considered. The safety of the people is determined by the length of the time for evacuation.

5. When there is an emergency in any public building, the safety period (from the occurrence of the emergency to the hazard of the safety of the public) is determined by the nature of the emergency and the building itself, taking the value in the interval $[0, T_0]$

6. The people out of the building leave the entrance immediately and they do not affect the evacuation of the succeeding people. During the emergency, no one is allowed to get into the building. Therefore, it is only considered that the people get out of the building. This suggests that the system is a closed one.

**Ⅱ Notations**

For the convenience of illustration, the definition of "node" is given first, with its explanation made later.

Node: doorway, passage, ladder, corridor or other places where the people must pass because of immovable obstacles.

$[0, T_0]$: the safety period of a given public building;

$C_{\max}$: the maximal number of the people that a building can have(without considering the safety period when an emergency takes place);

$t$: The time from the start of the emergency;

$L(t)$: the number of people who can arrive at the "node" before time $t$;

$N(t)$: the number of all the people passing through a "node" before time $t$;

$M(t)$: the number of people in the crowding state at time $t$;

$E$: the maximal number of people who can pass through a "node" in a unit time;

$kE$: the number of people who can pass at a unit time when an emergency takes place, $k \in (0,1)$ is termed blocking coefficient;

$C_0$: the lawful capacity of a building, i. e. the solution we want to get;

C: the number of people in the building;

$T$: the evacuate time required when the total number of the people in the building is $C$;

$S$: the area of the flour where the people are scattered in the building;

$P$: the value is $C/S$, that is, the scattering density of the people before the emergency occurs;

$V_0$: the moving speed of the people when there is no crowded state and blocking within the building.

**Ⅳ Analysis of the problem**

The problem requires us to give the lawful capacity of a building on the condition that the lawful capacity is determined in a common way by numerous criteria such as the "crowd safety", "environmental safety", "financial income", "effect", "convenience", "mood", etc. Moreover, the magnitude of influence of the different factors on the number of the people within the building is different Thus, what we must do is find an optimal holding capacity C under the condition satisfying the above criteria as much as possible. This is actually a constrained multi-object optimization problem.

In order to describe the algorithm conveniently, we introduce the concept of "node" to simplify the escaping line of the people, and to get the tree diagram by "reverse branching" structure with "nodes". First, we discuss the situation of whether or not the people are crowded when passing through the "initial node" and how large is the number of the people who pass through the "node", etc. Then we discuss the situation of the lower node in order to obtain the time $T$ when all the people are evacuated. The algorithm and its description will be given in the next part.

## Ⅴ Model making

### 1. Preparation

When an emergency takes place in a building, the evacuation of the people may be affected by obstacles or doorways, corridors, passages, ladders, exits, etc., We consider the influence of the above obstacles by define them as "nodes" of evacuation.

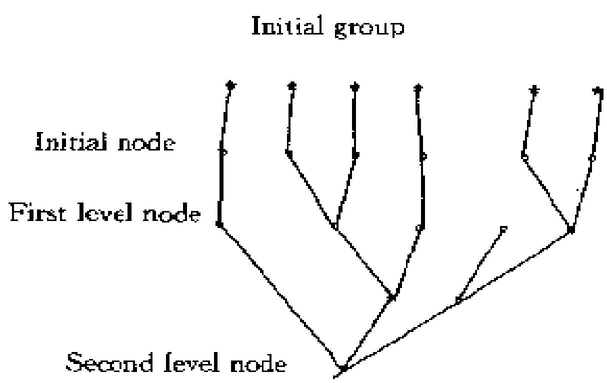
During the course of model making, we have made different treatments on the movability of the facilities within the building:

(1) The movable facilities only delay the moving speed of the people when evacuating, but does not change the moving line of the people.

(2) The immovable facility determines the moving line of the people. We convert its influence into a "node" for consideration. For instance, when two immovable facilities co-exist to enable part of the people to pass between them, the space between them can be looked at as a "node".

Grading rules of "nodes":

When an emergency takes place, the group of people separated by the seats or other obstacles must move to a "node". Such a group is taken as the "initial group" and the "node" as the "initial node"; the number of the "initial group" depends on the magnitude of the area occupied and $t$.




