Controlling Departing Airport Traffic

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Summary

The current system of first-come-first-served used in setting departing airplane queues is fair but inefficient. We develop a system that uses a fast database to determine the most efficient way to clear planes for takeoff, where efficiency is measured in terms of airline cost and traveler satisfaction.

Our method uses an assignment-type linear programming model to determine the optimal ordering for pushback and subsequent departure of planes. According to our method, airplanes do not line up at the head of the runway. Instead, they are held at the gate and released one at a time in regular intervals, according to the optimal ordering and in such a way that the runway is always in use. This method allows the order of the planes that have not yet departed to be rearranged, if necessary, after a change in the database, without any physical reordering.

1. Restatement of the Problem

Determine an efficient, practical method for airports to assign multiple departing planes to a single runway.

2. Assumptions

We make the following assumptions.

1. The control tower has a fast database with all the information needed. The necessary parameters will be numerated later.
2. All planes require exclusive use of the runway to take off, and this time is the same for all planes. This assumption allows

Table 1.
Notation of the model.

Δ	width of a departure window
t_0	time of the first departure window
t_d	scheduled departure time
T_A	scheduled arrival time
t	amount of time a plane has been delayed
τ	longest time a given plane can be delayed and still arrive on schedule
K	a constant for each type of plane that determines the fuel cost of delay
v_{av}	average speed of a type of plane
v_{max}	maximum speed of a type of plane
r	cost of rerouting passengers who have missed connections
π	number of passengers who have connecting flights
P	total number of passengers
α	rate of increase of dissatisfaction among delayed passengers.
a	converts delayed passengers' dissatisfaction into dollars
b	converts into dollars the dissatisfaction of passengers who have missed connections

us to divide time into discrete units, or departure windows, of duration Δ.

3. The cost associated with plane i departing through window j does not depend on which planes departed earlier. This assumption allows us to express the total cost of a given sequence of planes as a linear function.

4. All planes take the same amount of time to reach the head of the runway after departing the gate.

5. Let τ be the time before which a plane must depart if it is to reach its destination on schedule. If a plane leaves after time τ, it will fly at its maximum safe speed.

6. If a plane leaves after time τ, all passengers with connecting flights will miss their connections.

7. The cost of rerouting passengers who have missed their connections is the same for every passenger.

Assumptions 2 and 3 are the key assumptions that allow us to model the system as a linear programming problem. The others result in simplifications of varying degrees.

3. Analysis and Design of the Model

Suppose n planes request clearance for takeoff at time t_0; and for the time being, suppose that all the planes have immediate access to the head of the runway. We associate a cost with each plane as it departs, depending on actual airline expenses and traveler satisfaction. We wish to assign each plane to exactly one of n departure windows so that each window is used by exactly one plane, in such a way that the total cost is minimized.

Let c_{ij} be the cost for plane i to take off through window j. Define x_{ij} by

$$x_{ij} = \begin{cases} 1, & \text{if plane } i \text{ is assigned to window } j \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

Then the total cost of a given assignment is

$$C = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}.$$

To ensure that each plane is assigned to exactly one window and each window to exactly one plane, we subject the decision variables x_{ij} to the constraints

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \ldots, n; \qquad \sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, 2, \ldots, n. \tag{2}$$

By Assumption 3, the c_{ij} are independent of the x_{ij}, so the total cost is clearly a linear function.

Finding the values for the x_{ij} which minimize C, subject to the constraints given (2), is an example of an *assignment problem*. Sophisticated computer algorithms have been developed to take advantage of the special structure of the problem. Spivey and Thrall [1970] implemented one of these, the *Graves-Thrall primal* *algorithm* (adapted from a more general algorithm developed at the RAND Corporation in 1965), on an unspecified machine. For a randomly generated assignment problem of up to 16 variables, it required at most 2.9 seconds.

Contemporary computers being orders of magnitude faster, we could solve the problem for any reasonable number of airplanes in a fraction of a second. Our model will specify simple formulas for the cost coefficients that depend on quantities available from the database, so setting up the problem would take another fraction of a second. The whole model then requires on the order of a second to set up and solve. Thus a control tower faced with a barrage of departure requests could determine takeoff priorities almost immediately.

