Author-Judge's Commentary: The Outstanding Bicycle Wheel Papers

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Introduction: The Problem

Professional bicycle racers have a wide variety of wheel types available to them. The types of wheels range from the familiar spoked wheels, to wheels with three or four blades, to solid wheels. The spoked wheels have the lowest mass but have the highest friction forces due to interactions with air. The solid wheels have the most mass but have the lowest friction forces. The question posed was to demonstrate a method to determine what kind of wheel to use for a given race course.

The problem focused on the two most basic types of wheels, the spoked wheel and the solid wheel. Three tasks were given:

- Find the wind speeds for which one wheel has an advantage over the other for particular inclines.
- Demonstrate how to use the information in the first task to determine which wheel to use for a specific course.
- Evaluate whether the information provided in the first task achieved the overall goal.

The UMAP Journal 22 (3) (2000) 253–256.

General Remarks on the Solution Papers

As is the case each year, many fine papers were submitted. The papers were judged on both their technical merit and on their presentation. The submissions in which both aspects were superior received the most attention. The problem required that many assumptions had to be made; because of the severe time restrictions, it was extremely important to be able to choose assumptions that did not make the problem too simple but still relevant.

For example, a number of submissions concentrated on the *yaw angle*, the angle that the wind makes with respect to the direction of movement of the bicycle. While some of these submissions were quite good, it appeared that others spent so much time trying to figure out how to deal with this complicated aspect that sufficient progress was not made with respect to the other parts of the problem. Moreover, it was often difficult to read and interpret the resulting descriptions of the teams' efforts.

While the assumptions were important, it was also important in developing a mathematical model for this problem to stay consistent with the basic definitions of mechanics. There were a number of entries in which Newton's Second Law, the torque equations, or the power was not correctly identified. There was also some confusion about units. Such difficulties represented a key division between the lower and higher rankings.

Approaches

Overall, there were two different approaches:

- The first approach focused on the mechanics of a bicycle moving on an incline. The forces acting on the bicycle and rider were used to find the equations of motion from Newton's Second Law and the torque equations. The equations could then be used to isolate the force acting to move the bicycle forward.

  > The main difficulty with this approach was in isolating and identifying the relevant force based on the equations from Newton's Second Law and the corresponding torque equations. In many cases, it was difficult to identify exactly how the system of equations was manipulated and how the equations were found. The submissions in this category that were highly rated did an excellent job of displaying and referring to the free-body diagrams, as well as discussing how the relevant force was isolated by manipulating the system of equations.

- The second approach focussed on the aerodynamic forces acting on the wheels, then calculated the power to move the wheels forward. For the spoked wheel, the total force acting on the wheel was found by adding the effects on each spoke (along its entire length) with its respective orientations. For the solid wheel, the forces acting on the whole wheel were found with respect to the wind yaw angle.

> This second approach turned out to be a difficult one. In some cases, it was hard for the judges to identify the approach and what assumptions were being made. The submissions in this category were also more likely to concentrate on the yaw angle and its associated complications. The teams that carefully structured their approach and clearly identified each step stood out.

For either approach, there were different assumptions that could be made about the motion of the bicycle and rider. The most common approach was to make some assumption about either the acceleration or the steady-state velocity as the bike and rider moved along the hill. The second most common assumption was to assume that the rider provided a constant power output and then work backwards to isolate the forces acting on the wheels. For the most part, the judges did not question the technical merits of these kinds of assumptions. The judges concentrated instead on whether or not the submissions presented a clear and consistent case based on the given assumptions.

Fulfilment of the Tasks

There were many fine entries in which the first task (provide a table) was addressed. The first task was the most specific and straightforward part of the problem. The factor that set the entries apart was in how the two remaining tasks (use the table in a time trial, determine if the table is adequate) were addressed and presented. The majority of submissions discussed the second task by dividing the race course into discrete pieces; the total power could then be found by adding up the power requirements over each piece. This part of the submissions often seemed to have received the least amount of attention by the different teams and was often the hardest part to read and interpret.

The analysis and qualitative comparisons within each submission were crucial in determining how a team's efforts were ranked. Many teams provided an adequate formulation for the first task in the problem but addressed the other tasks in a superficial manner. The real opportunity to express a deeper understanding of the problem and show some creativity lay in how the remaining aspects of the problem were approached.

The entries that most impressed the judges went further in their analysis. In particular, a small number of entries approached the third task by noting that the real goal was to minimize the time spent on a particular race course. By assuming that the rider would expend a constant power output, the equations of motions from Newton's Second Law could then be found. The position of the rider on the course at any given time could then be approximated through a numerical integration of the resulting system of equations. For a given racecourse, the total time on the course could be found for different wheel configurations. A simple comparison of total times determined which wheel to use for the course.

The submissions that went beyond the stated problem and stayed true to the original goal received the most attention from the judges. Such papers showed creative and original thought, and they truly stood apart from the rest. Moreover, they showed the deepest understanding of the task at hand.

About the Author

Kelly Black is visiting Utah State University and is on sabbatical leave from the University of New Hampshire. He received his undergraduate degree in Mathematics and Computer Science from Rose-Hulman Institute of Technology and his Master's and Ph.D. from the Applied Mathematics program at Brown University. His research is in scientific computing and has interests in computational fluid dynamics, laser simulations, and mathematical biology.