A Close Look at Leaves

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Abstract

We construct four models to study leaf classification, relationships between leaf shape and leaf distribution, correlations between leaf shape and tree profile, and total leaf mass of a tree.

Model 1 deals with the classification of leaves. We focus primarily on the most conspicuous characteristic of leaves, namely, shape. We create seven geometric parameters to quantify the shape. Then we select six common types of leaves to construct a database. By calculating the deviation index of the parameters of a sample leaf from those of typical leaves, we can classify the leaf. To illustrate this classification process, we use a maple leaf as a test case.

Model 2 studies the relationship between leaf shape and leaf distribution. First, we simplify a tree into an idealized model and then introduce the concept of solar altitude. By analyzing the overlapping individual shadows through considering the relationship between leaf length and internode length under different solar altitudes, we find that the leaf shape and distribution are optimized to maximize sunlight exposure according to the solar altitude. We apply the model to three test types of trees.

Model 3 discusses the possible association between tree profile and leaf shape. Based on the similarity between the leaf veins and branch structure of trees, we propose that leaf shape is a two-dimensional mimic of the tree profile. Employing the method of Model 1, we set several parameters reflecting the general shape of each tree and compare them with those of its leaves. With the help of statistical tools, we demonstrate a rough association between tree profile and leaf shape.

Model 4 estimates the total leaf mass of a tree given size characteristics. Carbon dioxide (CO_2) sequestration rate and tree age are introduced to establish the link between leaf mass and tree size. Since a unit mass of a leaf sequesters CO_2 at a constant rate, the CO_2 sequestration rate has a quadratic relationship with the age of the tree, and the size the tree experiences logistic growth.

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Introduction

We tackle four main subproblems:

- classification of leaves,
- the relationship between leaf distribution and leaf shape,
- the relationship between the tree profile and the leaf shape, and
- calculation of the total leaf mass of a tree.

To tackle the first problem, we select a set of parameters to quantify the characters of the leaf shape and use the leaf shape as the main standard for our classification process.

For the second question, we use the overlapping area that one leaf s shadow casts on the leaf directly under it as the link between the leaf distribution and the leaf shape, since the leaf shape affects the overlapping. We assume that the leaf distribution tries to minimize the overlapping area.

As for the third question, we set parameters for the tree profile and compare those with the parameters for the tree s leaf shape to judge whether there is a relation between tree profile and leaf shape.

We use age to link the size of tree and the total weight of its leaves, because the tree size has an obvious relationship with its age and the age affects a tree s sequestration of carbon dioxide, which affects the total weight of a tree s leaves.

Assumptions

- The trees are all individual (“open grown) trees, such as are typically planted along streets, in yards, and in parks. Our calculation does not apply to densely raised trees, as in typical reforestation projects where large numbers of trees are planted close together.
- The shape of the leaves does not reflect special uses for the trees, such as to resist extremely windy, cold, parched, wet, or dry conditions, or to produce food.
- The type of the leaf distribution (leaf length and internode distance relation) reflects the tree s natural tendency to sunlight.
- The tree profile that we consider is the part above ground, including the trunk, the branches, and leaves.
- All parts of a leaf can lie flat, and the thickness or protrusion of veins can be neglected.
- Leaves are the only part of the tree that reacts in photosynthesis and respiration, so that the carbon dioxide sequestration of a tree is the sum of the sequestration of the leaves.

- The sequestration of a tree or a leaf is the net amount of CO_2 fixed in a tree, which is the difference between the CO_2 released in respiration and the CO_2 absorbed in photosynthesis.
- The trees are in healthy, mature, and stable condition. Trees of the same species have same characteristics.

Model 1: Leaf Classification

Decisive Parameters

To classify the shape of a leaf, we set seven parameters and establish a database for comparison.

Rectangularity

We define the ratio of the area of the leaf to the area of its minimum bounding rectangle as the leaf s *rectangularity* (**Figure 1**).

Figure 1.

Figure 2.

Figure 3.

Figure 4.

The photographs of leaves in **Figures 1 4** are reproduced (with overlays by the authors of this paper) from Knight et al. [2010], by kind permission of that paper s authors.

