The Snowplow Problem

Ronald Chernak
Laura E. Kustiner
Larry Phillips
Southern Oregon State College
Ashland, OR 97520
Advisor: K.R. Yates

Summary

Our problem is to find an efficient way for two plow-trucks to sweep snow from all the county roads of a snow-removal district in Wicomico County, MD. We combine the original assumptions of the stated problem with several new assumptions, to clarify as well as simplify the problem. We postulate and analyze various plowing schemes, with the goal of minimizing plowing time and total distance traveled by the plows.

Our first scheme, the priority scheme, is more realistic in that it plows main thoroughfares first and minor roads last. There are two cases, which depend on blade size and the number of passes needed to clear the roadway.

Our second scheme, the north/south scheme, divides the area to be plowed into two essentially equal regions, each of which is plowed by a single truck with minimal backtracking. Again, we analyze by cases.

Based on our analysis and use of efficiency statistics, we conclude that the north/south scheme is more efficient. It divides the plowing task equally between the two trucks, and it minimizes the amount of time spent traveling on previously cleared roads, a key measure in our definition of efficiency.

Assumptions

- There are 2 state highways: one west of Wicomico County, and one south of Wicomico County. The dotted line north of Wicomico County is a county line, not a highway or thoroughfare.
- The state highways are clear of snow and remain clear of snow while the plows are at work.
- Every county road is a paved, two-lane, two-way road.
- There is no new snowfall after the plows begin their work until their task is complete.

- The weather is clear and cold while the plows are sweeping, so the trucks can be seen in plenty of time by any traffic, in order to avoid accidents or interference.
- All roads will need to be plowed; that is, any snow on the roads will not melt completely before the plows arrive.
- The topography of Wicomico County is essentially flat, based on a 1969 US Geodetic Survey map of Salisbury, MD.
- All roads can be plowed with the same ease. We need not consider mountainous areas, etc.
- Plowing will begin early in the morning, say 4 A.M., to minimize traffic interference.
- Light traffic on unplowed roads will be moving slowly, if at all, to minimize the risk of accidents.
- The plows will have traction devices. Hence, we need not consider any problems with ice or snow on the roads.
- The plows leave from the garage at the same time and, upon completion of their tasks, return to the garage.
- There will be two workers per plow, a driver and a navigator. Both plows will have maps of Wicomico County as well as specific instructions on the routes to be followed. The plows are also in constant radio contact with each other and the garage.
- The plows will neither break down nor get stuck.
- The trucks may use the state highways to access the county roads.
- The road intersections require no special plowing techniques. The trucks do not need to stop at intersections and may turn right, left, or around in the intersections without constraint.
- On clear roads, the trucks will travel at a constant speed. We assumed this to be 40 mph. On unplowed roads, the trucks will travel at half of this speed (20 mph) due to snow, ice, and speed constraints of the traction devices. This is slow enough to allow the trucks to negotiate turns without difficulty. The rate is justified by the weather conditions (ice, snow), recommendations for the use of traction devices, the possibility that the trucks will be sanding/graveling the road as well as plowing, and the fact that we want to maximize fuel efficiency.
- The trucks can change speeds instantaneously.

- Each truck has a 60-gallon fuel tank and averages 3 miles per gallon. This allows each truck to cover 180 mi before stopping to refuel.
- If refueling is necessary, we will also change the transportation workers manning the plow. Assuming the truck has traveled its maximum distance (180 mi) at its minimum speed (20 mph), this would mean that the original workers will have had a (maximum) 9-hour shift, justifying our need to replace them. We assume that time spent at the garage for refueling, maintenance, shift change, etc., is approximately 0.5 hours.
- The trucks work with the flow of traffic.
- Each truck is equipped with a plow blade oriented at an angle of 45° with respect to the front of the truck, which sweeps snow to the right side of the road. This angle will maximize both path width and the amount of snow cleared to the right of the truck.