It is unlikely that at a real airport we would have as the modus operandi a number of planes clustered at the runway head requesting clearance. Equivalently, and more realistically, we can assign planes takeoff priorities before they leave the gate. If we assume all planes require the same amount of time to travel from the gate to the runway, and that the planes at the gate are released in the order they are supposed to take off, they will arrive at the runway and take off in the correct sequence.

Now suppose a number of planes have left the gate but not yet departed, forming a line at the head of the runway, and another plane requests permission to leave the gate. The optimum order of the queue may change as a result of this new plane, so we may have to shuffle the planes physically to maintain optimum efficiency. To avoid doing so, we allow only one plane to leave the gate in each period Δ. This way, the runway is always in use, but the takeoff sequence of planes that have not yet left may be changed without a physical reordering.

In the next section, we show how to use scheduled pushback times to avoid having to recompute the optimal departure sequence every time another plane becomes ready. Changes in the database (e.g., because a plane is late) will require the problem to be resolved, but only a second or so is involved.

3.1 Determining the Cost Coefficients

The cost of a plane departure is the sum of the real costs to the airline and the cost associated with passenger dissatisfaction. We will take our base costs to be zero; we are only concerned with the additional expenditures of delaying the planes. The real costs to the airline of delay include the extra cost of fuel if a plane must fly faster than its most efficient speed and the cost of rerouting passengers who have missed their connecting flights. Additional real costs could be added without difficulty.

If a plane is delayed, it must fly faster in order to arrive at its destination on schedule. It will not use its fuel as efficiently at these greater speeds because of greater wind resistance, engine design, and other factors, so the airline's cost for fuel will increase every time unit that a plane is delayed. Since the pilot will want to arrive as soon possible, even if the plane is late, and will not want to endanger the passengers, the plane will fly at its maximum safe speed after accelerating to it. Thus, the cost of fuel will be constant after a certain time. We do not know how fuel costs increase with speed, so we choose a linear function for simplicity. Thus the cost of fuel will have the form

$$F(t) = \begin{cases} Kt, & t < \tau \\ K\tau, & t \geq \tau. \end{cases}$$

Here t is the amount of time a plane has already been delayed, i.e., $t = 0$ at the scheduled time of departure.

Let

t_0 be the time of the first departure window,

t_d be a plane's scheduled departure time,

Δ be the width of a window, and

j be the number of a window.

Then

$$t = t_0 - t_d + (j-1)\Delta.$$

Since τ is the longest time a plane can be delayed and still arrive on schedule after time $t = \tau$, the plane will be late even if it flies at maximum speed. We see that the cost of fuel increases the longer the plane is delayed, until it reaches a maximum, where it levels out. We can calculate τ as

$$\tau = T_A - \frac{d}{v_{\max}},$$

where

T_A is the scheduled arrival time of the plane,

d is the distance to the destination, and

$v_{\max}$ is the maximum speed of the plane.

We may estimate d by

$$d \approx (T_A - t_d)v_{\text{av}},$$

where

t_d is the scheduled departure time and

v_{av} is the speed of the plane if it leaves on time.

These speeds are experimental constants that would differ for each type of plane.

The constant K takes into account the price of fuel and the rate of increase of fuel consumption for each time unit of delay. This constant will probably be different for different types of planes (since some designs are more fuel-efficient than others) and would have to be supplied by the database. The farther a plane's destination, the greater the overall cost for inefficiencies. To take this into account, K should also include the maximum speed of the plane, and it should be multiplied by the flight time of the plane at maximum safe speed for the plane under prevailing conditions.

