

# Air Traffic Control

Samuel Westmoreland Malone

Jeffrey Abraham Mermin

Daniel Bertrand Neill

Duke University

Durham, NC

Advisor: David P. Kraines

## Introduction

We propose five models. Each gives a metric for measuring *eventual danger* and a threshold such that controller intervention becomes necessary. An *immediate danger* metric prioritizes problems for a controller.

We test the models on several different sample cases. The Close-Approach model matches most closely our intuitive understanding of the situations.

We present an algorithm that models the decision process of a controller detecting and solving conflicts. The time-complexity of scanning for potential conflicts varies quadratically with the number of airplanes, though this could be reduced by clustering the airplanes by proximity. We argue that conflict resolution for a cluster of  $n$  airplanes is an NP-complete problem with worst-case complexity  $2^{n(n+1)/2}$ .

## Assumptions and Hypotheses

- A *near mid-air collision* (sometimes called a *near miss*) is defined by the FAA as an incident in which two airplanes pass within 500 ft of each other [Federal Aviation Administration 2000].
- The airspace can be represented by a convex subset of  $R^3$ .
- Air-traffic controllers have established protocols to prevent airplanes from colliding when crossing airspace boundaries in opposite directions.
- We know the position and velocity of every airplane in the airspace, with negligible error.

---

*The UMAP Journal* 21 (3) (2000) 241–256. ©Copyright 2000 by COMAP, Inc. All rights reserved. Permission to make digital or hard copies of part or all of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice. Abstracting with credit is permitted, but copyrights for components of this work owned by others than COMAP must be honored. To copy otherwise, to republish, to post on servers, or to redistribute to lists requires prior permission from COMAP.

- Every airplane has sufficiently negligible acceleration that linear models for its movement make sense over at least the next 2 min unless it is:
  - accelerating under the direction of a controller (so the controller has already determined potential conflicts),
  - taking off (so it is not in the airspace used by cruising airplanes), or
  - attempting to land (so it is not in the airspace used by cruising airplanes).
- Airplanes can accelerate through a 2-minute turn, either clockwise or counterclockwise, parallel to the  $xy$ -plane [Denker 2000].
- Airplanes travel within vertically well-separated planes, called “cruising altitudes” [Mahalingam 1999, 27]. Airplanes in different cruising altitudes pose no danger to each other.
- Airplanes can accelerate at a given maximum rate in the direction of their velocity. (In particular, they do not cruise at their maximum speed.)
- Two airplanes present an *eventual danger* when, if their velocities are allowed to go unchanged indefinitely,
  - they will collide,
  - they will pass “near” each other at some time, or
  - they will pass through “nearby” points in space at “nearby” times.

We later discuss appropriate values of “near.”

## Possible Solutions to the Danger Problem

Two considerations dominate in determining the danger of two airplanes to each other: the proximity that they will attain and the time until it occurs. We present several approaches:

- The Trivial Model provides yes-or-no answers to the question “Will the airplanes collide?”
- The Probabilistic Model determines risk based on the probabilities of collisions and near misses.
- The Close-Approach Model calculates the closest approach of two airplanes and the time until it occurs.
- The Space-Time Model considers the closest approach in four-dimensional space-time and the time until it occurs.
- The Logarithmic Derivative Model approximates a human observer’s intuitions about how fast the airplanes are approaching each other.

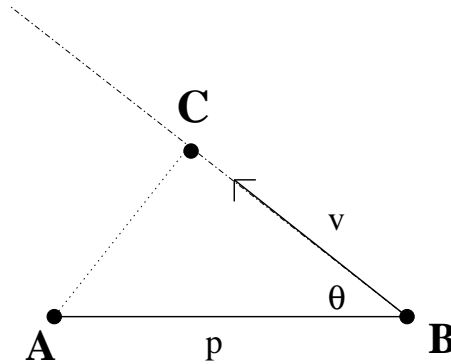
## The Trivial Model

### How It Works

The Trivial Model ignores effects such as wind, measurement uncertainty, or piloting imperfection, which could make an airplane's course deviate from the linear projection of its current position and velocity.

Large commercial aircraft have lengths and wingspans of about 200 feet. Thus, *a collision occurs only if the centers of airplanes pass within 200 ft of each another, and a near miss occurs only if they pass within 700 ft.*

Suppose airplanes  $A$  and  $B$  have position and velocity vectors  $p_A, v_A$  and  $p_B, v_B$ . Set  $p = p_B - p_A$  and  $v = v_B - v_A$ , the position and velocity of  $B$  relative to  $A$ . The distance of closest approach is the altitude from  $A$  to  $v$  (**Figure 1**). Its length is  $d = |p| \sin \theta = |p \times v| / |v|$ .



