Runway Management: A Mathematical Model

Greg Abbas
Krissy Culler
Jeff Vanderkam
North Carolina School of Science and Mathematics
Durham, NC 27705

Advisor: D. Teague

Summary

We devise an index to measure the dissatisfaction of a plane. The index takes into account both the airline's and the passengers' dissatisfactions, as well as the dissatisfaction of those waiting for the plane at the next airport.

Using the index, each time a runway becomes free, we give the plane with the highest index (and thus the most dissatisfaction) the next clearance for takeoff.

We also implement a maximum-waiting-time restriction, arguing that no plane should have to wait in the queue for longer than a specified amount of time.

We examine three other models that an airport might have to use because of its runway layout and compare these in turn to our own, using a computer-generated simulation of an airport's operation.

We then do a sensitivity analysis on the model by altering some of the parameters.

We discuss the strengths and weaknesses of our model, and look at some possibilities for modifying it in the event more information should be available in the original database.

1. Restatement of the Problem

As air traffic controllers, we are given the following information for each plane scheduled to take off at our airport:

- Scheduled pushback time;
- Actual pushback time;
- Number of passengers on board;
- Scheduled arrival time at next airport;

Table 1. Variables used in the model

Variable	Type	Description
TP_{SCH}	DB	Time scheduled for pushback
TP_{ACT}	DB	Actual pushback time
P_P	DB	Original number of passengers on plane
P_W	DB	Number of passengers waiting to make a connection
T_C	DB	Time available to make connection
T_A	DB	Scheduled arrival time
P_{CAP}	DB	Capacity of airplane
L	C	Time it takes one plane to lift-off
R	C	Number of runways
G	C	Time required by ground crew for maintenance
T	C	Time when computation is made
TP	F	Pushback delay
TQ	F	Time waited in queue
W	F	Time between two consecutive takeoffs
DS_{INDIV}	F	Dissatisfaction of one person on a plane
DS_P	F	Total dissatisfaction of passengers on a plane
DS_A	F	Total dissatisfaction of airline for one plane
DS_{WI}	F	Dissatisfaction of one person waiting at next airport
DS_W	F	Total dissatisfaction of all people waiting at next airport for one plane
DSI	F	Total dissatisfaction index of a plane

Key:
- DB found in the database
- C a constant for each airport
- F calculated from other variables

- Number of people waiting to board at the next airport;
- Scheduled time of takeoff from next airport;
- Maximum capacity of the plane, ranging from 100 to 400 people in steps of 50.

Using this database, we are to design a system to manage runway traffic so as to satisfy both passengers and airlines as much as possible.

2. Assumptions

1. The queue referred to in the problem is a physical line, wing to wing (that is, the planes are out on the runway waiting area, not in their terminals).

2. Travel time is constant for a flight, i. e., we cannot speed up or slow down a plane to keep it on schedule. Thus, a plane that leaves the airport 15 min late will arrive at its destination 15 min late.

3. The controllers will know the model of the plane and hence its maximum capacity.

4. Maximizing satisfaction is the same as minimizing dissatisfaction.

5. Dissatisfaction of the passengers is directly related to the amount of time they must wait.

6. Since the goal of the airlines is to move as many people as possible (to increase ticket sales), dissatisfaction of the airlines is directly related to the number of seats being delayed, as well as to the length of the delay.

7. Over any sufficiently long period of time, the number of incoming planes at an airport is no greater than the number of planes the airport can allow to leave (backlog is avoidable over sufficient periods of time).

8. The people waiting at the next airport to board the incoming plane are as important in figuring dissatisfaction as those on the plane itself, and thus they too should be considered by the control tower.

9. The dissatisfaction of a person on a plane is the same for everyone on the plane, and consideration will be given to each person's dissatisfaction when figuring the total for the plane. Corollary: The dissatisfaction of each person on each plane goes up by the same amount every minute.

3. Analysis and Model Design

We began by establishing notation for quantities involved with the dissatisfaction of the passengers and the airlines (Table 1).

