Make Your Way Faster!

Introduction

We create a double queues system, which including both the QuickPass line and the regular line. These factors have an extremely affection on guest grievance: the returning time for QuickPassers, the standby waiting time in line for QuickPassers, and the proportion of the returning time for QuickPassers to the waiting time for the standers in regular line. To increase people's enjoyment of the amusement park, when we constructing our model, we run the justice principle through. That is "first come, first served". In order to minimize the difference of returning time stated in close QuickPass coupons, the length of QuickPass line can be too long.

We divide all the time that the attractions open into several time intervals according to a certain granularity. Besides, we restrict the maximum number of QuickPass coupons issued in each time interval. After taking these measures, the customer flow distributes evenly instead of random distribution. Then we do several simulating experiments, the results confirm that QuickPass coupons restriction do shorten the maximum length of QuickPass line and avoid the disequilibrium display of the QuickPass Clock.

At last, we conclude criteria to evaluate alternative schemes from the analysis of the model and simulating experiment: a good model should make the customer flow distributes as evenly as possible.

Interpretation of the Problem

In a traditional amusement park, you may observe that there is an extremely long line in front of a popular ride. Everyone is waiting in the same queue. It seems that nothing is abnormal. However, you are prone to be tired and impatient. To increase people's enjoyment of the amusement instead of useless wait, the modern amusement parks offer a new way of approaching your favorite rides: "QuickPass".

"QuickPass" is an innovative system designed to reduce the time customers wait in line in order to give them more time to enjoy the amusement park they are visiting. It is an advanced reservation system giving you a slip to a popular ride during a specific frame after inserting your daily park entrance ticket in a kiosk (we call it a booth) near that ride. When you come back during the interval which is given by the "QuickPass" system, you can have a priority to those customers who standing in a regular line.

Terms and Definitions

Table 1

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T_S	the time a customer insert his/her park entrance ticket in the “QuickPass” machine
T_{now}	the current time
$\mu_{regular}$	the proportion of customers waiting in regular line to ride the attraction per time
$\mu_{QuickPass}$	the proportion of customers waiting in QuickPass line to ride the attraction per time
T_{game}	how long the ride lasts
N_{game}	the number of customers to the ride per time
$N_{granularity}$	the granularity of two closest QuickPasses
$N_{regular}$	the number of customers who are waiting in the regular line
$N_{QuickPass}$	the number of customers who have a valid QuickPass slip and haven’t used it at the current time
N_i	the number of the QuickPass coupons issued in each time interval
$T_{interval}$	Time-window
N_{period}	the maximum number of QuickPass coupons issued in every time interval

Assumptions

- The formulae “first come, first served” is the basis of “QuickPass” system.
- Customers must arrive at the attraction during the time interval which is given by the system, otherwise, the “QuickPass” coupon will be prohibited.
- Customers can only obtain one “QuickPass” coupon of an attraction. Until two hours after they insert their entrance park ticket, can they get another.
- No customers give up once they are waiting in the regular line.
- Each customer spends the same certain time on the same attraction.
- The popular rides operate well all the time, and no accidents happen.
- During the peak time of riding the attraction, maximum customers ride it at a time.
- All the customers who have obtained the “QuickPass” coupon reach the attraction during the time frame stated by the system.

Analysis of the problem

To attract more customers, amusement parks invest a large amount of money and human

resource to build some exciting and attractive rides. Presumably, the amusement parks reach their achievement, but other problems arise: the rides are so amused that every customer comes to the amusement just for them. In the end, maybe most of them fulfill their dreams, but what they spend is not only money, but also a long long time waiting in line. Guest grievance flows to the amusement executives' office. How to deal with it then? "QuickPass" appears! The greatest advantage of "QuickPass" is staggering the customer flow. And "QuickPass" is not a packed service, but an extra choice the amusement park offers.