Supposing there are $P_0$ "Initial nodes". There are a number of "nodes" for the people getting out from these $P_0$ nodes to choose as the evacuating line for next step evacuation. According to the idea of a common person, he should rush to the nearest "node". If the distance between him and two "nodes" is the same, we specify that the person will choose the right-handed "node". If there are people at "initial nodes" A of these $P_0$ nodes who have chosen the same nodes "$B$" using the above method, then the, "initial nodes" A are called the "upper-level nodes" of the "node" $B$. The "node" $B$ are called the "lower-level node" of the "initial nodes" A, or the "first-level nodes", and so on and so forth. In this way, there can be a lot of levels. The outmost exits can also be termed "final nodes".

In the earliest model, we don't consider the impacts on the evacuation of the people at the upper "nodes" owing to the blocking of "nodes". We will make comparison and analysis the impact in the modified model.

**2. Tile making of a concrete model**

**Model(I)**

**i) Procedure**

1) Analysis of the initial node

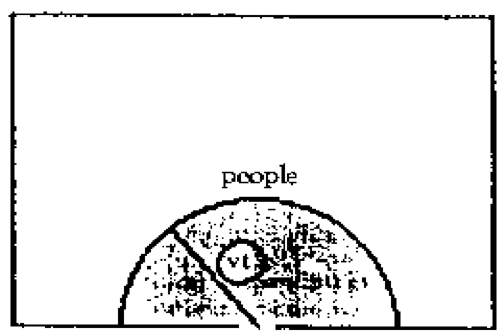
It is unencumbered to consider $N(t)$ and $M(t)$ as continuous variation and have derivative in its definite region. Assume that $t_0$ is the time "initial node" to be crowded. And suppose that once it is crowded, this condition will continue until all people that should pass this node have evacuated from it. Assume that the node isn't crowded at the time $t$. Make a circle with the node as center and $vt$ as radius. Then people in the common part of the disk and the interior of the structure (Area is $S(t)$) can pass from the node. At this time, we have

$$N(t) = L(t) = p\,S^*(t) \qquad 0 \leqslant t \leqslant t_0 \tag{1}$$

From the definition of above, we have

$$\mathrm{d}N(t)/\mathrm{d}t = kE \qquad t_0 \leqslant t \tag{2}$$

Couple with (1) and (2), we obtain a differential equation with initial condition.



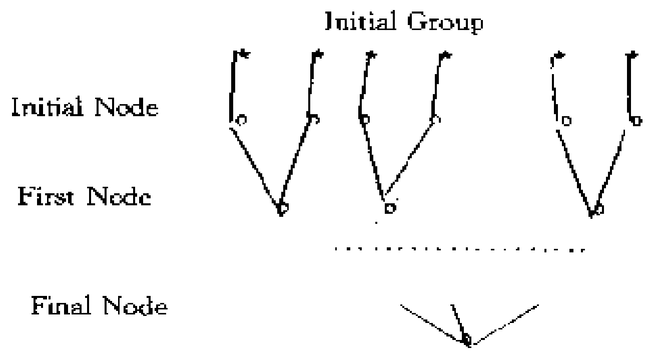

$$\begin{cases} N(t) = \rho S^*(t) & 0 \leqslant t \leqslant t_0 \\ \dfrac{\mathrm{d}N}{\mathrm{d}t} = kE & t_0 \leqslant t \end{cases}$$

If we can obtain the time $t_0$, we can solve this equation easily.

Let $p$ be the "initial node", the treatment method is as follows. Assuming that at time $t$, there is no crowdedness at the node.

$L(t) = N(t) \Rightarrow dl/dt = dN/dt \leqslant E$. As $dl/dt = \rho S^*(t)$ will increase with the increase of time $t$, then, when $dl/dt > E$, the node is in the crowded state. Obviously, the time $t_0$ satisfies $dl/dt|_{t=t_0} = E$, that is, the time when the crowdedness occurs. Substituting $t_0$ into the equation, $N(t)$ can be obtained.

