邓宇俊、凌宁、潘志明
中山大学
1996 Meritorious Winner

案例 2.2 利用环境噪声场探测无自噪声潜艇*

一、问题

同案例 1.2。

二、摘要

本案例按照海洋环境噪声场的一般特性，建立一空间均匀、各向同性的噪声场模型。利用水听器监测运动潜艇对周围声场的 Doppler 效应及吸收效应获取潜艇位置、大小、速度和航向的信息；并计算了在非均匀介质中的定位问题。最后，讨论了本模型的优点、不足以及展望。

三、问题的重述

由于地震、船只及生物等的影响，海洋中存在噪声场。怎样利用这个声场来探测大型的移动物体，如水中的潜艇？假设潜艇没有自噪声的情况下测定潜艇的位置、速度、大小及移动方向，且只能利用环境噪声场各种信息的变化。先考虑单一频率和单一振幅的情况。

四、模型的假设

1. 环境噪声场

在水深 0~200m 处，噪声场为均匀场，即空间均匀，点点场强等大，各向同性。

2. 水听器

*本案例指导教师陈仁术，队员邓宇俊、凌宁、潘志明。

假设可以利用水听器监测环境噪声场，如果水听器能接收沿某一方向的微小方向角传来的声音信号，并经过一系列的声音处理，使我们可以得到一张频谱图。譬如，测得单一频率、单一振幅的环境噪声场的频谱图，如图 2.2.1 所示。

假设我们所使用的水听器具有相当高的精度，可以探测到大型移动物体(如潜艇)的局部所反射回来的声波。这样，我们可以用一个水听器阵列探测到整艘潜艇所反射回来的声音。

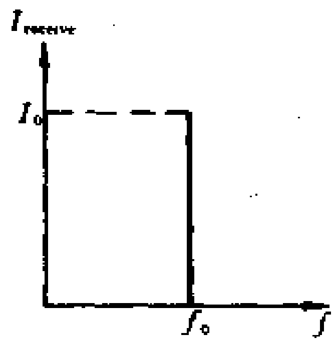

图 2.2.1

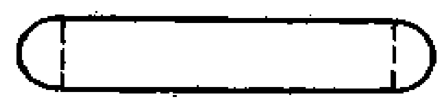

图 2.2.2

3. 潜艇

在模型中的潜艇为棒状，即两端为半球而中间为圆柱状的物体，如图 2.2.2 所示。

现实中，一般的潜艇对声波几乎是完全反射的，即不吸收声能(如果能吸收，它就可以吸收主动声纳发出的声波，从而使潜艇“隐形”。但即便如此，本文仍给出一个探测方法)。

4. 尾流

尾流是指潜艇航行时，由于螺旋桨的旋转而引起的气泡区域。尾流在海军战斗中起重要作用。首先声的散射和吸收影响水声仪器工作；其次，它为发现和追踪潜艇提供了可能。

5. 潜艇活动的海洋环境

假设潜艇的最大下沉深度为 200m，即模型考察的范围是从水

面至水深 200m 的海域。在这个范围内，海水可被看作为均匀介质，声波在其中能直线传播，即声线为直线，并且海域内只有一艘潜艇。

五、符号定义

I	单位面积上的声强
I_0	单频率时噪声声强
I_{0f}	多频率噪声声强分布
$I_{receive}$	水听器接收到的声强
f	频率
f_0	噪声频率
f_{head}	潜艇头部反射的频率
f_{tail}	潜艇尾部反射的频率
f_m	潜艇中部反射的频率
Ω	水听器张角
v	潜艇速度
c	声速
S_0	水听器的张口面积
S_1	环境噪声场中对水听器张角为 Ω 的某截面
θ_h	水听器与艇首连线与航向夹角
θ_t	水听器与艇尾连线与航向夹角
L_h (x_h, y_h, z_h)	艇首坐标
L_t (x_t, y_t, z_t)	艇尾坐标
λ	吸收率

六、基础知识及工具

1. 关于双水听器阵确定目标的计算

(1) 确定潜艇首尾坐标

假设坐标原点在两水听器中间，水听器间距离为 $2d$；坐标分别为(d，0，0)、($-d$，0，0)。

潜艇坐标的计算如图 2.2.3 所示。

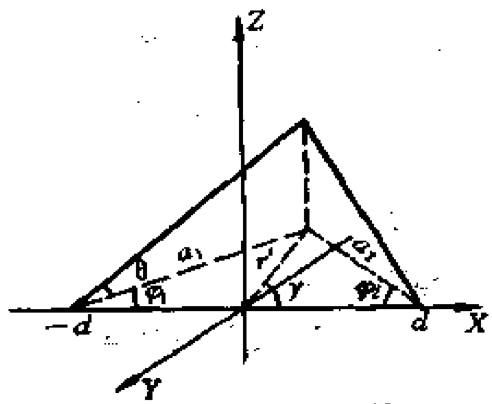

图 2.2.3

$$\frac{a_1}{\sin\varphi_2}=\frac{a_2}{\sin\varphi_1}=\frac{2d}{\sin(\varphi_1+\varphi_2)}$$

$\therefore$ $$a_1=\frac{2d\sin\varphi_2}{\sin(\varphi_1+\varphi_2)}\qquad a_2=\frac{2d\sin\varphi_1}{\sin(\varphi_1+\varphi_2)}$$

$\therefore$ $$z_h=a_1\tan\theta$$

$$r' = a_1^2 + d^2 - 2a_1d\cos\varphi_1$$

$\therefore$ $$\sin\gamma = a_2\frac{\sin\varphi_2}{r'}$$

$$x_h = r'\cos\gamma$$

$$y_h = r'\sin\gamma = a_2\sin\varphi_2$$

$\therefore$ $$\begin{cases}x_h=r'\cos\gamma\\ y_h=r'\sin\gamma\\ z_h=a_1\tan\theta\end{cases}\qquad(1)$$

用同样方法可求得另一端的坐标(x_t，y_t，z_t)，从而得到：

艇长：$L=\sqrt{(x_h-x_t)^2+(y_h-y_t)^2+(z_h-z_t)^2}$

位置：$\begin{cases} x=\dfrac{1}{2}(x_h+x_t) \\ y=\dfrac{1}{2}(y_h+y_t) \\ z=\dfrac{1}{2}(z_h+z_t) \end{cases}$

(2) 确定航向与潜艇和水听器连线的夹角

由上面可得潜艇两端的坐标，假设潜艇与其中一个水听器的关系如图 2.2.4 所示：

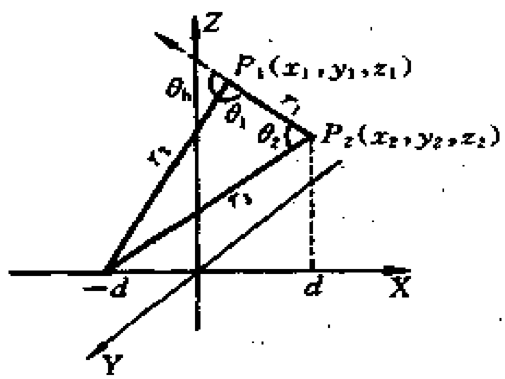

图 2.2.4

$$r_1^2=(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2$$
$$r_2^2=(x_1+d)^2+(y_1-0)^2+(z_1-0)^2$$
$$r_3^2=(x_2+d)^2+(y_1-0)^2+(z_1-0)^2$$
$$\cos\theta_1=\frac{r_1^2+r_2^2-r_3^2}{2r_1r_2}\qquad \cos\theta_2=\frac{r_1^2+r_3^2-r_2^2}{2r_1r_3} \tag{2}$$
$$\theta_h=\pi-\theta_1$$
$$\theta_t=\theta_2$$

