From Lake Murray to a Dam Slurry

Clay Hambrick
Katie Lewis
Lori Thomas
Harvey Mudd College
Claremont, CA

Advisor: Jon Jacobsen

Summary

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The South Carolina State Capitol is safe under even the most extreme circumstances, since it sits on a hill well above the highest possible water level. However, flood waters could still reach 17 m at Columbia and even higher upstream. Buildings in Columbia close to the water would be inundated, but there should be enough warning time for residents to escape. Both our model and local evacuation plans suggest that low-lying areas for miles around would be covered with water.

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The UMAP Journal 26 (3) (2005) 217.

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Introduction

In central South Carolina, a lake is held back by a 75-year-old earthen dam. What would happen if an earthquake breached the dam? The concern is based on an earthquake in 1886 at Charleston that scientists believe measured 7.3 on the Richter Scale [Federal Energy Regulatory Commission 2002]. The location of fault lines almost directly under Lake Murray [SCIway 2000; South Carolina

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Geological Survey 1997; 1998] and the frequency of small earthquakes in the area led authorities to consider the consequences of such a disaster.

Our task is to predict how water levels would change along the Saluda River, from Lake Murray Dam to Columbia, if an earthquake on the same scale as the 1886 breaches the dam. In particular, how far would the tributary Rawls Creek flow back and how high would the water rise near the State Capitol in Columbia, South Carolina.

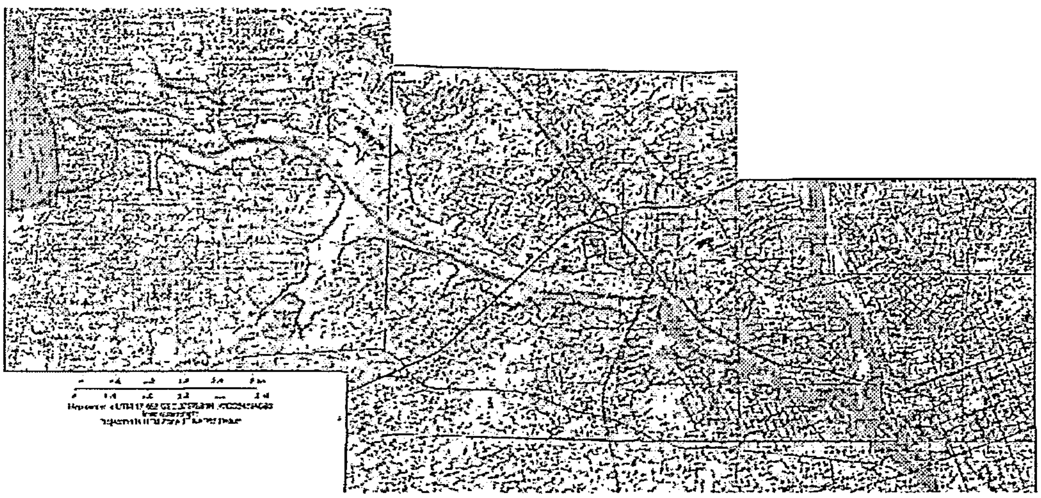

Figure 1. Topographical map of the Saluda River from the base of Lake Murray to the Congaree River [Topozone 2004].

We lay out our assumptions and set up a submodel of Lake Murray and the Lake Murray dam to simulate the overflow when the dam breaks.

We then build a model based on the Saint-Venant equations [Moussa and Bocquillon 2000], using conservation of water and momentum to capture the nature of a flood where the main water channel overflows into the surrounding area. We convert the model to a system of difference equations and feed the dam outflow into the beginning of the river.

To increase accuracy, we measure along the river the ratio of the floodplain to the river width and use these values to modify the equations. We then use data from Lake Murray and the Saluda river to model several scenarios.

Finally, we discuss the implications of our model, analyze its strengths and weaknesses, and discuss how the model could be extended.

