邹波涛、李治国、边道宗
华南理工大学
1997 Meritorious Winner

案例 4.5　有效讨论的最优混合解(英文)*

问题

为讨论重要问题，特别是长远规划问题而召开小组讨论会正变得越来越普遍。人们相信，很多人参加的会妨碍有成果的讨论，甚至一位占支配地位的人能控制并操纵会议的讨论。因此，公司的董事会在召集全体董事开会之前，会先开一些讨论有关事务的小组会议，这些规模较小的小组会议仍然有被某个占支配地位的人控制的危险。为降低这种危险，常用的办法是安排每个小组开几次会，每次会有不同的人参加。

An Tostal 公司的一次会议的参加者为 29 位公司董事会成员，其中 9 位是在职董事(即本公司的雇员)。会议要开一天，每个小组上午开 3 段，下午 4 段，每段会议开 45 分钟，从上午9：00到下午 4：00 每整点开始开会，中午 12：00 午餐。上午的每段会议都有 6 个小组讨论会，每个小组讨论会都由公司的一位资深高级职员来主持讨论，这些资深高级职员都不是董事会的成员。因此，每位资深高级职员都要主持 3 个不同的小组讨论会，这些资深高级职员不参加下午的讨论会，而且下午的每段会议只有 4 个不同的小组讨论会。

公司董事长要一份公司董事参加 7 段会议的每个小组讨论会的分配名单。这份搭配名单要尽可能多地把董事均匀搭配，理想的搭配应是每一位董事和另一位董事一起参加小组讨论会的次数相同，与此同时要使不同段的小组中在一起开过会的董事数达到最小。

* 本案例指导教师郝志峰副教授，队员邹波涛、李治国、边道宗。

名单中的搭配还应满足下列两个准则：

①在上午的讨论会上，不允许一位董事参加由同一位资深高级职员主持的两次会议。

②每个分组讨论会都不应有不同数目的在职董事参加。

给出一张 1～9 号在职董事、10～29 号董事、1～6 号公司资深高级职员的搭配名单，说明该名单在多大程度上满足了前面提出的各种要求和准则。因为有的董事可能在最后一分钟宣布不参加会议，也可能不在名单上的董事将出席会议，因此一个能使秘书在一小时前得到变更与会与否通知的情况下来调整搭配的算法定会得到赞识。如果算法还能涉及到不同水平的与会者参加的未来的会议中每类与会者搭配的情况的话，那就更理想了。

Abstract

In this paper, we solve the problem to design assignments of several sessions with a different mix of people in each group.

We use a (0, 1)-matrix C to record designed groups, and a matrix T to record currently known times of every two members' discussions. After concluding that the optimal attending times of each member are 6, we construct orthogonal Latin Squares to obtain the morning list. For the afternoon list, we use the ideas of Greedy Algorithm. According to the designed assignments, we design the current discussion group most efficiently.

On an average, each board member will meet 21.86 other board members, the number of common membership of groups for the different sessions is up to 2. By Mapping Transformation, we make a convenient algorithm to adjust assignments involving different levels.

Furthermore, we perform computer simulation to indicate

that the model makes sense. We summarize a set of general and practicable schemes. In generalization, we estimate the proper range by the empirical formular for a given number of members.

The method can be widely applied to experimental design.

Restatement

In corporate board meetings, the board will meet in small group with a different mix of people to discuss issues before meeting as a whole.

Our task is to design a model for determining an assignment, in which the 29 board members would be scheduled into 3 sessions of 6 groups in the morning and 4 sessions of 4 groups in the afternoon. And the assignment should also satisfy the following criteria as much as possible:

(1) Each board member would attend the same number of discussion groups.

(2) Minimizing common membership of group for different sessions.

(3) For the moring sessions, no board member should be in the same senior officer's discussion twice.

(4) No discussion group should contain a disproportionate number of inhouse members.

Basic Assumptions

(1) The board members at the same level are equal while the members at different levels should be distinguished.

(2) Senior officers only lead the group and don't join the discussion.

(3) The discussion is not influenced by accidental factors, such as, no member cancel at halfway.

(4) The time of each session is enough. It is sufficient for members in the same group to exchange their views.

(5) The first discussion of two members is productive discussion, repeated discussions are unproductive.

(6) The secretrary is just, she adjusts the assignments by the algorithm absolutely.

(7) If some board members cancel or some not scheduled show up undesignedly, they should tell the secretrary an hour before.

(8) Senior officers can't cancel in the meeting.