**Figure 1.** The position and velocity vectors of airplane  $B$  relative to airplane  $A$ .

There will be a collision if  $d$  is ever less than 200 ft, and a near miss if it is ever less than 700 ft. A measure of the eventual danger takes on three discrete values  $a$  ( $\gg 1$ ), 1, or 0 corresponding to a collision, a near miss, and no danger. The value of  $a$  is best determined empirically.

### Strengths and Weaknesses

This model is simple and efficient. However, it assumes that airplanes always travel at a constant speed in a straight line; but in fact airplanes are buffeted by changing winds and their actual trajectories vary significantly and chaotically from those predicted by a linear model. Additionally, this model considers only eventual danger, not how soon immediate danger will be present, though the model could be extended to rank collisions and near misses based on immediate danger (time to collision or near miss).

## Probabilistic Simulation Model

### How It Works

The Probabilistic Simulation Model calculates the probability that a given situation will result in a collision or near miss by using a Monte Carlo simulation. To do so, it performs a large number of random trials (each of which may result in a collision, near miss, or neither) and computes a measure of eventual danger:  $\text{danger} = c_1x + c_2y$ , where  $x$  and  $y$  are the probabilities of collision and near miss and, as in the Trivial Model, we set  $c_2 = 1$  and  $c_1 \gg 1$ .

We assume normal distributions of each airplane's speed and direction, with the mean the measured value and the standard deviation specified by the user. For each trial, a normally distributed random value of each quantity is chosen, then both airplanes' paths are extrapolated linearly; we use the minimum-distance formula from the Trivial Model to determine whether a collision, near miss, or neither occurs.

### Strengths and Weaknesses

Though a minimum time to collision or near miss is computed, this time is not taken into account in the measure of danger. Hence, we add an optional user-specified maximum time horizon; if two airplanes do not reach their minimum distance by then, their distance at that time is considered instead of the (later) minimum distance. Thus, we ignore conflicts that occur far in the future, focusing on more immediate dangers. According to one source [MAICA: MET improvement . . . , 124], conflict analysis tools that extrapolate based on current aircraft trajectories "operate over a short time horizon, generally less than two minutes."

## The Close-Approach Model

### How It Works

We expect eventual danger to be inversely related to closest approach and immediate danger to be inversely related to time until that closest approach. Thus, we have, as a first approximation,

$$\text{Eventual danger} \approx \frac{1}{(\text{distance of closest approach})^\alpha},$$

Immediate danger  $\approx$

$$\frac{1}{(\text{distance of closest approach})^\alpha (\text{time until closest approach})^\beta}.$$

Since danger could be averted by accelerating the airplanes away from each other, the extra separation achieved should be proportional to the square of the

time of acceleration. Since the time during which they can accelerate is bounded by the time until projected close approach, it seems reasonable to set  $\beta = 2\alpha$ . Since raising to a positive power doesn't affect ordering, we set  $\beta = 2$ ,  $\alpha = 1$ .

Such a simple formula runs into trouble in boundary situations:

- No matter how far away the airplanes will be at their closest approach, immediate danger goes to infinity as they come to closest approach.
- If the two airplanes are on a collision course, the formula gives infinite immediate danger, no matter how much time remains until collision.
- If the airplanes have nearly equal velocities, the formula rates immediate danger as near zero (unless the aircraft are practically on top of each other) when it should intuitively be inversely proportional to current separation.

We fix the formula as follows:

$$\text{Immediate danger} = \frac{1}{(\text{distance of closest approach} + c_1)(\text{time until closest approach} + c_2)^2} + \frac{c_3}{\text{current separation}},$$

where  $c_1$ ,  $c_2$  and  $c_3$  are positive constants, probably best determined empirically.

Now we calculate the ingredients in the formula. If the distance of closest approach of a pair of airplanes is sufficiently large (e.g.,  $d > 5$  nautical mi [Mahalingam 1999, 26–27]), they pose no danger to each other. If they will pass closer, we rate the level of eventual danger as  $d$ .