Background of Earthquake Effects

- Effect on the dam
    - How the dam is compromised (size and shape of the initial breach)
    - Interaction between the lake water and the initial breach
    - Breach size and shape over time

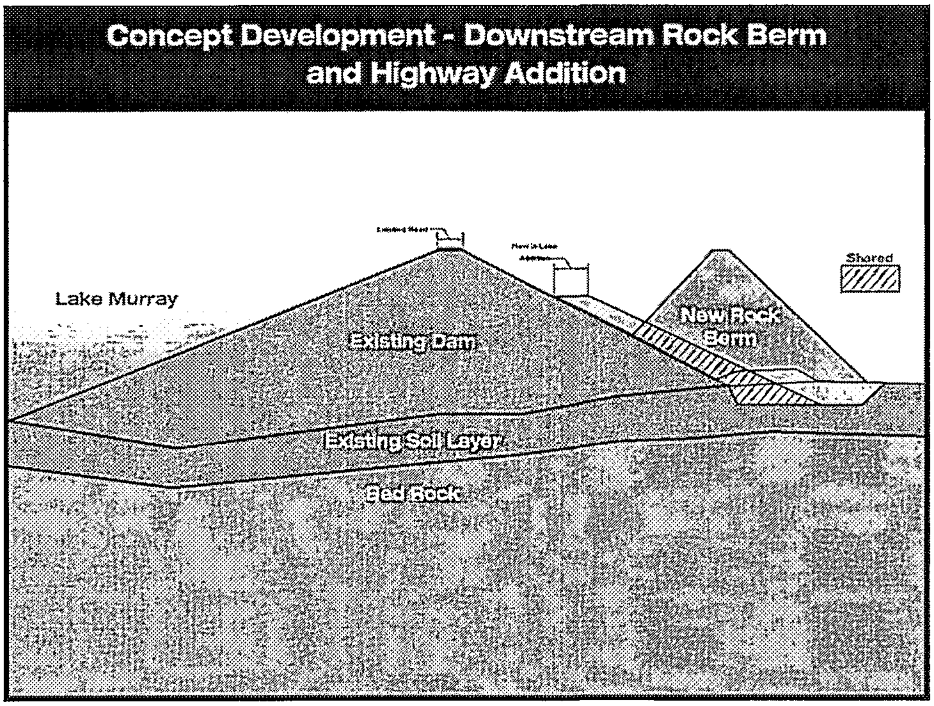

Figure 2. Schematic of the earth dam and the new planned dam at Lake Murray [Lake Murray 2005].

- Effect on the water
  - Earthquake's effect on the lake in so far as it affects the dam
- Effect on the surrounding countryside
  - Whether earthquake alteration of the landscape opens or closes available floodplains
  - Whether earthquake damage could divert the Saluda river
  - Whether earthquake damage would make the Saluda's path choppier and slow down the water

The situation looks something like this: A large earthquake compromises the Lake Murray dam. Earthen dams do not usually fail completely or instantaneously [U.S. Army Corps of Engineers 1997]. Instead, the dam begins to leak. Over time, the water causes further erosion, allowing more and more water to flow out of the lake, until the lake and dam reach a new equilibrium. Depending on the initial breach and the dam construction, the final equilibrium may take anywhere from a few minutes to a few hours [U.S. Army Corps of Engineers 1997] to reach. The fully formed breach usually has a width somewhere between half the height of the dam and three times the height of the dam [U.S. Army Corps of Engineers 1997; 1980]. At half a mile wide (1,609 m) and 208 ft (63 m) tall [SouthCarolinaLakes.net], the Lake Murray Dam is about 25 times as wide as it is tall, which suggests a breach width much smaller than the dam width.

Below the dam, the increasing flow of water puts stress on the countryside, with flooding and hillside carving. Water back-flows up smaller creeks such as Rawls Creek and pools in the flatter sections. Far downstream, either the water pools enough to stay within normal channels, or excess water creates its own channel, or excess water continues flowing from river to river to the sea.

Assumptions

Earthquake

- *Aftershocks disregarded:* Earthquakes generally consist of a main shock and smaller aftershocks. Although an aftershock is itself a significant event and might cause a spike in dam destruction, for simplicity we neglect aftershocks.

- *Dam breach only:* The earthquake could affect the dam, the water involved, and the landscape. The earthquake's effect on the water matters only if the water damages the terrain or escapes from the lake or riverbed. Thus, by assuming that the earthquake affects only the dam, we bundle any effect of the earthquake on the water into the water's effect on other things. The earthquake could significantly affect the terrain, but such changes are unpredictable and we assume no significant terrain changes take place.

Weather and Terrain

- *No effect from wind:* The effects of wind here are minuscule in comparison with other forces.

- *Low land near river would flood:* We assume that the river would overflow its banks and fill the surrounding floodplain.

Lake Murray and Dam

- *Lake has a simple shape:* We assume that the lake has perfectly vertical sides and a flat bottom.

