Fly With Confidence

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Abstract

We develop a model to design a pile of cardboard boxes to cushion the fall of a stunt motorcycle; the kinetic energy of the motorcycle is consumed through breaking down the boxes.

We ignore the scattering effect of the boxes and begin by analyzing a single box. The energy to break it down has three components: the upper surfaces, the side surfaces, and the vertical edges. When big boxes are used, the upper surface provides the main supporting force; when small ones are used, the vertical edges play a great role.

We extend our analysis to the pile of boxes. Taking into account the maximum impulse that a person can bear, along with the camera effect and cost concerns, we determine the size of a box.

We conceive several stacking strategies and analyze their feasibility. We incorporate their strengths into our final strategy. We also examine several modifications to the box to improve its cushioning effect.

To validate our model, we apply it to different cases and get some encouraging results.

Assumptions

- *The stunt person and the motorcycle are taken as a system,* which we refer to as the motorcycle system or for brevity as the motorcycle. We ignore relative movement and interaction between them.
- *The motorcycle system is a uniform-density block.* We consider only the movement of its mass center, so we consider the motorcycle system as a mass particle.
- *The cardboard boxes are all made of the same material*, single wall S-1 cardboard 4.5 mm thick [Corrugated fiberboard . . . n.d.]

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- *The cardboard box is cubic and isotropic.*
- *Cost is proportional to the total surface area of cardboard.*

Symbols and Terms

Table 1.
Symbols.

Motorcycle parameters	
$\bar{a}$	mean acceleration during the landing
b	drag coefficient for the motorcycle plus rider
E	kinetic energy of the motorcycle when it hits the pile of boxes
H	height of the stage from which the motorcycle leaves
m	total mass of the motorcycle system
S	cross-sectional area of the motorcycle system, which we assume is 1.5 m^2
σ_m	standard deviation of v_0
σ_d	standard deviation of the direction of v_0
θ	angle between motorcycle direction and the horizontal
v_0	initial projection speed of the motorcycle
v	speed of the motorcycle when it lands on the pile
Box parameters	
E_{box}	energy that a single box can absorb during its breakdown
ℓ	edge length of the box
p_{pole}	pressure needed to break down the pole of a box
p_{side}	pressure needed to break down the side of a box
τ	coefficient for transfer of pressure from side to pole
ρ	density of energy absorption (DEA) of a box
V_{box}	original volume of a single box
Pile parameters	
h	height of the pile
S_{total}	total combined surface area of all boxes in the pile
V_{pile}	combined volume of all boxes in the pile
Cardboard parameters	
b_s	bursting strength of the cardboard
e_s	edgewise crush resistance of the cardboard
Rider parameter	
$a_{\max}$	the maximum acceleration that a person can bear.

Pole: the vertical edge of the box

Edge: the horizontal edge of the box

Corner: the intersection of poles and edges

Bursting strength: the maximum pressure on the cardboard before it bursts

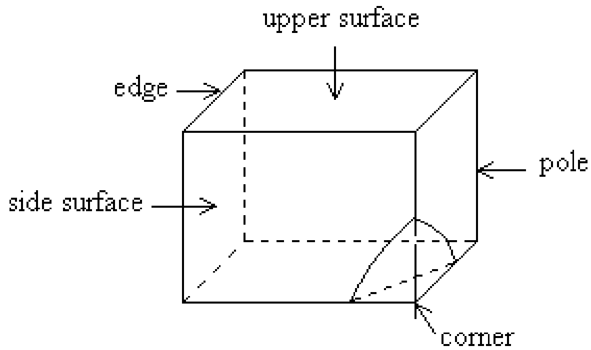

Figure 1. Illustration of terminology.

Edgewise crush resistance: the maximum force on unit length of the edge before the side surface crushes

Piercing Strength: the energy of an awl piercing the cardboard

Problem Analysis

Our primary goal is to protect the stunt person. Having ensured this, we should minimize the height of the pile (to get a good film effect) and minimize the total superficial area of boxes (to lower the cost).

From the analysis of the jump, we get the pile area and kinetic energy of the motorcycle system. Then we consider the cushion process. Since the maximal impulse a person can bear is 12 kN [UIAA Safety Regulations n.d.], the problem is to ensure that the force exerted on the person stays within the range during the cushioning.

Consider the motorcycle landing in a pile of cardboard boxes. The boxes provide a supporting force to decelerate it. We examine the cushioning effect of both big boxes and small boxes. In our modeling, we focus on energy and reduce the problem to analyzing the energy to break down the boxes. We search for the relation between this energy and the size and number of the boxes. We then improve the cushioning by changing stacking approaches and modifying the box.

Model Design

The Jump

Local Assumptions

- The speed v and direction of the motorcycle are random variables that are normally distributed.

- The standard deviation (σ_m) of the magnitude of v_0 is $0.05v_0$.
- The standard deviation (σ_d) of the direction of v_0 is $5°$.
- The stunt is considered safe if the probability of landing on top of the box pile is more than 99.7%.

Local Variables

- v_0: initial speed,
- θ: angle between velocity and horizontal direction,
- H: height of stage,
- m: mass of motorcycle system.