The time until closest approach is the time until airplane  $B$  reaches point  $C$ :

$$\frac{|\overrightarrow{BC}|}{|v|} = \frac{|p| \cos \theta}{|v|} = \frac{\frac{p \cdot v}{|v|}}{|v|} = \frac{p \cdot v}{|v|^2}.$$

Plugging in, we get

$$\text{Immediate danger} = \frac{|v|^5}{(|p \times v| + |v| c_1) (p \cdot v + |v|^2 c_2)^2} + \frac{c_3}{|p|}.$$

Since we get 0/0 in the first summand when  $v = 0$ , that is, when the two airplanes are flying parallel to each another, in this case we set the immediate danger equal to  $c_3/|p|$ .

## Strengths and Weaknesses

The danger can be computed with just over 50 basic numeric operations if there is eventual danger and about half as many to conclude that there is

not; thus, a personal computer could handle this computation every second for several thousand airplane-pairs, or about 500 airplanes every 2 min.

This model does not worry about airplanes that pass near each other in time but not in space. For example, it cannot distinguish between two airplanes flying the exact same route through space with a time-separation of 15 sec and two airplanes flying parallel with a physical separation of 2 mi in a direction orthogonal to their velocity; the first situation appears to us to be much more dangerous. The next model attempts to differentiate such situations.

## The Space-Time Model

### How It Works

The Space-Time Model uses similar reasoning to the Close-Approach Model but considers the airplanes' proximity in space-time rather than simply in physical space. Thus, intuitively, we have:

$$\text{Eventual danger} \approx \frac{1}{\text{closest approach in space-time}},$$

Immediate danger  $\approx$

$$\frac{1}{(\text{closest approach in space-time}) (\text{time until closest approach})^2}.$$

The same corrections for boundary conditions apply, leaving

Immediate danger =

$$\frac{1}{(\text{closest approach in space-time} + \gamma_1) (\text{time until closest approach} + \gamma_2)^2} + \frac{\gamma_3}{\text{current space-time separation}}.$$

These quantities are harder to compute than in the Close-Approach Model. We can represent the future of airplane *A* (initially at the origin) and airplane *B* by rays in  $R^4$ , parametrized by the vectors  $(v_{ax}t_a, v_{ay}t_a, v_{az}t_a, kt_a)$  and  $(v_{bx}t_b + p_x, v_{by}t_b + p_y, v_{bz}t_b + p_z, kt_b)$ , for  $t_a, t_b > 0$ , where  $k$  is a constant chosen so that one unit of time is as dangerous as one unit in one of distance. Mahalingam [1999, 26–27] equates a 15-min separation to a 5-nautical-mile separation; we assume that this equivalence scales. Then  $k$  equals 5 nautical mi per 15 min, or about 34 ft/s.

For any  $t_a, t_b$ , the space-time distance between airplane  $A$  at time  $t_a$  and the airplane  $B$  at time  $t_b$  is

$$\delta(t_a, t_b) = |(v_{a_x}t_a, v_{a_y}t_a, v_{a_z}t_a, kt_a) - (v_{b_x}t_b + p_x, v_{b_y}t_b + p_y, v_{b_z}t_b + p_z, kt_b)|.$$

This yields

$$(\delta(t_a, t_b))^2 = At_a^2 + Bt_at_b + Ct_b^2 + Dt_a + Et_b + |p|^2,$$

where

$$A = k^2 + |v_a|^2, \quad B = -2k^2 - 2v_a \cdot v_b, \quad C = k^2 + |v_b|^2, \quad D = -2p \cdot v_a, \quad E = 2p \cdot v_b.$$

The minimum space-time distance occurs at  $t_a$  and  $t_b$  that minimize this expression, that is, where  $\nabla \delta^2 = 0$ :

$$t_\alpha = \frac{2CD - BE}{B^2 - 4AC}, \quad t_\beta = \frac{2AE - BD}{B^2 - 4AC},$$

which are well-defined whenever the velocities are not equal, since

$$B^2 - 4AC = 4 \left[ (v_a \cdot v_b)^2 - |v_a|^2 |v_b|^2 + 2k^2 v_a \cdot v_b - k^2 |v_a|^2 - k^2 |v_b|^2 \right].$$

The first two terms add to less than zero by Cauchy-Schwarz, and the last three by AM-GM, with equality in both cases only when the velocities are equal. [The case when the velocities are equal is handled more simply: For every  $t_\alpha$ , there is a unique  $t_\beta$  satisfying  $Bt_\alpha + 2Ct_\beta + E = 0$  (since  $C$  is always positive) that yields the minimum space-time separation.] The minimum space-time separation is  $\delta(t_\alpha, t_\beta)$ , and the time until this separation is  $\min\{t_\alpha, t_\beta\}$ .