Aspect Ratio

The aspect ratio is the ratio of the height of the minimum bounding rectangle to its width. (**Figure 2**).

Circularity

To evaluate how round a leaf is, we consider that ratio if the ex-circle to the in-circle. (**Figure 3**).

Form Factor

Form factor, a famous shape description parameter, is calculated as

$$\mathrm{FF} = \frac{4\pi A}{P^2},$$

where A is the area of the leaf and P is its perimeter.

Edge Regularity Area Index

Although the aspect ratio and the rectangularity of two leaves may be similar, the contour or the exact shape of two leaves may vary greatly. To take the different contour of the leaf into consideration, we join every convex dot along the contour and develop what we call the *bounding polygon area*. The ratio between the leaf area and this bounding polygon area is the *edge regularity area index*. The closer this ratio is to 1, the less jagged and smoother the leaf s contour is (**Figure 3**).

Edge Regularity Perimeter Index

Similarly, we develop another parameter, the *bounding polygon perimeter*, the perimeter of the polygon when we join the convex dots of a leaf. We define the ratio of the bounding polygon perimeter to the perimeter of the leaf to be the *edge regularity perimeter index*. The smaller this ratio, the more jagged and irregular the contour of the leaf is (**Figure 3**).

Proportional Index

Since it is also highly critical to capture the spatial distribution of different portions of a leaf along its vertical axis, we divide the minimum bounding rectangle into four horizontal blocks of equal height, and then calculate the proportion of the leaf area in a particular region to the total leaf, which we refer to as the *proportional index (PI)* for that region (**Figure 4**). Hence, the PI is a vector of length four.

Common Types of Leaves

We develop a database of the six most common leaf types in North America (**Figure 6**), using the seven parameters discussed above. **Table 2** gives the values of the parameters for each leaf type, as measured from scans of photos of leaves in Knight et al. [2010].

Comparison

Given a specific leaf, we calculate the seven characteristics of it and compare them with our database by calculating the squared deviation of each parameter of the given leaf from the corresponding standard parameter of each category. We realize that some of the parameters are somehow more important than others. So in an effort to make our model more accurate

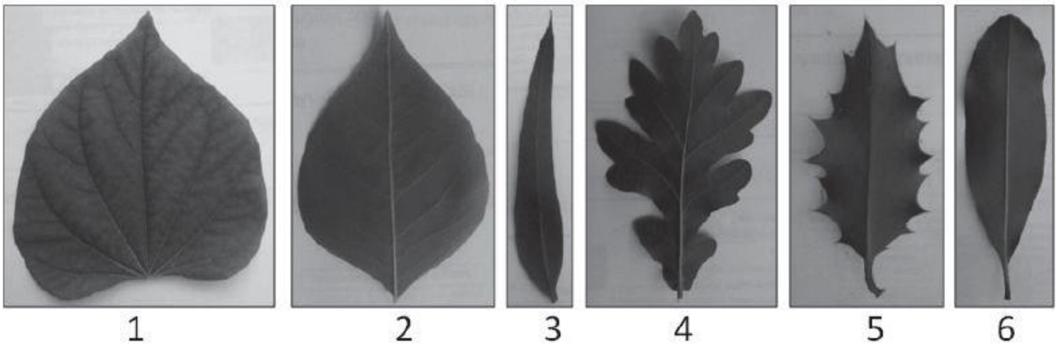

Figure 5. The six most common seen leaf types in North America. (The photos, from Knight et al. [2010], are reproduced by kind permission of that paper s authors.)

Table 1.
Parameter values for the six leaf types.