航向角用 β_x；β_y，β_z 表示（β_x 表示航向与 X 轴的夹角；β_y 表示航向与 Y 轴的夹角；β_z 表示航向与 Z 轴的夹角）。

$$
\begin{cases}
\cos\beta_x = \dfrac{x_h - x_t}{\sqrt{(x_h - x_t)^2 + (y_h - y_t)^2 + (z_h - z_t)^2}} \\
\cos\beta_y = \dfrac{y_h - y_t}{\sqrt{(x_h - x_t)^2 + (y_h - y_t)^2 + (z_h - z_t)^2}} \\
\cos\beta_z = \dfrac{z_h - z_t}{\sqrt{(x_h - x_t)^2 + (y_h - y_t)^2 + (z_h - z_t)^2}}
\end{cases} \tag{3}
$$

2. Doppler 效应

当波源和观察者同时或其中之一相对于介质运动时，观察者接受到的频率和波源的频率不同的现象称 Doppler 效应。当观察者相对参照系以速度 v 运动，而频率为 f_0 的波源相对参照系以速度 u 运动时，观察者测得波源的频率为：

$$
f' = \frac{c - v}{c - u} f_0 \tag{4}
$$

所以，如果环境噪声场中有潜艇在其中运动时，就会发生 Doppler 效应。我们只需测量频率变化，即可利用 Doppler 效应知道潜艇的位置和速度。

3. 环境噪声场中声的传播与反射

由于噪声场是恒定的，所以声场中各点都存在着沿各个方向具有同一频率的声波。如图 2.2.5 所示，考察 A 点，过 A 点存在各个方向传来的声波。现仅考虑其中的两个：1 和 3。由声线是直线的假设，知 1→2，3→4。若场中存在表面比较光滑的物体，对声波产生反射，即有 1→4，2→3。显然，如果物体对声波发生完全反射(不吸收)，则物体的存在与否并不影响声场的分布。

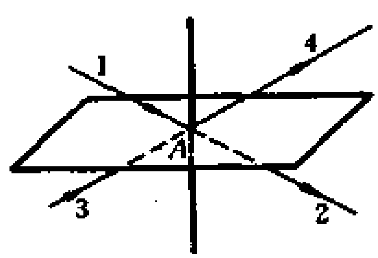

图 2.2.5

事实上，从能量的角度来看，若物体不吸收能量，则物体的存在对场能量的分布无影响（在后面部分"水听器与环境噪声场"中将给出更详细的解释）。换句话说，若物体能吸收声波的一部分，则可以借此判断场中是否有外来物。本模型中对尾流的分析正是以此为基础的。

4. 水听器与环境噪声场

下面将给出有关环境噪声场的数学化表述。

设通过空间内任意单位截面 ds 的声强为 Ids，由于声强在空间各个方向是均匀的，所以通过截面 ds，在某个方向 (θ, φ) 单位立体角上的声强为 $\frac{Ids}{2\pi}d\Omega$。于是，水听器可以看成面积为 S_0、张角为 Ω 的球面。在 t 时刻水听器接收到的声强，是 $(t-R/c)$ 时刻（c 为声速）通过 S_1 面的声强传到 S_0 上的叠加。

证明如下：如图 2.2.6 所示。

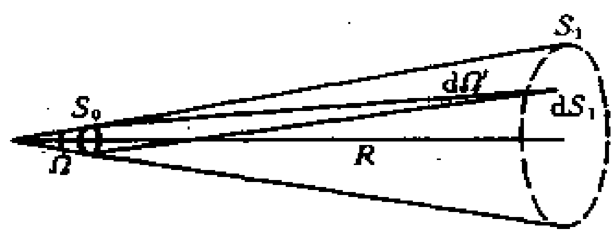

图 2.2.6

$$I_{receive} = \int_{S_1} \frac{IdS_1}{2\pi} d\Omega' \qquad (d\Omega' = S_0/R^2)$$

$$= \int_{S_1} \frac{IS_0}{2\pi R^2} dS_1$$

$$= \int_{S_1} \frac{IS_0}{2\pi} d\Omega \qquad (d\Omega = dS_1/R^2)$$

$$= \frac{S_0}{2\pi}\int_{S_1} I \mathrm{d}\Omega \tag{5}$$

(5)式给出了水听器接收声强的一般公式，当 I 是常数时，(5)式可以简化为：

$$I_{receive} = \frac{S_0 I}{2\pi}\Omega \tag{6}$$

七、模型的建立——单一频率、单一振幅情形下探测潜艇的方法

在本模型中，我们根据(5)式，利用水听器接收环境噪声场传来的信息，当场中没有外来物时，$I_{receive}$满足：

$$I_{receive} = \frac{S_0 I}{2\pi}\Omega = I_0$$

当测得 $I_{receive} \neq I_0$ 时，我们便知道场中有外来物体。应该指出，无论是 $I_{receive}$还是 I 都是一种频率分布，即包含着对应不同频率的不同声强的信息。由(5)式知，$I_{receive}$是决定于 S_1 面上 I 的分布，I 包含着哪些频率的声强信息，$I_{receive}$同样也包含着这些频率的声强信息。

我们发现，有两种情况可以促使 $I_{receive} \neq I_0$：潜艇的移动引起 Doppler 效应；潜艇的尾流对声音有吸收作用。

1. Doppler 效应

(1) 潜艇中部的 Doppler 效应

如图 2.2.7，声波 1 经反射为 2，所以 $\theta_2=\theta_3$，$\theta_1=\theta_4$，由 Doppler 效应频率公式得：

$$\begin{aligned} f_m &= \frac{c+v\cos\theta_4}{c}\cdot\frac{c}{c+v\cos\theta_1}f_0 \\ &= \frac{c+v\cos\theta_4}{c+v\cos\theta_1}f_0 \\ &= f_0 \end{aligned} \tag{7}$$

即潜艇中部不发生 Doppler 效应。

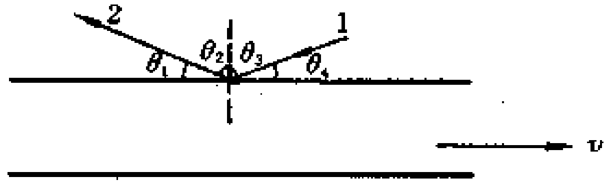

图 2.2.7

(2) 潜艇首尾的 Doppler 效应

由上述分析可知，在图 2.2.8 中，线段 AB、CD 中均不发生 Doppler 效应。由于首尾的分析方法类似，下面仅给出艇首的分析，然后类推出艇尾的结论。

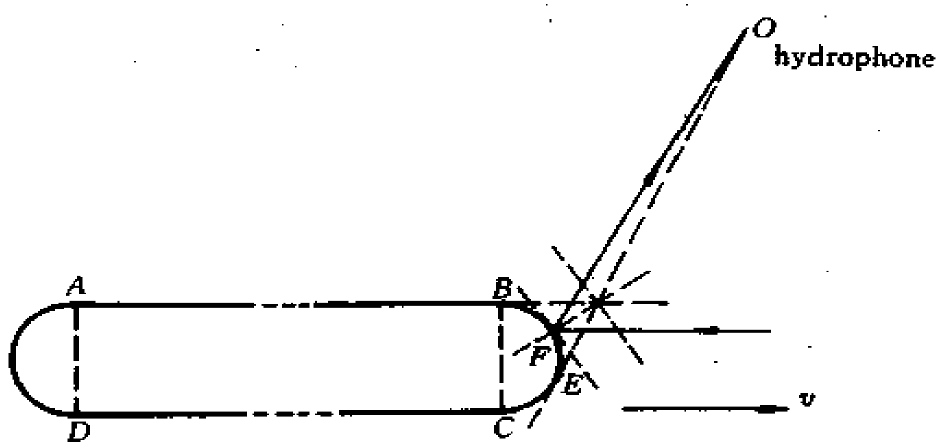

图 2.2.8

如图 2.2.8，OE 是切线，故只有圆弧 BE 上才会出现 Doppler 效应。从图中点 F 的确定方法知总存在一束迎头的声波经过艇首的反射，传到水听器阵，由 Doppler 效应知，这时的频率最大：

$$f_{head} = \frac{c+v}{c} \cdot \frac{c}{c - v\cos\theta_h} f_0$$

$$= \frac{c+v}{c - v\cos\theta_h} f_0 \qquad (> f_0) \tag{8}$$

θ_h 是水听器阵和艇首连线与潜艇航向的夹角。而圆弧 BE 中，除 F 点外的频率则介于 f_0 与 f_h 之间。

类似地，我们总可以找到一束与潜艇航向相同的，经潜艇后方反射后传到水听器阵的声线，其频率为：

$$f_{tail}=\frac{c-v}{c}\cdot\frac{c}{c-v\cos\theta_t}f_0$$
$$=\frac{c-v}{c-v\cos\theta_t}f_0 \qquad (<f_0) \tag{9}$$