The current space-time separation would appear to be the minimum of the space-time separations between the current position of airplane  $A$  and the future of airplane  $B$ , and the current position of  $B$  and the future of  $A$ .

Determination of danger is done as in the Close-Approach Model, except that the times associated to closest approach must be computed first. If either is negative, then any danger posed by this airplane-pair has already been avoided, and so the eventual and immediate dangers are set to zero. Then the space-time separation of the closest approach is computed; if it is sufficiently large (e.g., more than 5 nautical mi), then the airplanes pose no danger to each other.

## Strengths and Weaknesses

Every airplane-pair receives at least as high immediate- and eventual-danger measures from the Space-Time Model as from the Close-Approach Model, while the Space-Time Model recognizes as dangerous some cases that the Close-Approach Model does not. The Space-Time Model is not significantly slower in operation.

On the other hand, this model is much more opaque to any human who must try to work with it; human beings are not equipped to think in terms of extra dimensions.

## The Logarithmic Derivative Model

### How It Works

The model arises from the observation that, if the velocities of airplanes  $A$  and  $B$  remain constant, the time derivative  $dd/dt$  of the distance between the airplanes is monotonically increasing with time (unless the airplanes are traveling in the same line, in which case it is constant) and is bounded both above and below. Thus, the negative derivative is monotonically decreasing, and

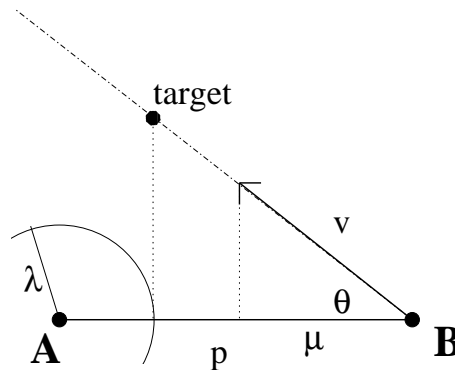
$$-\frac{d-\ell}{\frac{dd}{dt}} \bigg|_{t=t_0}$$

gives a lower bound on the time between  $t_0$  and any future time  $t$  at which the airplanes are separated by a distance less than some danger threshold  $\ell$ . Thus, the reciprocal of this quantity,

$$-\frac{\frac{dd}{dt}}{d-\ell}$$

(the negative derivative of the logarithm of  $d - \ell$ ), might work as a measure of immediate danger.

We investigate the behavior of this function. Suppose that airplane  $B$  has position and velocity vectors  $p$  and  $v$  in some frame of reference where  $A$  is stationary at the origin, as in **Figure 2**.



**Figure 2.** The logarithmic derivative.

Now

$$\frac{d}{dt} (|p|^2) = \lim_{h \rightarrow 0} \frac{|p + hv|^2 - |p|^2}{h} = \lim_{h \rightarrow 0} \frac{2hp \cdot v + h^2 |v|^2}{h} = 2p \cdot v.$$

But

$$\frac{d}{dt} (|p|^2) = 2|p| \frac{d}{dt} |p|,$$

so

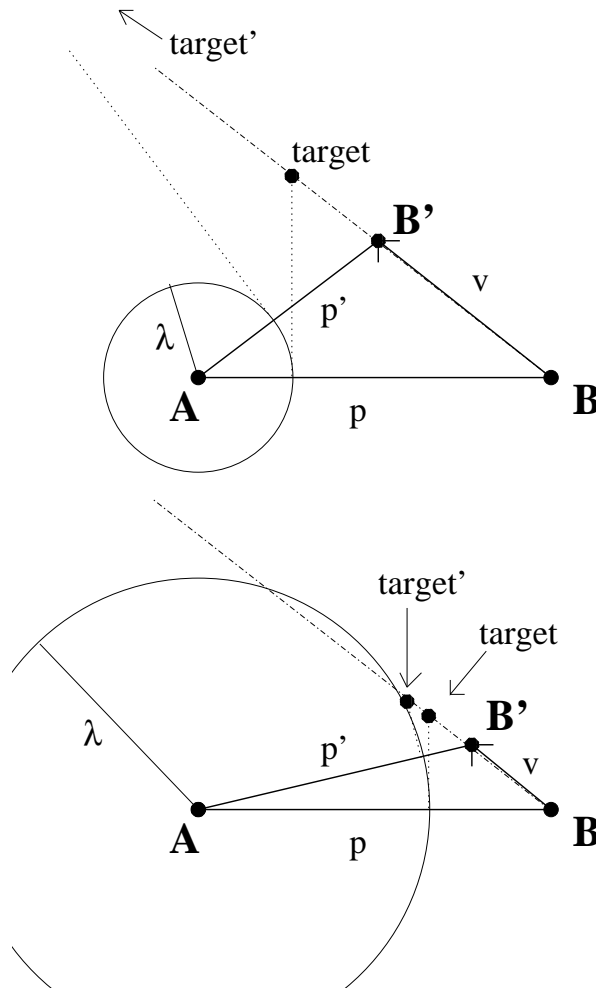
$$\frac{d}{dt} |p| = \frac{p \cdot v}{|p|} = |v| \cos \theta.$$