We first examine the path that the motorcycle follows. Taking the air resistance into account, we get two differential equations

$$\frac{dv_x}{dt} = -\frac{bv_x}{m}, \qquad \frac{dv_y}{dt} = \frac{bv_y}{m} - g, \tag{1}$$

where

$$v_x = v_0 \cos\theta, \qquad v_y = v_0 \sin\theta, \qquad \text{and} \qquad \frac{dx}{dt} = v_x, \qquad \frac{dh}{dt} = v_y. \tag{2}$$

The drag coefficient b can be obtained by computing the mass and terminal speed of a skydiver [Halliday et al. 2001]. For $v = 60$ m/s and $m = 200$ kg, we have $b \approx 30$ N/(m/s).

A typical elephant is 3 m high and 5 m long, with a trunk 2 m long [Estes 1999]. To jump it over safely, we assume that the stage is higher than the elephant's trunk reach, which is 5 m.

Solving equations **(1)–(2)** with Matlab, we get a quasiparabola. Air resistance makes a difference of no more than 5%, so we neglect it. With height difference H and initial speed v_0, neglecting air resistance, the landing point is $v_0\sqrt{2H/g}$ from the projecting point.

Determine the Area of the Pile

The area of the box pile must be large enough for the actor to land on, that is, the upper face of the pile must capture more that 99.7% of falls. (Very few may crush into the side face, which is still quite safe.) To meet this criterion, the surface must extend to cover six standard deviations (three on each side of the mean) of both the projection speed v_0 and its direction [Sheng et al. 2001].

We calculate the landing point in the combinations of $v_0 \pm 3\sigma_m$ for the speed and $\pm 3\sigma_d$ for deviation from straight. The resulting length is 3.03 m and the width is 5.23 m. Taking the size of the motorcycle into account, the length needs to be approximately 4.5 m and the width 6 m. Since the motorcycle has a high horizontal speed, the box pile must in fact be longer to cushion the horizontal motion.

The Cushioning Process

Definitions

Big box: The cross-sectional area is much larger than that of the motorcycle, so the motorcycle interacts with only one box when hitting the cushion. We ignore the deformation and resistance of edges and poles. Thus, we need to consider only the interaction between motorcycle and the upper surface of that box.

Small box: The cross-sectional area is smaller than or comparable to that of motorcycle, so the motorcycle interacts with a number of them simultaneously. In this situation, the edges and poles play a great role in cushioning.

Since big and small boxes have different interactions, we analyze the two situations separately and compare their cushioning effect to determine the box size.

Analysis of Big Box

The motorcycle has considerable velocity when it hits the upper surface of the box, exerting a force on the upper surface. Because the corrugated cardboard is to some extent elastic, it first stretches a little. After some elongation, it goes into the plastic region and finally ruptures. This process is too complicated for us to calculate the total energy that results in the final rupture, so we reduce it to the following extreme situation.

The area of the upper surface of the cardboard is infinitely large and the motorcycle can be taken as a point compared with the cardboard. The piercing strength of the corrugated paper is 4.9 J [Corrugated fiberboard ... n.d.]. The total energy of the motorcycle is $E = mv^2/2$; since the order of magnitude of v is 10 m/s, the energy is approximately 10^4 J. Thus, the kinetic energy of the motorcycle is about 10^3 times the energy needed to pierce the cardboard. So the cardboard is easily pierced and provides little cushioning.

Although we examine the extreme situation, we can safely reach the following conclusion: The bigger the box, the easier for the motorcycle to penetrate the upper surface. In fact, as the cardboard becomes smaller, the motorcycle cannot be taken as a point again, so the force distributes over the surface, making it more difficult to penetrate the upper surface.

When the box becomes even smaller, the edges and poles of the box provide great support. But the cost increases at the same time.

Small Box Model

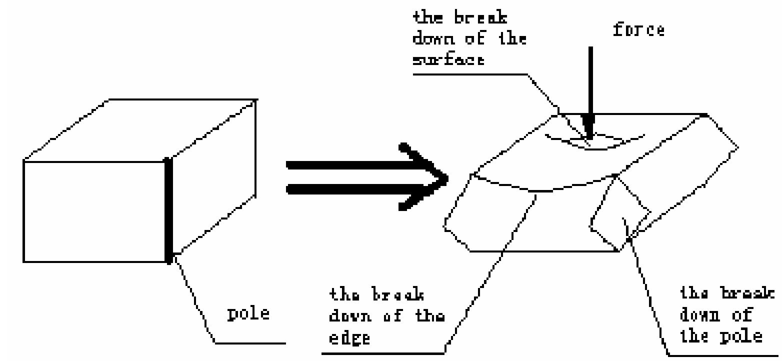

Figure 2. Energy to break down a box.

Energy to break down a box. The breakdown of a box consists of three processes:

- breakdown of the upper surface,
- breakdown of the side surfaces, and
- breakdown of the poles.

