Cardboard Comfortable When It Comes to Crashing

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Abstract

A scene in an upcoming action movie requires a stunt person on a motorcycle to jump over an elephant; cardboard boxes will be used to cushion the landing.

We formulate a model for the energy required to crush a box based on size, shape, and material. We also summarize the most readily available boxes on the market. We choose a maximum safe deceleration rate of $5g$, based on comparison with airbag rigs used professionally for high-fall stunts.

To ensure that the stunt person lands on the box rig, we analyze the uncertainty in trajectory and extract the landing point uncertainty.

We construct a numerical simulation of the impact and motion through the boxes based on our earlier energy calculations. After analyzing the sensitivity and stability of this simulation, we use it to examine the effectiveness of various configurations for the box stack (including different box sizes, types of boxes, and stacking patterns). We find that 200 kg is the most desirable combined mass of the motorcycle and stunt person, and a launch ramp angle of 20° is optimal when considering safety, camera angle, and clearance over the elephant.

A stack of $(30\text{ in})^3$ boxes with vertical mattress walls spaced periodically is optimal in terms of construction time, cost, and cushioning capacity. We recommend that this stack be 4 m high, 4 m wide, and 24 m long. It will consist of approximately 1,100 boxes and cost $4,300 in materials. The stunt person's wages are uncertain but fortunately the elephant works for peanuts.

Introduction

Airbag rigs are commonly used for high-fall stunts [M&M Stunts 2003], but they are designed only to catch humans. The alternative is a cardboard-box rig—a stack of boxes that crush and absorb impact.

Our objectives are:

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- to catch the stunt person and motorcycle safely, and
- to minimize the cost and size of the box rig.

Our approach is:

- We investigate the relationship between the size/shape/material of a box and the work (*crush energy*) required to crush it.
- We review the available cardboard boxes.
- By comparison with an airbag rig, we estimate the maximum acceptable deceleration that the stunt person can experience during landing.
- We analyze the trajectory of the motorcycle and the uncertainty in its landing location. This determines the proper placement of the box rig and how large an area it must cover.
- Using the crush energy formula, we estimate the number of boxes needed.
- We formulate a numerical simulation of the motorcycle as it enters the box rig. Using this model, we analyze the effectiveness of various types of boxes and stacking arrangements for low, medium, and high jumps.
- As an alternative to catching the stunt person while sitting on the motorcycle, we analyze the possibility of having the stunt person bail out in mid-air and land separately from the motorcycle.
- We make recommendations regarding placement, size, construction, and stacking type of the box rig.

Energy Absorbed by Crushing Cardboard

We estimate the energy required to crush a box, based on physical considerations and experimentation. We assume that the primary source of energy absorption is the breakdown of the box walls due to edge compressive forces.

Commercial cardboard is rated by the edge crush test (ECT), which measures edge compressive force parallel to the flute (the wavy layer between the two wall layers) that the cardboard can withstand before breaking. This can be interpreted as the force against the edge per unit length of crease created [Pflug et al. 1999; McCoy Corporation n.d.]. Once a crease has formed, very little work is required to bend the cardboard further.

To understand how the formation of wall creases relates to the process of crushing a box, we conducted several experiments (**Figure 1**). We found:

- The first wall-creases typically form in the first 15% of the stroke distance.
- These creases extend across two faces of the box; a schematic of one such crease is illustrated in **Figure 2**.

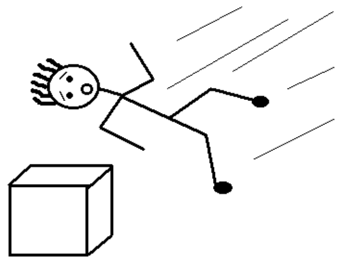

Figure 1. Experimental apparatus for crushing boxes: We dropped a *crush-test dummy* (i.e., team member) onto several boxes and observed how the structure (the box, not the dummy) broke down.

Figure 1a. Crush-test dummy in action. (Left: Jeff Giansiracusa; right: Simon Pai.)

Figure 1b. Crushed box with creases. (Photos courtesy of Richard Neal.)

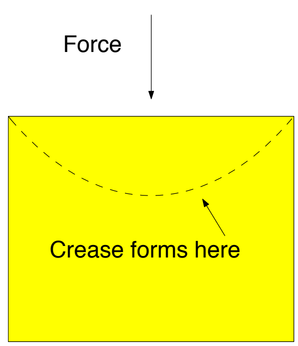

Figure 2. The first crease forms in a curve across the side faces as the box is compressed.

- Once these have formed, the box deforms further with comparatively little resistance, because additional creases are created by torque forces rather than edge compressive forces.
- The primary creases each have length approximately equal to the diagonal length of the face.