The *pushback delay* (TP) is the difference between the actual pushback time and the scheduled pushback time:

$$TP = TP_{ACT} - TP_{SCH}.$$

The *time waited in the queue* (TQ) is the difference between the time of computation and the actual pushback time:

$$TQ = T - TP_{ACT}.$$

The *time between two consecutive takeoffs* (W) is the quotient of the takeoff time and the number of runways. If there are two runways, a plane in the queue will have to wait only half as long to take off as if there were only one runway. This quantity is not involved in the dissatisfaction indices but is used in our simulation:

$$W = L/R.$$

For each person on a flight, the *individual index* (DS_{INDIV}) is the total time that the flight is delayed:

$$DS_{INDIV} = TQ + TP.$$

Since each person on a flight has the same waiting time, and thus the same index (DS_{INDIV}), we defined the *total passenger dissatisfaction* (DS_P) for a plane as the total number of travelers multiplied by the individual index:

$$DS_P = DS_{INDIV} * P_P.$$

The *dissatisfaction of the airline* (DS_A) is found in a similar way. Since the airline is concerned with the *total number of seats on the plane* (P_{CAP}), we multiply P_{CAP} by the total waiting time (see Assumption 6):

$$DS_A = (TQ + TP) * P_{CAP} = DS_{INDIV} * P_{CAP}.$$

The people waiting at the next airport to make a connection with the incoming plane have a different situation. Their main concern is whether or not their outgoing flight will leave on time, and they are dissatisfied if and only if their plane leaves after its *scheduled departure time* (T_C). The incoming aircraft will arrive $TQ + TP = DS_{INDIV}$ minutes late. After that, the ground crew will take G minutes to prepare the plane for the next flight. Thus the next flight will be ready to leave no earlier than $T_A + DS_{INDIV} + G$, and the *dissatisfaction for one person at the airport* (DS_{INDIV}) is given by:

$$DS_{WI} = \begin{cases} TA + DS_{INDIV} + G - T_C & \text{if this quantity is positive} \\ 0 & \text{otherwise (since there is no delay).} \end{cases}$$

Since each person waiting at the airport has the same delay, the *total dissatisfaction for those waiting* (DS_W) is given by the product of the number of people waiting (DS_P) and their individual dissatisfaction indices:

$$DS_W = DS_{WI} * P_W.$$

The *total dissatisfaction index for one plane* (DSI) must be a weighted average of DS_P, DS_W, and DS_A. By Assumption 8, we must consider those waiting at the airport to be just as important as those actually on the plane, so we weight DS_P and DS_W equally. Thus we now have the following general form for our dissatisfaction index:

$$DSI = a\,(DS_P + DS_W) + b\,DS_A,$$

where a and b are weights. For our original model, we set $a = b = 0.5$, but these weights could vary; in our analysis we will show that changing the weights does not significantly change the success of our model.

4. Runway Analysis

We considered three types of runways.

Runway A requires the first-in/first-out (FIFO) schedule, which we are trying to improve upon. As can be seen from Figure 1, once a plane is waiting in the queue, there is no way for it to get out of the line to pass another plane, and there is no place

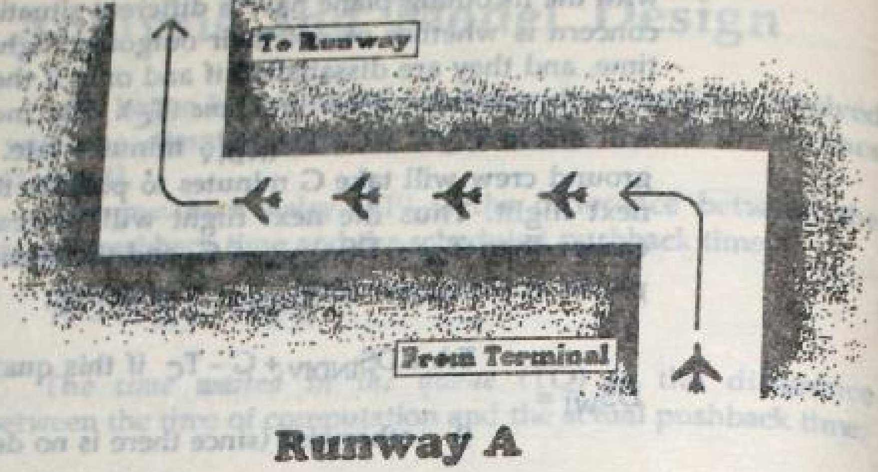

Figure 1. A runway that requires first-in/first-out queueing.

for a new plane to come into the middle of the line, even if it is in an extreme hurry. This configuration prevents us from moving an already delayed large plane ahead of an on-time small plane, thus drastically increasing the dissatisfaction of the large plane. It also prevents us from making alterations in the departure order after planes have already entered the queue. Thus we chose not to model this type of runway.