The vast spread of "QuickPass" leads to a revolution in the way customers visiting the park. Which is more important, it do increase people's enjoyment to some extent. However, you can't enjoy all the advantages at one time. The "QuickPass" can offer customers an extra time to enjoy other rides; meanwhile, the indeterminacy of the real return time also increases the probability of endless waiting for customers with "QuickPass" coupons. Besides, the exact returning time is changeable, which breaks our basic principle that "the one who come first is served first" to some extent.

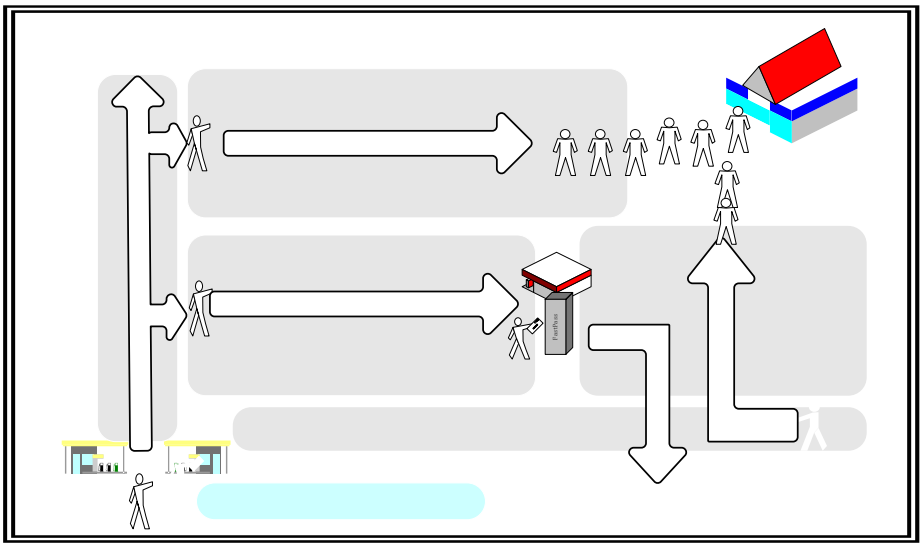
Does "QuickPass" always make it fast?

In most instances, "QuickPass" can shrink guests' waiting time effectively. However, due to some weaknesses of the system itself, within a short time, the system may give customers quite different returning time. In our opinions, the reason for a long "QuickPass" standby waiting line is the long returning time interval. In practice, if you insert your park entrance at 1:15pm, and the "QuickPass" states that you come back between 3:30 to 4:30pm. You can come back at either time during this period. What is the truth? Even if you return after 4:30pm, the system will accept your "QuickPass" slip as well. Then this trouble follows: assuming that the returning time frame of customer A is between 3:30 to 4:30pm, and customer B's is between 4:00 and 5:00pm, and customer C's is between 4:30 to 5:30pm. This instance exists: at 5:00pm, A, B, and C all come back and join in the "QuickPass" line. It wouldn't be surprising that the "QuickPass" line is as long and slow as the regular line.

Lines before the popular rides

When you want to ride an attraction, there are two lines in front of you to choose, the "QuickPass" line and the regular line. Which to choose is just as you like. If "QuickPass" is your final choice, you should insert your entrance ticket in the "QuickPass" machine near that ride, and out will come a slip that states when you can return to that ride. When you come back after a specific time, you can join the "QuickPass" line and head straight to the ride. If you choose the latter, you will face a long wait. Observe the sketches in figure 1.

Figure 1.sketch map of an attraction



The customer flow

Our tasks

Customers have been complaining about some anomalies in the test system, which may decrease their enjoyment of the amusement park. For example, when you return to the attraction, after three hours according to the “QuickPass” slip, you observe that the “QuickPass” line moves not so fast as the advertisement states, or even nearly as long and slow as the regular line. Certainly you will lose your temper and write a complaining letter to the amusement park executives. We find there are several weaknesses in the “QuickPass” system, and we are trying our best to improve the system in order to satisfy the customers. If necessary, we will probably change the system completely. Our tasks are:

- Minimize the longest returning time that the “QuickPass” system gives.
- Make the “QuickPass” lines move as fast as possible
- Make the “QuickPass” line as short as possible when the customers with “QuickPass” slips come back to join the line.
- Determine what criteria to use in evaluating alternative schemes.
- Propose other schemes which can increase people’s enjoyment of the amusement park basing on our original model.

