Wind and Waterspray

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Introduction

Given anemometer readings from a nearby building, the task is to devise an algorithm that controls the height of a fountain in an open square. Our mission is to keep passersby dry and yet have the fountain look as impressive as possible. With ever-changing winds, we must devise a scheme to regulate the flow of water through the fountain to ensure that the bulk of the water shot into the air falls back to the ground within the fountain basin boundary.

Our model considers many factors and is divided into five basic parts:

- The conversion of wind speed on top of the building to wind speed at ground level based on height and the force of drag.

- The determination of initial velocity, maximum height, and time of flight from fountain nozzle characteristics, using Bernoulli's equation and the rate of flow equation of continuity.

- The assessment of the displacement effects of the wind on the water's ascent.

- The assessment of the displacement effects of the wind on the water's descent.

- The calculation of the optimal flow rate by comparing the water's total horizontal displacement to the radius of the fountain basin.

After creating this model in a MathCAD worksheet, we solved every function involved in this model as a function of the water flow rate. This worksheet takes the input from several variables such as the nozzle radius, the maximum flow rate the fountain can handle, the dimensions of the building on which the

anemometer is placed, and the dimensions of the fountain. From the inputs, the model finds the maximum flow rate that keeps the water in the fountain basin. As wind speed and direction vary, the model reacts to produce the optimal flow rate.

Testing the model shows that while the results are reasonable, the main source of error results from our drag calculations due to the interaction between wind and the buildings. To solve this error, measurements should be taken at both the building roof and the fountain itself. Although future work would resolve this issue and improve the model, our current model still provides realistic results.

We provide in **Table 1** a list of symbols used.

Problem Approach

We break the overall problem down into several smaller pieces, solve the pieces separately, and put the pieces together to find the overall solution.

- How the wind is affected as it flows around the buildings.

  - How the wind varies with height off the ground.
  - How the buildings slow the wind.

- How the wind affects the water from the fountain.

  - How the wind affects the water on the way up.
  - How the wind affects the water on the way down.
  - How to contain that total displacement within the basin.

Assumptions

Overall Assumptions

- **The plaza has a fountain in the center with four surrounding buildings.** Other arrangements can be handled with slight modifications.

- **The buildings are rectangular and have the same dimensions.** Most buildings are rectangular; for same-size buildings, we can use a single constant drag coefficient.

- **The distances from each building to the fountain are the same,** so each building has the same effect on the fountain water.

- **The acceptable splash area is the radius of the fountain basin.** A basin surrounds the water jet, and people walking outside the fountain do not want to get wet.

Table 1.

Table of symbols.

| Symbol	Meaning (units)
R	rate of flow of the fountain (m^3/s)
R_e	Reynolds number
υ	flow speed (m/s)
d	a relevant dimension (m)
ν	kinematic viscosity of the fluid
$F_{\vec{D}}$	force of drag (N)
ρ	density of the wind (kg/m^3)
v_{bh}	speed of the wind before the building at height h (m/s)
C_d	drag coefficient
A	surface area interacting with the wind (m^2)
v_z	wind speed measured by the anemometer at the height z (m)
h	height above ground (variable) (m)
z	height of the building (m)
α	terrain constant number = 0.105
h_{max}	maximum height that the water reaches, a function of R (m)
K_i	kinetic energy of the wind-building system before the wind hits the building (J)
K_f	kinetic energy of the wind-building system after the wind passes the building (J)
W_{NC}	work done by nonconservative forces, drag of the building times the length over which it is applied (J)
$\vec{d}$	distance over which drag acts, length and width of the building (m)
b	width or half the length of one of the buildings (m)
v_h	speed of the wind after it passes the building at a height h (m/s)
m	mass of the air that interacts with the building in 1 s if the speed v_{bh} was constant over the face of the building (kg)
θ	angle at which the wind strikes the building (°)
A_p	cross-sectional area of the pipe at the nozzle tip (m^2)
v_f	speed of the water as it leaves the nozzle (m/s)
r_p	radius of the pipe at the nozzle tip (m)
g	acceleration due to gravity, 9.803 m/s^2
r_c	radius of the column of water at a time t after leaving the nozzle with a rate of flow R (m)
P	pressure on the water caused by the wind (N/s^2)
A_c	surface area of the column of ascending water(m^2)
ρ	density of air (kg/m^3)
m_T	total mass of the water in the air at a flow rate R (kg)
T_{Total}	total time that the water spends in the air with a flow rate R (s)
ρ_{water}	density of water (kg/m^3)
a_c	horizontal acceleration of the water in the column with a flow rate of R and a wind of speed v_h (m/s^2)
F_c	force on the column of water from the wind of speed v_h (N)
x_c	horizontal displacement of the ascending column of water with a flow rate R and wind speed v_h at a time t (m)
P_D	pressure on a drop of water from wind of speed v_h (N/m^2)
F_d	force on the drop from wind of speed v_h (N))
A_d	area of a drop (m^2)
m_d	mass of a drop of water (kg)
a_d	horizontal acceleration of the drop of water as a function of rate of flow R and time in air t (m/s^2)
a_{avg}	average horizontal acceleration of a drop during its descent at a rate of flow R and wind of speed v_h (m/s^2)
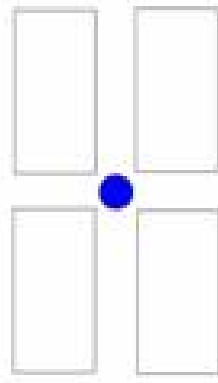
Figure 1. The fountain in the center of four buildings.