The work done in crushing the box is given by the average force applied times the distance through which it is applied. This and our experimental qualitative results lead us to write the following equation for energy absorbed by a box of dimension $l_x \times l_y \times l_z$ crushed in the z-direction:

$$E = \text{ECT} \times 2\sqrt{l_x^2 + l_y^2} \times l_z \times 0.15 \quad \textbf{(1)}$$

As a reality check, we compute the crush energy for a standard 8.5 in $\times$ 17 in $\times$ 11 in box with ECT = 20 lbs/in and a C-flute (the type commonly used to store paper). With these numerical values, **(1)** gives an energy of 187 J. This corresponds roughly to a 140-lb person sitting on the box and nearly flattening it. Crush-test dummy results confirm this estimate.

Energy can also be absorbed in the process of flattening the flute within the cardboard walls. However, the pressure required to do this is approximately 150 kPa [Pflug et al. 1999] and the surface area involved is more than 1 m^2, so a quick calculation shows that the stunt person would decelerate too quickly if the kinetic energy were transferred into flattening boxes. We therefore ignore this additional flattening effect.

So, any successful box rig configuration must dissipate all of the kinetic energy of the stunt person and motorcycle through box-crushing alone.

Common Box Types

Minimizing cost is important. The cardboard box rig will consist of perhaps hundreds of boxes, and wholesale box prices can range up to $10 or $20 per unit; so we restrict our attention to commonly available box types (**Table 1**).

Table 1.

Commonly available box types [Paper Mart n.d.; VeriPack.com n.d.]

Type	Size (in)	ECT rating (lbs/in)	Price
A	$10 \times 10 \times 10$	32	$0.40
B	$20 \times 20 \times 20$	32	$1.50
C	$20 \times 20 \times 20$	48	$3.50
D	$30 \times 30 \times 30$	32	$5.00
E	$44 \times 12 \times 12$	32	$1.75
F	$80 \times 60 \times 7$	32	$10.00

Some Quick Estimates

Maximum Safe Acceleration

To determine acceptable forces and accelerations for the stunt person, we compare the box rig with other cushioning devices. In the stunt rigging business, it is common practice to use an air bag for high falls of up to 30 m; such airbags are approximately 4 m deep.

Assume that a stunt person falls from 30 m above the airbag. Gravity accelerates the performer from rest to speed v when the performer strikes the airbag and is decelerated completely, so we have

$$\sqrt{2gd_{\text{fall}}} = \sqrt{2a_{\text{bag}}h_{\text{bag}}},$$

where d_{fall} is the fall distance, a_{bag} is the deceleration rate the stunt person experiences in the airbag, h_{bag} is the height of the airbag, and g is the acceleration due to gravity. Thus,

$$a_{\text{bag}} = \frac{d_{\text{fall}}}{h_{\text{bag}}}\, g = \frac{30 \text{ m}}{4 \text{ m}} g = 7.5g.$$

We therefore conclude:

- When using an airbag, the stunt person experiences an average acceleration of at most $7.5g$. This provides an upper bound on the maximum acceleration that a person can safely withstand.
- With the airbag, the stunt person is able to land in a position that distributes forces evenly across the body. In our stunt, however, the stunt person lands in the box rig while still on the motorcycle, with greater chance for injury under high deceleration.
- We choose $5g$ as our maximum safe deceleration.

Displacement and Energy Estimates

If the deceleration is constant through the boxes, then we can estimate the distance required to bring them to rest. Since any deviation from constant acceleration increases either the stopping distance or the peak deceleration, this will give us a lower bound on the stopping distance and hence on the required dimensions of the box rig.

Suppose that the stunt person enters the rig at time $t = 0$ with speed v_0 and experiences a constant deceleration a until brought to rest at time $t = t_f$. The person's speed is $v(t) = v_0 - at$. Since the stunt person is at rest at time t_f, we have

$$t_f = v_0/a.$$

Let $x(t)$ be the displacement from the point of entry as a function of time. Since $x(0) = 0$, we have

$$x(t) = v_0 t - \tfrac{1}{2} a t^2$$

and so the total distance traveled through the boxes is

$$\Delta x = x(t_f) = \frac{v_0^2}{a} - \frac{1}{2}\, a \left(\frac{v_0}{a}\right)^2 = \frac{v_0^2}{2a}.$$

Therefore, we arrive at:

- Given an impact velocity $v_0 \approx 20$ m/s and deceleration bounded by $5g$, the stunt person requires *at least* 4 m to come to rest.

The energy that must be dissipated in the boxes is roughly equal to the kinetic energy that the motorcycle and stunt person enter with. (Since the box rig should be only 3–4 m high, the potential energy is a much smaller fraction of the total energy.) Thus, for $v_0 = 20$ m/s and a mass of 200 kg, the change in energy is 40,000 J. From **(1)**, we calculate that the crush energy of a standard (30 inch)^3 box is 633 J, so we need. $40,000/633 \approx 60$ boxes.

Trajectory Analysis and Cushion Location

Cardboard boxes won't dissipate any energy unless the stunt person lands on them. It is therefore important to consider the trajectory, so we know where to place the box rig and what the uncertainty is in the landing location.

