A Myopic Aggregate-Decision Model for Reservation Systems in Amusement Parks

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Summary

We address the problem of optimizing amusement park enjoyment through distributing QuickPasses (QP), reservation slips that ideally allow an individual to spend less time waiting in line. After realistically considering the lack of knowledge faced by individuals and assuming a rational utility-oriented human-decision model and normally-distributed ride preferences, we develop our Aggregate-Decision Model, a statistical model of waiting lines at an amusement park that is based entirely on the utility preferences of the aggregate.

We identify in this model general methods in determining aggregate behavior and net aggregate utility and use these methods, along with complex but versatile QP accounting and allocation systems to develop the Aggregate-Decision QuickPass Model. We develop criteria for judging QP schemes based on a total utility measure and a fairness measure, both of which the Aggregate-Decision QuickPass Model is able to predict. Varying the levels of individual knowledge, the QP line-serving rates, the ability to cancel one's QP, and the QP allocation routines, we obtain a variety of different schemes and test them using real life data from Six Flags: Magic Mountain as a case study. We conclude that the scheme in which individuals are able to cancel their QPs, know the time for which a QP will be issued, and are allocated to the earliest QP spot available provides park-goers with the greatest total utility while keeping unfairness levels relatively low.

The UMAP Journal 25 (3) (2004) 281–300.

Introduction

As the number of park-goers increases, so do the waiting lines. In a QP system, rather than standing in a regular line, people can opt for a ticket to come back later and join a presumably faster line to the ride. People may hold only one active ticket at any given time.

We develop a means to evaluate QP systems. We first develop a working economic understanding of myopic human decision-making in amusement parks. Applying this to all park-goers, we develop our Aggregate-Decision model, which predicts the statistical behavior of groups faced with queueing choices. We then include QP lines and develop the Aggregate-Decision Quick-Pass model, which describes large-group statistical decisions about joining a regular line or obtaining a QP ticket. We test various QP distribution schemes and compare them on the criteria of maximizing utility while maintaining an acceptable level or fairness.

Definitions and Key Terms

- An *amusement park* is a collection of n rides $R_1, \ldots, R_n$ associated with a number P_T, representing the total population of the park (people in the park who are either looking, waiting, or are on a ride).
- For the ith ride, l_i is the number of people in line to get on the ride and k_i is the rate (persons/min) at which the line moves.
- The *fluid population* P_F of an amusement park is the number of people actively looking for a ride.
- The *utility* of a ride is measured by how long people are willing to stand in line for it, given that the alternative provides them with zero utility. Individuals have utilities t_i, while the utilities for each ride have distributions $\mathcal{T}_i$ with expected values μ_i and standard deviations σ_i.
- The *preference* that an individual has for a ride is normalized to $r_i = t_i / \sum_k t_k$.
- The *popularity* of a ride R_i is determined by its popularity rating $\rho_i = \mu_i / \sum_{k=1}^{n} \mu_k$ (we use here a first-order approximation of $E[\mathcal{T}_i / \sum_{k=1}^{n} \mathcal{T}_k]$ [Brown 2001]).
- A *QuickPass system* is a line-management scheme that allows a person to obtain a ticket for return later to a presumably faster QuickPass line.
- A QuickPass is *live* when it can be used by a holder to gain access to the QuickPass line.
- A QuickPass is *active* from the time of issue to the end of the time interval in which it is live.

Table 1.
Symbol table.

Symbol	Definition	Units
	Variables	
l_i	Number of people waiting in the regular line for R_i (such that $l_{i,QP} + l_{i,\text{NOQP}} = l_i$)	people
$l_{i,QP}$	Number of people waiting in the regular line for R_i with an active QuickPass	people
$l_{i,\text{NOQP}}$	Number of people waiting in the regular line for R_i without an active QuickPass	people
q_i	Number of people waiting in QP line for R_i	people
$w_{i,ex}$	Expected free waiting time	min
k_i	Regular line speed (in non-QP model equal to c_i)	people/min
d_i	QP line speed	people/min
t_i	Measure of the utility for R_i	min
$\vec{t}$	Collection of t_i	vector
r_i	Preference for R_i an individual has based on the t_i	unitless
ν	Impatience measure for individual based on t_i	unitless
T_i	Random variable representing based on distribution for t_i about μ_i with standard deviation σ_i	min
$\vec{T}$	Collection of T_i	vector
ρ_i	Aggregate popularity for R_i based on the μ_i	unitless
χ	Expected impatience measure based on μ_i	unitless
$U_{j,i}$	Expected net utility provided by R_j, gauged with variables from R_i	min
$U_{j,i}^{qp}$	Expected net utility provided by R_j (including QP waiting), gauged with variables from R_i	min
$U_{k,s}^{qp}$	Expected utility provided by R_k (including QP waiting) for people with QP for R_k which becomes live between sI and $(s+1)I$ time steps, gauged with variables from R_k	min
P_F	Fluid population of the park (such that $P_{f,QP} + P_{f,\text{NOQP}} = P_F$)	people
$P_{F,QP}$	Part of fluid population with an active QuickPass	people
$P_{F,\text{NOQP}}$	Part of fluid population without an active QuickPass	people
$QP_{i,t}$	Number of people with a QuickPass for R_i which becomes live in t time steps	people
QP_i	Number of people with a QuickPass for R_i which becomes live in any number of steps	people
ϕ_i	Preference distribution function	function
	Constants	
n	Number of rides	unitless
m	Number of QuickPass machines	unitless
R_i	Ride	unitless
μ_i	Expected value of t_i over the population	min
σ_i	Standard deviation of t_i over the population	min
c_i	Serving rate of R_i	people/min
P_T	Total park population	people
e	Ratio of the disutility for free waiting to line waiting	unitless
κ	Ratio of the time for free waiting to regular line waiting	unitless
I	Length of interval for which a QuickPass is issued	min
Cancel	Whether active QuickPasses can be canceled	boolean
DisplayTime	Whether QuickPass kiosk displays to the public when the next QuickPass issued with become live	boolean
$\delta_{i,s}$	Maximum number of QuickPasses issued for R_i in interval s	unitless