- **The fountain does not squirt water higher than the buildings,** although shooting water over the roofs would indeed be spectacular.

- **The fountain shoots water straight into the air.** This is important for our model so that we can predict how the water will flow up, how it will fall, and where it will fall.

- **The fountain nozzle creates a single sustained stream of water.** This assumption enables us to neglect drag as the water reaches its peak height. Furthermore, most fountains have a continuous flow of water.

Wind

- **The pertinent wind flow is around the sides of the buildings, not over them.** Since the fountain does not exceed the height of the buildings, it does not interact with wind that passes over the tops of the buildings. This assumption is important in calculating the drag caused by the buildings.

- **The flow of the wind continues in the same direction across the entire plaza.** The wind flows through the plaza in a constant direction, goes around obstacles, and resumes the same direction of motion. The wind does not get stuck in the plaza nor react to cars, people, doors, or windows in the plaza.

- **Wakes caused by buildings are not factors.** The wake that results when wind hits a building and goes around it does not change the velocity after the wake, so the wake force does not influence the wind's speed or direction.

- **The fountain is not in the wake of the buildings.** With this assumption, there is no need to worry about wake in our model. This is important because wake is too complex to be modeled.

- **The change in wind velocity is due solely to drag.** The reason that the wind decreases before and after hitting the building is because of drag. This

assumption allows us to use the law of conservation of energy to predict the change in velocity.

- **The anemometer measures wind speed and direction at the top of the building before any effects of drag.** The anemometer must be at the top of the building on the windward side, elevated above the height of the building so as not to measure any of the effects of the building. To simplify, we assume that it is at the height of the building.

- **The wind pattern is the same across the entire plaza as measured at the anemometer.** If the pattern changed, the anemometer reading would be invalid

- **The fountain is in a city or urban area.** This assumption allows us to determine the effect of the ground on wind speed a given height.

- **The drag applied to wind at a certain height is equal to the average effect of drag,** that is, to the total drag caused by the building at the velocity at that height divided by the height of the building. This is slightly inaccurate but still produces a reasonable model.

Water Height

- **Water has laminar flow.** Water has a constant velocity at any fixed point, regardless of the time. A fluid may actually have various internal flows that complicate the model, but we consider the flow as the jet of water ascends as constant so that we can model it as an ideal fluid.

- **Water has nonviscous flow.** The water experiences no viscous drag force in the pipe or in the air. The outer edge of the column of water actually interacts with the air and loses some energy to due to the viscosity of both fluids; but since air and water both have a low viscosity, this loss is negligible.

- **Water is incompressible.** The density of water is constant and does not change as the water moves up into the air and back down again.

Water Movement Sideways

- **The water jet upward flows as a cylinder.** Since the surface tension of the water holds it together unless it is acted upon by a force, the water should somewhat retain the dimensions of the nozzle from which it emerges.

- **The pressure of the wind is a force per area on the water column and on water drops.** Wind and water are both fluids, so the interaction between them is a complex relationship of their viscosities; but we also know that wind creates a pressure difference that we can model. We model the force on the water as the pressure caused by a certain velocity of wind multiplied by the surface area of the body of water.