2) Analysis of the "$k$th-level node" ($k = 1, 2, \ldots$)




Suppose there are $m$ nodes of "$(k-1)$th-level". They are upper-level nodes of the "$k$th-level node" $b$ and we mark them as $A_i (i = 1, 2, \ldots, m)$. In fact, we can obtain the time $t_i$ that should be spent for a person to rush from $A_i$ to be when there are no jam. (Note: Here, movable furniture has been already considered as a factor slowing down the people's speed.) Assume that $N_i(t) (i = 1, \ldots, m)$ is the corresponding number of people coming from node $A_i$ who have passed node $b$ before moment $t$. We mark $M(t)$, $L(t)$, $N(t)$ and $E$ corresponding to node b as $M_b(t)$, $L_b(t)$, $N_b(t)$ and $E_b$. Obviously, when $t < \min\{t_1, \ldots, t_m\}$, $M_b(t) = N_b(t) = 0$. Node $b$ isn't crowded at moment 0. This condition will continue until $dL/dt > E_b$. Because

$$L(t) = \sum_{i=1}^{m} N_i(t - t_i)$$

Hence, the condition of jam become to

$$\sum_{i=1}^{m} \frac{dN_i(t - t_i)}{dt} > E_b$$

This moment is marked as $t_{b0}$ Then, when $t \in [0, t_{b0}]$, we have

$$\begin{cases} N_b(t) = \sum_{i=1}^{m} N_i(t - t_i) \\ M_b(t) = 0 \end{cases}$$

Since node $b$ is crowded at moment $t_{b0}$, $N_b(t)$ satisfy $dN_b(t)/dt = kE_b$. The initial value is

$$N_b(t_{b0}) = \sum_{i=1}^{m} N_i(t_{b0} - t_i)$$

The expression of $M_b(t)$ is

$$M_b(t) = \sum_{i=1}^{m} N_i(t - t_i) - N_b(t)$$

The jam will not disappear until $M_b(t) = 0$ and the left-derivatives of $M_b(t)$ is smaller than 0. This moment is marked as $t_{b1}$. Since $t_{b1}$, we can use the inequation

$$\sum_{i=1}^{m} \frac{dN_i(t - t_i)}{dt} > E$$

to judge when the node will be in jam again. Repeat as what we do above, we can obtain the interval when node $b$ is in jam or fluency. And from equation

$$\begin{cases} N_b(t) = \sum_{i=1}^{m} N_i(t - t_i) & \text{"}b\text{" is not crowded} \\ \dfrac{dN_b}{dt} = kE_b & \text{"}b\text{" is crowded} \end{cases}$$

we can obtain $N_b(t)$.

3) In the step of 1 and 2, we give the analysis of the initial node and the $k$th-level node. Similar to "mathematical induction", we can get

the $N(t)$ corresponding to the final node. If the number of the final node is n, and the corresponding $N(t)$ is $N_j(t)$ $(j=1, \ldots, n)$, then when $\sum N_j(t)=C$, all people has evacuated from the facility. From this, we can obtain the evacuating time $T$ corresponding to $C$.

4) We went to find the optimal "$C$", "$C$" must make $f(C)+1/h \cdot C$ to be minimum under the binding condition of

$$\rho < \rho_0, \ \rho > \rho_1 \rho < \rho_2 \ \rho < \rho_3$$

Here
$$f(C)=\begin{cases} T(C)-T_0 & \text{when } T \leqslant T_0 \\ +\infty & \text{when } T > T_0 \end{cases}$$

$hC$ means the sum of "financial income"

$\rho_0$: The maximum of density only thinking about safety criteria of the surrounding environment

$\rho_1$: The minimum of density only thinking about criteria of effect

$\rho_2$: The maximum of density only thinking about criteria of convenience

$\rho_3$: The maximum of density only thinking about criteria of mood

This is a problem of constrained multi-object optimization.

We define "punishing function" is

$$Q(c)=f(c)+1/h \cdot c+\lambda_0(\rho-\rho_0)+\lambda_1(\rho_1-\rho)+\lambda_2(\rho-\rho_2)+\lambda_3(\rho-\rho_3)$$

In the expression $\lambda_i(i=0,1,2,3)$ is the weight of the criteria to the capacity. Using the "Golden section"[1], we can find the minimum of $Q(c)$ and "$C$" according to it.