Symbol System

V	total number of the board members
V_i	Member $i(i=1, 2, \cdots, 29)$, $V_1, \cdots, V_9$ for in-housemembers
X	$X=\{V_1, V_2, \cdots, V_{29}\}$
Y_i	Officer $i(i=1, 2, \cdots, 6)$
AM_{ij}	set of board members at Group j, Session i in the morning$(i=1, 2, 3;\ j=1, 2, \cdots, 6)$
PM_{ij}	set of board members at Group j, Session i in the afternoon$(i=4, 5, 6, 7;\ j=1, 2, 3, 4)$
r	times of each member joining discussion
k_{ij}	number of board members at the Group j, Session i.
P_{ij}	the proportination of in-house member at Group j, Session i.

λ_{ij}	times of member i, j in common groups
λ_{max}	the maximum of λ_{ij}
b	total number of the small groups (in this problem $b=6\times 3+4\times 4=34$)
C	matrix of already designed small discussion groups
T	$T=CC^T$ (matrix of currently known λ_{ij})
e_i	number of board member who have discussed with member i
N_{pd}	total number of productive discussions
N_{upd}	total number of unproductive discussion
F	$F=\frac{2N_{pd}}{29}$, average number of meeting other members

Explain of Symbol

Use matrix C to decribe attending relationship, define C_{ij} by:

$$C_{ij}=\begin{cases}1 & \text{member } i \text{ have attended group } j\\ 0 & \text{member } i \text{ have not attended group } j\end{cases}$$

Since $b=34$, So:

$$\begin{array}{c|c} & AM_{11}\ldots AM_{16}AM_{21}\ldots AM_{36}PM_{41}\ldots PM_{44} \\ \begin{matrix}V_1\\V_2\\\vdots\\V_{29}\end{matrix} & \begin{bmatrix} & & \\ & C_{ij} & \\ & & \end{bmatrix}_{29\times 34}\end{array}$$

Let $T_{29\times 29}=CC^T$ then t_{ij} means the common membership between V_i and V_j. And the elements in diagonal line means board member attending times. In fact $t_{ij}=\lambda_{ij}$.

Analysis of Problem

Our goal is to give a list of board member assignments to discussion groups for each of the seven.

Firstly, a member can join a discussion group at most in one session, i.e.,

$$AM_{i1} \cap AM_{i2} \cap AM_{i3} \cap AM_{i4} \cap AM_{i5} \cap AM_{i6} = \varnothing \quad (i=1,\ 2,\ 3)$$

$$PM_{i1} \cap PM_{i2} \cap PM_{i3} \cap PM_{i4} = \varnothing \quad (i=4,\ 5,\ 6,\ 7)$$

Secondly, the assignments should achieve as much of board member as much as possible. We interpret this into three points:

(1) Each board member should meet as much of board members as possible. The list should include as much of productive discussions as much as possible. That, we should minimize the number of $\lambda_{ij}=0$.

(2) Unproductive discussions should be minimized. That is, we should minimize the number of $\lambda_{ij}>1$.

(3) If two members meet in too large groups, it is easy that one is controled by the other. Hence λ_{max} should be minimum.

Each member has equal rights, so they should join the discussion groups the same r times. It is easily seen $r \leqslant 7$. As r is rising, the total productive discussions can be raised. But when r is too large, the unproductive discussions will appear more and more. Hence, it is very important to determine r.

Furthermore, each member meet many members. The number of other member who be met by one member should be equal. Thus,

(1) There are more than 2 members in each group.

(2) The differences among numbers of each group's mem-

bers are up to 1. We use the theory of combinatial analysis to solve these problems. Each discussion may be viewed as a block. Each element of X appears r times in different blocks. Where, cardinality k and the number of meet λ are varied.

Each morning session consists of six discussion groups while each afternoon session consists of four. No board member should be in the same senior officer's discussion group twice in the morning sessions. That is:

$$AM_{i1} \cap AM_{i2} \cap AM_{i3} = \varnothing \qquad (i=1,\ 2,\ 3,\ 4,\ 5,\ 6)$$

So we divide our design to two steps. In the first step, we design the list of board members assignments in the morning. Then, according to the morning list, we design the afternoon list which contain as much of productive discussions as much as possible.

Model Design

1. The times r of board member attending discussions

By solving a Diophantine equation, we conclude: (See appendix)

①Each member attends 6 times discussions;

②All groups in the morning have 4 members;

③10 groups have 6 members and 6 groups have 7 members in the afternoon.

2. The morning list

Let us arrange the small discussion groups of the morning as 3 matrix vectors:

$$AM_i = \begin{bmatrix} AM_{i1} \\ AM_{i2} \\ AM_{i3} \\ AM_{i4} \\ AM_{i5} \\ AM_{i6} \end{bmatrix}_{6\times 4} \qquad (i=1, 2, 3)$$

In the above 3 matrix vectors, since

$$AM_{1j} \cap AM_{2j} \cap AM_{3j} = \varnothing \qquad (j=1, 2, \cdots, 6)$$

and $AM_{i1} \cap AM_{i2} \cap AM_{i3} \cap AM_{i4} \cap AM_{i5} \cap AM_{i6} = \varnothing$ $(i=1, 2, \cdots, 3)$

The morning assignment may be changed into the Latin Squares' designs. In fact, we shall find out three suitable crippled Latin Squares to form AM_1, AM_2, AM_3.