θ_t 是水听器和潜艇尾的连线与潜艇航向的夹角；同样，艇尾也会反射介于 f_0 与 f_{tail} 的频率的声波到水听器阵。这样，艇首与艇尾反射的声波不再是 f_0，而分别是 $[f_0, f_{head}]$ 与 $[f_{tail}, f_0]$ 的两列波。

(3) 结论

潜艇运动时，只有艇首与艇尾会发生 Doppler 效应，即在艇首与艇尾的声强 I 会发生变化，从而使对准艇首与艇尾的水听器接收的 $I_{receive}$ 相应地变化，如图 2.2.9 所示。

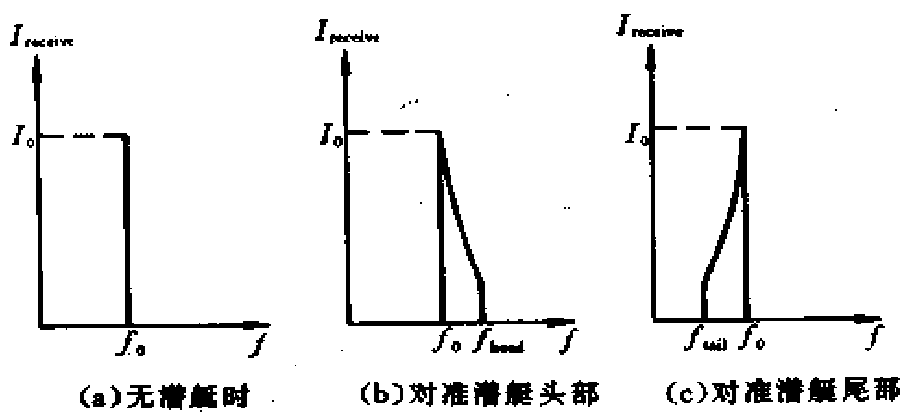

图 2.2.9

所以，如果我们的水听器接收到类似图 2.2.9b 或 c 的频谱图，即说明我们发现了潜艇的踪迹，微调方向即可发现整艘潜艇的踪迹，即在水听器阵中，同时捕捉到图 2.2.9b、c 的图像。

利用前述方法，用两个水听器阵，即可分别确定艇首坐标 $L_h(x_h, y_h, z_h)$ 和艇尾坐标 $L_t(x_t, y_t, z_t)$，并得到艇长、位置、θ_h、θ_t 及航向。

下面计算航速。

①当 $\theta_h>90°$，$\theta_t>90°$ 时，水听器阵位于潜艇后方。从艇尾取得的信息较丰富，故利用 f_{tail}、θ_t 计算航速较好。

$$\because \quad f_{tail}=\frac{c-v}{c-v\cos\theta_t}f_0$$

$$\therefore \quad v=c\cdot\frac{f_0-f_{tail}}{f_0-f_{tail}\cdot\cos\theta_t} \tag{10}$$

②当 $\theta_h<90°$，$\theta_t<90°$ 时，水听器阵位于潜艇前方。从艇首取得的信息较丰富，故利用 f_{head}、θ_h 计算航速较好。

$$\because \quad f_{head}=\frac{c+v}{c-v\cos\theta_h}f_0$$

$$\therefore \quad v=c\cdot\frac{f_0-f_{head}}{f_0+f_{head}\cdot\cos\theta_h} \tag{11}$$

③当 $\theta_h>90°$，$\theta_t<90°$ 时，则应按前面两种算法分别求出 v_{tail}、v_{head}，取

$$v=0.5(v_{tail}+v_{head}) \tag{12}$$

当然，也可通过指出一系列的潜艇运动轨迹，用路程除以时间求得。

2. 潜艇尾流对声音的吸收作用

(1) 探测隐形潜艇的方法

由前所述，如果潜艇是吸收声能的，假设吸收率为 λ，则用水听器测得的 $I_{receive}$ 不再是 I_0，而是：

$$I_{receive}=\frac{S_0}{2\pi}\int_{S_1}(I-\lambda)I\,d\Omega$$

$$=(I-\lambda)\frac{S_0 I}{2\pi}\Omega$$

$$= (I - \lambda)I_0 \qquad (13)$$

依然存在 Doppler 效应。

当 λ 不太大时，上述方法仍适用。

当 λ 较大时，要分辨出 f_{tail}和 f_{head}已经较难，而此时，I_0 本身的衰减也较大，但仍可以根据 $I_{receive}$是否等于 I_0 来判断潜艇的存在与否。同样，可以利用两个水听器阵来确定潜艇的首尾位置，求出艇长、航向等参数。而航速则可以利用潜艇的运动轨迹，用路程除以时间求得。

一般来说，潜艇是不吸收声波的。能吸收声波的称为“隐形潜艇”，故这里给出的方法仅对“隐形潜艇”适用。

(2) 利用尾流探测潜艇的方法

虽然一般潜艇不吸收声波，但如果潜艇使用螺旋桨，则必会产生尾流。尾流有以下两个特点可被利用：①尾流吸收声波；②尾流中空气泡的破碎发出某种频率 f_s 的声波。这样，一旦水听器对准尾流，则会出现如图 2.2.10 所示的频谱。

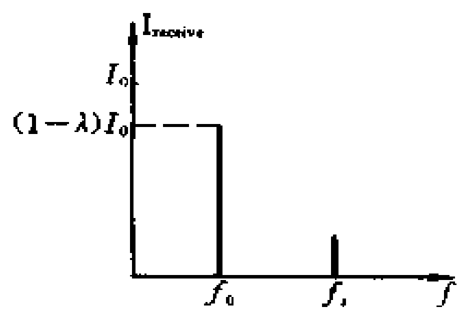

图 2.2.10

由于空气泡破碎发出声波的频率(f_s>10kHz)较高，而高于10kHz 的声波在海洋中传播衰减作用是较强烈的，所以，到底能否利用 f_s 的存在来确定尾流有待实验证实，但有一点可以确定，尾流的存在必然使 f_0 的声强由原来 I_0 降为$(1-\lambda)I_0$。利用水听器阵

来定位的方法仍然适用。

3. 潜艇没尾流的情况

据报道，现代潜艇的一个发展方向是以其他装置（如类似喷气式飞机的喷筒）代替螺旋桨以除去尾流。但是，这样的潜艇其速度也快，Doppler 效应也越明显，所以我们仍可用 Doppler 效应来探测潜艇。