- **The largest particle of water that we want to contain is the size of average drop of water 0.05 mL.** The column of water breaks into smaller particles at the peak of its ascent, and they descend individually. We estimate that particles smaller than that size would be acceptable to bystanders hit by them. Any larger particle would have more mass, hence a lower mass-to-surface-area ratio, so the pressure could not push it as far.

- **Water drop behaves as a rigid body.** Since a drop is small, internal currents have very little effect. Additionally, the pressure acts over the entire surface area of the drop and should accelerate it as a single body.

Model Design

Effects of Buildings on Wind Velocity

Because buildings surround the fountain, the wind velocity at the anemometer on top of a building is different from that at fountain level. Buildings disrupt wind currents, slow the wind, and change its direction [Liu 1991, 62]. Buildings create areas of increased turbulence, as well as a wake—an area of decreased pressure—behind the building. Thus, the behavior of wind after it passes a building is so complex as to be almost impossible to model. Hence, we assume that the fountain is located outside of the wakes of the buildings.

Wind Speed Reduction

The wind inside a group of buildings is less than that outside of the group; the interaction between the wind and the buildings causes a decrease in speed. The drag between the building and the wind decreases the kinetic energy of the wind and hence its speed.

Since the fountain is squirting water into the air in a symmetrical shape, the wind affects where the water lands in the same way regardless of the wind's direction; so there is no need to find the wind direction after it hits the building.

Drag

Nevertheless, wind direction before the wind hits the building is an important factor. The angle at which the wind hits the building changes the surface area that the wind interacts with, and drag changes with area. The drag force $\vec{F}_d$ is given by

$$\vec{F}_d = \tfrac{1}{2}\rho v_{\mathrm{bh}}^2 C_d A,$$

where ρ is the density of air, v_{bh} is the speed of wind at height h, C_d is the drag coefficient, and A is the surface area interacting with the wind. Therefore, we

must know from which angle the wind approaches the building and how this affects the surface area perpendicular to the direction of the wind.

For a rectangular building with the narrow face to the wind, $C_d = 1.4$ [Macdonald 1975, 80].

Figure 2 diagrams the plaza and fountain. No matter which way the wind blows, it interacts with a narrow edge of a building. Wind from due east or west create a problem for this model, because of discontinuity in the the drag coefficient. Instead, we assume that the coefficient remains constant.
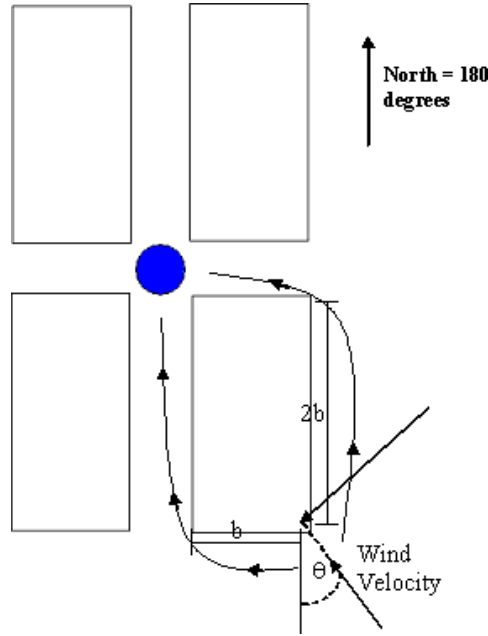
Figure 2. The plaza.

Wind Speed at Differing Heights

The speed of wind changes with the height from the ground because there is an additional force on the wind due to surface friction (dependent on the surface characteristics of the ground). The effect of this friction decreases as the wind speed is measured from a greater distance to the ground, creating faster speeds at greater heights.

Wind speed also varies because the temperature varies with height and location. However, if we assume that temperature and ground roughness are constant, a mean speed at a certain height can be modeled by

$$v_{\text{bh}} = v_z \left(\frac{h}{z} \right)^{\alpha} \qquad \text{[Macdonald 1975, 47]}, \qquad \textbf{(1)}$$

where v_{bh} is the speed of the wind before it hits the building, v_z is the wind speed measured by the anemometer at the height z of the building, h is the

variable height of the water, and α is the terrain constant number. We use $\alpha = 0.105$, the value for ground roughness of a city center [Macdonald 1975, 48].