- *Dam breach is rectangular:* We can thus model a variety of dam breaches, since we can vary the height and width independently.

- *Washed-out dam materials are negligible compared to water flooding:* Since we already assume that the breach does not erode, there is no new source of earth after that initial point. This assumption should work well when the breach is small but less so when the breach is large.

Saluda River

- *River channel has constant width:* The Saluda river widens slightly after 11.4 km [Topozone 2004]; but to model it simply, we assume that it has a constant width.
- *River has steady elevation loss:* Due to limits of our topographical data, we assume that the height of the river drops off steadily.
- *River has constant initial depth:* Because we assume that the river drops off steadily, there are no pockets where water could pool. Since the river starts in equilibrium, we assume that the depth is uniform from start to finish.
- *River is straight:* The curvature of a river contributes somewhat to slow the flow of water, and some models include a curvature parameter; but given how straight the Saluda is [Topozone 2004], it is reasonable to approximate it as a linear river.

Dam Model

We use a submodel to simulate what happens on the lake and at the dam after an earthquake causes a breach. The submodel provides information about the volume and speed of water leaving the dam at any given time. This information depends on the volume of water in the lake, the surface area of the lake, and the size of the breach in the dam.

We model the breach as a rectangular opening in the dam. We assume that water would flow out of the bottom of this breach and that its energy would be conserved. The potential energy is converted into kinetic energy, and so from the equation

$$\tfrac{1}{2}ms^2 = mgh$$

we get

$$s = \sqrt{2gh},$$

where

s is the speed of the water,

m is the mass of the water,

g is acceleration due to gravity, and

h is the height of the water—the difference in height between the lake and the bottom of the breach.

We assume that all water leaves at the maximum speed, a slight overestimate. We can write this equation in terms of our model as

$$s_{\text{water leaving}} = \sqrt{2g(h_{\text{lake}} - h_{\text{dam}})}.$$

The volume of water leaving in each time step is the area of the breach times the velocity of the water times the size of the time step:

$$v_{\text{water leaving}} = w_{\text{breach}}(h_{\text{lake}} - h_{\text{dam}}) s_{\text{water leaving}} t_{\text{time step}},$$

where v is volume, w is width, h is height, s is speed, and t is time.

We assume in effect that the lake is a large straight-sided holding tank, so its area doesn't change when the water height does. This means that the height of the lake is simply the volume divided by area, or

$$h_{\text{lake}} = \frac{v_{\text{lake}}}{a_{\text{lake}}}$$

where h is the height, v the volume, and a is the area. This assumption can be changed to make the area of the lake a function of the amount of water in it; for example, we could model the lake as a shallow cone.

We also assume that the breach in the dam stays the same size throughout the simulation, though it would be simple to make the width and depth of the breach increase as a function of the amount and speed of the water flowing through. Doing so would mimic erosion caused by the force of the water traveling through the gap.

Saint-Venant Model

Our primary model is based on the Saint-Venant system of (partial differential) equations. This choice was inspired by Moussa and Bocquillon [2000], who describes how to use them (slightly modified) to model floods. These equations govern open-channel fluid flow that is nonuniform and nonconstant, and they take into account variations in velocity, the topography of the river and surroundings, and friction with the ground. This makes the Saint-Venant system much preferable to simpler models, especially since friction is a dominant force in flood behavior (the floodwaters cover uneven ground with many obstacles—trees, houses, etc.).

The (modified) Saint-Venant system consists of a water conservation equation,

$$\eta \frac{\partial y}{\partial t} + y \frac{\partial V}{\partial x} + V \frac{\partial y}{\partial x} = 0,$$

and a linear momentum equation,

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} + g \left(\frac{\partial y}{\partial x} + S_f - S \right) = 0,$$

where

y is the height of the water;

x is the distance along the river;

t is time;

V is the speed of the water,

S is the river slope;

η , the new parameter introduced by Moussa and Bocquillon [2000], is the relative floodplain width (see below); and

S_f is the so-called *energy-line slope.*

The energy-line slope represents how much friction the flowing water must overcome; it is calculated from the velocity and flow radius of the water via the Manning formula [Moussa and Bocquillon 2000],

$$S_f = n^2 k V^2 R^{-4/3},$$

where R, the hydraulic or flow radius, is given by $R = W_1 y/(W_1 + 2y)$, where W_1 is the width of the channel. There are two constants: n is the dimensionless "roughness parameter" characterizing the land that the water flows over, while k is a constant equal to 1 $s^2/m^{2/3}$.