八、模型的推广

1. 多频率情形

原模型中，我们是假设环境噪声场中只存在单一频率，但实际上环境噪声场中包含各种不同频率的声波，例如：

0.1～10Hz	声源为地震、远处风暴、海洋和大气湍流等
50～300Hz	远处的航船运输
0.5～50kHz	波浪的破碎声
>100kHz	分子热噪声

图 2.2.11 为海洋环境噪声的平均频谱。

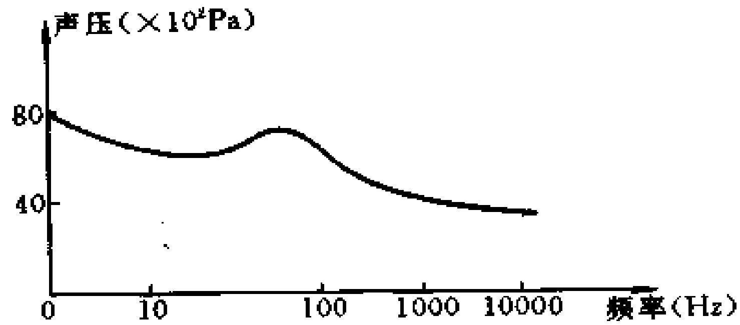

图 2.2.11

下面分析本模型多频率前提下的推广。在多频率的情况下，水听器对着环境噪声场时，

$$I_{\text{receive}} = \frac{S_0}{2\pi}\int_{S_1} I_f \mathrm{d}\Omega = \frac{S_0}{2\pi} I_f \Omega = I_{0f} \tag{14}$$

其中 I_f 是多频率时环境噪声场的声强分布（I_f 恒定或者周期性地按某一规律变化）。当环境噪声场中无外来物体时，I_{receive} 满足 $I_{\text{receive}} = I_{0f}$。当测知某区域 $I_{\text{receive}} \neq I_{0f}$，我们便可知，场中有外来物体。

（1）Doppler 效应

类比单一频率的讨论可知，在艇首产生的 Doppler 效应使每个频率均向高频端展宽；同理，在艇尾则向低频端展宽。

（2）尾流的吸收作用

类似单一频率的推导，知水听器对着尾流时，有

$$I_{\text{receive}} = (1-\lambda) I_{0f} \tag{15}$$

这里，λ 是吸收率，且 λ 是频率 f 的函数：$\lambda = \lambda(f)$，仍然可用 $I_{\text{receive}} \neq I_{0f}$ 判断。

总之，无论是利用 Doppler 效应还是利用尾流的吸收，均可用两个水听器阵确定潜艇首尾坐标，从而求出其他一系列参数。

2. 非直线情形

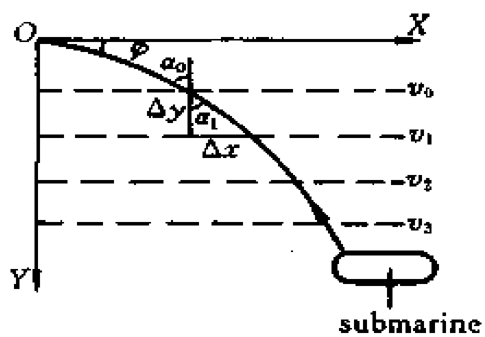

图 2.2.12

假设声速随深度变化，且 $\frac{dv}{dh} = G$ 为常数。由图 2.2.12 所示，可将海区分为若干个水平层，认为每一层中的声速是常数，并令各层的厚度趋近于零。第一层中的声速为 v_0，第二层中的声速为

v_1。从声源射出的声线与水平面的夹角为 φ。

$$\tan\alpha_1 = \frac{\Delta x}{\Delta y}$$

根据折射定律

$$\frac{\Delta x}{\Delta y} = \frac{v_1}{v_0} \frac{\cos\varphi}{\sqrt{1 - \frac{v_1^2}{v_0^2}\cos^2\varphi}}$$

取极限得

$$\lim_{\Delta y\to 0} \frac{\Delta x}{\Delta y} = \frac{dx}{dy} = \frac{v_1}{v_0} \frac{\cos\varphi}{\sqrt{1 - \frac{v_1^2}{v_0^2}\cos^2\varphi}} \tag{16}$$

第二层的声速为：

$$v_1 = v_0 + Gy$$

令

$$G/v_0 = a$$

则

$$v_1/v_0 = 1 + ay$$

代入(16)式得

$$\frac{dx}{dy} = (1 + ay)\frac{\cos\varphi}{\sqrt{1 - (1 + ay)^2\cos^2\varphi}} \tag{17}$$

则

$$x = \int \frac{(1 + ay)\cos\varphi}{\sqrt{1 - (1 + ay)^2\cos^2\varphi}}dy$$

$$= -\sqrt{\frac{1}{a^2\cos^2\varphi} - \left(y + \frac{1}{a}\right)^2} + C \tag{18}$$

积分常数 C 以 $y=0$，$x=0$ 的条件来确定，得

$$C = \sqrt{\frac{1}{a^2\cos^2\varphi} - \frac{1}{a^2}} = \frac{\tan\varphi}{a} \tag{19}$$

所以

$$\left(x-\frac{\tan\varphi}{a}\right)^2+\left(y+\frac{1}{a}\right)^2=\frac{1}{a^2\cos^2\varphi} \tag{20}$$

因

$$a=G/v_0$$

则

$$\left(x-\frac{v_0}{G}\tan\varphi\right)^2+\left(y+\frac{v_0}{G}\right)^2=\frac{v_0^2}{G^2\cos^2\varphi} \tag{21}$$

上式说明，在声速垂直梯度恒定的情况下，声线轨迹是半径为 R 的圆弧。

$$R=\frac{v_0}{|G|\cos\varphi} \tag{22}$$

而圆心的坐标 x_c，y_c 为：

$\varphi=0°$时，$x_c=0$，$y_c=-\dfrac{v_0}{G}$

$\varphi\neq0°$时，$x_c=v_0\tan\varphi$，$y_c=-\dfrac{v_0}{G}$

在一般的海域，声音的传播速度随深度的变化如图 2.2.13a：

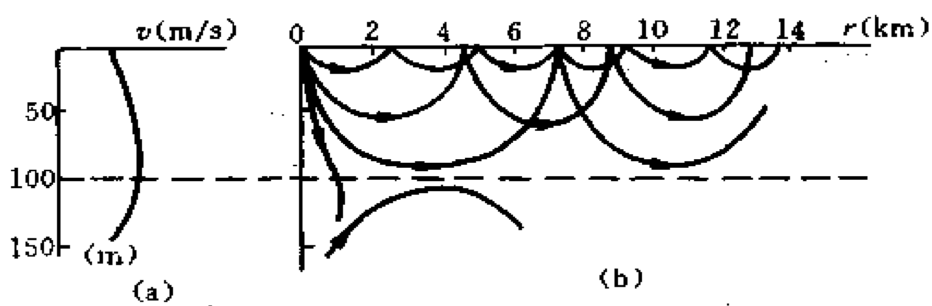

图 2.2.13

所以声线的传播路线如图 2.2.13b。

若已知 v_0、G、φ，可根据式(21)对单一频率的定位方法进行修正。

九、模型的评价

1. 模型的优点

①本模型用适当的数学形式，较好地把水听器接收到的声强与环境噪声场中对水听器张角为Ω的某截面声强联系起来。判断环境噪声场中存在大型移动物体(如潜艇)的依据为：

$$I_{receive} \neq I_0 \quad (单频率时)$$

或者

$$I_{receive} \neq I_{0f} \quad (多频率时)$$

正因如此，本模型从单频率过渡到多频率时，没有碰到多大的困难。另外本模型并没有对多频率时的声场作过多的要求。

②本模型充分利用了环境噪声场的信息：频率及该频率的声强，开发了两套测定大型移动物体位置、大小和速度等参数的方法。对于潜艇，这两套方法可同时使用(确定潜艇的首尾坐标及尾流的位置)。而对于其他大型移动物体，运动速度大的，主要利用Doppler效应；而对声波吸收能力强的则利用物体吸收声能的一套方法。