Now we can set up our model.

(1) Design OLS (Orthogonal Latin Square)

Since we have 6 small groups in a session and 4 board members in each group, we use slope method (see appendix) to design three 6×4 Orthogonal Latin Squares.

First, we use the orbit idea of block design to construst the following three 6×4 matrices:

$$\begin{array}{ccc} AM_1 & AM_2 & AM_3 \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \\ 4 & 4 & 4 & 4 \\ 5 & 5 & 5 & 5 \end{bmatrix} & \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 0 \\ 4 & 5 & 0 & 1 \\ 5 & 0 & 1 & 2 \\ 0 & 1 & 2 & 3 \end{bmatrix} & \begin{bmatrix} 2 & 4 & 1 & 3 \\ 3 & 5 & 2 & 4 \\ 4 & 0 & 3 & 5 \\ 5 & 1 & 4 & 0 \\ 0 & 2 & 5 & 3 \\ 1 & 3 & 0 & 2 \end{bmatrix} \end{array}$$

In these matrices, the same element can not repeatly appear in *the same place* of. So-called *a same place* is in the same row and in the same column, such as $AM_1(1, 1)=0$, but $AM_2(1, 1)=1\neq 0$, $AM_3(1,1)=2\neq 0$.

Second, we set each element a subscript(i.e. 0, 1, 2, 3). Then we could get three crippled Orthogonal Latin Squares.

$$AM_1 = \begin{pmatrix} 0_0 & 0_1 & 0_2 & 0_3 \\ 1_0 & 1_1 & 1_2 & 1_3 \\ 2_0 & 2_1 & 2_2 & 2_3 \\ 3_0 & 3_1 & 3_2 & 3_3 \\ 4_0 & 4_1 & 4_2 & 4_3 \\ 5_0 & 5_1 & 5_2 & 5_3 \end{pmatrix} \quad AM_2 = \begin{pmatrix} 1_0 & 2_1 & 3_2 & 4_3 \\ 2_0 & 3_1 & 4_2 & 5_3 \\ 3_0 & 4_1 & 5_2 & 0_3 \\ 4_0 & 5_1 & 0_2 & 1_3 \\ 5_0 & 0_1 & 1_2 & 2_3 \\ 0_0 & 1_1 & 2_2 & 3_3 \end{pmatrix} \quad AM_3 = \begin{pmatrix} 2_0 & 4_1 & 1_2 & 3_3 \\ 3_0 & 5_1 & 2_2 & 4_3 \\ 4_0 & 0_1 & 3_2 & 5_3 \\ 5_0 & 1_1 & 4_2 & 0_3 \\ 0_0 & 2_1 & 5_2 & 1_3 \\ 1_0 & 3_1 & 0_2 & 2_3 \end{pmatrix}$$

(2) Code 29 members

Now, we set different codes for 29 members, so as to get the list. In order to make each discussion group contain proportionate number of in-house members, we let each group contain only one in-house member. So we code 24 members as follows: (Tab. 4.5.1、Tab. 4.5.2):

Tab. 4.5.1

0_0	1_0	2_0	3_0	4_0	5_0	0_1	1_1	2_1	3_1	4_1	5_1	0_2	1_2	2_2	3_2	4_2	5_2
V_1	V_2	V_3	V_4	V_5	V_6	V_{10}	V_{11}	V_{12}	V_{13}	V_{14}	V_{15}	V_{16}	V_{17}	V_{18}	V_{19}	V_{20}	V_{21}

Tab. 4.5.2

0_3	1_3	2_3	3_3	4_3	5_3
V_{22}	V_{23}	V_{24}	V_{25}	V_{26}	V_{27}

Remainting 5 members will code as follows(Tab. 4. 5. 3)

Tab. 4. 5. 3

y_1	y_2	y_3	y_4	y_5
V_7	V_8	V_9	V_{21}	V_{20}

Now a new problem appears, only 24 members have been arranged. How to solve it? We get into the following step.