But what does the introduction of the parameter η do? The model assumes that outside the river channel there is a floodplain that has a very high fluid resistance (e.g., trees, houses). This means that the downstream flow of water in this area is negligible. However, the floodplain serves as a sink for water, so $\partial y/\partial t$ is modified by the factor η, the ratio of of the floodplain width to the channel width. This way, when the water rises, the actual height change in the channel is attenuated by η, since some water is absorbed by the floodplain. We make η a function of the distance along the river by measuring the width of the floodplain at various points.

To model numerically, we turn this PDE system into a difference-equation system. As is common with numerical PDEs, and in particular fluid dynamics problems, special care must be taken to ensure the stability of the algorithm [Trefethen 1996]. We use the Lax-Wendroff difference formula,

$$u_j^{n+1} = u_j^n + \tfrac{1}{2}\lambda(u_{j+1}^n - u_{j-1}^n) - \tfrac{1}{2}\lambda^2(u_{j+1}^n - 2u_j + n + u_{j-1}^n).$$

Here the upper indices represent time and the lower, space; λ is the ratio of the time to the space step size. (Our model converts distance and time to model units, so the step size in each is 1.) The second term acts to damp out spikes, since it looks at how much each point differs from the points on either side of it, and compensates.

We find that the model is highly sensitive to the roughness parameter n (note that this is the effective roughness in the channel only). When n is large (even at 0.03, the standard value for large rivers), there is high resistance to the water flow, and the floodwater tends to pile up. This leads to excessive steepness in the water-depth profile and tends to make the model break down. Fortunately, we can assume a smaller value for n, since we are considering only the water in the channel area, which is bounded on the sides not by rocks and grass (as a river is normally), but by other floodwater (covering the floodplain), which is moving a bit slower (in fact, we assume that it is stationary) but should be smoother than stationary rocks. Therefore, we take $n = 0.01$.

Further, the model is increasingly unstable at higher rates of lake outflow. This is presumably because the Saint-Venant equations are essentially perturbations about steady flow, so they tend to break down in massive flooding. We resort to periodic averaging of neighboring water depths (every 20 time steps, for the most part). This does not seem to affect the results much.

Rawls Creek Back-Flooding

Our initial idea for computing the back-flooding at Rawls Creek was to use the same Saint-Venant modeling technique as for Saluda, adjusted for the different parameters of Rawls, and using the water depths calculated at the creek's mouth for the "dam". However, it is unclear what the initial speeds should be, since the back-flow water moves more or less perpendicularly to the main flood. Moreover, the model displays severe instability with the relevant data. Hence, we take the water height at the mouth and use the topographical map Topozone [2004] to find the matching place upstream. Though highly simplistic, this method is consistent with a modified Saint-Venant system, since it assumes that there is no flow outside the main channel and that the floodplain area is filled instantaneously along with the channel. The Rawls Creek valley is simply a wider section of floodplain (and we include it in calculating the floodplain widths).

Parameters

Lake Murray Dam

We use the following parameter values:

- $g = 9.8$ m/s^2, the gravitational constant
- $h_{0_{\text{lake}}} = 60$ m [SouthCarolinaLakes.net]. This is the initial height of water in the lake.
- $v_{0_{\text{lake}}} = 3 \times 10^9$ m^3 [Publications 2004]. This is the initial volume of water in the lake.
- $a_{0_{\text{lake}}}$, the area of the lake assuming that the sides are exactly vertical.

Saluda River

- $\text{length}_{\text{river}} = 16200$ m, the length of the Saluda River as measured on the topographic map in **Figure 1**.
- $h_{\text{BedUpstream}} = 0$ m, the height of the stream bed just after the dam, compared to the base of the dam.
- $h_{\text{BedDownstream}} = -10$ m, the height of the stream bed as it joins the Congaree River outside Columbia. We obtain this value by comparing the height above sea level of the beginning and at the end of the Saluda river on the topographic map in **Figure 1**.
- $h_{0_{\text{water}}} = 1.2$ m [South Carolina Department of Natural Resources], the initial water depth along the river, assumed uniform.

