Positioning and Moving Sprinkler Systems for Irrigation

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1 Introduction

The problem of optimizing a schedule for a moving sprinkler system is broken into several parts. First, a pipe flow model is developed to predict the sprinkler nozzle velocity and flow rate; it is consistent with commercially available sprinklers. Second, a kinematic model for water droplet trajectories is proposed with a quadratic wind-resistance term. The solution technique is matches contemporary software, and the distribution of water is close to experimental sprinkler studies. Finally, a graph or network model for scheduling is suggested with a well-known solution algorithm.

2 Pipe flow model

Assuming the flow is Bernoulli (inviscid, irrotational, incompressible), the quantity $B = \frac{p}{\rho} + \frac{u^2}{2}$ is conserved from the flow inside the pipe to the flow through the sprinkler nozzle.

$$B_{pipe} = \frac{p_{pipe}}{\rho} + \frac{u_{pipe}^2}{2} = \frac{p_{atm}}{\rho} + \frac{u_{nozzle}^2}{2} = B_{nozzle} \tag{1}$$

The pressure in the pipe is 420 kPa; the flow rate in the pipe is 150 liters/minute, so the average velocity (u = flow rate/cross section area) is .1013 meters/second.

With the problem parameters as specified, we find the velocity through the nozzle to be $u_{nozzle} = 25.3\frac{m}{s}$. At this velocity, 42.9 liters/minute flow through the sprinkler nozzle.

It is reasonable to expect some frictional dissipation because of the sharp angles of pipe fittings, so this may be viewed as an upper bound for velocity and flow rate.

To determine if this is sensible, we refer to the specifications of current sprinkler heads. For large applications (farms and fields), this flow rate is within the standard range.

Note that the conservation of Bernoulli function is independent of nozzle size, which necessitates a mass conservation constraint. Since the fluid is constant in density, in-flow must be balanced by out-flow: $Q_{in} = Q_{out}$. The flow in is 150 liters/minute, so 3.5 sprinkler heads are needed.

3 Spray coverage model

To model the area that the sprinkler's spray covers, we use kinematic equations with a quadratic air drag term. The initial velocity is set by the nozzle velocity, v_0, discussed in the previous section.

$$\frac{d^2y}{dt^2} = g - \frac{1}{2m} C_D A \rho_{air} \left(\frac{dy}{dt}\right)^2 ; \quad \left.\frac{dy}{dt}\right|_{t=0} = v_0 sin(\theta) \tag{2}$$

$$\frac{d^2x}{dt^2} = -\frac{1}{2m} C_D A \rho_{air} \left(\frac{dx}{dt}\right)^2 ; \quad \left.\frac{dx}{dt}\right|_{t=0} = v_0 cos(\theta) \tag{3}$$

where g is the gravitational constant, m is the droplet mass, C_D is the drag coefficient, A is the cross-section area, and ρ_{air} is the air density. The droplets are assumed to be spheres, so $m = \frac{4}{3}\pi r^3 \rho_{water}$ and $A = \pi r^2$. The values used for air and water density are adopted from Kundu and Cohen ([2]).

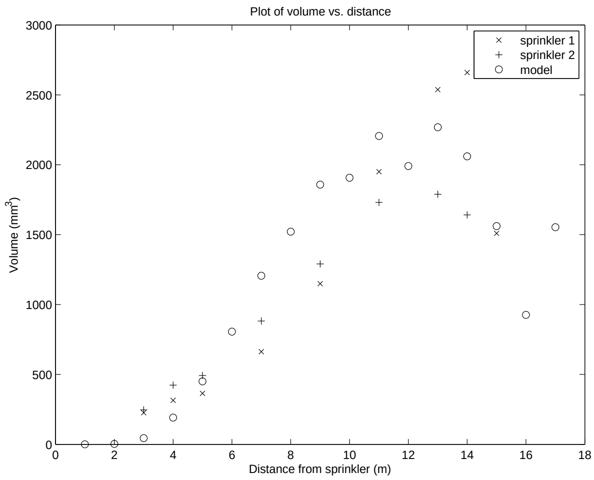
The drag coefficient for spheres has been empirically tested over a large range of Reynolds numbers ($Re = \frac{ul}{\nu}$). The range of Reynolds numbers for this problem is $Re \approx 10^3 - 10^5$. Within this range, the drag coefficient of a sphere is approximately a constant value of $C_D = 0.6$ ([2]).

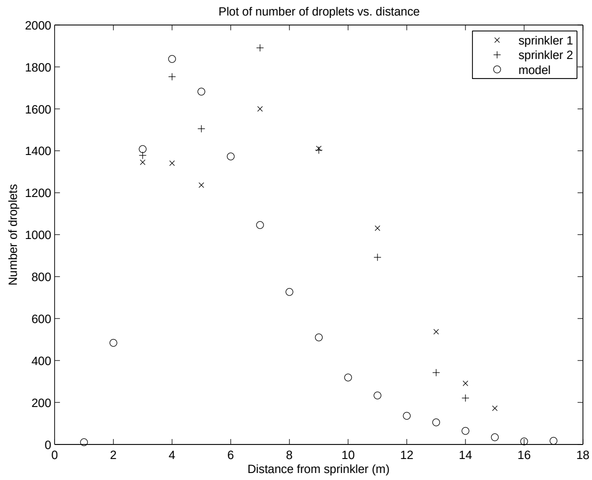
The kinematic equations are integrated using MATLAB's ode45, an explicit Runge-Kutta(4,5) formula. This is the same method used in a contemporary software package for simulating the trajectories of sprinkler droplets ([1]).

