# The World's Most Complicated Payroll

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## Introduction

We present a model for paying the faculty that we believe is fair and consistent with the requirements, as well as ways of dealing with cost-of-living increases, budget shortages, and the transition process.

We use a two-track salary model: All of the instructors are on one track while all of the Ph.D.s are on the other. To accommodate both seniority and rank elements into our model, we make salary a function of "quality points," an index that incorporates both seniority and rank for Ph.D.s but just seniority in instructors. We use part of a root function as our salary function for both tracks and make the slope of the instructor function half that of the Ph.D. function.

Furthermore, we designed a transition process for instructors promoted to assistant professor. To incorporate cost-of-living raises, our salary functions work in constant dollars. To deal with budget deficits, we let promotions happen as normally but also give out fractional quality points and make up the difference later. Our model does not provide for drastic deficits; these would require major changes, such as cuts or layoffs, which we leave as the business of the administration.

To create a transition process, we give all faculty making more money than they should (as determined by the model) a minimum \$100 raise each year (in real, not constant, dollars) until they are on the track as they should be, and divert the remaining money into extra raises for the financially challenged. Estimating some data that were not provided, we determined that 95% of the overpaid faculty would be on the proper payroll track within 4.6 years, meaning that with the exception of a few grossly overpaid faculty, the problem of unfairness would be solved.

Our model is highly flexible and adaptable; many of the parameters can be changed to fit the wishes of the administration. Unfortunately, many of the estimates that we produced are based on simulations of data that were not given. We do, however, include the templates we used, so better estimates can be made using data which should be in the college's files.

What follows is our recommendation to the Provost for the new faculty compensation system, including the of cost-of-living increases, the policy when there is not enough money to fully support the model, and the transition from the current system to the new, improved system.

# Major Assumptions

- *All other colleges from which faculty might be transferred have a faculty ranking system similar to ABC's.* Thus, when a faculty member transfers from another school, we have a good estimate both of her status in the system and of her experience.

- *An assistant professor or associate professor may not be promoted until she has spent seven years at her current rank.* The circumstances stipulate that an associate professor must work seven years before promotion but do not say anything about an assistant professor. Furthermore, we consider promotion after exactly seven years in a rank to be "on time."

- *We have enough money to implement our model, and there is no inflation or deflation.* ("The world is perfect.") Later we will consider the effect of limited funds and inflation.

- *Every year that a faculty member receives a promotion benefit, she also receives a normal year's raise* (the raise first and then the benefit). We assume that all promotions take place in between academic years.

- *With no inflation, the starting salaries are $27,000 for instructors and $32,000 for assistant professors.* In this model, a full professor who has worked at ABC for twenty-five years starting as assistant professor, with promotions after seven and fourteen years, will receive a salary that is twice the starting salary of an assistant professor (from Principle 3), or $64,000.

- *The number of faculty members in each rank does not change significantly from one year to the next.*

# Problem Analysis and Development of the Model

We graph the current salary data for each of the faculty ranks in **Figure 1**.

The most obvious problem with the data is that the number of years on the chart is only the number of years at ABC. Many faculty members may

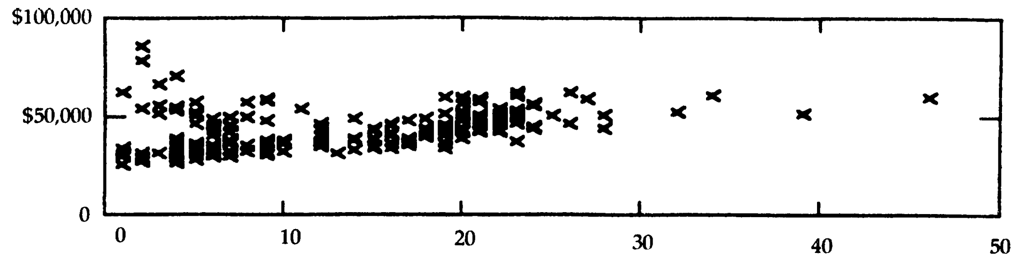



**Figure 1.** Scatterplot of salary vs. years at ABC.

have transferred in from some other institution with credit for up to seven years of experience, but we do not know how much credit they have been given. We also do not know the number of years spent in each rank for each faculty member moving through the system. We would expect that a faculty member with a 24-year career, for example, who spent 7 years as an assistant professor, 7 years as an associate professor, and 10 years as a full professor, would be making more than another faculty member who had spent 15, 8, and 1 years, respectively. In the data, however, both are listed just as 25-year full professors.