(3) Replace members to make model more suitable

Every member should attend 6 different session groups. On the contrary, one member can be absent only one session. We efficiently solve this problem by exchange the code of board member. The replacement rules are as follows:

①The replacing element and the replaced element are in the same level;

②One element can neither appear in one submatrix nor in one row twice;

③The appearing times of each element should be as equal as possible. In this problem, we exchange elements as rules like this:

$$AM_1 = \begin{pmatrix} 0_0 & 0_1 & 0_2 & 0_3 \\ 1_0 & 1_1 & 1_2 & 1_3 \\ 2_0 & 2_1 & 2_2 & 2_3 \\ 3_0 & 3_1 & 3_2 & 3_3 \\ 4_0 & 4_1 & 4_2 & 4_3 \\ 5_0 & 5_1 & 5_2 & 5_3 \end{pmatrix} \quad AM_2 = \begin{pmatrix} 1_0 & 2_1 & 3_2 & y_4 \\ 2_0 & 3_1 & 4_2 & y_5 \\ y_1 & 4_1 & 5_2 & 0_3 \\ y_2 & 5_1 & 0_2 & 1_3 \\ y_3 & 0_1 & 1_2 & 2_3 \\ 0_0 & 1_1 & 2_2 & 3_3 \end{pmatrix} \quad AM_3 = \begin{pmatrix} 5_0 & 4_1 & 1_2 & y_5 \\ y_1 & 5_1 & 2_2 & 4_3 \\ y_2 & 0_1 & 3_2 & 5_3 \\ y_3 & 1_1 & 4_2 & 0_3 \\ 3_0 & 2_1 & 5_2 & 1_3 \\ 4_0 & 3_1 & 0_2 & y_4 \end{pmatrix}$$

In fact, in the second session:

$V_4 -> V_7$, $V_5 -> V_8$, $V_6 -> V_9$, $V_{26} -> V_{28}$, $V_{27} -> V_{29}$

And in the third session:

$V_1 -> V_7$, $V_2 -> V_8$, $V_3 -> V_9$, $V_{24} -> V_{28}$, $V_{25} -> V_{29}$

Now, we obtain the small groups list in the morning (Tab. 4. 5. 4):

Tab. 4. 5. 4

	session 1				session 2				session 3			
y_1	V_1	V_{10}	V_{16}	V_{22}	V_2	V_{12}	V_{19}	V_{26}	V_6	V_{14}	V_{17}	V_{29}
y_2	V_2	V_{11}	V_{17}	V_{23}	V_3	V_{13}	V_{20}	V_{29}	V_7	V_{15}	V_{18}	V_{28}
y_3	V_3	V_{12}	V_{18}	V_{24}	V_7	V_{14}	V_{25}	V_{22}	V_8	V_{10}	V_{19}	V_{27}
y_4	V_4	V_{13}	V_{19}	V_{25}	V_8	V_{15}	V_{16}	V_{23}	V_9	V_{11}	V_{20}	V_{22}
y_5	V_5	V_{14}	V_{20}	V_{26}	V_6	V_{10}	V_{17}	V_{24}	V_4	V_{12}	V_{21}	V_{23}
y_6	V_6	V_{15}	V_{21}	V_{27}	V_1	V_{11}	V_{18}	V_{28}	V_5	V_{13}	V_{16}	V_{26}

Now, we give the Martrix T. $\lambda_{max}=1$, $P_{ij}=1/4$. The result is very well.

3. The afternoon list

Main idea

To make the afternoon list, we arrange small group one by one through "greedy algorithm". Let k be the number of members in a small group. Let S be the set of members still disarranged in one session.

Each session can be designed as the following two steps:

Step1: Let $S=\{V_1, V_2, \cdots, V_n\}$.

Step 2: Choose $V_{j1}, V_{j2}, \cdots, V_{jk}$ from S. In matrix T, row j_1, row j_2, $\cdots$, row j_k. And column j_1, column j_2, $\cdots$, column j_k form submatrix D:

$$D=\begin{bmatrix}\lambda_{j1,j1} & \lambda_{j1,j2} & \cdots & \lambda_{j1,jk}\\ \lambda_{j2,j1} & \lambda_{j2,j2} & \cdots & \lambda_{j2,jk}\\ \hdashline \lambda_{jk,j1} & \lambda_{jk,j2} & \cdots & \lambda_{jk,jk}\end{bmatrix}$$

Such that:

(1) All elements in the diagonal line are less than r (in this problem $r=6$).

(2) D consists of "0" as much as possible.

Then remove $V_{j1}, V_{j2}, \cdots, V_{jk}$ from S. Repeat this step until all small groups in this session are designed.

During the course of design, we find:

(1) We shall arrannge in-house members $V_1, V_2, \cdots, V_9$ beforehand. This can not only assure that each group contain a proportionate number of in-house members, but also lower the complexity of the algorithm.

(2) Sort decendingly elements in S by e_i in advance. To lower the time of realizing the algorithm, we select the front elements in priority.