Since T_A is the scheduled arrival time, the distance to the destination is $(T_A - \tau)$ times the maximum speed. The fuel cost function is now

$$F(t) = \begin{cases} K(T_A - \tau)t, & t < \tau \\ K(T_A - \tau)\tau, & t \geq \tau. \end{cases}$$

For simplicity, assume that the rerouting cost will be a constant r for each person who misses a connection. (If the rerouting cost actually does vary, then let r be the expected value of the rerouting cost.) Rerouting comes into play if the plane is late, so the cost is

$$R(t) = r\pi u(t - \tau),$$

where

π is the number of people who have connecting flights, and

u is the unit step function $u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0. \end{cases}$

We assume that if the plane is late, all π people will miss their connections. Otherwise, $R(t)$ would be a sum of step functions, where a given step function would become 1 when the corresponding passenger missed a connection.

The cost coefficients must also take into account the cost of dissatisfied passengers. The longer a plane is delayed, the more dissatisfied each passenger will be. If the delay is only a minute or two, each person will not be too dissatisfied. However, if the delay is long, each passenger may be *very* angry. This reaction may be adequately described by an exponential term. A passenger with a connecting flight will also feel anxiety about missing the connection and frustration if it is missed.

Taking these into account, we have the dissatisfaction cost function

$$D(t) = (e^{\alpha t} - 1)aP + b\pi u(t - \tau),$$

where

α, a, and b are constants,

P is the number of passengers, and

π is the number of passengers with connecting flights.

The -1 is included in the dissatisfaction term so that the minimum dissatisfaction level will be 0 at time $t = 0$, when the plane is exactly on time. The constant α determines how quickly passengers become dissatisfied with waiting, and a and b convert passenger dissatisfaction into dollars. The term with the step function takes into account the dissatisfaction of passengers who have m ssed their connecting flights. It does not come into play until $t \geq \tau$, or when the plane cannot arrive at its scheduled time. The constants a and b are probably not equal, since a passenger who misses a connection will be much more dissatisfied than one who has been delayed a small amount.

The cost coefficient c_{ij} is the sum of the fuel, rerouting, and dissatisfaction cost functions for plane i departing through window j. We have assumed that all planes take the same amount of time to reach the head of the runway. If this is not the case, then at the time of departure one would make t equal to the time needed for the given plane to reach the head of the runway, instead of equal to 0, as we do now.

We can take into account the departure schedule in the following manner: Include planes that are not yet ready to depart in the problem, but let the cost of the plane departing through a window before its scheduled departure time be infinite. This will prevent clearing a plane for takeoff before it is ready.

Our model cannot optimize both departures and arrivals. In order to do this, the cost coefficients would depend upon the ordering of the previous planes, and so the objective function would no longer be linear. Thus our method of solution would no longer apply.

The model can incorporate arrivals, however, if the arriving planes are permitted to land as soon as they are ready to do so. Let

t' be the time a plane will arrive, and

Δ' be the time it takes for the plane to land and clear the runway.

Since Δ is the width of a departure window, the plane will arrive during window $j' = (t' - t_0)/\Delta$. The cost coefficients c_{ij} for $j < j'$ are calculated normally, and those for $j \geq j'$ are calculated by adding Δ' to t.

Combining equations, we have each cost coefficient given by

$$c_{ij} = \begin{cases} \infty, & t < t_d \\ K(T_A - \tau)t + (e^{\alpha t} - 1)aP, & t_d \leq t < \tau \\ K(T_A - \tau)\tau + r\pi + (e^{\alpha t} - 1)aP + b\pi, & t \geq \tau \end{cases}$$

where

$$t = t_0 - t_d + (j-1)\Delta, \qquad \tau = T_A - \frac{(T_A - t_d)v_{av}}{v_{max}},$$

for $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, n$.

The free parameters in the model are α, a, and b. The parameters supplied by the database are t_d, T_A, π, and P. The predefined constants are $\Delta, K, v_{av}, v_{max}$, and r. The variables t and τ can be easily calculated, and t_0 can be supplied by a system clock on the computer.

4. Testing the Model

The most important test of the model is that it make sense. We wrote a program using the simplex method for linear programming to see how our model behaved in a few simple cases. As detailed later, our results agreed with our expectations. A more thorough test could be made by revising the parameters, trying more complicated examples, and then actually implementing the system. If implementation resulted in cost savings to the airlines while still maintaining customer satisfaction at an acceptable level, the model would clearly be a success.

