Room Capacity Analysis Using a Pair of Evacuation Models

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Introduction

We present two models for determining the amount of time for a given number of people to evacuate a given room. A room's maximum capacity can be derived from this by imposing a maximum evacuation time, which must take into account factors such as the fire resistance of the room.

We develop a graph-based network flow simulation. People are modeled as a compressible fluid that flows toward and out the exit. This model assumes people's interaction properties, based on industry research.

We also develop a discrete particle simulation. People are modeled as disks that attempt to reach the exits. In this model, people's interaction properties emerge from local, per-person assumptions.

We compare and evaluate the models' outputs and analyze the capacity of several local rooms.

Graph Flow Model

Overview

The graph flow model is a pool-flux model that operates on a graph representing areas of open space within a room. The graph consists of a set of nodes N and a set of directed edges E. Each node is valued as the number of people in a square patch of floor. Each edge represents the direction of traffic flow from one node to another node.

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The ability of occupants to exit a node is constrained by the congestion in the node and the bandwidth of the edge leading to another node. Bandwidth represents the rate that people can move between nodes. A higher bandwidth is used when there are no obstacles or doors between nodes, and a lower bandwidth is used when an exit constricts flow.

The number of occupants who enter a node is constrained by the number of people leaving the node and the tendency of a node to pack tighter. This tendency is referred to as *fill rate*.

Because there are interdependent relations, each time step of the model is calculated in a cascading pattern from the exits. After the flow rate out of a node is calculated, it becomes possible to calculate flow into the node. By this method the flow rate calculations can be determined for the entire graph.

Assumptions

1. All people are aware of the emergency and attempt to exit.
2. People move only toward a single exit.
3. People are safe, and removed from the simulation, when they reach an exit node.
4. People in crowds move at a speed determined by the density of the crowd.
5. People's movement is restricted by the width of the area that they are trying to move over.
6. People will move to the exit as quickly as possible, without regard to the effect on crowd density.
7. The increase in crowd density over time is limited.
8. People are treated as continuous populations, allowing for fractions.

Weaknesses of the Assumptions

Assumption 1 is not completely supported by the literature—not everyone is aware of or willing to leave during a real emergency.

Assumption 2 implies that people pick a single exit and head for it. In reality, people might observe that an exit is less congested and choose that exit as their new target. This assumption precludes the existence of barriers directing traffic flow or preset fire escape routes within a room.

Assumption 3 ignores the exit discharge capacity. In reality, the number of people leaving by an exit will affect the total evacuation time for a building. With this assumption, the model is limited to a single room.

Assumptions 4 and 5 are based on literature describing pedestrian movement in a transportation terminal [Benz 1986]. Movement in a transportation terminal is not an escape or evacuation situation, so it could involve different dynamics.

Assumption 6 does not take into account human intelligence or the possible presence of authorities regulating traffic flow.

Assumption 7 was included to reduce the tendency of nodes to fill from empty to maximum capacity in a very short period of time.

Assumption 8 can create situations that are contrary to reality. A person cannot split into fractional parts and flow in two different directions. This assumption is loosely justified by the fact that people can be partially across the boundary of two nodes. This assumption is required by the model mechanics when using small time steps.

Mathematical Structure of the Graph Flow Model

Graph Structure:

$$\begin{aligned} N_i &= \text{graph node } i\text{, representing a patch of floor space} \\ E_i &= \text{exits: the set of all nodes } N_i \text{ may exit to} \\ I_i &= \text{inputs: the set of all nodes that exit to } N_i \end{aligned}$$

Spatial Values:

$$\begin{aligned} P_i &= \text{number of people at } N_i \text{ [persons]} \\ A_i &= \text{area of } N_i \text{ [ft}^2\text{]} \end{aligned}$$

Constants:

$$\begin{aligned} W_{ij} &= \text{bandwidth: flow rate from } N_i \text{ to } N_j \text{ [persons/ft] (parameterized)} \\ W_{ij} &= 0.541 \text{ between two nodes [persons/ft]} \\ W_{ij} &= 0.325 \text{ to an exit node [persons/ft]} \\ s^\alpha &= \text{base movement constant for } S_i \text{ [58.678, dimensionless]} \\ s^\beta &= \text{movement multiplier constant for } S_i \text{ [58.669, dimensionless]} \\ s_{\min} &= \text{minimum movement at maximum compression [2.5 ft/s]} \\ T &= \text{maximum (terminal) compression of an area [3 people/ft}^2\text{]} \\ r^\alpha &= \text{fill rate constant [4.333 ft/s] (parametrized)} \\ t &= \text{the time step of the model [1 s] (parametrized)} \end{aligned}$$

Parameter values are derived from industry research. We omit the derivations for brevity.