This quantity is represented in **Figure 2** by  $\mu$ , the projection of one time-unit of velocity onto  $p$ . Dividing by  $|p| - \ell$  gives the number of time units until the projection onto  $p$  intersects the circle of radius  $\ell$  about  $A$ .

Consider also the behavior of this function as time passes (**Figure 3**):

- If  $B$  will not pass within  $\ell$  of  $A$ , the target point goes to infinity as  $B$  approaches the point of least separation from  $A$ ; the measure simultaneously goes to zero, then becomes negative as that point is crossed. This is fine; all danger is past once the point of least separation is reached.
- If  $B$  will pass through the circle of radius  $\ell$ , the target point converges to the intersection of  $B$ 's trajectory with the circle; the measure goes to infinity as  $B$  approaches the circle, then becomes negative once it passes inside. If  $\ell$  is sufficiently small (e.g., 700 ft, the threshold for a near miss), this is also fine: Once the two airplanes are within  $\ell$  of each other, it is already too late.



**Figure 3.** Motion of the target point over time; two cases.

## Strengths and Weaknesses

The immediate-danger measure can be computed with only about 10 basic operations, even faster than the Close-Approach Model. Furthermore, it offers other useful information: The reciprocal of the measure is how much time remains until the airplanes play out whatever danger they face from each other.

This measure behaves correctly in two of the situations where the previous two measures required ugly fixes:

- If the two aircraft are on or very near a collision course, it acts as a countdown.
- If the airplanes are near in time to their closest approach, it gets large only if they are actually close to each other but remains small otherwise.

A flaw is that this measure always gives an immediate danger near zero when the airplanes have nearly identical velocities. As before, we'd like the danger in this case to be roughly inversely proportional to their separation. We can solve this problem by adding a term  $c/|p|$  to the measure; unfortunately, doing so eliminates the nice relationship between this measure and the time left for the controller to act.

A potentially more serious problem is the inability of this algorithm to project far into the future. It detects almost no difference between, for example, two pairs of airplanes with the same relative velocities, the first of which are on course to collide in 5 min, and the second to pass with 1 mi of separation.

## Testing the Models

In the situations of **Table 1**, all airplanes move at 480 knots (811 ft/s). Airplane *A*'s initial position is at the origin.

**Table 1.**  
Test situations.

Situation	<i>A</i> heading	<i>B</i> heading	<i>B</i> location (in ft)
1. Impending head-on collision	0°	180°	(6000, 0)
2. Impending oblique collision	60°	120°	(3000, 0)
3. Tailgating	0°	0°	(2400, 0)
4. Flying alongside	0°	0°	(0, 2400)
5. Same point, nearby time	0°	90°	(2400, -3200)
6. Same point, nearby time	0°	120°	(4400, -2100)
7. Passing at a distance	0°	180°	(18000, -6000)
8. Far-future head-on collision	0°	180°	(600000, 0)
9. Flying parallel	0°	0°	(0, 18000)
10. Right angles	0°	270°	(18000, 0)
11. Receding	0°	180°	(0, 6000)
12. Receding	120°	60°	(3000, 0)
13. Receding	180°	0°	(6000, 0)

We use our intuition and all of our models (except the Space-Time Model, for which we could not find appropriate constants in the allotted time) to rank the immediate dangers presented by each situation. With the Probabilistic Model, we use the metric

$$\text{danger} = \frac{\text{collisions}}{10000 \text{ trials}} + \frac{1}{20} \cdot \frac{\text{near misses}}{10000 \text{ trials}}.$$

The Close-Approach Model uses  $c_1 = 50$  ft,  $c_2 = 5$  s, and  $c_3 = .05$  Hz<sup>2</sup>.