We assume that the greatest height of the water that the fountain hits, $h_{\max}$, does not exceed the height of the building, so we can neglect the drag from the building's roof (since the wind that goes over the building does not interact with or affect the water in the fountain).

Converting Drag to Work

We need to convert the drag force into a form that will enable us to determine the actual loss of speed. Since drag is a nonconservative force (energy is lost during its application), we can use conservation of energy in the form that says that the initial kinetic energy K_i minus the work W_{NC} done by the nonconservative force equals the final kinetic energy K_f, or

$$K_i = K_f + W_{NC}. \tag{2}$$

For the K terms, we use the kinetic energy equation $K = \frac{1}{2}mv^2$. For K_i, we have v_{bh}; for K_f, we have v_h.

Work is the dot product of the force and the distance that the force is in contact with the surface, or

$$W_{NC} = \vec{F}_d \cdot \vec{d}.$$

The work done is the drag force exerted by the building on the wind times the distance that the wind travels along the sides of the building.

With substitution, we find

$$W_{NC} = \frac{1}{2}\rho v_{bh}^2 C_d A d. \tag{3}$$

The drag coefficient C_d is for the entire building. However, we cannot have the entire building's drag force working on the speed at a specific height or we will overestimate the influence of the drag. Instead, we find the average drag per meter of the building. To do this, we divide (3) by the height z of the building, then substitute the result into (2):

$$\frac{1}{2}mv_{bh}^2 = \frac{1}{2}mv_h^2 + \frac{\frac{1}{2}\rho v_{bh}^2 C_d A d}{z}.$$

Using (1), we can find v_{bh} at any height h. But the equation still has several unknowns that stop us from solving for v_h: the mass m, the area A, and the distance d.

Mass of Air

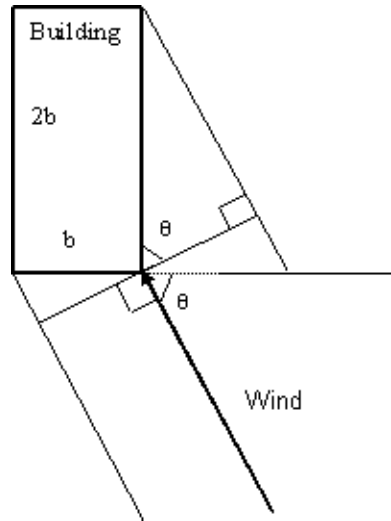The mass of wind that interacts with the building per second at height h is

$$m = v_{bh} A \rho t.$$

It is reasonable for convenience to use the average mass over 1 s.

Surface Area Interacting with Wind

As shown in **Figure 3**, the surface area as it relates to the drag due to wind is the cross section of the building perpendicular to the wind.

Figure 3. Orientation of wind to building.

Therefore, the surface are of the building based on the angle θ at which the wind strikes the building of width b, is found using trigonometry and gives

$$A = (b|\cos\theta| + 2b|\sin\theta|)z,$$

where z is the height of the building. We take the absolute value of the cosine and sine because we use the direction of the wind measured by the anemometer in terms of a 360° compass.

Distance

The distance d that the wind goes over the building is $3b$, the length of one side plus the width of the building, because the wind will curve around the building.

Combining the Equations

Combining, solving for v_h, and using $\alpha = 0.105$ gives the speed v_h at height h. [EDITOR'S NOTE: We do not reproduce the complicated expression here.]

Height of the Fountain

We find a function to model the maximum height $h_{\max}(R)$ of the fountain as a function of the rate of flow R. We assume that the water acts as an ideal fluid

and that the fountain shoots water straight into the air in a single sustained stream.

Volume Flow Rate and Bernoulli's Equation

We have from Halliday et al. [2001, 334]

$$R = A_p v_f, \quad \text{or} \quad v_f(R) = \frac{R}{A_p} = \frac{R}{\pi r_p^2},$$

where R is the rate of flow, v_f is its speed, A is the cross-sectional area of the pipe, and r_p is the radius of the pipe.

Based on the effect that we want the fountain to have, we make the water column (the radius of the pipe at the tip of the nozzle) have a 6-cm diameter, hence a radius of 0.03 m.