- A *time interval* is the period of time over which a QuickPass is live.
- The *free waiting time* is the time from when a QuickPass becomes active to when the QuickPass becomes live.
- The *net utility* of a ride is the utility gained by taking the ride added to the disutility associated with waiting for it, plus (in the case of a QuickPass) the disutility associated with free waiting time.

General Assumptions

Time Stepping

- The park runs continuously for a fixed amount of time (such as a working day), split into discrete time steps of length τ. We use $\tau = 5$ min.
- In one time step, a person can either take one action or make one decision: move to a ride to consider it (we assume that all rides are an equal distance apart and people can on average cover that distance in time τ) or actually be on a ride. While considering a ride, a person can decide to get in line, get a QP, or wander on to another ride in one time step τ.
- A person in line will not leave before completing the ride.
- The serving rate k_i of a ride is constant and independent of time and the length of the waiting line.
- Every ride takes one time step to run and lets out a batch of served people at the end of the time step in accordance with k_i. Thus, the number of people let out per time step is τk_i.
- People cannot trade QPs. (This assumption simplifies analysis.) Even if trading were allowed, it seems practically unfeasible. For a different perspective on reservation trading, see Prado and Wurman [2002].
- The park population size "ramps up" to its target size over a small number of time steps and then stays constant. The ramp-up is consistent with arrivals at the park; it follows an exponential distribution (**Figure 1**). After reaching its maximum, the park population stays constant until the end of the popular period. In reality, a ramp-down period follows; but as populations dwindle and lines shorten, QPs play a lesser role.

Individual Behavior

- All things are measured in utilities, and individuals seek to maximize their utilities. We measure all utilities in terms of time. Thus, the utility t_i for

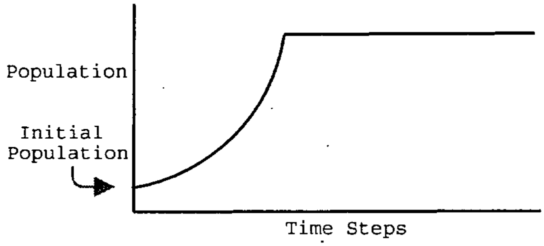

Figure 1. Population levels as a function of time.

taking a particular ride is measured by the length of time that it is worth waiting for the ride, given that all alternatives provide zero utility. We also measure the disutilities of waiting, as the total waiting time.

- An individual's enjoyment of a ride is fixed, not affected by waiting time.
- Disutility is linearly proportional to time waiting in a line.
- Individuals are *myopic*: They know information only about the ride where they are and thus determine their expectations for other rides based on their own preferences r_i and the line-serving rates c_i, d_i, and k_i. This is reasonable because in reality rides are a significant distance apart.
- An individual can immediately and accurately gauge how long a line is and its serving rate.
- Each individual knows r_i, the individual's own preferences.

Aggregates

- The population's preference distribution ϕ_i for riding each R_i is normal. This is reasonable, because for large populations the central limit theorem [Weisstein 2004] applies—the time that a person is willing to wait for a ride is a function of many random variables. Moreover, we assume further that in any subset of the population, the distribution ϕ_i applies with equal validity. This is reasonable if the population (and the aggregate) is sufficiently large. Thus, it makes sense to discuss the random normally-distributed preference variables T_i of an aggregate. The actual specifics of the distribution, such as $\mu_i = E[T_i]$ and σ_i, can be estimated empirically, e.g., by taking surveys.
- The random preference variables T_i are independent of one other: Each person considers different rides independently. In reality, preferences might be correlated based on type of ride, age, and enjoyment of amusement parks in general, as well as other factors; but since there are many other factors within each ride, this assumption is reasonable.

- The population preference distributions ϕ_i are temporally invariant. That is, aggregate preferences do not change with time and with ride experience; in essence, there is no aggregate memory. An individual's preference for a ride is likely a function of the number of visits [Prado and Wurman 2002], but we can either assume that this function is the constant function (that people have unchanging utility functions) or that preference changes over time cancel out in the distribution. If one person prefers a ride less after riding it, another will prefer it more.

- The popularity ρ_i of a ride corresponds to the fraction of the fluid population that goes to and considers R_i at any given time step. Effectively this is the fraction of the population for whom R_i is their favorite ride. This assumption is a result of defining ρ_i as a first-order approximate to $E[\mathcal{T}_i / \sum_{k=1}^{n} \mathcal{T}_k]$.