③本模型还讨论了声波非直线传播时的简单情形，这样模型更符合实际。

2. 模型的不足

①本模型只引入了噪声场空间均匀及各向同性的特征而忽略了噪声场的随机性起伏。如起伏的平均周期τ很小，则仪器可在时间$t(\gg\tau)$内测得噪声场的平均值，消去起伏的影响。当然，我们只能假设t不太大。

②当噪声场不是单一频率时，f_{tail}与f_{head}淹没在噪声场中不能分辨，这时Doppler效应测速失效，只能用$\Delta s/\Delta t=v$近似求得速度。

3. 模型的展望

①建议采用简洁适当的统计模型来描述环境噪声场的涨落，使模型更符合实际。

②建议先求出单一频率下 $[f_{tail}, f_0]$ 和 $[f_0, f_{head}]$ 的频率分布曲线 F，这可以利用

$$I_{receive} = \frac{S_0}{2\pi}\int_{S_1} I d\Omega$$

结合实验的方法求出。事实上，分布曲线 F 是与潜艇的级别、艇首尾的形状有关。所以，不同的潜艇其分布曲线 F 不同。由此，我们可以确定潜艇的大概类型。

多频率时的分布则可以将单频率所得的分布叠加而成.同样，最好与实验结合。

Example2. 2 Detect Submarine without Intrinsic Noise by Using Ambient Noise Field

Abstract

The following paper is based on the general characteristic of ocean ambient noise. We assume that the sphere of the ocean is even, the noise is isotropy. In this model, we suggest using hydrophones to detect Doppler's effect and absorption effect caused by the moving submarine. Analyzing these effects, we can get the information of the location, the size, the speed and the direction of the submarine. Further more, we make a discussion on how to locate a submarine in none-even medium—— a more practical problem. At last, we show the strengths, weaknesses and further development of our model.

Restatement of the Problem

Due to seism, ships and oceanic mammals, the world's ocean contain an ambient noise field. We wish to use this ambient noise to detect large moving objects, e.g. submarines without intrinsic noise locate below the ocean surface (detect the presence, speed, size, and direction of travel), using only the information obtained by measuring changes to the ambient noise field.

Model Assumption

Ambient Noise Field Model

Generally, submarine travels no deeper than 200m. In this domain, there are a lot of sound source. After reflection, diffraction and scattering, the noise can be considered even. That's to say it is even throughout the space, and the intensity of noise is isotropy.

At first we assume that the ocean water is even medium, so sound travel straight.

Hydrophone

We assume that the hydrophone can be used to detect the ambient noise field. It can receive sound signals from a certain direction and after analysing the signal, we can get a spectrum. For instance, if the hydrophone is put in an ambient noise field with single amplitude and single frequency, we can get a spectrum like Fig. 2. 2. 1.

We also assume that the hydrophone is good enough to distinguish sound reflected by a certain part of a large moving object (a submarine). So, we can use a group of hydrophones to detect the sound reflected by the whole submarine.

Submarine

We assume that the submarine is a stick-like object. Something like Fig. 2. 2. 2, whose extremes are semispheres and the body is a cylinder. In the reality, a normal submarine can reflect sound wave totally(this is why a active sonar can detect a submarine). The method in this paper can detect both this kind of submarine and even a submarine with total absorption of sound. We discuss both occasion in this paper.

Wake Flow

Wake flow is caused by the rotation of the propeller blade. It's made of a large amount of bubbles. Wake flow is very important in navy campaign. It can cause the absorption and scattering of sound wave. So it can provides some useful information of a submarine.

Symbols

I	intensity of sound
I_0	intensity of noise with single frequency
I_{0f}	intensity of noise with multi-frequency
$I_{receive}$	intensity of noise the hydrophone receive.
f	frequency
f_0	noise frequency
f_{head}	noise frequency reflected by the head of the submarine
f_{tail}	noise frequency reflected by the tail of the submarine
f_m	noise frequency reflected by the middle of the submarine
Ω	field angle of hydrophone

v	speed of submarine
c	speed of sound in sea water
S_0	field area of hydrophone
S_1	cross area detected by the hydrophone
θ_h	angle between the direction of the submarine's velocity and the line joining hydrophone and head of the submarine
θ_t	angle between the direction of the submarine's velocity and the line joining hydrophone and tail of the submarine
$L_h(x_h, y_h, z_h)$	location of the head of submarine
$L_t(x_t, y_t, z_t)$	location of the tail of submarine
λ	absorption coefficient
$e=(1-\lambda)$	reflecting coefficient

Tool Pack and Basic Knowledge

Hydrophone and Noise Field

We define I as the sound intensity. It is even in every direction, so the sum intensity which go through a cross section dS in the direction (θ, φ) is:

$$I_\Omega(x,y,z,t) = \frac{I(x,y,z,t)dS}{2\pi}d\Omega$$

We can calculate it by:

$$\begin{aligned} I_\Omega(x,y,z,t) &= \int_{S_1} \frac{I(x_1,y_1,z_1,t-R/c)dS_1}{2\pi}d\Omega' \quad (d\Omega' = dS/R^2) \\ &= \int_{S_1} \frac{I(x_1,y_1,z_1,t-R/c)dS}{2\pi R^2}dS_1 \\ &= \int_{S_1} \frac{I(x_1,y_1,z_1,t-R/c)dS}{2\pi}d\Omega \end{aligned}$$

$(\because dS \text{ is very small}, \therefore d\Omega = dS_1/R'^2 = dS_1/R^2)$

$$=\frac{dS}{2\pi}\int_{S_1} I(x_1,y_1,z_1,t-R/c)d\Omega$$

S_1 is a sphere. As I doesn't change rapidly, we can take S_1 as an area on the surface of the submarine.

Then, we take the hydrophone as a small sphere S_0, with field angle Ω, $I_{receive}(t)$ is the sound intensity received by the hydrophone at time t, we can calculate it by

$$I_{receive}(x,y,z,t) = \frac{S_0}{2\pi}\int_{S_1} eI(x_1,y_1,z_1,t - R/c)d\Omega \qquad (1)$$

Take S_1 as an area on the surface of the submarine. Because the reflect coefficient e is the same everywhere of the submarine.

$$\therefore \qquad I_{receive}(x,y,z,t) = \frac{S_0}{2\pi}eI\Omega \qquad (2)$$

Actually, the S_0 can not be too small, otherwise the received signal will be too weak. And in order to prevent the S_0 receiving the sound which is not in direction (θ, φ), we must surround the S_0 with a pyramidal face M (as shown in Fig2. 2. 14a), which made of sound absorbing material.

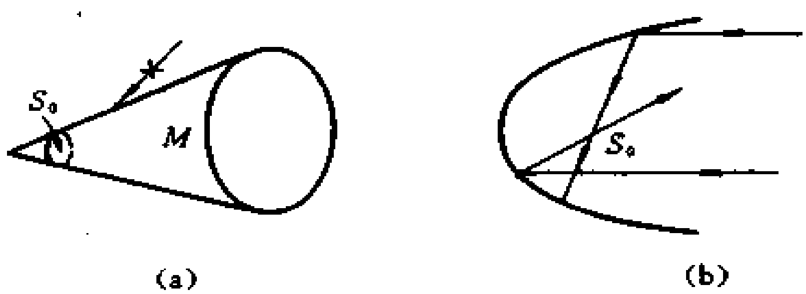

Fig. 2. 2. 14

We can improve it (as shown in Fig. 2. 2. 14b) with a hydrophone locate in the focus of a paraboloid made of total reflecting material. Then we can easily obtain the signal of one direction.