We use Bernoulli's equation [Halliday et al. 2001, 336], which relates forms of energy in a fluid, to calculate the maximum height of the water as it shoots into the air:

$$p_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2,$$

where p_1 and p_2 are the pressure of the water (both are zero since we are looking only at the water in the air) and g is the acceleration due to gravity. At the initial point, we consider the height of the nozzle as having zero gravitational potential energy, so the pressure head $\rho g y_1$ equals zero. Additionally, the speed v_1 is the speed from **(1)**. At the endpoint, the water has height $h_{\max}$ and the kinetic energy is zero. Substituting and simplifying gives

$$h_{\max}(R) = \frac{\left(\dfrac{R}{\pi r_p^2}\right)^2}{2g}.$$

With the radius r_p constant, the height of the top of the water stream varies directly with the square of the rate of flow R. **Figure 4** shows the heights for values of R between 0 and 0.04 m^2/s of water. Whatever mechanism s the water must be able to vary the flow rate by small amounts, particularly for large R, to maintain the maximum height allowable for the wind conditions.

The Effect of Wind on the Water Ascent

Radius Change in Ascent

Photos of fountains show that the water ascends as a slowly widening column until it reaches its maximum height, then falls back on itself and scatters. We can derive an expression that shows the change in the radius as the cylinder of water ascends; but since the change is very small, on the order of 1 mm, we use the initial radius at the nozzle, r_p, in our calculations.
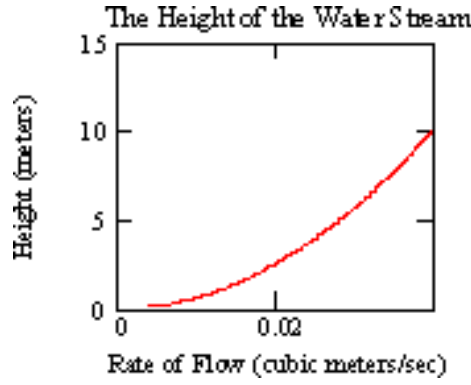
Figure 4. The effect of rate of flow on height of the fountain.

Wind Effects in Ascent

The other contributor to the water's horizontal movement is the wind, whose force can be determined from pressure. Pressure is force exerted over an area, so pressure multiplied by area gives the force:

$$P = \frac{F_c}{A_c},$$

where P is pressure, F_c is force, and A_c is area. The cylinder of water has height $h_{\max}$ and width twice the radius r_p, so $A_c = 2r_p h_{\max}$.

We find pressure in terms of wind speed using

$$P = \tfrac{1}{2}\rho v_h^2,$$

where ρ is the density of air and v_h is the speed of wind at height h. So we have

$$\tfrac{1}{2}\rho v_h^2 = \frac{F_c}{A_c} = \frac{F_c}{2r_p h_{\max}}.$$

Solving for F_c gives

$$F_c(R) = \rho r_p h_{\max} v_h^2 = \rho r_p \frac{\left(\dfrac{R^2}{\pi r_p^2}\right)^2}{2g} v_h^2.$$

Since we have the F from $F = ma$, finding mass should lead us to acceleration. Discovering the mass equation is pleasantly simple. If you take the flow rate and multiply it by the time that the water is in the air, then you know exactly how much water is in the air. Multiplying that volume by water's density gives the total mass, m_T:

$$m_T(R) = R \frac{t_{\text{total}}(R)}{2} \rho_{\text{water}},$$

where $\rho_{\text{water}} = 1000 \text{ kg/m}^3$. We solve for the acceleration:

$$a_c(R) = \frac{F_c(R)}{m_T(R)}.$$

We use kinematics yet again to find how far the center of the water cylinder shifts, x_c, by the time it reaches the top of its ascent:

$$x_c(R) = \tfrac{1}{2} a_c(R) \left(\frac{t_{\text{total}}(R)}{2} \right)^2 .$$

The Effect of Wind on Water Descent

The water's surface tension holds it in a very cylinder-like column during its accent; but when the water reaches the top of its path, it runs out of kinetic energy and begins falling. Modeling the erratic behavior of the fall is somewhat difficult. At that point, turbulence caused by the competition between the gravitational force and the momentum of the ascending water overcomes the water's surface tension and smaller bodies of water descend individually.

Since we are concerned about the bystanders level of dryness, we want the fountain to shoot to a height that will keep within the fountain's basin all particles with potential to dampen the onlookers. Anything smaller than a drop from a common eyedropper, about 0.05 mL, will not considerably moisten a person. Therefore, we want to spray the fountain to a height that will not let the wind carry this size drop outside the fountain's basin.