The Aggregate-Decision Model

Expectations of Our Model

- After the ramp-up period, the marginal utility for each time step for small-population amusement parks will be greater than for larger amusement parks. In large parks, there is more crowding and thus longer lines and more disutility at each time step.

- Increasing the sum of the μ_is should increase the cumulative utility over the course of the day.

- Increasing the popularity of a given ride should increase its line length.

- At a small park, people tend disproportionately toward the most popular rides. This expectation is suggested by the second-order expansion for expected value [Brown 2001]. At a large park, people tend toward popular rides less than expected, because of increased disutility from waiting in longer lines.

Individual Behavior with No QuickPass

Let $T = (t_1, \ldots, t_n)$. We define $\nu = 1/\sum_k t_k$ and $r_i = t_i \nu$. The r_i measure the individual's preference for a ride over the alternatives. We call ν the *impatience measure,* since multiplying by it normalizes utility (willingness to wait) and thus neutralizes differences in patience. We assume that people seek rides that they prefer most. Thus, a person considers the ride with the highest r_i.

An individual's net utility from a ride is the utility that the ride provides minus the disutility from waiting. As before, let k_i be the rate at which the line moves and l_i the length of the line; the approximate waiting time is then l_i/k_i. Thus, for ride R_i, a person's utility is

$$U_i(t_1, \dots, t_n) = t_i - \frac{l_i}{k_i}.$$

Since individuals cannot know the lengths and speeds of the other rides, they must estimate the utilities of those rides. Let $U_{j,i}$ be the utility that the individual estimates for R_j, using variables from R_i (we assume that the individual is considering R_i). The person estimates k_j and l_j from their preferences towards R_j and information about R_i. In our model, a person assumes that the population has the same preferences as their own; that is, the number of people at a ride is proportional to the individual's preference for that ride, so that the person estimates that $l_j = r_j P_T$. Furthermore, the person predicts the speed of the lines to be roughly be the same. Thus, an individual at R_i would reason that

$$U_{j,i}(t_1, \dots, t_n) = t_j - \frac{r_j P_T}{k_i} = t_j - \frac{P_T}{k_i} \frac{t_j}{\sum_k t_k}.$$

Then the person, comparing the utility from R_i with the expected utilities of the other rides, stays at R_i if U_i exceeds all the other expected utilities $U_{j,i}$. Thus, a person joins line for the ride R_i if $U_i \geq U_{j,i}$ for all $j \neq i$.

Aggregate Behavior

In dealing with an aggregate, randomly selected members considering ride R_i have as preference the random normally-distributed variables $\mathcal{T}_i$ instead of t_i, the case for the individual. Thus, because the utility functions U_i, and $U_{j,i}$ are functions of the t_i, they induce the following utility distribution variables on the aggregate population:

$$\mathcal{U}_i(\mathcal{T}_1, \dots, \mathcal{T}_n) = \mathcal{T}_i - l_i/k_i, \qquad \mathcal{U}_{j,i}(\mathcal{T}_1, \dots, \mathcal{T}_n) = \mathcal{T}_i - \frac{\mathcal{T}_i}{\sum_j \mathcal{T}_j} \frac{P_T}{k_i}. \quad \textbf{(1)}$$

The Formal Model

We develop an iterative process for determining how lines and utilities change as a function of time.

In our model, P_T (and P_F) "ramps up" until approximately 20 5-min time steps are completed and the park is at full capacity. At each time step, $\rho_i P_F$ people consider entering the line for ride R_i. (This quantity may not be an integer; at our final calculation, we round.) The aggregate population considering R_i has the utility distributions given by **(1)**.

An individual stays if $U_i \geq U_{j,i}$ for all $j \neq i$. Since the $\mathcal{T}_i$ are normal, the probability distribution function for each $\mathcal{T}_i$ is

$$\phi_i(t_i) = \frac{1}{\sigma_i \sqrt{2\pi}} e^{-(x-\mu_i)^2/2\sigma_i^2}.$$

We would like to find the probability that $\mathcal{U}_i(\mathcal{T}_1, \dots, \mathcal{T}_n) \geq \mathcal{U}_{j,i}(\mathcal{T}_1, \dots, \mathcal{T}_n)$ for all i, j. Define the domain $\Omega \subset \mathbb{R}^n$ as follows:

$$\Omega = \{(t_1, \dots, t_n) \in \mathbb{R}^n : U_i(t_1, \dots, t_n) \geq U_{j,i}(t_1, \dots, t_n) \text{ for all } j \neq i\}.$$

Then the probability that a person prefers ride R_i to all other rides is

$$\tilde{P} = P(\mathcal{U}_i \geq \mathcal{U}_{j,i} \text{ for all } j) = \int_\Omega \phi(\vec{t})d\vec{t},$$

where the distribution function $\phi(t_1, \dots, t_n) = \prod_{i=1}^n \phi_i(t_i)$ because the ϕ_i are independent of one another. So the number of people who get in line is the rounded value of the product of this probability and the number of people, or $\lfloor \tilde{P}\rho_i P_F \rfloor$. Since Ω may be a complicated domain, direct integration is impossible. We compute this integral numerically, using a variant of the Monte Carlo method. The average utility gained for these people is

$$\bar{U} = \langle U_i \rangle = \int_\Omega U_i(\vec{t})\phi(\vec{t})d\vec{t},$$

and the total utility gained is the product of this with the (rounded) number who get in line, or $\bar{U}\lfloor \tilde{P}\rho_i P_F \rfloor$. In a similar manner, we calculate the variance

$$\sigma^2 = \langle U_i^2 \rangle - \langle U_i \rangle.$$

The model proceeds by adjusting the line counts, removing back into the fluid population people who do not enter the line, and increasing the total (and fluid) populations if in the ramp-up period. **Figure 2** gives a flowchart.