Floodplain

Water flowing out from the dam would not stay entirely within the Saluda River bed. To model accurately the ratio of the river channel to the floodplain surrounding it, we examine topographical maps. The river has an elevation of approximately 170 ft (52 m). The area near the river rises gradually to approximately 200 ft (61 m), before nearby hills start. We assume that this area between the river and the hills is the approximate floodplain. We measure the width of this plain every 600 m. We assume that the width varies linearly between these measurements and interpolate plain widths for distances downstream that we didn't measure directly. This assumption allows us a much more accurate model than if we simply assume that the floodplain has constant width.

Results

The Lake Murray Dam is roughly 800 m long (in the highest region) by 60 m high [Topozone 2004], so any breach up to this size is at least theoretically possible.

No Breach

Breach width: 0 m, breach height: 0 m

Tested with no breach, the model performs as expected, with the water level staying very nearly constant, since replacement water from the ordinary hydroelectric pipes is included in the model.

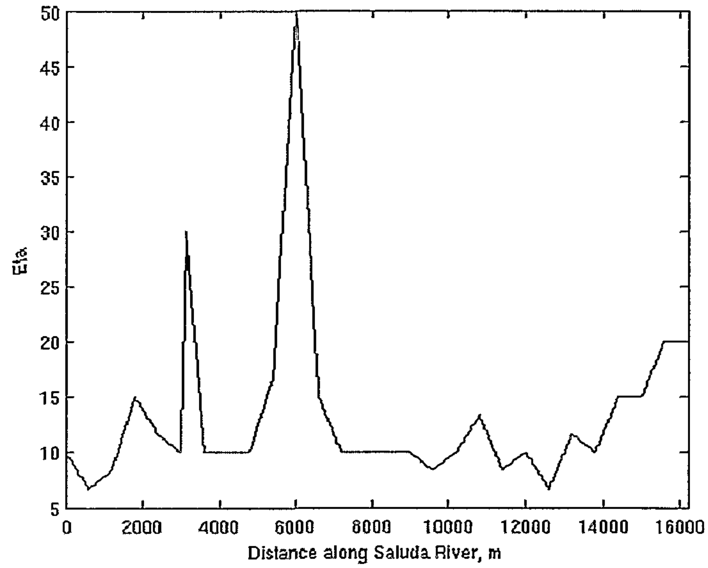

Figure 4. Ratio of the width of the river channel and the flood plain as a function of distance along the river. The two spikes are tributaries; the left one is Rawls Creek. The widening at the end is the mouth of the Saluda where it enters the Congaree.

Realistic Breach

Breach width 800 m, breach height 10 m

The most common earthquake failure mode for an earthen dam is for the underwater side to simply landslide down, producing a wide but shallow breach.

In this scenario, flooding crests in the Rawls after 1.1 h at a height of 7.1 m. This means that the creek backfloods for some 2.4 km along its course, as measured on the topographical map Topozone [2004]. Crest at Columbia (where the Saluda flows into the Congaree) is reached after 7.5 h at a height of 4.15 m. Since the Capitol sits some 50 m above the river, it is in no danger.

Alternative Breach

Breach width 133 m, breach height 60 m

To explore the effect of breach shape as well as size, we run a scenario with a breach of the same cross-section as the previous case but with the opposite rectangular shape. Since the breach is deeper, the speed of the escaping water is higher than before and more water escapes also, since the lake can drain to a lower level.

The water crests at the Rawls after 1.4 h at 9.11 m. The backflooding extends for 3.0 km. Crest at Columbia occurs after 7.0 h at 6.23 m.

