Selection of a Bicycle Wheel Type

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Introduction

We present a model that compares the performance of various wheels over a user-determined course. We approach the modeling problem by beginning with Newton's Second Law of Motion: The sum of the forces acting on an object equals the mass of that object multiplied by its acceleration.

We identify the four principal forces that contribute to a cyclist's motion: applied force, drag force, gravity, and rolling resistance. We further classify drag force into three components: the cyclist and bicycle frame, the front wheel, and the rear wheel.

Drag force is dependent on a cyclist's velocity, and the force of gravity is dependent on a cyclist's position. Thus, our force equation is a function of cyclist position and cyclist velocity.

We can then arrange Newton's Second Law to yield a second-order differential equation. Given position S, velocity dS/dt, acceleration d^2S/dt^2, mass m, and a force function F, the differential equation is

$$\frac{d^2S}{dt^2} = \frac{F\left(S, \frac{dS}{dt}\right)}{m}.$$

To implement our model, we created a computer software program that allows a user to input numerous pieces of data, including course layout, elevation profile, wind, weather conditions, and cyclist characteristics. The software iterates the differential equation using the fourth-order Runge-Kutta method. The software reports the preferred wheel choice based on the data.

As a real-world application of our model, we analyze the 2000 Olympic Cycling time-trial race. Over that course, a disk wheel provided a considerable advantage over a spoked wheel.

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Problem Analysis

We must determine whether a spoked wheel (lighter but less aerodynamic) or a disk wheel (more aerodynamic but heavier) is more power-efficient. The choice depends on many factors, including the number and steepness of hills, the weather, wind speed and direction, and type of competition.

We are not striving to discover which wheel is best for all situations. To the contrary, we are interested in which wheel outperforms other wheels given specific conditions. Given accurate data concerning factors such as hills, weather, wind, and competition type, a good model will be able to determine and recommend which wheel is preferred.

Our Approach

We determine equations for each force that acts on the cyclist and the bicycle. These forces are not constant throughout a race. For example, air drag increases with the square of velocity.

We apply Newton's Second Law of Motion to the cyclist-bicycle system: The sum of the forces acting on an object equals mass of the object times its acceleration. The sum of the forces is a function of both position on the course and velocity of the cyclist. Because the forces are different at different positions, the power required with a given type of wheel is also different. Our approach is to apply the same power to both types of wheels and determine how long it takes to traverse the course. The wheel that allows the cyclist to complete the course in the shortest time requires less power, that is, is more power-efficient.

A more comprehensive model would incorporate wind based upon a probabilistic function. However, to do so would be at odds with our goal: We wish to determine performance times for two types of wheels given a constant set of conditions. If we included probabilistic functions, differing performance times would be due to both wheel differences and random fluctuations instead of just to wheel differences.

Let the position of the bicycle on the course be S with components S_x, S_y, and S_z. The second-order differential equation for acceleration is

$$\frac{d^2S}{dt^2} = \frac{F\left(S, \frac{dS}{dt}\right)}{m},$$

where d^2S/dt^2 is the acceleration, $F(S, dS/dt)$ is the total force, and m is the total mass of the system.

We determine the time that it takes to complete a course by solving this equation numerically.

Assumptions

- Weather conditions (temperature, humidity, wind direction, and wind speed) are uniform over the course and constant throughout the race. Because wind varies over time, and terrain significantly changes wind speed and direction, larger courses with greater variability in elevation are most affected by this assumption. However, due to the unpredictability of the weather, a general wind direction and speed is probably the most detailed information to which the rider will have access.
- Both wheel types use the same tire.
- Turning does not significantly affect power efficiency of the wheel, speed of the rider, or acceleration of the rider.
- Based on the previous assumption, we assume that the bicycle moves in a linear path in 3-D space.
- The cyclist applies power according to the function that we develop in the **Model Design** section below, where we introduce other assumptions associated with developing this function.
- The drag coefficient for the rider plus bike frame is the same for all riders and does not change as a function of yaw angle. The drag coefficient is 0.5 and the cross sectional area is 0.5 m^2 [Analytic Cycling 2001a].
- The wheels do not slip in any direction as they roll over the course.
- Other riders have no effect on the aerodynamic characteristics of the bike-rider system. This means that we ignore the effects of drafting (which can reduce drag by up to 25%).
- The rider uses a conventional 36-spoke wheel on the front of the bike.
- The rotational moments of inertia for disc wheels and spoke wheels are approximately 0.1000 kg-m^2 and 0.0528 kg-m^2, respectively. In reality, these must be determined experimentally.