Model Validation

We programmed our model into a computer simulation. We simulate three different sized parks, with different μ_is (thus creating different preference distributions and measures of impatience). Then we process the resulting distributions of people and the cumulative utility throughout the day. We find that our model meets our expectations for a basic human-behavior model.

The Aggregate-Decision QuickPass Model

We add QP machines at $R_1, \dots, R_m$. QPs are given out for a constant time interval of I units of our time step: A QP becomes live in one of the time intervals $[0, \tau I]$, $[\tau I, 2\tau I]$, etc.

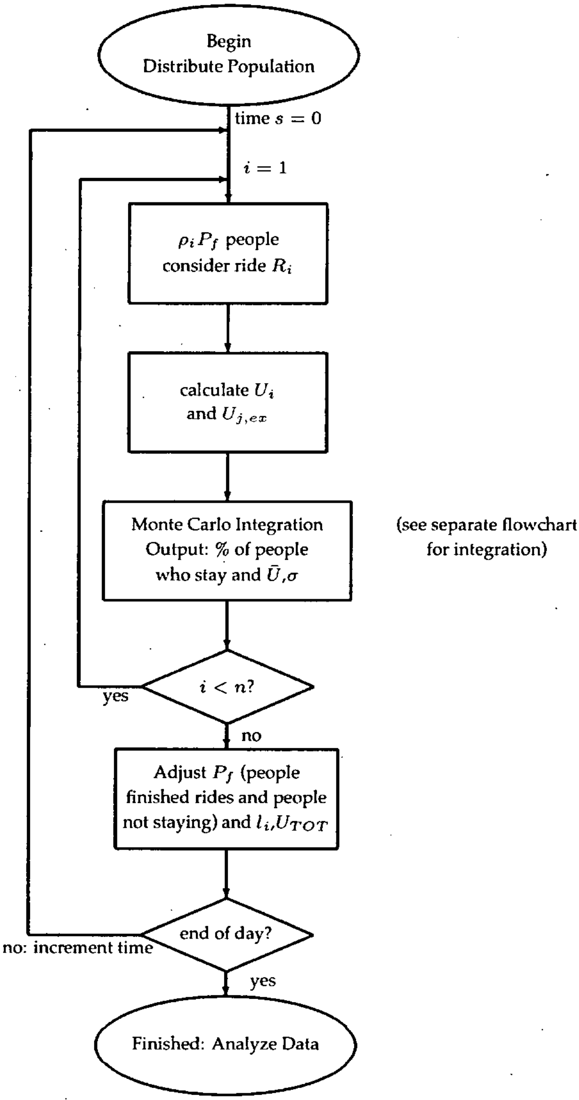

Figure 2. A schematic depicting the steps occurring in the Aggregate-Decision Model.

Additional Definitions

- A QP *expires* if
  - the individual does not enter the QP line during the QP's time interval (the individual *forfeits* the QP).
  - The individual accepts another QP (in models that allow doing so).

Since our framework presents a different decision model for QP holders vs. non-QP holders, we track the population of QP holders.

- Let q_i be population of the QP line at ride R_i. Let d_i be the rate at which R_i draws from q_i (people/min) and let c_i similarly be the rate at which l_i shortens—clearly, $d_i + c_i = k_i$.
- $l_{i,QP}, l_{i,\text{NOQP}}$ are the number of QP- and non-QP-holders in the line to R_i, respectively.
- Similarly, $P_{F,\text{QP}}$, $P_{F,\text{NOQP}}$ are the fluid populations of QP- and non-QP-holders, respectively.
- We do not track individuals but we track the QPs handed out for each time interval. Let $\text{QP}_{i,s}$ be the number of QP users with QPs for ride R_i at the sth time interval $[sI, (s+1)I]$. The $\text{QP}_{i,s}$ can decrease through forfeiting and increase according to the *allocation routine* of the QP scheme.

Assumptions about the QuickPass System

We assume that $\text{QP}_{i,s}$ is uniformly distributed throughout all lines and rides; that is, for any line, the people in that line with a QP are uniformly distributed throughout the line. However, the proportion of people with a QP can vary from ride to ride.

There is a limit δ for the total number of QPs for a ride.

Formal Development of the Model

The model relies on examining the populations with and without QPs, determining the proportion of each who take certain actions, and updating the populations and line counts. **Figure 3** presents the intuitive summary of how our model works. We describe the model illustrating two different scheme factors: the case where QP holders may cancel their QP for another QP (the cancel model), and the case where they cannot (and must wait until their QP expires to obtain a new QP).