Anything larger than such a drop has a greater mass and and surface area to mass ratio, so it will not accelerate as much nor travel as far. So, we need model the flight of only such a drop.

We assume that the drop behaves as a rigid body. This assumption neglects the forces that act internally in the fluid and thereby overestimates the effect of the wind. Thus, this assumption may lower the maximum height of the water but will not result in any excessive water hitting bystanders.

Since the drop behaves as a rigid body, Newton's second law applies: The sum of all the external forces equals the mass of the drop of water, m_d, times the net acceleration:

$$\sum \vec{F}_d = m_d \vec{a}_d.$$

The only significant forces are the the wind (parallel to the ground) and gravity (vertical). The wind force F_d we know from $P = F_d/A_d$; we can calculate the surface area of the drop, and we know the pressure of the wind at height h. We can calculate the force on the drop as a function of height. Dividing that by the mass of the droplet gives its acceleration a_d parallel to the ground as a function of height h:

$$a_d(h) = \frac{P(h)AS_d}{m_d}.$$

Unfortunately this acceleration depends on height, which is a function of time t in the air and the rate of flow R; so we cannot use the constant-acceleration kinematics equations. Also, the nature of the equation for acceleration makes integration with respect to time an unwieldy task. We can, however, get the

average acceleration by integrating the acceleration from the time at the peak to the total time in the air and dividing by half of the time in air:

$$a_{\text{avg}} = \frac{\int_{t_{\text{total}}/2}^{t_{\text{total}}} a_d\big(h(t,R)\big)\, dt}{t_{\text{total}}/2}.$$
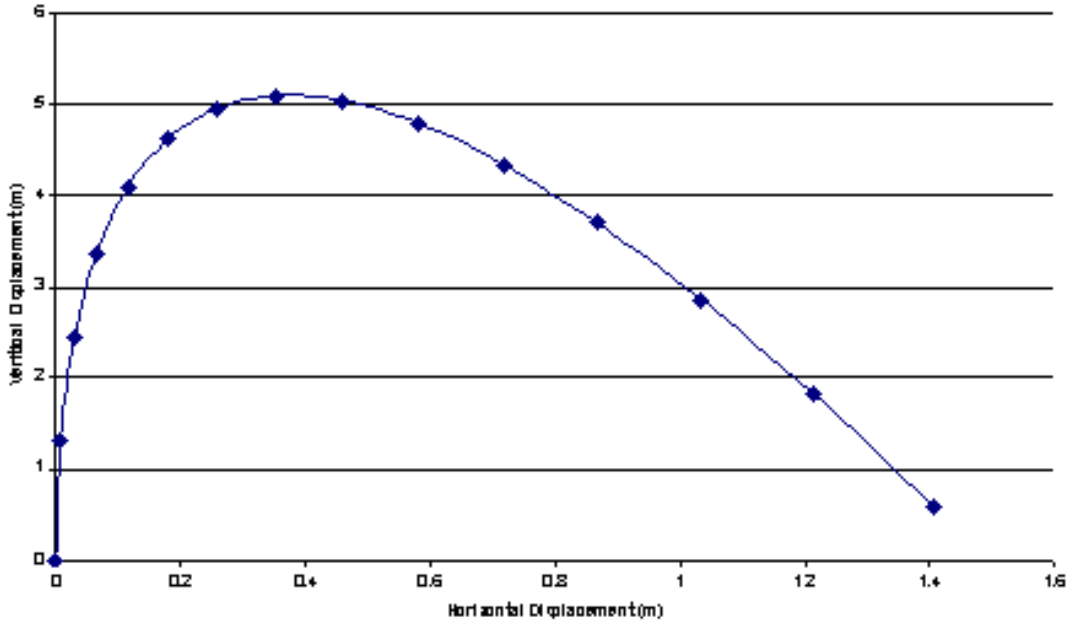
Using a_{avg} as a constant, we can find the displacement x_d of the drop in the horizontal direction. We know that

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2.$$

Applying this equation to the motion of the drop, we see that

$$x_d(R,t) = \left[\frac{d}{dt}r\left(R, t_{\text{total}}/2\right) + \frac{d}{dt}x_c\left(R, t_{\text{total}}/2\right)\right] t + \frac{1}{2}a_{\text{avg}}(R)\left(\frac{t}{2}\right)^2 +$$
$$x_c\left(R, t_{\text{total}}/2\right) + r_p.$$

Combined with the y position of the droplet, we get the flight path of **Figure 5**.