On Runway B (Figure 2), planes being added to the queue can enter it whenever and wherever needed; but a plane already in line is not allowed to jump in front of another plane; because of lack of space. We call this the *no-jump* situation. It can cause problems, as in the following example.

Example: A 100-person-capacity Boeing 737 with 50 people on board has been waiting for 20 min. The plane has only 20 min of "breathing room" before its flight at the next airport, where there are 100 people waiting. It has a dissatisfaction index (DSI) of 1500 by our method. Suppose a 400-seat Boeing 747 is ready to enter the queue. It has 400 people on board, nobody waiting at the next airport, and is running perfectly on time. By our index, this plane currently has a DSI of 0, and thus is placed behind the smaller plane.

Suppose there is a long line ahead of each plane, and they both wait another 15 min. The 737 now has a DSI of 3375, and the 747 has a DSI of 6000. Therefore the 747 should now be ahead of the 737; but it is not, and we cannot rectify the situation.

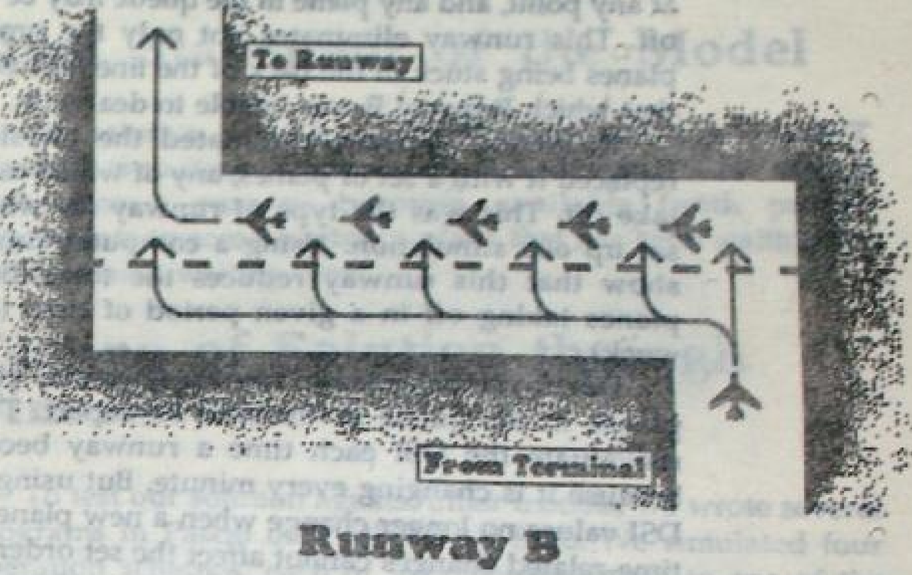

Figure 2. A "no-jump" runway.