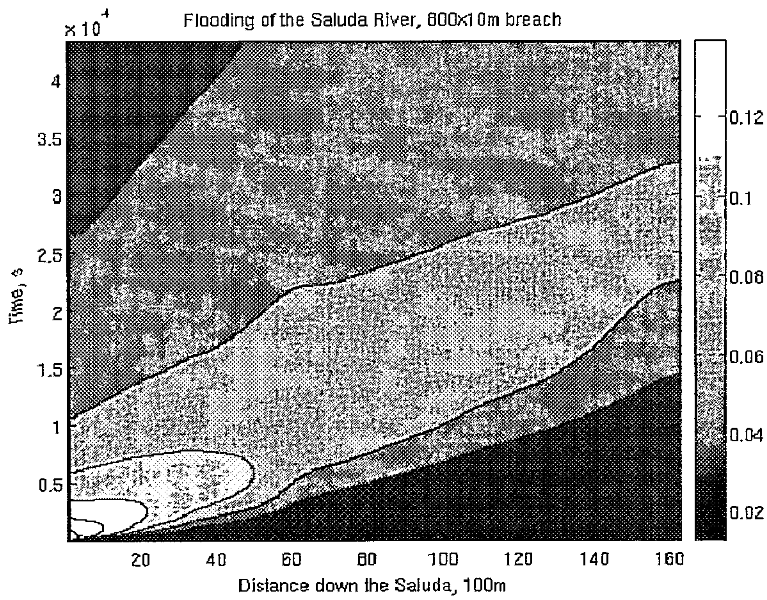

Figure 5. Contour map of the water level along the river from the start of the simulation to the end. The x-axis is distance (m) along the river and the y-axis is time (s) into the simulation. The color bar gives the scale for the height (100s of m) of the water.

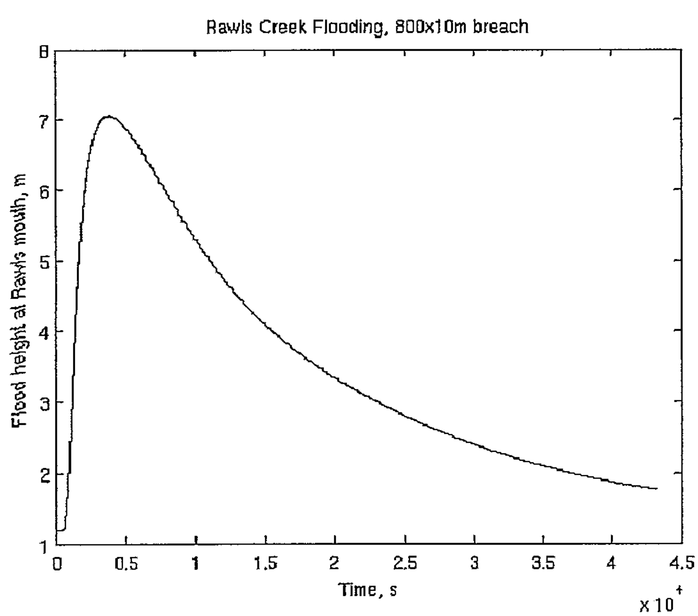

Figure 6. Water level where Rawls Creek joins the Saluda River, from the start of the simulation to the end. The x-axis is time (s) into the simulation and the y-axis is the height (m) of the water.

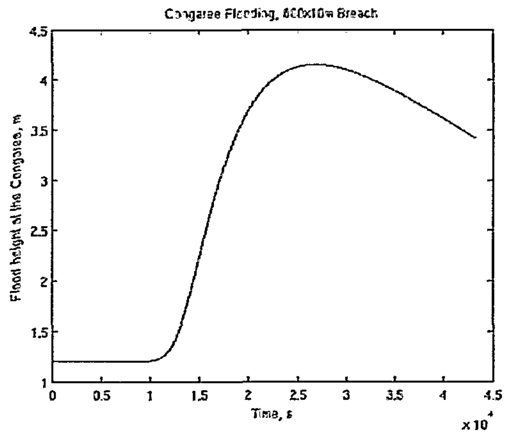

Figure 7. Water level where the Saluda River meets the Congaree River in Columbia, from the start of the simulation to the end. The x-axis is time (s) into the simulation (in seconds) and the y-axis is the height (m) of the water.

Maximum Breach

Breach width: 800 m, breach height: 60 m

What if the entire dam simply vanishes? Both the model and our assumptions are overextended by this scenario. Despite more frequent smoothing (every 5 time steps), the numbers consistently exploded after 3 h of simulated time. Fortunately, this was long enough for cresting at the Rawls and for a pretty good guess at the Columbia crest. Unfortunately, the water rises so high in the early sections of the river that our values for η are no longer valid—the flood simply expands outside the normal floodplain. This means that the water would not actually be as high as the model indicates.

The Rawls crest occurs after 0.4 h 34.35 m. This height of water causes backflooding as much as 5 km upstream (a strong indication that our η values are indeed too low for this level of flooding). The Columbia crest appears after 4 to 5 h and is no more than 17 m. The Capitol is still safe, by a large margin.

Interpretation

While the Capitol is safe in all scenarios, massive flooding nonetheless occurs in low-lying areas and in the homes and businesses along the Columbia. Happily, based on the flood scenarios above, if a warning system is in place, there should be enough time to escape before the flood water arrives.

Analysis of Model

Strengths and Weaknesses

Our model is built on trade-offs. One weakness is the transformation of PDEs into difference equations; the latter are prone to instability in extreme scenarios.