Derived Constants:

$$A_i T = \text{maximum occupancy of node [persons]}$$

Flux Capacity Equations

Let S_i denote the walking speed inside a node due to congestion [ft/sec]:

$$S_i = S(P_i, A_i) = \max\left[s^\alpha + s^\beta \ln\left(\frac{A_i}{P_i}\right),\ S_{\min}\right].$$

Let FR_i denote the fill rate: the maximum number of people who can be added to N_i over time t [persons]

$$\mathrm{FR}_i = \mathrm{FR}(N_i, t) = r^\alpha t\,\frac{A_i T - P_i}{A_i T}.$$

Let OF_{ij} be the desired (maximal) outflow: the number of people capable of moving out of N_i into N_j [persons]

$$\mathrm{OF}_{ij} = \mathrm{OF}(N_i, N_j, t) = tS_iW_{ij}.$$

Let IF_i be the maximum inflow: the number of people who can enter a node from any direction in t [persons]. Note that IF_i cannot be calculated until FFA_{iEj} is calculated for all N_j in E_i.

$$\mathrm{IF}_i = \mathrm{IF}(N_i, t) = \sum_{E_i} \mathrm{FFA}_{iE_i} + \mathrm{FR}_i.$$

Let FF_{ij} be the final flow: the number of people capable of moving from N_i to N_j that N_j is capable of accepting [persons]. Note that FF_{ij} cannot be calculated until IF_j is known.

$$\mathrm{FF}_{ij} = \mathrm{FF}(N_i, N_j, t) = \begin{cases} \mathrm{OF}_{ij}, & \text{if } N_j \text{ is an exit;} \\ \mathrm{OF}_{ij}\,\dfrac{\mathrm{OF}_{ij}}{\sum_{N_k \in I_j} \mathrm{OF}_{kj}}, & \text{if } \mathrm{IF}_{ij} < \sum_{N_k \in I_j} \mathrm{OF}_{kj}; \\ \mathrm{OF}_{ij}, & \text{if } \mathrm{IF}_{ij} \geq \sum_{N_k \in I_j} \mathrm{OF}_{kj}. \end{cases}$$

Let FFA_{ij} denote the actual number of people who move from N_i to N_j. Note that FFA_{ij} cannot be calculated until FF_{iE_j} is calculated for all N_j in E_i.

$$\mathrm{FFA}_{ij} = \mathrm{FFA}(N_i, N_j, t) = \begin{cases} \mathrm{FF}_{ij}, & \text{if } P_i \geq \sum_{k \in E_i} \mathrm{FF}_{ik}; \\ P_i\,\dfrac{\mathrm{FF}_{ij}}{\sum_{k \in E_i} \mathrm{FF}_{ik}}, & \text{otherwise.} \end{cases}$$

This is a pool-flux model. The P_i are pools, and FFA_{ij} is the only flux that is ever applied to a pool.

Development of the Flow Functions

For constant t and W_{ij}, OF_{ij} is linearly proportional to walking speed due to congestion (S_i). For constant t and S_i, OF_{ij} is linearly proportional to bandwidth (W_{ij}).

Because IF_i is a function of the actual final flows out of a node, IF cannot be calculated until these actual final flows have been calculated first. These final flows are a function of the IF for the nodes that N_i flows into. Because of this dependency, the node graph must be acyclic. If the graph contains a cycle, then no IF_i for any node that is a member of the cycle can be calculated because it is dependent on IF for another node that is a member the cycle. Then IF equals to the total number of people that flow out of a node plus the fill rate for that node.

The final flow FF_{ij} (the number of people capable of moving from N_i to N_j that N_j is capable of accepting [persons]) is a function of IF_j, unless N_j is an exit.

The relationship between final flow and actual final flow is straightforward. Final flow calculates the number of people that can flow out of a node. However, if final flow is more than the population of the node, then this population is divided evenly among the available final flows. Otherwise, actual final flow is equal to final flow.