**ii) Specification**

"$kE$" is the number of people passing the crowded node in unit time. The wider "the node" is, the bigger "$k$" is, the minimum is 0 and the maximum is 1. "$k$" is related with the width of the node and can be measured practically to get reasonable value.

Because we think that movable furniture is only to make people slower, we treat this condition simply. How much they hamper the runners depends on the concrete building.

For the other type of buildings: when the building is small but "$p$" is big, such as elevator, we consider it to be crowded when an emergency happened. Because the atmosphere of rock conceals and soccer tournaments is glowing, we can reduce the weight of the Criterion of mood. But "$T$" what we get should be magnified considering that the mood of people is agitated and it infects the ability of people to evacuate from the facility. Also, for this reason, we should increase the weight of the safety factor of the surrounding environment. And that the high density will make the block come earlier is another thing that should be considered when solve the problem in these environments.

**iii) Solve**

Solve idea: Using the "Grading rule of node", we can get the "tree diagram" corresponding to each "final node", which is called a "branch". After "$C$" was given, we can get the time corresponding to a branch when people in it have evacuated. Let the maximum of them be $T$.

Affirm $C_0$ by "Golden section".

**Algorithm**

Input: ① $m_0$—the number of branches

② The expression $N(t)$ of every "in initial node" (These can be get from the "Analysis of the initial node".)

③ the number of nodes in every level.

④ $n_0$—The number of levels every "branches"

⑤ the corresponding relation between "the upper node" and "the lower node"

⑥ $E$ and $k$ of every node

Direction: from "the upper node" to "the lower node"

Sequence: from the left node to the right one in one level.

Calculation:

1) Give a reasonable cadence $\tau$ (such as 0.2s), $n = 0$

2) Give a branch, $n \leftarrow n + 1$.

3) $n_l^{(i,j)}$ is the $j$th node of the $i$th level. "$l$" indicates that $n_l^{(i,j)}$

corresponds

to the $l$ th node in the lower-level, $l$ is zero according to "the final node"

4) $t \leftarrow t + \tau$

From up to down work out every $N_l^{(i,j)}(t)$

The begin of block $\Leftarrow \frac{\mathrm{d}}{\mathrm{d}t}\sum_{j=1}^{m} N_l^{(i-1,j)}(t) > E^{(i,l)}$

The end of block $\Leftarrow M^{(i,l)}(t) = 0$ and the left detivative of $M^{(i,l)}(t)$ is zero.

$$\begin{cases} N^{(i,l)}(t) = \sum_{j=1}^{m} N_l^{(i-1,j)}(t) & \text{"node } n_l^{(i,j)}\text{" is not crowded} \\ \frac{\mathrm{d}N^{(i,l)}(t)}{\mathrm{d}t} = k^{(i,l)}E^{(i,l)} & \text{"node } n_l^{(i,j)}\text{" is crowded.} \end{cases}$$

5) Scan every "node": IF $\sum_{j=1}^{m_0} N_0^{(n_0,j)}(t) = C$, GOTO 6 else GOTO 3

6) $S_n = t$, if $n = m_0$ GOTO 7 else GOTO 2 (Note: we can

improve the precision of $S_n$ through reduce the value of $\tau$)

7) $T = \max\{S_1 \ldots S_{m_0}\}$

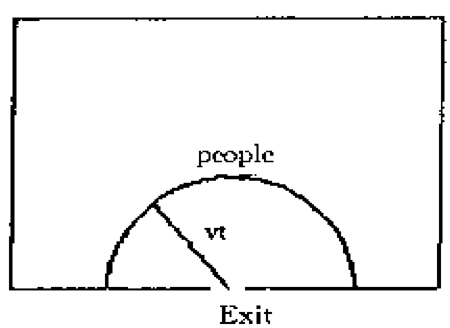
It gives an algorithm to calculate $T$ in use of $C$ from 1 to 7 in above, how to do the next work has been explained clearly in the "Procedure"