The Model

We divide the model into two cases, based on the size of the plow:

- **CASE A**

  The blade is 14 ft wide, so the width of the path cleared is approximately 9.9 ft. We assume this to be sufficiently wide for all vehicles, given that an average lane is approximately 10 ft wide. So we will make only two passes per road, one in each direction. The first pass will cover the inner portion of the lane, ignoring an approximately 1-ft segment of roadway on the right.

- **CASE B**

  The blade is 12 ft wide, so the width of the path cleared is approximately 8.5 ft—wide enough for an automobile or small truck, but not for a large truck. Hence, we will need to make four passes per road, two for each lane. The first pass will cover the inner portion of the lane, while the second will cover the outer (shoulder area) portion of the lane. We also assume that the second pass will be made at the same speed as the first: 20 mph if the shoulder area is snow-covered, and 40 mph if the plow has already swept the shoulder area. Although there will be only about 1.5 ft of roadway left to be cleared on a second pass (given that a lane is approximately 10 ft wide), the driver must take care so as not to run off the road, hit any obstacles, etc., thus justifying that the rates are the same for the second pass. A final assumption is that after the first pass over the plow route, the trucks will return to the garage, refuel, and then begin the second pass. This procedure is to avoid the possibility that one or both of the trucks run out of fuel during the second pass.

Problem Analysis

Our task is to maximize the efficiency of the snowplows. But how does one define efficiency? We decided on three definitions:

1. Minimum time needed to plow the roads.
2. Minimum distance covered in order to plow all roads.
3. Open up main arteries and then clear the remaining roads.

Because the speed of a plow on snow-covered roads is constant and because the total number of miles of roads to be plowed is constant, we know that the time it will take to plow only these roads will be constant. So time is only a consideration for traveling on roads that are clear, refueling, etc. Similarly, distance is only a factor in that we want to minimize the number of miles of "backtracking," i.e., driving on cleared roads. Hence, definitions 1 and 2 are related.

But do some roads have a higher priority than others? With this question in mind, we thought about our task in a more realistic light. Wouldn't the plows clear main thoroughfares first, in order to allow access across the county to emergency vehicles, buses, and other special types of vehicles? In this event, we would want to plan our route in order to open up major north-south/east-west arteries as our first priority and then clear the other roads, minimizing time and backtracking for both parts.

In order to explore these different definitions of efficiency, we develop two submodels. We need to compare the efficiencies of the submodels and their corresponding cases. We proceed from our definitions of efficiency.

For definition 1, we can set up a proportion to express efficiency in the following manner:

$$t_w = \text{percentage of wasted time} = \frac{\text{time spent driving on clear roads} + \text{time spent at garage}}{\text{total time to complete route}}$$

According to definition 1, our *time efficiency* is $1 - t_w$, or the percentage of non-wasted time.

For definition 2, we can set up another proportion to express efficiency:

$$m_w = \text{percentage of wasted mileage} = \frac{\text{number of miles on clear roads}}{\text{total distance traveled}}$$

According to definition 2, our *mileage efficiency* is $1 - m_w$, or the percentage of non-wasted mileage.

Definition 3 relies on many subjective observations. For one, we must hypothesize which roads are main arteries. Higher street density on the map could indicate an urban area, to and from which we need access as soon as possible. In order to compare efficiency under this scheme, we must first determine whether or not our hypothesized main arteries are cleared first and most efficiently under definitions 1 and 2. Second, we must determine overall efficiency, also under definitions 1 and 2. This problem prompts us, under our initial assumptions and with our definitions of efficiency, to examine various plowing routes (our submodels and their cases) and compare their efficiencies.

Design of the Model

Our model is designed with our general assumptions, additional assumptions, and overall efficiency goals in mind. We develop two different schemes, one emphasizing our main-artery efficiency goal and the other emphasizing our minimal time/distance efficiency goal. In this section analyze the cases of these models with regard to our efficiency definitions; we offer error analysis in the next section.