Model Design

We identify the forces that act on a bicycle and rider:

- The forward force that the rider applies with pedaling.
- The drag force that opposes the motion of the bicycle. Since we are concerned with analyzing wheel performance, we divide the total drag force into three components:

- The drag force F_f on the front wheel.
- The drag force F_r on the rear wheel.
- The drag force F_B on the bicycle frame and rider.

- The force of gravity F_g that either opposes or aids motion due to the road grade.
- The force of rolling resistance F_{rr} due to the compression and deformation of air in the tires.

Because we assume that the bicycle travels in a linear path, we need consider only components of these forces that act co-axially (i.e., parallel to the bicycle's direction of movement). This will remain realistic so long as the assumption that the wheels do not slip remains true, because the static frictional force between the wheels and the ground prevents movement normal to the velocity.

Consequently, we do not need to treat the forces as vectors; we must note only whether they aid or oppose the bike's movement.

The Force that the Rider Applies

The rider applies a force to the pedals, which the gears translate to the wheels of the bicycle. Competitive bicycle racers generally shift gears to maintain a constant force on the pedals as well as a constant pedaling rate (cadence) whenever the sum of the other forces oppose the motion (i.e., going up a hill or into the wind) [Harris Cyclery 2001]. In other words, the cyclist attempts to exert constant power.

However, as the rider moves downhill, gravity aids the effort. As the bicycle gains speed, the rider's pedaling results in a diminished effect on speed because drag forces increase with the square of speed. Eventually, at a speed v_{cutoff}, the rider ceases pedaling.

We model the power P_a that the rider inputs as a function of speed v_g:

$$P_a = \begin{cases} 0, & \text{if } v_g \geq v_{\text{cutoff}}; \\ P_{\text{avg}}, & \text{if } v_g < v_{\text{cutoff}}, \end{cases}$$

where P_{avg} is the average power that the rider can sustain; its value varies from rider to rider and with the type of race. Typical values range from 200 W for casual riders to between 420–460 W for Olympic athletes on long-distance road races, to as much as 1500 W in sprint races [Seiler 2001].

If we assume that there is no energy lost in the translation of the power between the pedal and the wheel (i.e., in the gears), then by conservation of energy the power goes into either rotating the wheels or moving the bicycle forward:

$$P_a = P_w + P_f, \tag{1}$$

where P_w is the power to rotate the wheels and P_f is the power to drive the bicycle forward.

From elementary physics, the rotational kinetic energy of an object is $\frac{1}{2}I\omega^2$, where I is the rotational moment of inertia and ω is the angular velocity of the object. For the front and rear wheels, we have

$$K_f = \tfrac{1}{2}I_f\omega^2, \qquad K_r = \tfrac{1}{2}I_r\omega^2,$$

where I_f and I_r are the rotational moments of inertia of the front and rear wheels. The total rotational energy K_T of the wheels is then

$$K_T = K_f + K_r.$$

The power due to the rotation of the object is the time derivative of the rotational energy:

$$P_w = \frac{dK_T}{dt} = \frac{d}{dt}\left(\tfrac{1}{2}I_f\omega^2 + \tfrac{1}{2}I_r\omega^2\right) = (I_f + I_r)\omega\omega'.$$

The angular velocity ω of an object equals its ground speed v_g divided by its radius R, while its angular acceleration ω' is a/R.

Substituting these into the above equation yields

$$P_w = (I_f + I_r)\frac{v_g a}{R^2}.$$

If we solve for the power P_a that pushes the bike forward, then divide both sides of **(1)** by v_g, we obtain the applied force F_A that pushes the bicycle forward:

$$P_f = P_a - P_w, \qquad F_A = \frac{P_a}{v_g} - (I_f + I_r)\frac{a}{R^2}.$$

The Drag Forces on the Bicycle

The drag force acting on an object moving through a fluid is

$$F = \tfrac{1}{2}C\rho AV^2,$$

where ρ is the density of the fluid (air), A is the cross-sectional area of the object, V is its velocity relative to the fluid, and C is the coefficient of drag that must be determined experimentally.