Figure 5. Water path from the fountain, due to wind.

The initial speed is the derivative of two position functions, $r_c(R,t)$ (the radius of the column) and $x_c(R,t)$ (the displacement of the column due to the wind). Taking the rate that those distances change when the drop separates from the column gives the initial speed of the drop. In addition, the equation shows that the initial displacement, x_0, is the original width of the column plus the distance that the wind pushes the center of the cylinder, $x_c(R,t)$ for the time t that it takes for a particle of water to reach its peak, $t_{\text{total}}/2$.

The displacement depends on the rate of flow. This is useful(!) since we must moderate the rate of flow to control the amount of water that escapes the basin. We can now set the displacement $x_d(R, t)$ equal to the maximum allowable displacement—the radius of the basin—and solve for the rate of flow.

The Optimal Rate of Flow

Our computer algebra system choked on solving for R exactly in terms of the other parameters. Instead, we adapted an incremental approach with a simple program in MathCAD. The maximum value for R could be anything; but an available off-the-shelf industrial pump has a maximum value of 0.04 m^3/s [Fischer Process Industries 2002]. We set that as the upper limit for R. We set $R = 0.001$ m^3/s and increment it in steps of 0.001 m^3/s until the displacement is greater than the radius of the basin.

Results and Discussion

We discuss how well our model handles each of the six variables that affect the solution:

- the fountain nozzle radius,

- the height of the building,

- the wind speed,

- the angle between the wind and the building,

- the building width, and

- the fountain radius.

Wind Speed

As the wind speed increases, the flow rate must decrease inside the fountain basin. Since the flow rate decreases, the height should also decrease because less water is forced through the nozzle, causing a lower initial speed. Does our model reflect these phenomena? Yes, it does.

Angle Between Wind and Building

The angle has no apparent effect on the solution our model produces. How is this possible? It is possible because our fountain is surrounded by buildings.

The way we calculated the buildings' effect on the wind created similar effects at any angle.

There were indeed variations present when we calculated how much the wind was slowed down by the building depending on the angle; however, these variations were too small to affect the fountain's setting.

Nozzle Radius

A smaller radius at a given flow rate means a higher speed. As the radius increases, so does the flow rate until the maximum rate is reached.

What does this do to the height of the fountain? Height increases as the radius increases (because the rate of flow increases as well) until the maximum rate is reached. If the radius still keeps increasing, the flow remains constant through a larger opening, causing a lower speed and therefore a lower height.

Height of Building

As the height of the building increases, the height of the fountain could increase as well. Our model doesn't accurately reflect this. The problem most likely lies in our drag calculations, the only place where building height shows up. Both the wind angle and the building height depend on the accuracy of our drag assumptions, and both have produced questionable results.

Building Width

We finally find some data that suggest that our drag equation is at least partially correct. As the width of the building increases, more surface area is created for wind/building interaction. The increased surface area leads to more drag, a lower wind speed by the time the wind reaches the fountain, and therefore a higher flow rate and higher height of the fountain.

Fountain Radius

Our goal is to keep the water contained in the fountain basin. If the radius increases, we can shoot the water higher up into the wind and still have it land in the fountain basin. Both the rate of flow and height produced increase within basin radius.

Summary and Conclusions

Our task was to develop a model to take inputs of wind speed and direction measured on a rooftop and use them to regulate flow through a nearby fountain.

By breaking the problem down into parts, we developed a model that produces believable results; and we have shown how our model responds to different inputs.

The effects of wake formation and wind interaction against a building are the two biggest problems. We assumed that the wake has no effect, and we dealt with the building interaction—but our findings raise questions. Clearly, the best test would be a fountain in a wind tunnel.

It would make sense to install an anemometer into the fountain structure and use wind-speed readings from the fountain itself. Direction and surroundings would be insignificant; only the wind speed at the fountain would be important.

We assume that wind gusts do not occur. There must somehow be a warning for the fountain that a gust is coming. Perhaps the rooftop anemometer could gauge wind speed change and send a signal to the fountain to reduce flow until the wind speed returns to normal.

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