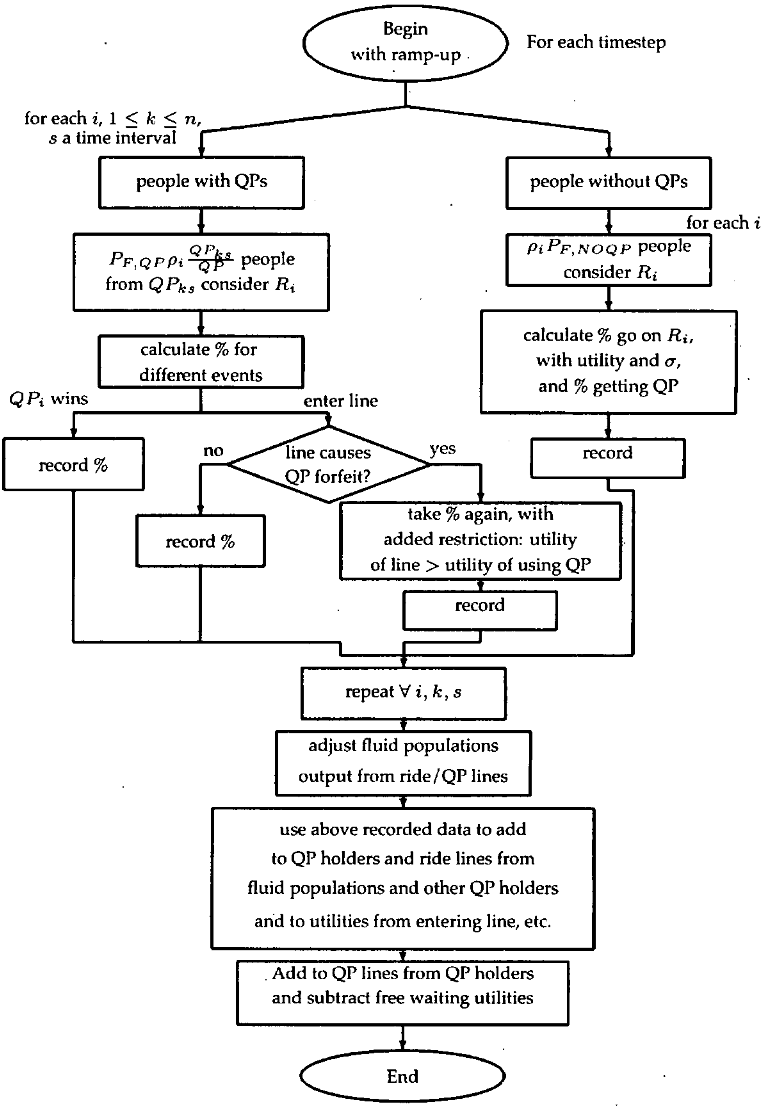

Figure 3. A flowchart detailing the Aggregate-Decision QuickPass Model.

Aggregates without QuickPasses

An individual without a QP must decide not only which ride to go to but also whether to get a QP for some ride, hence will examine the expected utility from staying at a particular ride vs. the expected utilities of other rides—just as in the non-QP model. However, with QPs, an individual also compares that with the expected utilities of obtainable QPs.

As in the non-QP model, the individual estimates the utility of ride j as

$$\mathcal{U}_{ji}(\mathcal{T}_1,\ldots,\mathcal{T}_n) = \mathcal{T}_j - \frac{\mathcal{T}_j P_T}{c_i \sum_k \mathcal{T}_k}.$$

Now the person compares these utilities to the utility provided by a QP. Because the person cannot go on the ride immediately, there is a disutility proportional to the wait. We assume that the constant of proportionality e is approximately the same for all individuals. Further, an individual assumes that the length of the QP line later will be approximately the current length. Therefore, the utility for a person getting a QP at ride R_i (if available) is

$$\mathcal{U}_{ii}^{qp}(\mathcal{T}_1,\ldots,\mathcal{T}_n) = \mathcal{T}_i - \frac{q_i}{d_i} - ew_i,$$

where w_i is the time of the wait on the QP ticket. If there is free waiting-time clairvoyance, the individual will know w_i in advance of getting the ticket; if not, and the person does not know what w_i will be, the person approximates w_i as proportional to the current length l_i of the line. This is reasonable if we regard the l_i and w_i as correlated with the popularity of the ride. Thus, a person not knowing w_i approximates it as $\kappa l_i/c_i$, for some constant of proportionality κ; for rides other than R_i, the approximation is $\kappa P_T r_j/c_i$. Thus, we have

$$\mathcal{U}_{ii}^{qp} = \mathcal{T}_i - \frac{q_i}{d_i} - e\kappa l_i/c_i, \qquad \mathcal{U}_{ji}^{qp} = \mathcal{T}_j - \frac{q_i}{d_i} - \frac{\mathcal{T}_j}{\sum_k \mathcal{T}_k}\frac{e\kappa P_T}{c_i}.$$

So an individual decides to go on ride R_i if the utility U_{ii} exceeds each U_{ji} and U_{ji}^{qp}. Similarly, a person gets a QP at ride R_i if U_{ii}^{qp} exceeds everything else (that is, if U_{ii} exists—if not, then the individual cannot opt for a QP). To find the proportions of the aggregate that make these decisions, we must integrate over the domains $\Omega_0, \Omega_1 \subset \mathbb{R}^n$ in which U_{ii} is greater than all alternatives and U_{ii}^{qp} is greater than all alternatives, respectively. Similarly, we calculate the average utility gained by entering the line.