Air density, which can have a significant effect on drag forces, depends on temperature, pressure, and humidity. Pressure depends on altitude and weather. Most baseball fans can attest to the significant affects of air density on drag forces: Baseballs carry much farther in Coors Field in Denver, Colorado, because the high altitude leads to a low pressure, which means that the air density is less as well.

We consider all of these factors that affect air density in our model, via calculations in the **Appendix** [EDITOR'S NOTE: We omit the appendix.]. The most important aspect is that the air density is a function of the bike's elevation, S_z. The bike's acceleration d^2S/dt^2 depends on the drag forces, which depend on air density, which depends on the bike's position. This means that the differential equation that we develop will be second-order, because the second derivative of position depends on the position.

The Movement of the Air and Bicycle

The air through which the bicycle moves is not stagnant—wind blowing over the course has a significant effect. We represent the wind as a vector field $\vec{V}_A$ with a magnitude and direction that are uniform over the racecourse and constant for the duration of the race.

The cyclist's speed over the ground is v_g; thus, the velocity is $v_g\vec{u}$, where $\vec{u}$ is a unit vector in the bicycle's direction of movement. We now consider the air's velocity relative to the bicycle instead of the bicycle's velocity relative to the air, because this is an easier way of thinking about the problem and the magnitudes of these two velocities are equal. Since the bike's velocity over the ground is $v_g\vec{u}$, the air's velocity relative to the bike due to the bike's motion is $-v_g\vec{u}$.

The total velocity of the air moving relative to the bike is a function of two motions: the air's movement relative to the ground in the form of wind, $\vec{V}_A$, and the air's movement relative to the bike due to the bike's movement over the ground, $-v_g\hat{u}$, where we use the hat over u to denote that u is a unit vector. The total speed is then the magnitude of the sum of these two velocities (**Figure 1**),

$$v_T = \|\vec{V}_A - v_g\hat{u}\|.$$

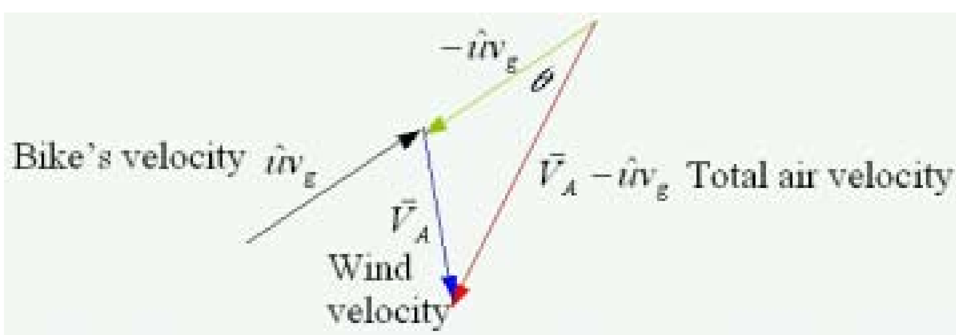

Figure 1. Components of total air velocity.

The yaw angle θ is the angle between the bicycle's axis of movement and the air direction, which we find from the dot product, $\vec{a}\cdot\vec{b} = \|\vec{a}\|\|\vec{b}\|\cos\theta$, where

θ is the angle between the vectors. We have

$$-v_g \hat{u} \cdot (\vec{V}_A - v_g \hat{u}) = v_g \|\vec{V}_A - v_g \hat{u}\| \cos \theta,$$

$$\theta = \arccos \left[\frac{-v_g \hat{u} \cdot (\vec{V}_A - v_g \hat{u}}{v_g \|\vec{V}_A - v_g \hat{u}\|} \right].$$

The bicycle's aerodynamic characteristics, and thus the drag forces, change with θ. Furthermore, because the bicycle does not always head into the wind, the overall drag force has components both normal and axial to the rider's path. We assume that the normal component is negligible and consider only the axial component (the component parallel to the cycle's axis of travel).

The Wheels

The axial drag force on the wheels largely depends on the yaw angle of the air moving past them and must be calculated experimentally for different types of wheels. For experimental results, we rely on Greenwell et al. [1995], who used a wind tunnel to determine the axial drag coefficient at different yaw angles for various commercially available wheels.