Realize the thought like this:

Use "slope method" to minimize common in-house membership of groups. First we arrange the in-house members as follows (Tab. 4.5.5):

Tab. 4.5.5

	session 4	session 5	session 6	session 7
PM_{j1}	V_1 V_2	V_4 V_5	V_2 V_5	V_5 V_8 V_9
PM_{j2}	V_5 V_6	V_6 V_7	V_1 V_6 V_9	V_2 V_7
PM_{j3}	V_7 V_8	V_2 V_3 V_9	V_4 V_7	V_1 V_4
PM_{j4}	V_3 V_4 V_9	V_1 V_8	V_3 V_8	V_3 V_6

By the concrete algorithm(see appendix), we design the list of the afternoon as follows(Tab. 4. 5. 6):

Tab. 4. 5. 6

PM_{41}	V_1	V_2	V_{26}	V_{27}	V_{28}	V_{29}	
PM_{42}	V_5	V_6	V_{19}	V_{22}	V_{23}	V_{24}	V_{25}
PM_{43}	V_7	V_8	V_{12}	V_{16}	V_{17}	V_{20}	
PM_{44}	V_3	V_4	V_9	V_{10}	V_{11}	V_{14}	V_{15}
PM_{51}	V_4	V_5	V_{17}	V_{18}	V_{22}	V_{27}	V_{29}
PM_{52}	V_6	V_7	V_{11}	V_{13}	V_{24}	V_{26}	
PM_{53}	V_3	V_8	V_9	V_{16}	V_{21}	V_{28}	V_{29}
PM_{54}	V_1	V_8	V_{14}	V_{18}	V_{19}	V_{20}	
PM_{61}	V_2	V_5	V_{10}	V_{12}	V_{13}	V_{15}	
PM_{62}	V_1	V_6	V_9	V_{18}	V_{20}	V_{23}	V_{28}
PM_{63}	V_4	V_7	V_{16}	V_{24}	V_{27}	V_{29}	
PM_{64}	V_3	V_8	V_{17}	V_{22}	V_{25}	V_{26}	
PM_{71}	V_5	V_8	V_9	V_{16}	V_{21}	V_{24}	V_{29}
PM_{72}	V_2	V_7	V_{14}	V_{23}	V_{25}	V_{27}	
PM_{73}	V_1	V_4	V_{11}	V_{12}	V_{19}	V_{26}	
PM_{74}	V_3	V_6	V_{10}	V_{13}	V_{21}	V_{28}	

Here is the matrix T of the whole day:

6 1 0 1 0 1 0 1 1 1 2 1 0 1 1 1 0 2 2 2 0 1 1 0 1 2 1 2 1

1 6 1 0 1 0 1 0 1 1 1 2 1 1 1 1 1 0 1 0 1 0 2 0 2 1 2 2 2

0 1 6 1 0 1 0 1 2 2 1 1 2 1 1 1 1 1 0 1 2 1 0 1 2 1 0 1 2

1 0 1 6 1 0 1 0 1 1 2 2 1 1 1 1 1 1 2 0 1 1 1 1 1 1 2 1 1

0 1 0 1 6 1 0 1 1 1 0 1 2 1 1 1 1 2 1 1 1 2 1 2 1 1 1 2 1
1 0 1 0 1 6 1 0 1 1 1 0 2 1 1 0 1 1 1 1 2 1 2 2 1 1 1 2 1
0 1 0 1 0 1 6 1 0 0 1 1 1 2 1 2 1 1 0 1 1 1 1 2 1 2 2 0 1
1 0 1 0 1 0 1 6 1 1 0 1 0 1 2 2 2 1 2 2 1 1 1 1 1 1 1 0 1
1 1 2 1 1 1 0 1 6 2 2 0 0 1 1 1 1 2 0 2 2 1 1 2 1 0 0 1 2
1 1 2 1 1 1 0 1 2 6 1 1 2 1 2 1 1 0 1 0 1 1 0 1 0 0 1 1 0
2 1 1 2 0 1 1 0 2 1 6 1 1 1 1 0 1 1 1 1 0 1 1 1 1 2 0 0 0
1 2 1 2 1 0 1 1 0 1 1 6 1 0 1 1 1 1 2 1 1 0 1 1 0 1 0 1 0
0 1 2 1 2 2 1 0 0 2 1 1 6 0 1 1 0 0 1 1 1 0 0 1 1 1 0 2 1
1 1 1 1 1 1 2 1 1 1 1 0 0 6 2 0 1 0 1 2 1 1 1 0 1 1 1 0 1
1 1 1 1 1 1 1 2 1 2 1 1 1 2 6 1 0 1 1 1 1 0 1 0 0 1 1 0 0
1 1 1 1 1 0 2 2 1 1 0 1 1 0 1 6 1 0 0 1 1 1 1 1 1 0 1 1 2
0 1 1 1 1 1 1 2 1 1 1 1 0 1 0 1 6 1 0 1 0 2 1 1 1 1 1 1 1
2 0 1 1 2 1 1 1 2 0 1 1 0 0 1 0 1 6 0 1 1 1 1 2 1 1 1 2 1
2 1 0 2 1 1 0 2 0 1 1 2 1 1 1 0 0 0 6 1 0 1 1 1 2 1 1 1 0
2 0 1 0 1 1 1 2 2 0 1 1 1 2 1 1 1 1 1 6 0 1 1 0 0 1 0 1 1
0 1 2 1 1 2 1 1 2 1 0 1 1 1 1 1 0 1 0 0 6 1 1 1 1 0 1 1 2
1 0 1 1 2 1 1 1 1 1 1 0 0 1 0 1 2 1 1 1 1 6 1 1 2 1 1 1 0
1 2 0 1 1 2 1 1 1 0 1 1 0 1 1 1 1 1 1 1 1 1 6 1 2 0 1 1 0
0 0 1 1 2 2 2 1 2 1 1 1 1 0 0 1 1 2 1 0 1 1 1 6 1 1 1 0 2
1 2 2 1 1 1 1 1 1 0 1 0 1 1 0 1 1 1 2 0 1 2 2 1 6 1 1 0 1
2 1 1 1 1 1 2 1 0 0 2 1 1 1 1 0 1 1 1 1 0 1 0 1 1 6 1 1 1
1 2 0 2 1 1 2 1 0 1 0 0 0 1 1 1 1 1 1 0 1 1 1 1 1 1 6 2 2
2 2 1 1 2 2 0 0 1 1 0 1 2 0 0 1 1 2 1 1 1 1 1 0 0 1 2 6 1
1 2 2 1 1 1 1 1 2 0 0 0 1 1 0 2 1 1 0 1 2 0 0 2 1 1 2 1 6