Doppler's Effect

When either the source of sound or the detector has movement relevant to the medium, frequency of the sound received by the detector will be different from the frequency of source. This is called Doppler's effect. The frequency received will be:

$$f' = \frac{c - v}{c - u} f_0 \qquad (3)$$

u and v are the velocities of the source and the detector respectively, f_0 is the frequency of sources. So the submarine moving in the noise field will cause Doppler's effect. From the change of the frequency we can get the submarine's velocity and location.

Reflection of Sound in the Noise Field

When a total reflecting object is station in the noise field, we can not observe any change of the noise.

As shown in Fig. 2. 2. 5, we analyze the point A, there are sound transmit to A from every direction, such as 1 and 3. If there is nothing at point A, then $1 \to 4$, and $2 \to 3$. Because the noise intensity is even in all direction and the plane is total reflecting, we got:

$$I_1 = I_2 = I_3 = I_4$$

Obviously after reflecting, the noise field has no change.

On the other hand, if the object can absorb sound, we got:

$$I_1 = I_3 > I_2 = I_4$$

So the noise field change and the object is detectable. Our analysis of wake flow is based on that.

Lock Object with Two Hydrophones

1. Locate the submarine

In this section, we assume that hydrophone No. 1 (D_1) detect the submarine is in Area 1 and hydrophone No. 2 (D_2) detect that the submarine is in Area 2(Fig. 2. 2. 15):

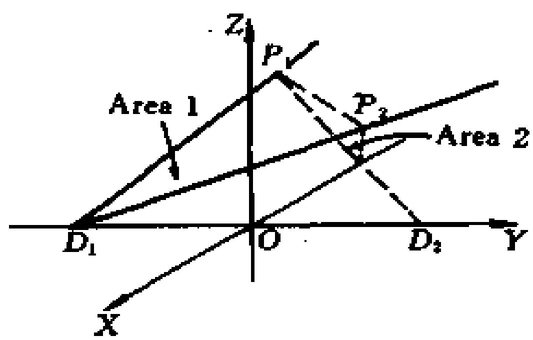

Fig. 2. 2. 15

Two areas have a cross line segment P_1P_2, it's just the submarine!

We assume that the two hydrophones locate on same depth Dm of the ocean, and the plane below the surface for Dm is defined as A. So we build up a frame whose Y axis is the line passed through two hydrophones, the origin is the middle of two hydrophones and X-Y plane is Plane A. The following Fig. 2. 2. 16 shows how to locate P_1. Because the hydrophones can only give out an elevation-angle and a rotation-angle, so we assume that the two hydrophones have located P_1 as $(\theta_{11}, \varphi_{11}, \varphi_{21})$ which θ represent the elevation-angle; φ represent the rotation-angle; the first subscript refer to the number of the cross points (P_1, P_2) and the second one refer to the number of the hydrophones (D_1, D_2).

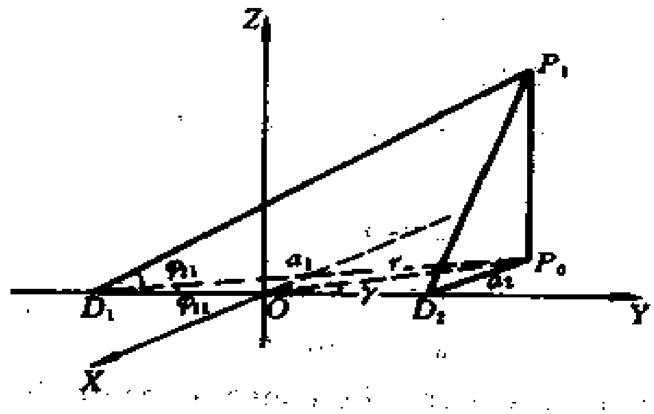

Fig. 2.2.16

From $\Delta P_0D_1D_2$ (P_0 is the shadow of P_1 on X-Y plane) we got:

$$\frac{P_0D_2}{\sin\varphi_{11}} = \frac{P_0D_1}{\sin\varphi_{21}} = \frac{D_1D_2}{\sin(\pi - \varphi_1 - \varphi_2)} = \frac{D_1D_2}{\sin(\varphi_1 + \varphi_2)}$$

Let $P_0D_1 = a_1$, $P_0D_2 = a_2$, $OP_0 = r$, $\angle P_0OD_2 = \gamma$, $D_1D_2 = 2d$

So

$$\frac{a_1}{\sin\varphi_2} = \frac{a_2}{\sin\varphi_1} = \frac{2d}{\sin(\varphi_1 + \varphi_2)}$$

$$\therefore \qquad a_1 = \frac{2d\sin\varphi_{21}}{\sin(\varphi_{11} + \varphi_{21})}, \qquad a_2 = \frac{2d\sin\varphi_{11}}{\sin(\varphi_{11} + \varphi_{21})}$$

From ΔP_0OD_1 we got:

$$r^2 = a_1^2 + d^2 - 2a_1d\cos\varphi_{11}$$

$$\therefore \qquad \sin\gamma = a_2\frac{\sin\varphi_2}{r'}$$

In ΔP_0OD_2 we got:

$$\frac{\sin\varphi_{21}}{r} = \frac{\sin\gamma}{a_2} \Rightarrow \sin\gamma = a_2\frac{\sin\varphi_{21}}{r}$$

Assume that P_1 is head of the submarine, so:

$$y_h = r\cos\gamma$$
$$x_h = r\sin\gamma$$

and in $\Delta P_0P_1D_1$, we got:

$$z_h = |P_1P_0| = a_1\tan\theta_{11}$$

so the results:

$$\begin{cases} x_h = r\sin\gamma \\ y_h = r\cos\gamma \\ z_h = a_1\tan\theta_{11} \end{cases} \tag{4}$$

with the same method we got the coordinates of the tail:

$$\begin{cases} x_t = r'\sin\gamma' \\ y_t = r'\cos\gamma' \\ z_t = a_1'\tan\theta_{12} \end{cases} \tag{5}$$

which:

$$a_1' = \frac{2d\sin\varphi_{22}}{\sin(\varphi_{12} + \varphi_{22})}$$

$$a_2' = \frac{2d\sin\varphi_{12}}{\sin(\varphi_{12} + \varphi_{22})}$$

$$r' = a_2'^2 + d^2 - 2a_2'd\cos\varphi_{12}$$

$$\sin\gamma' = \frac{a_2'\sin\varphi_{22}}{r'}$$

So now we got the length of the submarine:

$$L = \sqrt{(x_h - x_t)^2 + (y_h - y_t)^2 + (z_h - z_t)^2}$$

and position of the middle of submarine:

$$\begin{cases} x = \frac{1}{2}(x_h + x_t) \\ y = \frac{1}{2}(y_h + y_t) \\ z = \frac{1}{2}(z_h + z_t) \end{cases}$$

2. Angle between direction and line joining submarine and hydrophone

Assume that the direction is along the line of the submarine's body. We use $\beta_x, \beta_y, \beta_z$ to describe where the submarine heading for which β_x represents angle between the direction and X-axis. So are β_y, β_z.

$$\begin{cases} \cos\beta_x = \dfrac{x_h - x_t}{\sqrt{(x_h - x_t)^2 + (y_h - y_t)^2 + (z_h - z_t)^2}} \\ \cos\beta_y = \dfrac{y_h - y_t}{\sqrt{(x_h - x_t)^2 + (y_h - y_t)^2 + (z_h - z_t)^2}} \\ \cos\beta_z = \dfrac{z_h - z_t}{\sqrt{(x_h - x_t)^2 + (y_h - y_t)^2 + (z_h - z_t)^2}} \end{cases} \tag{6}$$

In the discussion below, two varies θ_h, θ_t show in Fig. 2. 2. 17 are very useful.