For each type of wheel, we plotted axial drag coefficient vs. yaw angle. From the plots, we constructed a polynomial regression of axial drag coefficients as a function of the yaw angle. Greenwell et al. considered the reference cross-sectional area of each wheel S_{ref} to be the total side cross-sectional area of the wheel,

$$S_{\text{ref}} = \pi R^2.$$

Thus, the effects of the cross-sectional area changing with yaw angle are included in the axial drag coefficient. Consequently, to use their results, we must use the same reference area. The axial drag force on the front wheel is then

$$F_f = K_W C_F v_T^2,$$

where $K_W = \frac{1}{2}\rho S_{\text{ref}}$ and C_F is the axial drag coefficient of the front wheel at yaw angle θ.

An interesting result of Greenwell et al. is that drag forces on the rear wheel are generally reduced by about 25% due to aerodynamic effects of the seat tube. This means that the axial drag force on the rear wheel is

$$F_R = (0.75) K_W C_R v_T^2,$$

where C_F is the axial drag coefficient of the rear wheel at yaw angle θ.

The Bicycle Frame and Rider

Unfortunately, we were not able to obtain data relating the drag coefficient of the rider and frame to the yaw angle, so we could consider only the effects of the component of the wind that is parallel to the bicycle's velocity.

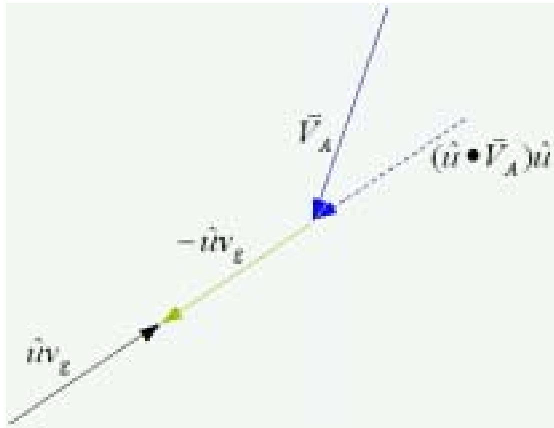

Figure 2. Drag force on the frame and rider.

In other words, for the drag force on the frame and rider, we consider only the vector projection of the wind onto the rider's velocity (**Figure 2**). The total air velocity is then the sum of this projection and the negative of the rider's velocity. We find the total drag force on the frame and rider to be

$$F_B = K_B \|(\hat{u} \cdot \vec{V}_A)\hat{u} - v_g \hat{u}\|^2,$$

where $K_B = \frac{1}{2} C_B \rho A$, A is the cross-sectional area of bicycle frame and rider, and C_B is the drag coefficient of bicycle frame and cyclist.

Force of Gravity

If the bicycle is on a hill, the component of gravitational force that is in the direction of the motion is

$$F_g = m_T g \sin \phi,$$

where m_T is the total mass of the bicycle and rider, g is the acceleration of gravity, and ϕ is the angle at the bottom of the hill. But since the road grade is $G = \sin \phi$, we have$F_g = m_T g G$.

Force of Rolling Resistance

Because the wheels have inflatable tires, the compression of the air within the tires causes a resistance to their rolling. This rolling resistance is a reaction to the rolling of the tires, which means that it will be 0 so long as the tires are not rotating but proportional to the total weight of the bicycle and rider when they are. Thus,

$$F_{rr} = \begin{cases} C_{rr} m_T g, & \text{if } v_g \neq 0; \\ 0, & \text{if } v_g = 0, \end{cases}$$

where C_{rr}, the coefficient of rolling resistance, is about 0.004 for most tires.

Summing the Forces

We sum the forces that act in the model:

$$F_A - F_B - F_f - F_r - F_g - F_{RR} = m_T a, \tag{2}$$

where

$F_A = \dfrac{P_a}{v_g} - (I_f + I_r)\dfrac{a}{R^2}$ (forward force that rider exerts),

$F_B = K_B \|(\hat{u} \cdot \vec{V}_A)\hat{u} - v_g \hat{u})\|^2$ (drag force on bike frame and rider),

$F_F = K_W C_F v_T^2$ (drag force on front wheel),

$F_R = 0.75 K_W C_R v_T^2$ (drag force on rear wheel),

$F_g = m_T g G$ (force of gravity),

$F_{rr} = C_{rr} m_T g$ if $v_g \neq 0$, 0 if $v_g = 0$ (force of rolling resistance).