After all three components break down, the box is completely damaged and cannot provide any cushioning. The the total energy required to break down a box is

$$E_{\text{box}} = E_{\text{upper}} + E_{\text{side}} + E_{\text{pole}}.$$

After some analysis (see **Appendix**), we find that E_{upper} and E_{side} are rather small compared withE_{pole}, so

$$E_{\text{box}} \approx E_{\text{pole}}. \tag{3}$$

We cannot find any data for calculating E_{pole}, so we make a rough estimate. Our analogy is to steel, for which we have data. We obtain the relationship between the maximum pressure to break the pole and the side:

$$p_{\text{pole}} = \tau p_{\text{side}},$$

where p_{pole} is the breakdown pressure for the pole, p_{side} is the breakdown pressure for a side surface, and τ is the transfer coefficient.

The breakdown pressure for a side surface is inversely proportional to the length ℓ of a side, so with edgewise crush resistance of the cardboard e_s we have

$$p_{\text{side}} = \frac{e_s}{\ell}. \tag{4}$$

Height of the pile. The motorcycle lands in a pile with an initial velocity and ultimately decelerates to zero, trapped in this pile. During that process, the force exerted on the motorcycle must be smaller than the maximum force that a person can bear; otherwise, the stunt person would be injured. Since 12 kN is the threshold, we consider 6 kN the safety bound. Thus, a 60 kg person can bear a maximum acceleration of $a_{\max} = 6000/60 = 100\ \mathrm{m/s}^2$. We want the mean acceleration to be smaller than this: $\bar{a} \leq a_{\max}$; we use mean acceleration because the cushion process has approximately constant deceleration. Thus, using kinematics, we obtain

$$\bar{a} = \frac{v^2}{2h} \leq a_{\max}, \qquad \text{or} \qquad h \geq \frac{v^2}{2a_{\max}}.$$

Thus, we let the pile height h be $v^2/2a_{\max}$, so that the motorcycle just touches the ground when it stops. In terms of the kinetic energy $E = mv^2/2$ of the motorcycle, we have

$$h = \frac{E}{ma_{\max}}. \tag{5}$$

Size of Boxes

To see how a box cushions the motion of the motorcycle, we define the *density of energy absorption* (DEA) of a box as

$$\rho = \frac{E_{\text{box}}}{V_{\text{box}}},$$

where E_{box} is the energy that the box can absorb during its breakdown and V_{box} is the original volume of the box. This density reflects the average cushioning ability of the box, and ρ can be thought of as the proportion of energy that is absorbed.

In a homogenous pile, all the boxes have the same DEA. The total energy that the pile absorbs is ρV_{pile} for the collapsed boxes. The height of the stack of boxes is h and the cross-sectional area of ones collapsed by the motorcycle is S, so

$$E = \rho V_{\text{pile}} = \rho S h = \rho S \cdot \frac{E}{ma_{\max}} \tag{6}$$

from **(5)**. Cancelling the Es, we get

$$\rho = \frac{ma_{\max}}{S}.$$

We assume that the work done in breaking down a single box is proportional to p_{pole}, with proportionality coefficient k. In breaking down the pile of boxes, this pressure is exerted across an area S and through a distance h, for total work $kp_{\text{pole}}Sh$. Equating this work to absorbed energy $E = \rho Sh$, we have

$$\rho Sh = E = kp_{\text{pole}}Sh,$$

so

$$\rho = \frac{ma_{\max}}{S} = kp_{\text{pole}};$$

and substituting $p_{\text{pole}} = \tau e_s/\ell$, we get

$$\ell = \frac{k\tau e_s}{ma_{\max}}\,S, \qquad \text{together with the previous} \qquad h = \frac{v^2}{2a_{\max}}. \tag{7}$$

Substituting

$$m = 200\text{ kg}, \quad a_{\max} = 100\text{ m/s}, \quad v = 13\text{ m/s},$$
$$k = 0.5, \quad e_s = 4 \times 10^3\text{ N/m}, \quad S = 1.5\text{ m}^2, \quad \text{and} \quad \tau = 1.5,$$

we get

$$\ell = 0.225\text{ m}, \qquad h = 0.845\text{ m}.$$

Number of Boxes

The landing area, 4.5 m by 6 m, needs to be extended to take the horizontal motion into account. The maximum length h that the motorcycle system can penetrate is sufficient for cushioning horizontal motion, so the pile should be $(4.5 + h) \times 6 \times h$ (meters) in dimension. From this fact, we can calculate the number of boxes needed in the pile.

From **(7)**, we have

$$h = \frac{mv^2}{2k\tau e_s S}\,\ell.$$

Thus, we know the dimensions of the pile:

$$N_h = \left\lceil \frac{h}{a} \right\rceil = \left\lceil \frac{mv^2}{2k\tau e_s S} \right\rceil = 4, \qquad N_w = \left\lceil \frac{6}{\ell} \right\rceil, \qquad N_l = \left\lceil \frac{4.5 + h}{\ell} \right\rceil. \tag{8}$$

Numerically, we get

$$N_h = 4, \qquad N_w = 27, \qquad N_l = 24;$$

and the total number of boxes needed is $N = 4 \times 27 \times 24 = 2592$, with total surface area

$$S_{\text{total}} = 6a^2N = 6 \times 0.225^2 \times 2592 \approx 780\text{ m}^2.$$

Next, we analyze the change in cost if we alter the edge length of the boxes. As an approximation, we use

$$N = \frac{V_{\text{pile}}}{V_{\text{box}}} = N_h \cdot \frac{6 \times (4.5 + h)}{\ell^2} = \frac{108 + 24h}{\ell^2} = \frac{108 + \frac{12m\ell v^2}{k\tau e_s S}}{\ell^2}.$$

We have calculated for minimum h and ℓ. If we increase ℓ, that is, use bigger boxes, we need fewer boxes but the total cost increases, since

$$S_{\text{total}} = 6a^2 N = 6 \cdot \left(108 + \frac{12m\ell v^2}{k\tau e_s S}\right) \approx 648 + 540\ell.$$

The number of layers is 4 regardless of edge length; but for increased ℓ, the pile is lengthened to ensure that the motorcycle will not burst out of the pile due to the reduction of DEA.