We calculate trajectories by solving the following differential equation, where v is the speed, k is the drag coefficient, and $\vec{x}$ is the position:

$$(\vec{x})'' = -g\hat{z} - \frac{k}{m}|v|^2\hat{v}$$

We used Matlab's ODE45 function to solve an equivalent system of first-order equations. We use an air drag coefficient of $k = 1.0$ [Filippone 2003]. We see from **Figure 5** that it would be unwise to ignore air resistance, since it alters the landing position by up to several meters.

It is unreasonable to expect the stunt person to leave the ramp with exactly the same initial velocity and angle every jump. We therefore need to allow for some uncertainty in the resulting trajectory and ensure that the cardboard cushion is large enough to support a wide range of possible landing locations. The ramp angle ϕ is constant, but the motorcycle might move slightly to one side as it leaves the ramp. Let θ be the azimuthal angle between the ramp axis and the motorcycle's velocity vector. Ideally θ should be zero, but small variations may occur. The other uncertain initial condition is the initial velocity v_0.

In modeling possible trajectories, we assume the following uncertainties:

- Initial velocity: $v_0 = v_{\text{intended}} \pm 1$ m/s

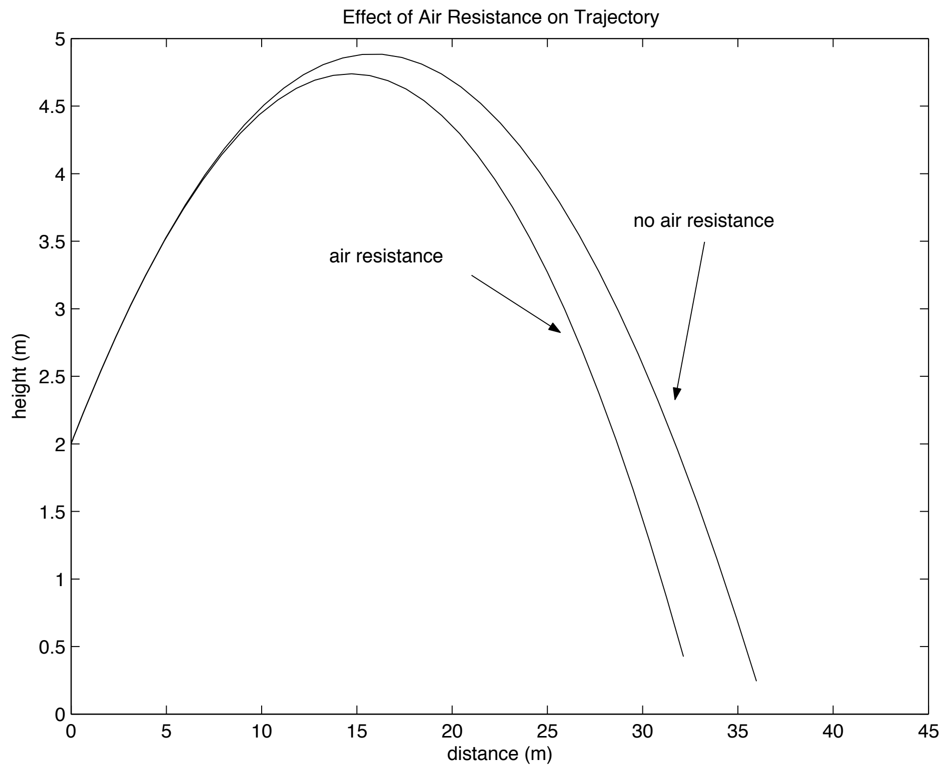

Figure 3. Air resistance significantly changes the trajectory.

- Azimuthal angle: $\theta = 0° \pm 2°$

We use this to identify the range of possible landing locations by plotting the trajectories that result from the worst possible launches (**Figure 6**).

If the intended initial velocity is 22 m/s, the ramp angle is 20°, and the mass of the rider plus motorcycle is 200 kg, then the distance variation is $\pm$ 2.5 m and the lateral variation is $\pm$ 1.5 m.

Impact simulation

To evaluate the effectiveness of various box rig configurations, we construct a numerical simulation of the motion of the stunt person and motorcycle through the box rig.

Assumptions

The full physics of the box rig is far too complex to model accurately. We make the following assumptions to approximate and simplify the problem.

- *The problem is two dimensional.* We restrict out attention to the plane of motion of the stunt person.