Aggregates with QuickPasses

People who already hold QPs provide an additional complication. As **Figure 3** demonstrates, a QP-holder considering a ride can forfeit the QP and enter a line, simply enter the line, obtain a new QP (in our cancel model), or do nothing. Because the decision to forfeit depends on the time interval in which the user holds the QP, we must split up our aggregates into proportions corresponding to each $\mathrm{QP}_{i,s}$. Since each $\mathrm{QP}_{i,s}$ is uniformly distributed throughout the population of QP-holders, which in turn is uniformly distributed in line and in the fluid populations, the subset of $QP_{i,s}$ that is fluid is $P_{F,\text{QP}}\text{QP}_{i,s}/\text{QP}_{TOT}$; hence, we can say as before that $\rho_i P_{F,\text{QP}}\text{QP}_{i,s}/\text{QP}_{TOT}$ people consider ride R_i.

Then these people consider the expected and actual utilities of obtaining a QP for each ride and for entering any ride line. Via integration, we calculate the proportion of people who choose another QP, with the added comparison against the remaining utility of the existing QP $U^{qp}_{k,i,s}$, which only accounts for the remaining free waiting time (the lost time is sunk).

We calculate the proportion of people who enter a line (and the respective average utility) as the previous section, with one caveat: If members of the QP population would forfeit their QPs by entering a line, then we must also ensure that the utility that they would gain by entering line is greater than the remaining utility of their existing QP $U^{qp}_{k,i,s}$—an additional constraint on the domain of integration.

Adjustments to Populations, Lines, and Utility

- At every ride, we must move people from lines and QP lines into the fluid population $P_{F,\text{NOQP}}$ and $P_{F,\text{QP}}$. To account for the fact that our lines l_i and q_i may have fewer individuals than the rates at which they are drawn from $c_i\tau$ and $d_i\tau$, respectively, we create a function that incorporates checking whether (q_i or $d_i\tau$) and (l_i or $c_i\tau$), respectively, is the minimum. Let $x_i = \min(q_i, d_i\tau)$, $y_i = \min(l_i, c_i\tau)$, η_i = the number of people who leave from l_i, and ζ_i = the number who leave from q_i. Then

$$\eta_i = \min\left(c_i\tau\,\frac{x_i}{d_i\tau} + k_i\tau\,\frac{d_i\tau - x_i}{d_i\tau},\ l_i\right),$$

$$\zeta_i = \min\left(d_i\tau\,\frac{y_i}{c_i\tau} + k_i\tau\,\frac{c_i\tau - y_i}{c_i\tau},\ q_i\right).$$

Then from this ride, the new line and fluid population lengths become:

$$\begin{aligned}
P_{F,\text{NOQP}} &\leftarrow P_{F,\text{NOQP}} + \lfloor \frac{l_{i,\text{NOQP}}}{l_i}\eta_i \rfloor + \lceil \zeta_i \rceil \\
P_{F,\text{QP}} &\leftarrow P_{F,\text{QP}} + \lceil \frac{l_{i,\text{QP}}}{l_i}\eta_i \rceil \\
l_{i,\text{NOQP}} &\leftarrow l_{i,\text{NOQP}} - \lfloor \frac{l_{i,\text{NOQP}}}{l_i}\eta_i \rfloor \\
l_{i,\text{QP}} &\leftarrow l_{i,\text{NOQP}} - \lceil \frac{l_{i,\text{QP}}}{l_i}\eta_i \rceil \\
q_i &\leftarrow q_i - \lceil \zeta_i \rceil.
\end{aligned}$$

To see why this is so (up to our rounding), suppose that $q_i < d_i\tau$ (the argument for $l_i < c_i\tau$ is symmetrical). Then, since

$$d_i\tau\frac{y_i}{c_i\tau} + k_i\tau\frac{c_i\tau - y_i}{c_i\tau} > d_i\tau$$

(because $k_i \geq d_i$), note that $\eta_i = q_i$, and ζ_i is as we would like, because R_i takes people from l_i with rate c_i until q_i is depleted, after which it takes from l_i at the full rate k_i—that is, unless this number is less than l_i.

- For all of the QP populations free-waiting another unit, subtract a unit of utility $e\tau$.

- Once this process has been completed, for each ride R_i, for the population $\mathrm{QP}_{i,s}$ where s is an interval that is currently live, move a certain number of people into the QP line q_i from R_i. This number need not be the entire population $\mathrm{QP}_{i,s}$ but rather a random number predetermined by our model such that after the live period is over, the entire population has entered the QP line. This certainly corresponds to real life, in which the arrival time of humans in queues is erratic and can affect the dynamics of the queue. For each collection of individuals added to the queue, add the remaining utility of their QP—essentially $(t_i - q_i/d_i)$.

- Multiply each of the average utilities by the number of people entering the line, and add, to get approximate total net utility, and add these total net utilities to the total utility.

