Judge's Commentary: The Outstanding Scanner Papers

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Each of the participating schools is to be commended for its fine effort. The judges did not witness a wide range of mathematical modeling by the participants to obtain their solutions. Most teams recognized this problem as an image processing problem.

According to the problem statement, the current family of MRIs slice a three-dimensional scanned image, vertically or horizontally. One component of the problem required teams to obtain an oblique slice. Teams used one of three basic methods to obtain their oblique slice:

- The method seen most often was to create a plane, $Ax + By + Cz = D$, and then rotate it using a standard matrix transformation.
- Selecting two points in three-space and defining a plane between those points.
- Selecting one point and two angles to define their plane.

Teams realized that a critical element was mapping the coordinates of their oblique plane through their three-dimensional data set in order to obtain a gray scale scheme (0–255) for the elements in the plane. The three-dimensional data set was defined by three integers, while the points in the oblique plane were real numbers. Methods had to be developed to interpolate the gray scale values for all the points in the oblique plane.

Methods chosen by the teams included:

- a nearest neighbor algorithm, using eight or more points;
- a weighted point algorithm;
- splines (linear through cubic); and

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- Lagrangian polynomials.

Teams usually tried more than one method to obtain the grayscale values. Comparisons of methodologies were generally sketchy and lacked analysis. Teams that critically compared and analyzed their methodology and results, and reached valid conclusions, impressed the judges.

The problem statement required teams to design and test an algorithm to produce sections of three-dimensional arrays made by planes in any orientation in space, preserving as closely as possible the original grayscale values. The use of grayscale was a distinguishable characteristic. Some teams used color to enhance their presentation; this was acceptable provided that the grayscale was not totally removed. Several teams suggested color because their grayscale resolution could not detect certain diagnostic elements; this was viewed as a fatal flaw, since the problem statement required the use of grayscale.

The judges felt that to distinguish teams better, they would focus more closely on the MUST and SHOULD requirements of the problem statement.

- The team's algorithm MUST produce a picture of the slice of the three-dimensional array by a plane in space. This became one critical element for teams to move beyond the Successful Participant category. Judges wanted to see a picture, not a matrix portrayal. Pictures were closely scrutinized to see if they appeared to be oblique slices.

- The teams SHOULD:

  - Design data sets to test and demonstrate their algorithm.
  - Produce data sets that reflect conditions likely to be of diagnostic value.
  - Characterize data sets that limit the effectiveness of their algorithm.

  Thus, judges looked for a good description of the data sets chosen and a description of the elements of diagnostic value. Verbal descriptions stating teams were looking for tumors or anomalies in body parts were acceptable. Teams also created spheres inside cubes as their representative data set. Provided teams put something of diagnostic value inside their lager 3-D elements, their data sets were still acceptable.

  The characterization of data sets that limited the effectiveness of the team's algorithm was the most avoided "SHOULD" requirement. A verbal description of any data set content that limited effectiveness was acceptable by the judges in order to separate the top quality papers.

Another important element not uniformly accomplished was some kind of error analysis. Very few teams even checked for accuracy their integer values in the plane against the corresponding integer point in their three-space data set. The judges praised those teams that accomplished that. Almost every team referred to their pictures—"outputs"—to explain or attempt to show accuracy. Teams, as their only basis for analysis, used the "blurry versus sharp" edges.

Style and clarity of presentation was viewed as another critical element. Teams' organization and ability to explain their methodologies separated participants. Good organization and a solid layout helped distinguish teams.

The judges who evaluated this problem were impressed by the quality and completeness of the solutions presented. We were amazed at how much work was accomplished during that weekend.

About the Author

William P. Fox is Chairman of the Dept. of Mathematics and Professor of Mathematics at Francis Marion University. He received his M.S. in operations research from the Naval Postgraduate School in 1982 and earned his Ph.D. from Clemson University in 1990. He has served as a judge and as the associate contest director of the MCM. Bill will be the contest director for the new High School Mathematical Contest in Modeling under a grant through COMAP.