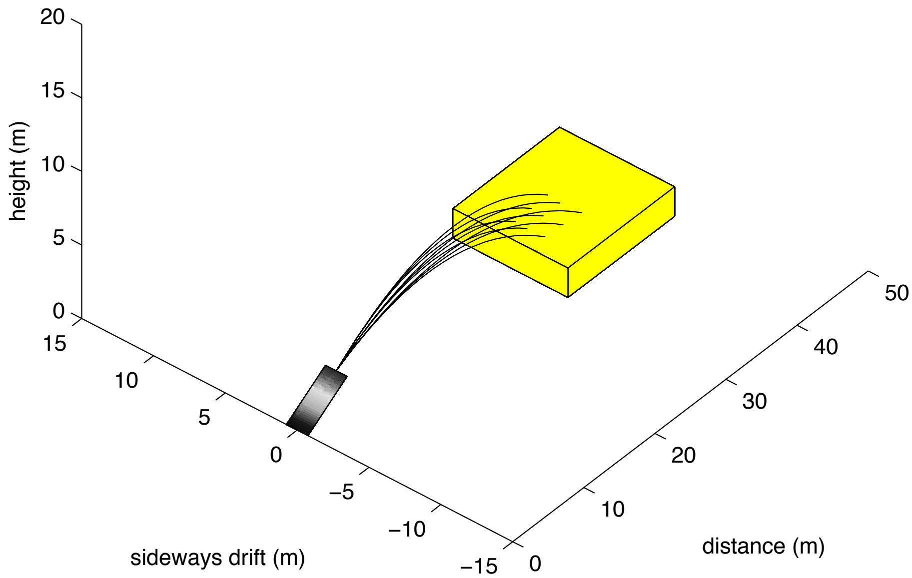

Figure 4. Trajectory uncertainty due to launch uncertainties (box rig is not to scale).

- *As the motorcycle plows through the boxes, a thick layer of crushed boxes accumulates against its front and lower surfaces*. These layers increase the effective size of the motorcycle and cause it to strike a larger number of boxes as it moves. This assumption captures the effects of internal friction and viscosity within the boxes.
- *In striking a large number of boxes, the velocity magnitude is reduced but the direction is unchanged.*
- *Boxes are crushed rather than pushed out of the way.* In practice, this can be ensured by placing a strong netting around the three sides of the box rig that face away from the incoming stunt person.
- *Boxes are crushed to a uniform level.* Some boxes may be crushed only slightly while others are completely flattened, but these effects disappear when we average over a large number of collisions.

Formulation

We formulate the simulation as follows:

- The motorcycle with stunt person is represented by a bounding rectangle that is initially 1.2 meters long, 1.2 m high and 0.7 m wide.
- The box rig is represented by a two-dimensional stack of boxes.
- We numerically integrate the motion in discrete time steps of 0.05 s. The only object in motion throughout the simulation is the stunt person plus motorcycle—all boxes are stationary.

- When the bounding rectangle intersects a box, the box is considered crushed. We modify the stunt person's velocity according to the kinematics described later and ignore further interactions with the crushed box.
- For each box crushed, we add a layer of additional thickness to either the front or the bottom of the motorcycle bounding rectangle. We assume that boxes are crushed to 20% of their length or height. We allow the front layer to extend above and below the original bounding rectangle (and likewise for the bottom layer), so that the force of the motorcycle striking a tall box is effectively distributed along the length of the box. These debris layers increase the effective size of the motorcycle and therefore cause it to strike a larger number of boxes as it moves. We use this process to account for the effects of friction.
- The vertical component of the velocity is set to zero when the bounding rectangle strikes the ground.

Kinetics

As the stunt person with motorcycle falls into the rig, each box collided with collapses and absorbs a small amount ΔE of kinetic energy, thereby slowing the descent. The crushed box is then pinned against the forward moving face of the stunt person and motorcycle and must move with them, contributing an additional mass of m_{box}.

We calculate the change in this velocity using conservation of energy and assuming that the velocity direction remains unchanged (this is a good approximation in the average of a large number of collisions):

$$\tfrac{1}{2}(m_0 + m_{\text{box}})v_{\text{new}}^2 = \max\left(\tfrac{1}{2}m_0 v_0^2 - \Delta E, 0\right).$$

We take the maximum to avoid imparting more energy to the box than the motorcycle has. Solving for v_{new} yields

$$v_{new} = \sqrt{\max\left(\frac{m_0 v_0^2 - 2\Delta E}{m_0 + m_{\text{box}}}, 0\right)} \tag{2}$$

We use this equation to calculate the new velocity after each collision.

Stability and Sensitivity Analysis

Given the crude nature of our collision detection, there is the danger of finding results that depend sensitively on the initial location of the motorcycle relative to the phase of the box-rig periodicity (typically less than 1.5 m). To show that these phase alignment effects are negligible we vary the initial location of the motorcycle by 0.4 m (37% of the rig periodicity) in either direction. Deceleration rates and stopping distance vary by less than 5%. The simulation is therefore insensitive to where the motorcycle lands relative to the period of the box rig.

As a second check, we vary the time step size from 0.025 s to 0.1 s (our standard value is 0.05 s). There are no distinguishable changes in results; the simulation is highly insensitive to the size of the time step.

Configurations Considered

We consider the following configurations for the stunt:

- *Seven different stacking arrangements.* Details are shown in **Table 2** and **Figure 7**.

Table 2.

The seven box rig configurations. Refer to **Table 1** for data on the lettered box types.