Our assumptions represent other trade-offs. The floodplain, though a vast improvement over an extremely simple model where all water stays in the channel, requires us to assume that the water instantaneously drains from the river and immediately stops moving. Extending the system to be be fully three-dimensional, with water flowing both downstream and outward from the riverbed, would represent a great improvement (and indeed, is performed admirably by various commercial software packages).

On the other hand, we implement equations designed specifically to model situations like the one on the Saluda River and use data specific to Lake Murray and the Saluda River.

Comparison to Other Predictions

The company that owns the dam provides an evacuation map that shows where the water is expected to go during a flood. This map seems to agree roughly with our worst-case model predictions.

Future Work

- We could model an expanding trapezoidal breach, representing erosion of the original breach, using values from the literature [U.S. Army Corps of Engineers 1980] to select appropriate slope and time intervals.
- We could acquire data on the normal width of the Saluda River at intervals along its course between Lake Murray and the Congaree, rather than assuming a constant stream width.
- We could collect data on the elevation of the stream at regular intervals. For instance, the river might have a waterfall, which could affect the flood pattern.
- We could consider information on the distribution of the lake's water. In real life, the lake has large areas that are shallow, with a smaller deep region.
- We could move from the straight-stream assumption to a two-dimensional analysis; some momentum is lost in bends in the river.

- Our assumptions (the earthquake affects just the dam, aftershocks can be disregarded, wind has no effect, and washed-out dam materials can be disregarded) are sturdy enough that an upgrade of the water-flow modeling technique used (Saint-Venant) should be attempted before correcting these assumptions.

Conclusion

Dam-breach flooding is a rare but very serious problem, especially when the dam sits less than 20 km above a major city. We create a hydrodynamical model that gives the downstream results of both likely and possible earthquake-driven dam breach scenarios. Since the city of Columbia sits mainly on a hill, the predicted flood levels of 4 m to 17 m would flood only the few blocks closest to the river. However, upstream areas such as Rawls Creek would experience levels 7 m to 34 m higher. The water could arrive at Rawls in as little as half an hour, and flood 2.5–5 km upstream; so an early-warning system for dam breaches along the Saluda is a vital protective measure.

Our model produces results that make intuitive sense when we vary the parameters: The flooding increases with a larger breach, and a deeper breach floods more than a shallower one of the same area. The water height falls off downstream, as some of the water is held in the floodplain, and this attenuation varies with the width of the floodplain.

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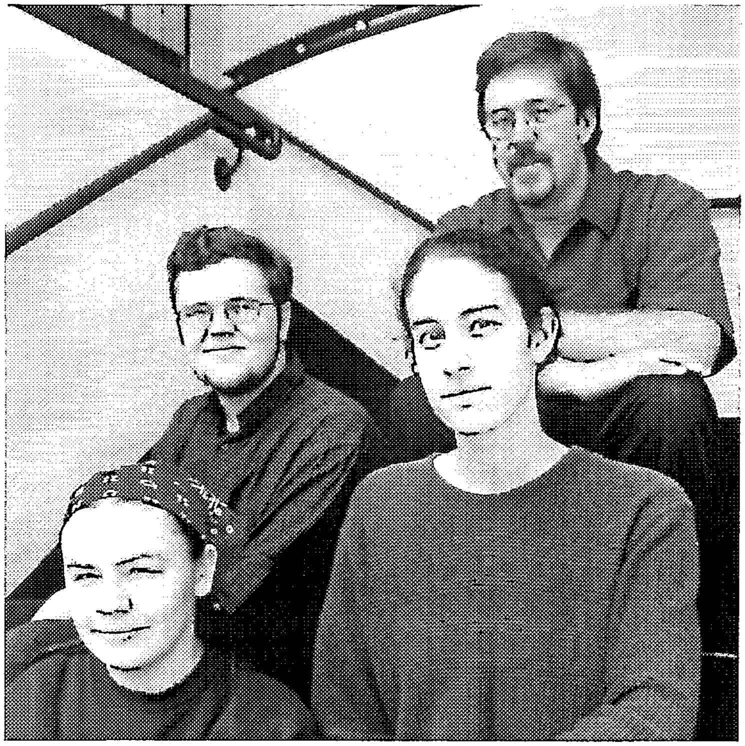

Lori Thomas, Clay Hambrick, Katie Lewis, and Jon Jacobsen (advisor).