**iv) Example and results (with cafe as an example)**

This is a special example for a room with movable furniture its shape shown in figure. Because tables and chairs are movable, there is not node between tables or chairs, people can move to exit in a straight line. So we consider exit as "initial node". According to the analysis we make a circle with the "exit $P$" as center and $Vt$ as radius, The area of common part of the disk and the interior of the room is $S(t) = \frac{\pi}{2}(Vt)^2$

So when $\frac{dL(t)}{dt} = \rho\frac{ds(t)}{dt} = \rho\pi V^2(t) = E$, the Exit begins to be crowded, further we can get the time when crowded: $t_0 = E/\rho\pi V^2$

$$\text{So}\begin{cases} N(t) = \rho S(t) = \rho\frac{\pi}{2}(Vt)^2 & 0 \leqslant t \leqslant t_0 \\ \dfrac{dN(t)}{d(t)} = kE & t_0 \leqslant t \end{cases}$$



$$\text{We can get } N(t)=\begin{cases}\rho\frac{\pi}{2}(Vt)^2 & 0\leqslant t\leqslant t_0\\ kEt+\frac{E^2}{2\rho\pi V^2}(1-2k) & t_0\leqslant t\end{cases}$$

For example:

$E=9$ people/s, $k=0.85$, $T_0=20$s, $S=200\text{m}^2$

$V_0=4$m/s (The following results can get by a simple C program)

Firstly, we only considered "criterion of safety"

and got $C_0=150$ peoples

Secondly, we also consider "Criterion of Financial income", "Criterion of convenience" and "Criterion of mood"

and get $C_0=87$ peoples (binding condition: $\rho\leqslant 0.5$)

4) Analysis of results

The first result of former exampes is larger than "the actual results" in cafeteria. The main cause is that we think that the people distributed evenly in cafe and only consider the handicap of tables and desks, but we don't consider its own occupied area.

The second result is close to the reality because we consider some more criteria.

**Model (Ⅱ)**: (of buildings with the structure similar to lecture hall)

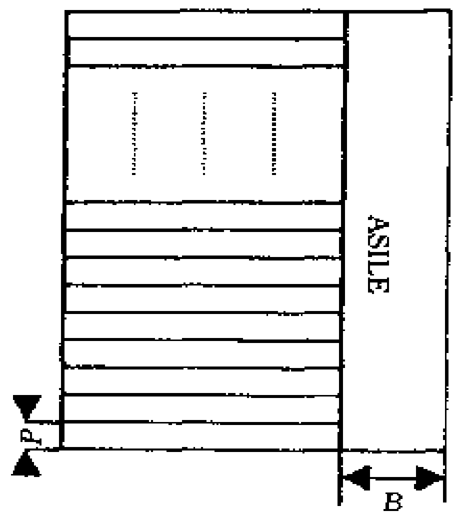
Becuase the former model is rather complicate, moreover, because we assume that groups of people gathered in one point will not affect the flow of people in upper nodes. The conclusion, however, contravene the fact obviously in certain buildings such as lecture halls. Therefore, we set up another model to this kind of buildings.

**i) Procedure**

The structure of a lecture hall is shown as the figure.

People can pass through the passages between every two rows.

Let's provide that there are $N$ rows, the distance between two rows is $d$, the width of aisle is $B$, the room of a people is $D$. The end of the passages are upper-level nodes to the exit of the aisle. And we mark them as $A_1$, ..., $A_N$ from the first passage to the last. Let's provide that the speed of people tun out of the passage is $E_0$ without any block. After the passage is crowded, the speed changes to $k_0E_0$.




Accordingly, $N(t)$ could be marked as $N_i(t)=0$, $t<0(i=1,2,\ldots,N)$. The coefficients and functions aisle are marked as $E^a$ (fluency), $k^aE^a$(block), $N^a(t)$, $M^a(t)$, The amount of people of every row is $C_i(i=1,2,\ldots,N)$

$$\begin{cases} N^a(t) = \sum\limits_{i=1}^{m} N_i(t-t_i) & \text{The exit of aisle is in fluency} \\ \dfrac{\mathrm{d}N^a(t)}{\mathrm{d}t} = k^aE^a & \text{The exit of aisle is in blockage} \end{cases}$$

Judge whether the exit of aisle is crowded in real time, and figure out the $N^a(t)$. If $C=N^a(t)$, $(C=\sum C_i)$, $T$ is got.