In conclusion, smaller boxes lower the pile and are cost efficient; from computation, we should choose the minimum size 22.5 cm on a side, and need 2592 boxes.

Stacking Strategy

In the above discussion, we assume that we stack the boxes regularly stacked (no overlapping). We examine several other stacking strategies.

Pyramid Stack

Pyramid stacking stacks fewer boxes on top and more boxes at the bottom. When a stress is exerted on the pile, it is divided into the normal stress and shearing force along the slopes so that the downward stress diverge [Johnson 1985]. Furthermore, a pyramidal stack is more stable than a regular stack.

Mixed Stack

Mixed stacking is to stack boxes of different sizes in the pile; a common practice is to lay big boxes on the top and small boxes at the bottom.

For a regular stack, we considered the cushioning a motion with constant acceleration because we assumed that the supporting force provided by boxes is constant. However, this is not the case; generally speaking, the force is larger in the first few seconds, so decreasing the supporting force in the beginning is good for cushioning.

We have shown that big boxes provide less support than small boxes. So the mixed stack can be characterized as softer on the top and stiffer at the bottom. In fact, this kind of pile is similar to a sponge cushion, which is often used in stunt filming, high jump, pole vault, etc. In addition, cardboard is superior to a sponge cushion in this situation; a sponge cushion is too soft, so the motorcycle may lose balance.

Sparse Stack

Sparse stacking reserves a space c between adjacent boxes. Because each box absorbs a constant amount of energy, the spaces decrease the density of energy absorption ρ. Thus, given the initial kinetic energy, the height of the pile must increase to compensate for the decrease in ρ. Since $\rho Sh = E$ from **(6)**, we have

$$\frac{h_{\text{new}}}{h_{\text{old}}} = \frac{\rho_{\text{old}}}{\rho_{\text{new}}} = \frac{1/\ell^2}{1/(\ell+c)^2} = \left(1 + \frac{c}{\ell}\right)^2.$$

So, the cushioning distance h is proportional to the square of $(1 + c/\ell)$. The sparse stack saves some material at the cross section but increase the cushioning distance. With no change in base area, for rectangular stacking the surface area is constant, while in a pyramid stacking it decreases.

Crossed Stack

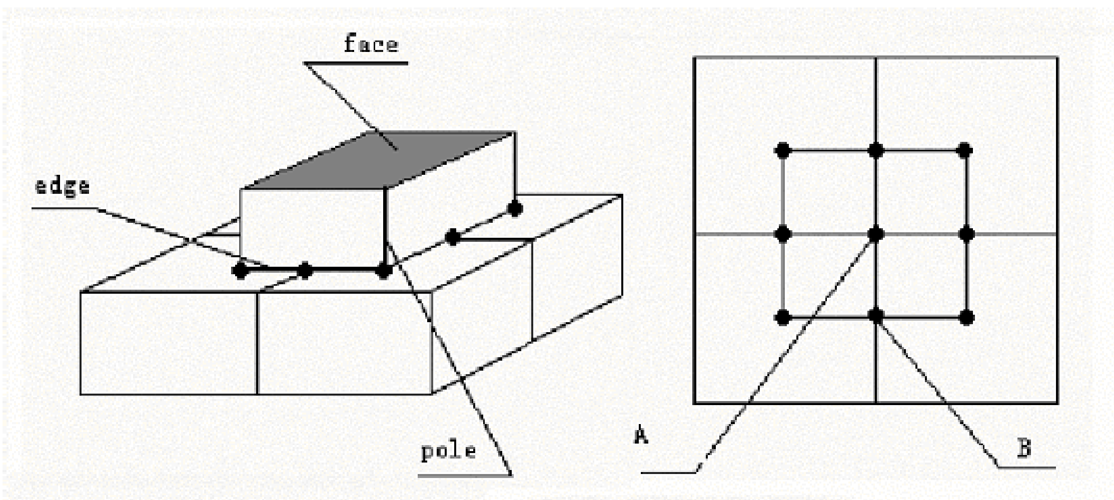

Figure 3. Crossed stacking: side and top views.

Crossed stacking is to lay the upper boxes on the intersection of the lower boxes, as shown in **Figure 3**. There are two kinds of interactions on the surface:

- vertex-to-face: the interaction between pole of the upper box and the surface of the lower box. Because the pole is much stronger than the edge and surface, the pole will not deform but the upper surface may break down.
- edge-to-edge: the interaction between edge of the upper box and the perpendicular edge of the lower box. The two edges both bend over. **Figure 4** shows the deformation of the boxes.