Since the data are incomplete, we designed a model independent of the data. The first model we tried involved using several salary tracks, one for each rank, each with the same slope. On promotion, an instructor would move an extra seven years worth of salary on her own track and then move up to the next one. This complicated system resulted in several problems; for example, faculty who were promoted earlier would earn less money after a while than those promoted later. We wanted faculty promoted earlier to earn more money. Furthermore, we wanted a model that would be easy to present, easy to implement, and reasonably flexible.

We opted for a different alternative. Since the salary system for assistant/associate/full professors is a single track with consistent rules, we deal with the instructor track separately, thus giving a two-track system. We tackle inflation by simply vertically stretching the curves by the next year's projected cost-of-living increase.

The next problem was what to do if there is not enough money for each faculty member's expected raise. We wanted to see the effects of this change on the actual faculty at ABC, but lack of the prior work history of each faculty member stops us from being able to calculate quality points and the corresponding salary. Instead, we generated a possible set of work histories for the faculty. We used a MathCAD template to generate at random 204 (the number of current faculty members) sets of four points (the numbers of years spent in each rank) and then ran these through our salary function to find the salaries. We calculated the amount needed to pay everyone, the expected raises, and amounts for each faculty member when there is not enough to give everyone a full raise. For application at ABC, actual histories could be entered into the template. We also created a special plan for the transition between the current system and the new one, involving giving a minimum raise to those making more than they should and dividing the extra among the others.

# The Ph.D. Track

For the ranks of faculty with a Ph.D., it is much clearer how the salary increases than in the instructor phase. We are explicitly told that a promotion is "on time" for a Ph.D. if it happens after seven years and that the benefit should be the same as seven years of raises. For an instructor, there is no clear definition of "on time" or minimum time to promotion. Someone could stay instructor for her entire teaching career by never getting a doctorate. Furthermore, being an assistant professor is a prerequisite to reaching the higher levels, while being an instructor is not.

We solve the problem of the difference by having the two salary schemes lie on different lines. Since we put all Ph.D.s on one salary track, we must find a way of dealing with promotion benefits and years of experience as two parts of the same variable, since salary can be a function of only these two factors. We introduce "quality points," which take into account the following rules:

- Faculty receive 1 point for every year worked at ABC.
- When a faculty member is promoted, she receives 7 points and the correspondingly higher salary if she is promoted on time (after seven years). If she is promoted later than the minimum amount of time, the reward should be correspondingly less, to satisfy the constraints. We increase the number of points by $49/t$, where $t$ is the number of years spent in the rank from which she is promoted.
- Employees hired from other colleges receive 14 points if they have reached the rank of associate professor and an additional 14 points if they have reached the rank of full professor when they transfer in. These are the minimum numbers of points for these ranks. Furthermore, in whatever rank the employee enters, we give her up to seven years (quality points) of credit in that rank. For example, if an employee comes in with 30 years of assistant professor experience and 2 years of associate professor experience, we make her an associate professor with $14+2 = 16$ points; if another employee comes in with 30 years as an assistant professor and 9 years as an associate professor, we make her an associate professor with $14 + 7 = 21$ points.

Given these conditions, we observe that a new Ph.D. who has worked 25 years and has made all of her promotions on time has earned 39 points and thereby should earn $64,000. We should therefore fit our function $f(x)$, where $x$ is the number of quality points and $f(x)$ is dollars in thousands, to the following constraints:

- The function must always be increasing, to satisfy the constraint that more experience in a rank results in more money: $f'(x) > 0$.
- The function should always be concave down, so that its slope decreases. Therefore, the effect of a difference in experience will narrow over time, as prescribed: $f''(x) < 0$.
- $f(0) = 32$ and $f(39) = 64$.
- The slope should be reasonably large (but not too large) at the beginning, so new employees get reasonably large (but not huge) raises.