- Use proportions calculated and the aggregate size to determine the number of people entering a given line l_i, from the fluid members of each $\mathrm{QP}_{i,s}$, (and consequently $P_{F,\mathrm{NOQP}}$—these are added to $l_{i,\mathrm{QP}}$), and from the non-QP fluid populations (which are added to $l_{i,\mathrm{NOQP}}$). If the fluid members of $\mathrm{QP}_{i,s}$ entering the line are forfeiting their QPs, add them to $l_{i,\mathrm{NOQP}}$ instead, and remove that number of members from $\mathrm{QP}_{i,s}$.

- Finally, use the proportions calculated to determine how many people are given a new QP, whether from the fluid non-QP population or from another QP population $\mathrm{QP}_{j,s}$. Distribute this number among the QP_i using the *assignment routine* of the scheme.

QuickPass Schemes

We propose alternatives to the four factors below:

- **Free-Waiting-Time Clairvoyance** We allow people to know the free waiting time prior to getting a QP ticket, so they can better gauge whether to get a QP or to wait in line for the ride.

- **Cancellation Flag** People can get a new QP while a previous one is active; doing so deactivates the old QP.

- **Service Protocol** The QP line is served at a rate proportional to the length of the QP line, instead of at a constant rate.

- **Assignment Routine** A QP is *front-end-loaded* if the time interval on the ticket is the closest time interval with an available spot open (whether from cancellation or otherwise). Such loading is efficient but leads to anomalies such as two people getting QPs within minutes but the second person having a shorter wait. A QP is *queue-loaded* if it goes to the next time interval that is not fully populated and assigns a person to that time. This scheme is fair but does not take into account cancellations.

Case Study

To test the schemes, we study the amusement park Six Flags Magic Mountain, in Los Angeles, CA. Even though Magic Mountain does not use the Fastlane technology (an electronic modified version of QP), many other Six Flags parks of comparable size and type do [Six Flags Theme Parks 2004]. We estimate R_i, k_i, μ_i, σ_i, and P_T.

Magic Mountain has six rides with long waits during the busiest time, 3:00–4:00 [Ahmadi 1997]; we give these rides QP lines. We do not give QP lines to two other rides with medium-length lines, and we combine the other 20 rides into two generic rides with minimal utility, high k_i (serving rates), $\mu_i = 5$, and $c_i = 50$. We calculate all k_i data (except for FreeFall) from the Roller Coaster DataBase [Marden 2004]; Freefall's k_i is calculated from the New Jersey Six Flags Freefall [Six Flags Great Adventure 2004]. Meanwhile, we estimate the expected utilities μ_i from the values for wait-line times between 3:00–4:00 [Ahmadi 1997]. Lastly, to introduce reasonable variation, we estimate the σ_i to be one-fifth of the μ_i. **Table 2** summarizes the values.

Table 2.

Line speeds and expected utility values for Case Study rides. The first group are lines with QP lines and the second group are without such lines.

Ride	Name	k_i (people/min)	μ_i (min)	σ_i (min)
QP				
R_1	Ninja	26.67	48	9.6
R_2	Colossus	43.33	80	16
R_3	Flashback	18.33	60	12
R_4	FreeFall	3.75	72	14.4
R_5	Psyclone	20	46	9.2
R_6	Viper	28.33	54	10.8
Not QP				
R_7	Goldrusher	29.17	25	5
R_8	Revolution	6.67	35	7
R_9	Generic Ride 1	50	5	1
R_{10}	Generic Ride 2	50	5	1

We also must estimate P_T. Magic Mountain has daily attendance from 9,000 to 35,000 [Ahmadi 1997]; we estimate an expected P_T of 20,000 people/day. From surveys, we estimate $e = 0.2$ (ratio of the disutility for free waiting to line waiting) and $\kappa = 3$ (ratio of the time for free waiting to regular line waiting). We assume that $\delta = d_i \tau I$ (maximum number of QPs issued for R_i in an interval), which is the number of people that the QP line can handle in one time interval.

Criteria for Judging Schemes

- **Enjoyment:** Measured in minutes, this quantity is the net total utility. A QP system should enhance total utility.
- **Fairness:** Fairness plays a major role in people's satisfaction [Larson [1987]. In a fair system, a person obtaining a QP before someone else should be serviced first. We keep track of all relevant data by recording the allocation of QPs to different time intervals for each time step.

Representative Schemes and Simulation Results

We choose five representative schemes (plus a control case based on the Aggregate-Decision Model), to which we apply our two criteria for success. The various features of the test schemes are indicated in **Table 3**.

Table 3.
Overview of properties of test schemes.

Scheme #	1	2	3	4	5	Control
Waiting-time clairvoyance	No	No	No	Yes	Yes	Control
Cancellation allowed	No	No	Yes	No	Yes	Control
Service protocol (A: Const. Am't., P: Const. Prop'n)	A	P	A	P	P	Control
Assignment routine (Q: Queue, FE: Front-end)	Q	Q	FE	FE	FE	Control

Conclusions and Extensions of the Problem

Solving the Problem

In **Figure 4** and in **Table 4** we give the results, in terms of our criteria of enjoyment and fairness, for the five representative schemes plus control case.

In enjoyment, Scheme 5 out-performs all of the other schemes (**Figure 4**). This result meshes with our expectations that increased knowledge and choices, as well as a more-efficient service protocol and assignment routine, result in higher utility.