**Table 2.**  
Rankings of dangers of situations.

Situation	Intuition	Triv	Prob	Close-App	Logarithmic
1	1	2	1	2	3.5
2	2	2	2	1	3.5
3	3	9.5	4	3.5	10.5
4	4	9.5	7	3.5	10.5
5	5.5	4.5	5	5	2
6	5.5	4.5	3	6	1
7	8.5	9.5	11	9	5
8	7	2	6	10	7
9	8.5	9.5	8	8	10.5
10	10	9.5	11	7	6
11	12	9.5	11	12	10.5
12	12	9.5	11	12	10.5
13	12	9.5	11	12	10.5

## Recommendations

### Which Danger Model Should We Use?

The Probabilistic and the Close-Approach Models match well our intuitive rankings, though not particularly each other. The Trivial Model and the Logarithmic Derivative Model both compare much less favorably.

The Close-Approach Model agrees with our intuition almost exactly, except for switching the rankings of situations 8 and 10. As we expect very little eventual danger from situation 10, and very little immediate danger from situation 8, this is perhaps not so bad. The Space-Time Model would probably do even better: It would certainly rank situation 3 as more dangerous than situation 4 (agreeing with our intuition), and, as it is closely related to the Close-Approach Model, might well rank all the other situations identically.

We recommend the Close-Approach Model.

## How Close Is Too Close?

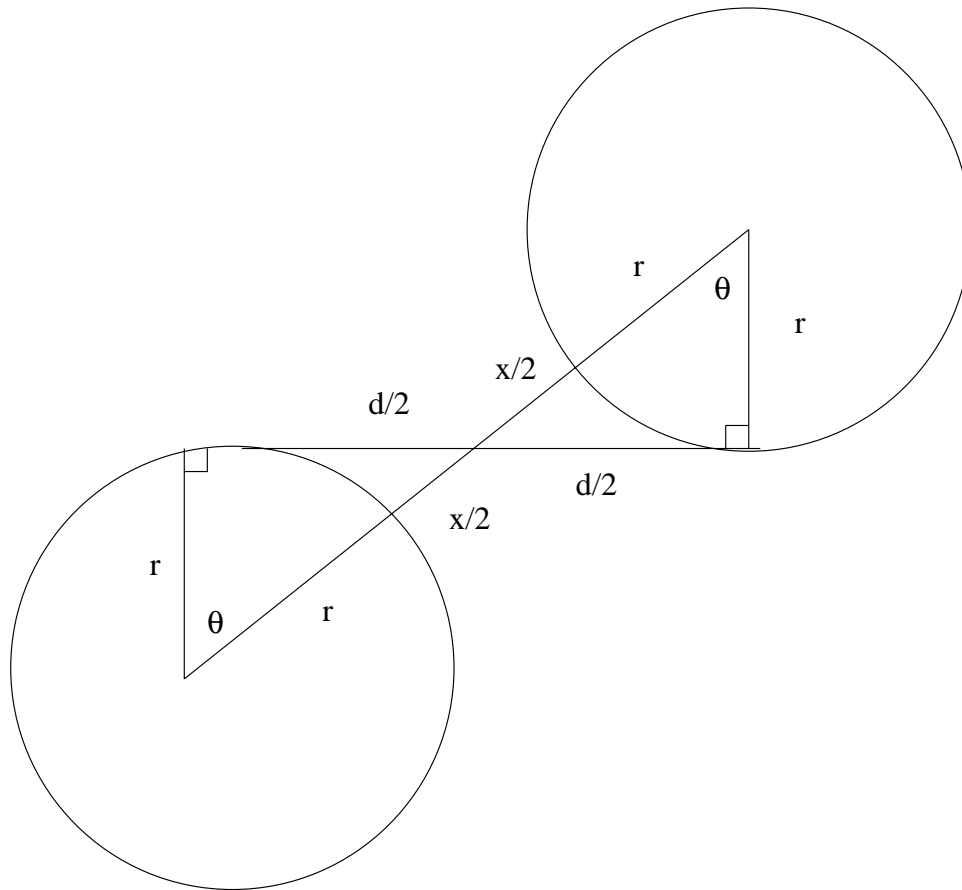
In the Close-Approach Model, there is a natural map from the eventual danger to distance (by taking the reciprocal). Mahalingam [1999, 26–27] argues that airplanes should be horizontally separated by 3 nautical miles. But is she right?