We use part of a power function to determine our function. In general form, the equation is part of a $p^{\text{th}}$ root equation, where $p$ is some (not necessarily integral) number greater than 1. Performing an affine transformation of sorts, we map $(0, 32)$ to $(1, 1)$, and $(39, 64)$ to $(2, 2^{1/p})$. This was accomplished via the function (see **Figure 2**)

$$f(x) = 32\left[1 + (2^p - 1)\,\frac{x}{39}\right]^{\frac{1}{p}}.$$

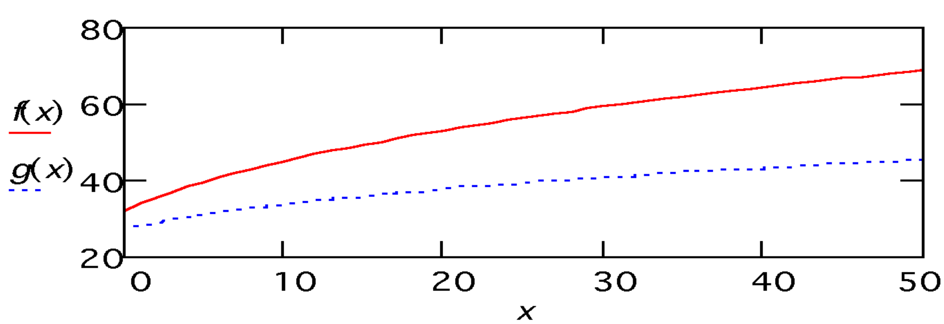



**Figure 2.** Salary vs. quality points: Ph.D. track ($f(x)$, solid line) and instructor track ($g(x)$, dotted line), with $p = 2$.

This family of functions has the advantage that we can adjust $p$ to produce a salary function that is very steep initially and later very narrow, or one that is close to a line.

# The Instructor Track

The instructor track cannot be defined as easily as the Ph.D. track, because we do not have any information about the expected value of the yearly raises. Arbitrarily, we set the raise for an instructor to be half of that of a Ph.D. with the same number of quality points. So the instructor salary $g(x)$ for $x$ quality points is

$$g(x) = \tfrac{1}{2} f(x) + 11 = 16 \left[1 + (2^p - 1)\, \tfrac{x}{39}\right]^{\frac{1}{p}} + 11$$

(see **Figure 2**). We add the 11 to force the function to go through $(0, 27)$. The multiplier of one-half can be changed easily if necessary. The number of instructor quality points equals the number of years as an instructor, and an instructor moving into ABC from another institution can receive as many as 7 quality points. Note that between the initial salaries for instructor and for assistant professor, we initially have a \$5,000 gap, which grows with quality points. We assume that if an instructor becomes an assistant professor very quickly, then her salary should be very close to that of someone who has had the same length of career and has always been an assistant professor, whereas an instructor who takes very long to get her Ph.D. should not be very close in salary to an assistant professor who has worked for the same number of years. Therefore, we suggest that upon the completion of a Ph.D., we give the new assistant professor an immediate salary raise of \$5,000 (not enough to completely make up the difference) and then reevaluate her quality points on the Ph.D. scale. The function

$$q_{\text{Ph.D.}} = f^{-1}(g(q_{\text{Inst.}}) + 5)$$

calculates the position on the Ph.D. curve for an instructor receiving a promotion and receiving a \$5,000 benefit.

# What Will It Cost?

For small $p$, the salary functions look too much like lines and salaries do not converge much; for high $p$, the cost of paying the faculty is high. We feel that $p = 2$ is a reasonable value that ABC might want to use.

Since we do not have the actual histories of the employees, we cannot calculate the actual sum of the salaries on the new system. Instead, we created randomized histories to get an idea of the monetary demands of the new system. This required a simulation satisfying a number of constraints, the details of which are in the **Appendix**.

Several iterations of the simulation on different random faculties estimate that a highly experienced faculty with approximately the same number of faculty members in each rank would cost about $\$10.0 \pm 0.1$ million to pay for a value of $p = 2$. This is not an average case estimate, but more of a "worst of the average" case estimate, because the simulated faculty have all been at ABC for their entire careers. Since most of the actual faculty have only been at ABC for a few years, they are making less money as a whole than the simulated faculties, so \$10.0 million is more than the college should expect to pay under our system.