Type	1	2	3	4	5	6
Rectangularity	0.6627	0.5902	0.6250	0.4772	0.4876	0.6576
Aspect Ratio	0.8615	0.6600	0.1800	0.6383	0.4792	0.3111
Circularity	0.8140	0.5432	0.4564	0.3454	0.3123	0.3311
Form Factor	0.9139	0.6206	0.2823	0.2470	0.3662	0.4956
ER Area Index	0.9322	0.8780	0.9091	0.8500	0.7880	0.8895
ER Perimeter Index	0.8727	0.8889	0.9384	0.8602	0.8231	0.9903
PI_1	0.0649	0.0769	0.1179	0.1909	0.1299	0.2920
PI_2	0.2958	0.3555	0.2208	0.3892	0.3606	0.4187
PI_3	0.3439	0.4243	0.4139	0.3047	0.4123	0.2677
PI_4	0.2954	0.1433	0.2474	0.1152	0.0970	0.0220

and reliable, we introduce a weighted *index of deviation* I_D, with

$$I_D = \sum_{i=1}^{7} w_i I_i,$$

where each I_i is the squared deviation, except that

$$I_7 = \frac{1}{4}\sum_{j=1}^{4}(\mathrm{PI}_j \quad \mathrm{PI}_{\mathrm{new},i})^2.$$

We determine the weights via the Analytical Hierarchy Process (AHP) [Saaty 1982]. We build a 7×7 matrix reciprocal matrix by pair comparison:

$$
\begin{array}{cc}
 & \begin{array}{ccccccc} \mathrm{R} & \mathrm{AR} & \mathrm{C} & \mathrm{FF} & \mathrm{ERAI} & \mathrm{ERPI} & \mathrm{PI} \end{array} \\
\begin{array}{l} \mathrm{R} \\ \mathrm{AR} \\ \mathrm{C} \\ \mathrm{FF} \\ \mathrm{ERAI} \\ \mathrm{ERPI} \\ \mathrm{PI} \end{array} &
\begin{pmatrix}
1 & 1/3 & 1 & 1/4 & 1/2 & 1/2 & 1/7 \\
3 & 1 & 3 & 1 & 2 & 2 & 1/3 \\
1 & 1/3 & 1 & 1/4 & 1/2 & 1/2 & 1/7 \\
4 & 1 & 4 & 1 & 3 & 3 & 1/2 \\
2 & 1/2 & 2 & 1/3 & 1 & 1 & 1/4 \\
2 & 1/2 & 2 & 1/3 & 1 & 1 & 1/4 \\
7 & 3 & 7 & 2 & 4 & 4 & 1
\end{pmatrix}
\end{array}.
$$

The meaning of the number in each cell is explained in **Table 2**. The numbers themselves are based on our own subjective decisions.

Table 2.
The multiplication table of D_{10}.

Intensity of Value	Interpretation
1	Requirements i and j have equal value.
3	Requirement i has a slightly higher value than j.
5	Requirement i has a strongly higher value than j.
7	Requirement i has a very strongly higher value than j.
9	Requirement i has an absolutely higher value than j.
2, 4, 6, 8	Intermediate scales between two adjacent judgments.
Reciprocals	Requirement i has a *lower* value than j.

We then input the matrix into a Matlab program that calculates the weight w_i of each factor, as given in **Table 3**.

Table 3.
AHP-derived weights.

Factor	R	AR	C	FF	ERAI	ERPI	PI
Weight	0.0480	0.1583	0.0480	0.2048	0.0855	0.0855	0.3701

We test the consistency of the preferences for this instance of the AHP. For good consistency [Alonso and Lamata 2006, 446 447]:

- The principal eigenvalue $\lambda_{\max}$ of the matrix should be close to the number n of alternatives, here 7; we get $\lambda_{\max} = 7.05$.
- The consistency index CI = $(\lambda_{\max} \quad n)/(n \quad 1)$ should be close to 0; we get CI $= 0.009$.
- The consistency ratio CR = CI/RI (where RI is the average value of CI for random matrices) should be less than 0.01; we get CR $= 0.006$.

Hence, our decision method displays perfectly acceptable consistency and the weights are reasonable.

Model Testing

We use a maple leaf of **Figure 6** to test our classification model. Visually, it resembles Category 4 most.

Figure 6. Test maple leaf.

Now we test this hypothesis with our model. First, we process the image of the leaf, calculating rectangularity, aspect ratio, circularity, form factor, edge regularity area index, edge regularity perimeter index, and the proportional index, with values as in **Table 2**. The values of the seven parameters are shown in **Table 4**.

Table 4.
Parameter values for the sample maple leaf.