**ii) The solution of the model ( Ⅱ )**

**algorithm:** We still use the signs in i). Assume that some people who located at n rows near to the exit of aisle are passing it at the beginning of block. So $n=[E^a/E_0]$. Assume that the exit begin to block at time $t$, at this time, $M(t)=0$.

(1) Give a reasonable cadence $\tau$ (for example $\tau=0.2\text{s}$)

(2) Compute $q=[M(t)^* D/(B^* d)]$ ($q$ is the number of the exits between rows in the crowded state at time $t$).

(3) Compute the variation of peoples in crowded state in interval $[t,t+\tau]$.

$$\Delta M \leftarrow \left(\sum_{i=1}^{k+1} k_0 E_0 + \sum_{i=k+2}^{n} E_0 - k^a E^a\right)\tau \quad (C_i \neq 0, i = 1,\ldots,n)$$

(4) $M^a(t) \leftarrow M^a(t) + \Delta M \quad C_i \leftarrow C_i k_0 E_0 \,\tau \quad i=1,\ldots,k+1$

$N^a(t) \leftarrow N^a(t) + k^a E^a \tau \quad C_i \leftarrow C_i E_0 \,\tau \quad i=k+2,\ldots,N$

(5) IF $n<N$ $\quad r \leftarrow d/v - \tau - \Delta MD/Bv$

IF $r>0$ GOTO (2).

ELSE $n \leftarrow n+1$ GOTO (2).

ELSE IF $C \geqslant N(t)$ GOTO (2)

ELSE $C=N(t)$ END

## Ⅲ. The enhancement to the model.

We have shown two models in the former text. Model ( Ⅰ ) discuss a method to general public buildings. No matter the layout in the buildings is regular or irregular. And all kinds of obstructions' affection can be regard as "nodes". People can get the result on a microcomputer through the algorithm that we apply. Model ( Ⅱ ) set up a practical and complete model to the buildings with row and aisle. The model can be applied to cinema auditorium, classroom with fixed desks and chairs and so on. We also give out the algorthm that might be applied to computers in detail.

During the course of setting up the models, we made some presumption, and omitted some condition. Therefore we can do some work to improve them:

1. In the grading rules of "nodes", we haven't considered that the people will select the last one in the right if there are some nodes that have the same distance. In fact, if there are $n_0$ nodes as that,

people will run to every node with the possibility of $1/n_0$, or, more sensible, people run to the node where the value of $M(t)$ is the least.

2. In condition of several levels, we haven't considered that every people will occupy some space, then we omitted the influence of the shape of gathering people between the upper-level nodes and lower-level nodes. If we can get the shape of area where people gather, of course, the model will be more complicate. In fact, model (Ⅱ) is a improvement of model (Ⅰ), suppose the builds is similar to lecture hall.

3. At "final nodes", we can add one possibility if one final node is empty, while other are still busy, people may run to the empty nodes.

4. We always presume the people distributed evenly. However, in buildings such as cinema we presume people distribute as normal distribution, because the density at the middle region is higher than the other area.

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# 论 文 点 评

本文在抓住安全因素的同时，还考虑了一些其它的因素，比如环境因素、人们的情绪、效益因素、方便性因素、效果因素等。并将这些因素对建筑物的容纳量的影响以一种比较统一的方式表达出来。在设计模型时，针对"建筑物中发生紧急情况时人们的撤离主要受到建筑物中行动瓶颈的影响"这一事实，将建筑物中的行动瓶颈抽象成了节点，并重点分析了人群在通过这些节点时的时间特性。通过对节点的计算给出了建筑物中合法容纳量的计算方法，这实际是一个多目标优化问题。该模型还考虑了建筑物中可移动家具和不可移动家具对人们行动的影响。最后应用这一模型，解决题目中提到的若干类型的建筑物的合法容纳量问题。论文的不足之处在于未给出一个典型的建筑物，运用所建的模型给出相应的结果，并与实际标出的结果相比较，进一步说明模型的特点。

本论文获得了 1999 年美国大学生数学建模竞赛(MCM)二等奖。