Particle Simulation Model

Overview

The particle simulation models humans one at a time as discrete, independent entities, instead of treating a flow of people as an undifferentiated group.

The simulation begins with a single, 2-D room at the start of an emergency. People in the room each choose a visible nearby exit and walk toward it. People navigate obstacles such as furniture, and, if crowded together, interact with one another. The simulation continues until everyone has reached an exit.

Individual humans (especially during an emergency) are concerned primarily with getting to an exit, greedily maximizing their own chance of survival. This model thus operates on a local level, allowing the overall global properties (such as total exit time and walking speed vs. congestion) to emerge.

Assumptions

Although human behavior is in general very complex, the modeling task is substantially simplified in a crowd during an emergency. Still, the primary weaknesses of this model lie in its restrictive and somewhat arbitrary assumptions.

1. All humans are aware of the emergency, and all attempt to exit.

2. People pick exits based on congestion (number of people near that exit), distance, and visibility—people cannot see through walls. Occasionally, people check for a better exit.

3. People are safe, and removed from the simulation, when they reach an exit.
4. People walk at 4 ft/s.
5. People may change direction and speed instantly.
6. If a person's intended path would pass through another person, that person stops and tries to go in some other direction.
7. People cannot walk through walls or furniture. For these purposes, people are treated as disks.
8. People plan a path around furniture to reach an exit.

Weaknesses of the Assumptions

Assumption 1 is not completely supported by the literature—not everyone is aware of or willing to leave during a real emergency.

Assumption 2 is more restrictive than reality—humans remember the location of out-of-view exits, and often "follow the crowd" to an exit that they can't see.

Assumption 3 neglects the finite person-handling capacity of many exits (e.g., narrow stairwells).

Assumption 4 neglects the very young, old, or handicapped, who may move more slowly, as well as the panic-stricken, who move more quickly.

Assumption 5 is contrary to basic physical principles, but significantly simplifies interactions.

Assumption 6 neglects people's sophisticated path planning, which allows us (usually!) to avoid walking into each other without stopping.

Assumption 7 treats people as hard, inelastic 2-D disks.

Assumption 8 neglects the panic-stricken, who may in fact run directly into furniture.

Example

Despite their disadvantages, these assumptions produce behavior that is remarkably crowd-like and consistent with research data.

We simulated 400 people leaving a 110 ft $\times$ 120 ft gymnasium, with the people initially distributed uniformly across the room. Moving independently, people quickly form groups near the exit. As people near the exit flow out, the groups shuffle around to bring more people to the exit. The model produces loose clumping around the exits, a natural result of people's desire to go towards the exit but with aversion to running into one another.

Human Interaction in Crowds

The overall result emerges only from our single assumption about how people interact: If your intended path will intersect another person, stop and try another (random) direction. We analyzed other potential ways for people to interact, but they produced decidedly non-human behavior.

We would have preferred to pick a deterministic interaction, because we would rather not have the results of our model change with each execution of the model. For each deterministic interaction we considered (e.g. if your path will intersect someone, go around them to your right), we could always find cases that created a circular-wait condition. This situation, in which object A waits for object B, who in turn waits on object A, is known to computer scientists as *deadlock*.

We can make the model give us the same results each time by using a pseudorandom (deterministic, but uniformly distributed and statistically uncorrelated) number generator to pick directions. Thus our randomized interaction scheme runs the same way each time, yet produces behavior that is reasonably similar to that of actual people—for example, they don't deadlock.

Test Scenarios and Model Validation

We used both the particle model and the graph flow model to evaluate exit time from test rooms. Each test room has one exit. One test room is 15 ft × 15 ft and has a 3-ft-wide exit in the center of the left wall. Each model was run repeatedly, using a different occupancy for each run.

The graph flow model was applied to a space that was equal in size to the particle flow model space.

After both models were executed repeatedly for different room occupancies, we obtained the results of **Figure 1**.

The results of both models (for this room and for others not shown here) appear to be very nearly linear for the rooms tested. However, the slope of the nearest linear approximation of each model differs. Since both models are driven by arbitrary parameters, specifically bandwidth for the graph-flow model and person radius for the particle model, it is not surprising that this difference exits.

We consider it significant that both models display similar trends. Each model was derived from an independent set of driving assumptions and data, but the behavior trends of the models are strongly correlated. This reflects positively on both models.