Stack type	Cost/m^2	Description
1	\$40	Standard rig, box type B (20-in cube).
2	\$94	Standard rig, heavy-duty box type C (20-in cube, ECT 48).
3	\$43	Standard rig, box type D (30-in cube).
4	\$47	Like type 3, but type-A boxes (10-in cube) inside the D boxes.
5	\$46	Modification of type 3: additional vertical walls of type F mattress boxes.
6	\$41	Like type 5, but horizontal mattress box walls.
7	\$46	Mattress boxes (type F) stacked horizontally, with periodic vertical walls

- *Three values for the total mass of the motorcycle and stunt person:* 200 kg, 300 kg, and 400 kg.

- *Three flight trajectories for the motorcycle and stunt person: low, medium, and high.* These provide three different entry angles and velocities for the simulation. Each trajectory is designed to clear an elephant that is roughly 3 m tall [Woodland Park Zoo n.d.]. Details of these trajectories are given in **Table 3** and are shown to scale in **Figure 8**.

Table 3.

The three test trajectories.

Jump type	Initial speed (m/s)	Ramp angle angle	Jump distance (m)
Low	29	10°	30.0
Medium	22	20°	28.5
High	20	30°	30.4

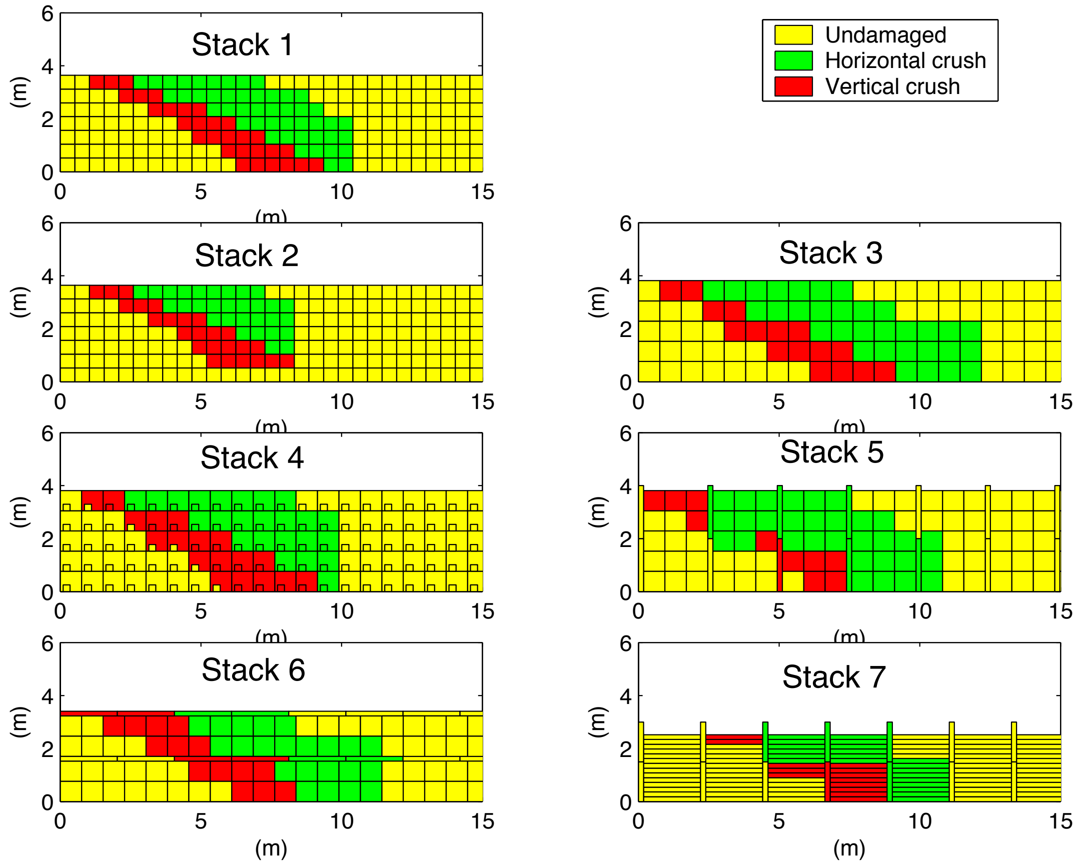

Figure 5. Box stacking configurations. Crush patterns are the result of simulated impacts of a 200-kg mass in the low trajectory.

Data Analysis

The simulation provides us the velocity as a function of time. The plots appear jagged and step-like because of the discrete way in which our simulation handles collisions. We obtain the acceleration by fitting a straight line to the velocity vs. time plot and measuring the slope (**Figure 9**).

In examining the plots for the runs, we look at:

1. deceleration experience by the stunt person, averaged over the entire time from impact to rest;
2. maximum of deceleration averaged over 0.1 s intervals; and
3. whether or not the boxes completely arrest vertical motion before the stunt person hit the ground.

If either (1) or (2) ever exceeds the maximum safe deceleration threshold of $5g$, or (3) fails, we consider the configuration unsafe.