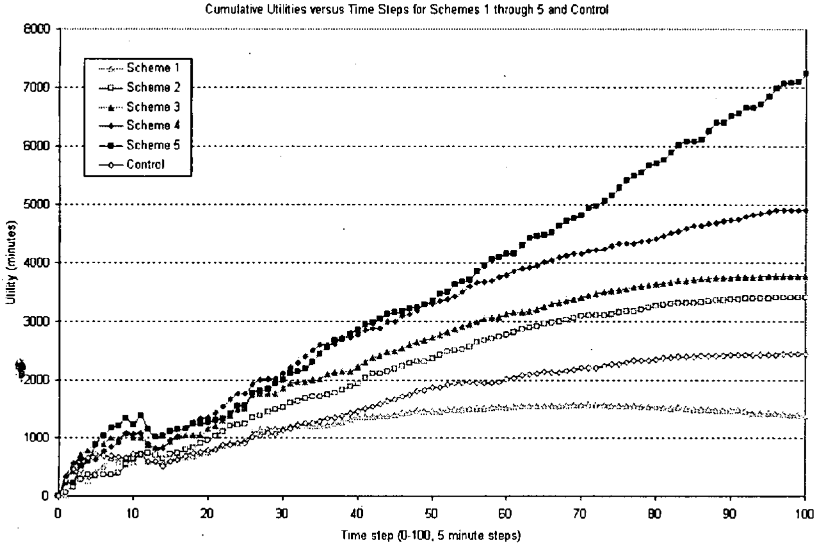

Figure 4. A graph demonstrating cumulative utilities and thus enjoyment levels associated with representative test schemes. Scheme 5 dominates after 50 time steps.

Table 4.
Overview of test scheme fairness as measured by QuickPass anomalies to assignments.

Scheme #	1	2	3	4	5	Control
Ratio of QuickPass anomalies to QuickPass assignments	0.00	0.00	0.36	0.03	0.04	0.00

Queue-loading systems do not allow anomalies in QP assignments and therefore are the most fair by that standard. In addition, to prevent unfair line-length distributions, we should use the constant-proportion service protocol. **Table 4** shows that all schemes except Scheme 3 result in about equal fairness levels.

On both criteria, Scheme 5 performs nearly as well as (if not better than) the alternatives. Ideally, then:

- the waiting time should be displayed, and
- people should be able to cancel a QP by activating another,
- the serving rate for the QP line should be proportional to the QP line length,
- QPs should be assigned via front-end loading (i.e., for the soonest time interval with available space).

Further Study

Our results have a recurring theme: The more knowledge and greater number of choices an individual has, the higher the cumulative utility tends to be.

We propose that every ride should have an electronic display of the waiting times for normal and QP lines for *all* rides.

Our model does not allow QP selling and trading, but allowing it might improve cumulative utility and fairness, as discussed in Prado and Wurman [2002].

Strengths of Model

The Aggregate-Decision model is a realistic and robust probabilistic framework that, for large population sizes, is statistically accurate at calculating aggregate behaviors. The utility functions are simple and realistic and reflect the processes of individual decision-making. The decisions, based on utility comparison, are reasonable and reflect the interest of the individual.

Weaknesses of Model

The probabilistic nature of our model and related statistical flaws are the main source of weakness. Our model for the aggregate breaks down for small population sizes and small numbers of rides, where issues such as memory and changing preferences influence preference distributions. We assume that the distributions remain constant over time, but preferences change with experience (whether riding or waiting). Our model also assumes that preference distributions are independent of one another, while in reality we expect that more of the people who enjoy one roller coaster also enjoy a similar one.

Whereas most humans would plan out a series of rides rather than considering just one ride at a time, our model does not have such forethought capabilities.

Lastly, our assumptions on uniform distances neglect the actual geometric configuration of the park and the effect of that geometry on ride considerations.

Conclusion

Our model takes into account the limited knowledge that influences the decisions of park-goers, based on economic assumptions. We apply our understanding of individual decision-making to develop a versatile model of aggregate decision-making for parks with and without QPs.

We tested different QP schemes systems using data from the Six Flags Magic Mountain. Factors in a QP scheme include:

- whether an individual has foreknowledge of the time interval for which a QP will be issued;
- whether people can cancel an active QP and obtain a new one;
- how people are fed onto the ride from the queues; and
- how QP times are allocated.

Our criteria for successful schemes were:

- Cumulative utility, summed over all people throughout the entire day, from taking rides and waiting in lines.
- Fairness, as measured by the ratio of the number of anomaly QP allocations to the total number QPs.

We compared QP schemes and found that the scheme with the greatest utility has the following properties:

- People have foreknowledge of when QPs are being issued for (perhaps by way of an electronic sign posting this information),
- people can cancel their QPs by switching to another QP,
- the QP line moves at a rate proportional to its length, and
- when people are allocated QP tickets, they receive tickets for the first available time interval.

This scheme provides a cumulative utility (measured in minutes) of 7,000, while the next highest cumulative utility was only 5,000 (the control had cumulative utility of 2,500 minutes). This scheme had an anomaly-to-allocation ratio of 0.04, while other schemes had values as high as 0.36.

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