Factor	R	AR	C	FF	ERAI	ERPI	PI_1	PI_2	PI_3	PI_4
Measured value	0.355	0.908	0.269	0.157	0.625	0.379	0.097	0.463	0.431	0.009

Finally, we use our weights to calculate the index of deviation I_D of the maple leaf from each of the six categories of leaves considered earlier. We show the results in **Table 5**.

Table 5.
Index of deviation of maple leaf from six common leaf categories.

Category	1	2	3	4	5	6
Index of deviation I_D	0.27	0.12	0.23	**0.08**	0.24	0.18

Since the index of deviation between the given maple leaf and Category 4 is smallest, the model predicts that the maple leaf falls into Category 4 which conclusion is consistent with our initial hypothesis.

Conclusion

Our model is robust under reasonable conditions, as can be seen from the testing above. However, since our database contains only the six commonly-seen leaf types in North America, the variety in the database has room for improvement.

Model 2: Leaf Distribution and Leaf Shape

Introduction

Genetic and environmental factors contribute to the pattern of leaf veins and tissue, thereby determining leaf shape. In this model, we investigate how leaf distribution influences leaf shape.

Idealized Leaf Distribution Model

We construct an idealized model that immensely simplifies the complex situation: The tree is made up of a branch perpendicular to the ground surface, and two identical leaves grown on the branch ipsilaterally (on the same side) and horizontally. The leaves face upward and point toward the sun in the sky. We suppose that the tree is at latitude L (Northern Hemisphere). Let the greatest average solar altitude in a year, which is attained at noon on the vernal equinox, be α.

Figure 7 illustrates our primitive model of a tree at noon on the vernal equinox.

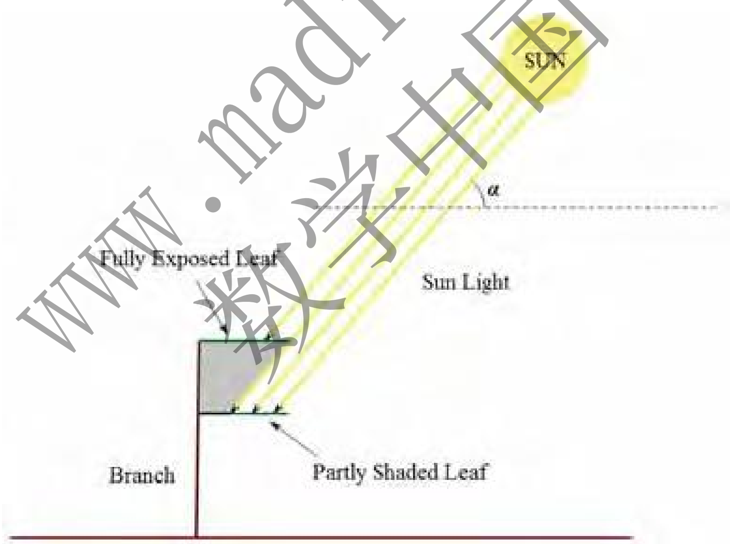

Figure 7. Primitive model of a tree, at noon on the vernal equinox.

Analysis of Overlapping Areas

Our focus is the partly shaded leaf in **Figure 7**. The output of the model is what proportion of the leaf (PL) is shaded. We divide the situation into three scenarios, depending on the influence of the angle α on PL.

Solar Altitude Near 90°

This situation usually takes place in tropical regions, where leaf shapes are typically broad and wide and the tree crown usually contains only one layer of leaves. This can be explained in terms of **Figure 7**: With α near 90°, the shaded part of the lower leaf would be too big to supply enough solar energy for photosynthesis, and the greatest absorption of energy can be achieved by a broad leaf shape.

Solar Altitude Near 0°

This situation usually takes place in frigid zones, where leaves are typically acicular (needle-shaped) and the tree crown contains dense layers of closely-grown leaves. In terms of **Figure 7**: With α near 0°, the shaded part of the lower leaf would approach zero, allowing a much more concentrated distribution of leaves than in other situations. In addition, the maximum absorption of energy can be best achieved by needle-like leaves.