Before doing so, however, it would be wise to see if the model is stable with respect to perturbations in the parameters. In other words, we want to make sure that if we have determined a value for one of the model parameters that is slightly off from the true value, then the results of our model will not be off wildly from the true solution. In addition, we want to identify which parameters the model is most sensitive to, so that more resources could be devoted to determining their values accurately before the model is actually implemented.

The assignment problem formulation, though useful for a solution, is not as useful for performing a sensitivity analysis. The problem as stated is *not* a linear programming problem, because the decision variables are constrained to assume only *integer* values. A linear programming model would facilitate analysis.

If we replace the integer-value requirement for the x_{ij} expressed in (1) with the set of non-negativity constraints

$$x_{ij} \geq 0, \qquad i = 1, 2, \ldots, n,$$

and replace C with $-z$, we have the following transportation problem:

$$\text{Maximize } z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \ldots, n; \qquad \sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, 2, \ldots, n; \qquad (3)$$

and

$$x_{ij} \geq 0, \qquad i = 1, 2, \ldots, n \text{ and } j = 1, 2, \ldots, n.$$

It is well known that if this problem has any feasible solution, then it has an optimal solution in which all the decision variables take on integer values. Notice that the constraints restrict the possible integer values of each variable to 0 or 1, so the transportation formulation is equivalent to the original formulation.

The transportation problem may be written more succinctly in vector form as

$$\text{Maximize } z = \mathbf{c}^t \mathbf{x} \text{ subject to } \mathbf{A}\mathbf{x} = \mathbf{b} \text{ and } \mathbf{x} \geq 0,$$

where the components of $\mathbf{x}$ are the x_{ij}, the components of $\mathbf{c}$ are $-c_{ij}$, the components of $\mathbf{b}$ are all just 1, and the elements of $\mathbf{A}$ are 1 or 0 according to the equality constraints of (3).

We now wish to determine how sensitive the solution to our problem is to some type of change. Clearly $\mathbf{A}$ and $\mathbf{b}$ are fixed for a given number of airplanes, so let us consider only changes in $\mathbf{c}$. Suppose we have solved the problem and obtained an optimal solution, $\mathbf{x}^*$, and suppose also that we have solved the dual problem

$$\text{Minimize } w = \mathbf{b}^t \mathbf{y} \text{ subject to } \mathbf{y}\mathbf{A} \geq \mathbf{c},$$

obtaining the dual optimal solution $\mathbf{y}^*$. Now suppose we replace $\mathbf{c}$ by $\mathbf{c}'$, where $\mathbf{c}' = \mathbf{c} + \Delta\mathbf{c}$. What are the resulting changes in the solution?

Since the constraint equations $\mathbf{A}\mathbf{x} = \mathbf{b}$ are unaffected, $\mathbf{x}^*$ is still a feasible solution, but it may not be optimal any longer. An important result of duality theory states that the old solution $\mathbf{x}^\star$ is still optimal if and only if $\mathbf{y}^\star\mathbf{A} \geq \mathbf{c}'$, i.e., when $\Delta\mathbf{c} \leq \mathbf{y}^\star\mathbf{A} - \mathbf{c}$.

It is clear, however, that even if $\mathbf{x}^\star$ is no longer optimal, the new value of the objective function is

$$\begin{aligned} z' &= \mathbf{c}'^t\mathbf{x}^\star \\ &= (\mathbf{c} + \Delta\mathbf{c})^t\mathbf{x}^\star \\ &= \mathbf{c}^t\mathbf{x}^\star + (\Delta\mathbf{c})^t\mathbf{x}^\star \\ &= z + (\Delta\mathbf{c})^t\mathbf{x}^\star. \end{aligned}$$

So the change in z is linear in the components of $\Delta\mathbf{c}$; thus, the uncertainty in the cost of an assignment may be made arbitrarily small by reducing the size of the uncertainty in the components of $\mathbf{c}$.