Assume that each airplane has velocity  $v$  ft/s and that each can turn  $z$  radians/s. We find the distance  $d$  (in ft) at which each airplane must start turning to ensure that the aircraft do not pass within  $x$  feet of each other at any time.

Each turn (assuming constant turning rate) forms an arc of a circle of radius  $r$ . Since the length of an arc subtended by an angle  $\theta$  is given by  $s = r\theta$ , we take the derivative to obtain

$$r = \frac{ds/dt}{d\theta/dt} = v/z.$$

Next, we note that  $x$  (the shortest distance between the two circles) is equal to the sum of the distance  $k$  between the centers of the two circles and the two radii:  $k = x + 2v/z$  (**Figure 4**).



**Figure 4.** Analysis of avoidance of head-on collision.

Finally, we observe that the initial line of flight of the two airplanes is tangent to both circles and hence two right triangles are formed. In each triangle, the lengths of the sides are  $r = v/z$ ,  $d/2$ , and hypotenuse  $k/2 = v/z + x/2$ . We apply the Pythagorean theorem to obtain  $(d/2)^2 = (v/z + x/2)^2 - (v/z)^2$ , so that  $d = \sqrt{x^2 + 4xv/z}$ .

We consider two airplanes with velocity 480 knots (811 ft/s) and turning rate  $3^\circ = \pi/60$  radians/s (a standard “two-minute turn”). To avoid a near miss, the centers of the airplanes must be 700 ft apart. We calculate  $d = 6,621$  ft = 1.09 nautical miles. So, both pilots must start a turn at a distance of at least 1.09 nautical miles apart, and the controller must identify the problem and communicate to the pilots before this point. Assuming a maximum delay of 15 s between the controller’s discovery of the problem and the pilots’ response gives a “safety distance” of about 5 nautical miles, or 19 s.

## Measuring Complexity

To measure the complexity of the workload faced by an air traffic controller (ATC), we need a basic understanding of the tasks performed by the ATC and how ability to perform these tasks is affected by the number of airplanes in the airspace sector. We present the following algorithm as a model for the decision process of an ATC in detecting and solving conflicts. The order of the steps is drawn from a synthesis by Endsley and Rodgers [1994] of reports on the factors identified by experienced air traffic controllers as relevant to conflict prevention.

### ATC Decision Algorithm

1. Scan the radar screen (and other sources of information) for airplanes located close to each other or currently at a safe distance but whose projected paths cross.
2. If a pair/group of airplanes at a given time instant appear close to each other, evaluate velocity and heading information to determine whether the airplanes will move to within a minimum separation distance of each other within the “near future” (2 min?).
3. If a potential conflict is detected, scan for other more pressing conflicts.
4. If there are no more-urgent conflicts, alert the pilots of the airplanes detected in Step 2 and formulate alternative routes for them.
5. Assess whether the alternative routes will cause conflicts with projected routes of nearby aircraft.
6. If the alternative routes will cause conflicts, reformulate other alternatives.
7. If there are no impending conflicts, or if the most recent conflicts have been resolved successfully, then take care of other tasks.

8. When the items in Step 7 have been adequately dealt with, return to Step 1.

## Complexity of Step 1

For  $n$  airplanes in the airspace, there are  $\binom{n}{2} = n(n-1)/2$  pairs, so this operation also has complexity of order  $O(n^2)$ .<sup>1</sup> More realistically, we could divide airplanes into clusters and analyze the complexity of each cluster, though clusters are not completely independent of one another.

## Complexity of Steps 2–6

We incorporate the danger presented by each aircraft pair into a single danger metric  $D$  for the airspace by summing the dangers of the individual pairs. It is useful first to assess each danger against a threshold level for ATC intervention; then the measure of danger  $D$  for the airspace becomes the number of interventions that must be made.

For each pair of endangered airplanes, the ATC must resolve the situation while making sure that the solution does not conflict with the constraints of any previously solved conflicting pair. Thus, each conflict constrains the choices to resolve each of the other conflicts; in some cases, after the first  $k$  conflicts are solved, no solution for conflict  $k+1$  may exist under the constraints, hence backtracking may be necessary. In other words, this problem is a form of the *general constraint satisfaction problem*, which is known to be NP-complete [Vardi 1999]. We guess that the worst-case complexity of our problem varies exponentially with the number of pairs:  $O(k^{n(n-1)/2})$ , for some constant  $k$ .