To determine whether crossed stacking is better than regular stacking, we compare the pile height of the two approaches.

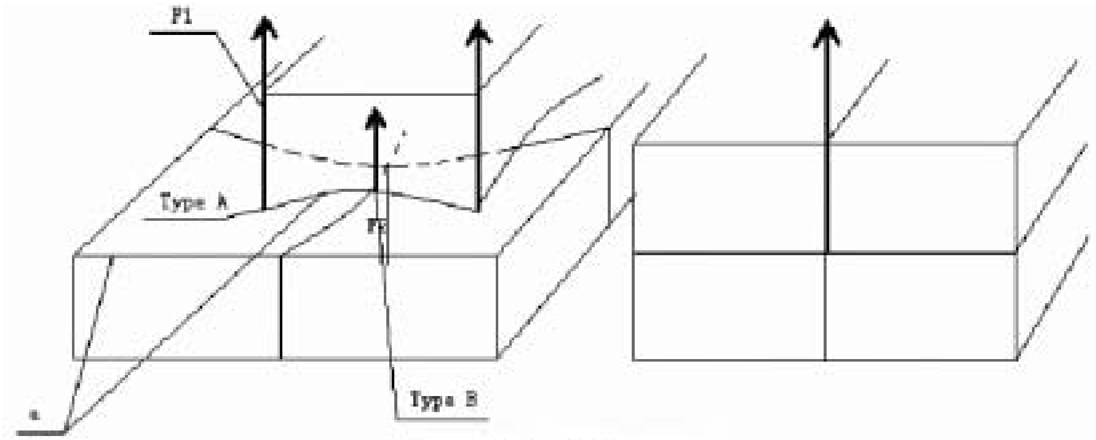

Figure 4. Deformation of boxes in edge-to-edge interaction.

The force provided by vertex-to-face structure is $F_1 = b_s a^2$; the force provided by edge-to-edge structure is $F_2 = e_s a$. Thus, introducing the same analysis as above, the pile height of the crossed stack is

$$h = \frac{mv^2\ell^2}{8kS(F_1 + F_2)} = \frac{mv^2\ell^2}{8kS(e_s\ell + b_s\ell^2)} = \frac{1}{8(e_s + b_s\ell)} \cdot \frac{\ell mv^2}{kS},$$

compared to the pile height of the regular stack,

$$h' = \frac{m\ell}{2k\tau e_s S} \cdot v^2 = \frac{1}{2\tau e_s} \cdot \frac{\ell mv^2}{kS},$$

For all edge lengths, we have $h < h'$: Crossed stacking needs a smaller pile height than regular stacking.

The Final Stacking Strategy

Synthesizing the analyses of several stacking approaches, we present our final stack approach (**Figure 5**). The far end of the pile is a quasislope, and big boxes are on the top while small boxes are cross-stacked at the bottom. The slope is in the same direction as the initial velocity of the motorcycle. Therefore, the stress that the motorcycle exerts on the pile dissipates along this direction.

Modifications to the Box

In search for a better solution, we may make some modifications to the boxes to make the pile more comfortable, as well as lower and cheaper.

Changing Edge Ratio

We have assumed the box to be cubic; we now investigate the energy absorbing properties of a box that is not cubic.

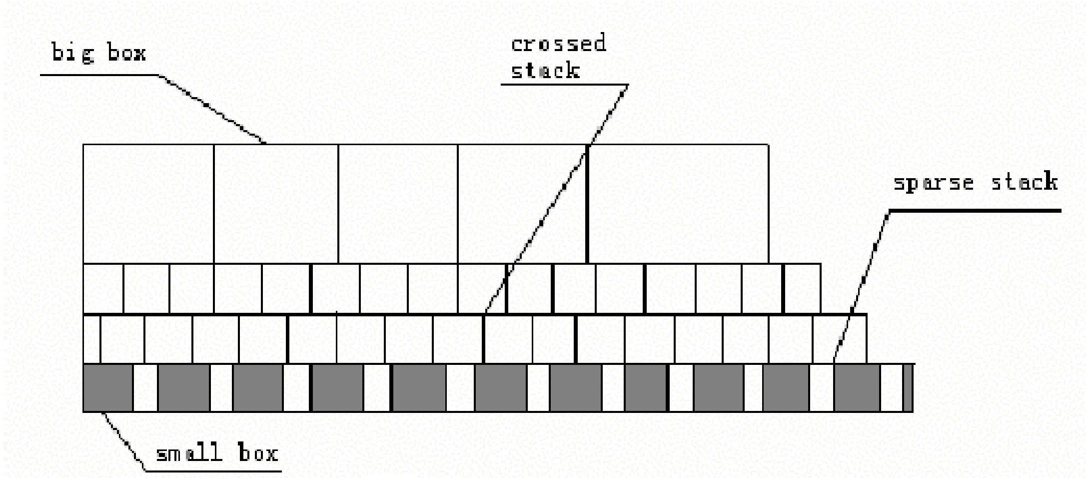

Figure 5. The final stacking strategy (cross-sectional view).