Since $\mathbf{c}$ is the vector of cost coefficients, uncertainties in it arise from two sources. One source is the parameters that determine the real cost to the airline of sending plane i through window j. These costs are the fuel costs (which should be a precisely known figure with little uncertainty) and the rerouting costs associated with passengers who miss their connections. Although we have used gross estimates of these numbers for our illustrative examples, presumably these costs are precisely known to the airline. Other costs that we may have overlooked may be added to the model as a trivial modification; so that, overall, the uncertainties in the real costs are negligible.

The other part of the uncertainty in $\mathbf{c}$ comes from the cost associated with dissatisfied passengers, which depends on the three parameters a, b, and α. Clearly, these parameters are the source of error in the model; and the solution is most sensitive to α, since α is in an exponent, whereas a and b are merely multiplicative constants.

Let us determine exactly how the uncertainty in the psychological parameters a, b, and α affects our determination of z. Suppose we have the following estimates and associated uncertainties:

$$\hat{a} \pm \sigma_a, \quad \hat{b} \pm \sigma_b, \quad \text{and} \quad \hat{\alpha} \pm \sigma_\alpha.$$

The partial derivatives of the c_{ij} with respect to a, b, and α are

$$\frac{\partial c_{ij}}{\partial a} = (e^{\alpha t} - 1)\,P, \quad \frac{\partial c_{ij}}{\partial b} = \pi, \quad \text{and} \quad \frac{\partial c_{ij}}{\partial \alpha} = te^{\alpha t}aP.$$

If we assume the measurements are uncorrelated, then the uncertainty $\sigma_{c_{ij}}$ of c_{ij} is given by

$$\sigma_{c_{ij}}^2 = \sigma_a^2\left(\frac{\partial c_{ij}}{\partial a}\right)^2 + \sigma_b^2\left(\frac{\partial c_{ij}}{\partial b}\right)^2 + \sigma_\alpha^2\left(\frac{\partial c_{ij}}{\partial \alpha}\right)^2$$

$$= \sigma_a^2 \left[(e^{\alpha t} - 1) P \right]^2 + \sigma_b^2 x^2 + \sigma_\alpha^2 \left(t e^{\alpha t} a P \right)^2 .$$

Similarly, since $z = \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij}$,

$$\frac{\partial z}{\partial c_{ij}} = x_{ij};$$

and so the uncertainty σ_z in our determination of z is given by

$$\sigma_z^2 = \sum_{i=1}^n \sum_{j=1}^n \left(\frac{\partial z}{\partial c_{ij}} \right)^2 \sigma_{c_{ij}}^2 = \sum_{i=1}^n \sum_{j=1}^n (x_{ij}^2)\, \sigma_{c_{ij}}^2 .$$

The values of a, b, and α could be determined by an airline manager, who would know, or be able to find through a commissioned study, the relative importance of customer dissatisfaction when compared to real costs. Of course, customer dissatisfaction may well translate into real costs in the long run, through lost patronage.

4.1 Computer Model

We wrote a computer program to illustrate some of the properties of the model, using the simplex algorithm to identify the optimal solution.

The following examples demonstrate that the model behaves reasonably so far. It could be tested rigorously by actual implementation, noting whether operating costs decrease, and by conducting customer surveys to note any change in satisfaction.

The theoretical model is much less restrictive than the computer model. For programming simplicity, the following assumptions are made, in addition to those already stated in the theoretical model.

1. There are at most three planes ready to depart through any departure window. If there are only two planes ready to take off, we insert a dummy plane with all of its cost coefficients equal zero. This assumption allows us to use the IMSL subroutine ZX4LP to solve our linear programming model.
2. We assign values to the parameters of our model by intuition. In practice, the values of these parameters should be determined by experiment and survey.
   - (a) Each departure window is one minute long. That is, any plane requires at most one minute to take off. No other plane can occupy the runway until the one minute is over.
   - (b) No planes may land on the runway.

Table 2.
Example 1. All else being equal,
the plane with the most passengers goes first.

Plane	Passengers	Cost matrix			Solution		
A	350	0.00	0.48	0.97	0	1	0
B	100	0.00	0.41	0.83	0	0	1
C	400	0.00	0.50	1.00	1	0	0

Table 3.
Example 2. One plane has a long delay.
The second column shows (on board)/(connecting).