We designate as Truck 1 the plow-truck that enters the county at Station 1 at Ocean City Road and the state highway. Truck 2 will be the plow-truck that enters the county at Station 2 at Dagsboro Road and the state highway.

We explain our two plowing schemes:

- **Scheme 1 (Priority Scheme)**

  We first clear roads we hypothesize to be major arteries. In **Figure 1**, Truck 1's primary run is shown as a dashed line and Truck 2's as a solid line. There is no backtracking on these primary runs. As the figure shows, we divide the county into 10 disjoint regions. Upon completion of its primary run, each truck returns to its designated station. From these points, the trucks begin their secondary runs. Truck 1 will cover Regions I, II, III, IV, and IX, in that order, returning to the garage when the task is complete. Truck 2 will cover Regions V, VI, VII, VIII, and X, in that order, then return to its garage. There is some backtracking on these secondary runs. Directions for the primary and secondary runs of both trucks are in the Appendix. [EDITOR'S NOTE: Omitted for space reasons.]

- **Scheme 2 (North/South Scheme)**

  We focus not on main arteries but on minimizing time and distance traveled. Since Station 1 is located at the southwestern end of Wicomico County and Station 2 is located at the center of the western border of Wicomico County, we divide the county into two regions, north and south (see **Figure 2**). Truck 1 will make only one run in the southern region; it will make essentially a beeline to the east, and then it will work its way westward clearing all roads as it goes. Truck 2 will make only

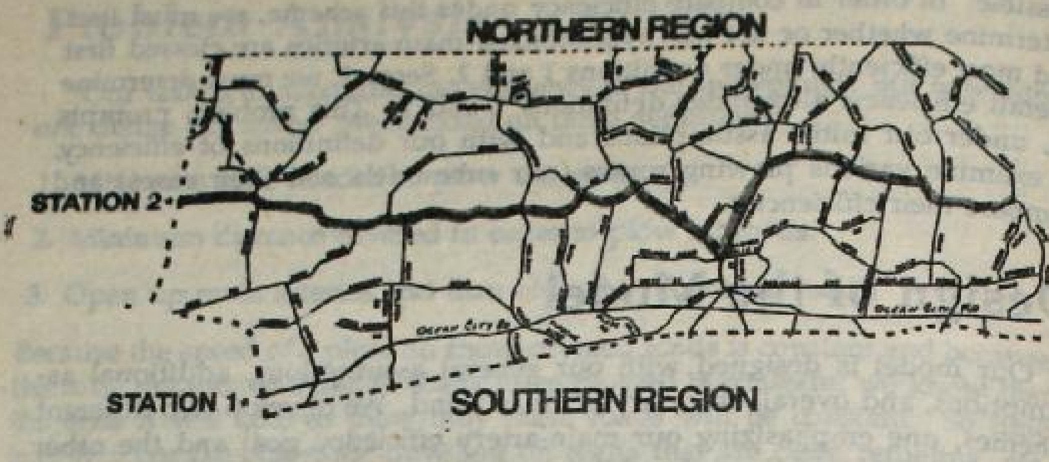

Figure 1. Routes under Priority Scheme.

one run in the northern region; it also will make a beeline to the east and then work its way westward, clearing all roads as it goes. There is no backtracking in this scheme. Directions for each of the trucks' runs are in the Appendix [EDITOR'S NOTE: Omitted].

In the next section, these schemes will each be analyzed twice, once for each of the Cases A and B, giving rise to the four variations we denote as Schemes 1A, 1B, 2A, and 2B.

Analysis of the Model

We explain our measurement techniques to determine distances. Our goal is to estimate the total mileage of roads in the snow-removal district. To that end, we measure the length of the road on the map with a scale, and use our given map scale of 2 inches to 3 miles to determine the distance between two points, to the nearest tenth of a mile. We measure the distances in small increments on the roads that are slightly curved, as the road would be approximately linear over these segments, and add up the length of each segment to give the total distance.