The forward force of the rider depends on acceleration. Since we want to have all the acceleration terms together, we first group them:

$$\left(\frac{P_a}{v_g} - (I_f + I_r)\frac{a}{R^2}\right) - F_B - F_f - F_r - F_g - F_{RR} = m_T a,$$
$$\frac{P_a}{v_g} - F_B - F_f - F_r - F_g - F_{RR} = a\left(m_T + \frac{I_f + I_r}{R^2}\right).$$

Substituting the other forces into **(2)** , we obtain

$$\frac{P_a}{v_g} - K_B \|(\hat{u} \cdot \vec{V}_A)\hat{u} - v_g \hat{u}\|^2 - K_W C_F v_T^2 - 0.75 K_W C_R v_T^2 - m_T g G = \left(m_T + \frac{I_f + I_r}{R^2}\right) a.$$

Solving for a, we find the second-order differential equation

$$a = \frac{d^2 S}{dt^2} = \frac{\dfrac{P_a}{v_g} - K_B \|(\hat{u} \cdot \vec{V}_A)\hat{u} - v_g \hat{u}\|^2 - K_W C_F v_T^2 - 0.75 K_W C_R v_T^2 - m_T g G}{m_T + \dfrac{I_f + I_r}{R^2}}.$$

Completing the Model

This differential equation is too complicated to solve analytically, so we solve it numerically using a fourth-order Runge-Kutta (RK4) approximation method [Burden and Faires 1997]. This method is generally more accurate than other numerical approximation methods such as Euler's method, especially at points farther away from the start point.

To make the approximation, we first define acceleration based on time t, position (elevation S_z), and speed v_g:

$$a = a(t, S_z, v_g)$$

The RK4 method uses a weighted approximation of the acceleration at a given time t_k with speed v_{g_k} to determine the speed at some time t_{k+1} in the future, where $t_{k+1} = t_k + h$, with h the time-step size.

$$(v_g)_{k+1} = (v_g) + \frac{1}{6}\, h \left(W_{K1} + 2W_{K2} + 2W_{K3} + W_{K4} \right),$$

where

$$\begin{aligned} W_{K1} &= a(t_k, v_{g_k}), \\ W_{K2} &= a\left(t_k + \tfrac{h}{2}, v_k + \tfrac{hW_{K1}}{2}\right), \\ W_{K3} &= a\left(t_k + \tfrac{h}{2}, v_k + \tfrac{hW_{K2}}{2}\right), \\ W_{K4} &= a(t_k + h, v_k + hW_{K3}). \end{aligned}$$

As with any numerical approximation method, we must know the initial speed v_{g_0}. Then, with the velocity computed at a given time, we calculate the position of the bike at time t_{k+1} as

$$\vec{S}_{k+1} = \vec{S}_k + h(v_g)_k \hat{u}$$

from the initial position of the bike $\vec{S}_0$. Again, because we consider only axial forces acting on the bike and ignore turning, we can model only a bike moving in a straight line; this means that $\hat{u}$, the unit vector in the direction of the velocity, is constant.

Model Validation

To validate our model, we developed a computer program to simulate traversal of a course (see screen display in **Figure 3**). We inputted a map of the 2000 Olympic Games time-trial course [NBC Olympics 2000a]. The prevailing wind speed and direction for Sydney on 27 September 2000 (the day of the Olympic finals for the road race) was 10 m/s at 315° [Analytic Cycling 2001]. We assume that the winner would maintain an average power of 450 W.

Using our model, we calculated that cyclists would complete 15 laps of the 15.3 km course in a time between 5 h 18 min and 5 h 47 min (depending on which wheel is used). In Sydney that day, Jan Ullrich of Germany won the race in a time of 5 h 29 min. As this result falls within the range of predicted values, we believe that our model is relatively accurate.