The radius of the droplet must be specified to compute its trajectory. We randomly draw radii from a log-normal distribution with parameters μ and σ. This choice of distribution is grounded in fire sprinkler literature ([4]); however, we refer to a sprinkler field test to determine the distribution's parameters.

To verify the validity of our droplet spray model and choose droplet distribution parameters, we compare to recent measurements of drop size distribution by optical spectropluviometer ([3]). In this paper, sprinkler nozzles with pipe diameter of 4.8 mm and pressure of 350 kPa are tested. One insight that the study offers is that as pressure decreases, drop size increases.

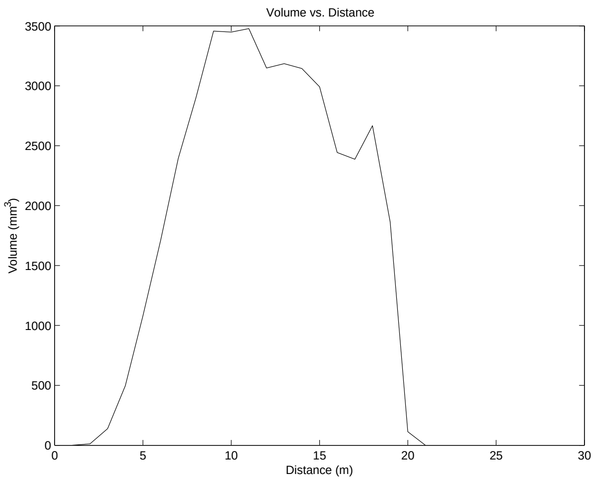
Below are plots comparing the results from our model and the real-world data.





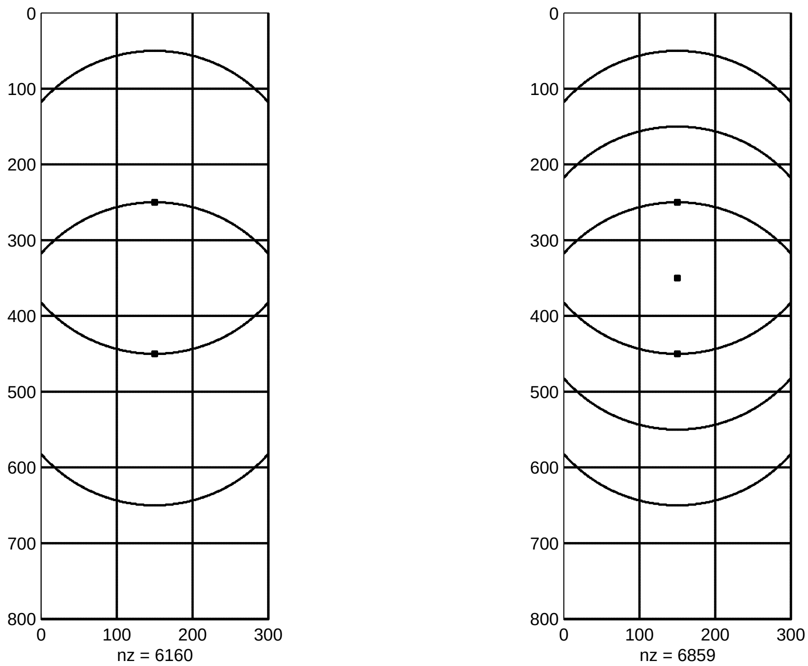
The discrepancies are most likely due to our uncertainty in the parameters of our droplet distribution. There is greater bias in the plot of number of drops at each distance, but this is less important than the volume at each distance.

For the calibration, the log-normal parameters are $\mu = 6$ and $\sigma = 0.7$. To adjust for the 20% higher pressure of the model problem pipe, we use $\mu = 5.8$ (20% less in a logarithmic sense) and $\sigma = 0.7$. With model problem parameters the spray of the sprinklers extends 20 meters away from the center.



4 The Field

Because the sprinklers cover so much area with the model problem parameters, it is reasonable to only use two or three sprinklers along the 20 meter pipe. Two such arrangements are shown below.



It is natural to break the possible locations for the sprinkler system into a discrete grid, with the costs of moving the irrigation pipe a linear function of distance. While a discrete grid does not truly represent the continuum of possible positions, it is adequate to the task because the spray distribution is smooth enough that it would be unreasonable to move the sprinklers less than one meter. The problem of scheduling moving the sprinkler system can be posed as a network or graph model and solved using the network simplex algorithm. The constraints are those in the problem statement, no part of the field should receive more than 0.75 cm of water per hour and each part of the field should receive at least 2 cm of water every 4 days, coupled with the sprinkler spray model.

5 Conclusion

Three mathematical models are proposed for different aspects of the problem: a Bernoulli solution to the sprinkler nozzle velocity, a kinematic model for the spread of water droplets, and a network simplex constrained optimization algorithm for finding an optimal moving schedule.

Though simplifications have been made to make the problem more tractable, each aspect of the model is consistent with experiments and existing literature.

References

[1] Carrion, P., J.M. Tarjuelo, and J. Montero (2001), SIRIAS: a simulation model for sprinkler irrigation I. Description of model, *Irrig. Sci.*, 20:73-84.

[2] Kundu, P.K. and I.M. Cohen (2004), *Fluid Mechanics*, 3rd ed., Elsevier, Amsterdam.

[3] Montero, J., J.M. Tarjuelo, and P. Carrion, Sprinkler droplet size distribution measured with an optical spectropluviometer, *Irrig. Sci.*, 22:47-56.

[4] Putorti, A.D. (2004), *Simultaneous Measurements of Drop Size and Velocity in Large-Scale Sprinkler Flows Using Particle Tracking and Laser-Induced Fluorescence*, Doctoral Thesis, University of Michigan.