It is obviously that: $\lambda_{max}=2$, $N_{pd}=317$, $F=21.8$.

How Well the Criteria are Met

First, in the list, each member participates in 6 sessions. It satisfies the lst criteria very well. In order to reduce λ_{max}, we may let $\lambda_{max}=1$, but even when $r=5$, we can't get the solution. Hence $\lambda_{max}=2$ is more suitable. We count the λ_{ij} for all the time, and obtain from Fig. 4. 5. 1:

From the Fig. 4. 5. 1, we can see that $\lambda_{ij}=1$ is the most, occupy 78. 9%.

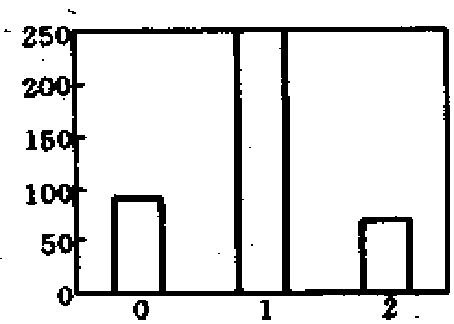

Fig. 4. 5. 1 meeting times

We get the list in the morning by using Latin square, which can not only suit criteria 3 but also make $\lambda_{max}=1$. It is ideal.

The average proportion of in-house member in each small groups is 0. 298 and the variance is 0. 060.

Furthermore, when we try to arrange the meeting without considering λ_{max}, we get $N_{pdmax}=320$, but in our given list, $N_{pd}=317$ ($N_{pd}/N_{pdmax}=0.99$), which is almost the maximum.

Algorithm for the Secretary

Let U be the set of members to join in discussion in one session scheduled, W be the set of members not to join in discussion in one session scheduled.

In one session, if there is a member in U (resp. W) and another member in W (resp. U), and the two members are of the same level and have joined the same times'session, we call them form a mapping.

We give the algorithm as follows:

(1) In one session, when member A in U cancel or member C in W show up, if there exits member B (resp. D) in W (resp. U) such that B (resp. D) and A (resp. C) form a mapping. We do a mapping transformation: let B join in groups that A should from now on, and let A join in groups that B should join from now on. So do C and D.

(2) If there is no mapping, adjust members of each group in the session or move a member between two sessions so that no group contain disproportional a number of each level.

For example:

①If V_2 cancels in AM_{21}, let V_4 join in AM_{21}, PM_{41}, PM_{53}, PM_{61} and PM_{72}. Let V_2 join in AM_{53}, PM_{42}, PM_{51}, PM_{63}, PM_{73}.

②If V_{12} shows up in PM_{41}, let V_{13} join in PM_{43}, PM_{61}, PM_{73}. Let V_{12} join in PM_{62}, PM_{61}, PM_{74}.

③If V_8 cancels in PM_{54}, let V_9 join in PM_{54}.

④If V_7 cancels in AM_{23}, let V_4 join in AM_{23}.

Test of Model

1. Computer simuliate

In this part of test, we use computer to simulate that the board member cancel or show up undesignedly.

The results of 100 times of simulations show that the algorithm can not only adjust the assignments well, but also make assignments for future meetings, involving different levels of participation for each type of attendee. see Tab. 4. 5. 7.