Constructing the Model

We construct a model including both QuickPass line and regular line in order to explain the problem better. In our model, there are two digital clocks for each line. Customer can observe a returning interval in the QuickPass booth. For example, you arrive at the attraction at 12:00pm.You may see the returning time interval displayed in the QuickPass Clock is between 3:10 to 4:10pm, while the waiting time stated by the Regular Clock is about one hour. The longer the waiting time for regular standers is, the more customers complain. So do the returning time for customers with QuickPasses. In conclusion, to increase people’s enjoyment of the amusement park,

our essential task is to find an appropriate way to calculate the waiting time. We are modeling with factors which presumably lead to guest grievance.

The Return Time of QuickPassers

The QuickPass system rather is based on boarding times. In practice, the QuickPass lines are always congested by massive customers, as they obtain the same returning interval. On the contrary, another returning interval is received by only few customers. As a result, the customers distribute in QuickPass line unevenly. Sometimes, guests crowd in the QuickPass line which is as long as the regular line. Sometimes there are few people in it. Block of return times are “released” in five minute batches, which insures a steady flow of people to the QuickPass return line. The biggest variable in the system that will affect you is how long the regular line for the ride is at the time you get your QuickPass. For example, if you arrive at an attraction’s QuickPass distribution area at some time between 10:25 and 10:30 AM and the wait to ride in the normal line is about 30 minutes, the your QuickPass will most likely be issued for between 11:00 and 11:30 AM or pretty close to that time window. Likewise, if you arrive sometime between 1:30 and 1:35 PM, the QuickPass you get then will say it can be used between 2:05 and 3:05 PM. etc.

Divide all the time the attraction runs a day into a certain number of time window in granularity of $N_{granularity}$. Assuming that an attraction opens from 9:00 to 21:00, the whole time will be divided into n time interval,

$$n = \frac{(21o'clock - 9o'clock) \times 60\min}{N_{granularity}}$$

Therefore, the time system issuing QuickPasses is divided in granularity of 5 minutes. In this case, you may observe these time intervals in your QuickPass slip: 9:00~10:00,9:05~10:05, 9:10~10:10,$\cdots$,14:35~15:35,$\cdots$. And we determine the time interval to be one hour. The maximum number of QuickPass coupons issued in every time interval is N_{period}, and $N_i, i \in (1,n)$ is the number of the QuickPass coupons issued in each time interval. We can get $N_i \le N_{period}$.

The waiting time displayed to the standers in regular line consists of two parts: the waiting time of the customers in regular line and the standby waiting time of the ones in QuickPass line. To make both lines move faster, the system admits a certain proportion of two lines to the attraction per time. The waiting time for standers in regular line is the product of the number in regular line per time interval (one hour) and the average time customers spend in riding.

$$Waiting\ Time\ in\ regular\ line = N_{regular} \cdot \frac{T_{game}}{N_{game}}$$

We couldn’t know the exact time the customers with QuickPasses return, for not all the customers return on time. So we give an expecting time for the customers with QuickPasses,