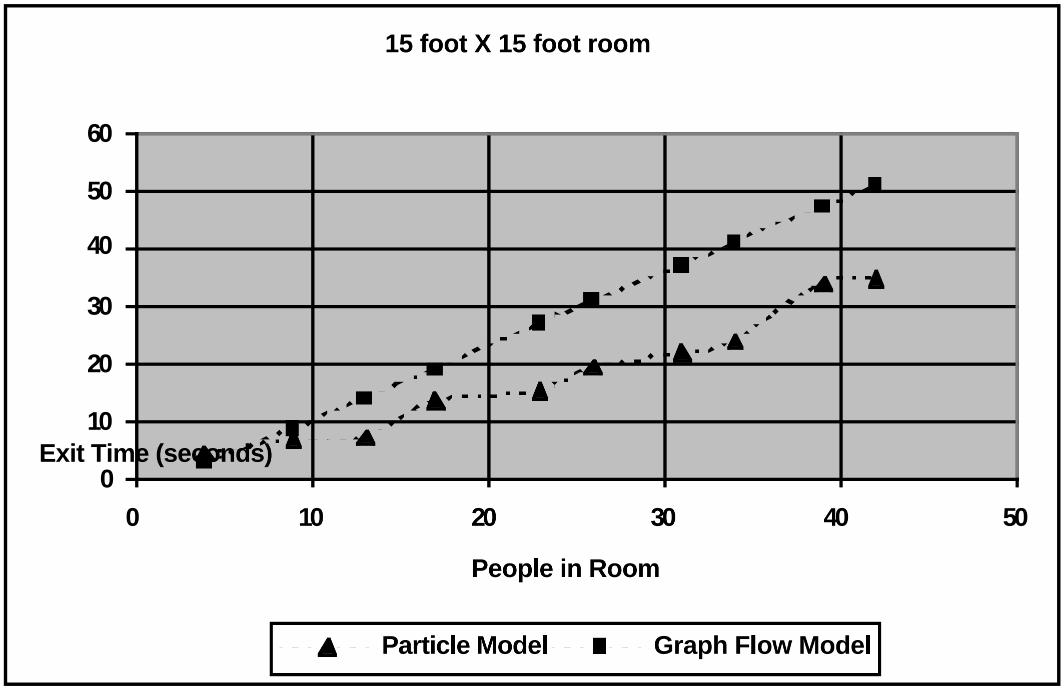

Figure 1. Results of simulations of the two models, for a 15 ft $\times$ 15 ft room.

Strengths/Weaknesses

The graph-flow model has several weaknesses. It treats people not as indivisible entities but as a fluid. Its results depend on an arbitrary choice of bandwidth, as well as on the source graph, and we did not address the problem of building this graph.

Human behavior in the graph-flow model is deterministic, but much of the mathematical structure of the graph-flow model is driven by actual research.

The particle simulator model has several weaknesses. Its results are a function of an arbitrary choice of radius. Its decisions are nondeterministic, so they can vary significantly for tiny input changes. The model is also occasionally subject to pathological, non-human behavior—for example, people occasionally lose sight of a nearby exit and travel a long distance to a visible exit.

The particle simulator model, however, also has several advantages. It models people as individual, indivisible entities. People can move independently of their neighbors. No assumptions need be made about the global flow in the room.

Conclusion

We have presented two models for determining how long it takes to evacuate a room. Despite their very different approaches and assumptions, both models substantially agree on our test cases.

Based on our test cases and the analysis of several local rooms, we find that a time-to-exit vs. initial population graph is nearly linear. The actual slope of the line depends on the layout of the room and the size and number of exits.

We expected the exit rate to decrease as more people tried to pack into the exits, but the actual exit rate (for both models) remains constant. We attribute this to the fact that exits become congested very quickly, even if only a few dozen people are attempting to exit. This is in agreement with our experience—it doesn't take many people (under a dozen) to block an exit.

The posted maximum occupancies of the local buildings that we simulated were adequate. At maximum occupancy, everyone evacuated in under 3 min, an acceptable time [Life Safety Code 1997, Section A–21–1.3].

To determine the maximum occupancy of a room, we suggest first consulting a fire marshal to determine the maximum acceptable time for evacuation. Then use the simulator to find the largest number of people who can escape in less than the maximum time. This is easy because the function relating the number of occupants to evacuation time is nearly linear.

References

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