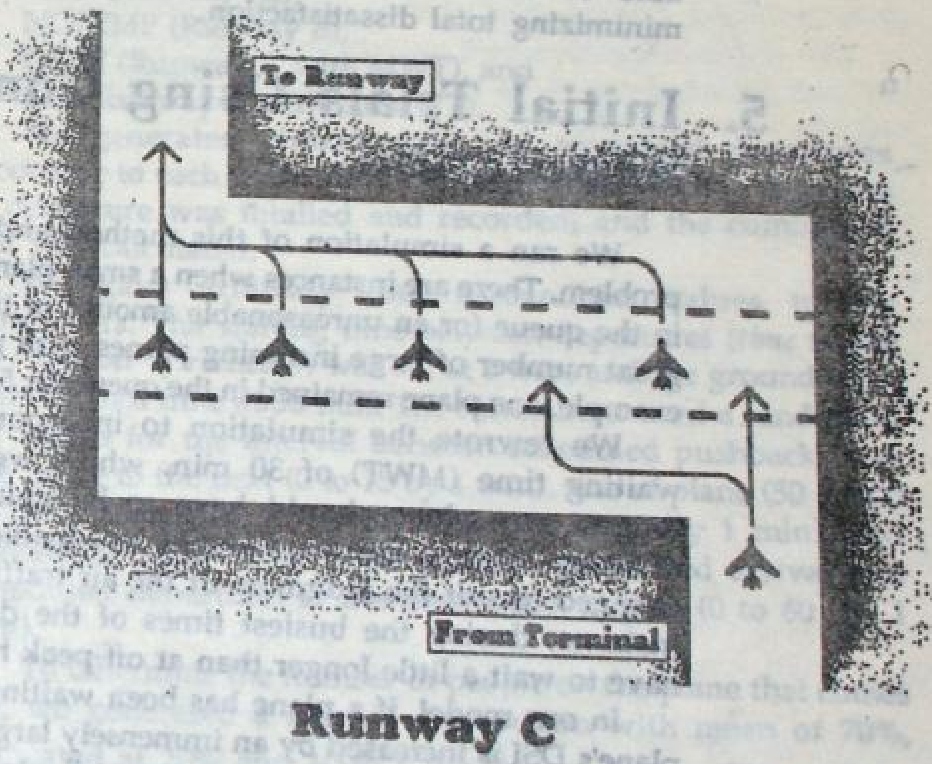

Figure 3. The best kind of runway to have.

Thus Runway B leads to problems with satisfaction, so we chose not to implement it.

We finally settled on Runway C (Figure 3). The advantages of this runway are obvious: Incoming planes may enter the queue at any point, and any plane in the queue may be the next to take off. This runway eliminates not only the problems of large planes being stuck in the back of the line, but also the situation that which Runway B was unable to deal with.

In effect, we have eliminated the physical queue and replaced it with a set of planes, any of which can be the next to take off. This was the type of runway for which we initially set up our simulation. Using a computer simulation,we will show that this runway reduces the total dissatisfaction of planes taking off in a given period of time (e.g., in a given week).

Due to our selection of runways, we may choose any of the planes in the queue as the next to depart. Thus, we should reevaluate the DSI each time a runway becomes available. because it is changing every minute. But using Runway C, the DSI values no longer change when a new plane is inserted; and time-related changes cannot affect the set order, because there is no set order to affect. So every time a runway opens, we are able to launch the plane with the highest DSI, thus minimizing total dissatisfaction.

5. Initial Trials Using Computer Simulation

We ran a simulation of this method and discovered one problem. There are instances when a small plane could get stuck in the queue for an unreasonable amount of time, because of a great number of large incoming planes with greater DSI's. For example, one plane remained in the queue for 54 min.

We rewrote the simulation to incorporate a maximum waiting time (MWT) of 30 min, which we considered the longest any plane should have to be delayed. This is an arbitrary interval that we thought reasonable, but it could be changed to suit the judgment of the air traffic controllers. For instance, during the busiest times of the day, planes might have to wait a little longer than at off-peak hours.

In our model, if a plane has been waiting for 30 min, that plane's DSI is increased by an immensely large number, to give the plane precedence to depart over all planes that have not waited as long. This large number is not incorporated into the total dissatisfaction for the week (so as not to throw off our comparisons between models); it just ensures that the waiting plane will take off immediately.

6. Final Statement of the Model

Our solution, using Runway C and incorporating the MWT criterion, is to clear the plane with the highest DSI for takeoff each time a runway becomes available (with proper adjustments to accommodate a plane that has been waiting for the maximum waiting time).

7. Testing of Solution through Simulation

To test our solution against other models, we wrote several programs in Pascal on a DEC VAX 8350. We simulated four operation schemes, each of which corresponds to one of the runways discussed above:

FIFO (Runway A),
NOJUMP (Runway B),
DSIW (Runway C with MWT), and
DSI (Runway C without MWT).

We generated aircraft data and organized departures according to each of the four schemes. Dissatisfaction of planes at departure was totalled and recorded, and the cumulative DSI was calculated.