According to Wolf's empirical formula [Sun 1995, 4] we have

$$P = \frac{1.1772 P_m \sqrt{tz}(0.3228 R_L - 0.127 R_L^2 + 1)}{100 H_0^{0.041}}, \tag{9}$$

where

- P is the stiffness of the box,
- P_m is the edgewise crash resistance of cardboard,
- t is the thickness of cardboard,
- z is the perimeter of the box,
- R_L is the ratio of the length and width of the box, and
- H_0 is the height of the box.

Because the exponent of H_0 is 0.041, the height makes little difference to the stiffness of the box. Thus, we should choose a small value for H_0, since a lower height means lower center of gravity and thus better stability for the pile.

Stuffing the Box

After the collapse process, the box no longer provides any supporting force. To lengthen the effective time, we may add elastic material in the box, such as foam and corrugated paper. They must be soft and loose enough, or else they may prevent the box from collapsing or occupy too much space when the box breaks down, defeating one of the reasons why a box pile is preferable to foam or a spring cushion.

Adding Supporting Structures

Apart from stuffing the box, we may also add supporting structures, such as vertical struts. They strengthen the box by preventing tangential displacement and supporting the upper surface of the box. There are different ways, such as triangular or square buttresses in the corners. These supporting structures bring considerable improvement to the mechanics while the total cost increases insignificantly. Because these structure significantly strengthen the box, the size of the box should be increased.

Other Considerations

To make the landing comfortable, the upper layers should be more elastic while the lower layers stiff. To accomplish this, we can put taller boxes with stuffing in the upper layers and shorter ones with supporting structures in the bottom layers.

Generalizing the Result

The General Process

We assumed that a 60 kg stunt person riding a 120 kg motorcycle jumps horizontally from a 5 m stage at 10 m/s. Now we offer a more general statement.

Let the masses of the rider and the motorcycle be m_r and m_m; the new a and m are $6kN/m_r$ and $(m_r + m_m)$. We also calculate the values of final speed v and kinetic energy E, using the initial speed v_0 and stage height H:

$$E = mgH + \tfrac{1}{2}mv_0^2, \qquad v = \sqrt{\frac{2E}{m}}.$$

We can then apply **(7)** to calculate the edge of box and the height of the pile, and we get the dimensions of the pile from **(8)**.

We can also make small adjustments by changing the structure of boxes and the pile, according to the approaches introduced in the section on stacking strategy.

Quick Reference Card for the Stunt Coordinator

To see how our model works with different circumstances, we suppose that a stunt team has an actor (65 kg) and an actress (50 kg) and four motorcycles, namely, Toyota CBX250S, Jialing JH125, Yamaha DX100, and Yamaha RX125 [Zhaohu n.d.].

We summarize the results in **Figure 6** for the stunt coordinators' quick reference in practical filming work. Note that the team needs only two types of boxes. Sensitivity analysis—varying v_0, H_0, m_r, and m_m by 10%—shows that h and ℓ are not sensitive to small changes in these variables.

The box size

Mass of Actor (kg)	Mass of Motorcycle (kg)	Basal Area (m^2)	Box size (cm)	Alternative	
				Box Size (cm)	Adjustment
50	129	1.342	18.7	22.5	Sparse Placement
50	105	1.500	24.2	22.5	Buttress
50	97	1.372	23.3	22.5	Stuffing
50	83.5	1.408	26.4	28.0	Sparse Placement
65	129	1.342	22.5	22.5	
65	105	1.500	28.7	28.0	Stuffing
65	97	1.372	27.5	28.0	Crossed Placement
65	83.5	1.408	30.8	28.0	Buttress

The pile height

Actor's Mass (kg)	Stage Height (m)	Initial Velocity (m/s)	Pile Height (m)		Actor's Mass (kg)	Stage Height (m)	Initial Velocity (m/s)	Pile Height (m)
65	15	15	2.81		50	15	15	2.16
65	15	5	1.73		50	15	5	1.33
65	5	15	1.75		50	5	15	1.35
65	5	5	0.67		50	5	5	0.51

Figure 6. Quick reference card for the stunt coordinator.

Model Validation

Validation of Homogeneity Assumption

Our main model is based on the assumption that when the boxes are small, we consider the box pile as a homogenous substance. Now we use Wolf's empirical formula **(9)** to validate our assumption.

Because we consider all the boxes as cubes and because the exponent of H_0, 0.041, is so small, the denominator can be considered as a constant 100. We simplify the expression to

$$P = \frac{1.1772 \cdot 5880 \cdot \sqrt{0.0045 \cdot 2\ell \cdot 100}(0.3228 - 0.127 + 1)}{100},$$

so that

$$p = \frac{P}{(100\ell)^2} = 7.85(100\ell)^{-\frac{3}{2}};$$

the derivative of p is $dp/d\ell = -11.78 \times 10^{-3} a^{-\frac{5}{2}}$. We graph the derivative in **Figure 7**. For $\ell > 5$ cm, we have $dp/d\ell \approx 0$, so p is independent of ℓ when $\ell > 5$ cm. So we have proved our assumption of homogeneity.