Solar Altitude within Normal Range

This scenario is typical in the temperate zone on earth, where sunlight irradiates the leaves in a tilted way. It is also the case in which our idealized model is the most suitable. Another crucial factor that we control in this case is the distance h between the two points connecting the leaves and the branch. We assume that a tree s leaf distribution tries to minimize the overlapping area between leaves, so our model investigates the quantitative relationship between the overlapping area and the shape of the leaf.

To simplify the model, we model the leaf as a rhombus, whose major axis has length L_{major} and whose minor axis has length L_{minor}. Also, we fix the area of the leaf as A, to ensure constant exposure area to the sun. With area fixed, now we only need to change the length of the major axis to change the shape of the leaf (see **Figure 8**).

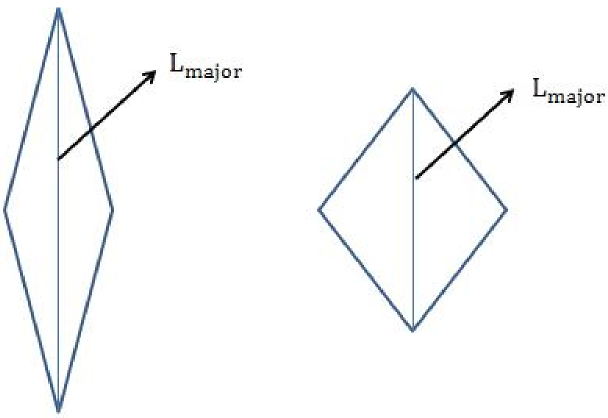

Figure 8. Two leaves of the same area but different lengths of major axis.

Also, since we have fixed the area of the leaf and just adjust its shape, the minimum proportion of the lower leaf shaded is

$$E = \frac{A_{\text{overlapping}}}{A},$$

where $A_{\text{overlapping}}$ is the smallest overlapping area.

The most efficient situation is for both leaves to be totally exposed to sunlight, as in **Figure 9a**: For some value $h = h_0$, we achieve $E = 0$.

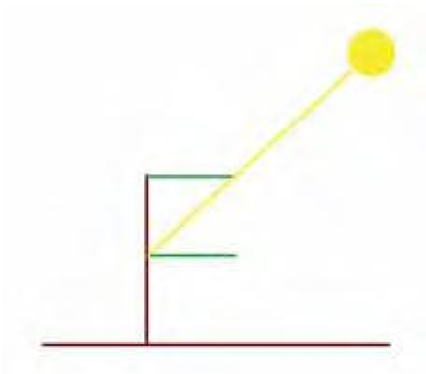

Figure 9a. Upper leaf does not overlap lower one.

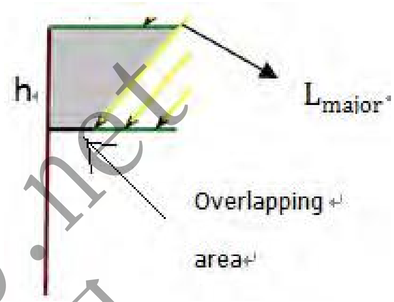

Figure 9b. Upper leaf overlaps lower one.

What if $h < h_0$, as in **Figure 9b**? We can easily give the relationship among h, L_{major}, and E for a given fixed solar altitude α:

$$E = \left(\frac{L_{\text{major}} \tan\alpha - h}{L_{\text{major}} \tan\alpha}\right)^2 = \left(1 - \frac{h}{L_{\text{major}} \tan\alpha}\right)^2.$$

For fixed h and α, the overlap area increases as the length of the leaf increase. The closer L_{major} is to $h/\tan\alpha$, the smaller the overlap.

From our discussion, the best leaf distribution occurs when $h = h_0$, which means $h = L_{\text{major}} \tan\alpha$.