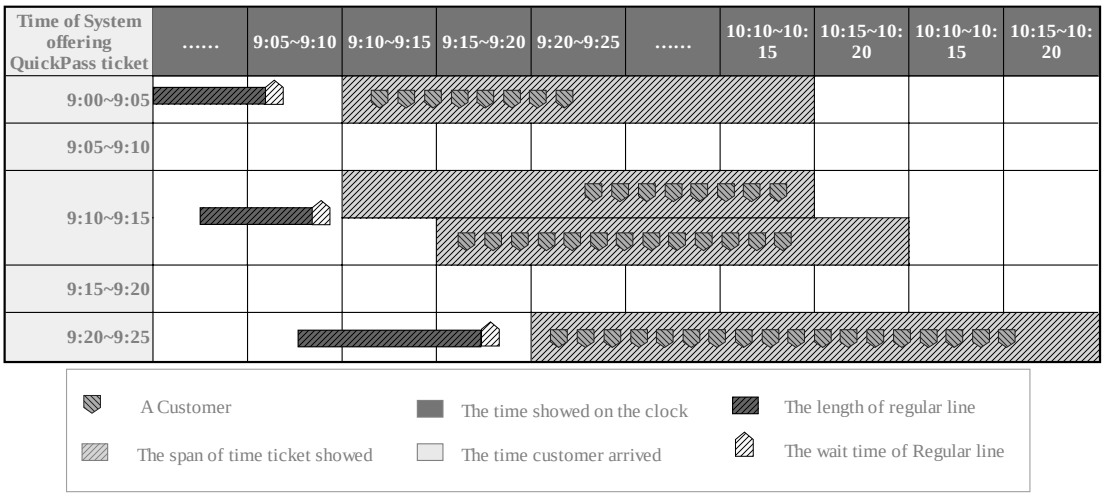
$$Expecting\ Waiting\ Time\ \ in\ \ QuickPass\ line = \frac{N_{QuickPass}}{\mu_{QuickPass}} \cdot \frac{T_{game}}{N_{game}}$$

In the equation, $N_{QuickPass}$ is the number of customers who have a valid QuickPass slip and haven't used it at the current time. At the time T_{now}, the waiting time for standers in regular line is T_{wait},

$$T_{wait} = (N_{regular} + \frac{N_{QuickPass}}{\mu_{QuickPass}}) \cdot \frac{T_{game}}{N_{game}} \quad (0.1)$$

QuickPass system follows this regulation when issuing the QuickPass slips: issuing the next interval slips until the current interval slips have be issued out. As shows in table 1, we explain an example to you. When Customer A reach the QuickPass booth between 9:00 to 9:05, and see the returning time interval displayed in the QuickPass Clock is between 9:10 to 10:10. You can get it from the table that the waiting time for the last one(Customer B) in regular line is about 9:07. Customer A arrives later than Customer B(when A arrives, B has already waiting in the regular line). According to our basic rule, A should be served later, and that is why he is arranged to return between 9:10 to 10:10. Then customer C arrives, maybe between 9:10 to 9:15. At that time, the coupons whose statement on returning window is 9:10~10:10 are not out, so C can receive the slip admitting him to come during that time. Now, the time is between 9:20 to 9:25, and the Regular Clock displays 9:17, the system will issue the slips in which the returning time interval is 9:20~10:20(closest time interval after 9:17).

Table 1.Returning time for QuickPassers



That is to say, T_{return} is a recursion variable:

$$T_{return}(i) = \begin{array}{l} if\ \ the\ slips\ of\ \ last\ time\ interval\ was\ sold\ out \\ \quad \max\left(T_{return}(i-1), \lceil T_{wait} \rceil_{gran}\right) \\ else \\ \quad \max\left(T_{return}(i-1), \lceil T_{wait} \rceil_{gran}\right)+1 \end{array} \tag{0.2}$$

In fact, not all the customers come back on time. As a result of the difference of the real returning time and the expecting returning time, the time displayed on both digital clocks is the longest waiting.

Longest Queue in QuickPass Line

After a long relaxing time visiting the other parts of the amusement park, customers come back with their QuickPass coupons. They can't wait another moment to ride their attraction, however, what they are facing is still an extremely long line. How disappointed they are! To make them satisfied, we try our best to restrict the length of QuickPass line.