We gave arbitrary (but reasonable) values to the parameters. The waiting time between departures (*time to take off / number of runways*) was set at 3 min, and the ground crew maintenance time at 30 min. The simulations used a random-to set values for the interval between scheduled pushback from one plane to the next (0 to 15 by 1 min), size of plane (50 to 400 by 50 passengers), delay at hangar (0 to 15 by 1 min), and "breathing room" (the time between scheduled arrival and scheduled departure) at the destination airport (0 to 60 by 1 min).

To determine the number of people on the plane that comes up, we generated a normal distribution with mean of 70%, truncated at 20% and 100%.[1] We figured that a plane will fly as long as 20% of its seats are filled.

[1] EDITOR'S NOTE: In fact, the authors use in their computer program a uniform continuous distribution from 20% to 100%.

Likewise, we assigned the number of people waiting at the next airport to range from 0% to 50% of capacity in a truncated normal distribution centered at 25%.[2]

The simulation also took into account the regular distribution of airplane arrival frequency, i.e., more arrivals at peak hours and fewer arrivals at midnight and 2 A.M. In doing so, however, we took care to ensure that the total number of incoming planes did not exceed the total number allowed to leave, in accordance with Assumption 9.

In order to make sure that chance was not a significant influence on our results, we ran the simulation 10 times. Each time the simulation was run, the same set of random numbers for the four airport situations was used, so each situation had exactly the same planes in its database. Figure 4 shows the results of the simulations.

Clearly, the most satisfactory methods are those involving Runway C, namely, DSI and DSIW. They have the lowest total dissatisfaction in every case. Our model with the MWT (Maximum Wait Time) restriction slightly raises the total DSI for each trial, as expected; but we chose the model with MWT because of the problems encountered earlier in the non-MWT model.

8. Stability

To test the stability of our solution, we considered what changes would result from alterations in the constants used in the simulation.

First we considered G, the amount of time taken by the ground crew to prepare a new arrival for takeoff. Since an airport would know this information about its own ground crew, we decided that it would adjust its schedules accordingly, thus making any changes in G immaterial in the overall results. For example, if a crew suddenly became 15 min slower, the airport would simply give an extra 15 min between incoming and outgoing flights: thus, no change in scheduling order.

We next considered the effect of changes in the number of runways R and the amount of time L needed for a plane to take off. Since these two numbers are used only in one equation, for

[2]EDITOR'S NOTE: Although the authors intended to use a truncated normal distribution, the formula in their computer code for the normal probability density function contains errors, resulting in fact in a nearly uniform distribution from 40% to 51%.

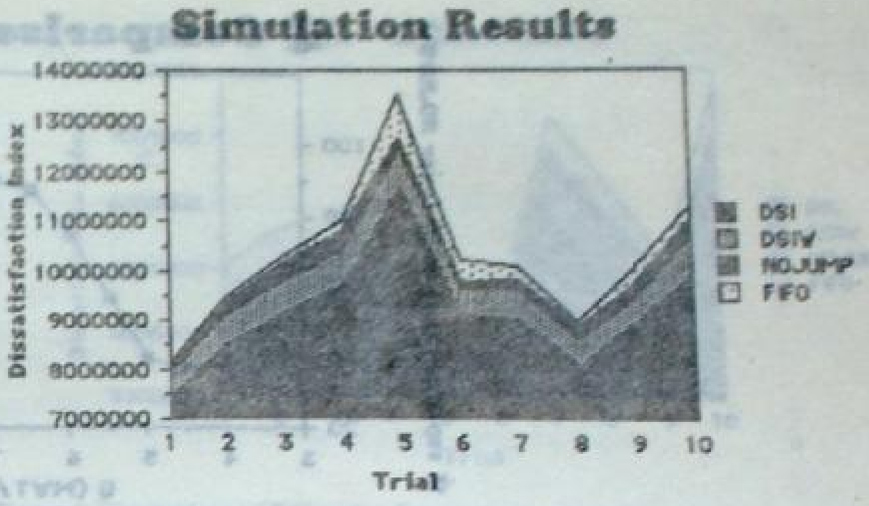

Figure 4. Results of 10 simulations for the four operations schemes.: FIFO on Runway A, NOJUMP on Runway B, DSI on Runway C, and DSIW (= DSI with MWT) on Runway C.

for the amount of time W = L/R waited between each departure), we simply looked at the effect of changes in W.