The least the college could expect to pay out in a year would correspond to replacing the entire faculty with people from other schools with no experience in the rank at which they enter ABC. The faculty would come in with no credit, so each faculty member would make the baseline salary for her rank. The total minimum salary for the new system with $p = 2$ is \$8.7 million, which is roughly the same as the current payroll (\$8.8 million).

The 1993 national average salaries for faculty members, from instructor to full professor, were \$28,300, \$38,600, \$46,900, and \$68,700 for a private college (as ABC must be with a name like that). For $p = 2$, the baseline salaries at ABC for each category with promotions on time (in the same order) would be \$27,000, \$32,000, \$46,100, and \$56,800. These values are all less than the national average, but they are just the baseline values. Because of years in rank, many faculty members will be making more, creating an average closer to the national one.

# The Cost of Living

We have determined the salary functions for faculty members with a Ph.D. and for instructors in terms of the number of quality points (we have not, however, determined a value for the parameter $p$). We need to incorporate annual increases in the cost of living into this model. The Consumer Price Index (CPI) measures well the year-to-year changes in the inflation rate [Parkin 1993, 628].

We propose to increase everybody's salary by a proportion equal to the increase in the cost of living every year. So, if after one year, the cost of living has increased 4%, then we just multiply $f(x)$ and $g(x)$ by 1.04.

Note that the \$5,000 salary increase that promoted instructors receive must also be adjusted for the cost of living.

# When Funds Run Short

Up until this point, we have been assuming that ABC has enough money to pay everyone on the new salary schedule. Currently, ABC is paying out \$8.8 million; unfortunately, according to our model, ABC has been paying out too little. This raises the question of what to do if ABC cannot afford the new system: how to pay everyone if funding is low for a year.

We must make several assumptions about priorities.

- Give the usual raise and a bonus to faculty who are promoted.
- Give the usual raise (if there is enough money) to faculty members not getting a promotion.
- Spend any excess money (if there is any after giving raises) on replacing faculty members who leave (about 7% each year; see the **Appendix**).
- If there is not even enough to give everyone their raise, do not hire replacements and give each faculty member a fraction of a quality point instead of a whole one.
- If the budget cut is very serious, other measures, such as cutting salary for some and laying off others, may need to be taken. We cannot dictate firing people, but the method for reducing salary that would be best is to subtract the same number of quality points from each faculty member.

Since we do not have the actual salary histories of the employees at ABC, we created another random data set to experiment with. This data set, unlike the previous one, creates a population of faculty members in which some have spent their whole career at ABC and some have not (see the **Appendix**). Using our random data, we then created a MathCAD template to calculate the salary of each faculty member when there is not enough money. The data can easily be changed to calculate these amounts based on the actual numbers.

If there is not enough money to pay everyone the full expected raise (not taking into account cost-of-living increases), we first pay everyone who is being promoted and then divide the remaining money among the remaining people by assigning each faculty member the same fractional quality point of a yearly increase. If the administration decides that it is not proper to hire someone during the money shortage, that faculty member's history should just be left out of the data file.

We ran a simulation with a budget of \$9.45 million for 1994–95. The amount needed to give everyone their raise and fulfill promotion raises was \$9.50 million. Since there is not enough money, the template calculates the appropriate number of quality points to give out after raises. The result was to give each faculty member 0.735 points.

# The Transition

We created the following procedure for moving current low-paid faculty members to the new scale without cutting anyone's salary. We must assume that there is enough money for the transition. The major points are the following:

- Anyone making more than they should receives a minimum raise (because the basic structure requires that each faculty member must receive a raise any year that money is available). We make this minimum raise \$100. We do not, however, include an increase for cost of living. In adding the minimum raise, we never allow an employee's actual salary to drop below her deserved salary.

- First, we pay everyone their previous year's salary, and people making more than they should are paid the minimum raise.

- We distribute the remaining money as raises to the people making less than they should, proportional to the amount they are below the salary that they deserve, including a cost-of-living increase.