Some roads are too curved for this linear approximation technique to be effective. For them we use a circular template to match the arc of the road, a protractor to determine the central angle of the hypothetical circle, and the relationship (in radian measure)

$$\text{arc length} = \text{radius} \times \text{angle}.$$

As an example of this technique, consider Carioca Road in the northwest corner of the map. The arc of a circle of radius 0.625 inches matches the

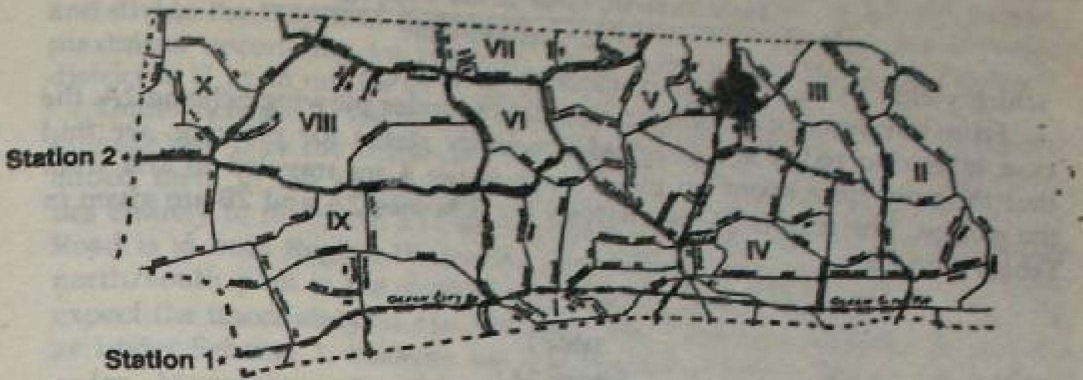

Figure 2. Truck routes under the North/South Scheme.

curvature of the road fairly closely. The central angle is 81° or 1.414 radians. The length of the road is then $(0.625)(1.414) \approx 0.88$ mi.

We calculate approximately 124 mi of road to be plowed in the district. Recall that each road requires one pass in each direction. Thus, the two plows must traverse a minimum of 248 mi to completely clear the roads.

Under Scheme 1A, Truck 1 has 145 mi of road to plow and Truck 2 has 103 mi. The plowing routes involve 23.9 mi of backtracking for Truck 1 and 15.8 mi of backtracking for Truck 2. Including distance to the district from the garage as distance on clear roads, we get $m_w = 18.3\%$, for a mileage efficiency of 81.7%.

Recall that we assume that the plows travel twice as fast on clear roads as on snow-covered roads. We get $t_w = 10.1\%$, for a time efficiency of 89.9%.

Under Scheme 1B, in which two passes are required in each direction to clear a road, the total mileage involved will require a truck to refuel after sweeping the roads once. We add the further assumption that refueling and any required driver change will take 0.5 hours. The total plowing times will be:

$$\text{Truck 1} : 2(8.1 \text{ hr}) + 0.5 \text{ hr at garage} = 16.7 \text{ hr},$$

and

$$\text{Truck 2} : 2(5.8 \text{ hr}) + 0.5 \text{ hr at garage} = 12.1 \text{ hr}.$$

The total distance traveled by the plows under scheme 1B will be exactly double that under 1A, so m_w under 1B will be unchanged. But t_w will now be

$$t_w = \frac{\text{total time on clear roads + garage time}}{\text{total elapsed time}},$$

which yields 13.2%, for a time efficiency of 86.8%

From our description of the routes taken under plowing Scheme 2A, the case where no roads are assigned a priority status for plowing, we find that the only time spent on clear roads is the 4 mi traveled to and from the garage. The results of the analysis for Schemes 2A and 2B are given in **Tables 1 and 2.**

Table 1.
Scheme 2A.

	Truck 1 (south)	Truck 2 (north)
miles plowed	126.6 mi	121.4 mi
to/from garage	8.0 mi	8.0 mi
time plowing	6.5 hr	6.3 hr

$m_w = \frac{8+8}{248} = 6.5\%$, mileage efficiency $= 93.5\%$.