However, to test truly the validity of the model, we would need to experiment with various riders and wheel types for which we know the drag

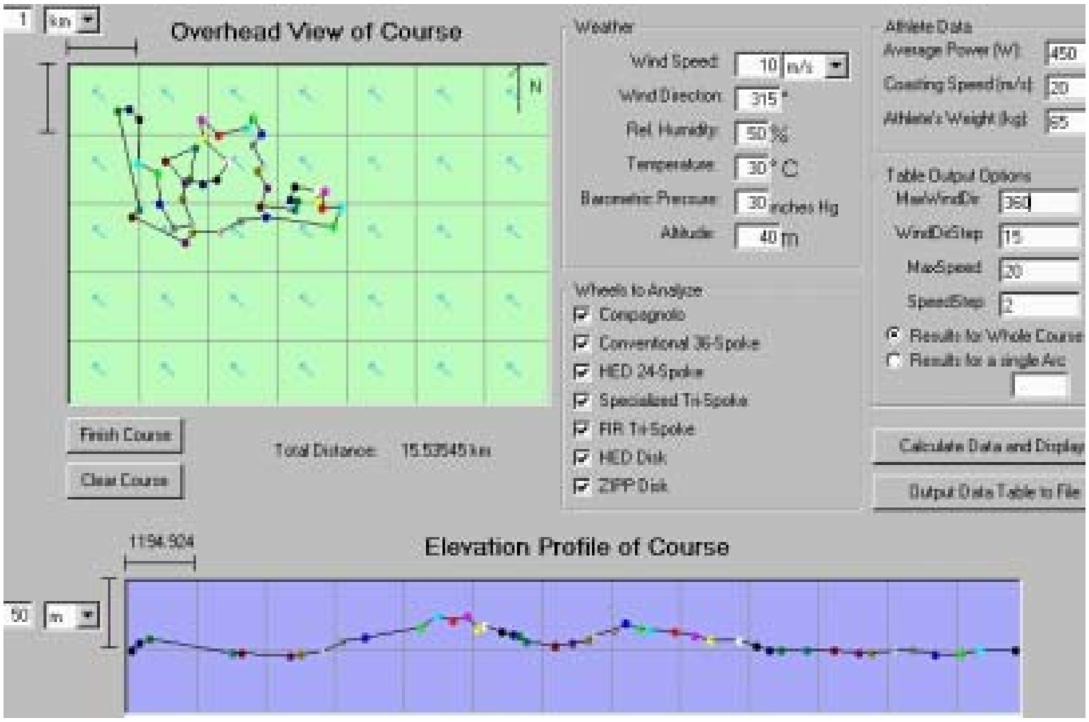

Figure 3. 2000 Olympic Games time-trial course.

coefficient as a function of yaw angle. The riders could use an exercise bike to determine their average power outputs. We would then have them traverse a course with different wheel types. After inputting the course in our model, we could compare the times that the model predicted with the experimental results.

Model Application

Table Creation

We construct tables for varying road grades and wind speeds. Since the direction of the wind has an appreciable effect on wheel performance, we create three tables: one for a headwind, one for a crosswind, and one for a tailwind. We applied our model to a hill 1.0 km long, beginning with a velocity of 10 m/s. The results in tabular form are in **Table 1**.

Table Analysis

Our model accounts for many additional factors other than wind speed and road grade; these contributions are lost if the results are pressed into table

Table 1.

Preferred wheel type for a hill 1.0 km long and starting speed 10 m/s.

Headwind

Preferred Wheel given Road Grade and Wind Speed

Road Grade	Wind Speed 0	4	8	12	16	20
0	Spoke	Either	Either	Disc	Disc	Disc
1	Spoke	Spoke	Disc	Disc	Disc	Disc
2	Spoke	Either	Disc	Disc	Disc	Disc
3	Spoke	Spoke	Either	Spoke	Either	Either
4	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
5	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
6	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
7	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
8	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
9	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
10	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke

Tailwind

Preferred Wheel given Road Grade and Wind Speed

Road Grade	Wind Speed 0	4	8	12	16	20
0	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
1	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
2	Spoke	Spoke	Spoke	Spoke	Spoke	Either
3	Spoke	Spoke	Spoke	Spoke	Spoke	Disc
4	Spoke	Spoke	Spoke	Spoke	Spoke	Either
5	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
6	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
7	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
8	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
9	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke
10	Spoke	Spoke	Spoke	Spoke	Spoke	Spoke

Crosswind

Preferred Wheel given Road Grade and Wind Speed

Road Grade	Wind Speed 0	4	8	12	16	20
0	Spoke	Disc	Disc	Disc	Disc	Disc
1	Spoke	Disc	Disc	Disc	Disc	Spoke
2	Spoke	Disc	Disc	Disc	Disc	Spoke
3	Spoke	Disc	Disc	Disc	Disc	Spoke
4	Spoke	Disc	Disc	Disc	Disc	Spoke
5	Spoke	Disc	Disc	Disc	Disc	Spoke
6	Spoke	Disc	Disc	Disc	Disc	Spoke
7	Spoke	Disc	Disc	Disc	Disc	Spoke
8	Spoke	Disc	Disc	Disc	Disc	Spoke
9	Spoke	Disc	Disc	Disc	Disc	Spoke
10	Spoke	Disc	Disc	Disc	Either	Spoke

form. Additionally, cycling races generally do not consist of one large hill of a uniform grade; there are typically many turns, hills, and valleys.