If the altitude of the airplanes cannot be changed, the ATC can either tell both pilots to bank to the right (from their point of view) or to the left. So, for every pair of aircraft in conflict, there are two possible solutions, hence  $2^D$  possible solutions for a system with  $D$  conflicts.

We must also take into account the decisions of the ATC in Steps 4 through 6, estimating how many operations are involved. For an airspace divided into  $C$  clusters, with  $n_i$  airplanes in cluster  $i$ , there are  $n_i - 2$  other airplanes to consider in resolving a conflict in cluster  $i$ . Thus there are (approximately)  $2(n_i - 2)(D(i))$  interactions that the ATC must consider in a given cluster  $i$ . The number of interactions added by this measure does not change the complexity of Step 2,  $O(k^{n(n-1)/2})$ .

---

<sup>1</sup>EDITOR'S NOTE: Several collision-detecting algorithms are known to be more efficient in practice than  $O(n^2)$  without having theoretical guarantees. For references, see Eppstein and Erickson [1999]. They also give an algorithm to solve the problem in time  $O(n^{0.6897})$  per collision using space  $O(n^{1.6897})$ , by means of ray shooting structures.

## Additional Factors in Workload Complexity

Complexity is also affected by factors such as the rate of airplanes entering and exiting the airspace, the volume of the airspace, and the presence of additional software tools.

The complexity of airplanes entering and exiting is linear in the total number.

Many of the operational errors by ATCs result from ignoring secondary conflicts for too long [Endsley and Rogers 1997], so the potential for accidents is higher for more airplanes per unit volume.

Software could identify conflicts and order them by danger level, thereby reducing the complexity in Step 1; nevertheless, the primary complexity comes from solving conflicts once they arise ( $O(n^2)$  for Step 1 vs.  $O(2^{n(n-1)/2})$  for Step 2). So programs to detect conflicts do not combat the primary complexity faced by an ATC, and they could cause an ATC to take a more passive attitude in searching for potential conflicts not identified by the software. In other words, software could worsen the problem of ignoring secondary conflicts for too long. *Programs designed to aid an ATC in identifying conflicts should be designed as guides to the ATC's judgment rather than as automation of ATC functions.*

## References

- Airbus Industries. 2000. <http://www.airbus.com/imperial.html>.
- Denker, John S. 2000. See How It Flies. <http://www.monmouth.com/~jsd/how/htm/maneuver.html>.
- Endsley, M., and M. Rodgers. 1994. Situation awareness information requirements for en route air traffic control. DOT/FAA/AM-94/27.
- \_\_\_\_\_. 1997. Distribution of attention, situation awareness, and workload in a passive air traffic control task. DOT/FAA/AM-97/13.
- Eppstein, David, and Jeff Erickson. 1999. Raising roofs, crashing cycles, and playing pool: Applications of a data structure for finding pairwise interactions. Extended abstract at *Proceedings of the 14th Annual ACM Symposium on Computational Geometry* (1998), 58–67. *Discrete and Computational Geometry* 22 (4): 569–592. <http://www.uiuc.edu/ph/www/jeffe/pubs/cycles.html>.
- Federal Aviation Administration. 2000. Welcome to the Office of System Safety. [http://www.asy.faa.gov/asy\\_internet/safety\\_data/default.htm](http://www.asy.faa.gov/asy_internet/safety_data/default.htm).
- Kremer, H., et al. 1997. Probabilistic vs. geometric conflict probing. National Aerospace Laboratory NLR. Amsterdam.
- Lauderman, L.V., et al. 1998. Dynamic density: An air traffic management metric. NASA, NASA/TM-1998-112226.
- Mahalingam, Sandra. 1999. *Air Traffic Control*. New Delhi: Kaveri Books.

- MAICA: Modeling and analysis of the impact of changes in ATM. *Transport Research* #71. Belgium.
- MICA: MET improvement for controller aids. *Transport Research* #72. Belgium.
- Rodgers, M., et al. 1998. The relationship of sector characteristics to operational errors. DOT/FAA/AM-98/14.
- Vardi, M. 1999. The descriptive complexity of constraint satisfaction. Implicit Computational Complexity 1999. <http://www.cs.indiana.edu/icc99/vardi.html>.
- Wiener, E., and D. Nagel. 1988. *Human Factors in Aviation*. San Diego: Academic Press.