The QuickPass system issues coupons in the granularity of $T_{ganularity}$, customers can reach the attraction at any time during the interval $T_{interval}$. We conclude that there are m kinds of QuickPass slips with a same time intersection.

$$m = \left\lceil \frac{T_{interval}}{T_{ganularity}} \right\rceil,$$

The longest waiting time for the customers in QuickPass is $T_{longest}$,

$$T_{longest} = \frac{N_{period} \cdot m \cdot T_{game}}{N_{game} \cdot \mu_{QuickPass}} \tag{0.3}$$

Model Simulation

Mentioned above, we have built a model including both QuickPass lines and the regular lines. We did some validation of the model for stability, sensitivity, and realism. In this part, we create an imitated environment to simulate how the customer crowd affects the satisfaction of the waiting customers.

To make our simulating environment as exact as the real world's, we propose these restrictions:

- The criteria whether to obtain a QuickPass slip or directly wait in the regular line for customers is the proportion of the returning time(the starting time of the returning time interval) for QuickPassers to the waiting time for regular standers. If the proportion is less than 3:1, we believe that customers will choose the QuickPass instead of waiting in

regular line. For example, when you approach an attraction, you observe that it will cost you one hour to wait in the regular line. If the returning time for QuickPassers is 2.55 (less than 3 hours)hours later, customers will choose the QuickPass system. If the returning time displayed in the clock is 3.05(more than 3 hours), they will choose waiting right now.

- The QuickPass service is not offered from the beginning to the end when the attractions open. QuickPass system runs only during the customers' peak period As the customers flow decrease to a certain extent, it will be stopped. For example, an attraction opens from 9:00am to 9:00pm, the operation time of the QuickPass system may be from 11:00am to 4:30pm.
- Taking the principle" customers comes first" into account, even if the QuickPass coupon is expired, the customers with it are still allowed to use it when they come back to the attraction.
- Some customers maybe not return to ride the attraction on time as the system required, then the system will clear up these "dead bugs" at intervals.

Numerical Simulation Experiment

In the simulating environment, we do several numerical simulation experiments. Assuming that the customers flow obey the normal school regulation. All the data are displayed in table 2.

Table 2. Initialize data

$\mu_{regular}$	0.2	$\mu_{QuickPass}$	0.8
T_{game}	5 min	N_{game}	60 people
N_{period}	6	$N_{granularity}$	5min
$T_{interval}$	1 hour	T_S	$Norm(\frac{5}{2},\frac{5}{6})$

From the experiments, we get figures of the length of QuickPass line and the length of Regular line(figure 2,figure 3).

Figure 2. The length of regular line

Figure 3. The length of QuickPass line

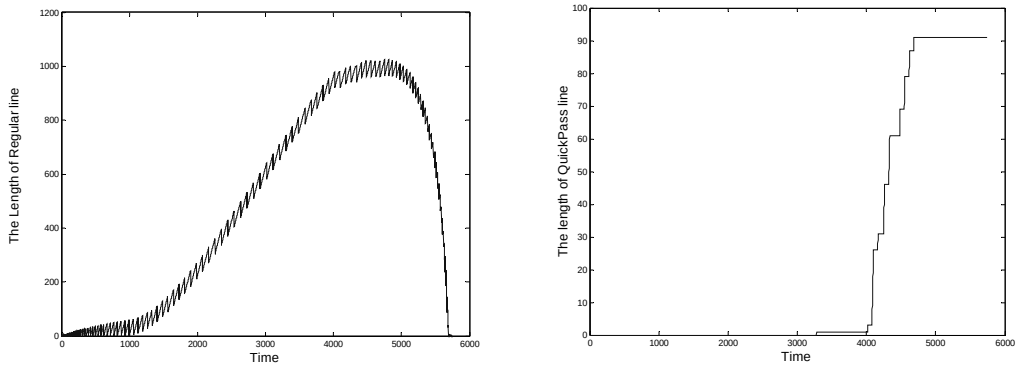
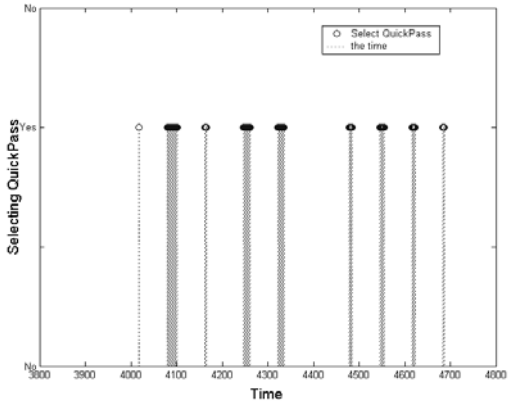