- We assume that the college budgets \$10 million in 1994–95 dollars each year until the current salaries catch up to the new payment system.

In order to create and test a template, we again need actual values. We use the current salaries and generate possible histories to go along with them. We calculate how long it would take for each overpaid faculty member to receive only as much money as she deserves. (We wanted to calculate how long it would take each underpaid faculty member to catch up, but we could not fix an error in our program.) Five runs of the program were combined. The number of people with a certain "catch-up" time appears to be an exponential distribution with a mean of 3.55. A quick computation shows that it should take 4.6 years for 95% of the overpaid people to catch up to their salaries, although some extremely overpaid people take much longer.

# Strengths

- Our model is simple. Using a single salary curve for all doctorates is much simpler than using a separate curve for each rank. The concept of quality points is a very convenient and simplifying assumption and is one that anybody could use. Furthermore, the cost-of-living adjustment that we make is simple.

- The model rewards those instructors who advance in rank quickly, as we believe is appropriate, without making the penalty for late promotions too drastic—late promotees still get a raise, just not as much.

- The template that we used with simulated data can easily be used with actual data.

- Our model is flexible. The parameters to which we assigned values (such as $p = 2$) can easily be changed,

- Our model conforms to all of the principles and circumstances of the problem statement.
- The model seems stable and consistent. Different random sets of data do not produce large changes in the results, and our results agree with values for average faculty salary in the U.S.
- If so desired, the concept of quality points could be extended to situations beyond the scale of this model. For example, if the administration wanted to give a faculty a special raise/cut for extraordinary performance, or to make especially good offers to desirable faculty from other schools, it could adjust a faculty member's number of quality points.

# Weaknesses

- The parameters used in our model are arbitrary, since we are missing data that would be useful in identifying them.
- We have no basis for setting the instructor slope at half of the Ph.D. slope.
- It is very difficult to perform sensitivity analysis on our model. We have no way of estimating how much our simulated data, upon which we should not rely too heavily, differs from the real data.
- We assumed that the number of instructors in each rank remains reasonably constant. It might not, which would alter our results.
- Implementing our model assumes adequate funding. If money is a problem, we could decrease $p$, but this may not totally remedy the situation.

# Appendix

## Creating Random Faculty

Since it does not seem to take very long to be promoted from instructor to assistant professor, the number of years as instructor is generated by taking a random number from an exponential distribution with a mean of 3. (This means that more than half of all faculty members spend 3 years or less as instructors, but a few spend much longer.) The number of years as an assistant professor is similarly taken to be 7 plus a random number from an exponential distribution with a mean of 3. The 7 must be added because it takes a minimum of 7 years to be promoted to associate professor. Similarly, the number of years as an associate professor is generated from the same distribution. The number of years as a full professor is generated by 1 plus a random number from an exponential distribution with a mean of 5.

Not very much information is given in the problem to support this model. However, it does have some basis. For assistant professors, the average number of years at ABC is roughly exponentially distributed, with a mean of 9.4 (see **Figure 3**). This suggests that exponentially distributed random numbers are decent estimates of this number. Most of the simulated times are between 7 and 10 years, which roughly agrees with the minimum and average time that faculty members spend as assistant professors. The time spent as an associate professor is roughly the same, so we use the same random variable for years as an associate professor.