Model Testing

We need to test whether this relation between leaf distribution and leaf shape is right. We offer data on leaf length L_{major} and internode distance h of several kinds of trees and use our formula to calculate the respective solar altitudes of the trees. By converting the solar altitude into latitude, we can predict the origin of a tree! We choose species native to China:

- *Ligustrum quihoui Carr.* (waxy-leaf privet or Quihou privet, a semi-evergreen to evergreen shrub);

- *Osmanthus fragrans* (sweet olive, tea olive, or fragrant olive, an evergreen shrub or small tree that is the city flower of Hangzhou, China); and
- *Camellia japonica* (Japanese camellia)

as our test trees. **Table 6** shows the results.

Table 6.
Test of model for leaf shape as a function of latitude.

Tree kind	L_{major}	h	Calculated $\tan\alpha$	Latitude Predicted	Latitude True
Ligustrum quihoui Carr.	2	2.5	1.25	38.7°	35 35°
Osmanthus fragrans	10	18.5	1.85	28.4°	23 29°
Camellia japonica	6	9	1.50	33.7°	32 36°

The predicted latitudes of origin are close to the true latitudes, confirming our hypothesis of a relationship between leaf distribution and leaf shape.

Model 3: Tree Profile and Leaf Shape

Hypothesis

Since

- the vein structure determines the leaf shape;
- the branch structure determines the tree profile; and
- to some degree, the leaf veins resemble branches,

we have a wild hypothesis that the leaf shape is two-dimensional mimic of the tree profile.

Comparison of Leaf Shape and Tree Contour

The leaf shape is two-dimensional, so it is relatively easy to study its parameters. However, the tree profile is three-dimensional, so it is important to find a two-dimensional characteristic of a tree to use for comparison. Since the longitudinal section of a particular tree reflects its general size characteristics, we focus on that.

Tree Profile Classification

In the leaf classification model, there are 6 general classes of leaves. Since we are comparing only the general resemblance between leaf and tree, we incorporate Class 5 (elliptic leaf with serrated margin) into Class 2 (elliptic leaf, smooth margin). As a result, we get 5 classes of leaves and 5 respective types of trees:

- Class 1: Cordate (Texas redbud)
- Class 2 and Class 5: Elliptic (camphor tree)
- Class 3: Subulate (pine)
- Class 4: Palmate (oak)
- Class 6: Obovate (mockernut hickory)

Parameters of the Tree

We appoint three parameters for the longitudinal section that can be compared with those of the leaf shape, namely, rectangularity, aspect ratio, and circularity.

Table 7 shows the measurements for both trees and leaves.

Table 7.
Comparison of leaf parameters and tree parameters.

Class	1	2 and 5	3	4	6
Rectangularity (R)					
Leaf	0.6627	0.5902	0.6250	0.4772	0.6576
Tree	0.6281	0.6846	0.5180	0.5292	0.6238
Aspect Ratio (AR)					
Leaf	0.8615	0.6600	0.1800	0.6383	0.3111
Tree	0.7914	0.7243	0.6601	0.7980	0.6750
Circularity (C)					
Leaf	0.6396	0.5698	0.1834	0.3069	0.2889
Tree	0.5800	0.5928	0.2895	0.4070	0.3866

For each of the parameter types, we drew a scatterplot, calculated the correlation, and investigated the statistical significance of the resulting line of best fit. Aspect ratio (AR) and circularity (C) were each statistically significant, pointing to linear relationships; rectangularity (R) was not.

Conclusion

The tests of aspect ratio and circularity support the theory that leaf shape is a two-dimensional mimic of the tree contour. Thus, the shape of leaf resembles the shape of tree to some extent.

Model 4: Leaf Mass

Introduction

A simple way to calculate the total leaf mass is to multiply the number of leaves by the mass of a single leaf. Our method is more accurate and less demanding, in that our model is (surprisingly!) independent of these two factors but dependent on a more reliable factor of a grown tree: photosynthesis. Our methodology of estimating the leaf mass of a tree is based on three variables:

- tree age;
- growth rate, which is determined by tree species; and
- general type (hardwood or conifer).

In other words, given the age and type of a tree, we can estimate the total mass of leaves. In this model, CO_2 is used as a calculating medium.

Leaf Mass and Tree Age

Leaf Mass and CO_2 Sequestration

Trees sequester CO_2 from the atmosphere in their leaves but mostly elsewhere in the tree. A tree s ability to sequester CO_2 is measured in terms of mass A_S of CO_2 (in pounds) per gram of leaf. Hardwood trees sequester more CO_2 per gram of leaf than conifers.