$t_w = \frac{1.0}{12.8} = 3.1\%$, time efficiency $= 96.9\%$.

Table 2.
Scheme 2B.

	Truck 1 (south)	Truck 2 (north)
miles plowed	253.2 mi	242.8 mi
to/from garage	16.0 mi	16.0 mi
plowing time	13.0 hr	12.6 hr
refueling time	0.5 hr	0.5 hr

$m_w = \frac{8+8}{248} = 6.5\%$, mileage efficiency $= 93.5\%$.

$t_w = \frac{0.8}{26.6} = 6.8\%$, time efficiency $= 93.2\%$.

Error Analysis

Our graphical method of mileage measurement introduces errors into our mileage estimates. Our scale is accurate to ±0.1 in, and thus our mileage for a given segment of straight road can be no more than ±0.15 mi. By segmenting a slightly curved road, we are even less accurate. We add an uncertainty of ±0.1 mi, and so could have as much as a ±0.25 mi error. Under Scheme 1, where we assign a priority for plowing certain streets and divide the map into 10 regions, 72 measurements are required. So the maximum uncertainty for the total street mileage with the snow removal district is 11 to 18 mi, for an error of 8.9% to 14.5%, in either direction.

Under Scheme 2, we divide the map into two halves. To calculate the mileage, we take the distances calculated in the region-priority plan and added them in the appropriate manner. For example, Tingle Road now lies entirely in the north half of our map, whereas under Scheme 1, Tingle Road is in both Region IV and V. The mileage of Tingle Road under our north/south plan is the sum of the mileage from Regions IV and V. We expect the uncertainty in our estimate of the total mileage to be the same as under Scheme 1, because we use the same data, added in a different order. Moreover, under Scheme 2, the mileage of roads plowed is much more evenly divided. There is no danger under Scheme 2 of either plow running out of fuel before it completes one sweep in each direction of all roads in its assigned area. So, under this plan, we can afford the luxury of less accurate measurement.

Evaluation of Assumptions

Weaknesses

- We assume all roads may be plowed with the same ease, so that curved roads and urbanized areas can be plowed at the same speed as rural areas.
- The assumption that plows change speeds instantaneously from snow-covered to clear roads is unrealistic, but it simplifies the problem considerably.
- Which roads receive priority for plowing is a subjective judgment.
- The lack of traffic is somewhat unrealistic, as some people are bound to venture out even if the roads have not been plowed.
- Another unrealistic yet simplifying assumption is our interpretation of "the road intersections require no special plowing techniques" to mean that the plows do not stop at intersections.

Strengths

- Our assumption of no further snowfall and no complete meltoff greatly simplifies the problem and allows us to ignore questions such as how much snow fell, whether every road needs to be plowed, and whether the snow will melt off.

- Assigning priority status to what appear to us to be main arterial routes adds an air of realism to our model. In the real world, roads to be plowed are not picked at random; main roads are plowed first, then secondary roads.
- Our treating the intersections as just another section of road, requiring no stopping, makes calculations of plowing times straightforward.
- All roads being two-way is reasonable, because we have no evidence of one-way roads. Since we do not permit plowing against the flow of traffic, one-way roads would further complicate our routing scheme.
- We consider two plow blade widths, 14 ft (which simplifies calculations) and 12 ft (which is more realistic).

Evaluation of Analysis

Weaknesses

- Under our priority scheme, the mileage of roads assigned to each plow is not evenly divided. With the uncertainty in our knowledge of the total mileage of roads, Truck 1 could run out of fuel before finishing its task.
- We have no solid evidence that either of our routing schemes is the most efficient possible.

Strengths

- Our proportion and efficiency statistics help quantify our measure of efficiency.
- The number of different schemes we propose for plowing allows us to incorporate more realistic assumptions and to compare them.
- We researched the topography of Wicomico County and obtained firm evidence that the land is relatively level, so a constant plow rate is more reasonable.
- We provide complete instructions for each plow driver.