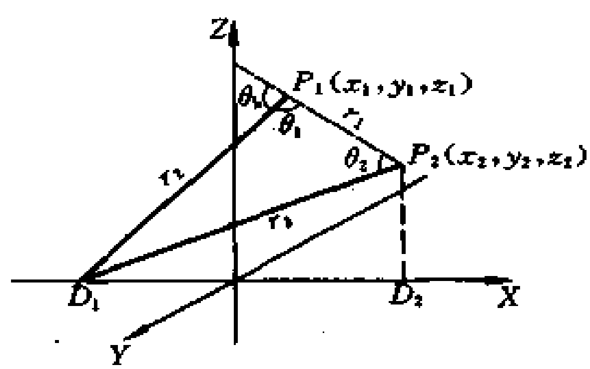

Fig. 2. 2. 17

In $\Delta P_2P_1D_1$, using the cosine principle:

$$r_1^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$$

$$r_2^2 = (x_1 + d)^2 + (y_1 - 0)^2 + (z_1 - 0)^2$$

$$r_3^2 = (x_2 + d)^2 + (y_1 - 0)^2 + (z_1 - 0)^2$$

and

$$\cos\theta_1 = \frac{r_1^2 + r_2^2 - r_3^2}{2r_1r_2} \qquad \cos\theta_2 = \frac{r_1^2 + r_3^2 - r_2^2}{2r_1r_3}$$

$$\theta_h = \pi - \theta_1 \qquad \theta_t = \theta_2 \tag{7}$$

Detect the Submarine in One Fixed Frequency and Amplitude Noise Field

Using Equ. (1) when there isn't any moving object in the direction of hydrophone I is a constance.

If there is a moving object in the direction of hydrophone, because of the Doppler's effect I is no longer a constant and the $I_{receive}$ will have a frequency distribution. And if the object has sound absorption $I_{receive}$ will less than I_0.

Doppler's Effect of Submarine

1. Doppler's effect of the body

The noise reflected by the body of submarine and received by the hydrophone is like.

As shown in Fig. 2. 2. 7, $\theta_2=\theta_3$, $\theta_1=\theta_4$, so

$$\begin{aligned} f_m &= \frac{c + v\cos\theta_4}{c} \cdot \frac{c}{c + v\cos\theta_1} f_0 \\ &= \frac{c + v\cos\theta_4}{c + v\cos\theta_1} f_0 \\ &= f_0 \end{aligned} \tag{8}$$

So it doesn't cause Doppler's effect.

So we conclude that if the normal direction of the reflecting plane is normal, the direction of the velocity Doppler's effect will not happen. We assume that the body of the submarine is a cylinder. As shown in Fig. 2. 2. 18, the body will not cause Doppler's effect.

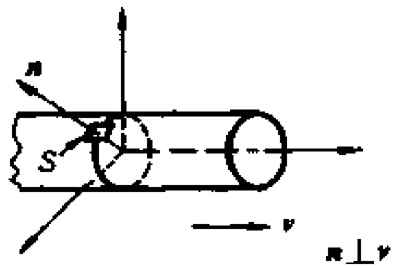

Fig. 2. 2. 18

2. Doppler's effect of the head and the tail

From the Fig. 2. 2. 18 above, we can easily see that the hydrophone can always receive a sound line which transmit opposite the submarine and reflected from point F. This sound line has the highest frequency.

$$f_{head} = \frac{c+v}{c} \cdot \frac{c}{c - v\cos\theta_h} f_0$$

$$= \frac{c+v}{c - v\cos\theta_h} f_0 \qquad (> f_0) \tag{9}$$

θ_h is the angle between the direction of hydrophone and the velocity. The frequencies of the sound reflected from the head (except point F) will be between f_0 and f_{head}.

Similarly, the hydrophone also can receive a sound which has the same direction of the submarine and reflected from the tail. Its frequency is:

$$f_{tail} = \frac{c-v}{c} \cdot \frac{c}{c - v\cos\theta_t} f_0$$

$$= \frac{c-v}{c - v\cos\theta_t} f_0 \qquad (< f_0) \tag{10}$$

and it is the lowest frequency. So after reflecting, the sound is not at one fixed frequency any longer, but has a frequency distribution.

3. Conclusion

In short, when the submarine is moving, Doppler's effect only take place on the head of the submarine and the stern, not on the body. So I reflected from the head and tail change, so that $I_{receive}$ changes accordingly. As shown in Fig. 2. 2. 9, we can see the head, the body and the tail of the submarine.

We use two groups of hydrophone, we can calculate θ_h, θ_t, the location, the size and the direction of the submarine.

Now we calculate the speed.

(1) When $\theta_h > 90°$, $\theta_t > 90°$, the hydrophone group is in the rear of the submarine. We got more information from the stern, so using f_{tail} and θ_t to calculate the speed is better:

$$\because \qquad f_{tail} = \frac{c - v}{c - v\cos\theta_t} f_0$$

$$\therefore \qquad v = c \cdot \frac{f_0 - f_{tail}}{f_0 - f_{tail} \cdot \cos\theta_t} \tag{11}$$

(2) When $\theta_h < 90°$, $\theta_t < 90°$, the hydrophone group is in front of the submarine. We got more information from the head, so using f_{head} and θ_t to calculate the speed is better:

$$\because \qquad f_{head} = \frac{c + v}{c - v\cos\theta_h} f_0$$

$$\therefore \qquad v = c \cdot \frac{f_0 - f_{head}}{f_0 + f_{head} \cdot \cos\theta_h} \tag{12}$$

(3) When $\theta_h > 90°$, $\theta_t < 90°$, we can calculate v_{head} and v_{tail} by the method discussion above, and

$$v = 0.5(v_{tail} + v_{head}) \tag{13}$$

In the above method, we use the information from one hydrophone group to calculate the speed. If we use two hydrophone groups then we get v_1 and v_2, the even speed is:

$$v = 0.5 \times (v_1 + v_2)$$

Of course, we can also calculate the track of the submarine first, then speed equal to the distance divided by time.

Absorption of Submarine's Wake Flow

1. Detecting hidden submarine

If the submarine can absorb sound, assume the coefficient of absorption is I, then $I_{receive}$ no longer equal to I_0, but

$$\begin{aligned} I_{receive} &= \frac{S_0}{2\pi}\int_{S_1}(1-\lambda)I\,d\Omega \\ &= (1-\lambda)\frac{S_0 I}{2\pi}\Omega \\ &= (1-\lambda)I_0 \end{aligned} \tag{14}$$

When λ is not too big, the method developed still works.

When λ is rather big, it is difficult to distinguish f_{tail} and f_{head}. But in this situation, $I_{receive}(=(1-\lambda)I_0)$ is obviously smaller than I_0. So we can judge the submarine presence by $I_{receive}=(1-\lambda)I_0 \neq I_0$. Use the method to calculate the parameters of the submarine. When it comes to speed, using the track of submarine, speed equal to the distance divided by time.

Generally speaking, the submarine does not absorb sound. The submarine that absorb sound is called hidden submarine. So the method developed in this section only apply to the hidden submarine.

2. Use wake flow to detect submarine

Although a general submarine does not absorb sound, if it is equipped with propeller, there is always wake flow. There are two characteristics of wake flow for us to use:

(1) Wake flow absorb sound.

(2) The burst of the bubble in wake flow produce sound of a certain frequency f_s.

f_s is rather high(f_s>10kHz) and the sound whose frequency is higher than 10kHz decay much when spreading in the oceans. So whether we can use the exist of f_s to judge the presence of wake flow should be affirmed by experiment. But there is one point we are sure, the absorption of wake flow makes $I_{receive}$ decay from I_0 to $(1-\lambda_w) I_0$. (λ_w is the coefficient of absorption of wake flow) Then we can use the method above to calculate the location of the wake flow.