A tree s ability to sequester CO_2 is different from its ability to absorb it, since the tree also releases CO_2 into the atmosphere as part of its respiration. In other words,

$$CO_2 \text{ sequestration} = CO_2 \text{ absorption} \quad CO_2 \text{ release}.$$

Now we need only to estimate the weight of CO_2 sequestered by the tree and then calculate the total mass of the leaves as the ratio of the mass of CO_2 sequestered to the mass of CO_2 sequestered per leaf:

$$m_{\text{leaves}} = \frac{m_{CO_2}}{A_S}.$$

CO_2 Sequestration and Tree Age

The relationship between the amount of CO_2 sequestered, the age of a tree, and the type of tree is given in a table by the Energy Information Administration [1998, Table 2, 8 9], which also divides trees based on their growth rate: fast, moderate, or slow.

For each growth rate, we graphed the annual sequestration rate vs. age of the tree and fitted a quadratic model (see **Figure 10** for conifer example).

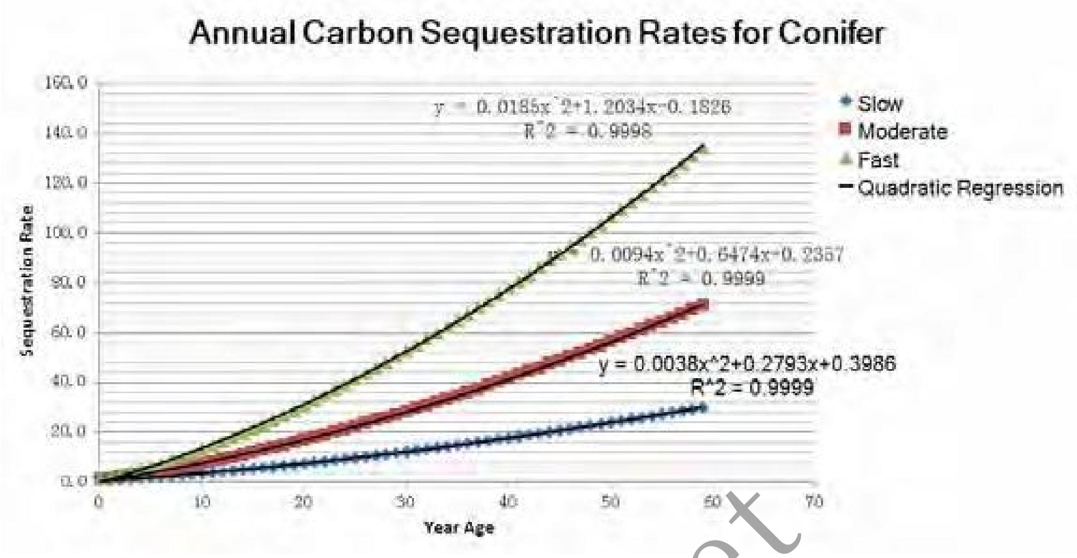

Figure 10. Annual CO_2 sequestration rates, in pounds of carbon per tree per year, for three rates of growth of conifer trees of increasing age.

We were surprised to find that the curves fit the data perfectly! (This fact strongly suggests that the original table values were not measured but calculated from such a model.) From the equations of the fitted curves, we can easily estimate the CO_2 sequestered for a tree of a given age and growth rate and consequently calculate the mass of the leaves.

Tree Age and Tree Size

Above, we used the age of a tree as a link between the two leaf mass and the size characteristics of the tree. Since we now know the relationship between the age of a tree (of a particular growth rate) and its total leaf mass, now we only need to work out the relationship between the age of the tree and the size characteristics of it. Tree size is the accumulation of growth, which is a biological phenomenon of increase with time.

In its life cycle, a tree experiences logistic growth, leading to a model for its “size, or profile, P (height, mass, diameter) as

$$P = k_1 \left(1 \quad e^{k_2 A}\right)^{k_3}, \qquad \text{hence} \qquad A = k_4 \ln\left(1 \quad k_5 P^{k_6}\right),$$

where A is the age of the tree and the k_i are constants that depend on the species of tree.