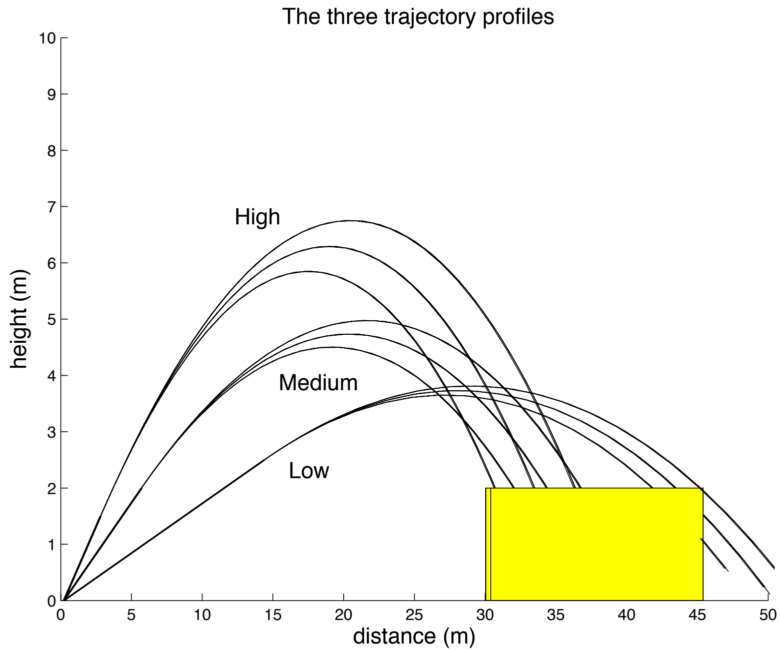

Figure 6. The three trajectories tested.

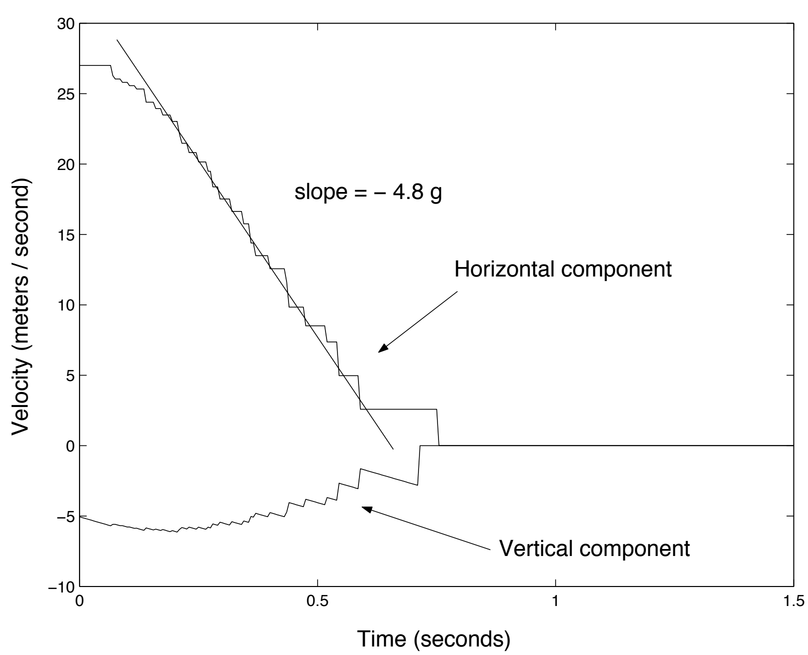

Figure 8. Velocity vs. time for a 200-kg low trajectory impact on a box stack of 20 in cubes stacked in standard style.

Results

Mass

For a mass of 400 kg, the boxes give way beneath the incoming motorcycle too easily; such a mass would require a stack of boxes nearly 6 m high. A mass of 300 kg is marginal, and 200 kg is optimal.

Stacking Types

We measure stopping distance from the point of impact to the furthest box damaged and report the stopping distance for the medium trajectory. (The motorcycle actually comes to rest in a significantly shorter distance, but it pushes a wall of debris several meters ahead of it.) The results for the stack types in **Figure 6** are:

1. Made from the cheapest and most common boxes, this stack resulted in $4.8g$ deceleration; it stopped the motorcycle in 11 m.
2. Rejected—deceleration of over $6g$ but brings the motorcycle to rest in only 7 m.
3. Very soft deceleration of $3.6g$ to $4.1g$. But this stack did not completely stop the vertical motion and took 13 m to bring the motorcycle to rest.
4. Marginally safe deceleration from $4.8g$ to $5.1g$, but this stack is the best at arresting the vertical motion; stopping distance of 9 m.
5. Behavior is similar to type (3), but stopping distance is reduced to 11 m.
6. The extra horizontal mattress boxes make very little difference. Deceleration is $4.1g$, and vertical motion is not slowed enough to prevent hitting the ground hard.
7. Rejected because deceleration ($5.2g$ to $5.7g$) is unsafe.

The difficulty of slowing the vertical motion enough might be overcome by stacking the box rig on top of a landing ramp.

Conclusion: Type (1) stacking is optimal without a ramp. However, with a landing ramp under the boxes, type (3) or type (5) stacking gives a much softer deceleration.

We tried additional variations on the type (5) stack. We conclude that

30-in boxes (type D) with mattress box walls spaced every 4 boxes is optimal.