As a result, we recommend against using the tables that we provide! Our software implementation of the model allows the entry of *all* factors relating to the course that affect wheel choice.

Typically, course layout and elevation profile are available well in advance of a cycling race. We recommend that users of our software input the course and run multiple scenarios based on varying wind speeds and directions.

Results and Conclusions

We analyzed 7 different wheels: 5 spoked (Compagnolo, Conventional 36-Spoke, HED 24-Spoke, Specialized Tri-Spoke, and FIR Tri-Spoke) and 2 disc (HED Disc and ZIPP Disc). We chose these wheels because we could find data relating the drag coefficients to the yaw angle of the air. After running our model over varying courses and conditions, we came to some interesting conclusions, which mesh well with what intuition would suggest.

Crosswinds

In crosswinds, disc wheels dramatically outperform spoked wheels, since the drag coefficients for disc wheels decreases sharply as the yaw angle increases. As yaw angle increases to around 20°, drag coefficients for disc wheels drop dramatically. For the HED Disc, the drag coefficients actually become negative at larger yaw angles, indicating that the wheel acts like a sail and helps propel the cycle forward instead of slowing it down! The ZIPP Disc drag coefficients do not become negative but drop very close to zero at larger yaw angles. Consequently, the difference in speed between discs and spokes in a crosswind is significant. If a course has a strong crosswind (greater than 20 mph), then a disc wheel can make time differences on the order of 20% or more. However, *even in a light crosswind, the two disc wheels that we analyzed outperformed every spoked wheel*.

Direct Head and Tail Winds

In direct head and tail winds, spoked wheels slightly outperform disc wheels, because both types have similar drag coefficients at small yaw angles and spoked wheels are lighter.

Spoked wheels generally outperform disc wheels when going uphill in the absence of wind, because they are lighter, whereas disk wheels outperform spoked wheels when going downhill, because they are heavier. When there is a head wind or a tail wind when going up a hill, spokes generally still outperform discs; however, the introduction of any cross wind with a yaw angle much greater than 15° or 20° causes the smaller disc drag coefficients to outweigh the mass differences. This means that disc wheels are more efficient than spoked wheels.

Long Races

In long races, disc wheels generally outperform spoked wheels, because the smaller drag forces on the disc have a more significant effect on overall performance. Generally, in a race of any length much over 5 km, discs are more efficient, because the wind acts on them for longer amounts of time than in short races, which means that aerodynamic characteristics are more important than mass differences.

Short Races

In short races, where acceleration is more important, the spoked wheels, with their smaller masses and moments of inertia, outperform the disc wheels.

Strengths of the Model

Robustness. We derive our model from basic physical relationships and limit our use of assumptions. In every case where an assumption is required, we substantiate it with evidence and reasoning that illustrates why the assumption is valid.

Grounded in theory and research. We constructed our model based on both theory (Newton's second law) and research (Greenwell et al. [1995]).

Ease of use. Although the physical concepts behind our model are relatively simple to understand, the mathematical derivations and calculations are difficult to perform. We created a user-friendly computer program with a graphical interface that allows anyone with a basic knowledge of computers to input the required data. The program performs the mathematical calculations and reports the preferred wheel choice, along with calculated stage times for the race.

Weaknesses of the Model

Representing the course. We represent the course as a sequence of points in three-dimensional space, with the path from point to point represented by a straight line. This is not an accurate representation of any course; the traversal of hills, valleys, and curves all create nonlinear movement. Accuracy could be increased through a detailed entering of the course with a larger number of location nodes.

Approximation of wheel drag data. Greenwell et al. [1995] reported data for yaw angle 0° through 60° only in 7.5° increments. We developed interpolating polynomials through this range. We would conduct further tests at various yaw angles.

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