Tab. 4. 5. 7

P_C(%)	P_S(%)	N_{pd}	F	$\lambda=0$	$\lambda=1$	$\lambda=2$	$\lambda=3$	$\lambda=4$	$\lambda=5$
1	1	311.0	21.45	95.0	242.6	67.4	0.96	0	0
2	2	306.7	21.15	99.3	237.8	67.4	1.50	0.01	0
4	4	301.7	20.81	104.3	231.2	67.5	2.30	0.03	0
8	8	284.0	19.59	122.9	210.35	67.17	5.36	0.15	0
10	10	276.1	19.24	129.9	203.0	66.7	6.00	0.23	0
15	15	263.3	18.14	142.6	191.7	63.8	7.51	0.25	0

Note: P_C: the posibility of canceling, P_S: the posibility. of showing up undesignedly

2. Stability test

We change the number of in-house members I to test the model's stability. See Tab. 4. 5. 8.

Tab. 4. 5. 8

I	P_{av}
8	0.268±0.052
9	0.298±0.060
10	0.335±0.066
11	0.369±0.072
12	0.399±0.079

Note: I: the number of in-house members;

P_{av}: the average value of P_{ij}.

Analyzing these data, we can see the in-house members' designment are proportinate.

Generalization

1. How to arrange efficient sessions and groups?

Given the number of attendence v and the limited number of members of small group n, how many sessions and officers should be arranged? Let's define:

J number of officer

X_i number of session

Y_i number of small group in a session

K_i number of small group member

($i=1$ means morning, $i=2$ means afternoon)

How can we arrange the least session?

Obviously $$Y_1=J$$

and $$K_1=\min\{n, \left[\frac{V}{J}\right]\},$$

Since the officer don't attend the afternoon session, so

$$K_2=n$$

$$Y_2=\left[\frac{V}{n}\right]$$

When N_{pd} is ideal $N_{pd}=X_1\times J\times C^2_{\min(n,[v/J])}+X_2\times\left[\frac{V}{n}\right]\times C^2_n$

Let $$N_{pd}=\alpha\times C^2_v \quad (0.5<\alpha<1.1).$$

The α is determined by the importance of the meeting. If the meeting is important, $\alpha=1$, otherwise α may be smaller.

Thus $$X_1\times J\times C^2_{\min(n,[v/J])}+X_2\times\left[\frac{V}{n}\right]\times C^2_n=\alpha\times C^2_v \qquad (1)$$

Assume that $X_1:X_2=t_1:t_2$ (t_1 is the total time of morning meeting, t_2 is the total time of afternoon meeting)

From equation (1), we can make some judgement if V and α has been given. See Tab. 4. 5. 9.

Tab. 4.5.9

V	a	J	X_1	Y_1	X_2	Y_2
20	1.0	3	2	3	2	3
29	1.0	4	3	4	3	4
50	1.0	7	4	7	4	7
100	1.0	14	8	14	8	14
20	0.6	3	1	3	1	3
50	0.6	7	2 or 3	7	2 or 3	7
100	0.6	14	5	14	5	14

Note: $t_1=t_2=3$ hours, $n=7$ persons.

2. How to adjust different type of members?

If the member are belonged to 3 or 4 levels even more, and require the number of each type should be proportionate, we can extend our model, apply "greedy" for every type. First, we use Latin squares to arrange the members of one type. Of course, the more especial type will do first. Then, for the other types, it will be well arranged by the greedy algorithm.

3. How to adjust different type of meetings?

For meetings as this one, there are many restrains, we can apply Latin square and greedy algorithm to solve them. For common meetings, they generally only need to mix members as much as possible. Then we can use greedy algorithm model to find a prompt, accurate arrange list (in this model, we only need to wait about 4 seconds while using 486 PC to get the list in the afternoon).

General Scheme for Small Groups Designing

We summarize our general measures like this:

①Uniformly distribute members among groups in one sessions.

② By the meeting scale, determine the times of each members' joining groups.

③Use OLS and replacement to design discussion groups of the morning.

④Use "greedy algorithm" to design other groups in the afternoon.

⑤Consider some level's proportion in groups design them in advance.

⑥ Use mapping transformation to design an algorithm for adjusting.

Strengths and Weaknesses

Strengths

① We provide a set of efficient schemes to design small groups.

②The list is satisfactory.

③Low the complexity of greedy algorithm.

④Design an efficient algorithm for the secretary by mapping transformation.

⑤Our model is flexible to be applied to different kinds of computer.

Weaknesses

We are unable to demonstrate that our model is optimum.

Appendix

1. Decide r

In accordance with criteria 4, the maximum number of a morning group's members is 5([29/6]+1). The maximum number of an afternoon group's member is 8([29/4]+1). But if there are 5 members in a morning group or 8 members in an afternoon group, then the in-house member in different groups will be disaproportional. Thus, number of each morning group's members are up to 4 and number of each afternoon group's members are up to 7.