An Alternative: Bailing Out

It may be desirable for the stunt person and motorcycle to separate before impacting the box rig, since doing so would reduce the chance of injury resulting from the stunt person being pinned against the motorcycle.

We assume that they separate after clearing the elephant and allowing for a clear camera shot, corresponding to a distance of about 25 m. We run the same simulation as before but alter the vertical velocity at the point of separation and then follow separately the two trajectories. An estimate of the change of momentum is necessary to figure out the corresponding changes in velocity. If the stunt person jumps vertically away from the motorcycle, it makes sense to consider the analogy of a person jumping on the ground. A decent jump corresponds to about 0.5 m. Since the initial velocity is $v_0 = \sqrt{2gd}$ where d is the height, we find that v_0 is roughly 3 m/s. Accordingly, we increase the stunt person's vertical velocity by 3 m/s. Then the corresponding change in velocity for the motorcycle is given by conservation of momentum. The resulting trajectories are plotted in **Figure 10**.

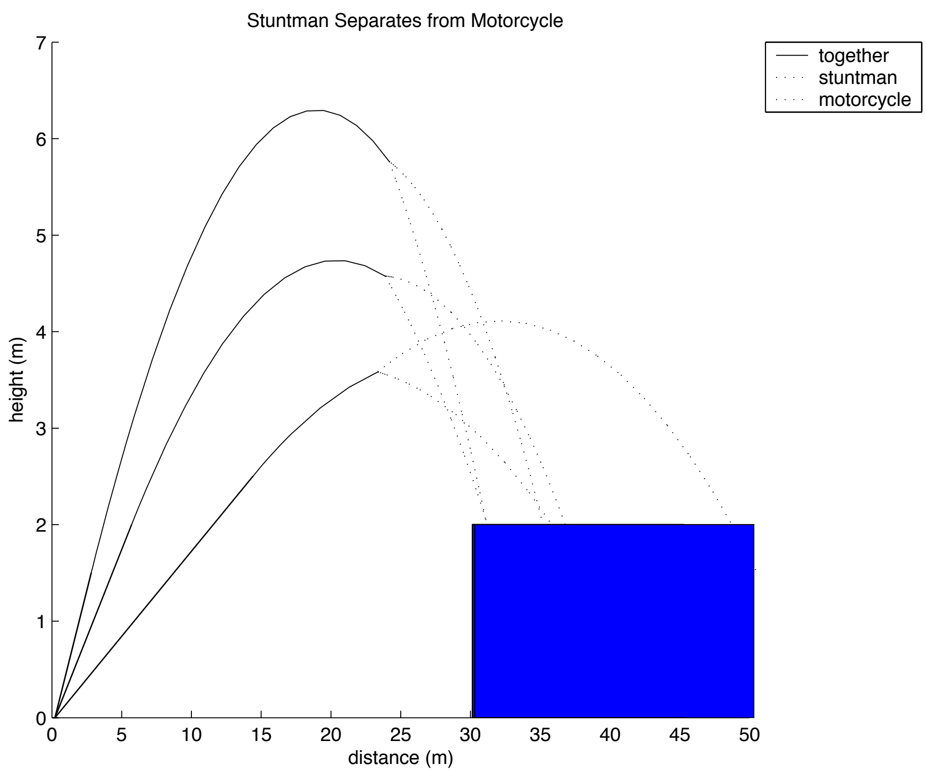

Figure 9. Stunt person separating from motorcycle in three possible trajectories.

When the trajectory is medium or high, stunt person and motorcycle are separated by only about 6 m at the landing point. When the trajectory is low, however, this separation increases to around 15 m. This presents a problem if we want to protect both the motorcycle and the stunt person. Naturally, the safety of the person is the most important, and it is simple to extend the box rig to the projected landing location of the stunt person.

Unfortunately, simulations show that a box configuration designed to decelerate the combined mass of motorcycle and stunt person smoothly doesn't work as well when there is just the mass of the person to contend with. In fact, it's possible that the stunt person will decelerate so quickly that our g-force tolerance is exceeded. Our simulations show that this is in fact the case for *all* of the box stacks that we considered. For the heights and speeds considered, a box rig is unsafe. However, if the boxes are stacked loosely enough with some spacing between the boxes as in **Figure 11**, then it is possible to decelerate the stunt person at a reasonable rate. Therefore, the best solution is to redesign the box rig, using a softer material and/or looser stacking.