Leaf Mass and Tree Size

Finally, we get to answer the question of whether there is a relationship between leaf mass and tree size characteristics. Putting together our earlier models, we have the relationships in **Figure 11**.

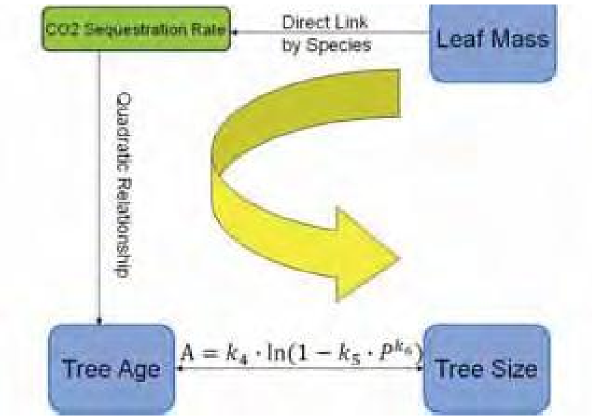

Figure 11.

According to our earlier results, leaf mass and tree age are related to each other through CO_2 sequestration, and we have just determined a function between tree age and tree size.

Strengths and Weaknesses

Model 1

Strengths:

Our model is based on quantitative analysis, so the classification process is both objective and efficient.

Our model is based on categories of leaf types that are the most typical and common.

Weakness:

We divide leaves into only six categories, which may not cover all leaf types.

Model 2

Strengths:

We have taken into consideration three climate conditions (tropical zone, temperate zone, and frigid zone) in discussing the relationship between the leaf distribution and the leaf shape.

The results of our model conform to the data that we found.

Weakness:

We consider the leaf distribution on a single branch but have not considered the inner-influence between different leaves of different branches.

Model 3

Strength:

The whole process uses data and quantitative analysis as foundations, so the output is objective and reasonable.

Weakness:

We have limited categories of tree profiles.

Model 4

Strength:

We use the carbon sequestration rate and age as the media to calculate the total mass of leaves, which is better than trying to estimate the number of leaves and the average weight of each.

Weakness:

The data are from a source that does not refer to the method of arriving at the data.

Letter to a Science Journal Editor

Dear Editor:

We present to you our key findings.

We first focus on the possible influence on leaf shape of the leaf distribution on the tree. For survival reasons, a tree should develop an optimal leaf distribution and shape pattern that adjust to the specific region of its origin, thereby gaining the most nutrients for photosynthesis by maximizing the exposure area to sunshine. We demonstrate a mathematical relationship among solar altitude, leaf shape, and leaf distribution. Based on this finding, we may be able to determine the best location for replanting or assisted-migration of a tree species by observing its leaf distribution.

Our second key finding is a rough relationship between the tree s profile and its leaves. In fact, we hypothesize that a leaf is a two-dimensional mimic of the tree. For several trees, we compared the shape of the leaf and the contour of the tree, finding similarities between certain characteristics.

This finding is another instance of the natural world containing examples of self-similarity, a mathematical concept that means that an object is approximately similar to a part of itself, as is the case for the mathematical objects of the Koch snowflake and the Mandelbrot set.

The third part of our study deals with the relationship between tree size characteristics and the total mass of the leaves. The two are linked by the CO_2 sequestration rate and the age of the tree. Hence, we can estimate the total mass of the leaves given some profile parameters of a tree, such as its height, diameter, volume, age, and type. This finding might have potential for agricultural and environmental uses, such as a new method to estimate tea production or wood production, or estimation of the CO_2 sequestration effect of a forest as an alleviator of global warming.

In hope of publishing our research in your journal, we enclose our research paper for you to examine and judge. We are convinced that our research on leaves promises to contribute to a variety of areas.

Sincerely yours,

Team 14990

Acknowledgment

The authors thank David Knight, James Painter, and Matthew Potter of the Dept. of Electrical Engineering at Stanford University for permission to reproduce photos of leaves from their paper Knight et al. [2010].

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