Plane	Passengers	Time delayed	Cost matrix			Solution		
D	210/140	10 min	0.82	0.91	1.00	1	0	0
B	100/100	1 min	0.07	0.15	0.22	0	0	1
A	350/100	1 min	0.09	0.17	0.26	0	1	0

(c) The cost to reroute any passenger is \$350.

(d) A traveler who misses a connecting flight is twice as frustrated as one who has been delayed 15 min.

Example 1. (Plane with the most passengers goes first)

Let's consider a simple example. At 6:00 A.M. three identical planes A, B, and C are ready to leave on schedule. They are flying to three different cities equidistant from the airport, and all three are scheduled to arrive at their destinations at 7:20 A.M. Further, plane A has 350 passengers on board, B 100, and C 400. Each plane has 100 passengers who have to connect with other flights.

Our computed result is summarized in Table 2.

The minimal cost is 1.31. The solution matrix shows that C, A, B is the optimal order for takeoff. This agrees with common sense, since given that everything else is equal, the best arrangement for takeoff is to send the airplane with the most passengers first.

Example 2. (Plane with the longest delay goes first)

As plane C is making its departure, plane D requests permission to depart. Plane D is already 18 min behind schedule. It has to leave within the next 2 min if it is to arrive at its destination on time at 7:06 A.M. Altogether there are 200 passengers on board, 150 of whom are making connections. Table 3 summarizes the result.

The minimal cost for this problem was computed to be 1.22. The optimal order of takeoff is D, A, then B, consistent with our intuition: We expect the plane delayed the longest to get top priority. Otherwise, the airline may have to reroute passengers. The

Table 4.
Example 3. Toss-up situation.
The second column shows (on board)/(connecting).

Plane	Passengers	Time Delayed	Cost Matrix			Solution		
B	100/100	3 min.	0.60	0.80	1.00	0	1	0
E	122/89	0 min.	0.00	0.28	0.56	1	0	0
X	0/0	0 min.	0.00	0.00	0.00	0	0	1

normalized cost matrix shows that plane D is very costly because it has been delayed for too long. The other planes have negligible costs compared to plane D.

Example 3. (Toss-up situation)

Suppose that 2 more minutes go by. Planes D and A have taken off. Plane B has been delayed for 3 min. Another aircraft, plane E, is ready for departure. Let plane E

- be ready on schedule,
- have 45 min to spare before it will be late (corresponding to 7:42 A.M. arrival time),
- have 122 passengers on board, 89 of whom have connecting flights, and
- cost $450/min for delay.

To use our program to solve this problem, we must make a dummy variable, plane X. This plane has zero cost associated with it (we simply set all of the input parameters equal to zero).

Table 4 shows our solution.

It is not obvious which order for takeoff is optimal. As a matter of fact, it may seem that it would be better to send plane B before plane E. However, because of the expensive operational cost of plane E at higher speed and the number of passengers on board, it is better to send plane E before plane B.

5. Strengths and Weaknesses

The principal weakness of our model is that the three parameters α, a, and b are difficult to measure accurately. However, they can be approximated by experiment.

Another weakness is that the model cannot optimize both departures and arrivals. It can, however, optimize departures while permitting arrivals.

Our model has several strengths. Implementation is easy: with today's computer technology and a quick data base, the problem can be solved within seconds, even for a large number of planes.

Its versatility is evident: The model determines the optimal ordering for departures with an arbitrary number of departing planes containing any number of passengers, and an arbitrary number of these passengers can have connecting flights. Also, the model can handle planes scheduled to depart at the same time and planes that have been delayed. It takes into account planes of different sizes and different fuel efficiencies.

Finally, our analysis shows that the model is stable, so we may have confidence in its predictions. Even if the estimates for some of the parameters deviate from their true values, the cost of a particular recommended assignment will differ from the actual minimum cost by an amount proportional to the deviations. Thus we are sure to save money for the airlines and keep the passengers happy as well.

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