We ran a simulation with W set at 6 min rather than the original 3, and discovered that NOJUMP was more efficient than DSIW roughly half the time. This was an unexpected result; but we realized that this had happened because the increase in W meant that the MWT restriction was used more often, since the planes have to wait for a longer time before they are cleared to depart. We realized that as W becomes larger, the priority becomes more and more based on getting each plane out as quickly as possible (for which FIFO is better suited than DSIW). As a result, DSIW starts to resemble FIFO as W becomes larger.

On the other hand, the smaller W becomes, the more DSIW resembles the regular DSI, since the MWT criterion (the one distinguishing factor between the two) is used less and less. We defined the variable Q as the quotient of MWT and W and attempted to find the optimal value of Q through another use of our simulation.

To do this, we created a simulation that would compare DSIW and NOJUMP over 20 weeks at several different values of Q. Since we had already done this with Q = 10 in our original simulation and found that DSIW is nearly always better than NOJUMP, we chose not to try values of Q greater than 10. We also chose not to try values of Q smaller than 4, since it is

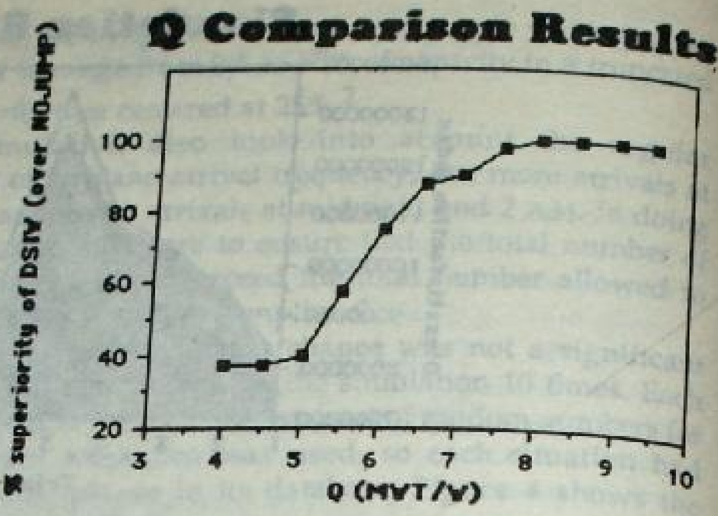

Figure 5. Comparison of DSIW and NOJUMP with varying Q.

unlikely that the maximum waiting time is less than the time required for four departures. The results of this experiment are in Figure 5.

We see that as long as Q is greater than 7, DSIW is consistently superior to NOJUMP; and that NOJUMP is not even close to DSIW in success rate unless Q approaches 5. Since 5 is rather low for a value of Q (remember, our original model used 10), we may safely say that our method is preferable for all reasonable values of Q.

In addition, we note that we are comparing DSIW with MWT to NOJUMP without MWT. Were we actually implementing NOJUMP, we would probably include an MWT restriction on it as well, thus increase its total DSI.

Finally, we took into account alterations on a and b, the weights assigned to the dissatisfactions of the airlines and the passengers in our DSI calculation. We would expect these to have a significant impact on the order of selection: If a is much larger, the planes will be sorted more by capacity; and if b is much larger, the bias will be towards planes with more passengers.

A change in Assumption 8, so that DS_P and DS_W would have different weights, would have similar effects on the model.

We ran further simulations to see how our method (DSIW) would compare to the other methods with different a and b values. Thus, we could see how our solution procedure might be changed. Figures 6 and 7 show the results of these simulations.