Further Development of the Model

In our initial model, we had assumed that there is only one fixed frequency in the ambient noise field:

0.1~10Hz	sound sources are seism, storm in distant places and the turbulence of oceans and atmospher
50~300Hz	ships traffic of distant places
0.5~50kHz	the burst sound of wave
>100kHz	thermal noise of molecule

Fig. 2.2.11 shows the even spectrum of the ambient noise field in the oceans.

Now let's analyse the development of our model when sound has a frequency distribution. In this situation

$$I_{receive} = \frac{S_0}{2\pi}\int_{S_1} I_f \, d\Omega = \frac{S_0}{2\pi} I_f \Omega = I_{0f} \qquad (15)$$

I_f is the distribution of sound intensity (I_f is unchanged or changes periodical or certain law). So when there is no moving object, $I_{receive}=I_{0f}$. When we've detected $I_{receive}\neq I_{0f}$ in certain region, we know there is a moving object in that region.

1. Doppler effect

By analogy of the discussion above, when sound has a frequency distribution, the Doppler effect taken place on the head of submarine make each frequency broaden to the higher frequency extreme, while on the stern, broaden to the low frequency extreme.

2. Absorption of wake flow

By analogy of the discussion above, when hydrophone aim at the wake flow

$$I_{receive} = (1-\lambda)I_{0f} \qquad (16)$$

λ is the coefficient of absorption, λ depend on frequency; $\lambda=\lambda(f)$. And $I_{receive}\neq I_{0f}$ is still the basis of judgment of finding moving object.

In brief, no matter we use Doppler's effect or the absorption of wake flow, we can use two hydrophone groups to ascertain the location of the head and stern of the submarine and other parameters.

3. Sound in the non-even medium

(1) First, we assume that the sound speed $c(y)$ only depend on the depth, and $\frac{dc(y)}{dh}=G$ (G is a constance). As shown in Fig. 2.2.12, we divide the ocean into several layers. In each layer speed is constant. In the first layers, the speed is v_0, the second is v_1. φ is elevation angle.

$$\tan\alpha_1 = \frac{\Delta x}{\Delta y}$$

Basing on the law of refraction

$$\frac{\Delta x}{\Delta y} = \frac{c_1}{c_0}\frac{\cos\varphi}{\sqrt{1-\frac{c_1^2}{c_0^2}\cos^2\varphi}}$$

$$\lim_{\Delta y\to 0}\frac{\Delta x}{\Delta y} = \frac{dx}{dy} = \frac{c_1}{c_0}\frac{\cos\varphi}{\sqrt{1-\frac{c_1^2}{c_0^2}\cos^2\varphi}} \tag{17}$$

the sound speed in the second layer:

$$c_1 = c_0 + Gy_1$$

let $\quad G/c_0 = a \qquad c_1/c_0 = 1 + ay$

So

$$\frac{dx}{dy} = \frac{(1+ay)\cos\varphi}{\sqrt{1-(1+ay)^2\cos^2\varphi}} \tag{18}$$

then

$$x = \int\frac{(1+ay)\cos\varphi}{\sqrt{1-(1+ay)^2\cos^2\varphi}}dy$$

$$= -\sqrt{\frac{1}{a^2\cos^2\varphi} - \left(y+\frac{1}{a}\right)^2} + d \tag{19}$$

integral constant d is determined by $y=0$ and $x=0$, so:

$$d = \sqrt{\frac{1}{a^2\cos^2\varphi} - \frac{1}{a^2}} = \frac{\tan\varphi}{a} \tag{20}$$

So

$$(x-\tan\varphi/a)^2 + \left(y+\frac{1}{a}\right)^2 = \frac{1}{a^2\cos^2\varphi} \tag{21}$$

$$a = G/c_0$$

then $$\left(x - \frac{c_0}{G}\tan\varphi\right)^2 + \left(y + \frac{c_0}{G}\right)^2 = \frac{c_0^2}{G^2\cos^2\varphi} \tag{22}$$

From Equ. (22) we can conclude that if G is a constant, the track of the sound will be a circle and its radius:

$$R = \frac{c_0}{|G|\cos\varphi} \tag{23}$$

the center of circle(x_c, y_c):

$x_c = c_0\tan\varphi/G$, $y_c = -c_0/G$

(2) If G is not a constant, but only depend on the depth y; and we know $G(y)$. we get

$$\frac{\mathrm{d}x}{\mathrm{d}y} = \frac{\left(1 + \frac{G(y)}{c_0}\right)\cos\varphi}{\sqrt{1 - \left(1 + \frac{G(y)}{c_0}\right)^2\cos\varphi}} \tag{24}$$

Assume a hydrophone receives a signal, we know v_0, φ. We can calculate the sound track through Equ. (24) with method of Runge-Kutta(Just like the function refixed (y, x_1, x_2, npoint, D)" for solving differential equation in Mcad 5.0 plus). With it we can modify the method for locating submarine.

In the normal ocean, how sound speed depend on the depth (As shown in Fig. 2.2.13a and how sound line transmit (As shown in Fig. 2.2.13 d).

Strengths and Weaknesses

Strengths

(1) Our model use proper mathematical form to associate $I_{receive}$ with the distribution of sound intensity of the ambient noise field.

(2) It is not difficult to develop the method from one fixed

frequency to multi-frequency.

(3) The model is easily adaptable to situation when the sound does not travel along a line.

(4) The model can use the information given by the ambient noise field efficiently.

Based on changes of frequency and amplitude, we develop two methods to detect a large moving object's location, size and velocity. For the submarine, these two methods can be used together (to locate the submarine and its wake flow). For detecting large moving object with high speed, we can use Doppler's effect; For the ones with high absorption rate, we can use the changes of the amplitude of reflected wave.

Weaknesses

(1) Our model only mention two characteristics of the ambient noise field; even in space and isotropy, but ignores the random undulation of the field.

(2) f_{tail} and f_{head} can not be distinguished in the noise field with multi-frequencies. In this occasion, we can not use Doppler's effect to detect the velocity. We can only get the speed value with $\Delta s/\Delta t = v$. So the deviation will increase in speed measuring.

Further Development

(1) We suggest using a proper statistic model to describe random undulation of the noise field so that the model will be more practical.

(2) We also suggest to work out the amplitude-frequency (in the domain $[f_{tail}, f_0]$ and $[f_0, f_{head}]$) distribution curve F with

$$I_{receive} = \frac{S_0}{2\pi}\int_{S_1} I d\Omega$$

and experiment. In fact, F is associated with the shape, the type frequency to multi-frequency.

(3) The model is easily adaptable to situation when the sound does not travel along a line.

(4) The model can use the information given by the ambient noise field efficiently.

Based on changes of frequency and amplitude, we develop two methods to detect a large moving object's location, size and velocity. For the submarine, these two methods can be used together (to locate the submarine and its wake flow). For detecting large moving object with high speed, we can use Doppler's effect; For the ones with high absorption rate, we can use the changes of the amplitude of reflected wave.

Weaknesses

(1) Our model only mention two characteristics of the ambient noise field: even in space and isotropy, but ignores the random undulation of the field.

(2) f_{tail} and f_{head} can not be distinguished in the noise field with multi-frequencies. In this occasion, we can not use Doppler's effect to detect the velocity. We can only get the speed value with $\Delta s/\Delta t = v$. So the deviation will increase in speed measuring.

Further Development

(1) We suggest using a proper statistic model to describe random undulation of the noise field so that the model will be more practical.

(2) We also suggest to work out the amplitude-frequency (in the domain $[f_{tail}, f_0]$ and $[f_0, f_{head}]$) distribution curve F with

$$I_{receive} = \frac{S_0}{2\pi}\int_{S_1} I d\Omega$$

and experiment. In fact, F is associated with the shape, the type