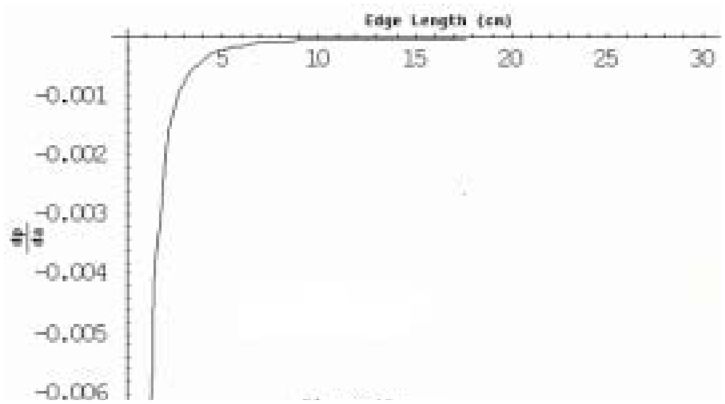

Figure 7. Derivative of p as a function of edge length ℓ.

Validation of Small Box Model

Consider a single box with generalized dimensions $\ell \times w \times h$. We assume that the stiffness is proportional to the average force that the box is subjected to when it is collapsed, with proportionality coefficient k. Thus the work it does, or the energy that it absorbs from the colliding object, is $W = kPh$. Equating W with E, we have $kPh = \rho Sh =$
$rho\ell wh$, or $kP = \rho\ell w$. We use Wolf's formula **(9)** for P and minimize the surface area of the box subject to the resulting constraint on ℓ, w, and h:

$$\begin{array}{ll} \text{minimize} & S = 2(\ell w + wh + \ell h) \\ \text{subject to} & k \cdot \left(\dfrac{1.772 P_m \sqrt{t(\ell + w)}(0.3228 R_L - 0.1217 R_L^2 + 1)}{100 h^{0.041}} \right) = \rho \ell w. \end{array}$$

This optimization model is nonlinear, so we cannot easily get an analytical solution. However, adding our assumption that the box is a cube significantly simplifies the constraint equation. We wrote a program to search for the optimum value and found that the minimum surface area is 2.1×10^3 cm^2 when the edge length is 19.1 cm.

This result is consistent with our earlier one, $S = 3.0 \times 10^3$ cm^2 and $a =$ 22.5 cm, confirming the correctness of our model.

Strengths and Weaknesses

Strengths

- We carefully built our model on the limited information that we could find; some of the data crucial to our solution are from cardboard company advertisements. Although there are not enough data available for us to justify our model fully, we compared our result with available data. We also visited such Websites as `http://www.stuntrev.com`, and examined stunt videos.

We found that most of their cushioning facilities agree well with our model. So, we believe our result has practical value.

- We abstract the pile of boxes into a simple homogenous model, which proves reasonable.
- We apply careful mechanical analysis in our model design. Given reasonably accurate data, the model can provide a good result.
- The model examines various stacking approaches and modifications to the boxes. It helps to find the best way to design a pile of box for cushioning.
- We generalize the model to different situations and get good results.

Weaknesses

- We ignore the scattering of boxes when they are crushed, which may contribute to cushioning.
- The model is only as accurate as the data used, but some data are dubious. We are forced to obtain a crucial data by analogy with a material (steel) of similar structure for which data are available.
- The number of boxes is very large (more than 2,000); this may be caused by our choice of the thinnest corrugated cardboard.
- We ignore air resistance; the error introduced is about 5%.

Appendix

Corner Structure

According to structure mechanics, the corner structure is a stable structure. That is why the "T shape L shape and I shape structure" are used widely in buildings and bridges [Punmia and Jain 1994].

We study the extension of the side surface when the crush happens. We consider a small bending of the pole after the outer force acts on the upper surface (see **Figures 8–9**)

The force F_p is composed of two equal forces F_{p1} and F_{p2} produced by the extension of the side surface, so $F_p = \sqrt{2}F_{p1} = \sqrt{2}F_{p2}$. The flexibility of the cardboard is described by Young's modulus. We have

$$E \cdot \frac{\Delta L}{a} = \frac{F_{p1}}{A},$$

where ΔL is the small extension of the side surface and A is the cross-sectional area (thickness plus side length) of the cardboard.

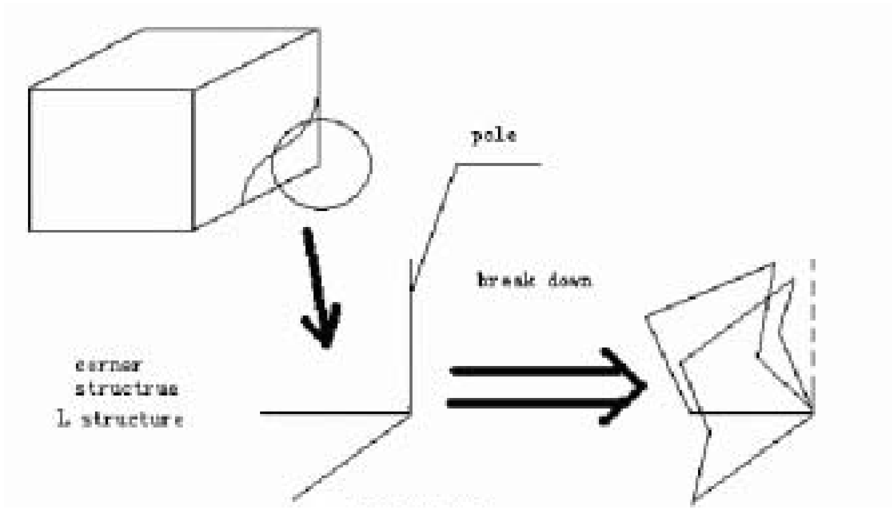

Figure 8. Breakdown of a corner structure.