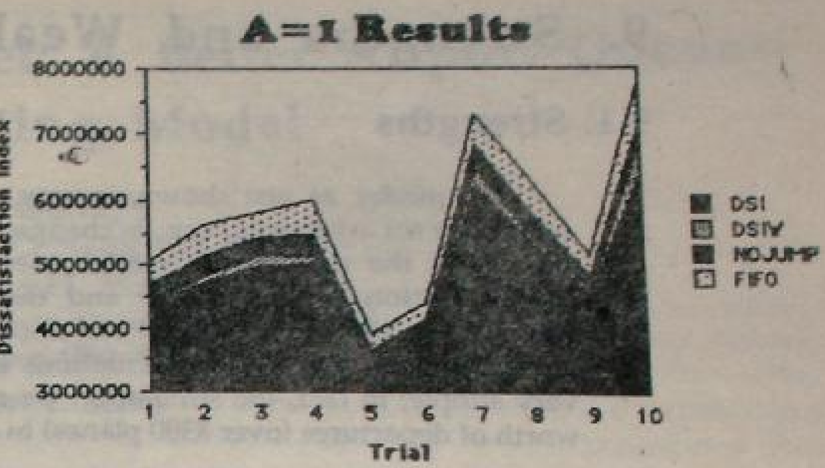

Figure 6. Comparison of DSI with other methods when $a = 1$.

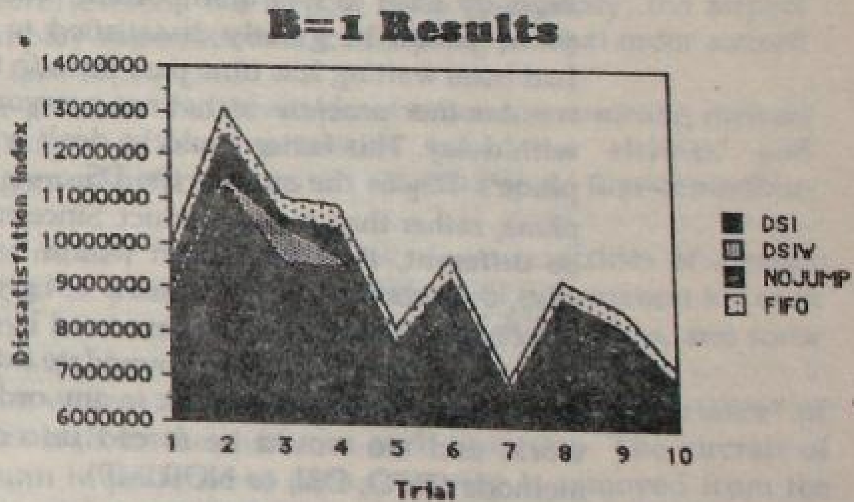

Figure 7. Comparison of DSIW with other methods when $b = 1$.

Again, there is little change in the performance of the methods. DSI still outperforms NOJUMP and FIFO, and compares well enough with DSI to justify including MWT. Thus, we conclude that our choice of DSIW as our solution is the correct choice under any circumstances.

9. Strengths and Weaknesses

9.1. Strengths

Our model, as just shown, is very stable. All parameters, which we set arbitrarily, can be changed without changing the quality of the model. The model does in fact minimize the dissatisfaction of the people and the airlines (taking into account the MWT). The model can also be easily implemented in any control tower. The calculations to determine order are very simple; in fact, the simulation we ran produced a week's worth of departures (over 3300 planes) in about 10 sec.

9.2. Weaknesses

One potential weakness in our model is human nature. Nobody likes to feel unimportant, thus the people in a small plane would be greatly dissatisfied to see a large plane that had been waiting less time pass them in the queue.

Another problem is that not everyone is equally concerned with delay. This factor could be dealt with by considering each plane's DS_P as the sum of the DS_{INDIV} of each person on the plane, rather than just a product. Since each of these would now be different, this calculation would be much more difficult. Also, the necessary information is not given to us nor could it be reasonably stored in a database used by air traffic controllers.

If an airport cannot accommodate our model (for instance, it cannot allow planes to depart in any order), our model may not work, and we would be forced into one of the other three methods (FIFO, DSI, or NOJUMP).

Our Assumption 9 is that everyone on each plane would have the same dissatisfaction increase every minute. But in most instances this would not be the case. In fact, if our method were put into place, some enterprising airline might want to assure high priority for its flights. This could be implemented in the model by multiplying a plane's DSI by a weighting factor; and the database would have for each plane one further piece of information, its *priority index*. The airline could market such *privilege planes*, to its on-the-go customers, such as executives and other jet-setters. The price of a flight would be much higher; but to those who believe in the well-known cliché "time is money," the loss of money would be worth the gain in time.