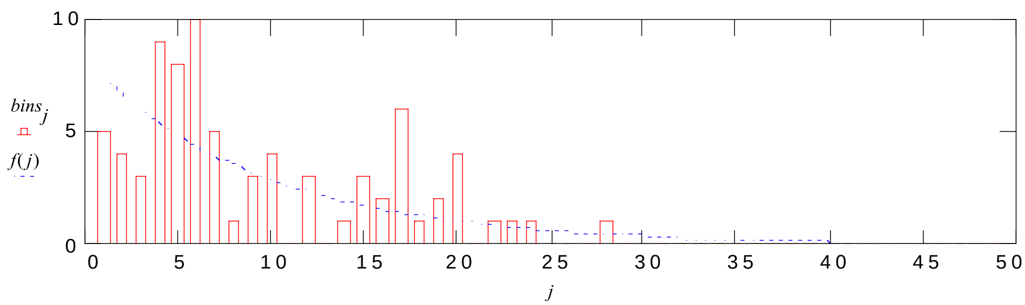
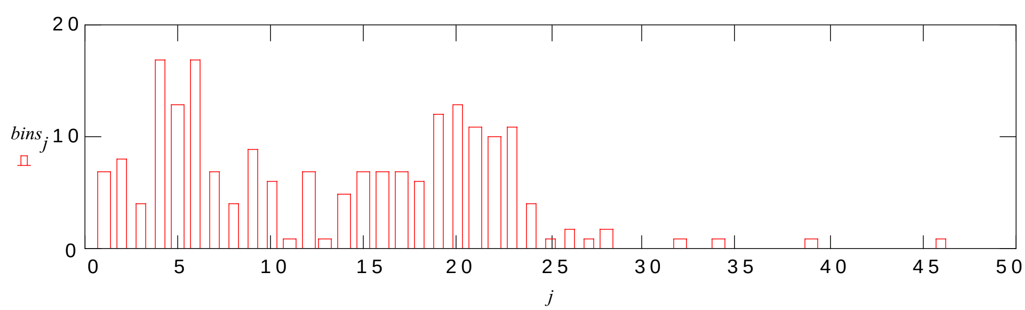



**Figure 3.** Number of assistant professors vs. years at ABC. The dotted curve is the function used in the simulations.

The mean of 5 for years-in-rank as a full professor comes from a total career of about twenty-five years: $7 + 3\text{-ish} + 7 + 3\text{-ish} + 5\text{-ish} = 25\text{-ish}$.

The simulation creates the same distribution of positions as the given salary list; we assume that the makeup of the faculty does not change.

The second simulation creates a random number of years that each instructor has been at ABC, based on the given data and using an exponential distribution with a mean of 13.3 (the mean of the data) (see **Figure 4**).




$mean(years) = 13.299$

**Figure 4.** Number of faculty members vs. years at ABC.

The number of faculty members who leave after $t$ years is the difference between the numbers there $t$ years ago and $t - 1$ years ago, which is approximately equal to the derivative of the density function at $t$. By explicit calculation, or from properties of the exponential distribution, the duration of stay is exponential with the same mean.

## How Many People Are Replaced Each Year?

We assume that the same fraction of the faculty is replaced each year and that a faculty member stays at ABC an average of 13.3 years.

For a continuous model, we use an exponential distribution with probability density function

$$f(t) = \frac{e^{-t/\lambda}}{\lambda}, \qquad t > 0,$$

which has mean $\lambda$. The area lying to the right of $\lambda$ is $e^{-1}$. We set $\lambda = 13.3$, so that after 13.3 years, the fraction of faculty remaining is $e^{-1} \approx .368$. For the exponential "decay" in the number of faculty, let $\mu$ be the half-life and $r$ the decay constant; $r$ is the fraction of faculty who remain from one year to the next. Let $N_0$ be the initial number of faculty in a cohort, and let $N_\mu$ be the number of those faculty remaining after one "half-life." We have

$$N_\mu = N_0 r^\mu = \frac{1}{e} N_0, \qquad r = e^{-\mu}.$$

For $\mu = 13.3$, we have $r = 0.928$, so roughly 93% of the faculty remain from one year to the next, and on average $.072 \times 204 = 15$ faculty members are replaced every year. Depending on the ranks involved, 15 faculty members cost anywhere from $405,000 (inexperienced instructors) to $900,000 (experienced full professors). If the college runs short of money, some money could be saved by not replacing these faculty members.

For a discrete model, we use a geometric distribution with probability mass function

$$f(n) = p(1-p)^{n-1}, \qquad n = 0, 1, 2, \ldots /,$$

where $p$ is the fraction of faculty who leave each year. The mean of this distribution is $1/p$. We set $1/p = 13.3$, getting $p = .075$, in good agreement with the continuous model.

# References

U.S. Department of Labor, Bureau of the Census. 1994. *Statistical Abstract of the United States: 1994.* 114th ed. Washington, DC: U.S. Government Printing Office.

Parkin, Michael. 1993. *Economics.* New York: Addison-Wesley.