Let p be the smaller number of each morning group's members, q be the smaller number of each afternoon group's members, s be the number of p members' morning groups, t be the number of q members' afternoon groups.

The (r, p, q, s, t) satisfy the following diophantine equation:

$$r\times 29=s\times p+(18-s)\times(p+1)+t\times q+(16-t)\times(q+1)$$

$$(p=3,\ 4;\ q=3,\ 4,\ 5,\ 6,\ 7)$$

If the times of every two members'discussions are up to 1, then N_{pd} will reach the ideal value N_{pdi}, and

$$N_{pdi}=s\times C^2_{p_1}+(6-s)\times C^2_{p_2}+t\times C^2_{q_1}+(4-t)\times C^2_{q_2}$$

N_{pdm}, the maximum of N_{pd}, is $C^2_{29_1}$.

As a result of the above two equations, we can obtain Tab. 4.5.10.

When $r=1$, 2, 3, 7, there is no result. Compare each N_{pdi} in the table with N_{pdm} and conclude $r=6$.

When $N_{pdi}=402$, groups with 3 members are 14 and groups with 7 members are 16. Unfornately, under this condition,

Tab. 4. 5. 10

r	N_{pdi}	N_{pdi}/N_{pdm}
4	144	0. 35
5	300	0. 74
6	402	0. 99

creteria 4 can't be satisfied. Hence, we arrange 18 groups with 4 members in the morning. We can design the assignments ideally as follows. At this moment, $N_{pdi}=384$, $p=4$, $q=6$.

2. Orbit method

This method is used to construct Latin Square.

Constitute $A^{(0)}=[a_{ij}^{(0)}]_{6\times4}$, $(i=0,1,\cdots,5;j=0,1,2,3)$

Let
$$a_{ij}=i_j$$

$$A^{(0)}=\begin{bmatrix}0_0 & 0_1 & 0_2 & 0_3\\ 1_0 & 1_1 & 1_2 & 1_3\\ 2_0 & 2_1 & 2_2 & 2_3\\ 3_0 & 3_1 & 3_2 & 3_3\\ 4_0 & 4_1 & 4_2 & 4_3\\ 5_0 & 5_1 & 5_2 & 5_3\end{bmatrix}$$

There are several kinds lines in different slope to fix four elements while one line at the same slope to constitute one matrix. So we can obtain 3 matrix like this:

$$A^{(2)}=\begin{bmatrix}1_0 & 2_1 & 3_2 & 4_3\\ 2_0 & 3_1 & 4_2 & 5_3\\ 3_0 & 4_1 & 5_2 & 0_3\\ 4_0 & 5_1 & 0_2 & 1_3\\ 5_0 & 0_1 & 1_2 & 2_3\\ 0_0 & 1_1 & 2_2 & 3_3\end{bmatrix}\quad A^{(3)}=\begin{bmatrix}2_0 & 4_1 & 1_2 & 3_3\\ 3_0 & 5_1 & 2_2 & 4_3\\ 4_0 & 0_1 & 3_2 & 5_3\\ 5_0 & 1_1 & 4_2 & 0_3\\ 0_0 & 2_1 & 5_2 & 1_3\\ 1_0 & 3_1 & 0_2 & 2_3\end{bmatrix}$$

Adjust the sequence of rows of $A^{(0)}, A^{(1)}, A^{(2)}$ and obtain the matrix we need.

3. The concrete algorithm

FOR $I=1$ TO 4 DO
 BEGIN
  $S=[V_{10}, V_{11}, \cdots, V_{29}]$;
  FOR $j=1$ TO 4 DO
   BEGIN
    sort element of S in decending;
    high$=0$;
    REPEAT
     select $V_{11}, V_{12}, \cdots, V_{1k}$ from S;
     $P=[V_{11}, V_{12}, \cdots, V_{1k}]$;
     form submatrix D of P from matrix T;
     IF $\max(\lambda_{11.11}\ \lambda_{12.12} \cdots \lambda_{1k.1k}) < r$ AND
      $\max(\lambda_{11.12}\ \lambda_{11.13} \cdots\ \lambda_{11.1k}\ \lambda_{12.13} \cdots \lambda_{12.1k} \cdots \lambda_{1(k-1).1k})$
      $<$max
     THEN count=how many "0" in D;
     IF high$<$count
      THEN
       BEGIN
        high=count; $Q=P$;
       END
     UNTIL hounting is over
   /* Element of Q are members of group being designed*/
    $S=S-Q$;
    END;

END.

Notice: We can change λ_{max} ascendingly to get the minimum of λ_{max} under the condition that the criteria is satisfied.

END.

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