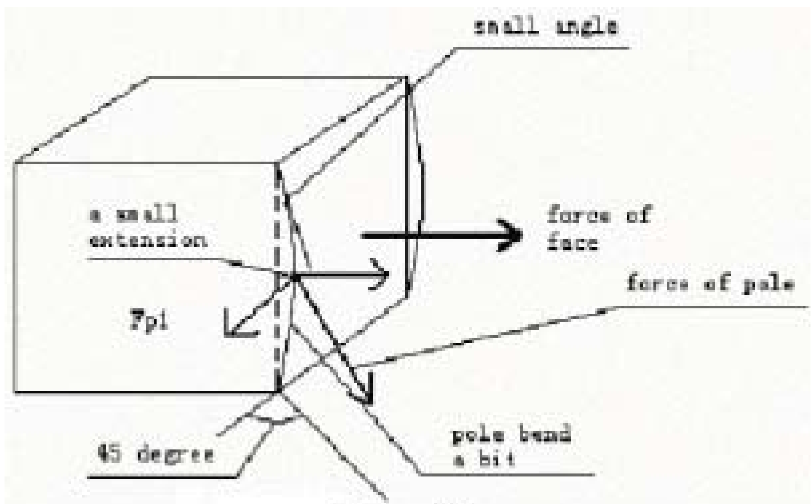

Figure 9. Bulging of a side under pressure on the top surface.

We do not find Young's modulus for corrugated cardboard on the handbook; but from the values for similar materials, we suppose that is is about 0.5 MPa. Building the force balance equation, we assume $F_p = F_f$; then

$$\Delta L = \frac{F_f a}{AE}; \qquad \max F_f = e_s a^2; \qquad \Delta L = \frac{e_s a^3}{AE}$$

For $a = 10$ cm, $A = 2.5$ cm^2, and $e_s = 4 \times 10^3$ Pa, we get $\Delta L = 3.2$ mm.

This result means that the maximum force F_f that can make the side surface deform can bring only a tiny (3.2-mm) extension to the pole. This is a strong support for our supposition that $F_p >> F_f$, which supports **(3)**, $E_{\text{box}} \approx E_{\text{pole}}$.

The Determination of τ

To compute the p_{pole} value of is an enormous difficulty because of the lack of data. As in the iron industry, we set $p_{\text{pole}} = \tau p_{\text{side}}$, where τ is a constant depending on the material.

We determine τ by analogy to the method in the iron industry, for which there is theory about the axial compression of column with cross-sectional shapes that are rectangular or L-shaped. The former has strength parameter 0.77 to 0.93 while the latter has strength parameter 0.56 to 0.61; the ratio τ of the two is between 1.26 and 1.66. The average value 1.46; for simplicity, we let $\tau = 1.5$. [Huang et al. 2002].

References

Ai, Zhaohu. 1999. *Imported and Domestic Motorcycles*. People's Posts and Telecommunications Publishing House.

Corrugated fibre board for export products GB 5034-85. 2001. `http://main.pack.net.cn/wlz/wlz_article/fl2.htm` .

Estes, Richard D., and Kathryn S. Fuller. 1999. Elephant, *Loxodonta africana*. `http://www.nature-wildlife.com/eletxt.htm` . In *The Safari Companion: A Guide to Watching African Mammals, Including Hoofed Mammals, Carnivores, and Primates*, by Richard D. Estes, Daniel Otte, and Kathryn S. Fuller. White River Junction, VT: Chelsea Green Publishing Company.

Halliday, David, Robert Resnick, and Kenneth S. Krane. 2001. *Physics*. 5th ed. Vol. 1. New York: Wiley.

Huang, Chengkui, Dan Wang, and Bo Cui. 2002. Study of ultimate axial compression of the special-shaped columns. *Journal of Dalian University Technology* 42 (2): 213–217.

Johnson, K.L. 1985. *Contact Mechanics*. New York: Cambridge University Press.

Leavitt, Steven. n.d. The Art of Stunts. `http://www.stuntrev.com` .

Punmia, B.C., and A.K. Jain. 1994. *Strength of Materials and Mechanics of Structure*. New Delhi: Lakshmi Publications.

Sheng, Zhou, Shiqian Xie, and Chenyi Pan. 2001. *Probability and Statistics*. Higher Education Press.

Sun, Cheng. 1995, *Package Structure Design*. China Light Industry Press.

UIAA Safety Regulations. n.d. International Mountaineering and Climbing Federation. `http://www.cc.nctu.edu.tw/~mclub/meichu/teach/equip/equipment2_1.html` .