Figure 4 shows the decisions which way customers choose when QuickPass is available. It indicates that not all the customers choose the QuickPass. They choose the way that fits them .

Figure 4. Decisions of choosing QuickPass



Strength and Weakness

Strength

- Our model incarnates the justice principle farthest. The one who come first is served first.
- By limiting the length of returning time interval, restrict the length of the QuickPass line and then shrink the standby waiting time effectively.
- QuickPass system operates basing on the limitation of the QuickPass slips issued in a time interval. Under this circumstance, the standby waiting line of QuickPassers will not be too long, meanwhile, a shorter time window will avoid a great difference of two returning time which is received within a short time.
- When the QuickPass system calculates the returning time, it calculates the expecting time of people in QuickPass line in a proportion. In this way, the system can reduce the error caused by those people with a QuickPass slip who never return to ride.

Weakness

- We don't take those people who have been waiting in the regular line but give up midway into account. However this situation possibly happens, and our ignoring of those people may lead an error.
- In our model, customers should return on time. Otherwise, the QuickPass coupon will be void. This restrict to some extent may lead to guest grievance.

Reference

Mouseplanet/Disneyland Information Guide: What is Fastpass?
http://www.mouseplanet.com/al/docs/fast.htm Accessed 7 February 2004
Mouseplant/ Walt Disney World Trip Planning Guide: All About Fastpass
http://www.mouseplanet.com/dtp/wdwguide/6_Parks/general/fastpass.htm Accessed 7 February 2004
Peter van Lith:Queue Management
AllEarsNet/Turingplans: Fastpass
http://allearsnet.com/tp/fastpass.htm Accessed 8 February 2004
DisneyDreams.net: Fastpass
http://www.disneydreams.net/FASTPASS.asp Accessed 8 February 2004
Disneyzone.net: Fast Pass Information
http://www.disneyzone.net/fastpass.htm Accessed 8 February 2004

Memorandum

As the QuickPass spreads apace, more and more customers choose this new way to reserve their favorite rides. Massive people obtain the QuickPass coupons, however, when they return as the coupon states, they still have to wait for an unacceptable time. Besides, some customers complain that the interval between the time they insert their ticket and the returning time(the starting time of the time window) is too long. To solve these problems, we give several suggestions below:

Our QuickPass system operation is easy. It works like this: when customers approach to the attraction, our system will offer two waiting time. One is the waiting time for customers in regular line, the other is a returning time interval(time window) for the ones who receive the QuickPass service. Customers can have alternative schemes to enjoy the attraction. Join the long waiting line right now or visit other rides first and then join in a shorter line? It depends on the customers.

The advantages of our model are:

Firstly, the system offers two ways to the customers who want to approach a popular ride. They can make a decision after a comparison of the two time offer by the system. The system can be improved by controlling the proportion of the waiting time for customers in the regular line to the returning time for QuickPassers.

Secondly, people can visit other rides before the returning time. In a traditional amusement park, waiting is the main activity of theme park visitors, in order to get to the rides. During a typical day, a customer will spend roughly one hour in attractions and about two hours walking about and in restaurants. The remainder is spent in queues or other unattractive locations. So more than 60% of the customer's time is lost in unattractive and boring activities.

The features of our model are:

- Reduce customers' waiting time as much as possible with the measure of restricting the QuickPass coupons.
- Minimize the difference of returning time stated in close QuickPass coupons
- Shorten the returning time window for QuickPassers.