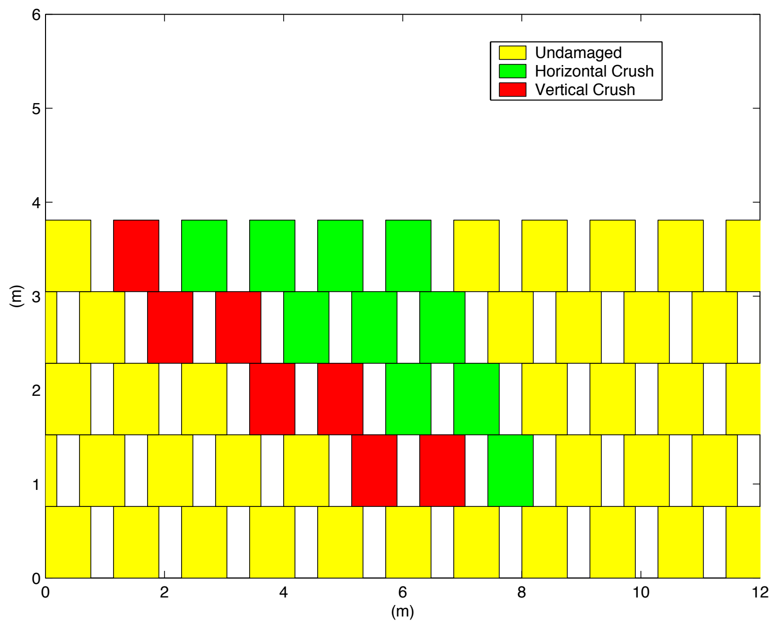

Figure 10. Box stacking arrangement suitable for catching a stunt person who has separated from the motorcycle.

Recommendations

- **Which mass is best?** A 400 kg mass is simply too much to be slowed adequately by a box stack less than 4 m high. The motorcycle invariably falls through the rig and hits the ground beneath at greater than 5 m/s; the motorcycle could easily tumble over in the boxes and crush the stunt person. A 300 kg mass is marginal, but the safest is 200 kg.

- **Which trajectory is best?** The low trajectory (10°) provides the least risk but allows only minimal clearance over the elephant (only 1 m for a tall elephant) and requires the highest speed (which increases the risk).

- **Which type of boxes and stacking is best?** The type (1) stack, made of $(20\text{ in})^3$ boxes, is best for landing without a ramp. With a ramp under the rig, type (3), made from $(30\text{ in})^3$ boxes, and type (5), which is type (3) with added vertical mattress box walls, are optimal. The added walls of type (5) decrease the landing distance by 2 m, so fewer boxes are required and construction cost is reduced.

- **What size must the rig be?** With the 200 kg mass, our simulation shows that 3 m height is usually enough for the low trajectory, but 4 m is necessary for the high trajectory. This can be reduced to as little as 2 m if the rig is stacked on top of a landing ramp. Stopping distance is between 10 and 13 m (measured from point of entry to the front of the debris wall) depending on stack type, and we estimate that the landing location uncertainty is 1 m laterally and 3 m forwards or backwards. We consider an additional 50% beyond these uncertainties to be necessary. Therefore our recommendations are:
  - Height: 4 m without landing ramp, 2 m with ramp.
  - Width: 4 m.
  - Length: 24 m for type (1) or (5) stacking, and 29 m for type (3) stacking.

- **How much does it cost?** The cost is between $4,300 for type (1) and $5,300, depending on the precise configuration; this is approximately the cost of renting an airbag rig for a day [M&M Film Stunt 2003].

- **How many boxes?** Type (1) stack requires 2,000 $(20\text{ in})^3$ boxes and type (3) requires 1,100 $(30\text{ in})^3$ boxes. Type (5) uses the same number as (3) and a few additional mattress boxes.

Final Recommendation

The overall best type of box rig uses $(30\text{ in})^3$ boxes stacked as usual, with vertical mattress box walls every couple of meters to distribute the forces over a larger number of boxes. This configuration gives the softest deceleration and requires the fewest boxes.

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Editor's Note

The authors' model emphasizes the important role that creases play in the breakdown of a box, based on their own experimental results with "crash dummies."

The authors' conclusions about this are confirmed in Peterson [2003], which summarizes some of the research on crumpling, and from which we quote and summarize:

> [T]he energy that goes into bending and stretching a material as it is crumpled is concentrated in the network of narrow ridges ... [C]rumpled sheets can be described in terms of the pattern of ridges and peaks that cover the surface. By adding together the deformation energies associated with individual ridges, scientists can estimate the total energy stored in a given crumpled sheet. . . .
>
> The researchers also discovered that increasing a sheet's size has an unexpectedly small effect on the total amount of energy required to crumple it. For instance, it takes only twice as much energy to crumple a sheet whose sides are eight times [as long] ...

A team of students from Fairview High School in Boulder, Colorado—Andrew Leifer, David Pothier, and Raymond To—won an award at this year's Intel International Science and Engineering Fair for their study of crumpling. They found that ridges in crumpled sheets show a fractal pattern, with a power law describing the lengths of ridges produced from buckling a single ridge and with a Weibull probability distribution describing the frequency distribution of ridge lengths.

> Their result supported the notion that paper crumpling can be viewed as a repetitive process of buckling multiple ridges and their daughter products.

Peterson gives extensive references to literature on crushing, crumpling, and buckling of thin sheets.

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Peterson, Ivars. 2003. Ivars Peterson's Math Trek: Deciphering the wrinkles of crumpled sheets. `http://www.maa.org/mathland/mathtrek_05_26_03.html` .